Studies of three-dimensional structure of nucleons by neutrinos

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Collaborators of IMP: X. Chen, S. Wu, Y.-P. Xie

References: X. Chen *et. al.*, Euro. Phys. J. A 60 (2024) 208.

第三届惠州大装置高精度核物理研讨会

Third Symposium on Huizhou Large Facility and High-Precision Nuclear Physics,

IMP, Huizhou, Guangdong, China, April 19-23, 2025

https://indico.impcas.ac.cn/event/74/timetable/#all.detailed

April 21, 2025

Self introduction

From April 1, 2025,
Quark Matter Research Center, Institute of Modern Physics
Chinese Academy of Sciences, Huizhou campus

Vitae and research history
https://research.kek.jp/people/kumanos/

1985.09: Ph. D, MIT theory center

1985.09--2022.03, U Illinois, U Tennessee, Oak Ridge National Lab, Indiana U,
U Mainz, Saga U, Graduate U for Advanced Studies, KEK

2022.03: Retired from KEK

2025.03: Retired from Japan Women's University

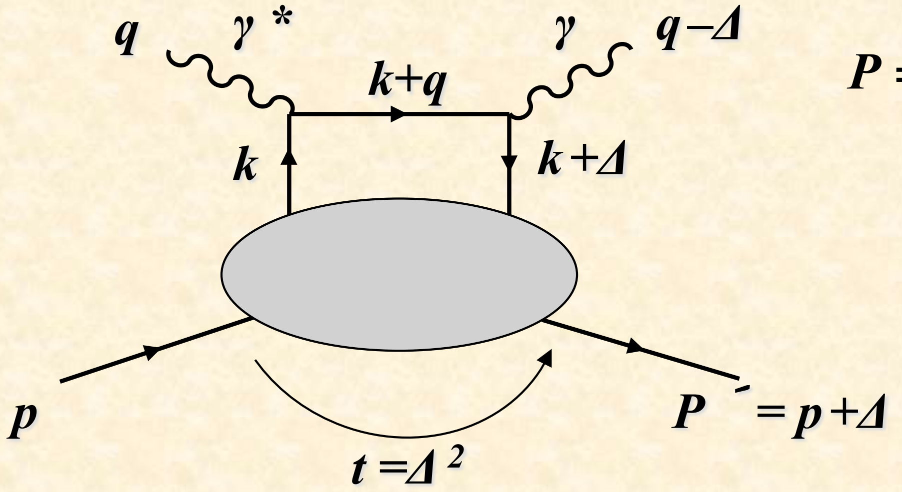
2025.04: IMP

Contents

HIAF (High Intensity heavy-ion Accelerator Facility)
Proton beam energy 9.3 GeV → 25 GeV → 100 GeV? in future.
High-energy neutrino nucleon interactions could be investigated!

Motivations for studying gravitational form factors and GPDs

Generalized Parton Distributions (GPDs)



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $\quad x = \dfrac{Q^2}{2p\cdot q}$

Momentum transfer squared $\quad t = \Delta^2$

Skewdness parameter $\quad \xi = \dfrac{p^+ - p'^+}{p^+ + p'^+} = -\dfrac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

Forward limit: PDFs $\quad H(x,\xi,t)\big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\quad \int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$

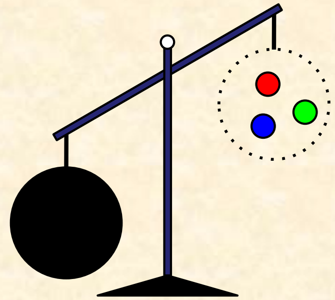
Axial and Pseudoscalar form factors G_A, G_P $\quad \int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$

Second moments: Angular momenta

Sum rule: $J_q = \dfrac{1}{2}\displaystyle\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0) \right], \quad J_q = \dfrac{1}{2}\Delta q + L_q$

$\Rightarrow$ probe L_q, key quantity to solve the spin puzzle!

Origin of hadron masses

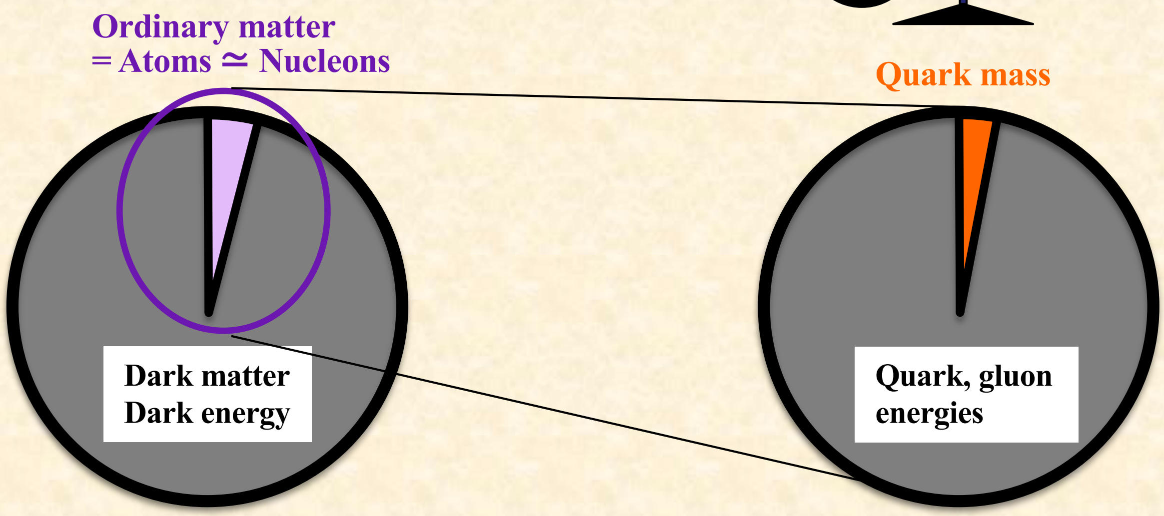


Mass and spin of the nucleon are two of fundamental quantities in physics.

Nucleon mass: $M = \left\langle p \left| \int d^3x \, T^{00}(x) \right| p \right\rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\ \alpha}(x)$$

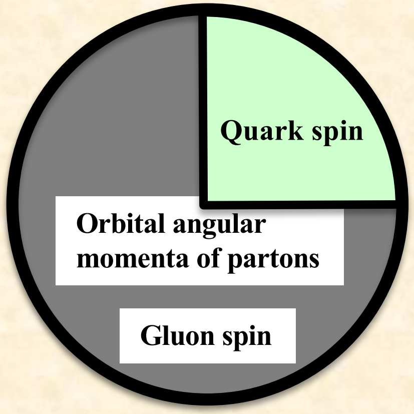


Dark matter **Origin of nucleon mass**

Nucleon spin: $\frac{1}{2} = \left\langle p \left| J^3 \right| p \right\rangle$

3rd component of total angular momentum: $J^3 = \frac{1}{2}\varepsilon^{3jk} \int d^3x \, M^{3jk}(x)$

Angular-momentum density: $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x) x^{\mu} - T^{\alpha\mu}(x) x^{\nu}$

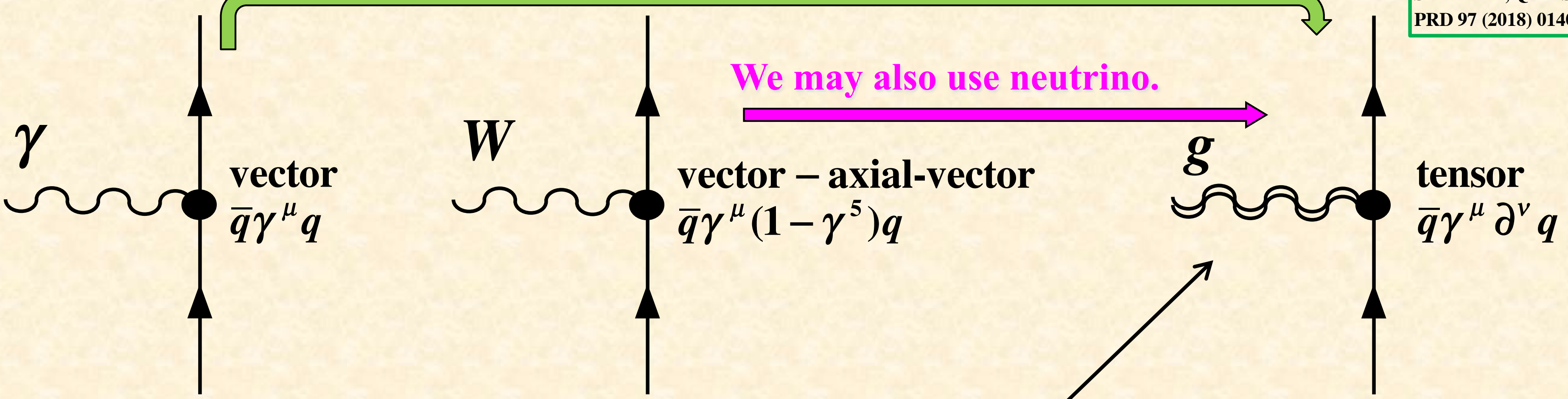


Origin of nucleon spin
("Dark spin")

Why "gravitational" interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev, PRD 97 (2018) 014020.



It is possible to probe gravitational sources in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

$$\underline{\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar q(-z/2)\gamma^+ q(z/2) | p\rangle \Big|_{z^+=0,\vec z_\perp=0}} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar u(p')\gamma^+ u(p) + E(x,\xi,t)\bar u(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

Non-local operator of GPDs/GDAs:

$$(P^+)^n \int dx\, x^{n-1} \underline{\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar q(-z/2)\gamma^+ q(z/2)\right]_{z^+=0,\vec z_\perp=0}}$$

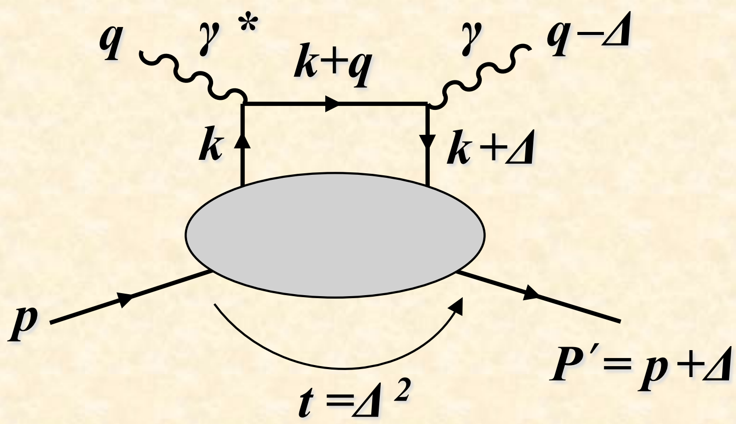
$$= \left(i\frac{\partial}{\partial z^-}\right)^{n-1} \left[\bar q(-z/2)\gamma^+ q(z/2)\right]_{z=0}$$

$$= \underline{\bar q(0)\gamma^+ \left(i\overleftrightarrow{\partial}^+\right)^{n-1} q(0)}$$

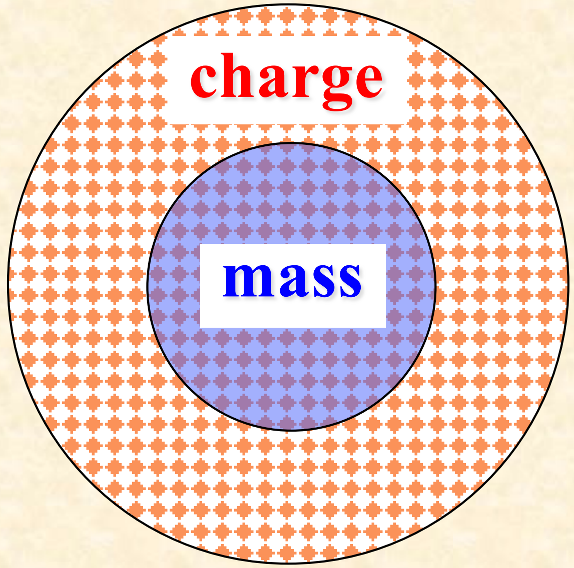
= energy-momentum tensor of a quark for $n = 2$
(electromagnetic for $n = 1$)

= source of gravitiy

Virtual Compton
or (timelike) two-photon process



Gravitational form factors and radii for pion



This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

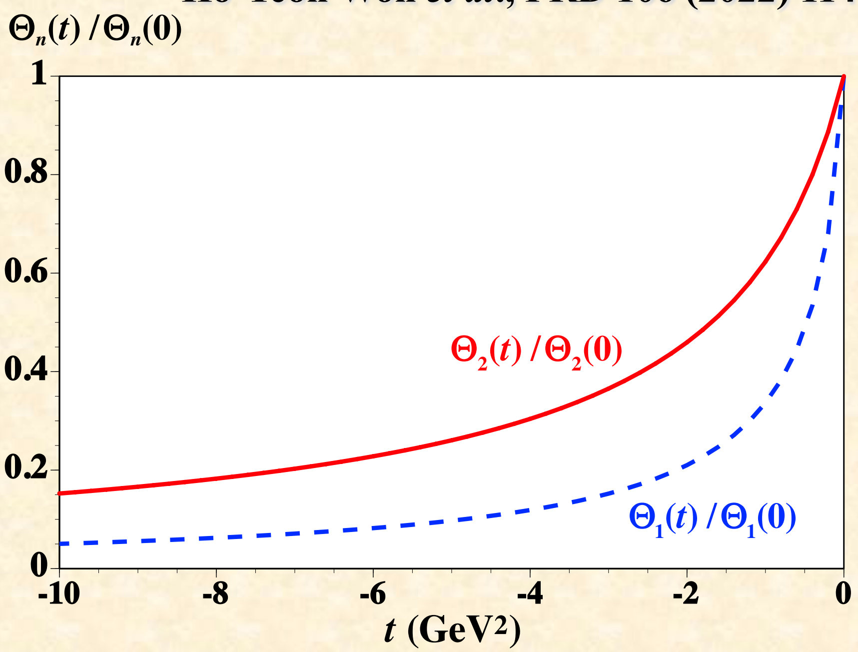
$$\Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

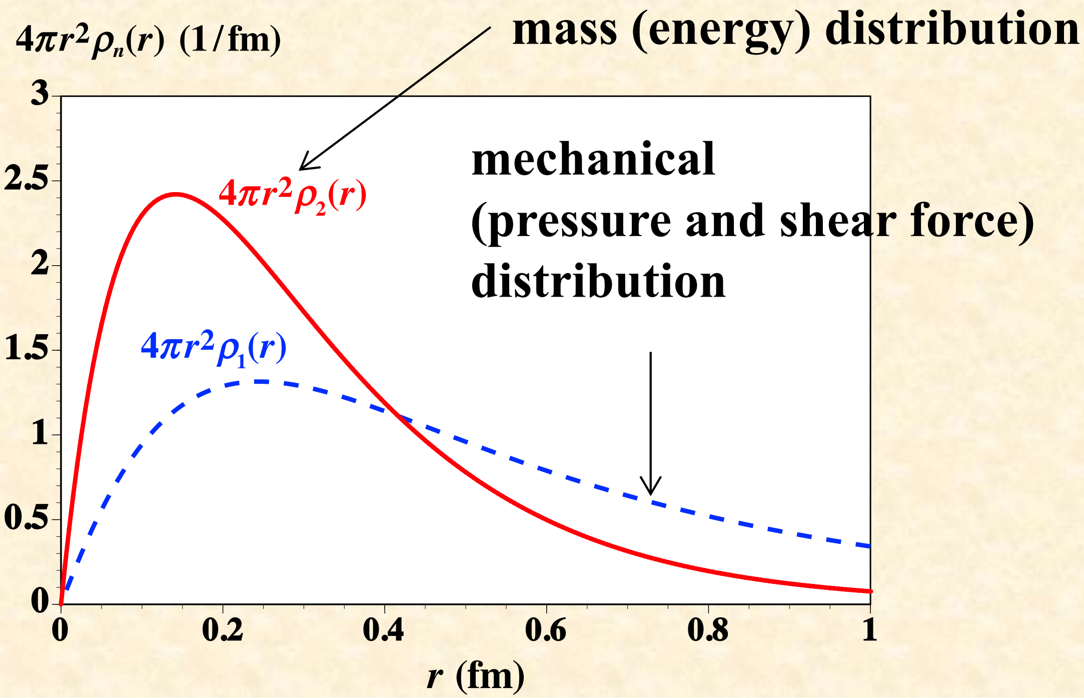
SK, Q.-T. Song, O. Teryaev
PRD 97 (2018) 014020.

Related theoretical studies:

A. Freeseand I. C. Cloet, PRC 100 (2019) 015201;
P. E. Shanahan and W. Detmold, PRD 99 (2019) 014511;
C. D. Roberts *et al.*, Prog. Part. Nucl. Phys.120 (2021) 103883;
J.-L. Zhang et al. PLB 815 (2021) 136158;
June-Young Kim and Hyun-Chul Kim, PRD 104 (2021) 074019;
Ho-Yeon Won *et al.*, PRD 106 (2022) 114009.

Proton mass radius:
R. Wang, W. Kou, Y.-P. Xie, X. Chen,
PRD 103 (2021) L091501.

