Muon-Proton Scattering Experiment

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第三届惠州高精度前沿会议

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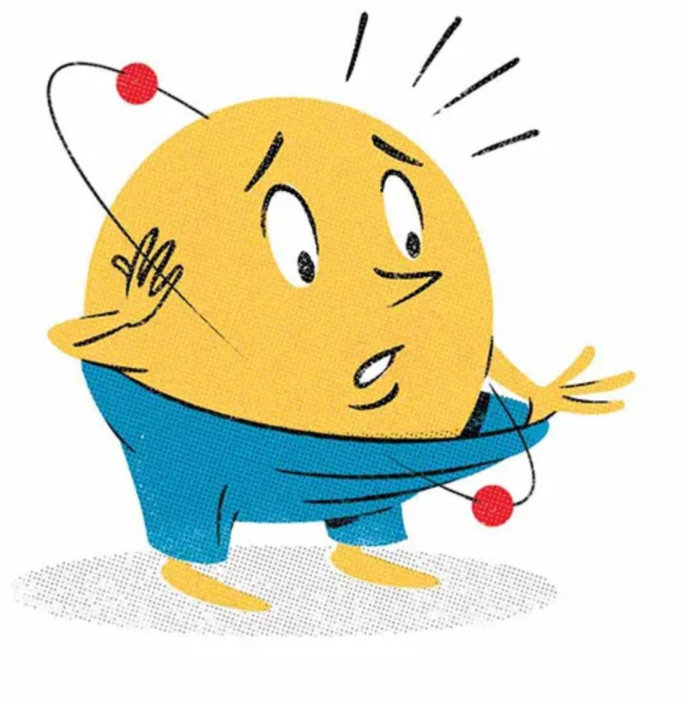
A fundamental property of the proton

- The proton radius is a basic quantity in nuclear and particle physics
  - Plays a crucial role in hydrogen atomic energy level calculation and QED tests

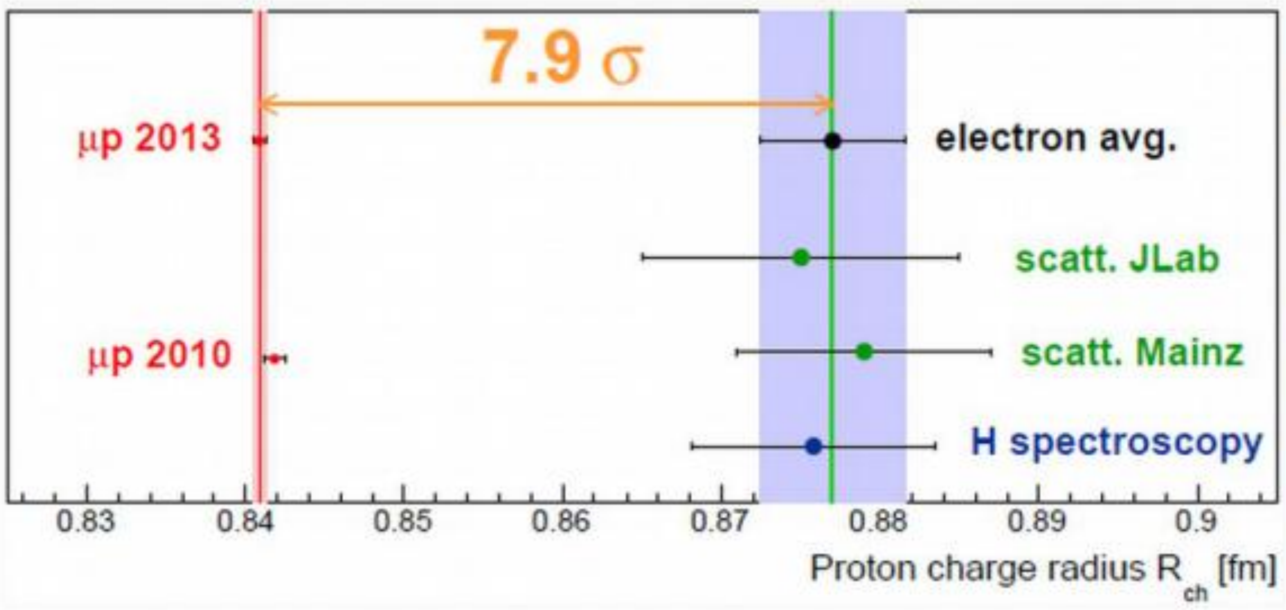
- The Proton Radius Puzzle (PRP):
  - Muon vs. electron: hints of potential flavour dependent interactions

- A global research frontier:
  - Ongoing efforts: PRad (JLab), MUSE (PSI), Amber (CERN) …
  - Still an open and active topic in both atomic and particle physics



Proton charge radius Puzzle

- Back to 2013, many interest on the proton charge radius
- The Proton Radius Puzzle (PRP)



μp 2013: Antognini *et al.*, Science **339**, 417 (2013)

JLab: Zhan *et al.*, PLB **705**, 59-64 (2011)

Mainz: Bernauer *et al.*, PRL **105**, 242001 (2010)

μp 2010: Pohl *et al.*, Nature **466**, 213 (2010)

Proton charge radius

- Two ways to measure it:
  - Atomic energy levels

$$\Delta E_1 = \frac{2\pi a}{3}\left|\phi^2(0)\right|\langle r_E^2\rangle$$

- Lamb Shift: finite size of proton changes hydrogen energy levels
- Extract from hydrogen spectroscopy

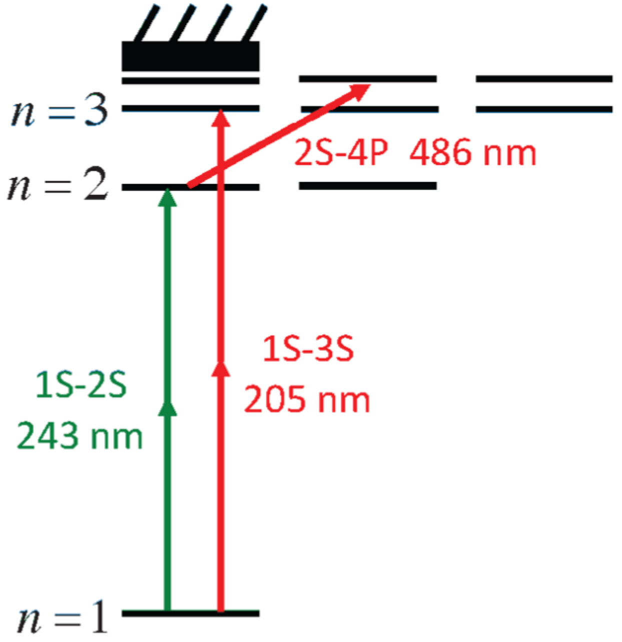
$$E_{nS} \cong -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

$$L_{1S}(r_p) = 8171.634(4) + 1.5645\langle r_p^2\rangle \text{ MHz}$$

Two measurements required to determine R_∞ and r_p

- A single narrow transition: $1S-2S$ ($\Delta\nu = 1.3\ Hz$)
- Other transitions: natureal width ~ MHz

Each measurement, combiend with $1S-2S$, yields a correlated pair (R_∞, r_p)



Proton charge radius

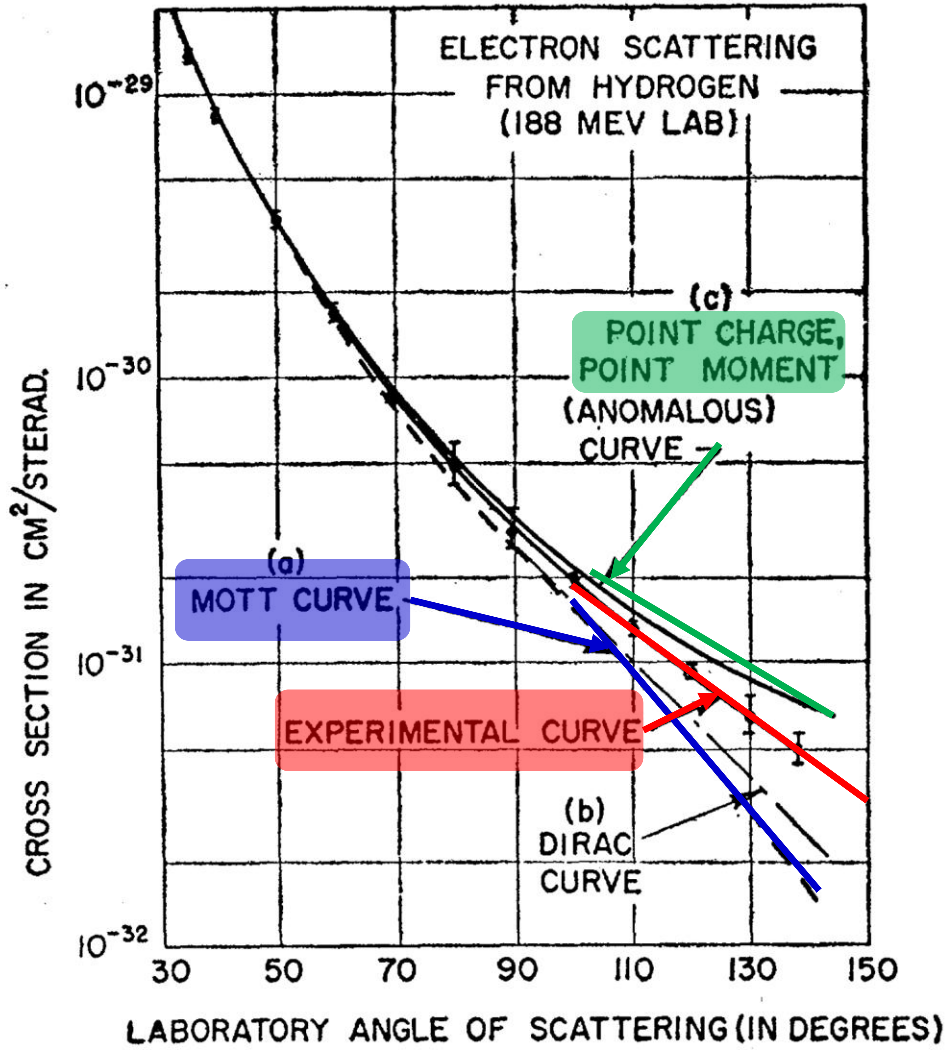
- Two ways to measure it:
  - Atomic energy levels
  - Scattering experiments

Electromagntic form factor

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{point} \times \left(G(Q^2)\right)^2$$

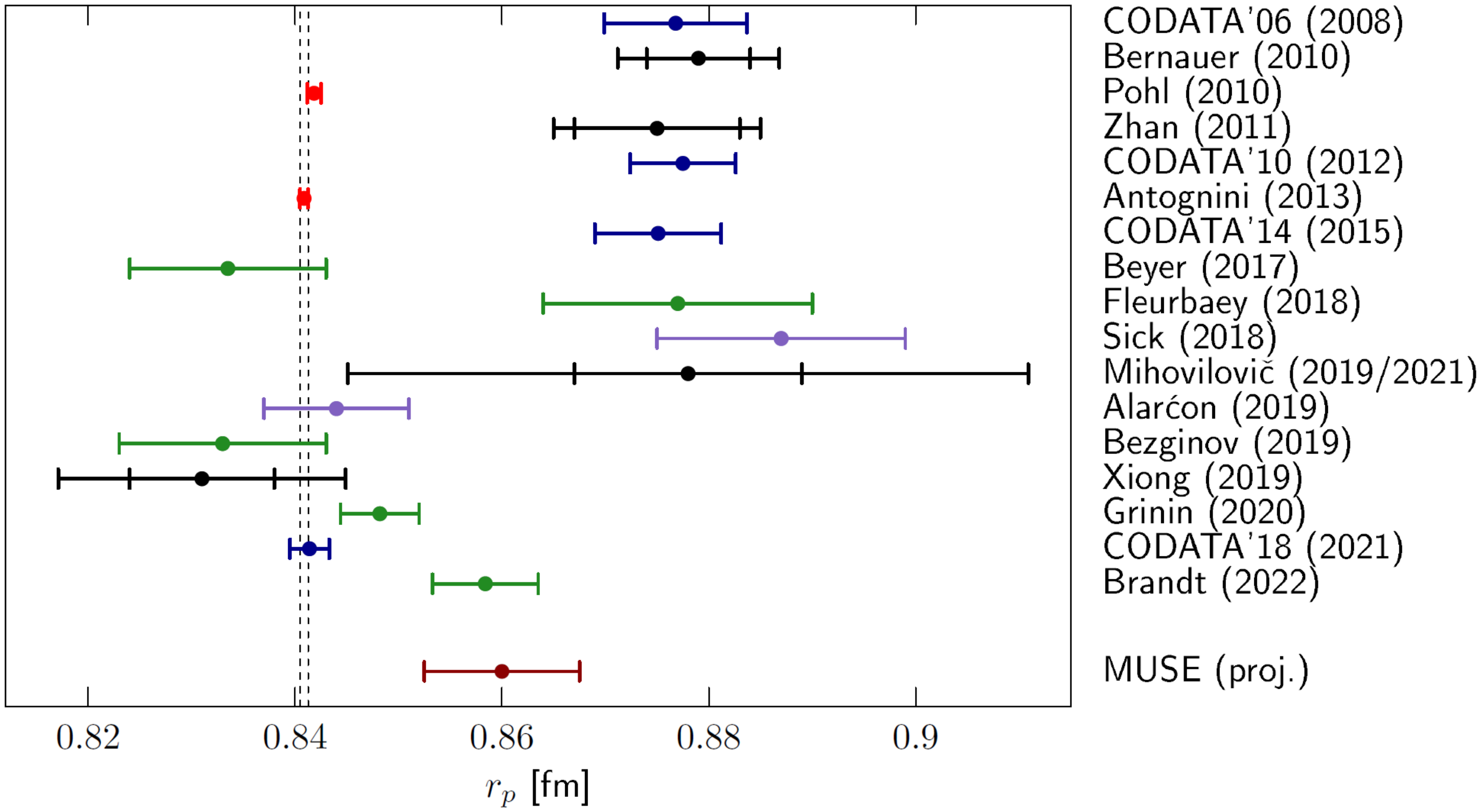
$$r_p^2 = -6\left.\frac{dG_E}{dQ^2}\right|_{Q^2=0}$$

Measure the differential cross-section



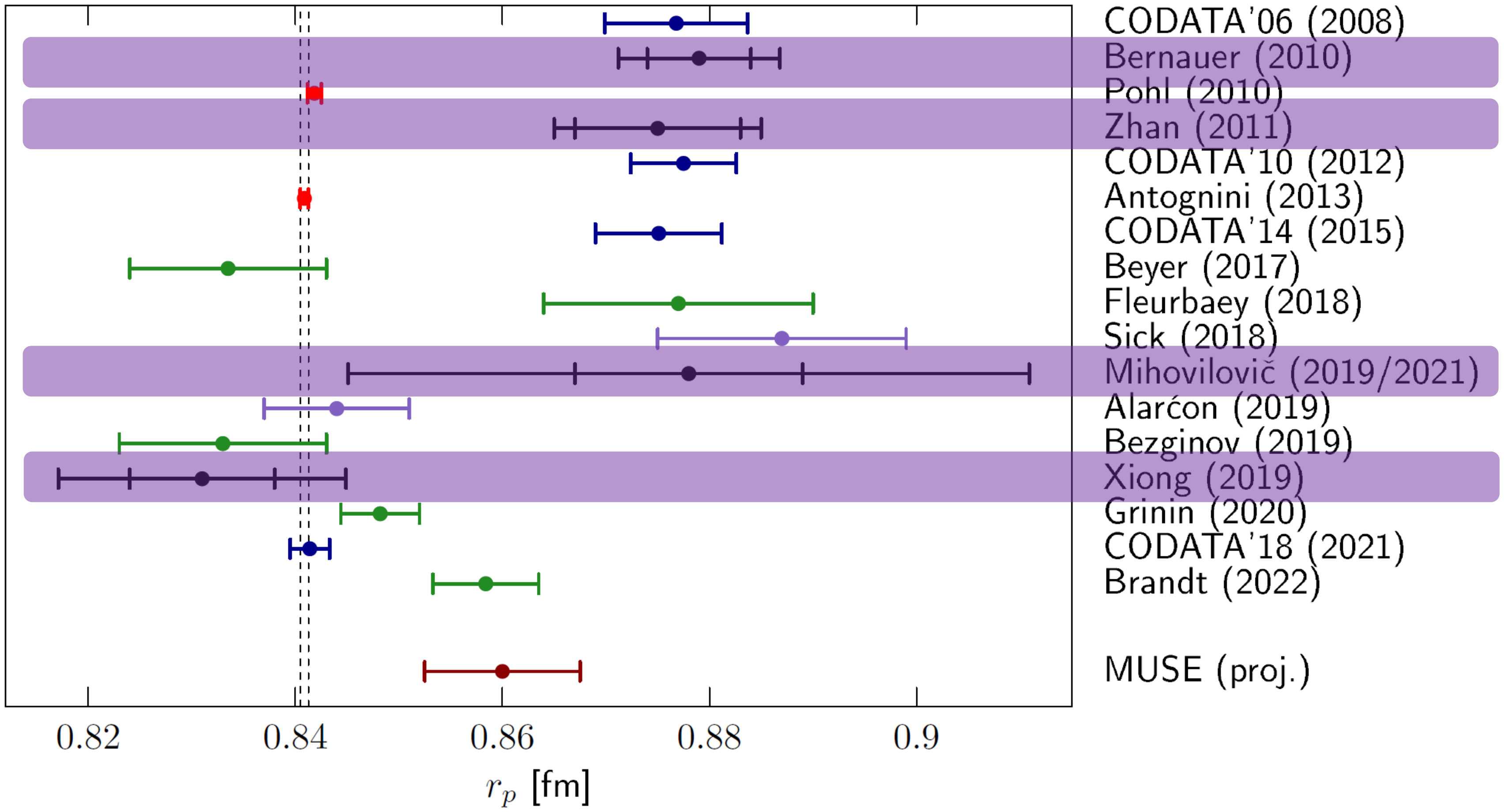
Phys.Rev. 103 (1956) 1454-1463

Current status of proton charge radius



Red: muonic hydrogen
Green: atomic hydrogen
Black: *ep* scattering
Blue: CODATA analysis
Purple: Other experiment

