



華中師範大學

CENTRAL CHINA NORMAL UNIVERSITY

由缪原子光谱学测量提取核结构信息 双光子交换核效应研究

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粒子物理研究所

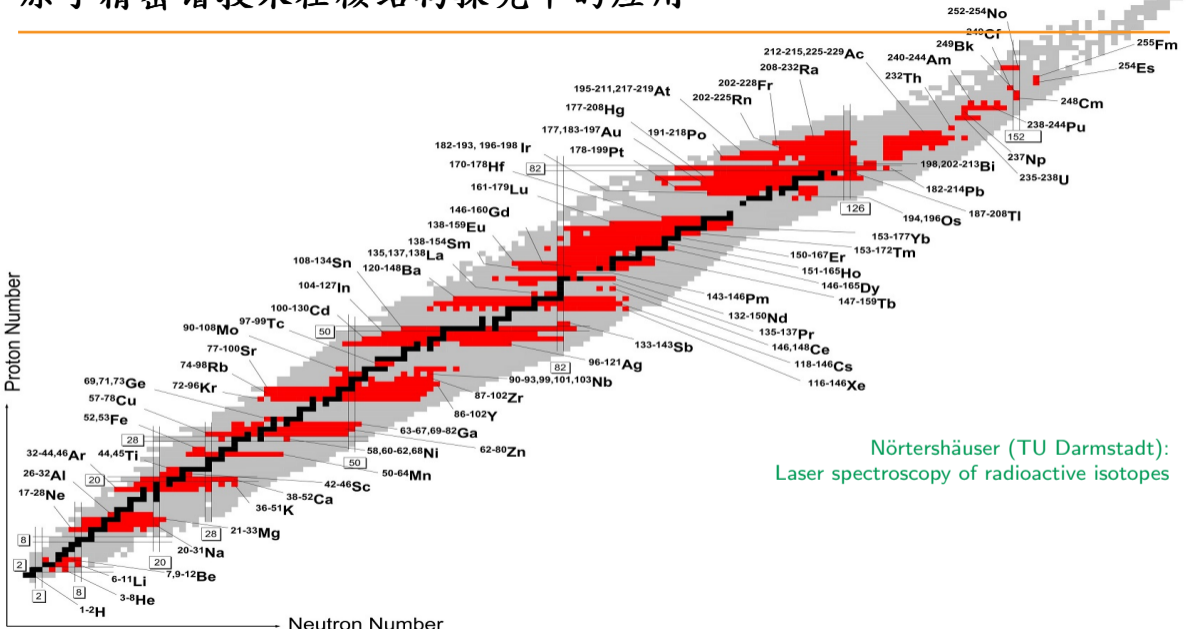


第三届惠州大装置高精度核物理研讨会
2025.04.20-22

原子精密谱技术在核结构探究中的应用

Proton Number

Neutron Number



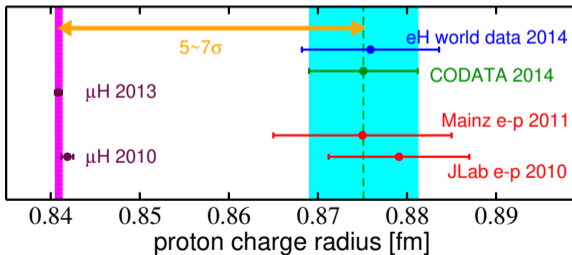
Nörtershäuser (TU Darmstadt):
Laser spectroscopy of radioactive isotopes

原子精密谱中的核结构

- 原子精密谱提供了丰富的核结构信息
 - 核结构物理量：自旋、电荷半径、磁矩、电四极矩、磁半径等
 - 核结构信息：壳层演化、 β 稳定线、滴线、晕结构、形变等
- 核结构的精密测量为构建核力模型和多体理论提供帮助
 - 确定张量力、介子交换电磁流、三体力等