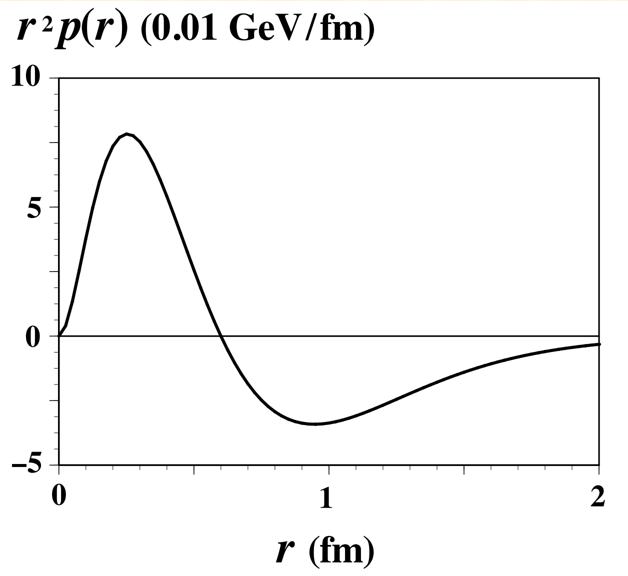




Nucleon pressure

$$\langle N(p')|T_q^{\mu\nu}(0)|N(p)\rangle = \bar{u}(p')\left[A\gamma^{(\mu}\bar{P}^{\nu)} + B\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}Mg^{\mu\nu}\right]u(p)$$

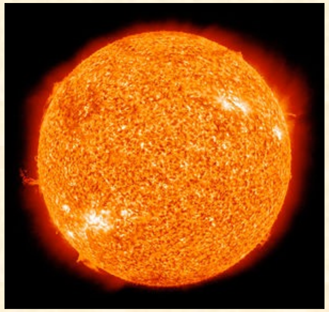
V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557 (2018) 396;
M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) 1830025;
C. Lorce, H. Moutarde, and A. P. Tranwinski, Eur. Phys. J. C 79 (2019) 89.



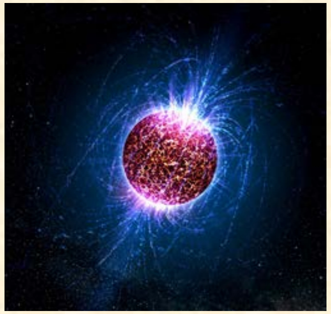
Highest pressure in nature   1 Pa (Pascal) = 1 N/m^2

Earth atmosphere
10^5 Pa =1000 hPa

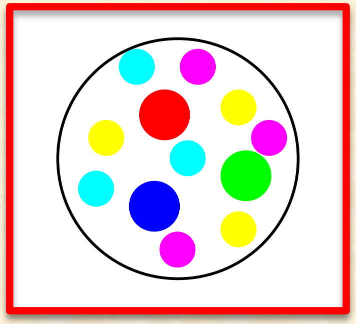
Center of earth
10^{11} Pa = 100GPa



Center of Sun
10^{16} Pa = 10 PPa



Neutron star
10^{34} Pa

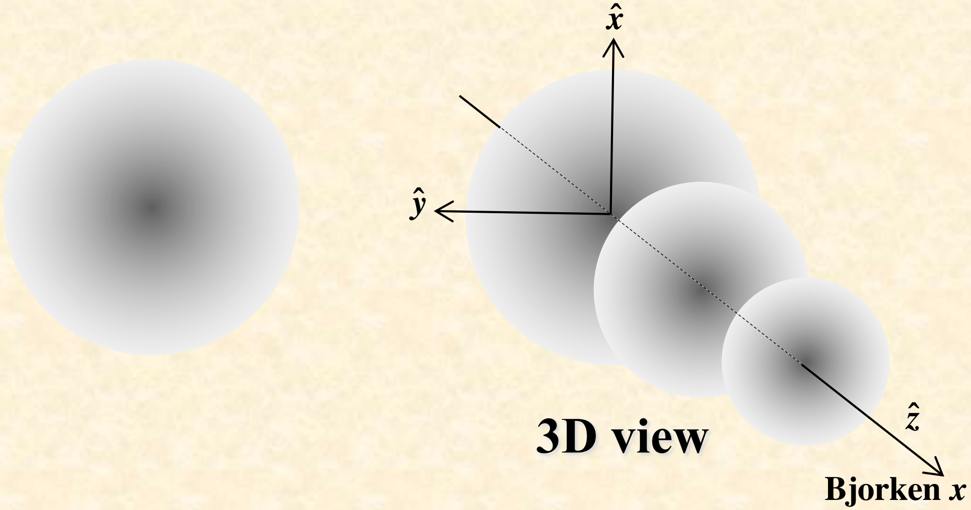


Hadron
10^{35} Pa

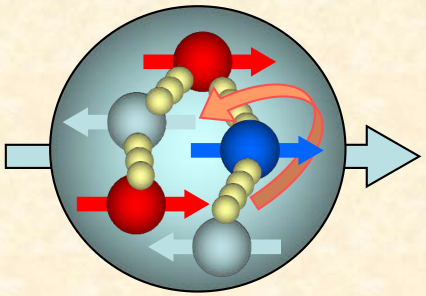
Proton (hadrons) puzzle studies by hadron tomography

Hadron tomography

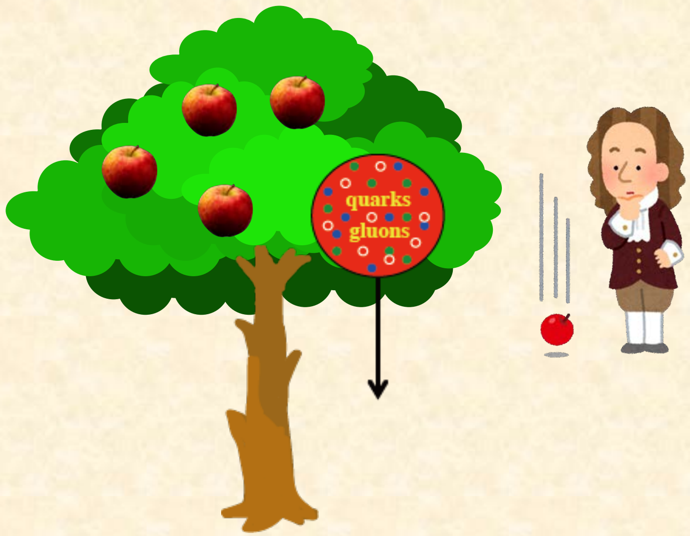
Proton radius puzzle



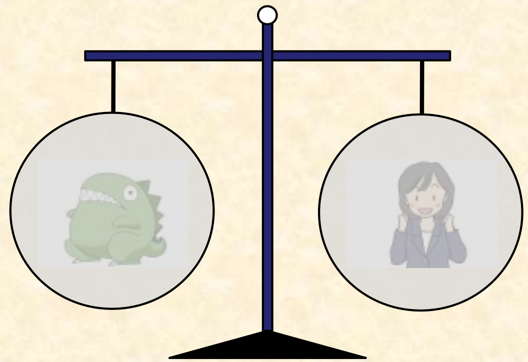
Origin of nucleon spin



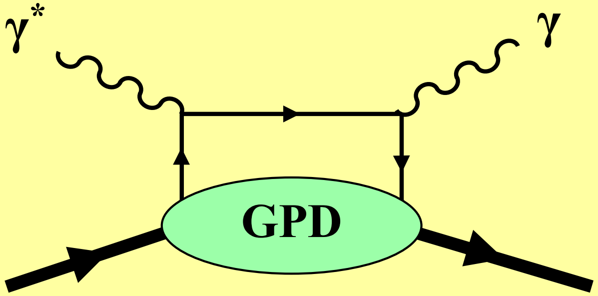
Source of gravity (mass)

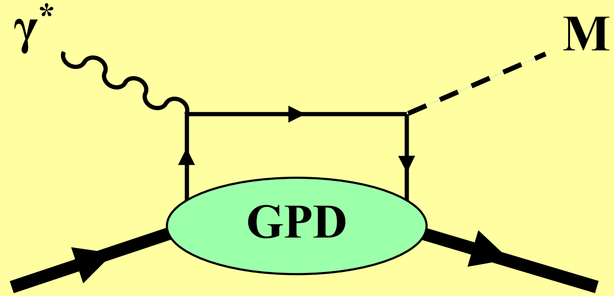


Exotic hadrons



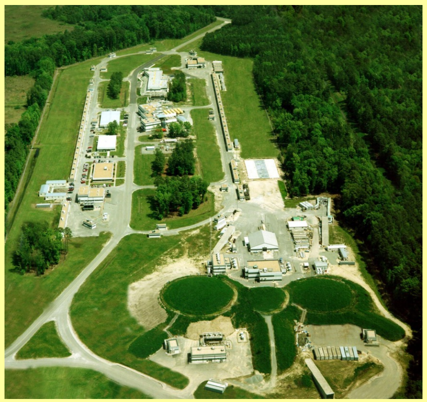
Charged-lepton scattering on spacelike GPDs



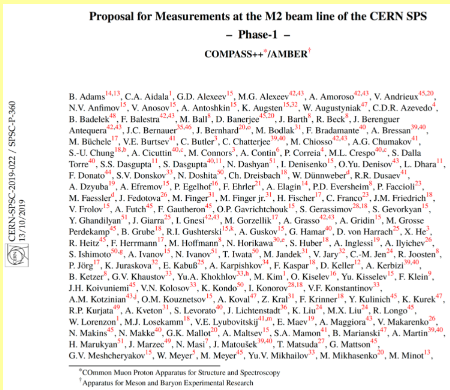


DVCS (Deeply Virtual Compton Scattering) **DVMP (Deeply Virtual Meson Production)**

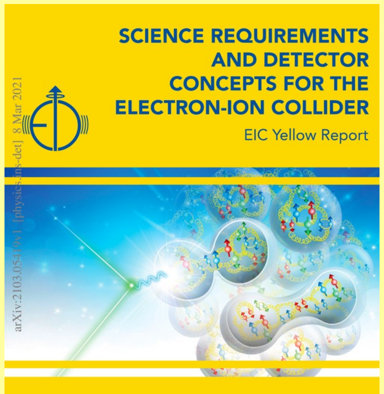
Jefferson Lab



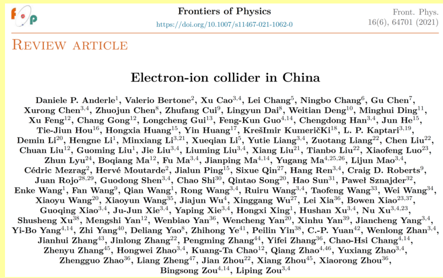
CERN-AMBER



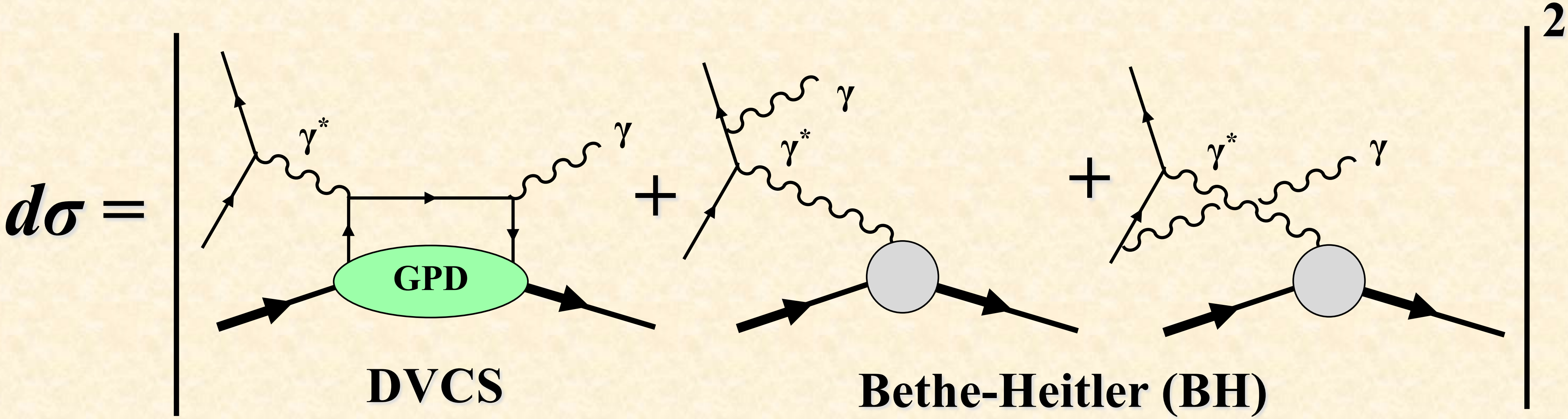
EIC-US



EicC



Deeply Virtual Compton Scattering (DVCS)



$$\frac{d\sigma(eN \to e'N'\gamma)}{dQ^2\,dx\,dt\,d\phi} \propto \left|T_{DVCS} + T_{BH}\right|^2$$

HERMES, JLab,
COMPASS/AMBER, EIC, EicC, $\cdots$

e.g. Polarized beam: $d\sigma(e^{\uparrow}) - d\sigma(e^{\downarrow}) \propto T_{BH} * \mathrm{Im}\left(T_{DVCS}\right)$

$$\mathrm{Re}\,\mathcal{H}_q = e_q^2 \mathcal{P}\int_0^1 dx \left[H^q(x,\xi,t) - H^q(-x,\xi,t)\right]\left(\frac{1}{\xi - x} + \frac{1}{\xi + x}\right)$$

$$\mathrm{Im}\,\mathcal{H}_q = \pi e_q^2 \left[H^q(\xi,\xi,t) - H^q(-\xi,\xi,t)\right]$$

- Polarized beam, unpolarized target: $\mathrm{Im}\left\{\mathcal{H}, \tilde{\mathcal{H}}, \mathcal{E}\right\}$
- Unpolarized beam, longigudinally-polarized target: $\mathrm{Im}\left\{\mathcal{H}, \tilde{\mathcal{H}}\right\}$
- Polarized beam, longigudinally-polarized target: $\mathrm{Re}\left\{\mathcal{H}, \tilde{\mathcal{H}}\right\}$
- Unpolarized beam, transversely-polarized target: $\mathrm{Im}\left\{\mathcal{H}, \mathcal{E}\right\}$

Recent measurement at JLab

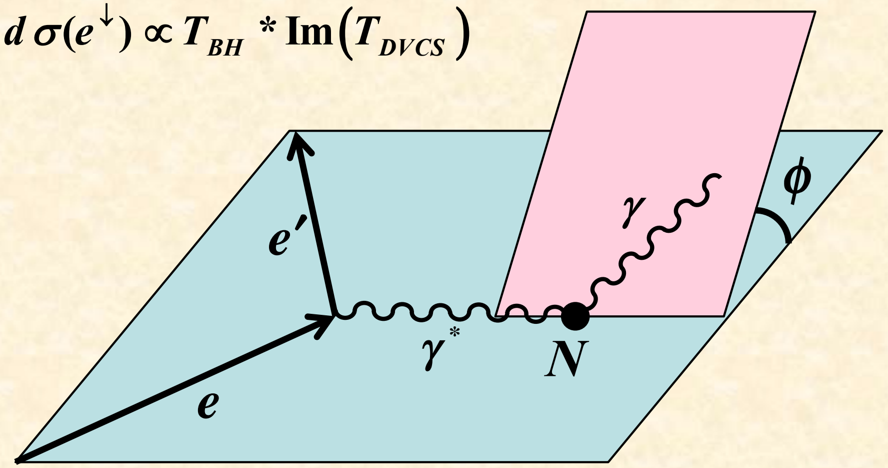
F. Georges et al., PRL 128 (2022) 252002.