Current status of proton charge radius



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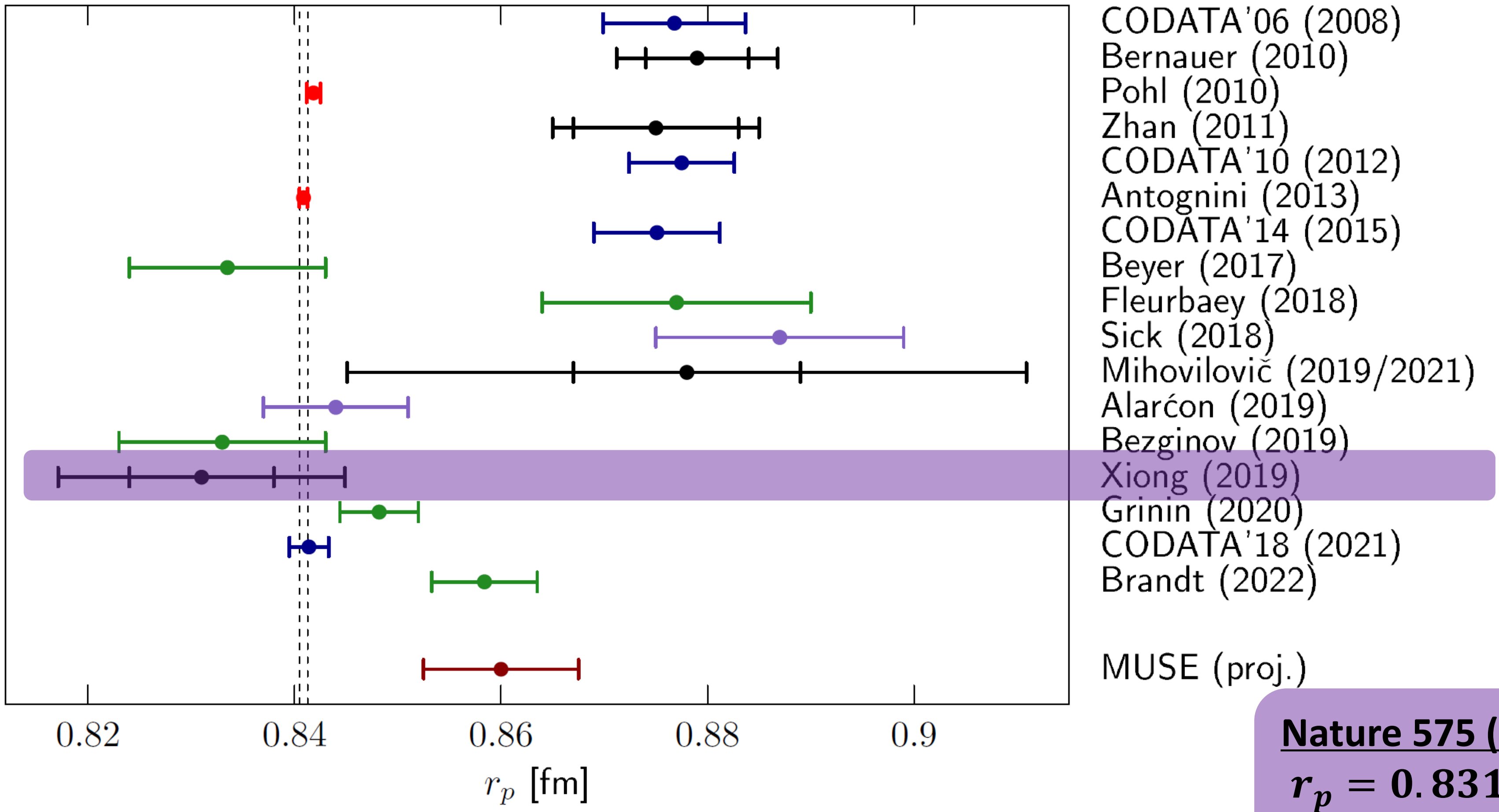
COMMITTEE ON DATA
CODATA
INTERNATIONAL SCIENCE COUNCIL

Current status of proton charge radius



Red: muonic hydrogen
Green: atomic hydrogen
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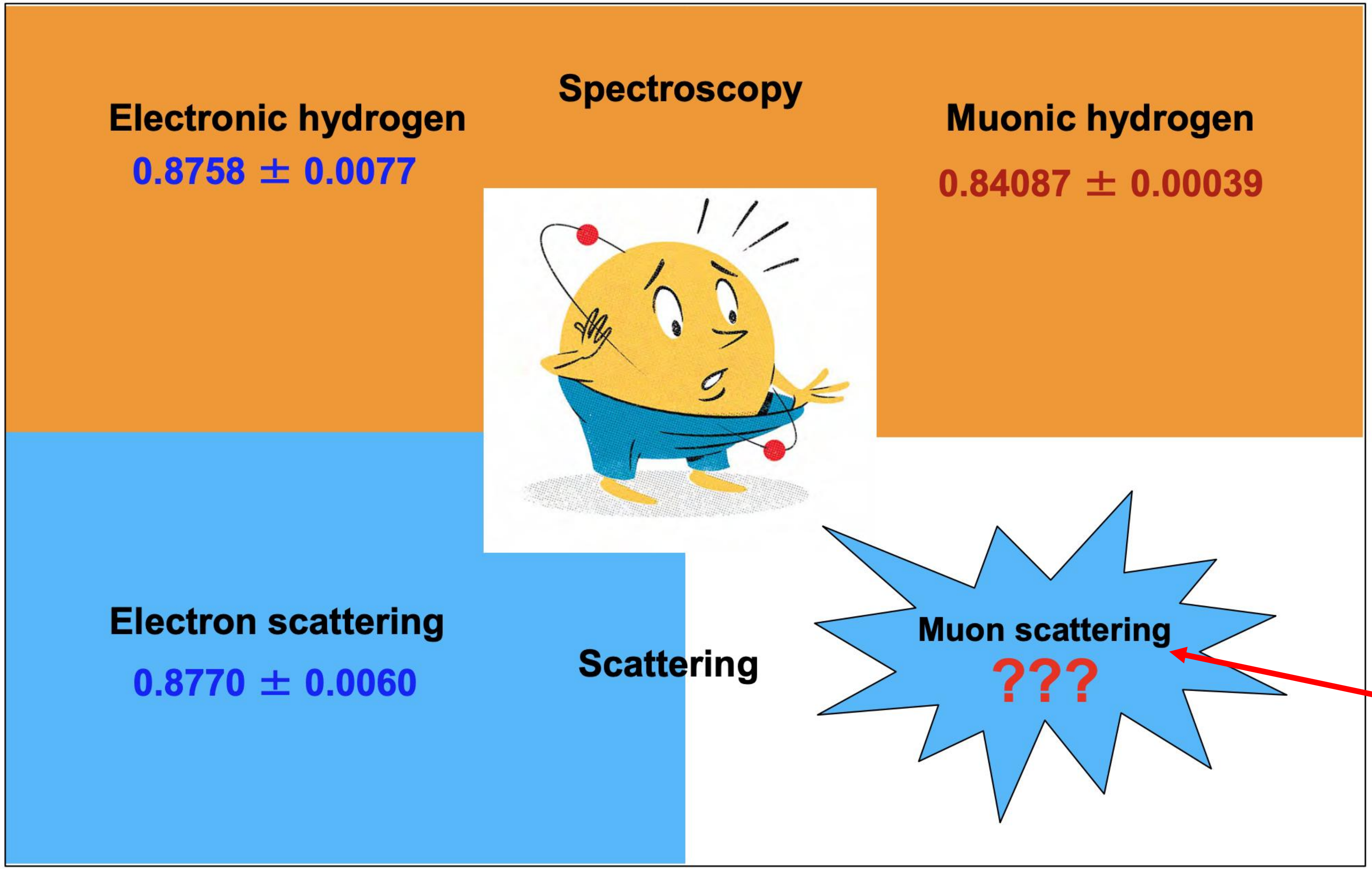
Blue: CODATA analysis
Purple: Other experiment

Nature 575 (2019) 7781, 147-150

$$r_p = 0.831 \pm 0.007(stat) \pm 0.012(syst)$$

Precision: 1.7%

μp scattering: essential inputs



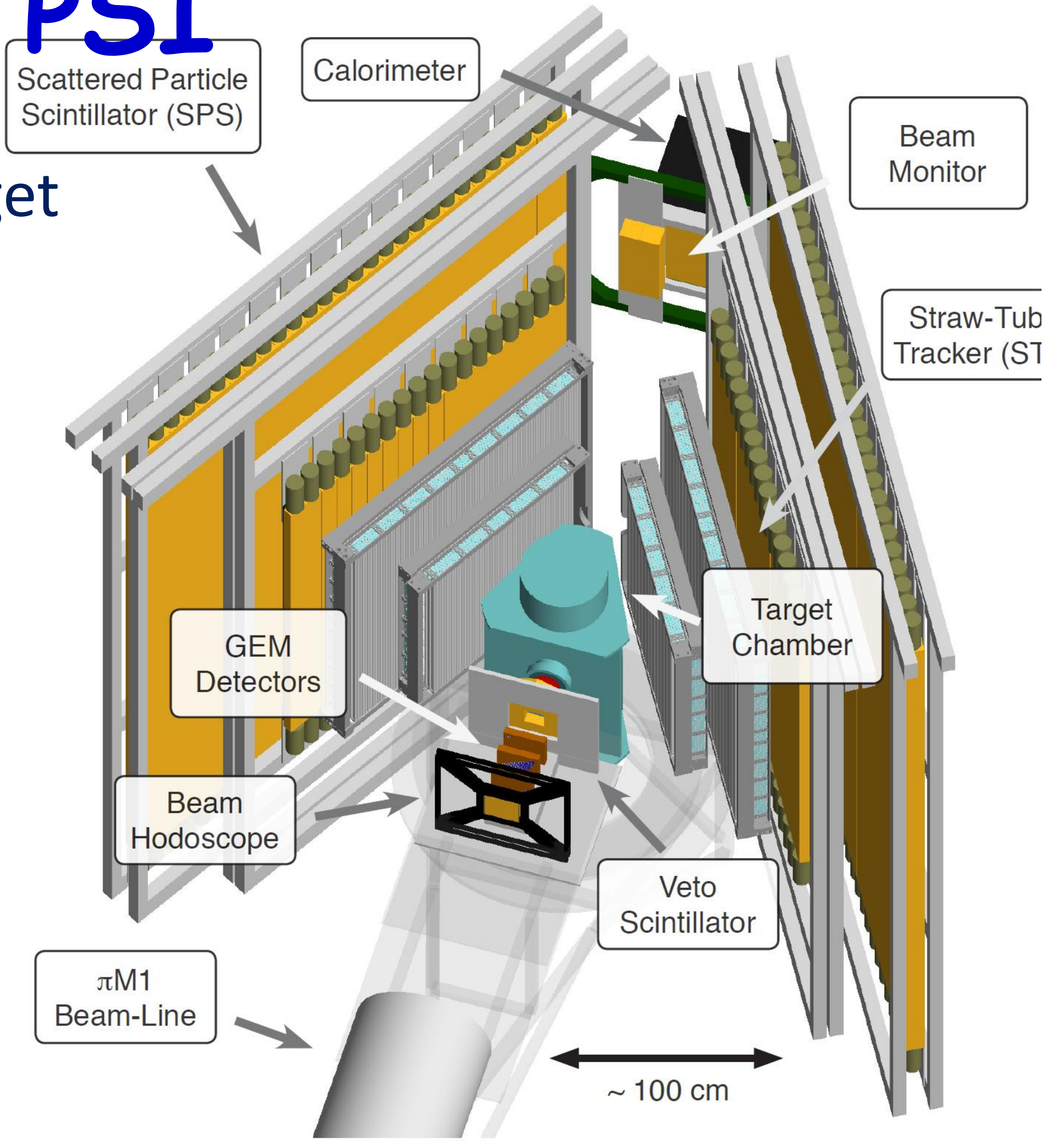
PSI: MUSE experiment

CERN: Amber experiment

MUSE experiment @ PSI

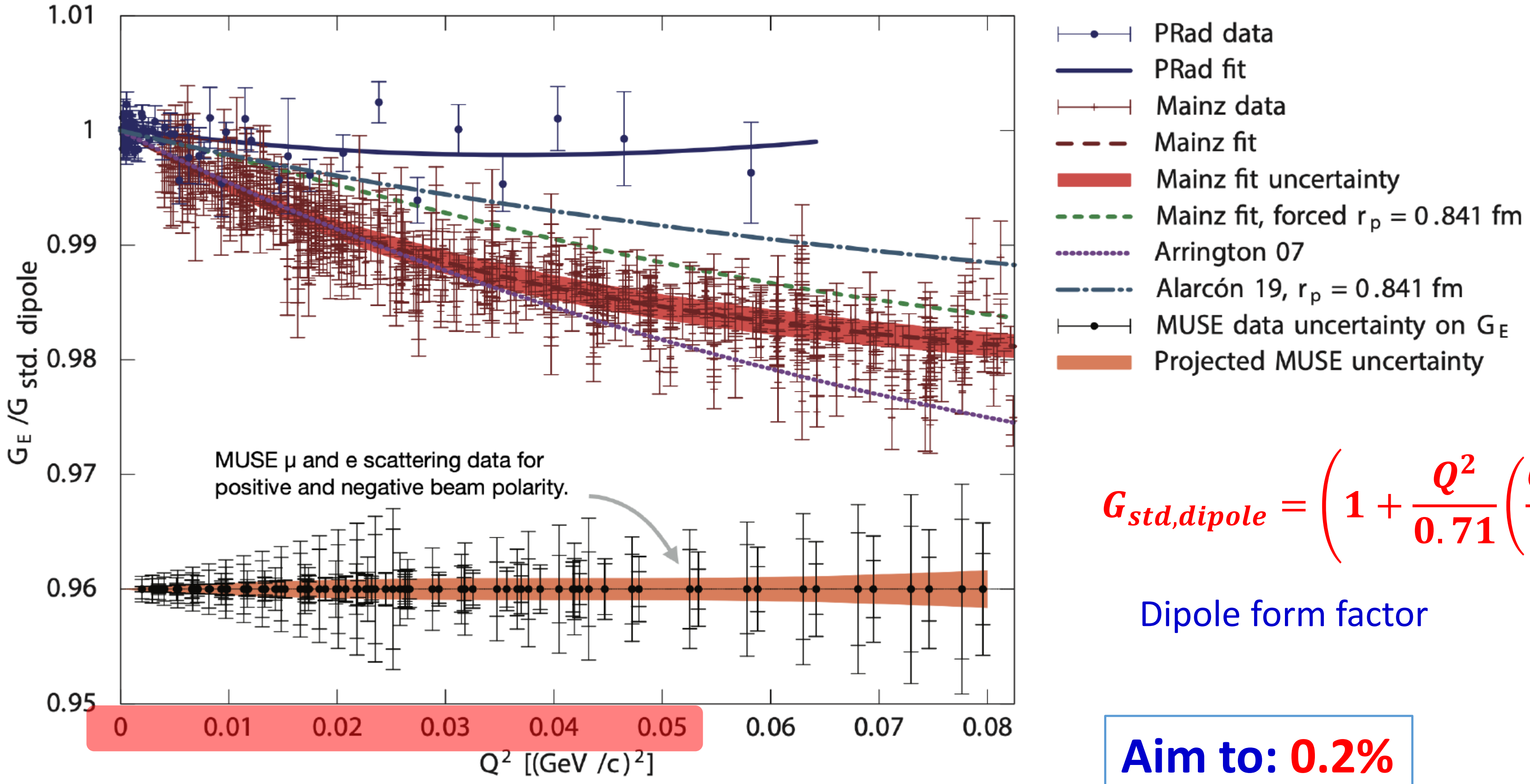
- Beams of $e^{\pm}, \pi^{\pm}, \mu^{\pm}$ on liquid H_2 target
  - Separate particles by TOF
  - Start to taking data: ~2022
- Muon momenta:
  - $115 - 210\ MeV/c^2$
  - $\theta: 20^o - 100^o$
  - $Q^2 = 0.002 - 0.07\ GeV^2$
- Extract G_E and G_M from fits to experiments cross-section data

$$Q^2 = \frac{2M_p E_1^2(1-cos\theta)}{M_p + E_1(1-cos\theta)}$$



Expected sensitivities from MUSE

Measure form factor as a function of Q^2



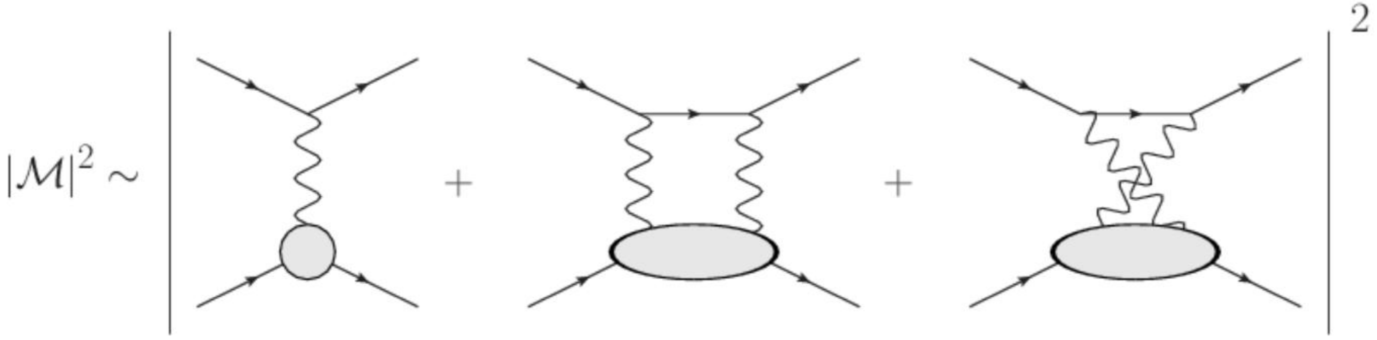
$$G_{std,dipole} = \left(1 + \frac{Q^2}{0.71}\left(\frac{GeV}{c}\right)^2\right)^{-2}$$