质子半径难题

- 电子-质子作用实验: $r_p = 0.8770(45)$ fm
 - eH 氢原子光谱
 - $e-p$ 质子散射
- μ 子-质子作用实验: $r_p = 0.8409(4)$ fm
 - μH 原子 兰姆位移 (ΔE_{2S-2P}) [PSI-CREMA]
Pohl et al., Nature (2010); Antognini et al., Science (2013)

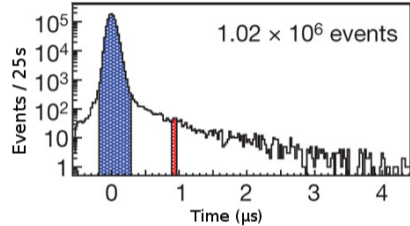
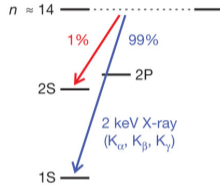


μ H Lamb shift experiment

Lamb Shift: 2S-2P splitting in atomic spectrum

Pic: Pohl *et al.* Nature (2010)

- **prompt X-ray** ($t \sim 0$ s): μ^- stopped in H_2 gases

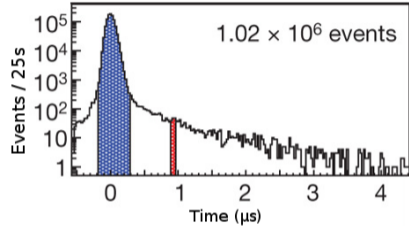
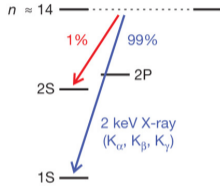


μ H Lamb shift experiment

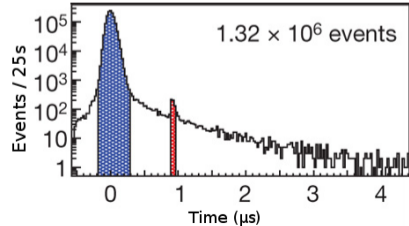
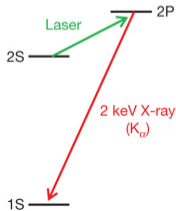
Lamb Shift: 2S-2P splitting in atomic spectrum

Pic: Pohl *et al.* Nature (2010)

- **prompt X-ray ($t \sim 0$ s):** μ^- stopped in H_2 gases



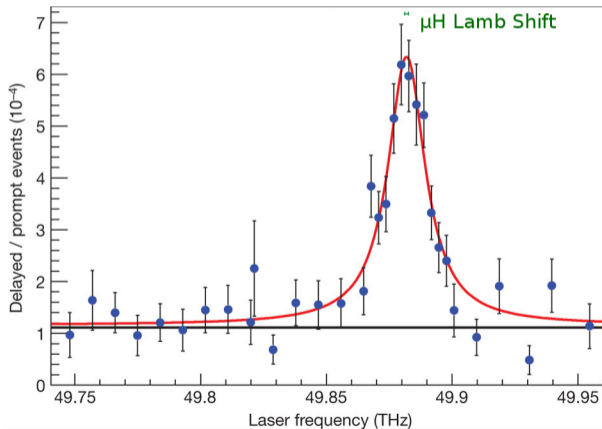
- **delayed X-ray ($t \sim 1 \mu$ s):** laser induced 2S \rightarrow 2P



μH Lamb shift experiment

- measure $K_{\alpha}^{\text{delayed}} / K_{\alpha}^{\text{prompt}}$
- $\delta E_{LS} = hf_{res}$