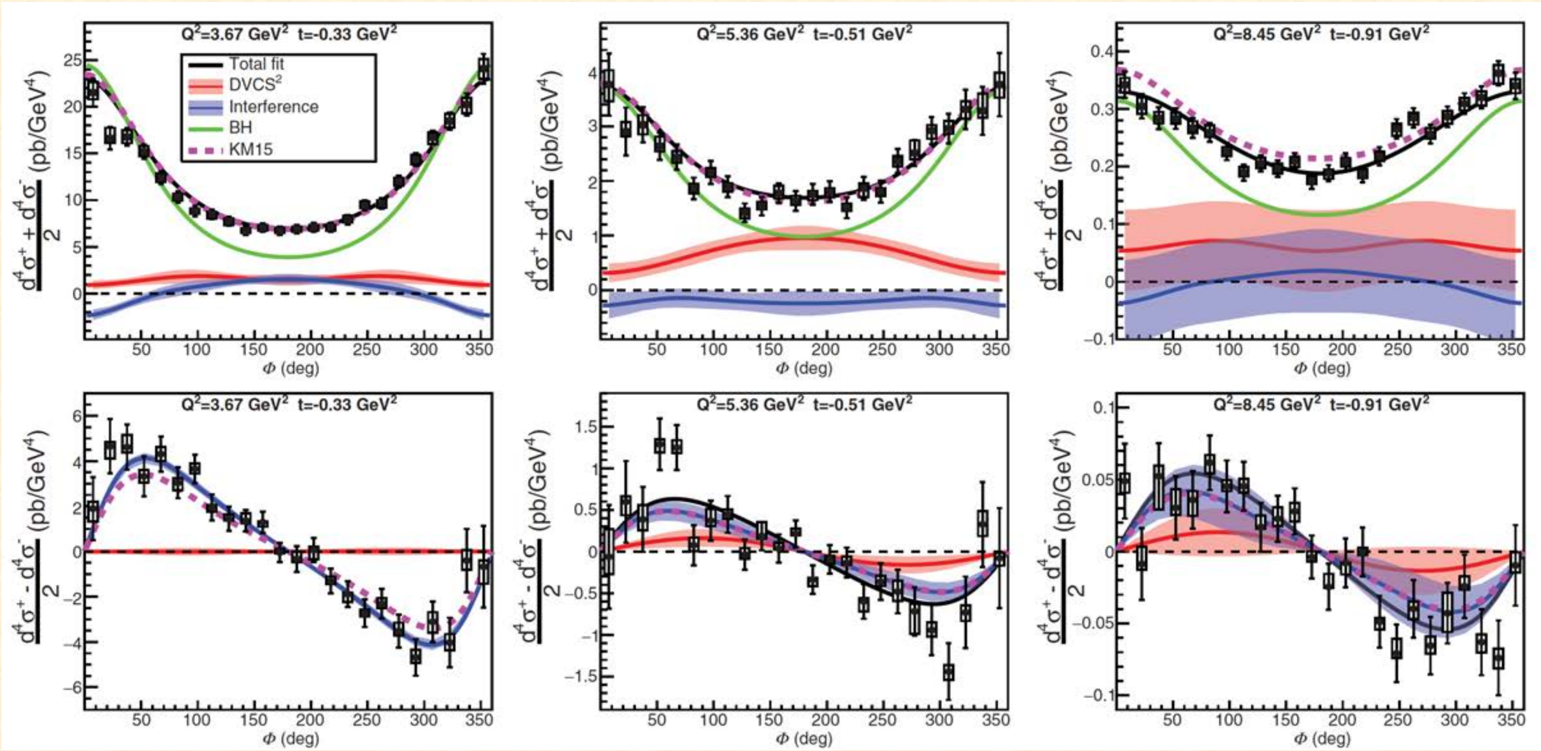
$$\frac{d\sigma(eN \to e'N'\gamma)}{dQ^2\,dx\,dt\,d\phi} \propto \left|T_{DVCS} + T_{BH}\right|^2,$$ e.g. Polarized beam: $d\sigma(e^{\uparrow}) - d\sigma(e^{\downarrow}) \propto T_{BH} * \mathrm{Im}\left(T_{DVCS}\right)$

$$\mathrm{Re}\,\mathcal{H}_q = e_q^2 \mathcal{P}\int_0^1 dx\left[H^q(x,\xi,t) - H^q(-x,\xi,t)\right]\left(\frac{1}{\xi - x} + \frac{1}{\xi + x}\right)$$

$$\mathrm{Im}\,\mathcal{H}_q = \pi e_q^2\left[H^q(\xi,\xi,t) - H^q(-\xi,\xi,t)\right]$$

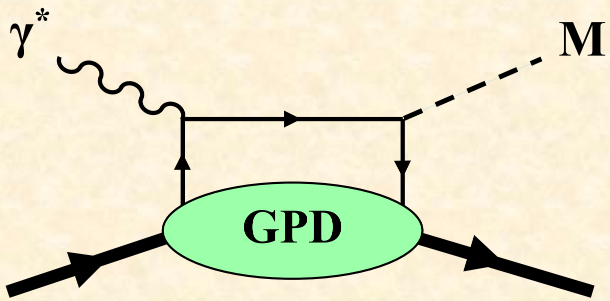
- Polarized beam, unpolarized target: $\mathrm{Im}\left\{\mathcal{H}, \tilde{\mathcal{H}}, \mathcal{E}\right\}$



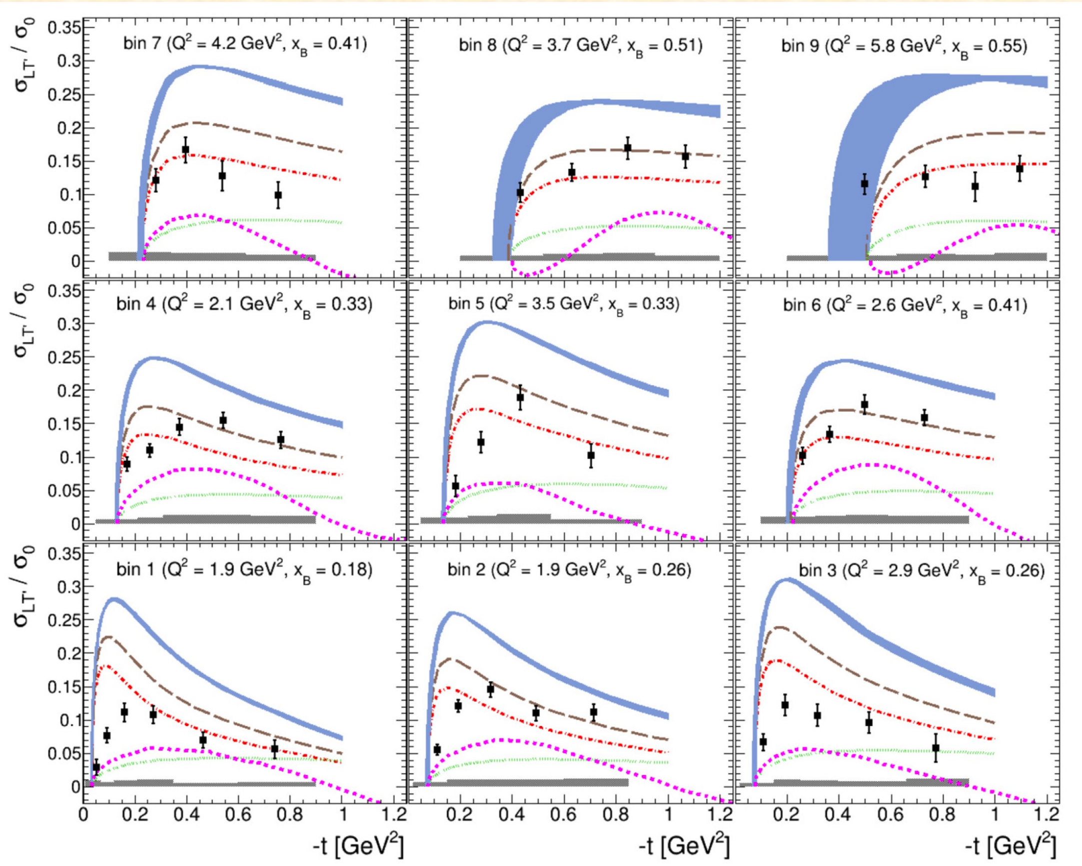


ϕ dependent cross sections

Deeply Virtual Meson Production (DVMP)



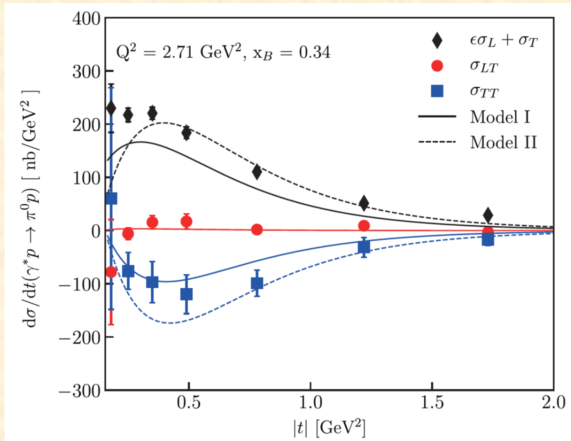
$$\frac{d\sigma(eN \to e'N'\gamma)}{dQ^2\, dx\, dt\, d\phi} \propto \sigma_T + \varepsilon\sigma_L + \varepsilon\sigma_{TT}\cos(2\phi)$$
$$+\sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT}\cos(\phi) + P_b\sqrt{2\varepsilon(1-\varepsilon)}\sigma_{LT'}\sin(\phi)$$



S. Diehl *et al.*, PLB 839 (2023) 137761.

	Meson	Flavor
$\mathcal{H}_T, \mathcal{E}_T$	π^+	$\Delta u - \Delta d$
	π^0	$2\Delta u + \Delta d$
	η	$2\Delta u - \Delta d + 2\Delta s$
$\mathcal{H}, \mathcal{E}$	ρ^+	$u - d$
	ρ^0	$2u + d$
	ω	$2u - d$
	ϕ	g

K. Joo, EIC Asia workshop (2024).



S. V. Goloskokov, Ya-Ping Xie, Xurong Chen,
Chin. Phys. C 46 (2022) 123101.

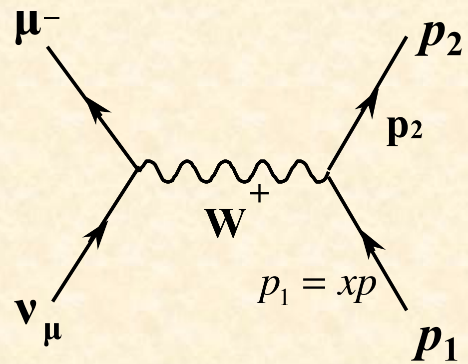
Nucleon structure functions in neutrino reactions

Neutrino deep inelastic scattering (CC: Charged Current)

Structure functions in parton model for neutrino-nucleon scattering (CC)

$$\frac{d\sigma^{CC}_{\nu,\bar\nu}}{dx\,dy}=\frac{G_F^2\,(s-M^2)}{2\pi\,(1+Q^2/M_W^2)^2}\left[x\,y^2F_1^{CC}+\left(1-y-\frac{M\,x\,y}{2E}\right)F_2^{CC}\pm x\,y\left(1-\frac{y}{2}\right)F_3^{CC}\right]$$

$$J_\mu^{CC}=\bar u(p_2,\lambda_2)\,\gamma_\mu\,(1-\gamma_5)[\,d(p_1,\lambda_1)\cos\theta_c+s(p_1,\lambda_1)\sin\theta_c]$$
$$+\bar c(p_2,\lambda_2)\,\gamma_\mu\,(1-\gamma_5)[\,s(p_1,\lambda_1)\cos\theta_c-d(p_1,\lambda_1)\sin\theta_c]$$



In parton model

$F_2=2xF_1$ (Callan-Gross relation)

$$F_2^{\nu p(\mathrm{CC})}=2x\left(d+s+\bar u+\bar c\right) \qquad F_2^{\nu n(\mathrm{CC})}=2x\left(u+s+\bar d+\bar c\right)$$

$$xF_3^{\nu p(\mathrm{CC})}=2x\left(d+s-\bar u-\bar c\right) \qquad xF_3^{\nu n(\mathrm{CC})}=2x\left(u+s-\bar d-\bar c\right)$$

$$F_2^{\bar\nu p(\mathrm{CC})}=2x\left(u+c+\bar d+\bar s\right) \qquad F_2^{\bar\nu n(\mathrm{CC})}=2x\left(d+c+\bar u+\bar s\right)$$

$$xF_3^{\bar\nu p(\mathrm{CC})}=2x\left(u+c-\bar d-\bar s\right) \qquad xF_3^{\bar\nu n(\mathrm{CC})}=2x\left(d+c-\bar u-\bar s\right)$$

Determination of valence-quark distributions

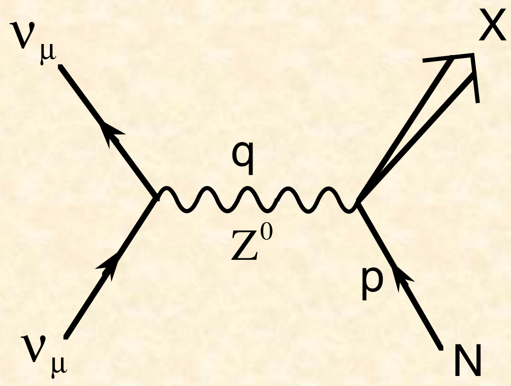
$$\longrightarrow\ \frac{1}{2}[F_3^{\nu p}+F_3^{\bar\nu p}]_{CC}=\underbrace{u_{\mathrm{v}}+d_{\mathrm{v}}}_{\text{valence-quark distributions}}+s-\bar s+c-\bar c$$

Neutrino deep inelastic scattering (NC: Neutral Current)

$$M = \frac{\rho}{1+Q^2/M_Z^2}\frac{G_F}{\sqrt{2}}\,\bar{u}(k',\lambda')\,\gamma^{\mu}\,(1-\gamma_5)\,u(k,\lambda) < X\,|\,J_{\mu}^{NC}\,|\,p,\lambda_p >$$

$$\frac{d\sigma}{dE'd\Omega} = \frac{\rho^2 G_F^2}{(1+Q^2/M_Z^2)^2}\,\frac{k'}{32\pi^2 E}\,L^{\mu\nu}\,W_{\mu\nu}$$

ρ is the strength of NC relative to CC

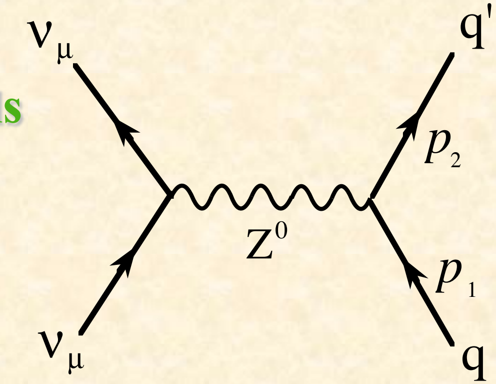


$$\frac{d\sigma^{NC}}{dx\,dy} = \frac{\rho^2 G_F^2\,(s-M^2)}{2\pi\,(1+Q^2/M_Z^2)^2}\left[x\,y^2 F_1^{NC} + \left(1-y-\frac{M\,x\,y}{2E}\right)F_2^{NC} + x\,y\left(1-\frac{y}{2}\right)F_3^{NC}\right]$$

$\nu_\mu + p \to \nu_\mu + X$

Neutrino-quark scattering (NC)

see next pages for details



$$J_{\mu}^{NC} = \sum_q \bar{q}(p_2,\lambda_2)\,\gamma_{\mu}\,[\,g_L^q\,(1-\gamma_5)+g_R^q\,(1+\gamma_5)\,]\,q(p_1,\lambda_1)$$

$$g_L^{u,c} \equiv u_L = +\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\ ,\quad g_R^{u,c} \equiv u_R = -\frac{2}{3}\sin^2\theta_W$$

$$g_L^{d,s} \equiv d_L = -\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\ ,\quad g_R^{d,s} \equiv d_R = +\frac{1}{3}\sin^2\theta_W$$

$\sin^2\theta_W$: weak mixing angle

$$F_1^{\nu p(NC)} = F_2^{\nu p(NC)}/2x$$

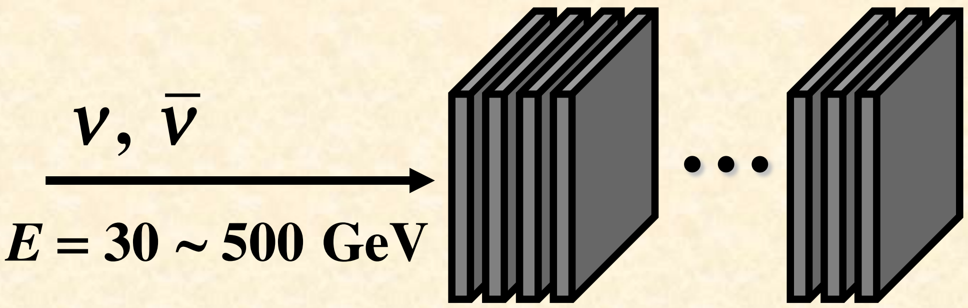
$$F_2^{\nu p(NC)} = 2x\,[\,(u_L^2+u_R^2)\{u^+(x)+c^+(x)\}+(d_L^2+d_R^2)\{d^+(x)+s^+(x)\}\,]$$

$$x F_3^{\nu p(NC)} = 2x\,[\,(u_L^2-u_R^2)\{u^-(x)+c^-(x)\}+(d_L^2-d_R^2)\{d^-(x)+s^-(x)\}\,]$$

$$q^{\pm}(x) = q(x) \pm \bar{q}(x)$$

Neutrino DIS experiments

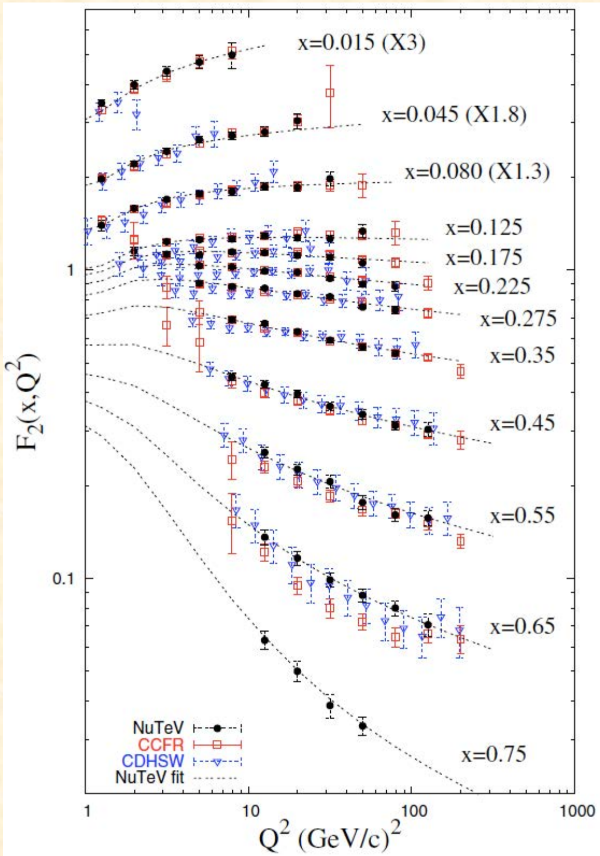
Experiment	Target	ν energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

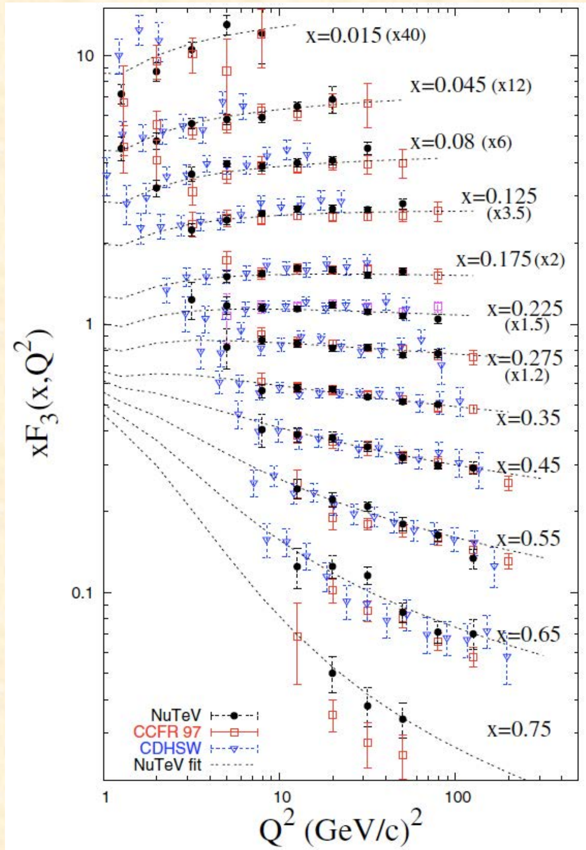


Huge Fe target (690 ton)

MINERνA (He, C, Fe, Pb), …

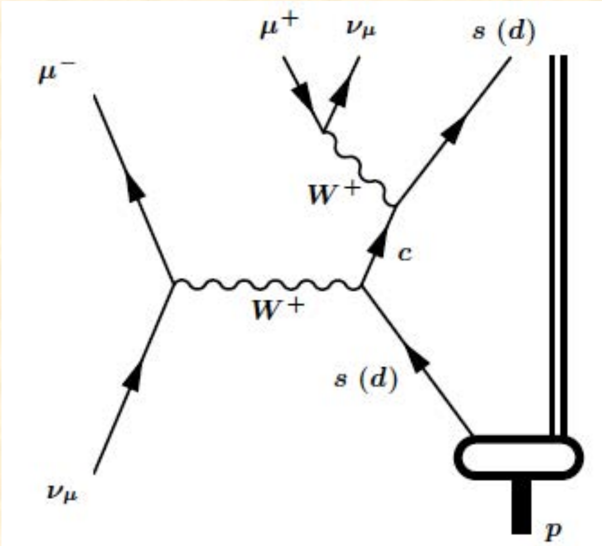
M. Tzanov *et al.* (NuTeV), PRD74 (2006) 012008.





Strangeness in the nucleon

Neutrino-induced opposite-sign dimuon events



A. Kayis-Topaksu *et al.*, NPB7 98 (2008) 1.
U. Dore, arXiv: 1103.4572 [hep-ex].

$$\kappa = \frac{\int dx\, x\,[s(x,Q^2)+\bar{s}(x,Q^2)]}{\int dx\, x\,[\bar{u}(x,Q^2)+\bar{d}(x,Q^2)]}$$

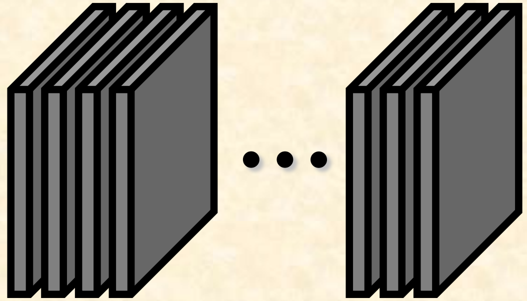
$Q^2 = 20\ \mathrm{GeV}^2$

Experiment	κ
This analysis	0.33 ± 0.07
CDHS [1]	0.47 ± 0.09
CCFR [2]	0.44 ± 0.09
CHARM II [3]	0.39 ± 0.09
NOMAD [4]	0.48 ± 0.17
NuTeV [5]	0.38 ± 0.08

CCFR, NuTeV

$\nu,\ \bar{\nu}$

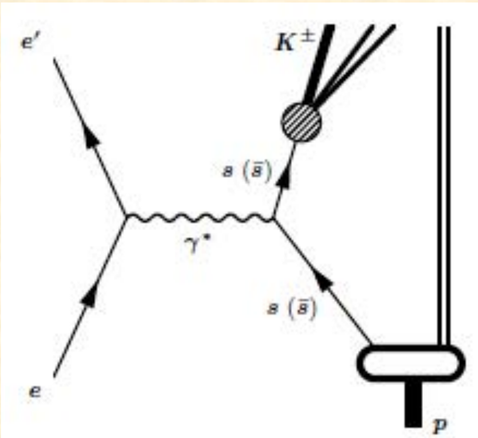
$E = 30 \sim 500$ GeV

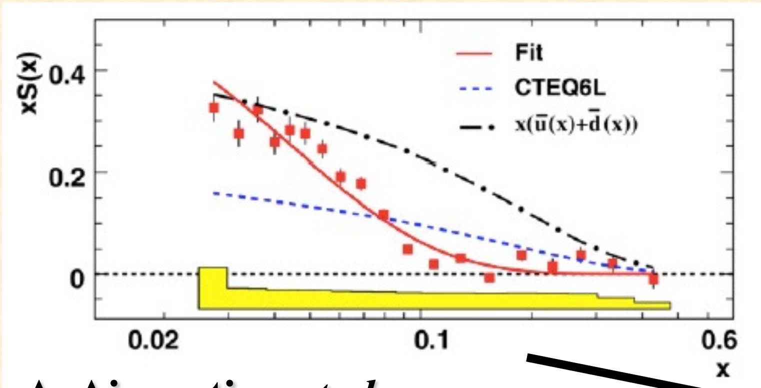


Huge Fe target (690 ton)

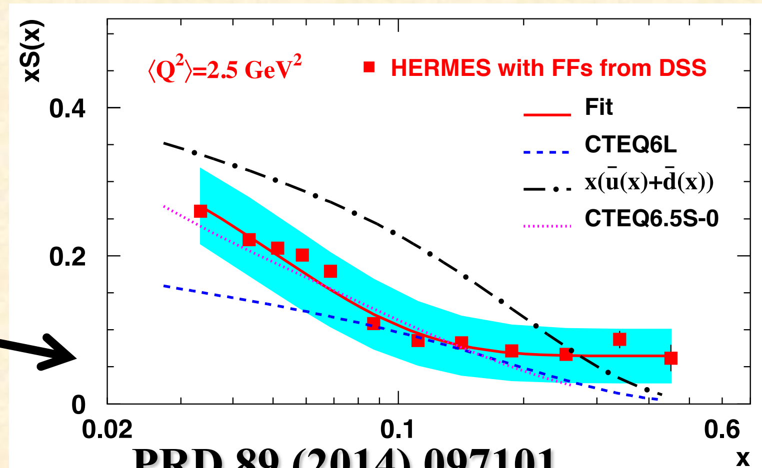
Issue: nuclear corrections

HERMES semi-inclusive measurement



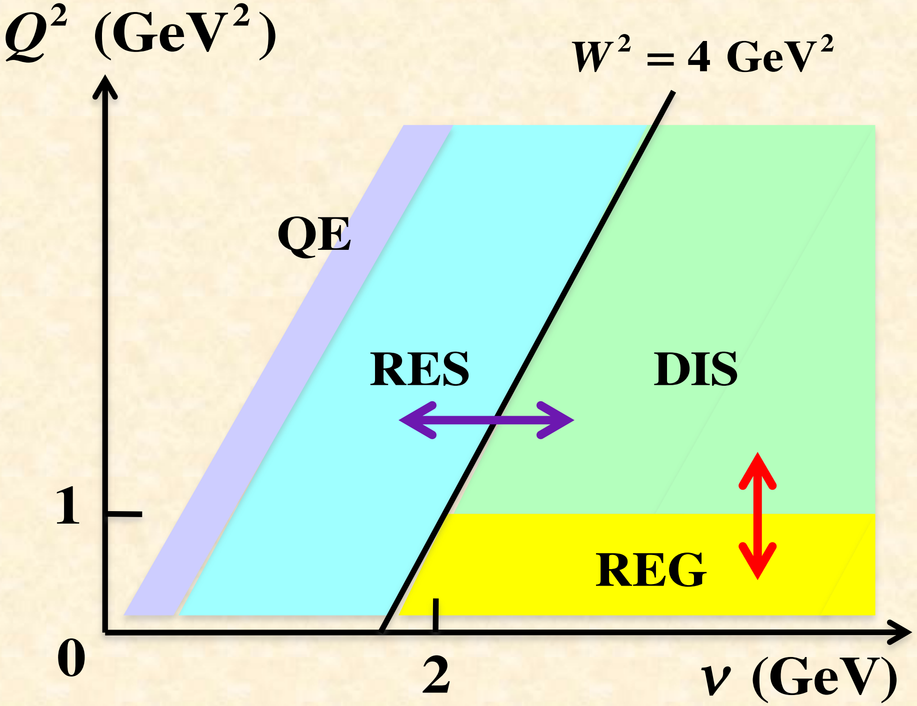


A. Airapetian *et al.*,
PLB 666 (2008) 446.



PRD 89 (2014) 097101.

Kinematical regions in ν reactions



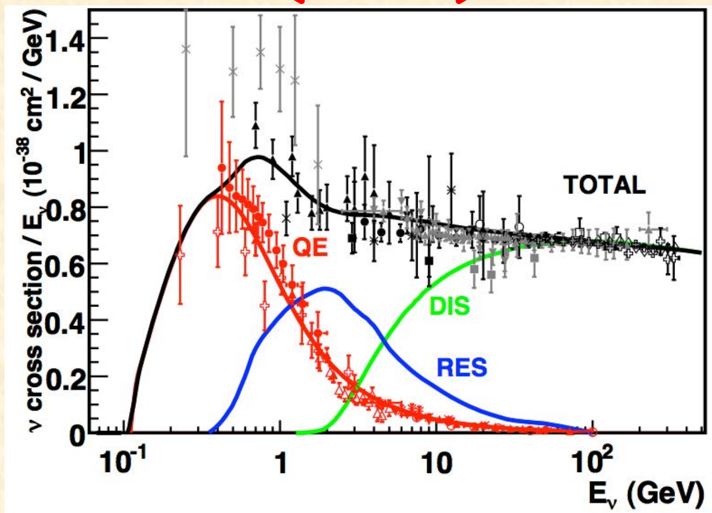
Depending on the neutrino beam energy, different physics mechanisms contribute to the cross section.

- QE (Quasi elastic)
- RES (Resonance)
- DIS (Deep inelastic scattering)
- REG (Regge)

MicroBooNE, NOvA

T2K

Minerva, DUNE



J.L. Hewett *et al.*, arXiv:1205.2671, Proceedings of the 2011 workshop on Fundamental Physics at the Intensity Frontier

ν interaction part is the major source of experimental errors in ν oscillation measurements.

Our work:
Towards a unified model of neutrino-nucleus reactions for neutrino oscillation experiments,
S. X. Nakamura *et al.*, Rep. Prog. Phys. 80 (2017) 056301.

Possible GPD studies at neutrino facilities

X. Chen, SK, R. Kunitomo, S. Wu, Y.-P. Xie,

Euro. Phys. J. A 60 (2024) 208.

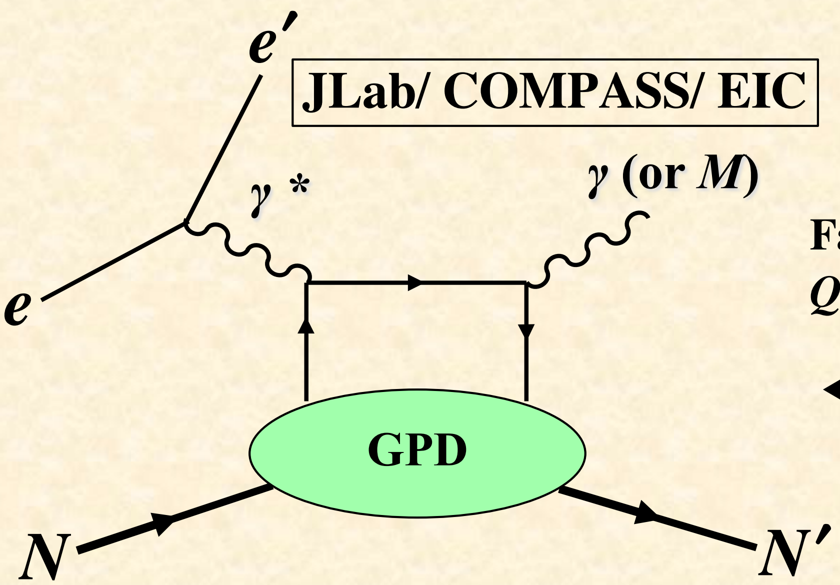
See also

SK, EPJ Web Conf. 208 (2019) 07003.

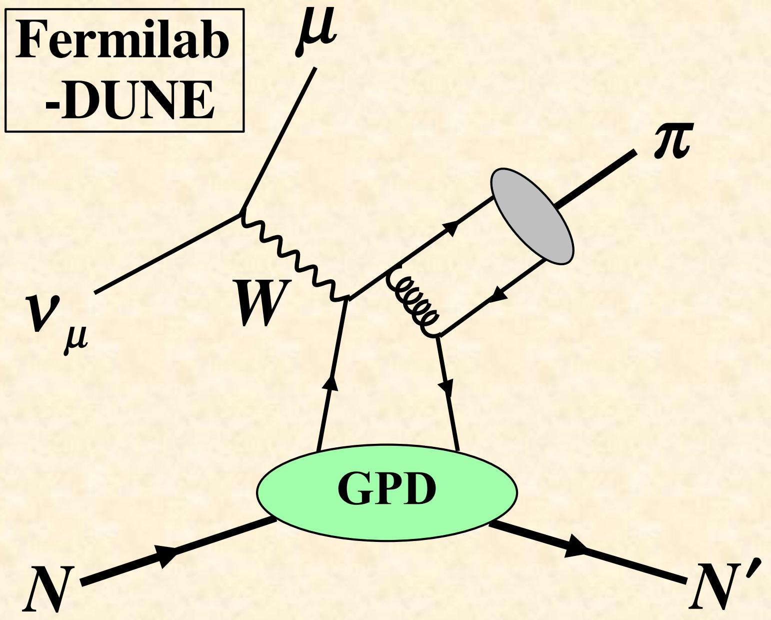
EIC yellow report, R. Abdul Khalek *et al*., arXiv:2103.05419, Sec. 7.5.2, Neutrino physics by SK and R. Petti.

SK and R. Petti, PoS (NuFact2021) 092.

Neutrino reactions for gravitational form factors @Fermilab-DUNE (Origins of hadron masses and pressures)

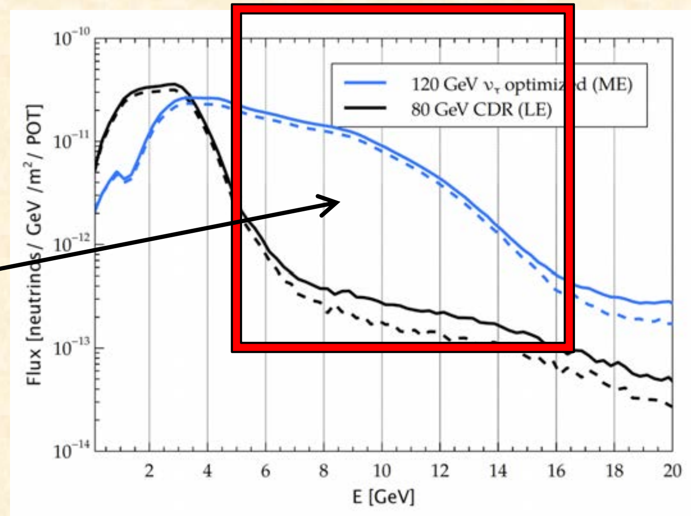


Factorization condition:
$Q^2 \gg |t|, \Lambda^2_{QCD}$



Deep Underground Neutrino Experiment (DUNE) at Long-Baseline Neutrino Facility (LBNF)

High-energy part of the LBNF ν beam can be used for the GPD studies.

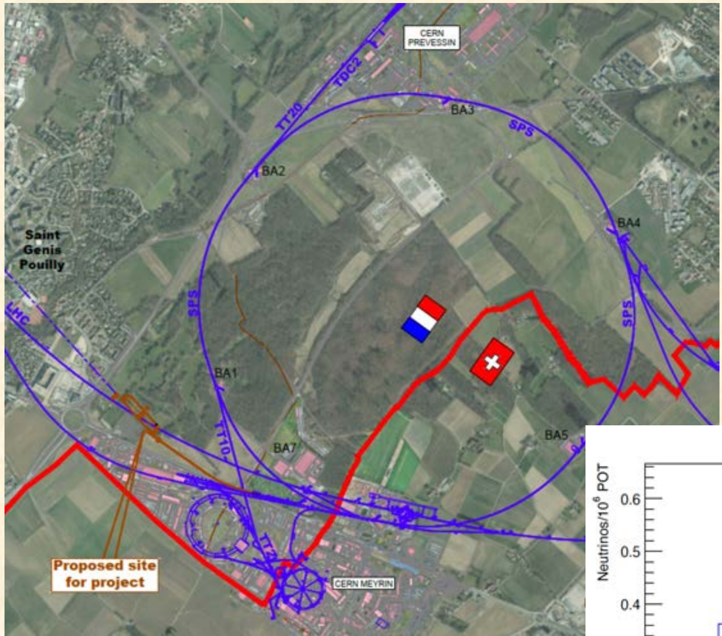


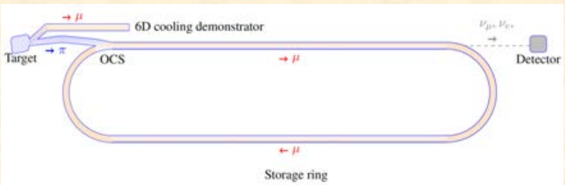
J. Rout *et al*., PRD 102 (2020) 116018

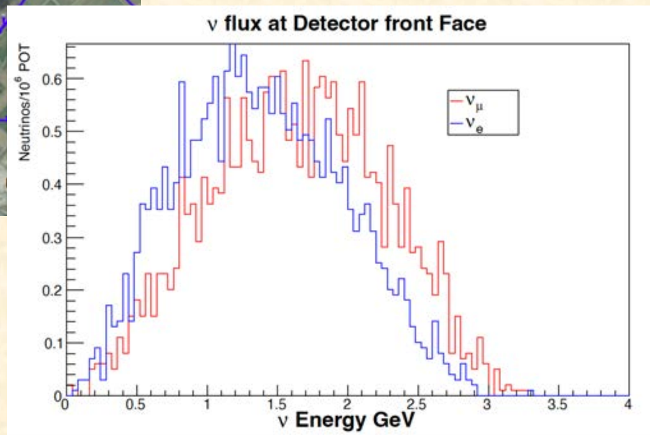
nuSTORM (Neutrinos from Stored Muons)

CERN

Feasibility Study, C. C. Ahdida *et al.*, (2020);
L. A. Ruso *et al.*, arXiv:2203.07545.



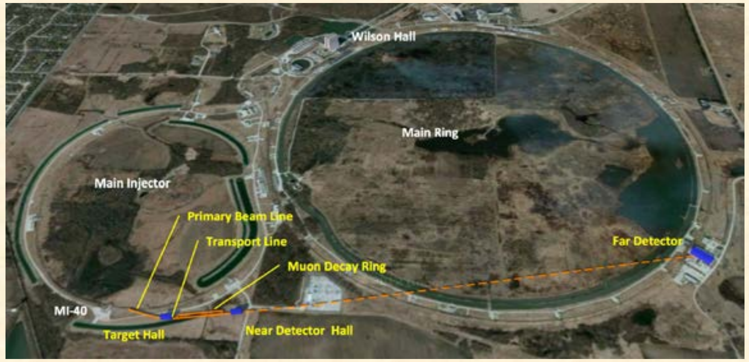


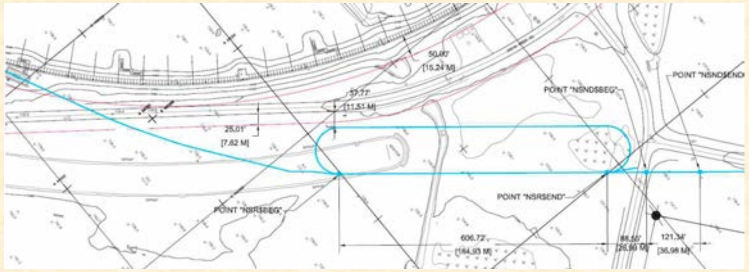


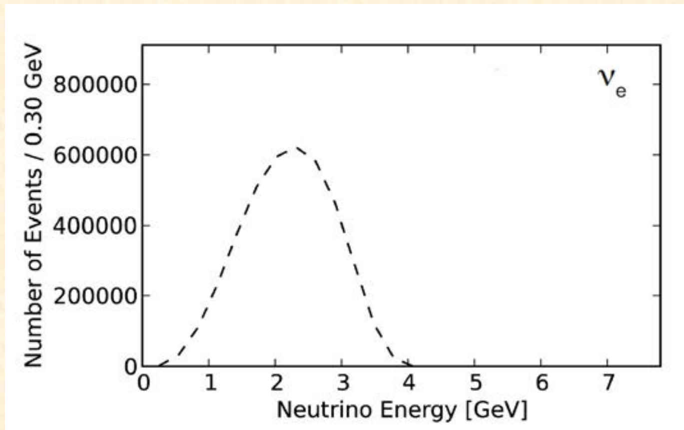
At this stage, the considered beam energy
is not high enough for structure-function studies;
however, high-energy option could be possible.
(personal communications: Xianguo Lu)
→ SK's talk at the nuSTORM-collaboration meeting
on July, 15, 2024
They could be interested in the higher-energy possibility.

Fermilab

Letter of Intent, arXiv:1206.0294,
P. Kyberd *et al*. (2012);
Proposal, D. Adey *et al*., arXiv:1308.6822.
No recent update.

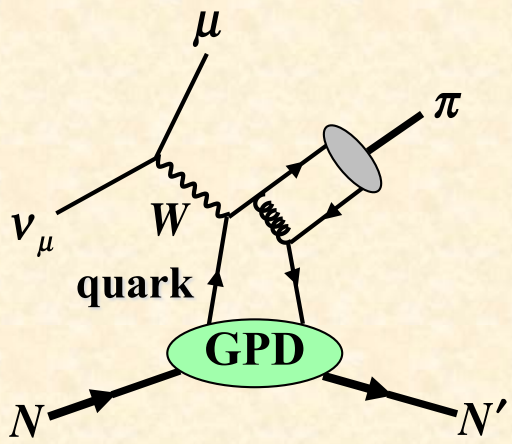






Cross section formalism

B. Pire, L. Szymanowski, J. Wagner,
Phys. Rev. D 95, 114029 (2017).



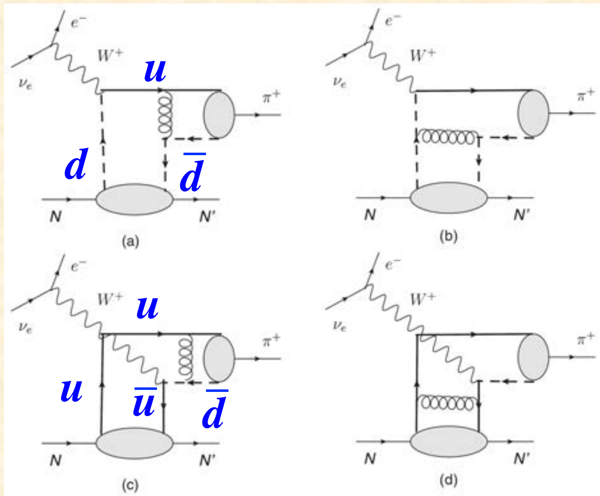
Cross section

$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} = \Gamma\,\varepsilon\,\sigma_L, \quad \varepsilon \simeq \frac{1-y}{1-y+y^2/2}, \quad \Gamma = \frac{G_F^2 Q^2}{32\,(2\pi)^4 (s-m_N^2)^2 y\,(1-\varepsilon)\sqrt{1+4x^2 m_N^2/Q^2}}$$

$$\sigma_L = \varepsilon_L^{*\mu} W_{\mu\nu} \varepsilon_L^{\nu} = \frac{1}{Q^2}\Big[(1-\xi^2)\Big\{\left|C_q\mathcal{H}_q + C_g\mathcal{H}_g\right|^2 + \left|C_q\tilde{\mathcal{H}}_q\right|^2\Big\} + \frac{\xi^4}{1-\xi^2}\Big\{\left|C_q\mathcal{E}_q + C_g\mathcal{E}_g\right|^2 + \left|C_q\tilde{\mathcal{E}}_q\right|^2\Big\}$$

$$-2\xi^2\,\mathrm{Re}\Big\{(C_q\mathcal{H}_q + C_g\mathcal{H}_g)(C_q\mathcal{E}_q + C_g\mathcal{E}_g)^*\Big\} - 2\xi^2\,\mathrm{Re}\Big\{C_q\tilde{\mathcal{H}}_q(C_q\tilde{\mathcal{E}}_q)^*\Big\}\Big]$$

Quark contributions



$$T_q = -i\,\frac{C_q}{2Q}\,N(p')\left[\mathcal{H}_q\hat{n} + \mathcal{E}_q\,\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2m_N} - \tilde{\mathcal{H}}_q\hat{n}\gamma_5 - \tilde{\mathcal{E}}_q\,\frac{\gamma_5 n\cdot\Delta}{2m_N}\right]N(p)$$

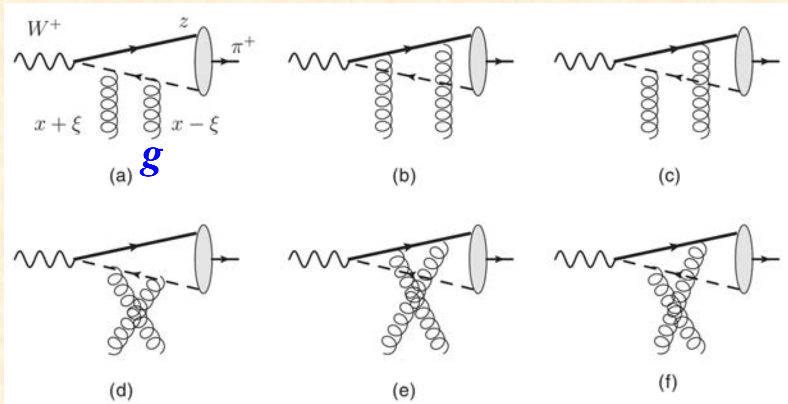
$$\mathcal{F}_q = 2f_\pi\int\frac{dz\;\phi_\pi(z)}{1-z}\int dx\,\frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

$$= \text{(pion distribution amplitude)}\cdot\text{(quark GPD)}$$

$$F_q(x,\xi,t) \equiv F_d(x,\xi,t) - F_u(-x,\xi,t)$$

$$F = H,\ E,\ \tilde{H},\ \tilde{E}$$

Gluon contributions



$$T_g = -i\,\frac{C_g}{2Q}\,N(p')\left[\mathcal{H}^g\hat{n} + \mathcal{E}^g\,\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2m_N}\right]N(p)$$

$$\mathcal{F}_g = \frac{8f_\pi}{\xi}\int\frac{dz\;\phi_\pi(z)}{z(1-z)}\int dx\,\frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$

GK (Goloskokov-Kroll) - 2013 parametrization

P. Kroll, H. Moutarde, F. Sabatie, Eur. Pjys. J. C 73 (2013) 2278.

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle p' \left| \bar{\psi}(-z/2)\gamma^+\psi(z/2) \right| p \right\rangle \Big|_{z^+=0,\vec{z}_\perp=0}$$

$$= \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle p' \left| \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) \right| p \right\rangle \Big|_{z^+=0,\vec{z}_\perp=0}$$

$$= \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

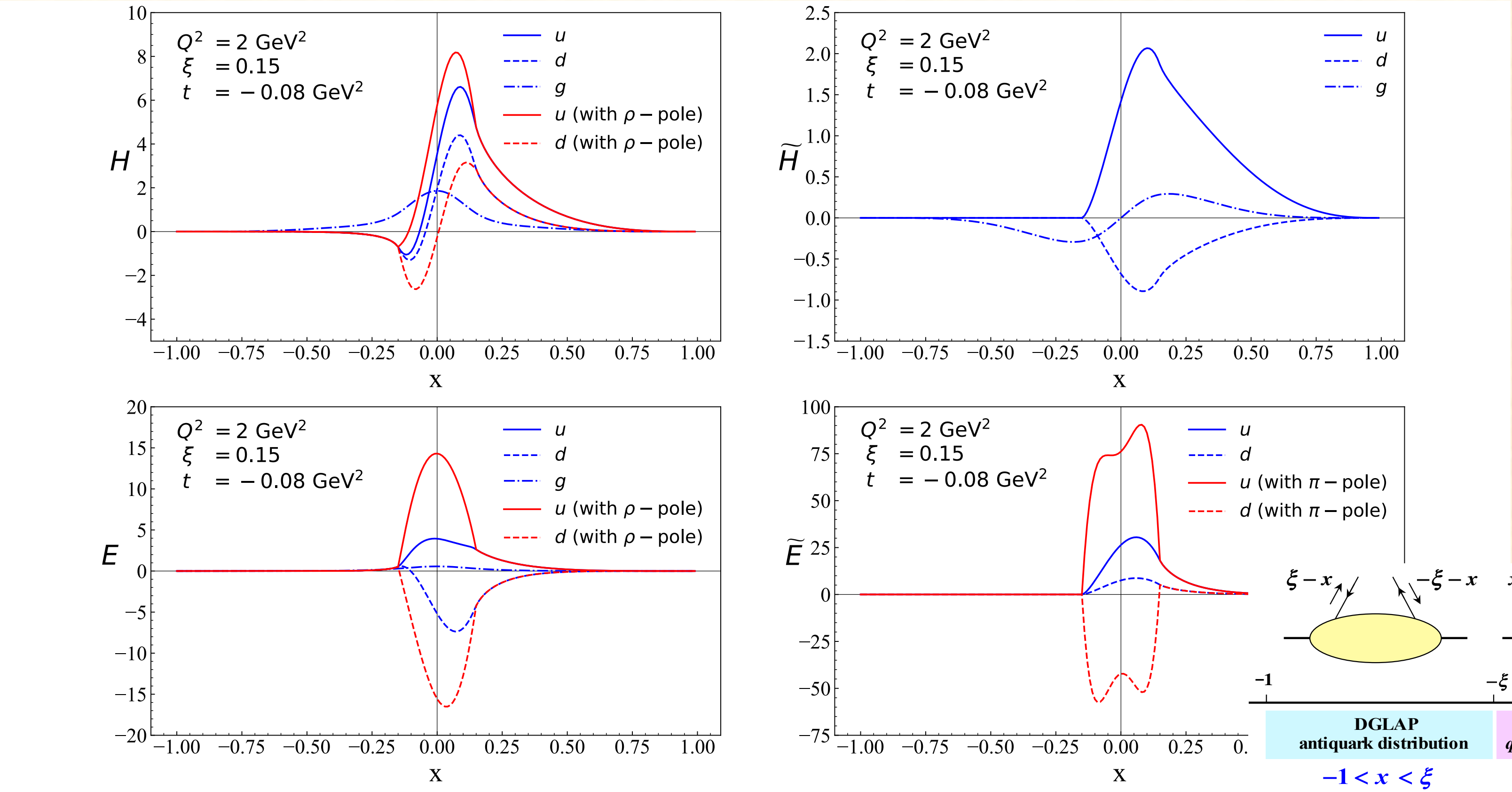
$$F_i(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-x) f_i(\beta,\alpha,t) + D_i(x',t)\Theta(\xi^2-x^2)$$

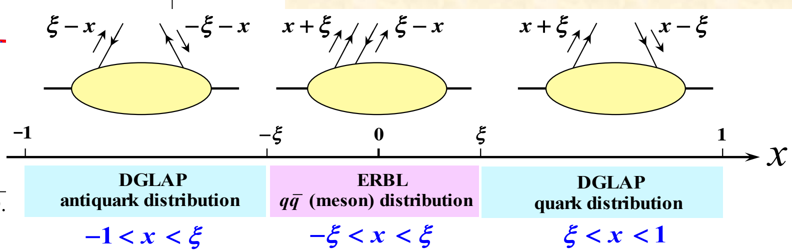
$$f_i(\beta,\alpha,t) = F_i(\beta,\xi=0,t=0)e^{tp_{h_i}(\beta)} \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)} \frac{\left[(1-|\beta|)^2-\alpha^2\right]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$\Theta(\xi^2-x^2) = \begin{cases} 1 & \xi^2 > x^2 \\ 0 & \xi^2 < x^2 \end{cases}, \quad p_{h_i}(\beta) = -\alpha'_{h_i}\ln\beta + b_{h_i}$$

$$F_i(\beta,\xi=0,t=0) = \beta^{-\delta_i}(1-\beta)^{2n_i+1}\sum_{j=0}^{3} c_{f_j}\beta^{j/2},$$

parameters determined by global analysis





Cross sections

π^+ production: $\nu p \to \ell^- \pi^+ p$

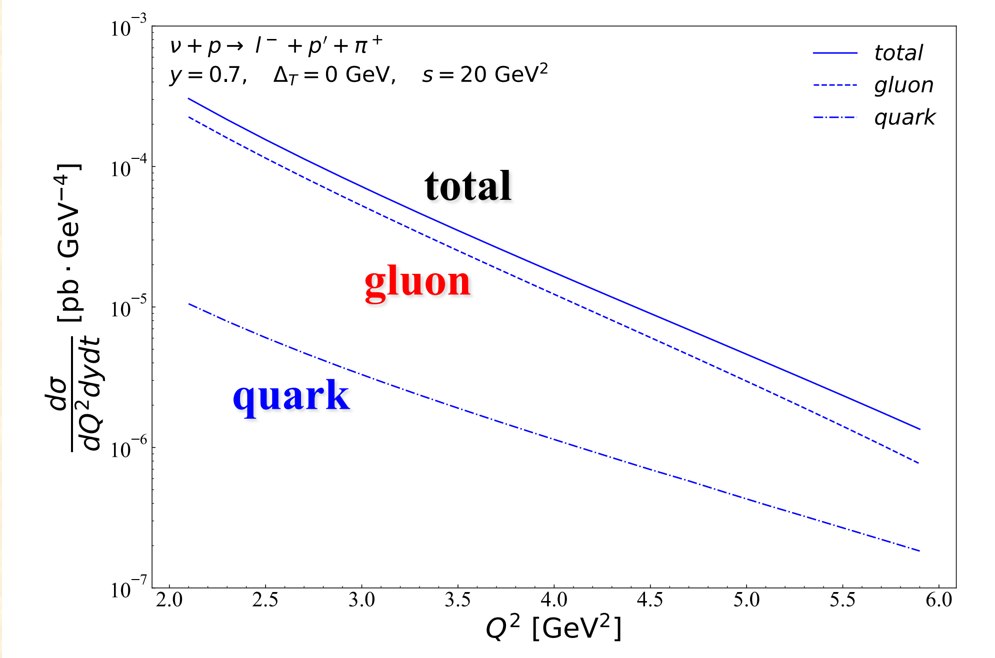
$$\mathcal{F}_q = 2 f_\pi \int \frac{dz\, \phi_\pi(z)}{1-z} \int dx \frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

$$\mathcal{F}_g = \frac{8 f_\pi}{\xi} \int \frac{dz\, \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$

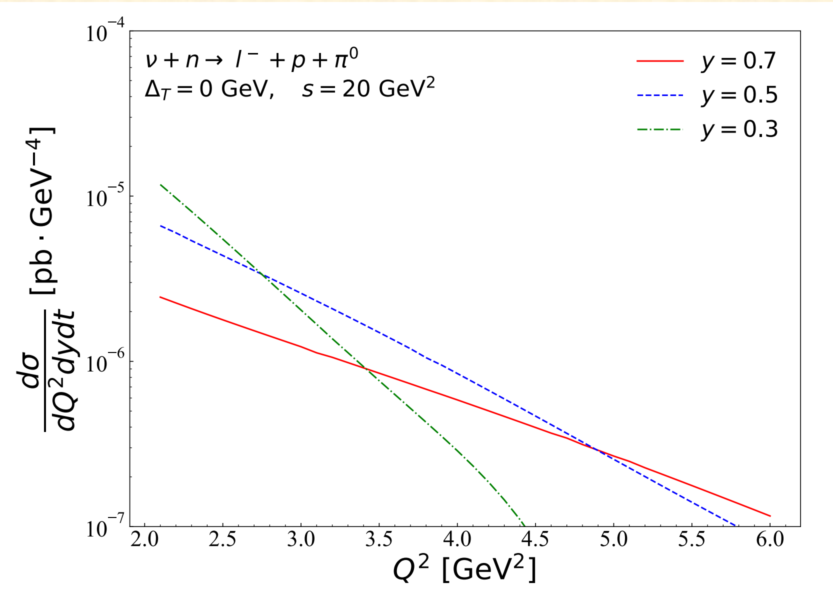
$$\frac{\mathcal{F}_q}{\mathcal{F}_g} \sim \frac{\xi}{8} = \frac{0.1 \sim 0.3}{8} = 0.01 \sim 0.04$$

$$= \text{ a few } \% \ll 1$$

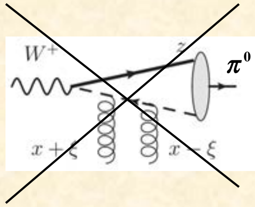
gluon $\gg$ quark



π^0 production: $\nu n \to \ell^- \pi^0 p$



no gluon



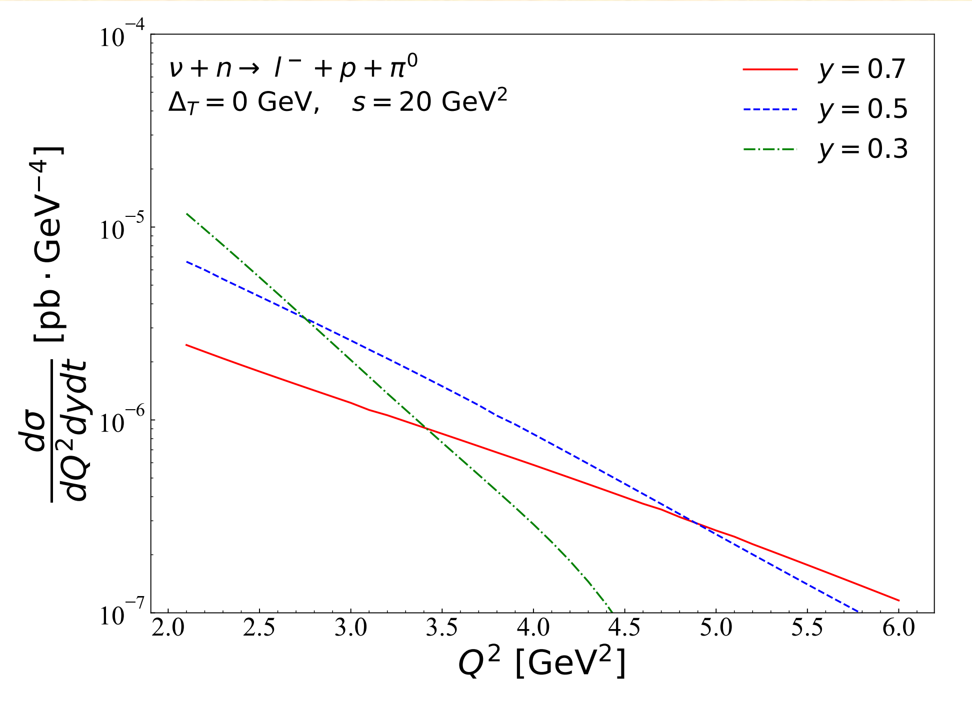
no gluon for π^0

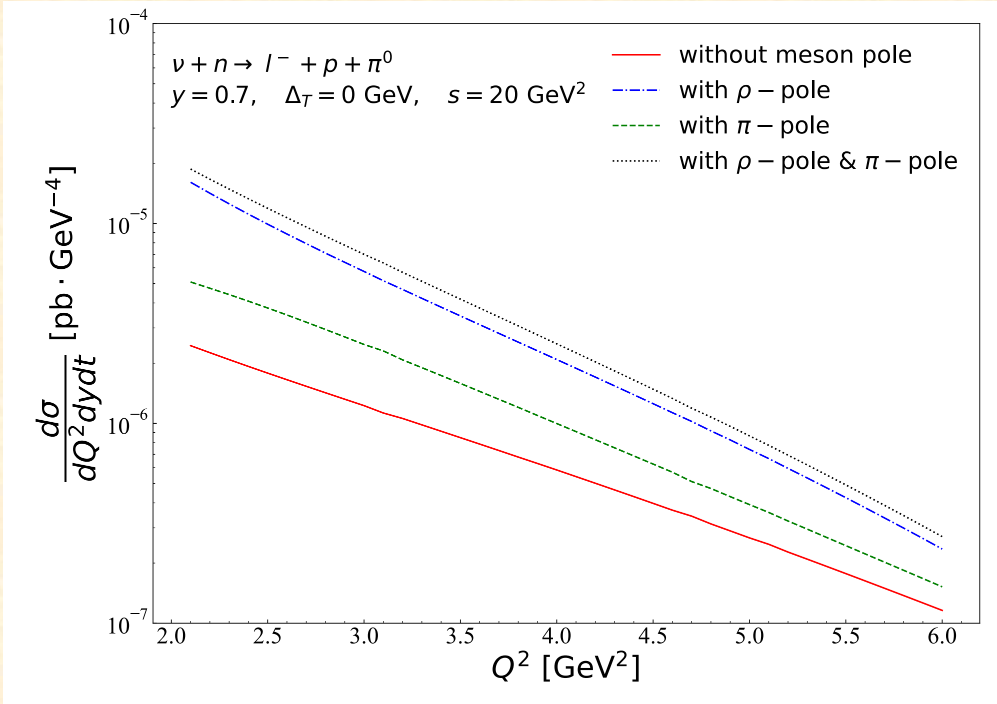
Neutrino GPD studies are complementary to the charged-lepton projects.

- Gluon GPDs could be probed in charged-pion production.
- Quark GPDs could be probed in π^0 production.
- Flavor dependece of quark GPDs could be investigated.

Contribution of each term to the π^0-production cross section

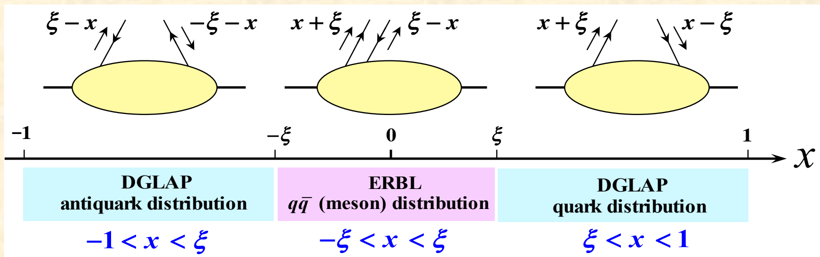
$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} \propto \frac{1}{Q^2}\Bigg[(1-\xi^2)\left\{\left|C_q\mathcal{H}_q + \cancel{C_g\mathcal{H}_g}\right|^2 + \left|C_q\tilde{\mathcal{H}}_q\right|^2\right\} + \frac{\xi^4}{1-\xi^2}\left\{\left|C_q\mathcal{E}_q + \cancel{C_g\mathcal{E}_g}\right|^2 + \left|C_q\tilde{\mathcal{E}}_q\right|^2\right\}$$
$$-2\xi^2\,\mathrm{Re}\left\{(C_q\mathcal{H}_q + \cancel{C_g\mathcal{H}_g})(C_q\mathcal{E}_q + \cancel{C_g\mathcal{E}_g})^*\right\} - 2\xi^2\,\mathrm{Re}\left\{C_q\tilde{\mathcal{H}}_q(C_q\mathcal{E}_q)^*\right\}\Bigg]$$





$$\cancel{H_g} > H_q > \tilde{H}_q > E_q,\ \tilde{E}_q,\ \cancel{E_g}$$

- π^0 production is senstive to quark $\mathcal{H}_q$.
- GPDs in the ERBL (Efremov-Radyushkin-Brodsky-Lepage) region could be probed.

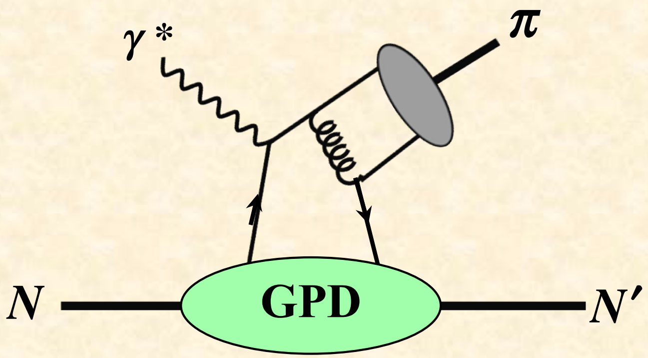


Possible studies on GPDs at hadron accelerator facilities

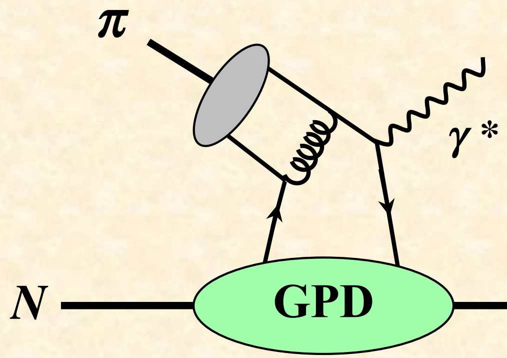
SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003;
T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka,
PRD 93 (2016) 114034.
J-PARC LoI 2019-07, J.-K. Ahn *et al*. (2019).
J-PARC proposal under preparation (2025),
Please get in touch with W.-C. Chang, N. Tomida
if you are interested in this project.

GPD projects at JLab / EIC / EicC and J-PARC / HIAF

JLab / EIC / EicC

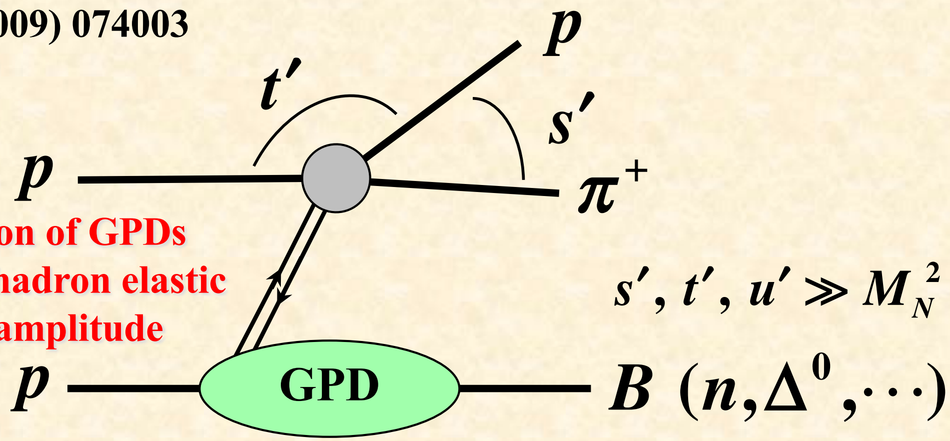


J-PARC / HIAF



$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003



Investigation of GPDs
with 2→3 hadron elastic
scattering amplitude

$$s',\ t',\ u' \gg M_N^2$$

J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

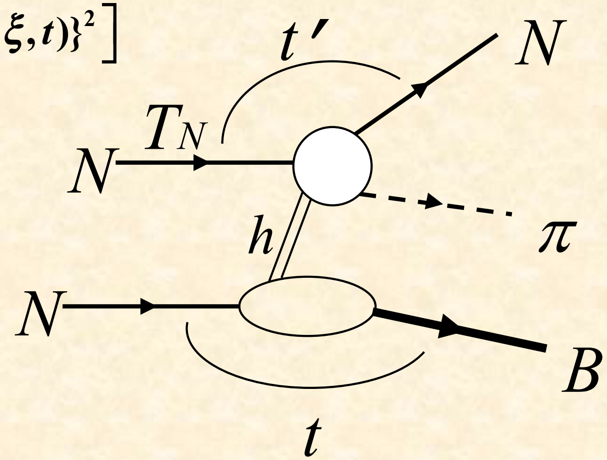
$$\pi + N \to \gamma + \gamma + N'$$

$$h + M_B \to h' + \gamma + M_D$$

$$h + M_B \to h' + M_C + M_D$$

Cross section estimate (ξ dependence)

$$\frac{d\sigma_{NN\to N\pi B}}{d\xi\, dt\, dt'} \propto \frac{d\sigma_{MN\to\pi N}}{dt'}\Big[8(1-\xi^2)\{H(x,\xi,t)\}^2+16\xi^2 H(x,\xi,t)E(x,\xi,t)-\frac{t}{m_N^2}(1+\xi)^2\{E(x,\xi,t)\}^2$$
$$+8(1-\xi^2)\{\tilde H(x,\xi,t)\}^2+18\xi^2\tilde H(x,\xi,t)\tilde E(x,\xi,t)-\frac{2t\xi^2}{m_N^2}\{\tilde E(x,\xi,t)\}^2\Big]$$

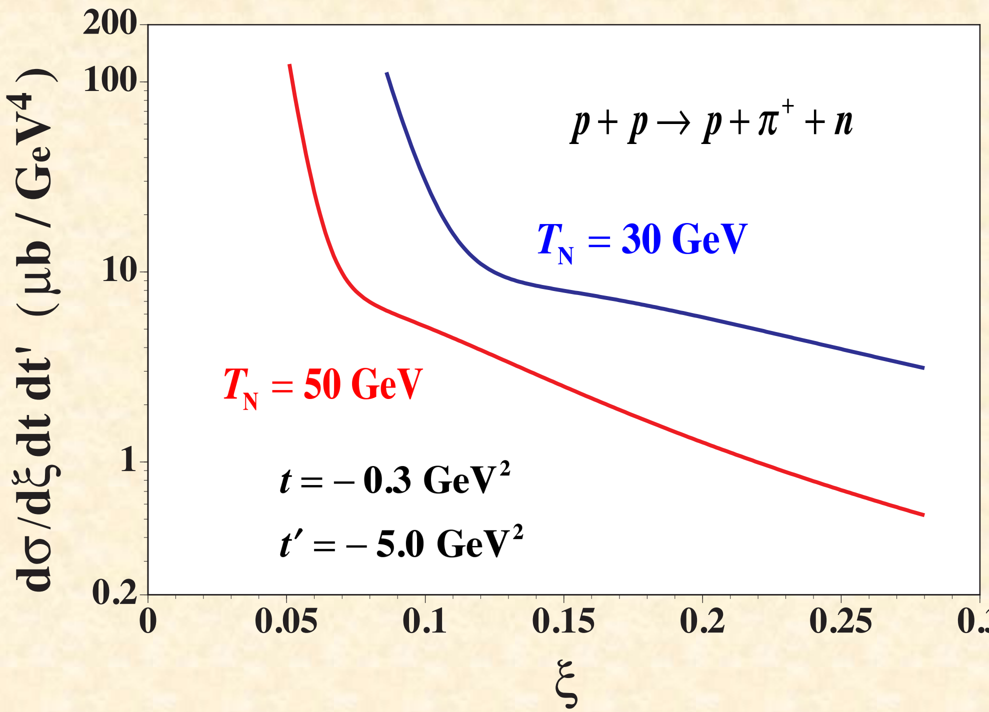


Skewdness parameter: $\xi = \dfrac{p_N^+ - p_B^+}{p_N^+ + p_B^+}$

$\dfrac{d\sigma}{d\xi dt dt'}\left(\dfrac{\mu\text{b}}{\text{GeV}^2}\right)$ as a function of ξ

at fixed $T_N = 30\ (50)$ GeV,

$t = -0.3\ \text{GeV}^2$, $t' = -5\ \text{GeV}^2$.



At this stage, our numerical results are for rough order of magnitude estimates on cross sections by assuming π- and ρ-like intermediate states.

For the details, please look at
SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003.

Exclusive Drell-Yan $\pi^- + p \to \mu^+ \mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \,\mathrm{Re}\left\{ \tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t) \right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]$$

$$Q'^2 = q'^2,\ \ t = (p-p')^2,\ \ \tau = \frac{Q'^2}{2p\cdot q_\pi} \simeq \frac{Q'^2}{s - m_\pi^2}$$

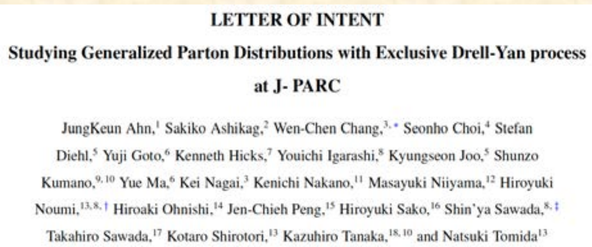
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle p(p') \right| \bar{q}(-z/2)\gamma^+\gamma_5 q(z/2) \left| p(p) \right\rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_p^q(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_p^q(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle n(p') \right| \bar{q}_d(-z/2)\gamma^+\gamma_5 q_u(z/2) \left| p(p) \right\rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

$$\tilde{H}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^{1} dx' \left[\frac{e_d}{x - x' - i\varepsilon} - \frac{e_u}{x + x' - i\varepsilon} \right] \left[\tilde{H}^d(x',\xi,t) - \tilde{H}^u(x',\xi,t) \right]$$

$$\tilde{E}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^{1} dx' \left[\frac{e_d}{x - x' - i\varepsilon} - \frac{e_u}{x + x' - i\varepsilon} \right] \left[\tilde{E}^d(x',\xi,t) - \tilde{E}^u(x',\xi,t) \right]$$

T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka, PRD93 (2016) 114034.

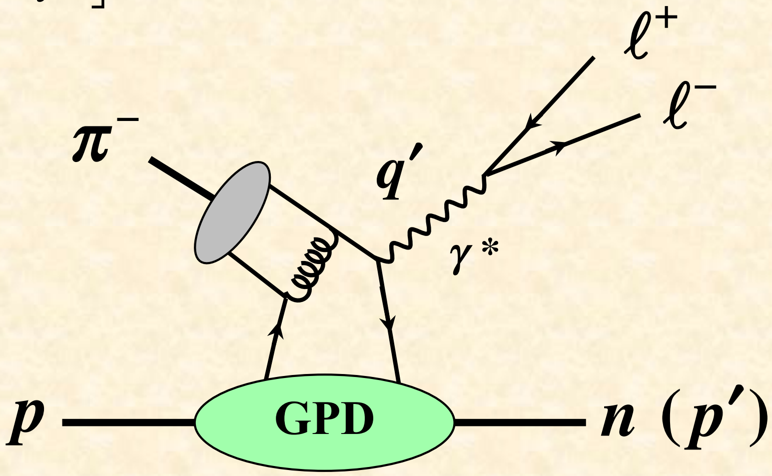


LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J- PARC

JungKeun Ahn,[1] Sakiko Ashikag,[2] Wen-Chen Chang,[3,*] Seonho Choi,[4] Stefan Diehl,[5] Yuji Goto,[6] Kenneth Hicks,[7] Youichi Igarashi,[8] Kyungseon Joo,[5] Shunzo Kumano,[9,10] Yue Ma,[6] Kei Nagai,[3] Kenichi Nakano,[11] Masayuki Niiyama,[12] Hiroyuki Noumi,[13,8,†] Hiroaki Ohnishi,[14] Jen-Chieh Peng,[15] Hiroyuki Sako,[16] Shin'ya Sawada,[8,‡] Takahiro Sawada,[17] Kotaro Shirotori,[13] Kazuhiro Tanaka,[18,10] and Natsuki Tomida[13]

LoI for a J-PARC experiment

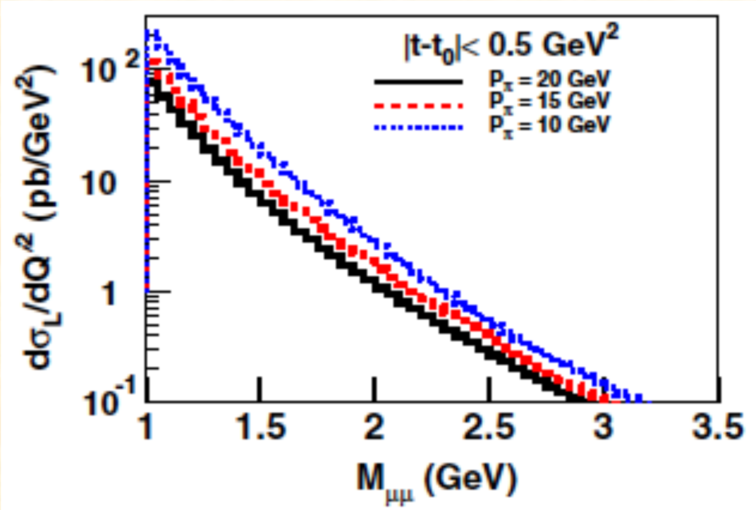


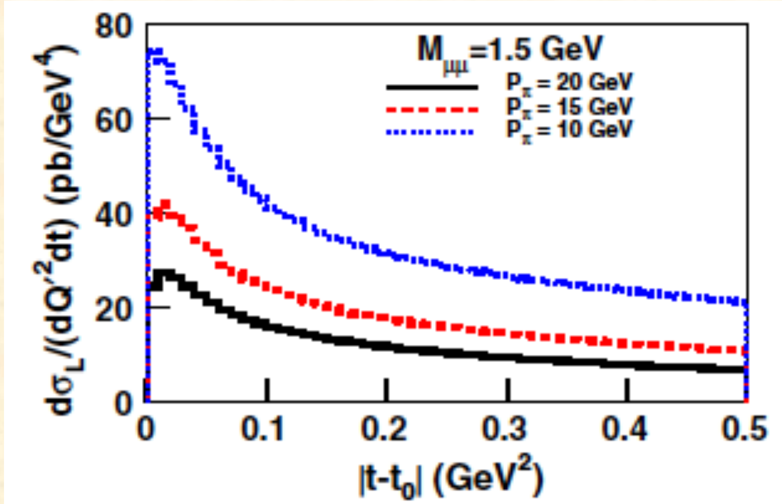
$$\pi^-(\bar{u}d) + p(uud) \to n(udd) + \gamma^*(\to \ell^+\ell^-)$$

Expected Drell-Yan events at J-PARC

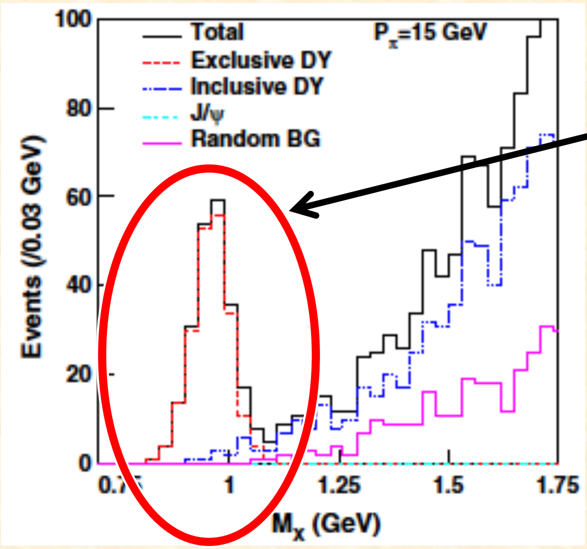
$$Q'^2 = q'^2,\ \ t = (p - p')^2,\ \tau = \frac{Q'^2}{2p\cdot q_\pi} \simeq \frac{Q'^2}{s - m_N^2}$$

$$\boxed{\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \,\mathrm{Re}\left\{ \tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t)\right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]}$$





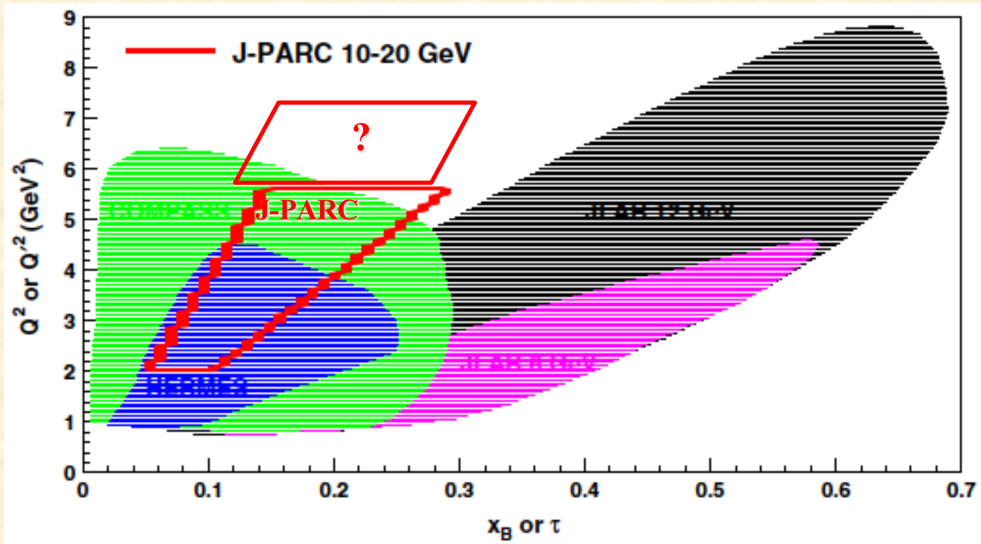
Missing mass



Exclusive Drell-Yan

$$M_X^2 = (q + p - q')^2$$

$$q = p_\pi,\ \ p = p_p,\ \ q = p_{\mu^+\mu^-}$$



Transition GPDs and exotic hadrons

S. Diehl *et al*. (S. Kumano, 15th author),
arXiv:2405.15386, submitted to Eur. Phys. J.

Transition GPDs

A. V. Belitsky, A. V. Radyushkin, Phys. Rept. 418 (2005) 1;
S. Kumano, M. Strikman, K. Suodh, Phys. Rev. D 80 (2009) 074003;
P. Kroll, K. Passek-Kumericki, Phys. Rev. D 107 (2023) 054009;
S. Diehl *et al.*, arXiv:2405.15386, submitted to Eur. Phys. J.

$N \to B$ (1/2$^+$) transition GPDs

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+ \gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

$$\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$$

$$\int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$$

$N \to B$ (3/2$^+$) transition GPDs

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP\cdot z} \left\langle \Delta^{++}(p', \lambda') \middle| \bar{u}\left(-\frac{1}{2}z\right)\gamma^+ d\left(\frac{1}{2}z\right) \middle| p(p, \lambda) \right\rangle_{z^+=0,\vec{z}_\perp=0}$$

$$= \frac{1}{2P^+}\bar{u}_\delta(p', \lambda')\left[\frac{\Delta^\delta n^\mu - \Delta^\mu n^\delta}{M_N}\left\{ \gamma_\mu G_1(x,\xi,t) + \frac{P_\mu}{M_N}G_2(x,\xi,t) + \frac{\Delta_\mu}{M_N}G_3(x,\xi,t) \right\} + \frac{\Delta^+\Delta^\delta}{M_N^2}G_4(x,\xi,t) \right]\gamma_5 u(p, \lambda)$$

$$\int_{-1}^{1} dx\, G_1(x,\xi,t) = 2G_M^{N\to B}(t), \quad \int_{-1}^{1} dx\, G_2(x,\xi,t) = 2G_E^{N\to B}(t)$$

$$\int_{-1}^{1} dx\, G_3(x,\xi,t) = 2G_C^{N\to B}(t), \quad \int_{-1}^{1} dx\, G_4(x,\xi,t) = 0$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP\cdot z} \left\langle \Delta^{++}(p', \lambda') \middle| \bar{u}\left(-\frac{1}{2}z\right)\gamma^+ \gamma^5 d\left(\frac{1}{2}z\right) \middle| p(p, \lambda) \right\rangle_{z^+=0,\vec{z}_\perp=0}$$

$$= \frac{1}{2P^+}\bar{u}_\delta(p', \lambda')\left[\frac{\Delta^\delta n^\mu - \Delta^\mu n^\delta}{M_N}\left\{ \gamma_\mu \tilde{G}_1(x,\xi,t) + \frac{P_\mu}{M_N}\tilde{G}_2(x,\xi,t) + \frac{\Delta_\mu}{M_N}\tilde{G}_3(x,\xi,t) \right\} + \frac{\Delta^+\Delta^\delta}{M_N^2}\tilde{G}_4(x,\xi,t) \right] u(p, \lambda)$$

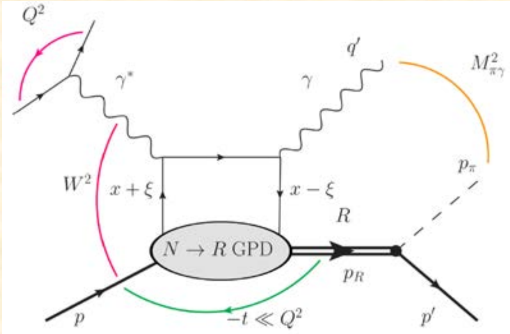
Rarita-Schwinger: $u^\alpha(p,\lambda) = \sum_{\rho,\sigma} \left\langle 1\rho; \frac{1}{2}\sigma \middle| \frac{3}{2}\lambda \right\rangle \varepsilon^\alpha(p,\rho) u(p,\sigma)$

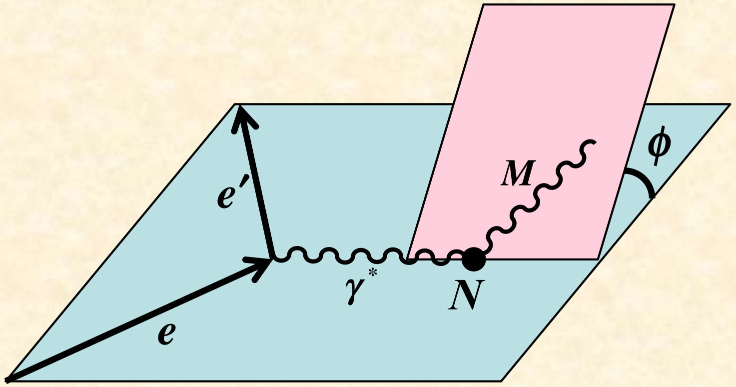
Adler form factors

$$\int_{-1}^{1} dx\, \tilde{G}_1(x,\xi,t) = 2C_5^{A,\,N\to B}(t), \quad \int_{-1}^{1} dx\, \tilde{G}_2(x,\xi,t) = 2C_6^{A,\,N\to B}(t)$$

$$\int_{-1}^{1} dx\, \tilde{G}_3(x,\xi,t) = 2C_3^{A,\,N\to B}(t), \quad \int_{-1}^{1} dx\, \tilde{G}_4(x,\xi,t) = 2C_4^{A,\,N\to B}(t)$$

First JLab results on DVMP for transition GPDs



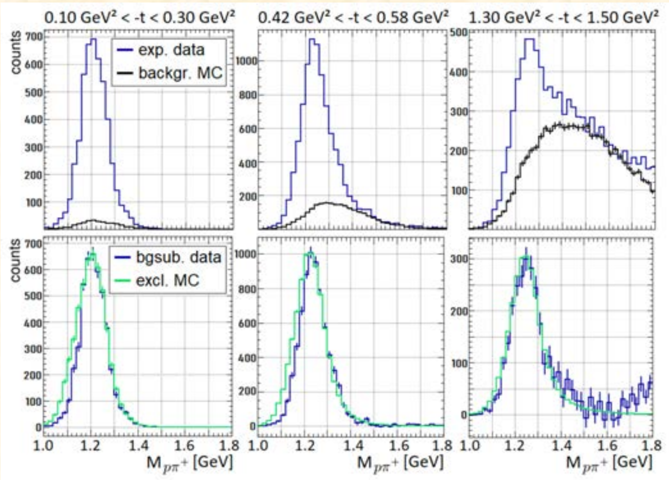


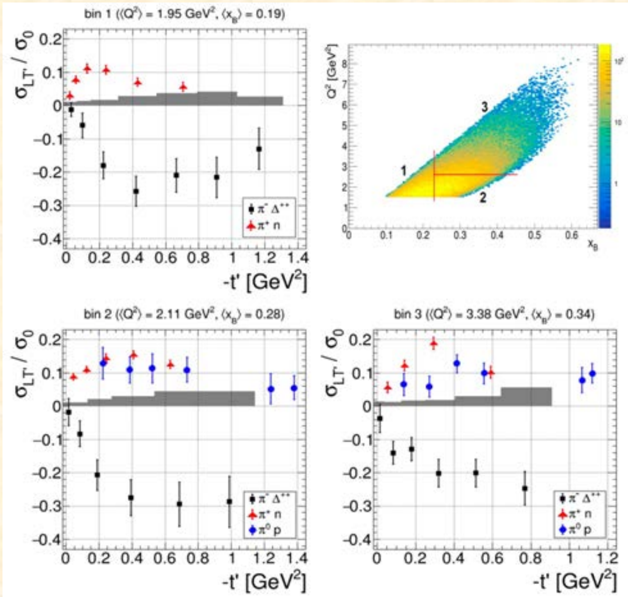
$$\frac{d\sigma(eN \to e'BM)}{ds\,dQ^2\,dt\,d\phi} \propto \frac{d\sigma_T}{dt} + \varepsilon\frac{d\sigma_L}{dt} + \varepsilon\cos(2\phi)\frac{\sigma_{TT}}{dt} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi\frac{d\sigma_{LT}}{dt} + h\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi\frac{d\sigma_{LT'}}{dt}$$

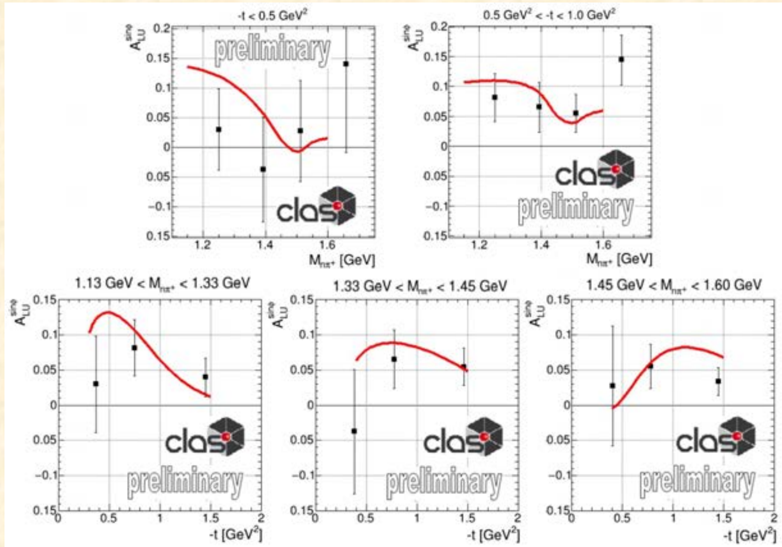
$\sigma_{T,L}$ = transverse and longitudinal photon cross section

$\sigma_{TT,LT,LT'}$ = transverse - longitudinal interference terms

ε = virtual-photon longidutinal flux / transverse flux

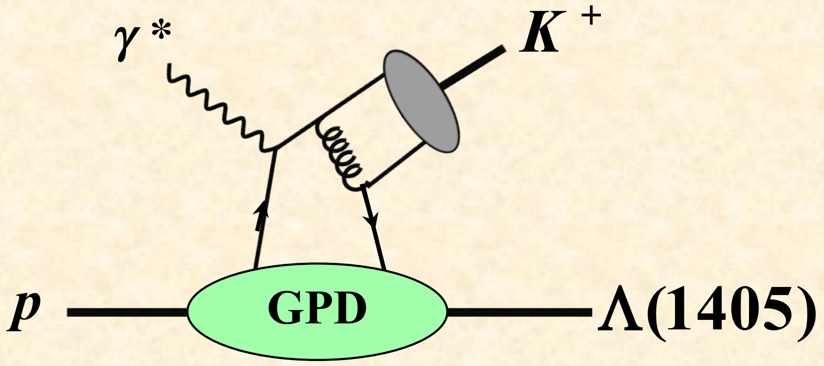




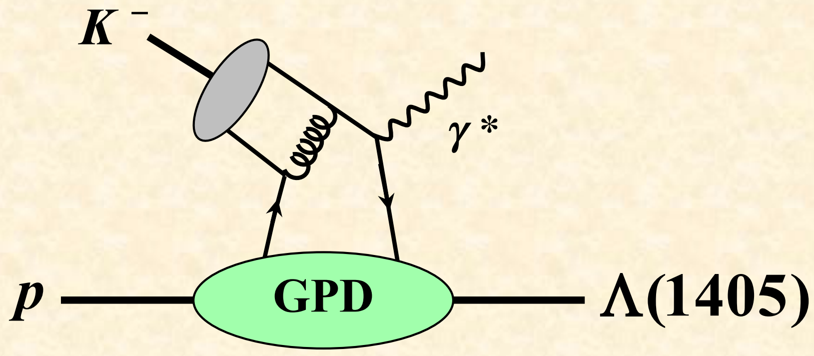


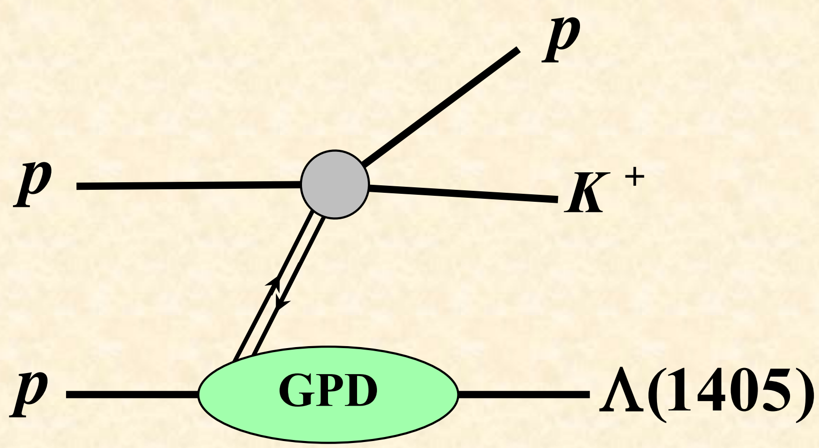
Transition GPDs for exotic hadrons

JLab / EIC



J-PARC

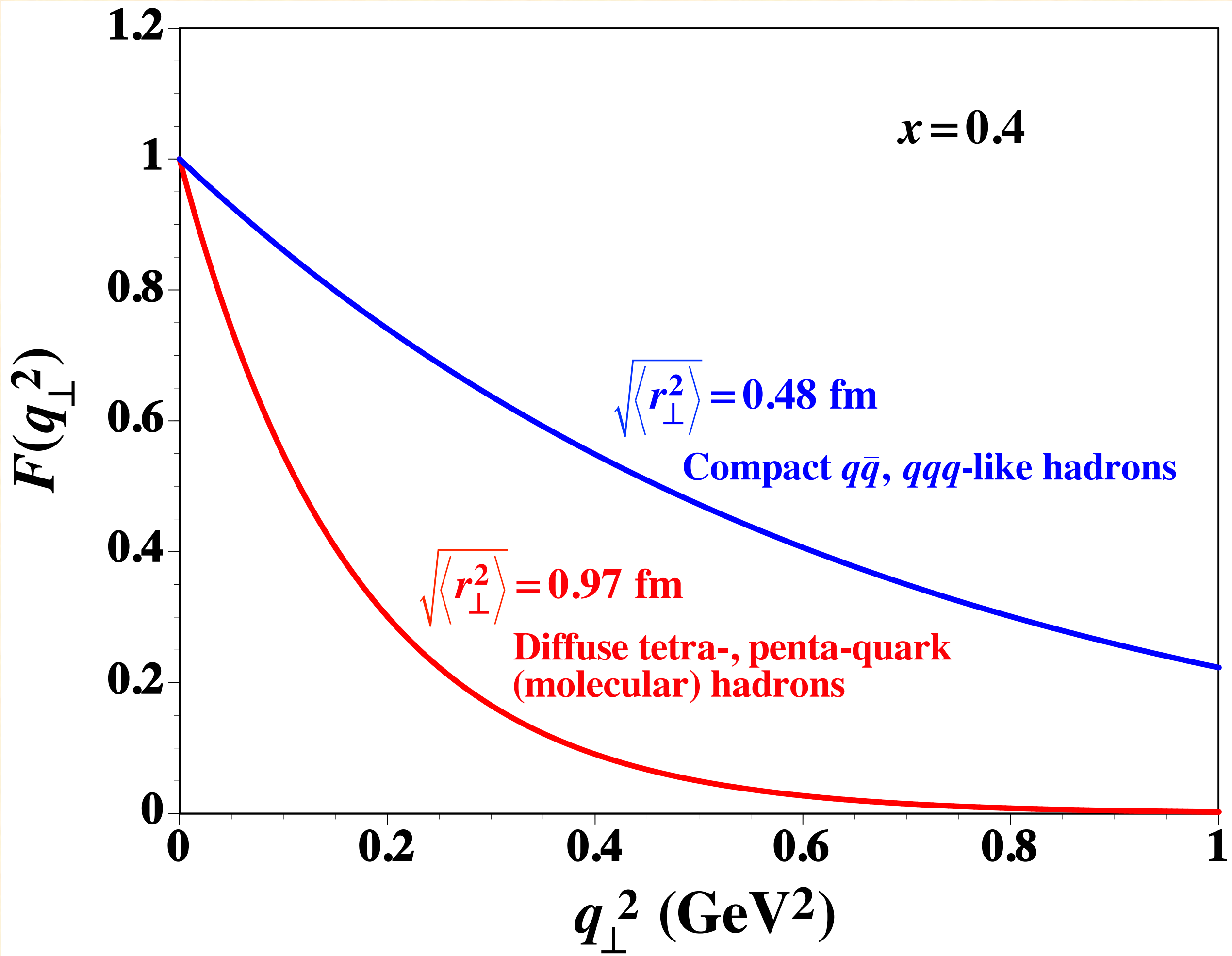


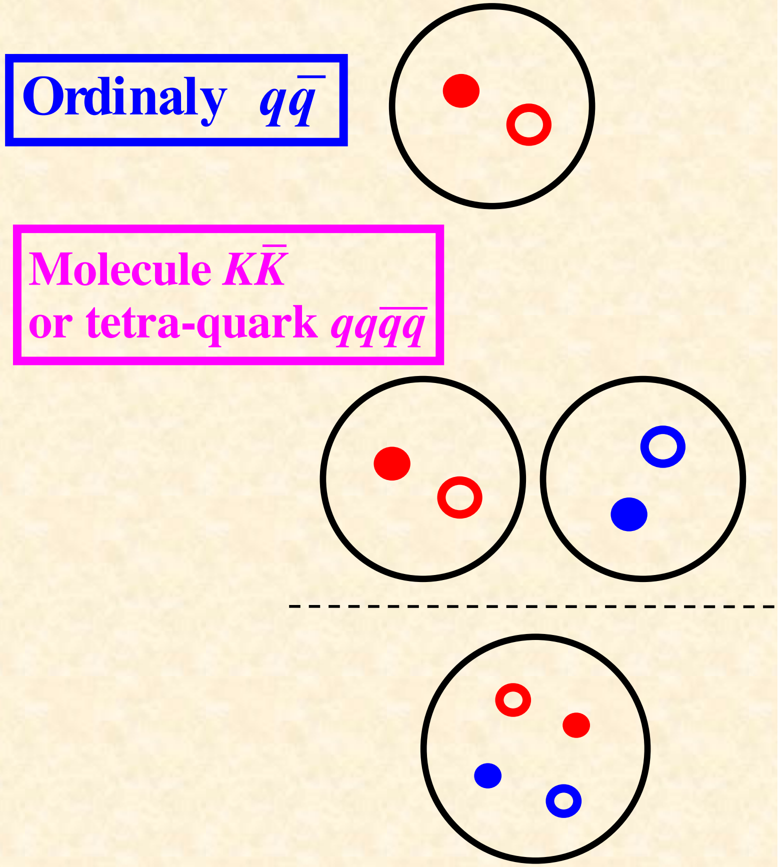


However, there is no theoretical study on the N→Λ(1405) transition GPDs at this stage.

Two-dimensional transverse form factor

$$F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$





Future prospects on GPD projects

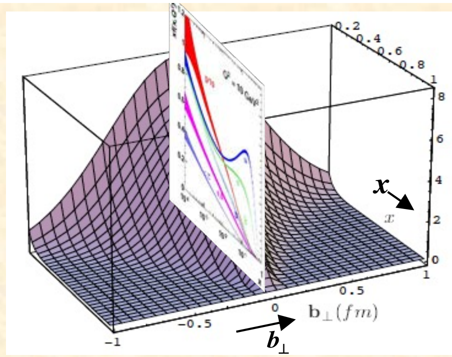
High-energy hadron physics experiments

Facilities on hadron structure functions on GPDs including future possibilities.

Hadron accelerator facilities. Lepton accelerator facilities.

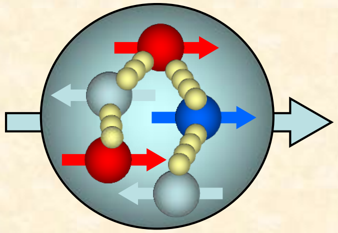
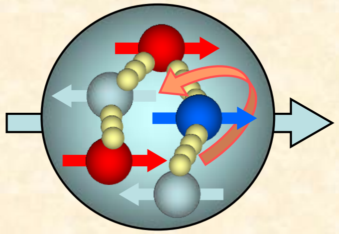
By hadron tomography

3D view of hadrons

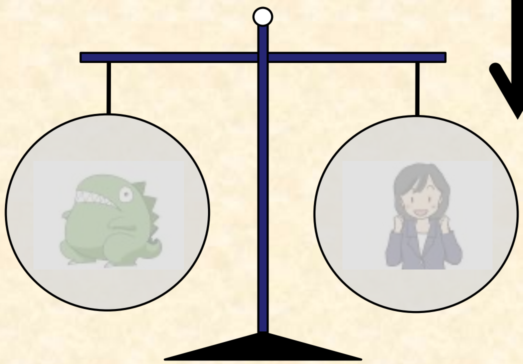


Origin of nucleon spin

By the tomography, we determine

 or  .

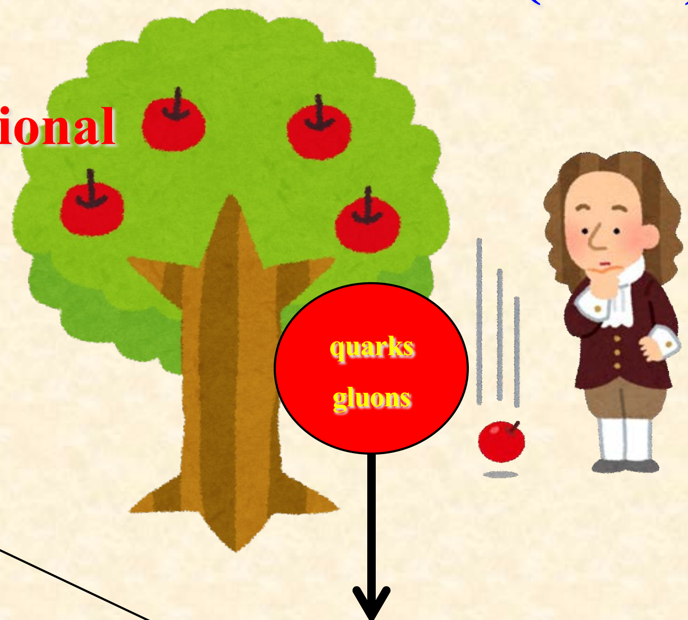
Exotic hadrons



By tomography, we determine  or  .

Origin of gravitational source (mass)

By tomography, we determine gravitational sources in terms of quarks and gluons.



Summary on the GPDs

Hadron-tomography and gravitational form factors by the GPDs.

- Puzzle to find the origin of hadron masses and pressures in terms of quark and gluon degrees of freedom
- Puzzle to find the origin of nucleon spin
- Exotic hadron candidates could be studied in the same tomography method.
- In addition to the electron scattering projects, the GPD studies are possible by neutrino- and hadron-beam facilities and e^+e^- colliders.
- If the HIAF will have a high-energy proton beam in future (9.3 GeV → 25 GeV → 100 GeV range?), a wider hadron-physics project is possible, such as the GPDs by proton and neutrino reactions.

The End

The End