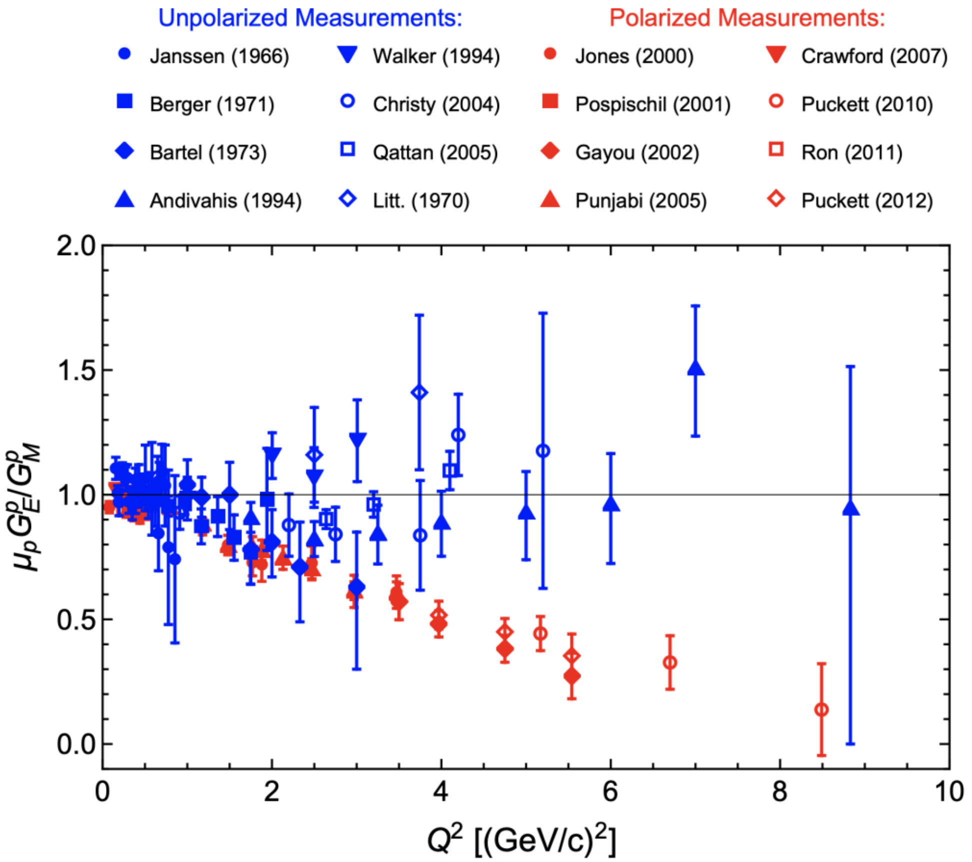
Dipole form factor

Aim to: 0.2%

MUSE Physics

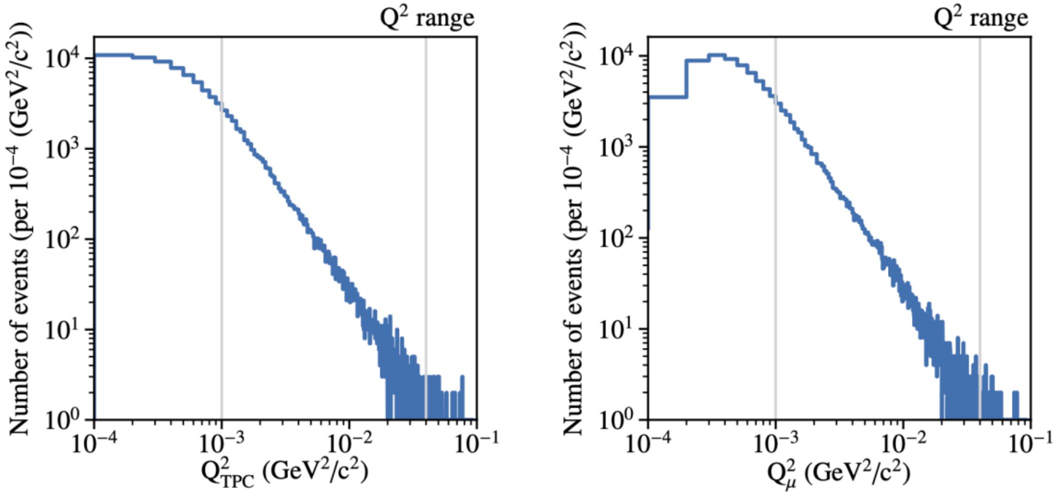
- Lepton-universality
  - Simultaneous electron and muon scattering experiment
  - Direct test of LU
- Radiative corrections
  - Muon is much heavier than electron
- Two photon exchange



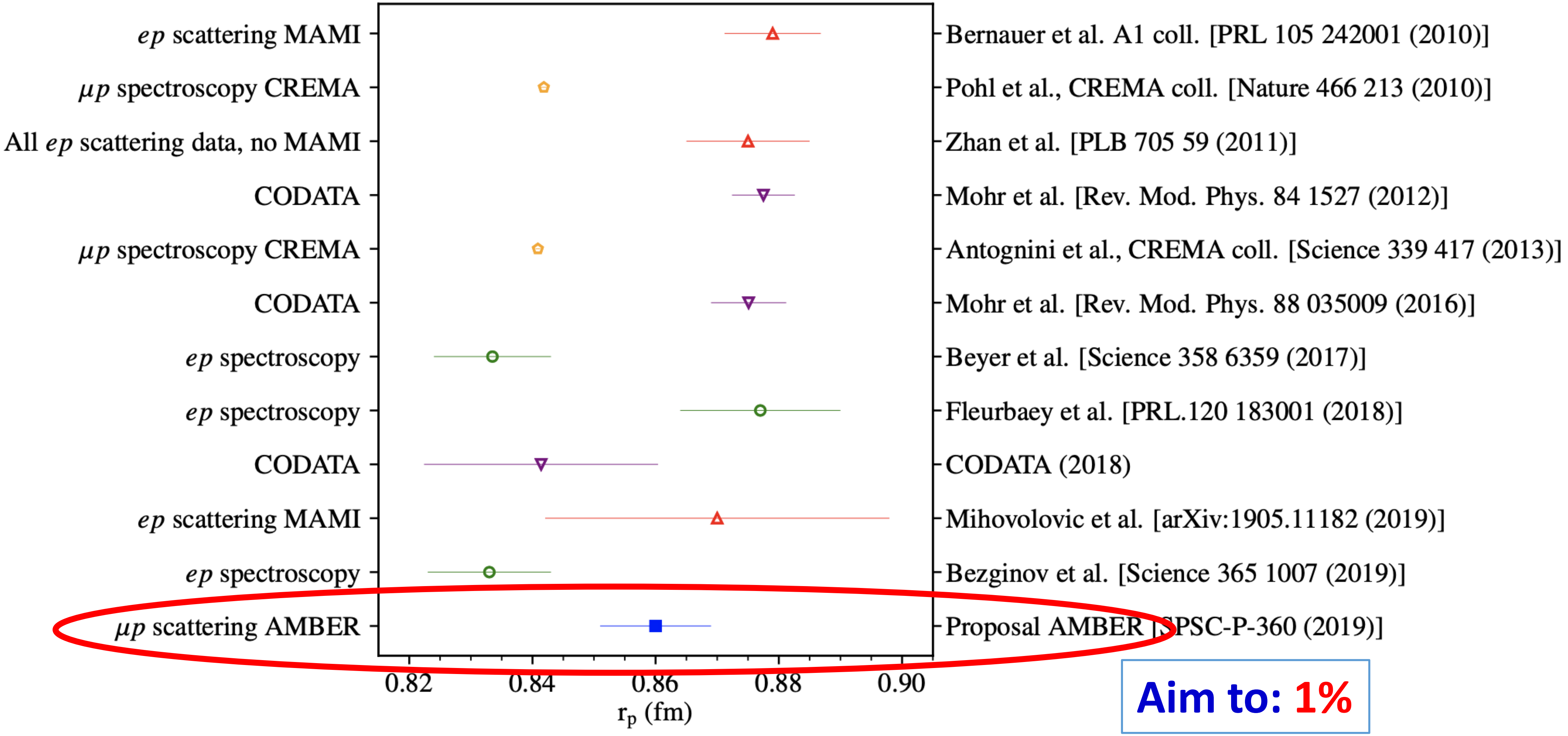


Amber experiment

- Compass -> Amber (NA66): Apparatus for Meson and Baron Experimental Research
  - A fixed target experiemtn at M2 beam line
  - Beam: muon, proton, pion, kaon from 50 GeV to 280 GeV
  - Physics:
    - Phase-I (2020-2025): dark matter, proton radius puzzle (100 GeV muon beam), pion and kaon PDFs
    - Phase-II: planning
  - Q^2: $\sim 10^{-3}$



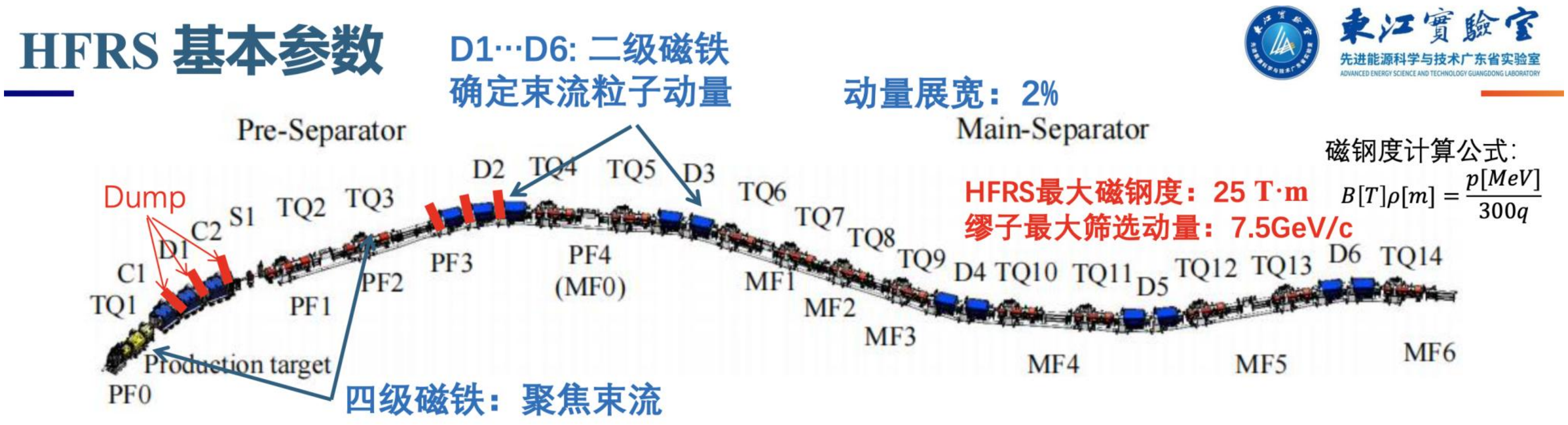
Amber Sensitivity



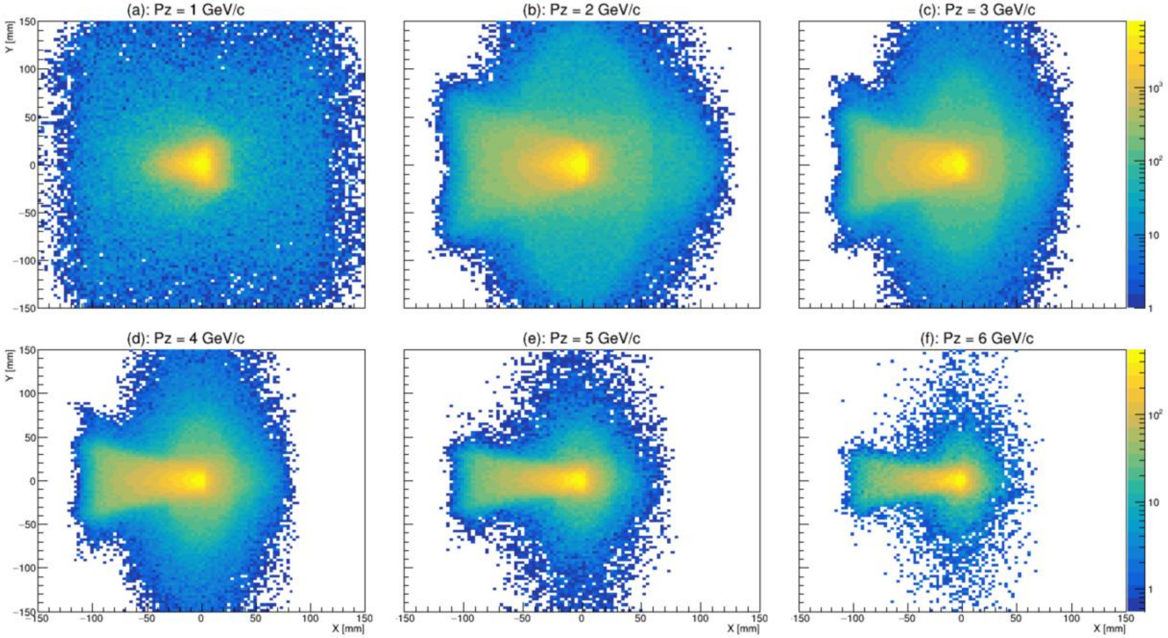
Aim to: 1%

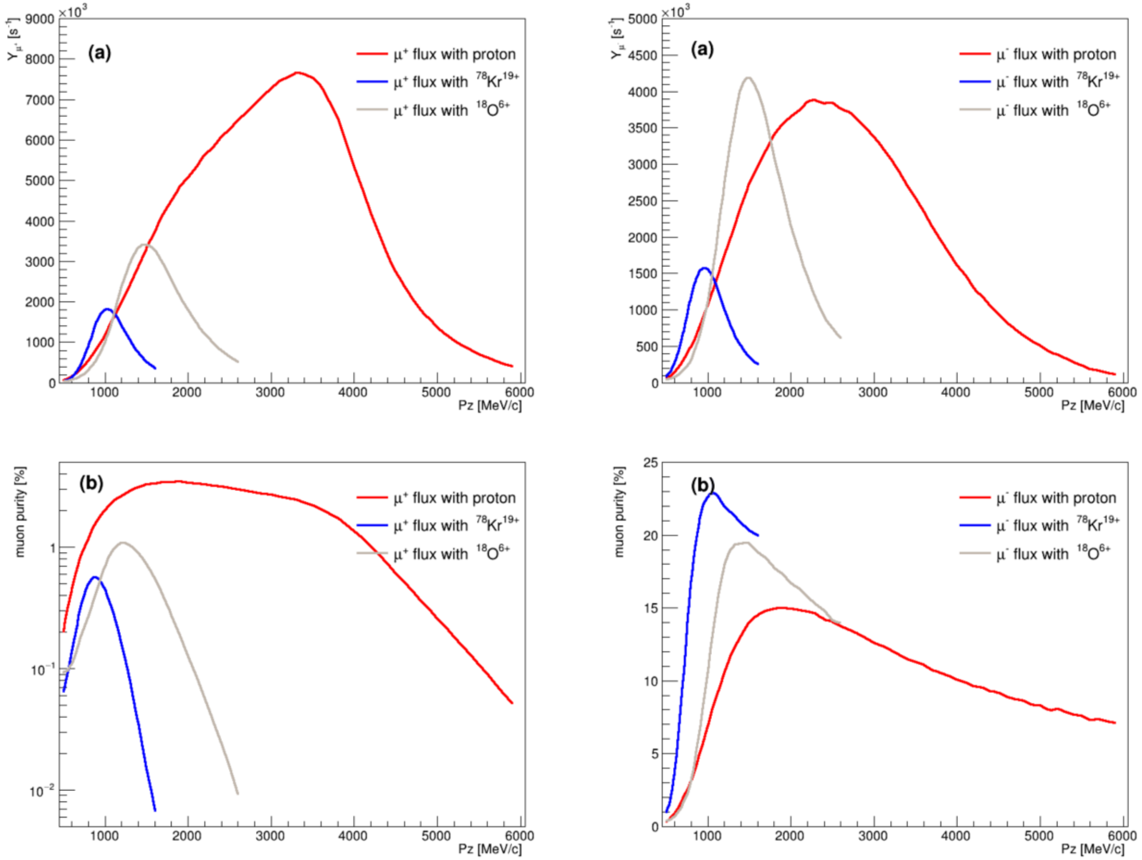
HIAF muon source

- HIAF可以利用HFRS束线产生缪子束流
  - 动量：500MeV/c – 7.5GeV/c
  - 产额：$10^6 - 10^7\ \mu/s$
  - 束斑大小：10cm*10cm
  - 经过特殊设置，纯度可以达到100%



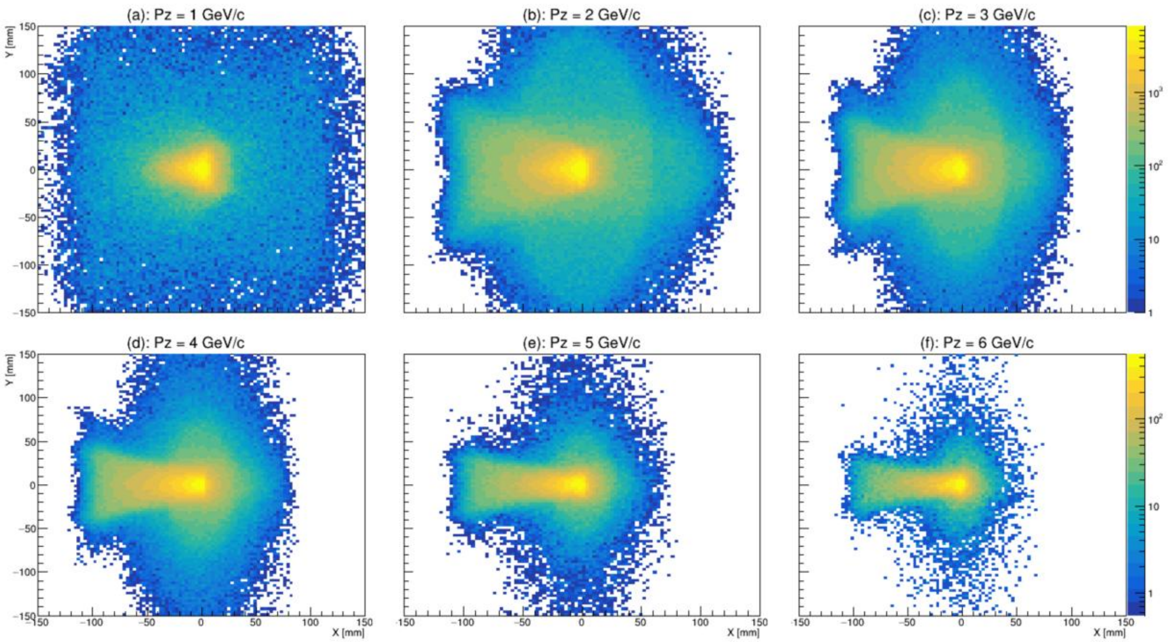
HIAF muon source

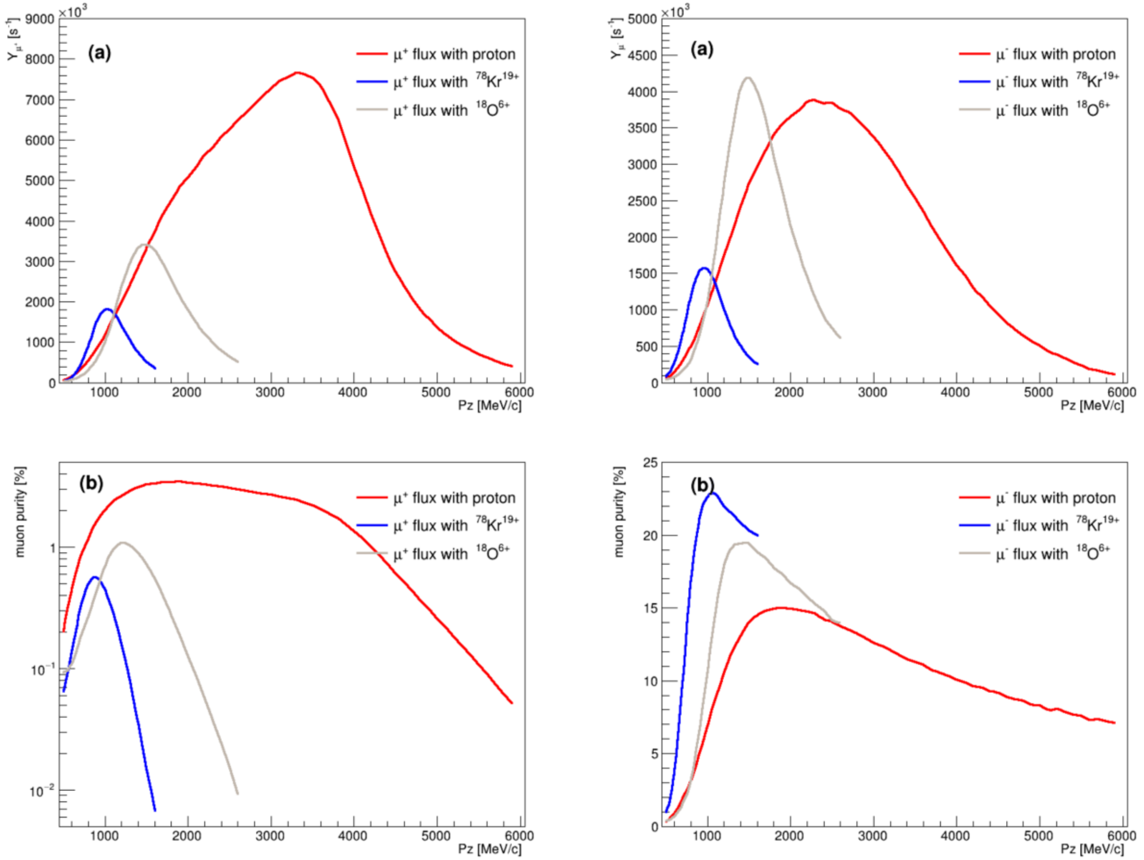




- After beam purification
  - $3 \times 10^5\ \mu^- / s$
  - $2.4 \times 10^5\ \mu^+ / s$

HIAF muon source





- After beam purification
  - $3 \times 10^5\ \mu^-/s$
  - $2.4 \times 10^5\ \mu^+/s$

How about a new muon-proton scattering experiment in China?