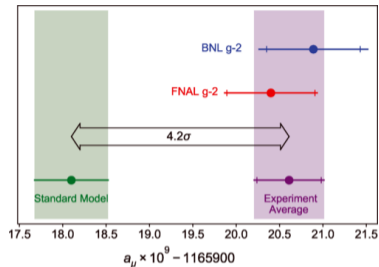
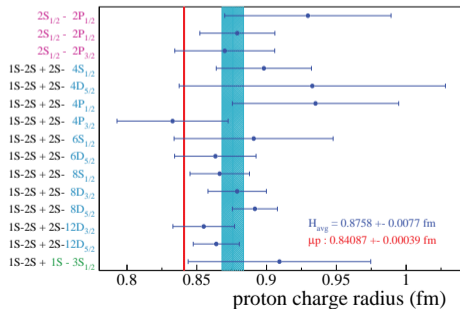
Pic: Pohl *et al.* Nature (2010)



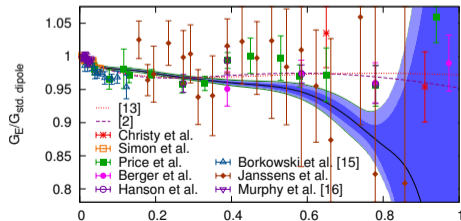
破解质子半径难题

- 半径难题的可能解释：
 - 轻子普适性破坏? ($g_\mu - 2$)
 - 奇异强子结构?
 - 被忽视的实验系统误差?

没有哪一种解释已被完全接受



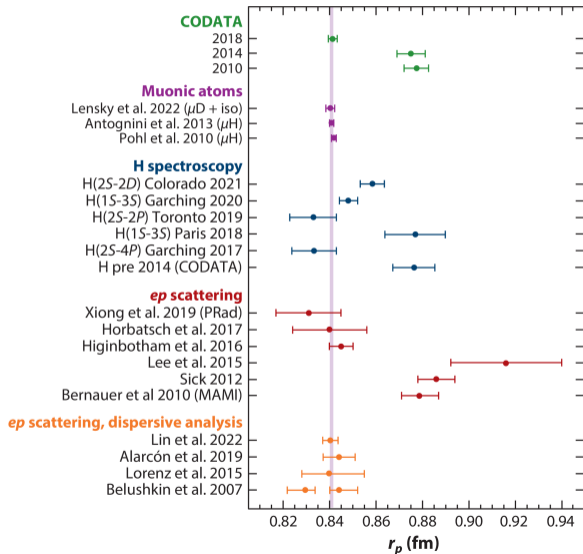
($g - 2$) $_\mu$ collaboration, PRL 126 (2021) 141801



破解质子半径难题

● 测量质子半径的新实验

- 电子-质子散射 (JLab, Mainz, Tohoku U.)
- μ 子-质子散射 (PSI-MUSE)
- 氢原子精密谱 (MPQ, LKB, York U.)



其他原子精密谱中的核半径测量

- μ 原子/离子兰姆位移谱 (PSI-CREMA)

- $\mu^2\text{H}$ [Pohl *et al.*, Science 2016]
- $\mu^4\text{He}^+$ [Krauth *et al.*, Nature 2021]
- $\mu^3\text{He}^+$ [K. Schuhmann *et al.*, arXiv:2305.11679]

测量原子核电荷半径

- $e^{3,4}\text{He}$ 原子精密谱

测量 ^3He - ^4He 核电荷半径同位素位移

- $e^{6,7}\text{Li}$ 原子超精细劈裂谱

测量 $^6,7\text{Li}$ 原子核磁矩分布

- μ 原子超精细劈裂谱 (PSI-CREMA)

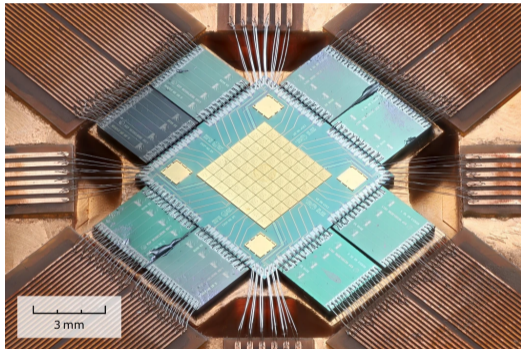
- $\mu^2\text{H}$, $\mu^3\text{He}^+$

测量原子核磁矩分布

QUARTET实验：高精度光谱

- 采用金属磁性微量能器测量X射线
- 高能量带宽、高分辨率、高探测效率
- 提升锂至氦轻核电荷半径测量精度

Method	Intrinsic Resolution	Typical Efficiency	Energy Range
HPGe	10^{-3} & ~ 500 eV	3×10^{-2}	>50 keV
Crystal spectrometers	10^{-6}	10^{-7}	<30 keV Narrow Band!
MCs	10^{-4}	10^{-4}	1-300 keV



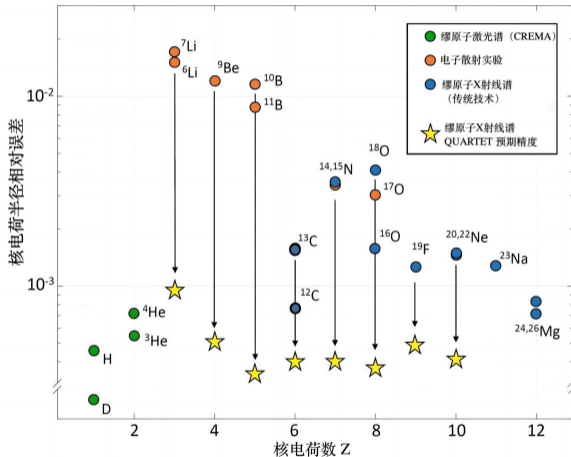
QUARTET实验：高精度光谱

- 技术：采用金属磁性微量能器测量X射线
- 优势：高能量带宽、高分辨率、高探测效率
- 目标：提升锂至氦轻核电荷半径测量精度

Isot.	E_{1S-2P} keV	δ_{exp} eV	δ_{NP} eV	fm/keV	$\delta_r \cdot 10^{-3}$ fm	Gain	Good for
^{11}B	52	5 → 0.10	0.2 → 0.05	-6.7	21 → 0.8	26	Mir., HLI, Chain
^9Be	33	10 → 0.07	0.1 → 0.03	-16	12 → 1.3	9	Mir., HLI, Chain
^7Li	19	60 → 0.05	0.03 → 0.01	-47	42 → 2.4	17	Mir., Chain

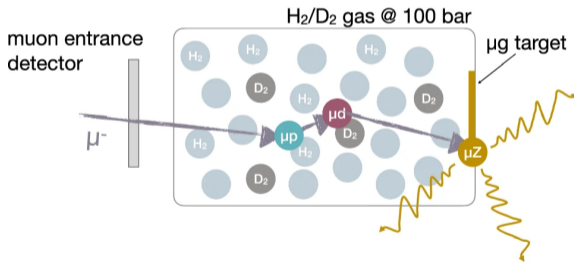
Isot.	E_{1S-2P} keV	δ_{exp} eV	δ_{NP} eV	fm/keV	$\delta_r \cdot 10^{-3}$ fm	Gain	Good for
^{12}C	75	0.5 → 0.15	0.3 → 0.15	-3.2	1.9 → 0.7	3	χ PT, HLI
^{11}B	52	5 → 0.10	0.2 → 0.05	-6.7	21 → 0.8	26	Mir., HLI, Chain
^9Be	33	10 → 0.07	0.1 → 0.03	-16	12 → 1.3	9	Mir., HLI, Chain

Pair	ΔE_{2P-1S} eV	δ_{exp} eV	δ_{NP} eV	fm/keV	$\delta_{\Delta r} \cdot 10^{-3}$ fm	Gain	Good for
$^{20,22}\text{Ne}$	230	6 → 0.5	2 → 0.5	-0.4	4.3 → 0.4	11	Δg
$^{16,18}\text{O}$	28	10 → 0.2	0.7 → 0.3	-1.0	9.8 → 0.3	33	Δg



μ X实验：微克量级靶材技术

- 技术：利用氘/氢混合气中缪子转移反应。
- 优势：靶材质量降至微克级，适合稀有同位素。
- 目标：测量镭-226电荷半径，研究宇称破缺。



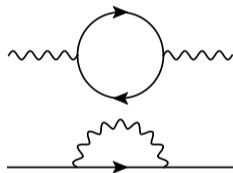
原子兰姆位移谱中的核结构

- 从 μ 原子兰姆位移提取核电荷半径

$$\delta E_{\text{LS}} = \delta_{\text{QED}} + \mathcal{A}_{\text{OPE}} R_E^2 + \delta_{\text{TPE}}$$

- **量子电动力学修正:**

- 真空极化效应
- 轻子自能修正
- 相对论反冲修正



原子兰姆位移谱中的核结构

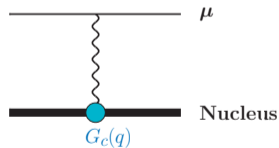
- 从 μ 原子兰姆位移提取核电荷半径

$$\delta E_{\text{LS}} = \delta_{\text{QED}} + \mathcal{A}_{\text{OPE}} R_E^2 + \delta_{\text{TPE}}$$

- 核结构效应修正:

- $\propto R_E^2 \implies$ 单光子交换中的核结构效应

$$\mathcal{A}_{\text{OPE}} \approx m_\mu^3 (Z\alpha)^4 / 12$$



原子兰姆位移谱中的核结构

- 从 μ 原子兰姆位移提取核电荷半径

$$\delta E_{\text{LS}} = \delta_{\text{QED}} + A_{\text{OPE}} R_E^2 + \delta_{\text{TPE}}$$

- 核结构效应修正:

- $\delta_{\text{TPE}} \implies$ 双光子交换中的核结构效应

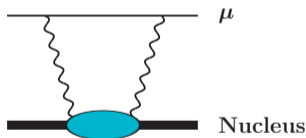
- 弹性贡献项: Zemach moment δ_{Zem}

- 非弹性贡献项: 核极化效应 δ_{pol}

- 提取核半径 R_E 的准确度依赖于 δ_{TPE} 的理论输入

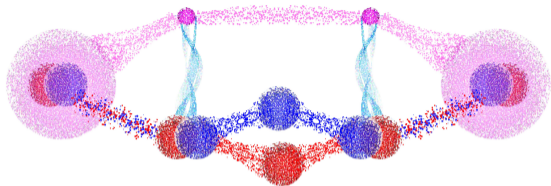
$\mu^2\text{H}$ 实验: δ_{pol} 需要1%的理论精度

$\mu^{3,4}\text{He}^+$ 实验: δ_{pol} 需要5%的理论精度



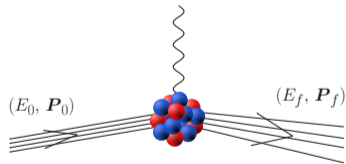
由光核反应求和规则计算核极化效应

$$\delta_{\text{pol}} = \sum_{g, S_{\hat{O}}} \int_{\omega_{th}}^{\infty} d\omega \underbrace{g(\omega)}_{\text{权重}} \underbrace{S_{\hat{O}}(\omega)}_{\text{核响应函数}}$$



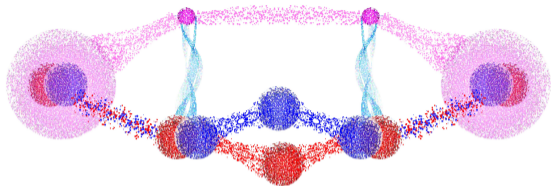
- 能量求和权重 $g(\omega)$
- 核响应函数 $S_{\hat{O}}(\omega)$

$$S_O(\omega) = \sum_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$



由光核反应求和规则计算核极化效应

$$\delta_{\text{pol}} = \sum_{g, S_{\hat{O}}} \int_{\omega_{th}}^{\infty} d\omega \underbrace{g(\omega)}_{\text{权重}} \underbrace{S_{\hat{O}}(\omega)}_{\text{核响应函数}}$$



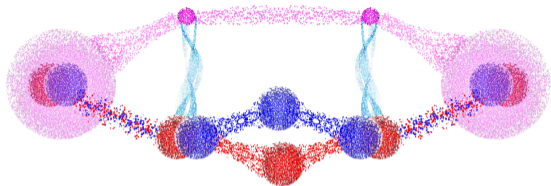
μ 原子中核极化效应 δ_{pol} 的贡献项:

- 电磁多极矩展开
 - E0, E1, E2响应求和规则
- 相对论效应与库仑扭曲效应修正
- 核子内部结构修正

[CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, JPG 45 \(2018\) 093002](#)

由光核反应求和规则计算核极化效应

$$\delta_{\text{pol}} = \sum_{g, S_{\hat{O}}} \int_{\omega_{\text{th}}}^{\infty} d\omega \underbrace{g(\omega)}_{\text{权重}} \underbrace{S_{\hat{O}}(\omega)}_{\text{核响应函数}}$$



$$S_{D_1}(\omega) = \frac{1}{2J_0 + 1} \sum_{N \neq N_0, J} |\langle N_0 J_0 | \hat{D}_1 | N J \rangle|^2 \delta(\omega - \omega_N)$$

$$S_{R^2}(\omega) = \frac{1}{2J_0 + 1} \sum_{N \neq N_0, J} |\langle N_0 J_0 | \hat{R}^2 | N J \rangle|^2 \delta(\omega - \omega_N)$$

$$S_Q(\omega) = \frac{1}{2J_0 + 1} \sum_{N \neq N_0, J} |\langle N_0 J_0 | \hat{Q}_2 | N J \rangle|^2 \delta(\omega - \omega_N)$$

$$S_{D_1 D_3}(\omega) = \frac{1}{2J_0 + 1} \text{Re} \sum_{N \neq N_0, J} \langle N_0 J_0 | \hat{D}_1^\dagger | N J \rangle \langle N J | \hat{D}_3 | N_0 J_0 \rangle \delta(\omega - \omega_N)$$

$$\hat{D}_1 = \frac{1}{Z} \sum_a R_a Y_1(\hat{R}_a)$$

$$\hat{R}^2 = \frac{1}{Z} \sum_a R_a^2$$

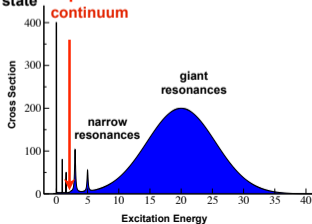
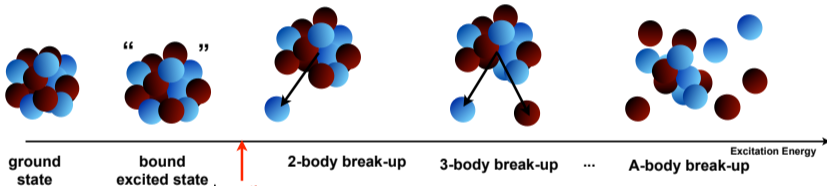
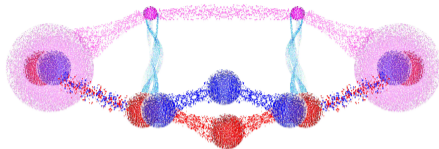
$$\hat{Q}_2 = \frac{1}{Z} \sum_a R_a^2 Y_2(\hat{R}_a)$$

$$\hat{D}_3 = \frac{1}{Z} \sum_a R_a^3 Y_3(\hat{R}_a)$$

核响应函数: 连续谱

- 原子核在双光子交换过程中被虚激发

$$S_O(\omega) = \sum_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$



$$\sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$

Exact knowledge limited in energy and mass number

由第一性原理计算核极化效应 δ_{pol}

- $\mu^{2,3}\text{H}$, $\mu^{3,4}\text{He}^+$:

- 第一性原理数值计算方法

Effective Interaction Hyperspherical Harmonics (超球简谐基展开)

Lorentz Integral Transform (核响应函数)

Lanczos Algorithm (求和规则)

束缚态 \rightarrow 共振/散射态

- 核子-核子间相互作用势

AV18+UIX

$\chi\text{EFT } NN(\text{N}^3\text{LO})+NNN(\text{N}^2\text{LO})$

$\not\chi\text{EFT } (\text{N}^2\text{LO})$

对比不同核力模型下 δ_{pol} 计算结果的差异, 分析核理论误差

CJ, Nevo-Dinur, Bacca, Barnea, [PRL 111 \(2013\) 143402](#)

Hernandez, CJ, Bacca, Nevo-Dinur, Barnea, [PLB 736 \(2014\) 344](#)

Nevo Dinur, CJ, Bacca, Barnea, [PLB 755 \(2016\) 380](#)

Hernandez, Ekström, Nevo Dinur, CJ, Bacca, Barnea, [PLB 788 \(2018\) 377](#)

CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, [JPG 45 \(2018\) 093002](#)

Hernandez, Ekström, Nevo, CJ, Bacca, Barnea, [PLB 788 \(2018\) 377](#)

Nevo, Hernandez, Bacca, Barnea, CJ, Pastore, Piarulli, Wiringa, [PRC 99 \(2019\) 034004](#)

Emmons, CJ, Platter, [JPG 48 \(2021\) 035101](#)

CJ, Zhang, Platter, [PRL 133 \(2024\) 042502](#)

有效相互作用超球谐基展开方法

- 质心系下求解三体薛定谔方程

$$[T + V]\psi(\vec{\eta}_1, \vec{\eta}_2) = E\psi(\vec{\eta}_1, \vec{\eta}_2)$$

- 超球坐标

$$\rho = \sqrt{\eta_1^2 + \eta_2^2}, \Omega = [\theta_1, \phi_1, \theta_2, \phi_2, \arctan(\frac{\eta_2}{\eta_1})]$$

$$T = T_\rho + \hat{K}^2/\rho^2$$

- 超球谐基展开

$$\psi(\vec{\eta}_1, \vec{\eta}_2) \sim \sum_{[K]}^{K_{max}} R_{[K]}(\rho) \mathcal{Y}_{[K]}(\Omega)$$

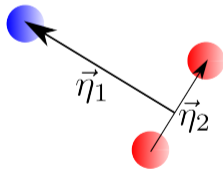
$$\hat{K}^2 \mathcal{Y}_{[K]}(\Omega) = K(K+4) \mathcal{Y}_{[K]}(\Omega)$$

- 哈密顿量有限基展开 $P: K = \{0, 1, \dots, K_{max}\}$

$$H = PH_0P + PV^{eff}P$$

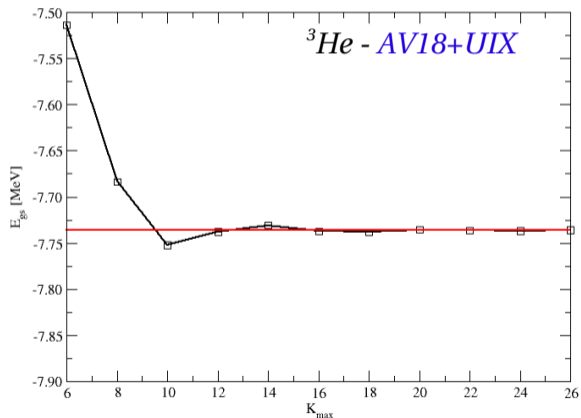
V^{eff} