核物理中的量子纠缠和Berry phase

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Entanglement

A classically forbidden form of correlation shared between separate local subsystems.

- **Unentangled** state: a product pure state with N parts (the wave function is make of N parts; not necessary N particles)

$$|\Psi_N\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_N\rangle,$$

with $|\phi_i\rangle$ being the wave function of the ith part.

- **Entangled** state: not unentangled.
- Qubit: **two-level** quantum system, with basis vectors given by $|0\rangle$ and $|1\rangle$, mapped into a **sphere**.

Bell's state: $|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ maximally entangled

$\neq (a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle)$ → entangled

Entanglement in atomic nucleus

- **Energy scale**: $\subset$ QIS at **MeV**; in comparison, the usual quantum QIS operates at μ**eV** to **eV**; **MeV**: the boundary between fundamental research and engineering application

- **Degrees of freedom**: nucleons (stable), atomic nucleus, and their decay/reaction products (gamma ray, etc).
  - QIS at/below eV: optical photon, etc.
  - QIS at keV: X-ray, etc.
  - QIS at GeV: X(3872) (unstable), etc.
  - QIS at TeV: Top quarks (unstable), etc.

- **Interactions**: strong (nonperturbative) + electroweak
  - QIS at eV and keV: electromagnetic
  - QIS at GeV and TeV: strong (perturbative at the parton level, nonperturbative in hadronization) + electroweak

Past and present

- **Chien-Shiung Wu** and I. Shaknov (1949): entanglement in pairs of **gamma-ray** photons (~0.5 MeV) produced by electron-positron annihilation, the first experimental evidence of quantum entanglement in laboratories.
- Bell tests in low-energy nuclear physics: Lamehi-Rachti and Mittig (1976), Sakai (2006).
- **Theoretical** investigations:

- Nuclear force: Beane (2018), DB (2022), Jinniu Hu (2024), etc
- Short-range correlation: Xurong Chen (2023)
- Fission: Junchen Pei/Yu Qiang (2024)
- Multi-nucleon transfer reactions: Pengwei Zhao (2024)
- Collective motion: Qibo Chen (2024)

- **Only a few** theoretical studies and **very few experimental** ones, in sharp contrast to QIS at eV.

Open questions

Understand **entanglement dynamics** at **MeV**: **generation**, **propagation**, **manipulation**, **detection**, and **decoherence**.

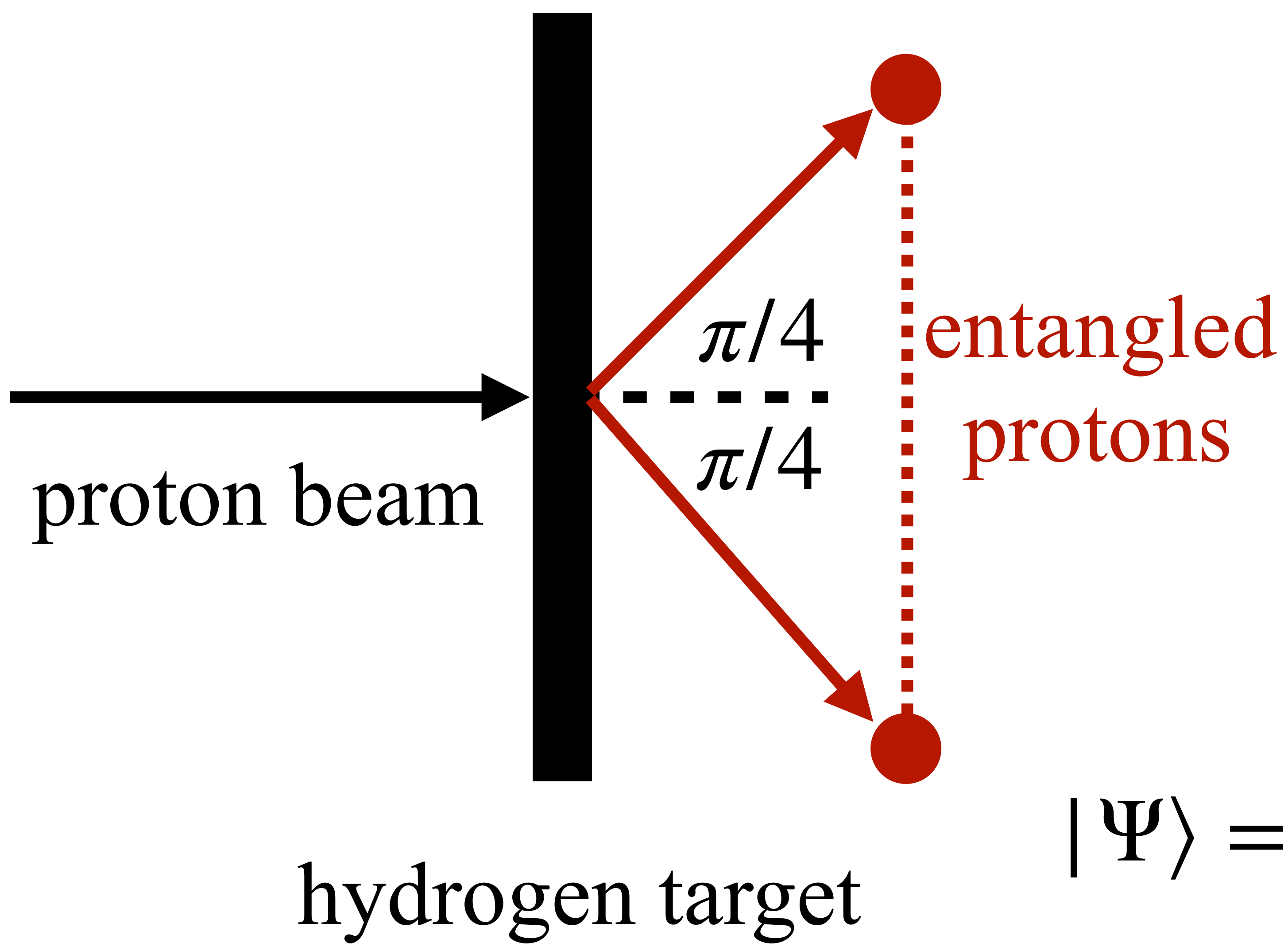
Spin entanglement of protons (spins as qubits) as an example:

- How to **generate** spin entangled protons?
- What happens to spin wave functions and spin entanglement when protons **propagate** through electromagnetic fields and detection materials?
- How to **manipulate** spin wave functions of entangled protons?
- How to **detect** spin entanglement of two protons? (working with **Wolfgang Mittig** at MSU)
- What physical laws govern **decoherence** of proton spin entanglement traveling through materials?

Generation of spin entangled protons

Large-angle generation vs **Small-angle generation:**

- **LAG:** Lamehi-Rachti and Mittig (1976), elastic proton-proton scattering at low energies (< 10 MeV).



- The relative angle between two protons is 90 degree.
- dominated by *S*-wave scattering, leading to the spin-singlet wave function

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Advantages:

- Elastic proton-proton scattering is well understood theoretically, simple process, small model-dependence, high reliability of theoretical predictions.
- Allow separate manipulation of the two protons.

Disadvantages:

- The unparallel motion of two protons is a problem for testing Bell's inequality [Lamehi-Rachti and Mittig (1976)]: "Because in the laboratory system the two proton directions form, 90 degree, …, when rotating **a** and **b** around the propagation direction of the protons to **a'**, **b'**, the vectors **a**, **b**, **a'**, and **b'** are not coplanar. … Using this correlation function directly one sees that it does not violate the inequality of Bell."

• **SAG:** Sakai (2006)

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Spin Correlations of Strongly Interacting Massive Fermion Pairs as a Test of Bell's Inequality

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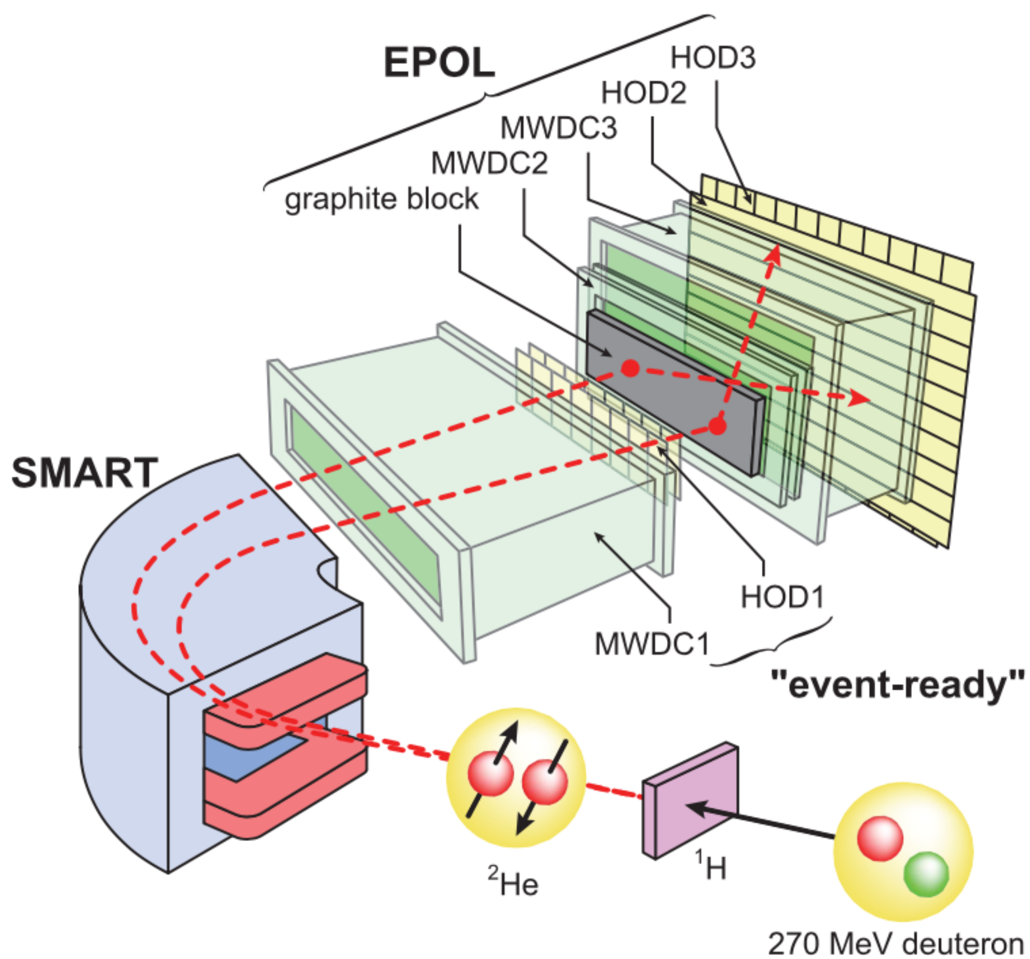

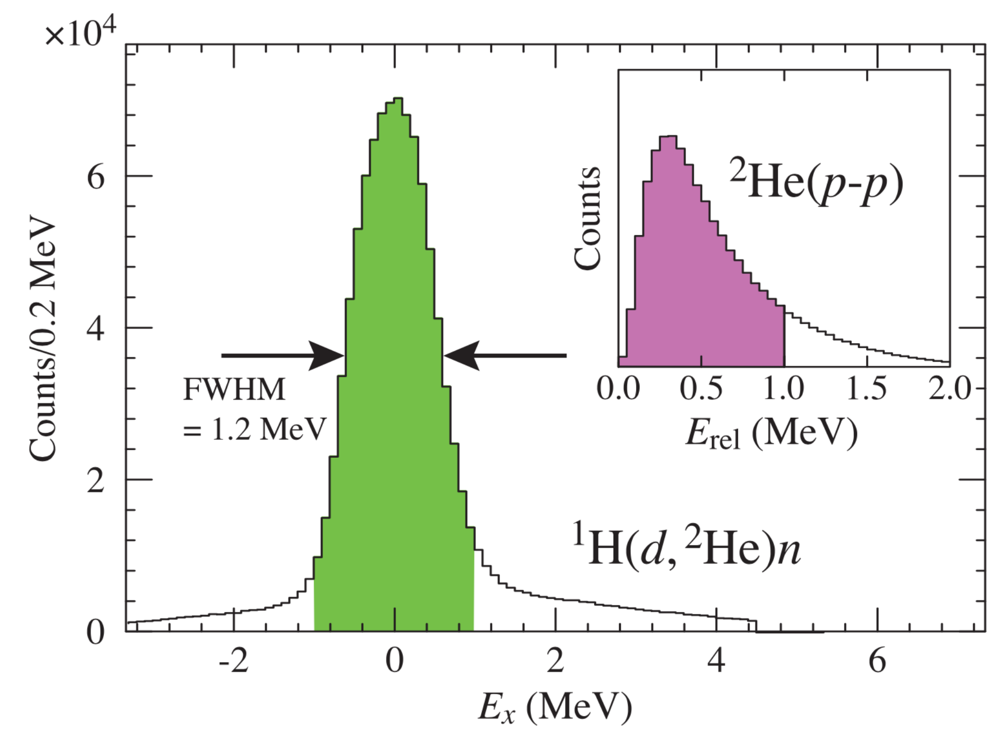

- Large lab-frame momentum, but small relative momentum
- Spin wave function = spin-singlet

Advantages:

- The two protons move in the same direction, friendly to the experimental test of Bell's inequality.

Disadvantages:

- Does not allow separate manipulation of the two protons, limit the application scope.
- The reaction process p+d->n+(2p) is more complicated than elastic proton-proton scattering.

In the following, I will mainly consider **LAG of spin-entangled protons**.

Manipulation of spin wave functions

Example: generate the **zeroth** component of **spin-triplet** wave function for protons

$$|\textbf{singlet}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\Rightarrow |\textbf{triplet}_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

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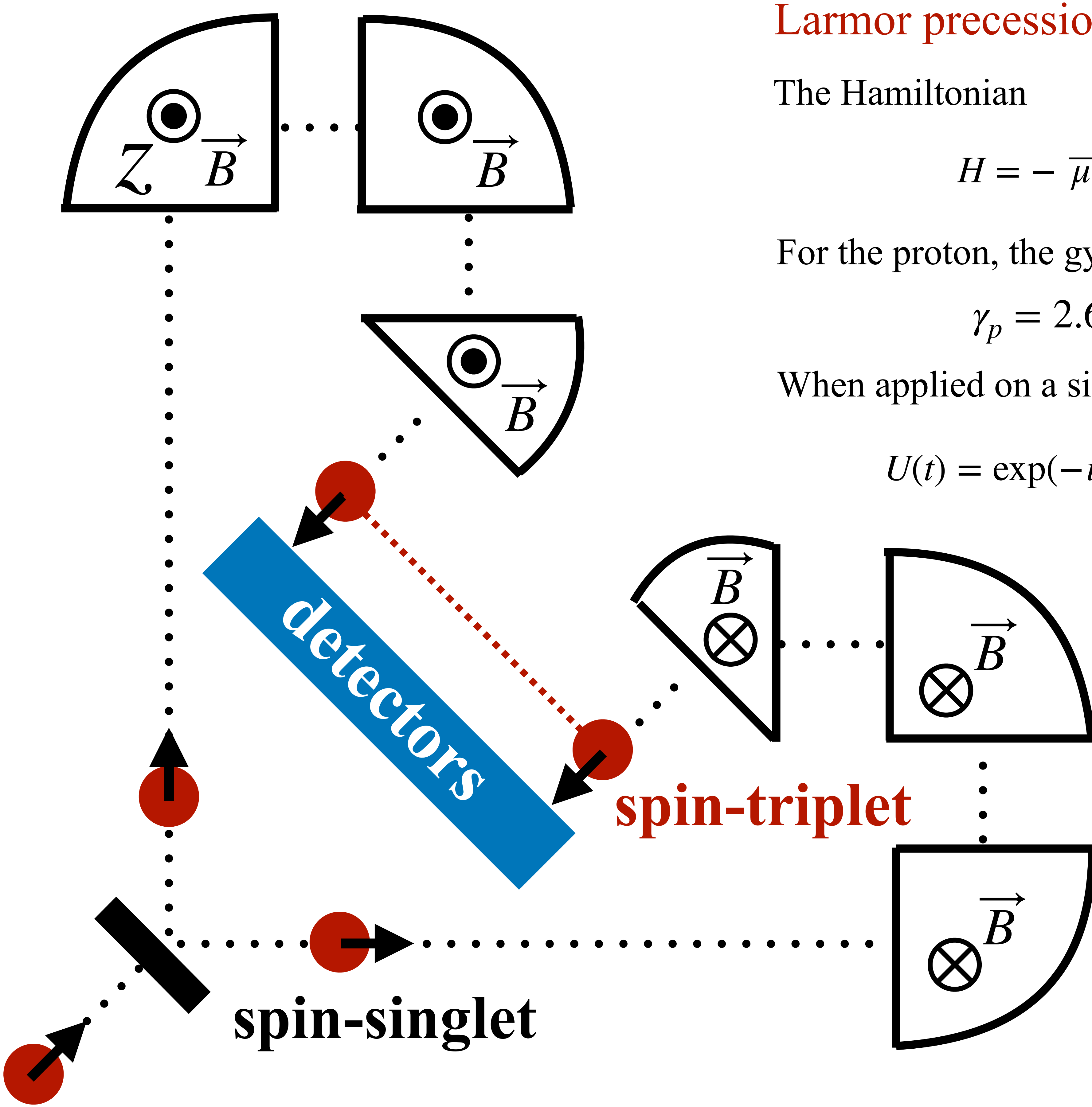
Bell correlations between spatially separated pairs of atoms

D.K. Shin[1], B.M. Henson[1], S.S. Hodgman[1], T. Wasak[2], J. Chwedeńczuk[3] & A.G. Truscott[1*]

2019, using the spin-triplet state



Larmor precession

The Hamiltonian

$$H = -\vec{\mu} \cdot \vec{B} = -\frac{\gamma_p}{2}\vec{\sigma} \cdot \vec{B}.$$

For the proton, the gyromagnetic ratio is

$$\gamma_p = 2.675 \times 10^8 \ \mathrm{S^{-1}T^{-1}}.$$

When applied on a singlet spin,

$$U(t) = \exp(-iHt) = \exp\left(i\frac{\gamma_p}{2}\vec{\sigma} \cdot \vec{B}t\right).$$

For the magnetic field along the z-axis,

$$U(t) = \exp\left(i\frac{1}{2}\gamma_p B t \sigma_z\right) = \exp(i\omega_L t \sigma_z)\,.$$

with $\omega_L = \frac{1}{2}\gamma_p B.$

$$U(t)\,|\uparrow\rangle = \exp(i\omega_L t)\,|\uparrow\rangle,$$
$$U(t)\,|\downarrow\rangle = \exp(-i\omega_L t)\,|\downarrow\rangle\,.$$

For the magnetic field in the opposite direction of the z-axis, the transformation matrix is given by $U(t)^\dagger$.

$$|\textbf{singlet}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$\Rightarrow \frac{1}{\sqrt{2}}(U(t)|\uparrow\rangle \otimes U(t)^{\dagger}|\downarrow\rangle - U(t)|\downarrow\rangle \otimes U(t)^{\dagger}|\uparrow\rangle)$$

$$= \frac{1}{\sqrt{2}}[\exp(2i\omega_L t)|\uparrow\downarrow\rangle - \exp(-2i\omega_L t)|\downarrow\uparrow\rangle]$$

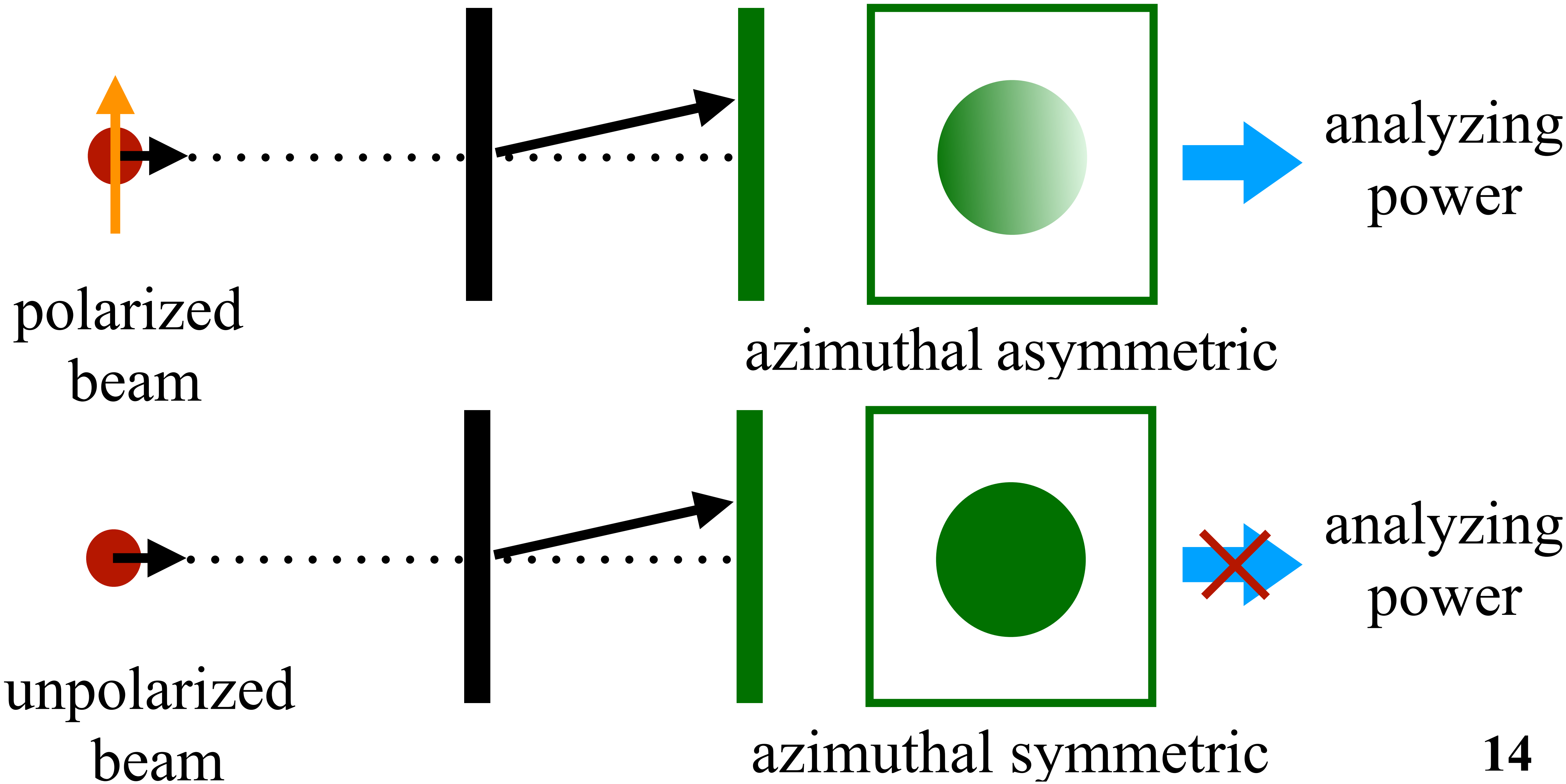
$$\propto \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle - \exp(-4i\omega_L t)|\downarrow\uparrow\rangle]$$

When $t = \frac{5\pi m_p}{4eB}$, $\exp(-4\omega_L t) = -0.9984 - 0.05612i$.

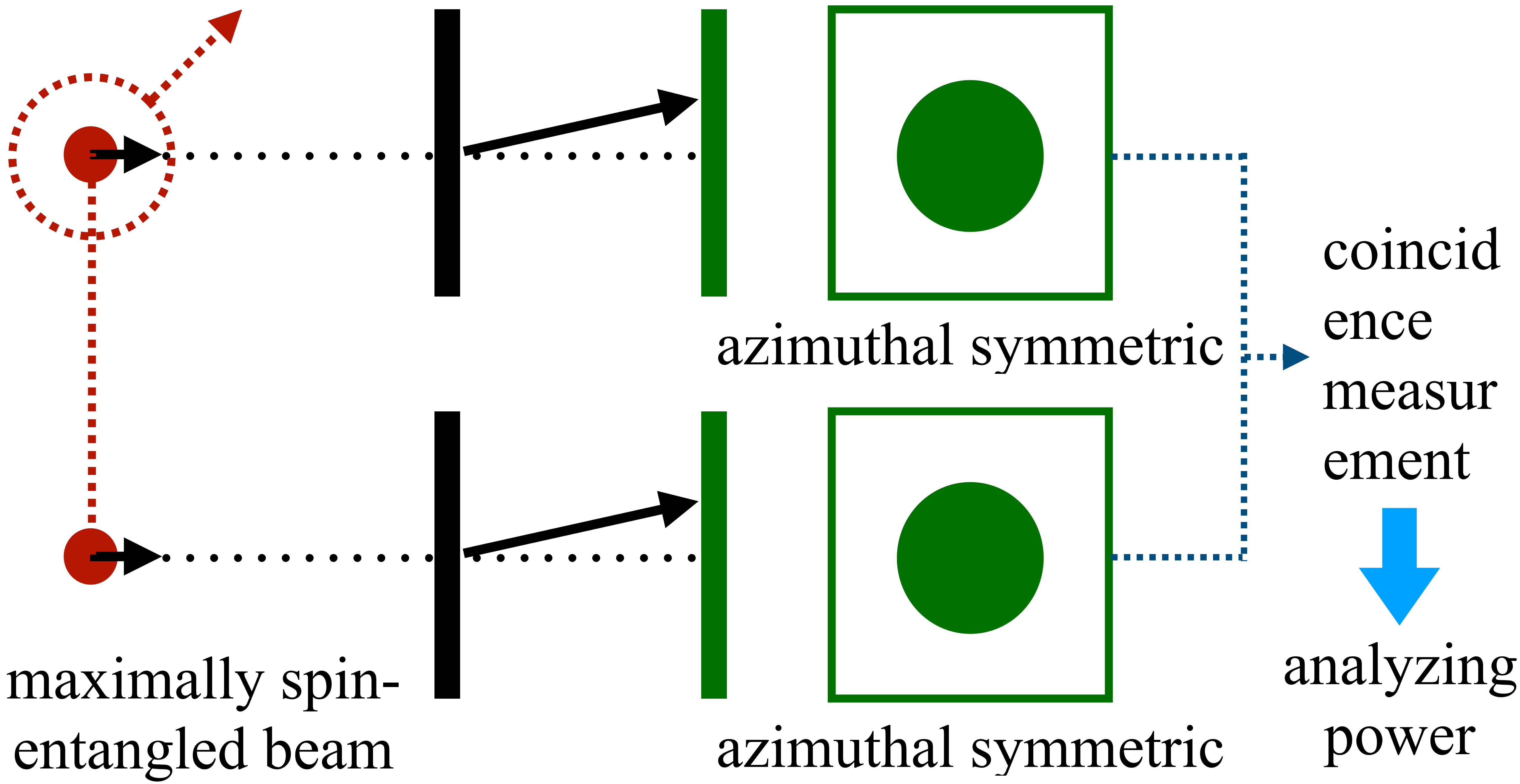
$$\approx \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\uparrow\downarrow\rangle) = |\textbf{triplet}_0\rangle$$

Polarization observables from spin-entangled beams

Traditionally, **analyzing power** is determined by firing a polarized proton toward the target (e.g., ^{12}C) and observing the **azimuthal** asymmetry of the cross section.



Each proton by itself is **completely unpolarized**.

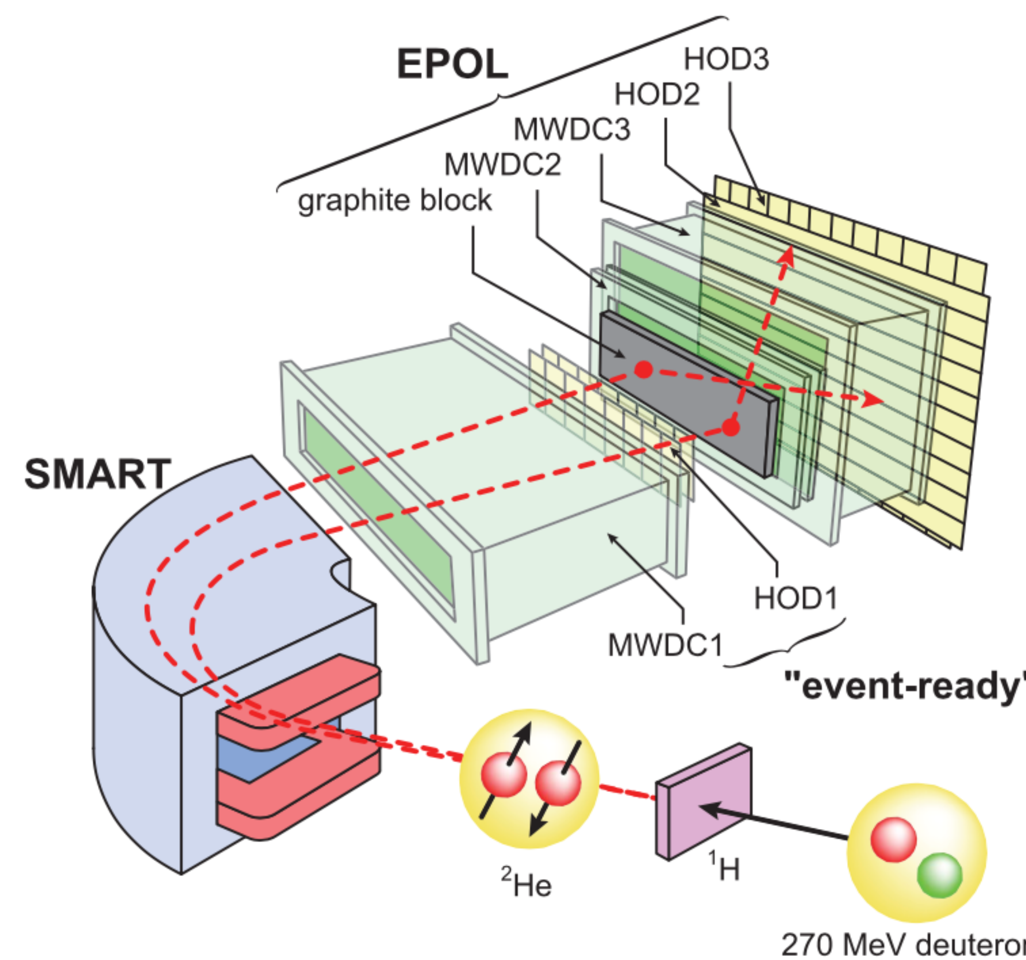


$$A_1A_2 = \frac{1}{\langle(\vec{\sigma}_1 \cdot \vec{n}_1) \otimes (\vec{\sigma}_2 \cdot \vec{n}_2)\rangle} \frac{(L_1L_2) - (L_1R_2) - (R_1L_2) + (R_1R_2)}{(L_1L_2) + (L_1R_2) + (R_1L_2) + (R_1R_2)}$$

Quantum decoherence of spin entangled protons

- Sakai (2006):

  "It is to be noted that the entanglement of the spin- singlet state was retained even when the two protons traversed through large amounts of material media (**50 cm thick argon + ethane gas in wire chambers, 1 cm thick plastic scintillators, and up to 5 cm thick graphite slab**). It is indeed remarkable that the strongly interacting pairs maintain the correlations for distances greater than 10^{13}–10^{14} times their coherence length, which was estimated to be 10^{-14} m (the size of a wave packet of two protons at production)."



spin-entangled protons in, e.g., spin singlet

spin entanglement will be weaken by mutual interaction between protons and matter

- Experimentally **detect** spin entanglement of two outgoing protons by quantum state tomography (working with **Wolfgang Mittig** at MSU)

Berry phase

In a quantum system at the *n*-th eigenstate, an **adiabatic** evolution of the Hamiltonian sees the system remain in the *n*-th eigenstate of the Hamiltonian, while also obtaining a phase factor.

$$|\psi(t)\rangle = \exp[i\Phi(t)]\,|n(\vec{R}(t))\rangle$$

$$H(\vec{R}(t))\,|n(\vec{R}(t))\rangle = E_n(t)\,|n(\vec{R}(t))\rangle$$

Consider a cyclical variation of the Hamiltonian $H(\vec{R}(T)) = H(\vec{R}(0))$,

$$\Phi(T) = -\int_0^T E_n(\vec{R}(t))\mathrm{d}t + i\oint d\vec{R}\,\langle n(\vec{R})|\nabla_{\vec{R}}|n(\vec{R})\rangle$$

dynamical phase → Present in all time evolutions, adiabatic or not. Governed by the Schrödinger equation's time-dependent phase.

Berry phase → Specifically requires a **cyclic adiabatic** process

Berry phase in atomic nucleus

Cyclic parameters:

- J. J. Valiente-Dobon *et al.*, Manifestation of the Berry phase in the atomic nucleus ^{213}Pb, Phys. Lett. B 816, 136183 (2021).

- Driven by **discrete** unitary transformation (**particle-hole conjugation**) rather than time;
- Valence shell filling fraction (n/Ω) in the single-j shell;
- The system evolves through a loop in parameter space where neutron number n in the single-j shell (here, the $1g_{9/2}$ orbital) is transformed to its particle-hole conjugate $\Omega-n$ ($\Omega=2j+1=10$ for $j=9/2$).
- At mid-shell ($n=\Omega/2=5$), the system returns to itself after particle-hole conjugation, creating a closed path. This symmetry induces a Berry phase in the quantum states.
- Berry phase provides an explanation of selection rules of *E2* transition.


Thank you!
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