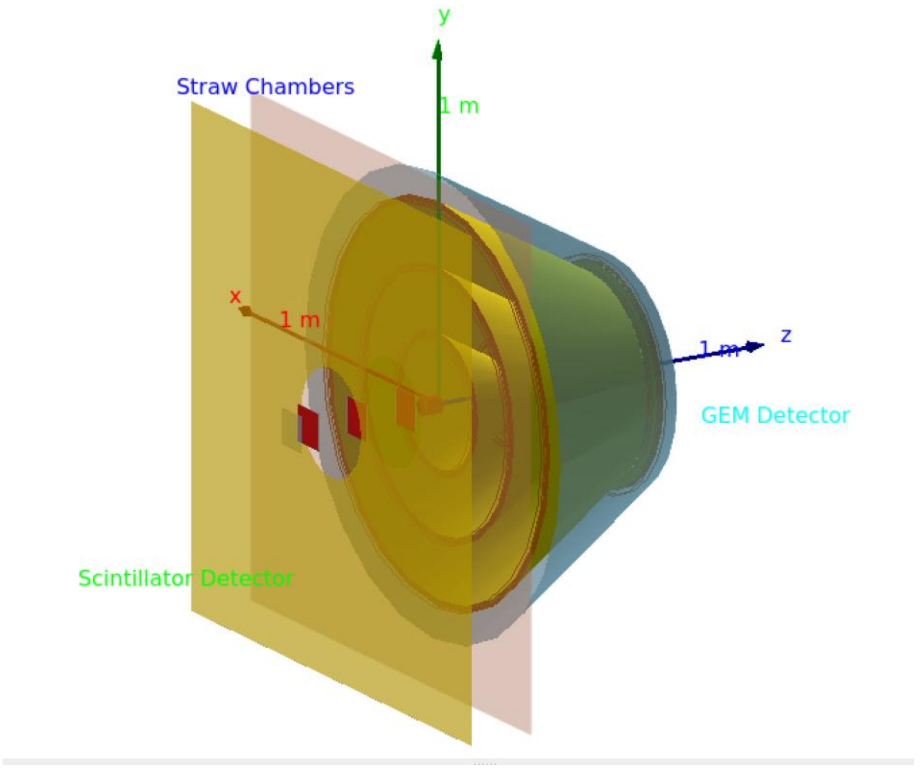
Precision of current results

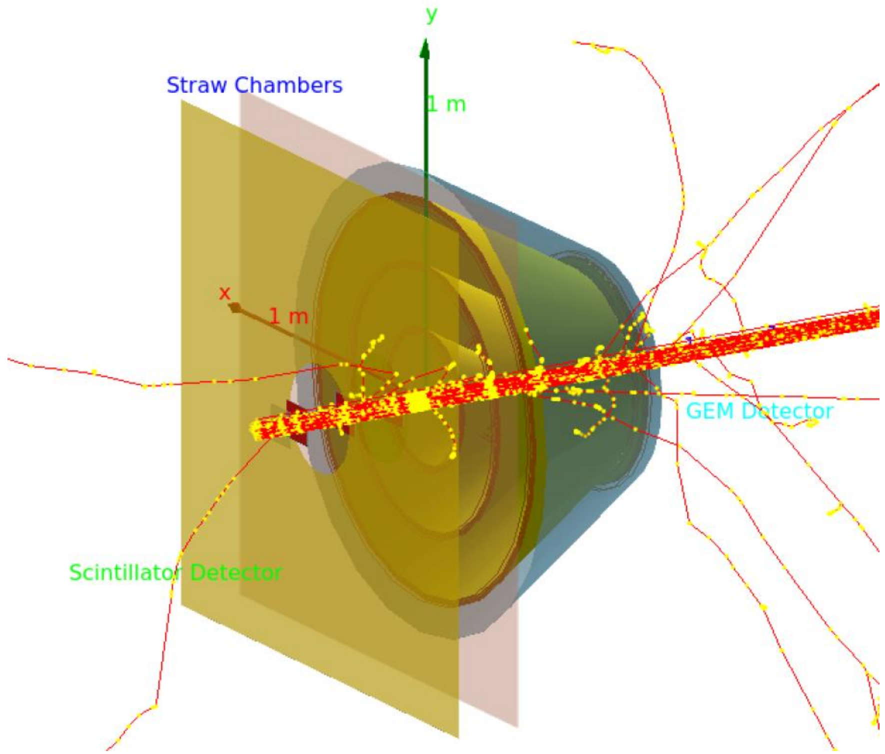
- Muonic hydrogen: 0.046%
- Electronic hydrogen: 0.88%
- Electron scattering: 0.68%
- Muon scattering: 0.2% (MUSE) , 1% (Amber)

- A new muon-proton scattering experiment in China:
  - 0.1% (our target)

A new proposal

- Based on the Geant4 simulation:
  - codes from Prof. Weizhi Xiong (熊伟志，SDU)



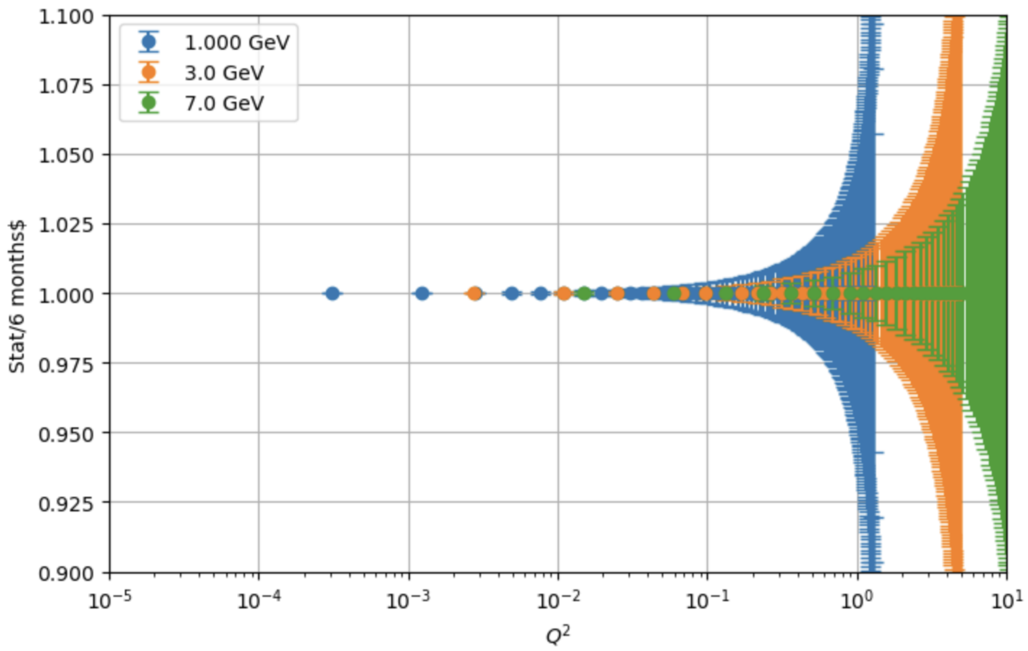


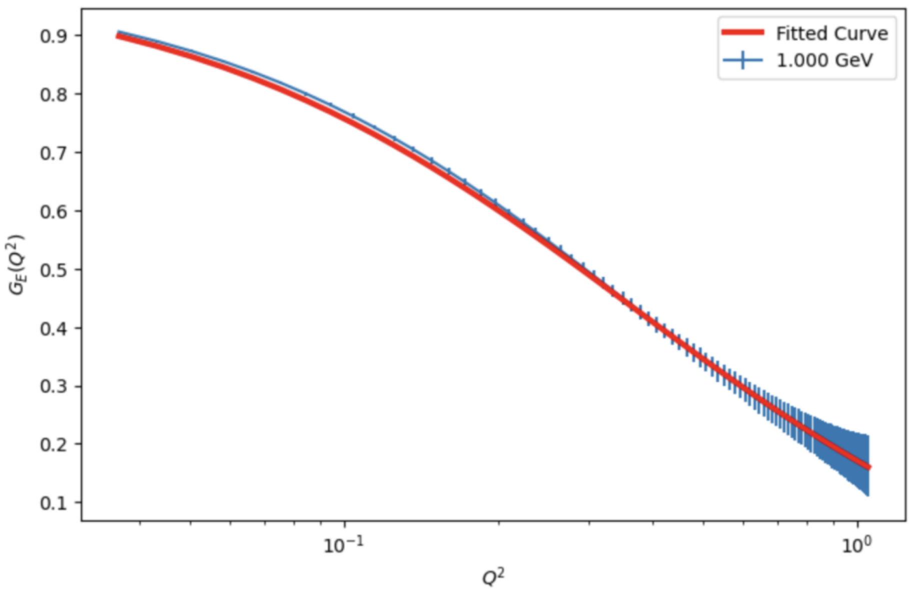
How to achieve this?

The proton radius measurement prefers a low energy muon beam!

- Acceptance: $10^o - 100^o$
- Running for 12 months
- Expected stat. uncertainty
  - dominated by syst. Uncertainty

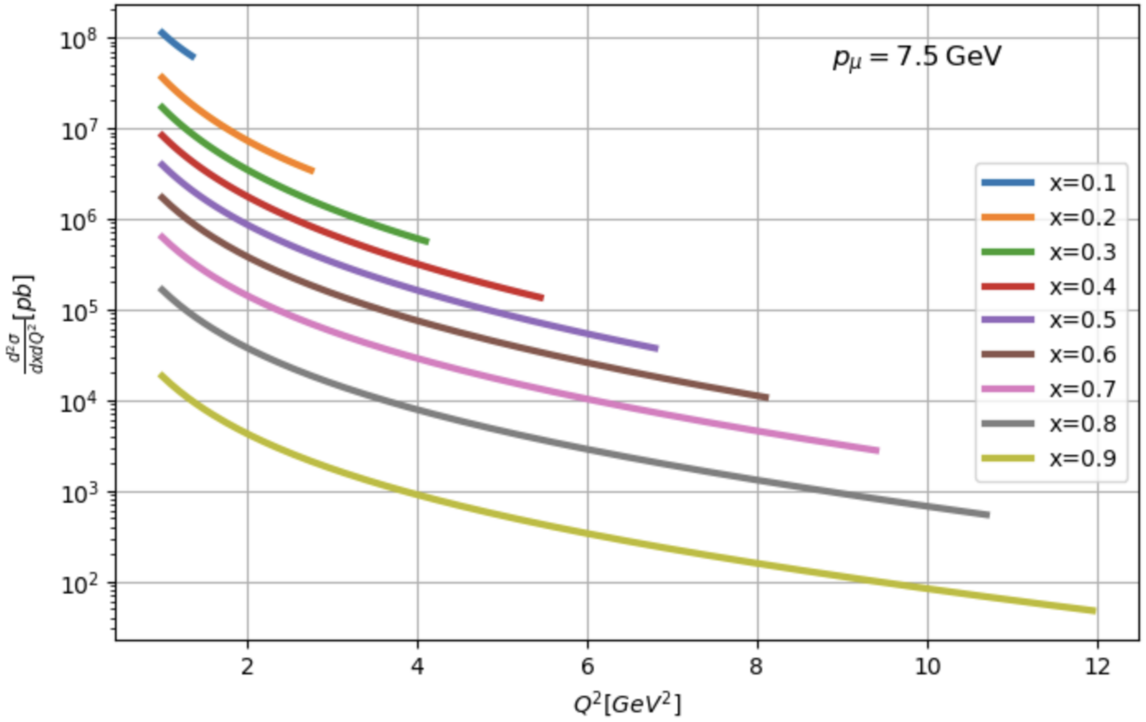
Beam	Absoluate uncer. (fm)	Relative uncer.
1 GeV	0.0021	0.24%
3 GeV	0.005	0.6%
7 GeV	0.008	0.7%

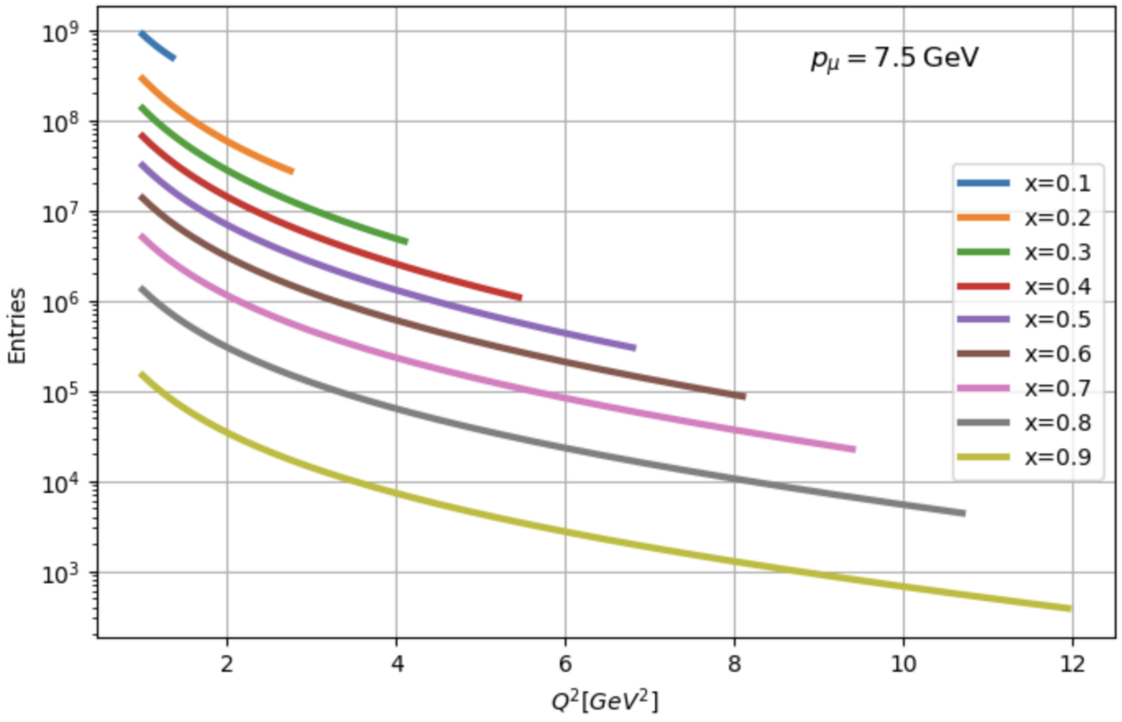




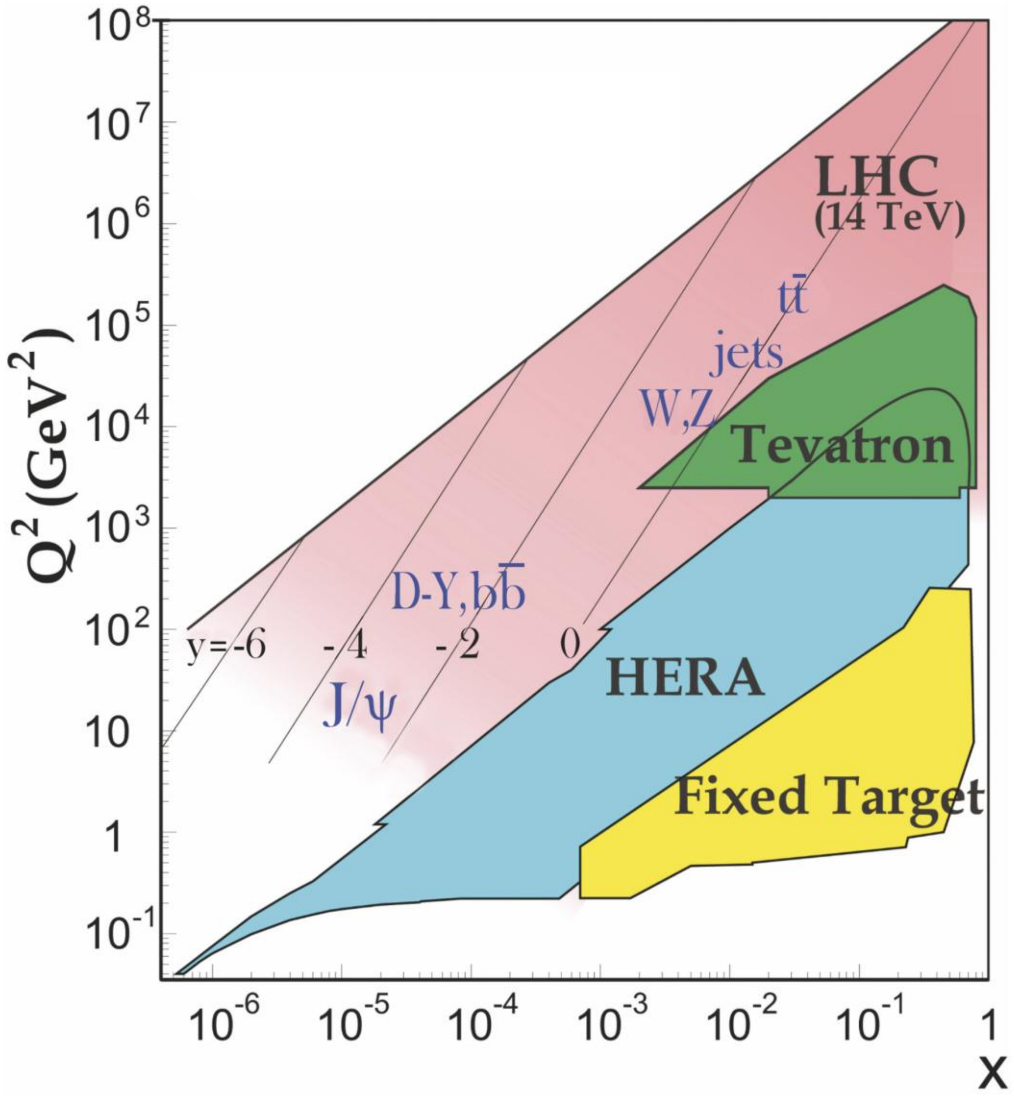
Large-x PDFs

- Also can be used to probe the proton PDFs with large-x



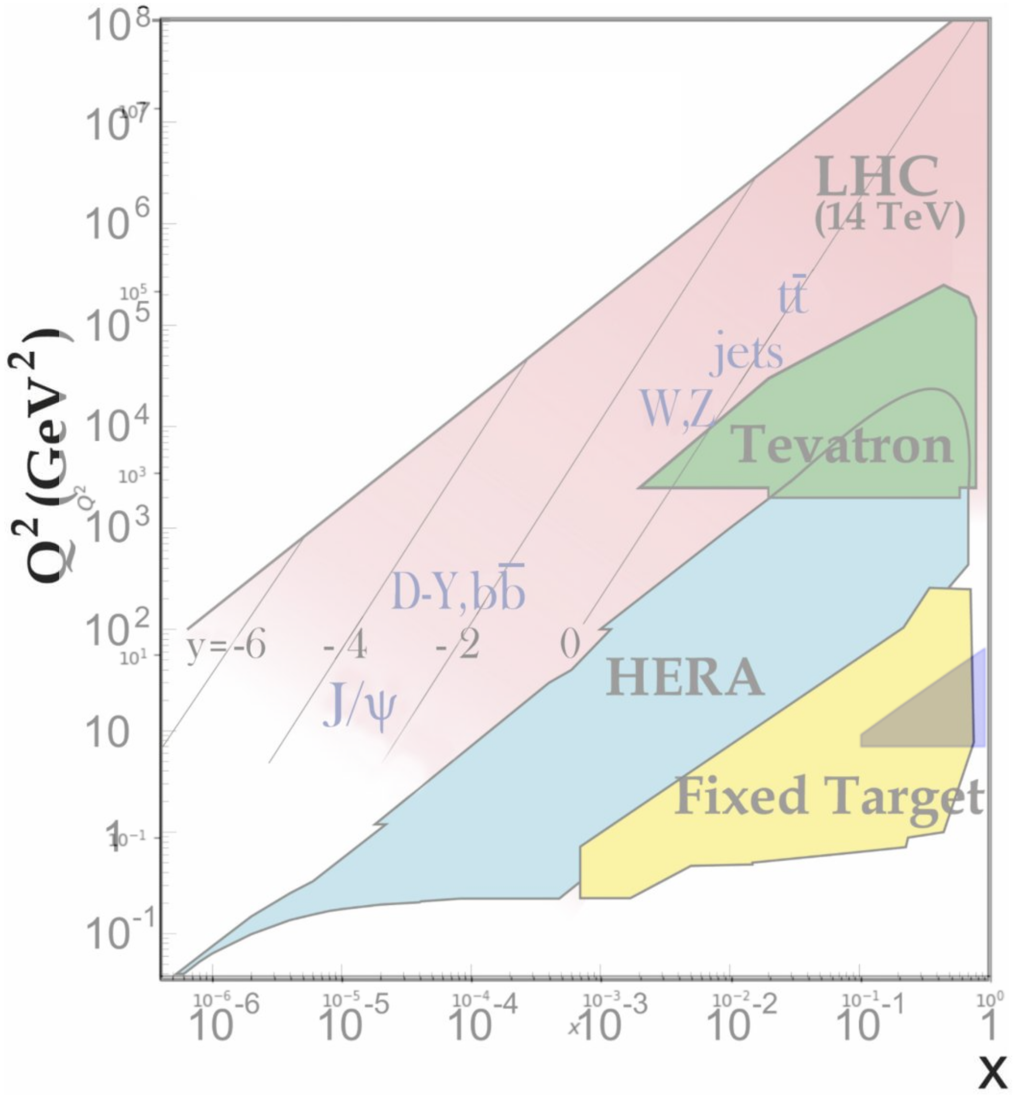


PDFs region



Q^2 (GeV^2)
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
LHC
(14 TeV)
$t\bar{t}$
jets
W,Z
Tevatron
D-Y,$b\bar{b}$
y = -6
-4
-2
0
HERA
J/ψ
Fixed Target

PDFs region



LHC
(14 TeV)
$t\bar{t}$
jets
W,Z
Tevatron
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y = -6
-4
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HERA
Fixed Target
Q^2 (GeV^2)
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

Other measurements?

- New Physics:
  - Lepton universality: electron and muon
  - Dark sector
- Neutron radius? (different targets)
  - With deuteron (proton+neutron)
  - With triton (proton+2 neutron)
- More?
  - Other targets?
  - $Be, C, Al, Ca, Fe, Ag, Au$ …

Conclusion + Discussions

- The proton charge radius needs more experimental inputs
  - MUSE @ PSI; Amber @ CERN; PRad @ Jlab …
- A new muon-proton scattering experiment in China
  - Measure the proton radius: 0.1% (most precise result from lepton scattering exp.)
  - The cost of detector looks reasonable: $\sim 10\ M\ CNY$
- More physics motivations:
  - Measure radius of other nucleons
  - Large x of proton PDFs
  - Dark matter
- **Letter of interest**: the end of this year

Conclusion + Discussions

us needs more experimental inputs

CERN; PRad @ Jlab …

cattering experiment in China

dius: 0.1% (most precise result from lepton scattering exp.)

oks reasonable: $\sim 10\ M\ CNY$

ons:

r nucleons

You are more than welcome to join us!

end of this year

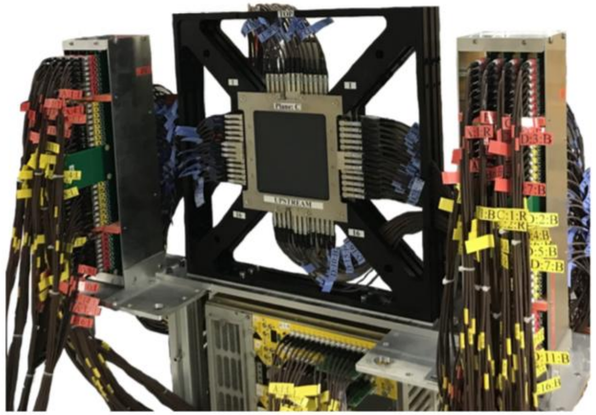
Backup

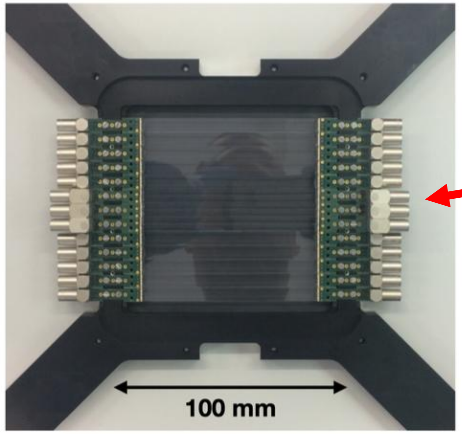
MUSE detector

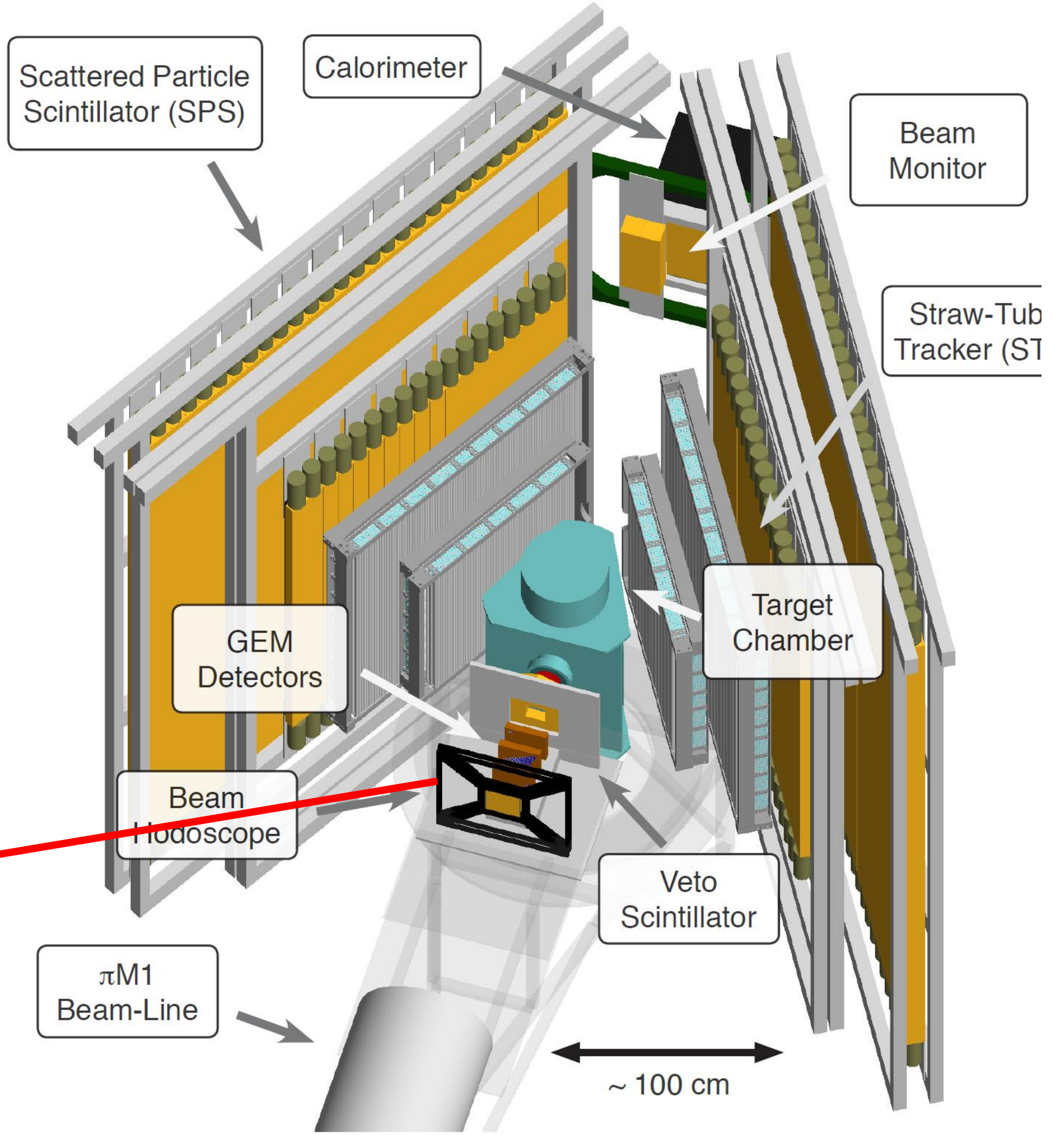
- Beam Hodoscope: 2 laryers
  - Thin plastic scintillators (BC-404)
    - Timing information of the incident particles
    - SiPM for readout

TABLE VI. Beam hodoscope detector requirements

Parameter	Performance Requirement	Achieved
Time Resolution	<100 ps / plane	✓ 80 ps
Efficiency	99%	✓ 99.8%
Positioning	≈1 mm, ≈1 mr	not attempted; easy – calibrated by data
Rate Capability	3.3 MHz / plane	✓ >10 MHz / plane





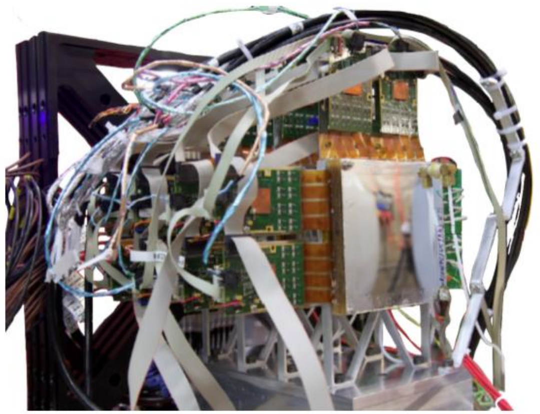


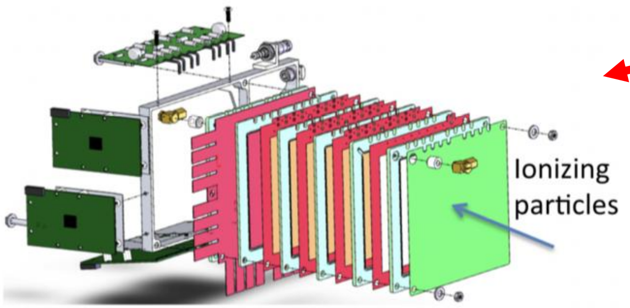
MUSE detector

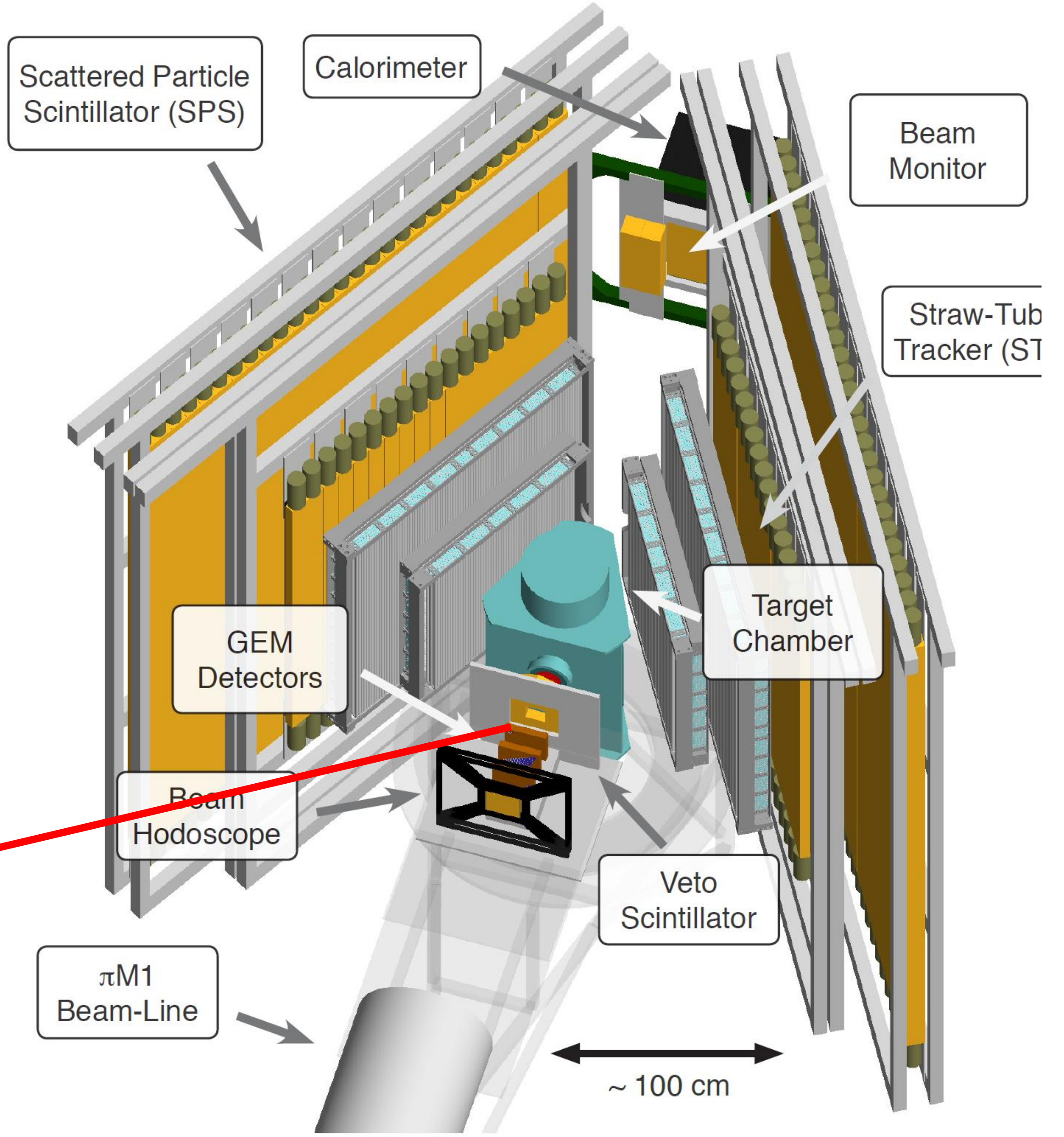
- GEM detectors: 2 layers
  - 5 μm copper-clad 50 μm Kapton-layers
  - Argon + CO_2, $10 \times 10\ cm^2$
    - incident particle tracking

TABLE VII. GEM detector requirements

Parameter	Performance Requirement	Achieved
Resolution	100 μm / element	✓ 70 μm
Efficiency	98%	✓ 98%
Positioning	≈0.1 mm, ≈0.2 mr	not attempted; easy
Rate Capability	3.3 MHz / plane	✓ 5 MHz
Readout Speed	2 kHz / 20% deadtime	1 kHz / 100% deadtime

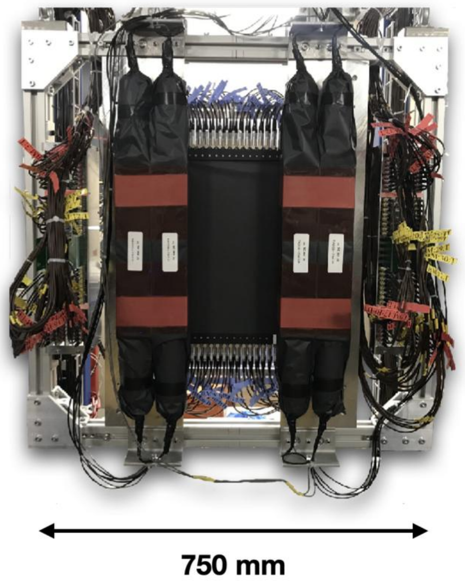


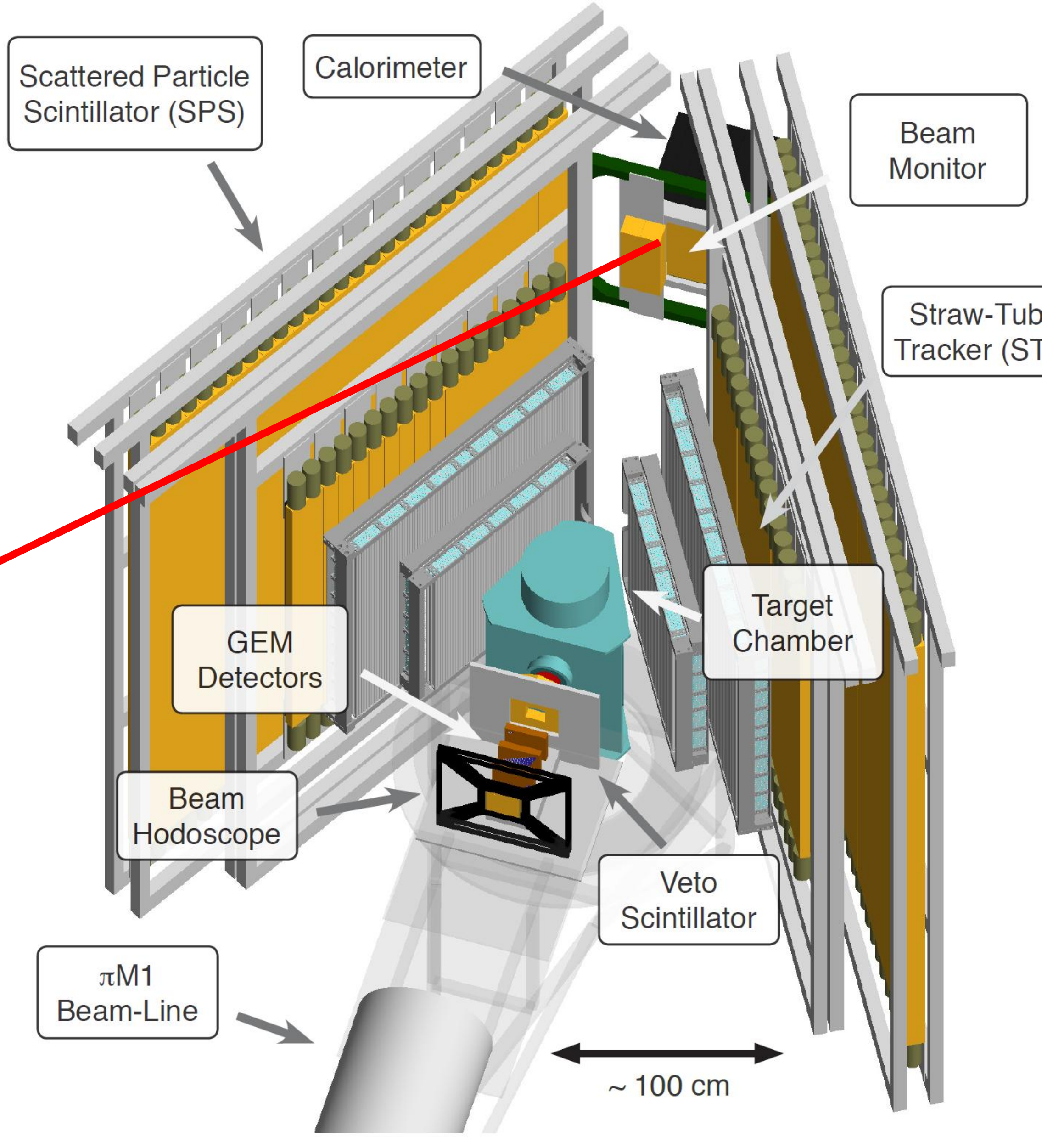




MUSE detector

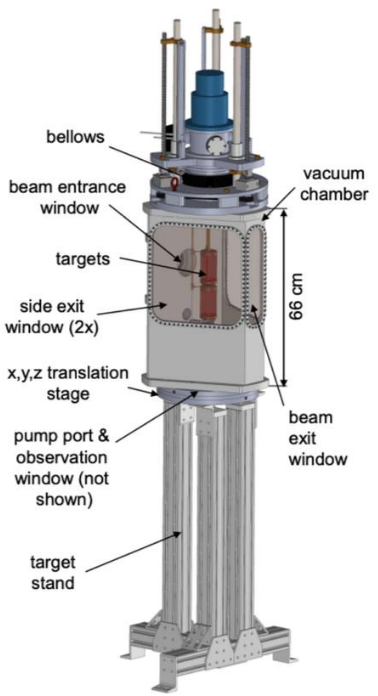
- Beam Monitor: 3 layers
  - plastic scintillators: BC-404
    - PMTs for readout
  - Timing: for PID

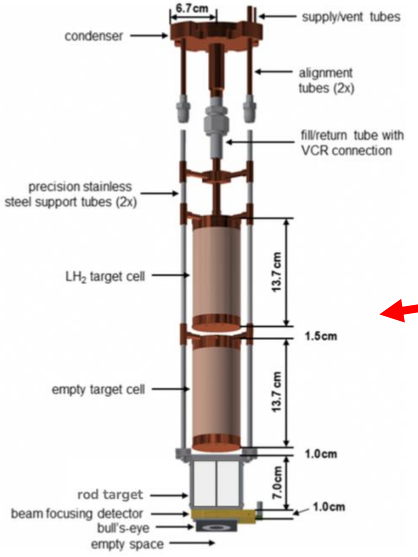


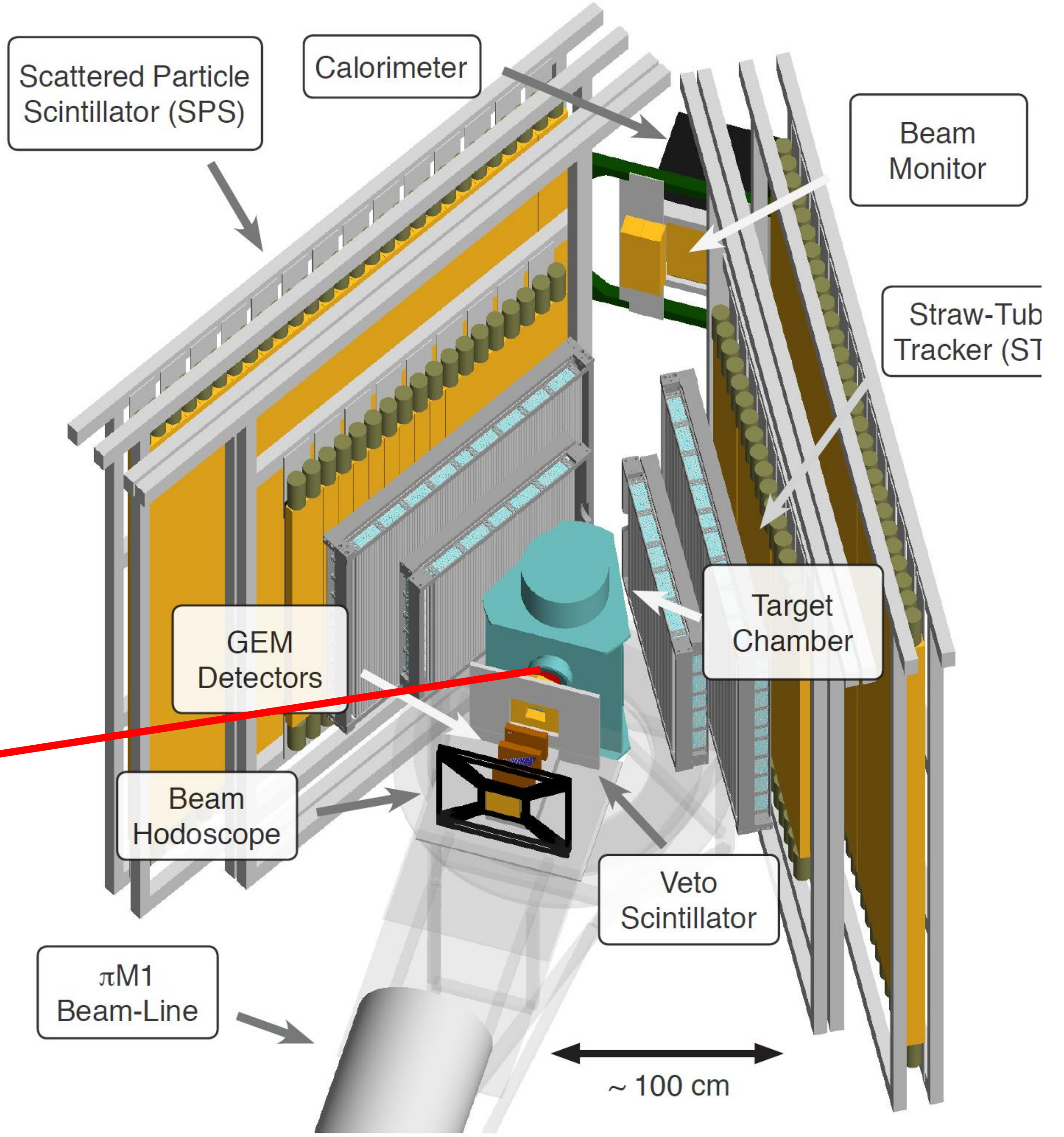


MUSE detector

- Target:
  - Liquid hydrogen: 20 K
  - 125 μm thick aluminized Kapton

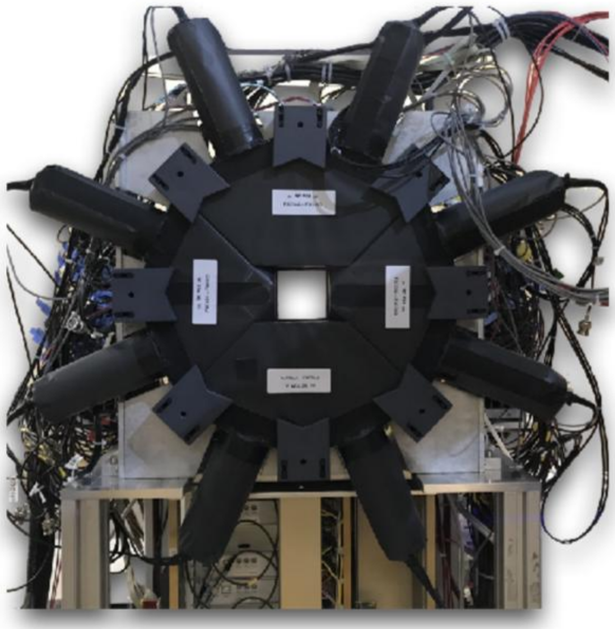




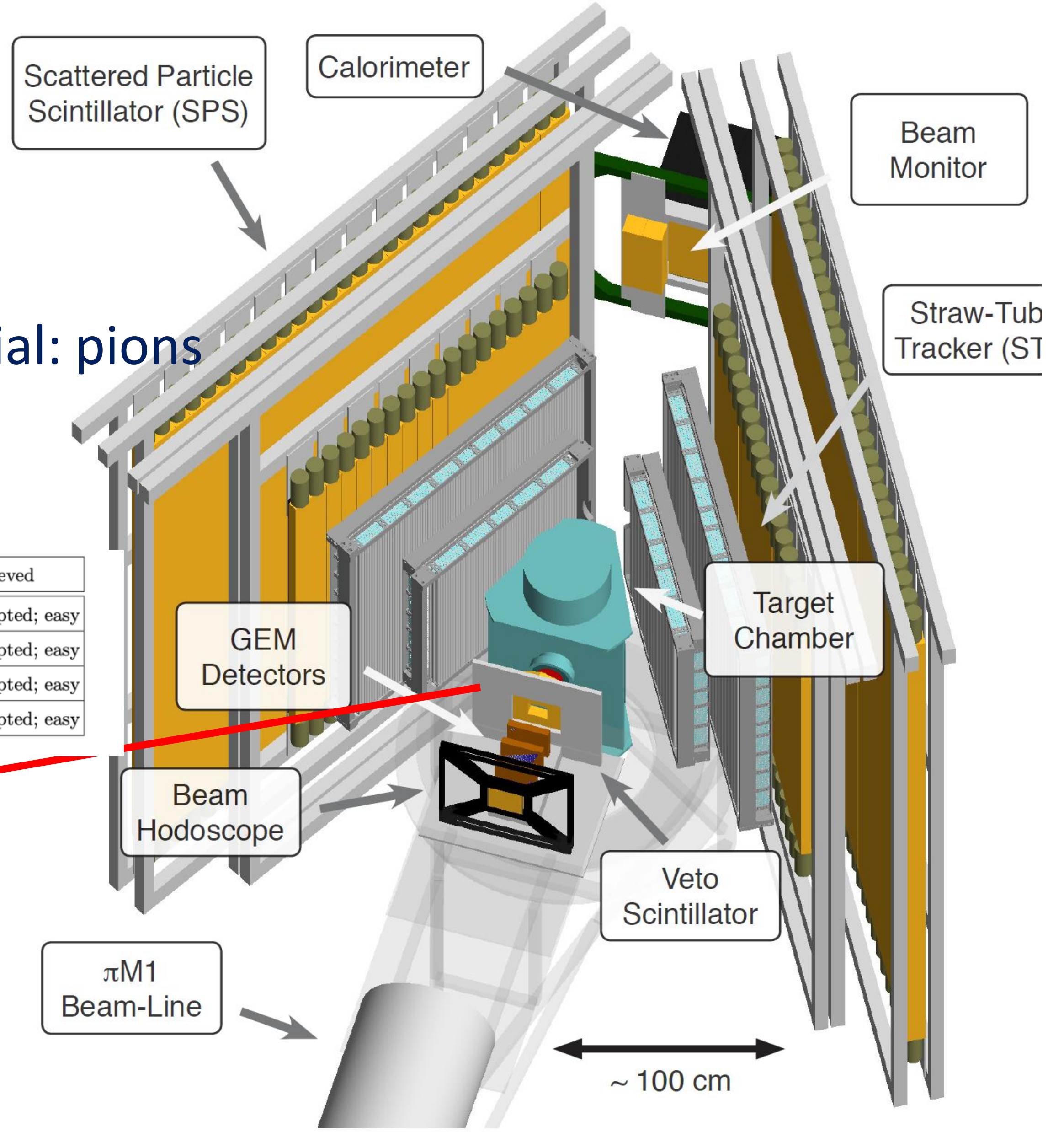


MUSE detector

- Veto detector:
  - Scintillators
  - Veto events hit entrance window material: pions



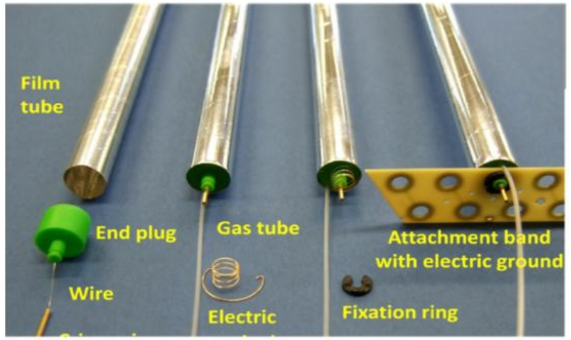
Parameter	Performance Requirement	Achieved
Time Resolution	1 ns / plane	not attempted; easy
Efficiency	99%	not attempted; easy
Positioning	≈1 mm, ≈1 mr	not attempted; easy
Rate Capability	1 MHz / plane	not attempted; easy

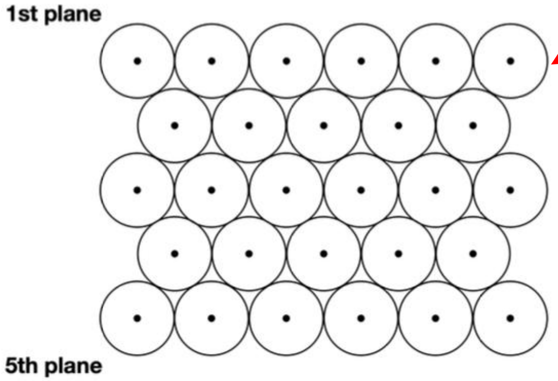


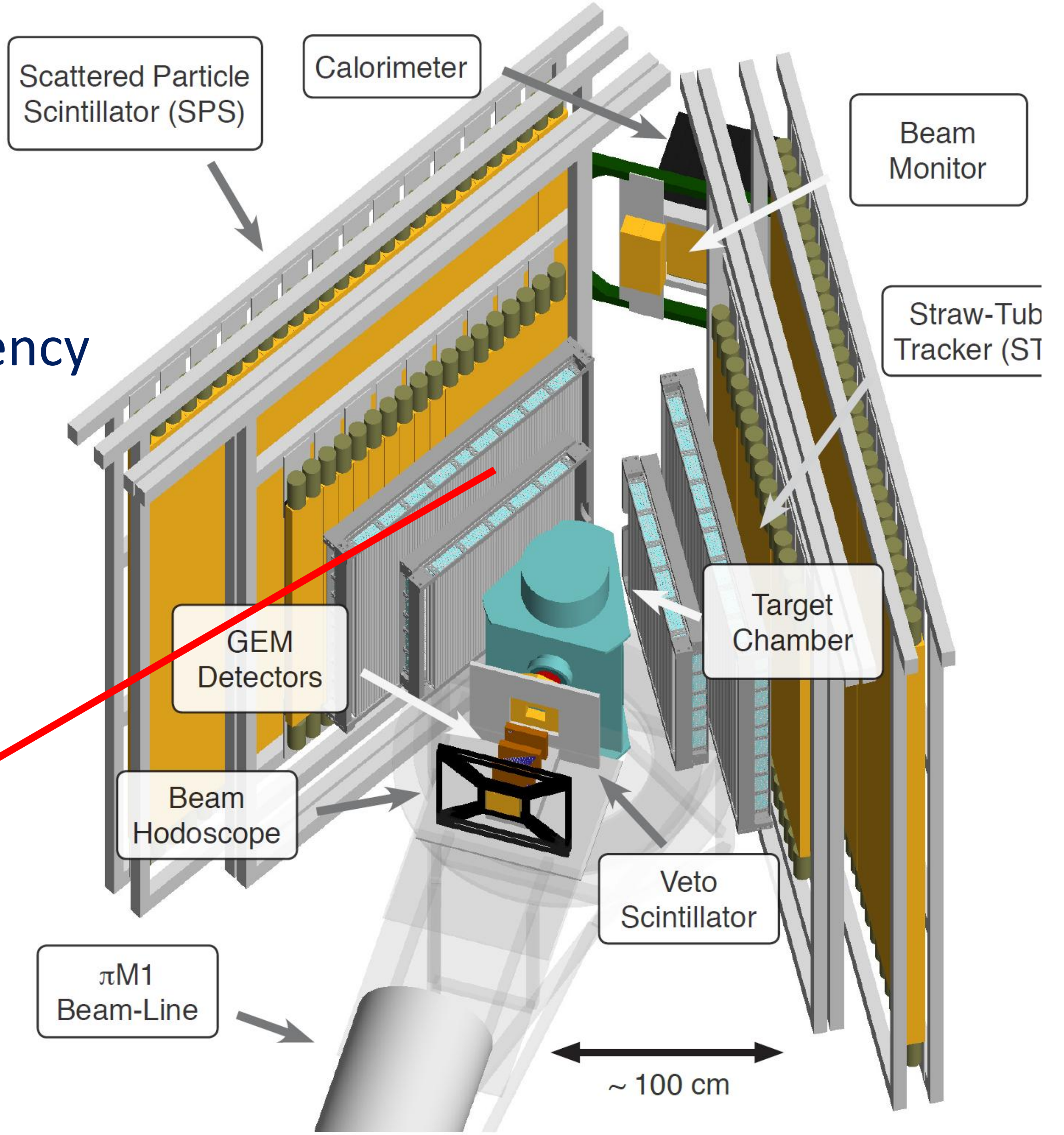
MUSE detector

- Scattered particle detector:
  - STT: Straw chambers
  - High position resolution, and high efficiency

Parameter	Performance Requirement	Achieved
Position Resolution	150 μm	✓ <120 μm
Efficiency	99.8% tracking	≈ 99% in prototype; moderate
Positioning	≈0.1 mm, 0.2 mr in θ	not attempted; moderate
Positioning	≈0.5 mr pitch, yaw, roll	not attempted; moderate
Positioning	50 μm wire spacing	not attempted; moderate
Rate Capability	0.5 MHz	not attempted; easy

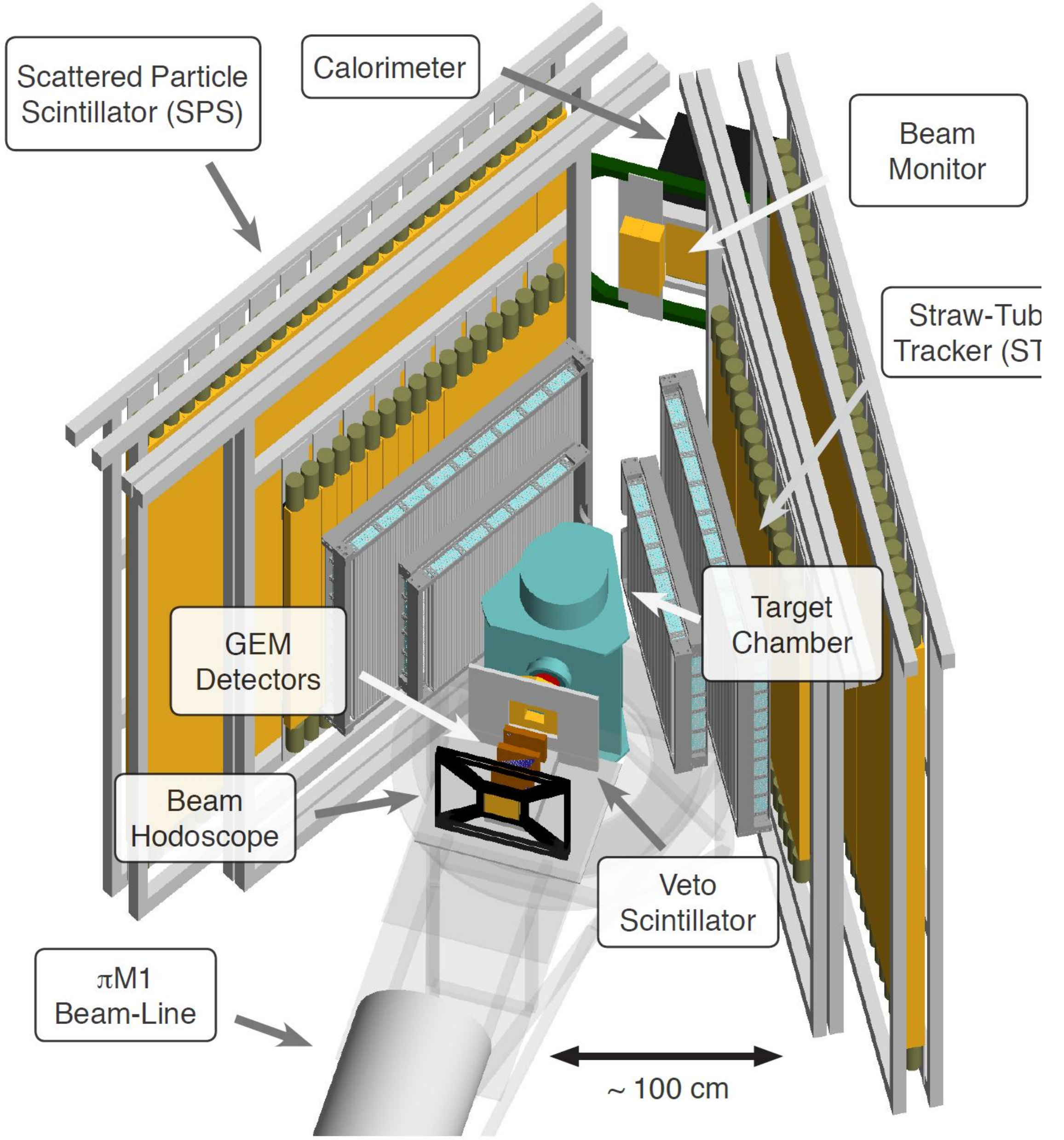




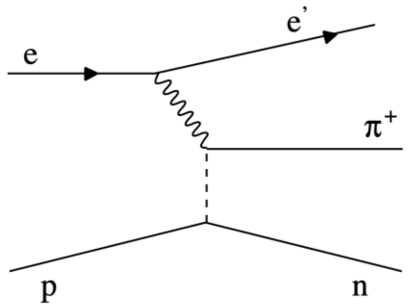


MUSE detector

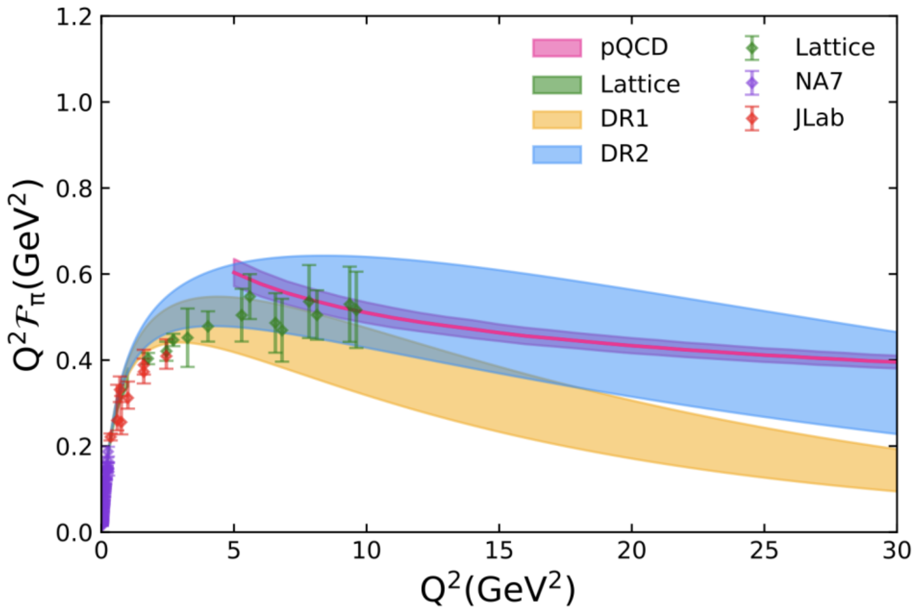
- Calorimeter:
  - Lead-glass
  - Stutdy radiative correction, PID

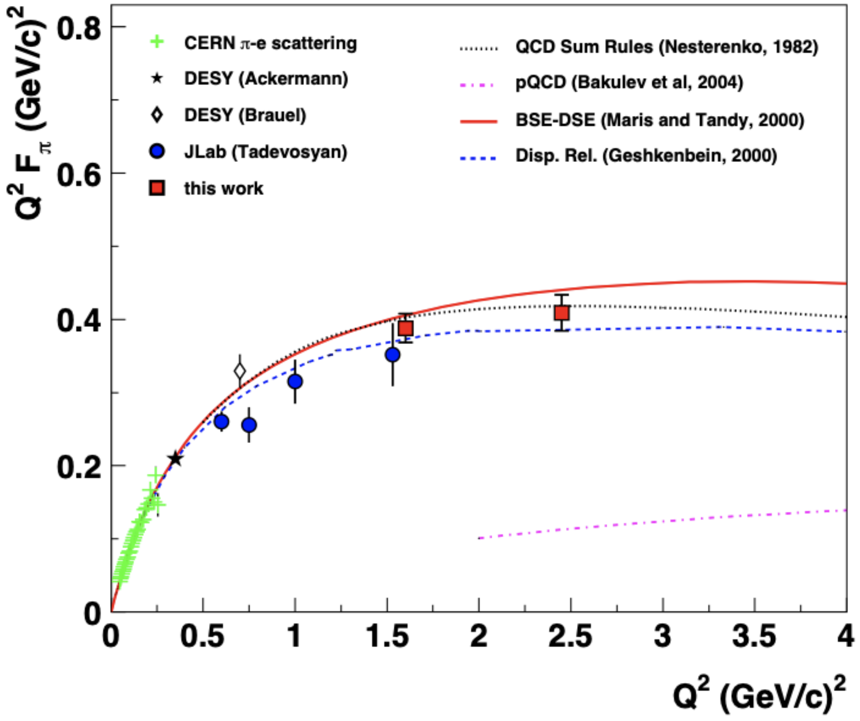


Pion form factor



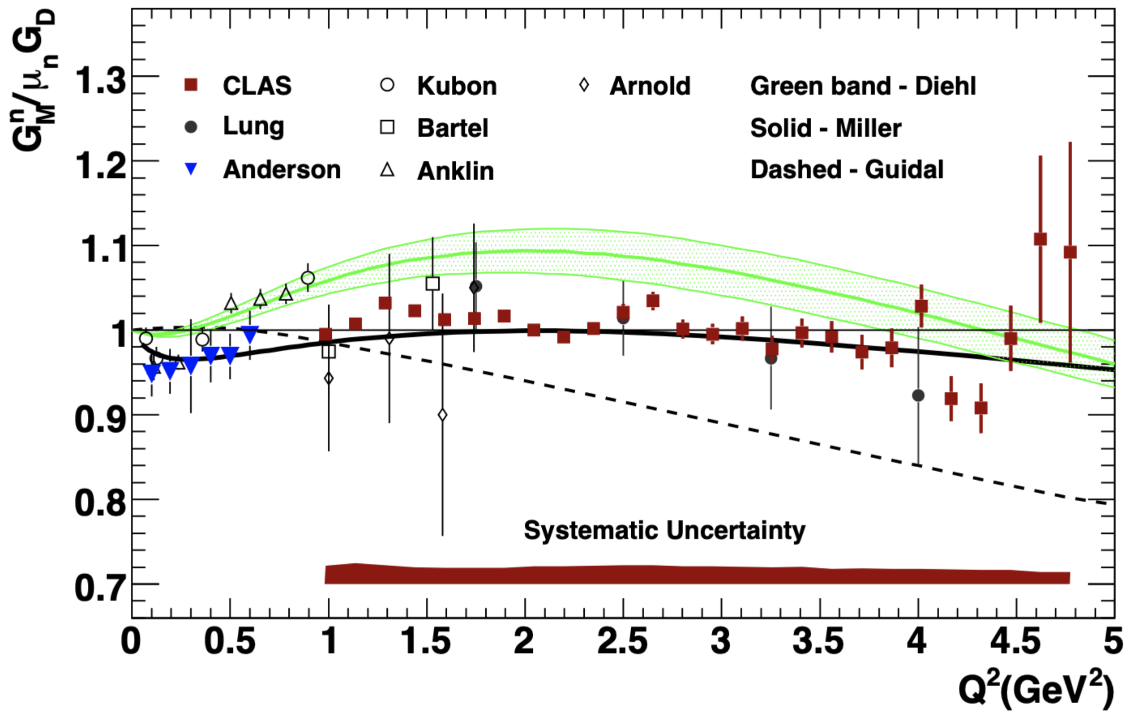
- To understand the hadronic structure
- Mesured use electron-proton scattering

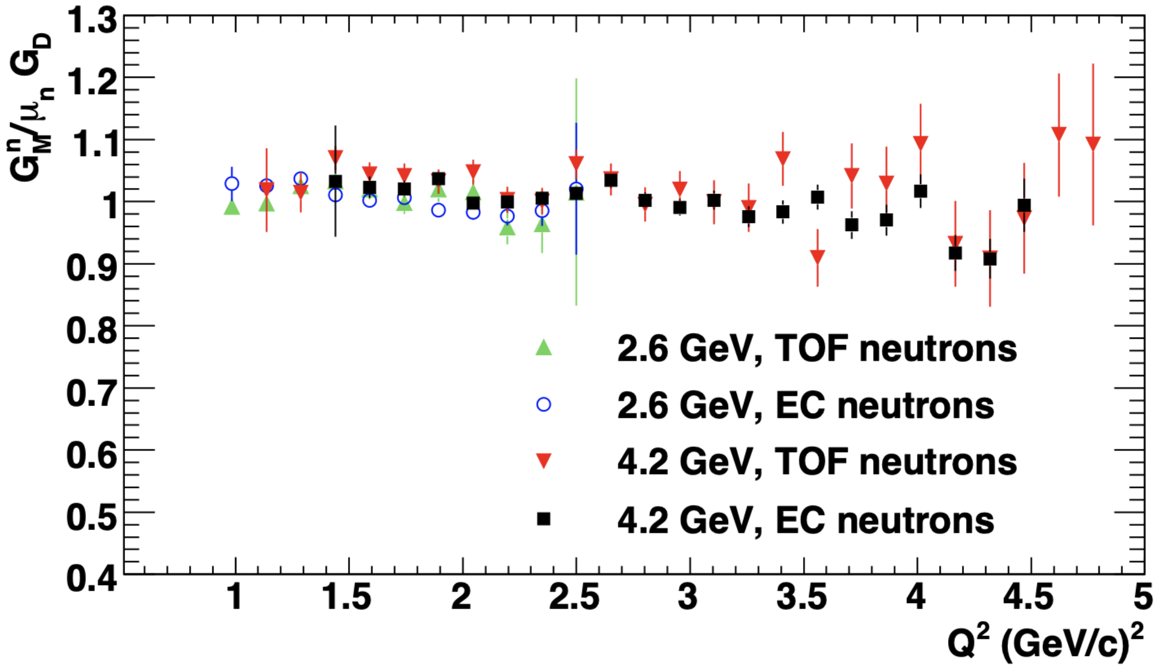




Neutron Magnetic form factor

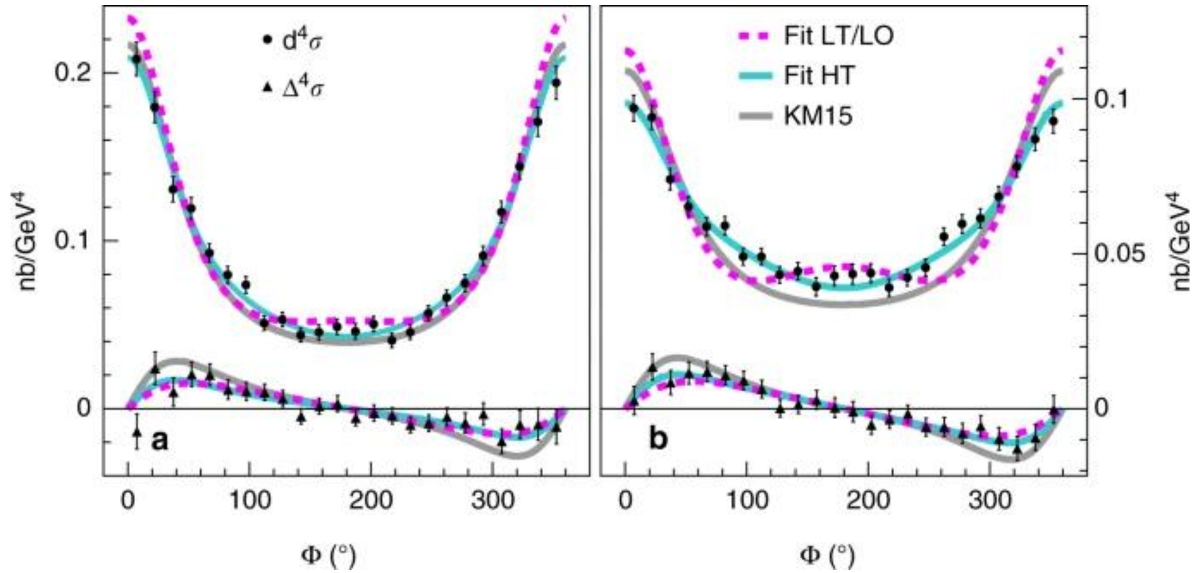
- Use polarized $^{3}He(ppn)$ gas target
- Electron Beam energy (Jlab): 2.6 GeV, 4.2 GeV

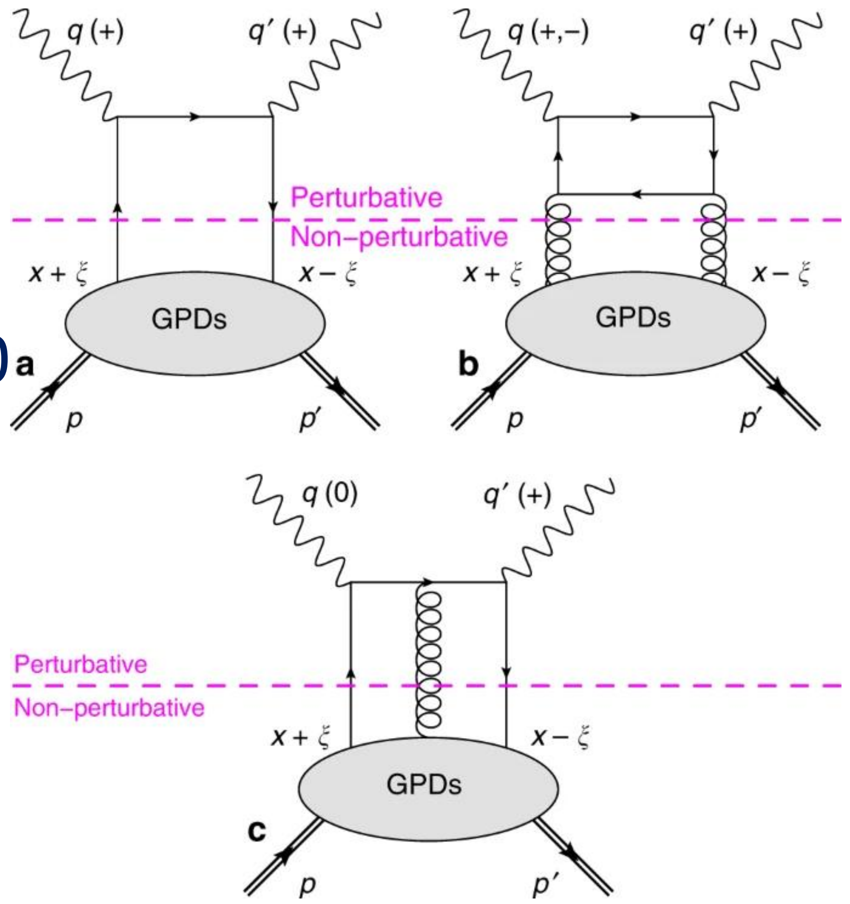




Virtual compton scattering

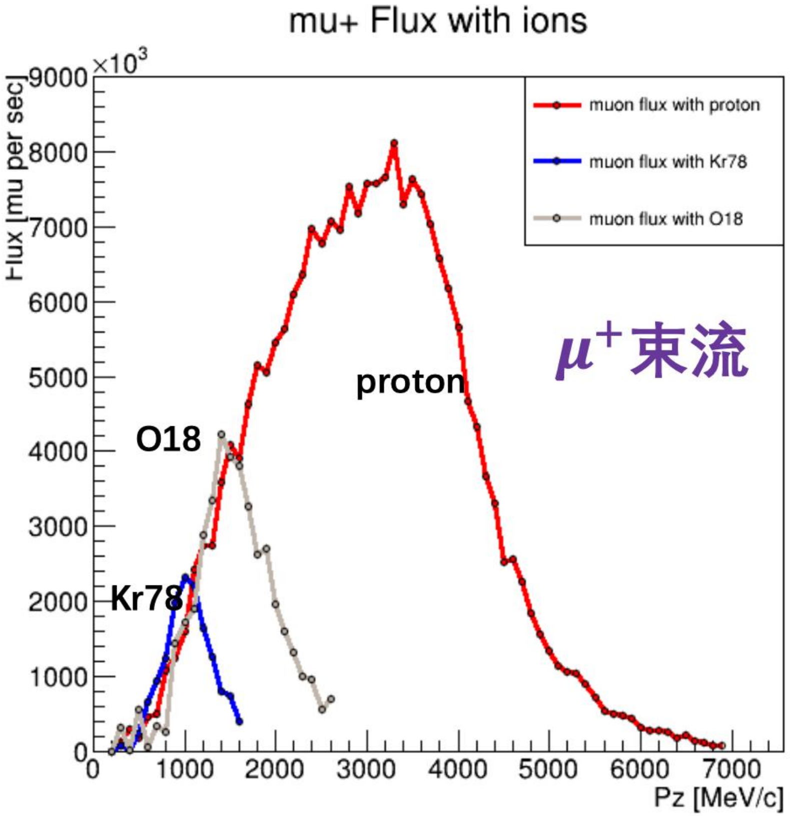
- Three-dimensional structure of nucleons
  - Generalized parton distributions
- Beam energy: 4.45, 5.55 GeV
- Two nature publications on this (2017, 2020)
  - One for proton, and another for neutron

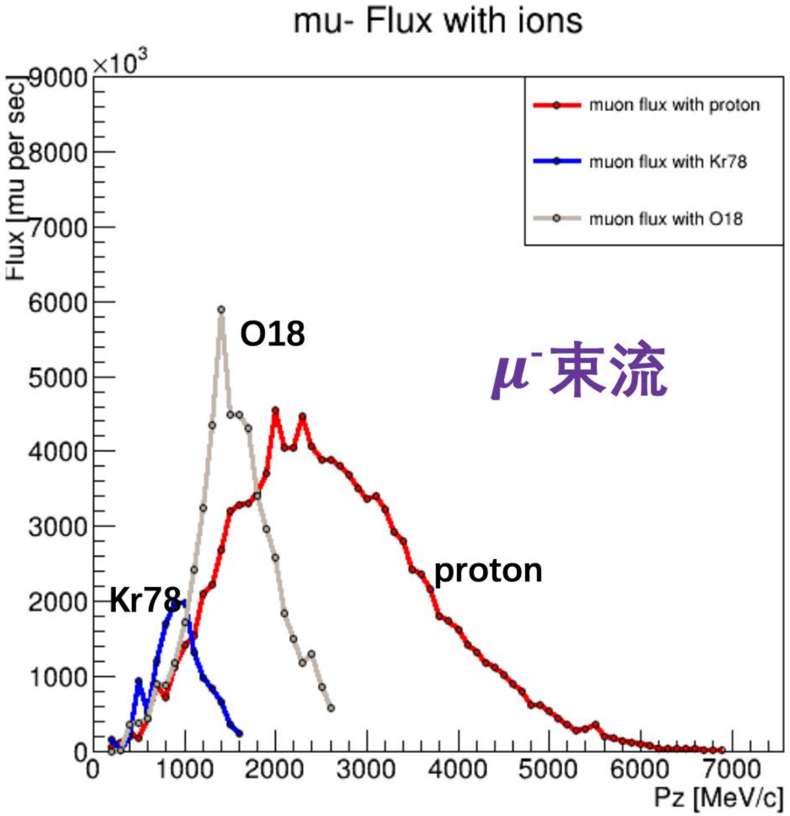




μ束流产额

- 对于μ^+束流：
  - 质子最佳
  - 最大产额：
    $8.5 \times 10^6\ \mu^+/s$@3.5GeV
- 对于μ^-束流：
  - Pz<1GeV:
    $^{78}Kr^{19+}$最佳
  - 1GeV<Pz<2GeV:
    $^{18}O^{6+}$最佳
  - Pz>2GeV:
    - 质子最佳
  - 最大产额：
    $6 \times 10^6\ \mu^-/s$@1.5GeV



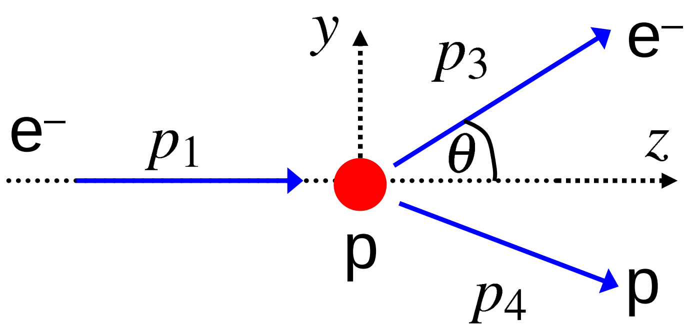


MUSE Beam flux

- Form factors for both electron and muon scattering
- Systematic uncertainties cancellation

Beam momenta MeV/c	Charge	Total Flux MHz	π Flux MHz	μ Flux MHz	e Flux MHz
115	+	9	0.6	2	6
153	+	18	8	2	8
210	+	70	60	5	6
115	-	6	0.06	0.2	6
153	-	9	0.8	0.2	8
210	-	12	6	0.5	6

Scatterings



- Rutherford scattering: $$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{\alpha^2}{16E_K^2 \sin^4\left(\frac{\theta}{2}\right)}$$

$$E_K = E \; (E \gg m_e)$$

- Mott scattering:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{\alpha^2}{4E^2 \sin^4\left(\frac{\theta}{2}\right)} cos^2\left(\frac{\theta}{2}\right)$$

From electron spin $\frac{1}{2}$

The interaction is electric rathter than magnetic (spin-spin) in nature

- Charge distribution of the proton: Form factors

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 sin^4(\theta/2)} \frac{E_3}{E_1}\left[cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{2M_p^2} sin^2\left(\frac{\theta}{2}\right)\right]$$

From proton spin $\frac{1}{2}$

Rutherford

Proton recoil

Electric/Magnetic scattering

Magnetic term due to spin

Form factors

- The Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 sin^4(\theta/2)} \frac{E_3}{E_1}\left[\frac{G_E^2 + \tau G_M^2}{(1+\tau)} cos^2\left(\frac{\theta}{2}\right) + 2\tau G_M^2 sin^2\left(\frac{\theta}{2}\right)\right]$$

With $\tau = \frac{Q^2}{4M_p^2}$

  - Charge distribution: $G_E(Q^2)$
  - Magnetic moment distribution: $G_M(Q^2)$

$$G_E(0) = \int \rho(\vec{r}) d^3\vec{r} = 1$$

$$G_M(0) = \int \mu(\vec{r}) d^3\vec{r} = \mu_p = +2.79$$

- At very low Q^2: $\tau = \frac{Q^2}{4M_p^2} \approx 0$

$$\frac{d\sigma}{d\Omega} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_0 \approx G_E^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4E_1^2 sin^4(\theta/2)} \frac{E_3}{E_1} cos^2\left(\frac{\theta}{2}\right)$$

Form factors vs. proton charge radius

- With low momentum transfer, as $qR \ll 1$
  - virtual photon: $\lambda \sim \frac{1}{q} \gg R$
- Tayler expansion of $e^{i\vec{q}\cdot\vec{r}}$

$$F(\vec{q}^2) = \int \rho(\vec{r})\, e^{i\vec{q}\cdot\vec{r}} d^3\vec{r}$$

$$F(\vec{q}^2) = 1 - \frac{1}{6}q^2 \int r^2 \rho(r)\, d^3r + \cdots$$

$$F(\vec{q}^2) = 1 - \frac{1}{6}q^2 \langle r^2 \rangle + \cdots$$

$$\langle r^2 \rangle \approx -6 \frac{dF(q^2)}{dq^2}\bigg|_{q^2=0}$$

Measure the form factor as a function of Q^2, coud used to extract the proton charge radius.

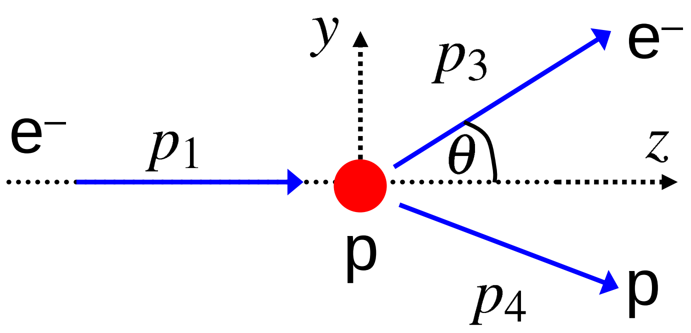
Q^2 close to 0 !!

The experimental observables

- The differential cross-section is expressed in terms of Q^2, E_3 and θ

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 sin^4(\theta/2)}\frac{E_3}{E_1}\left[\frac{G_E^2+\tau G_M^2}{(1+\tau)}cos^2\left(\frac{\theta}{2}\right)+2\tau G_M^2 sin^2\left(\frac{\theta}{2}\right)\right]$$

- Only one independent variable
- Both Q^2, E_3 can be expressed in terms of θ



$$E_3 = \frac{E_1 M_p}{M_p + E_1(1-cos\theta)}$$

$$Q^2 = \frac{2M_p E_1^2(1-cos\theta)}{M_p + E_1(1-cos\theta)}$$

- At very low Q^2
  - Count the number of scattered electron in a given angle !

The Mott scattering formula

$$\frac{d\boldsymbol{\sigma}}{d\boldsymbol{\Omega}} = \frac{(\boldsymbol{Z\alpha})^2 \boldsymbol{E}^2 \left(\mathbf{1} - \boldsymbol{\beta}^2 \sin^2\left(\frac{\boldsymbol{\theta}}{\mathbf{2}}\right)\right)}{\mathbf{4}\boldsymbol{p}^4 \sin^4\left(\frac{\boldsymbol{\theta}}{\mathbf{2}}\right)} (\hbar \boldsymbol{c})^2 (\boldsymbol{mb} \cdot \boldsymbol{sr}^{-1})$$

$p = 4\ GeV, \beta = 1, \theta = 10^o$:

$\sin\frac{\theta}{2} = 0.0872$

$\frac{d\boldsymbol{\sigma}}{d\boldsymbol{\Omega}} = \mathbf{0.357}\ (\boldsymbol{mb} \cdot \boldsymbol{sr}^{-1})$

$\boldsymbol{Z}$: the number of proton in a target nucleon. $= 8$ for Oxygen

$\boldsymbol{\alpha}$: the fine structure constant, $1/137$

$\boldsymbol{E}, \boldsymbol{p}$: energy of the incident muon, $E \approx p$, GeV

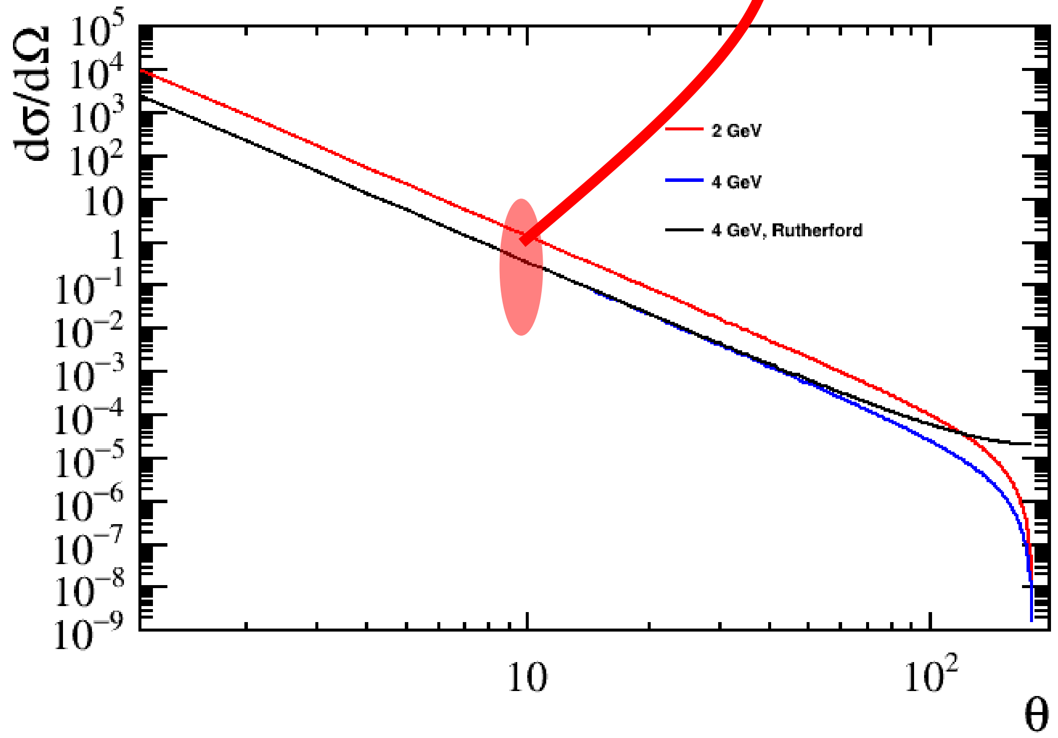
$\boldsymbol{\beta}$: $v/c \approx 1$

$\boldsymbol{\theta}$: scattering angle

$\hbar \boldsymbol{c}$: constant, $(\hbar \boldsymbol{c})^2 = \mathbf{0.389380}\ \boldsymbol{GeV}^2 \cdot \boldsymbol{mb}$

$\boldsymbol{mb}$: the cross-section $1\ mb = 10^{-27}\ cm^2$

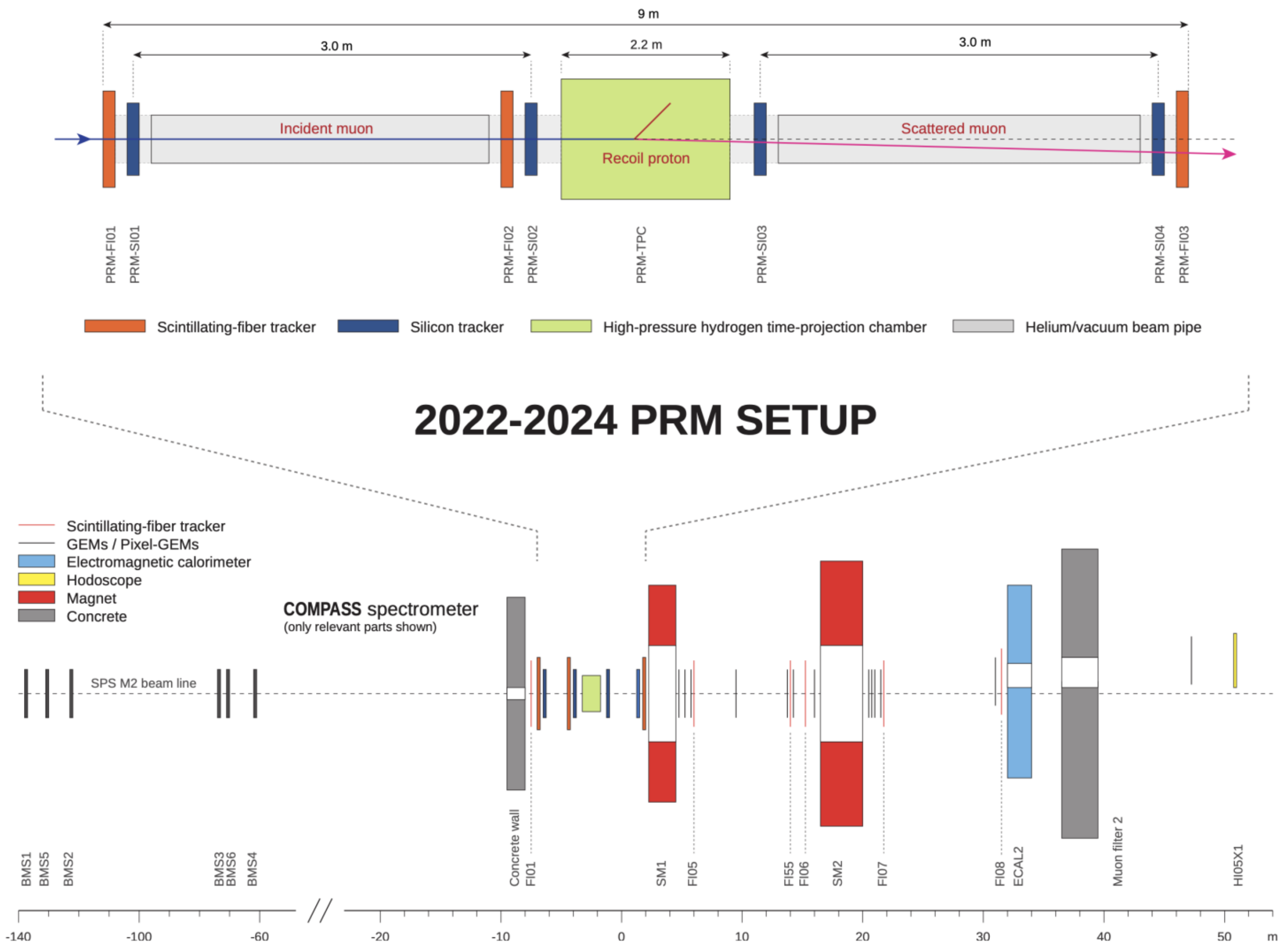
$\boldsymbol{sr}$: steradian, solid angle



PRad uncertainty

Item	r_p uncertainty (fm)	n_1 uncertainty	n_2 uncertainty
Event selection	0.0070	0.0002	0.0006
Radiative correction	0.0069	0.0010	0.0011
Detector efficiency	0.0042	0.0000	0.0001
Beam background	0.0039	0.0017	0.0003
HyCal response	0.0029	0.0000	0.0000
Acceptance	0.0026	0.0001	0.0001
Beam energy	0.0022	0.0001	0.0002
Inelastic ep	0.0009	0.0000	0.0000
G_M^p parameterization	0.0006	0.0000	0.0000
Total	0.0115	0.0020	0.0013

Amber experiment



Scattering angle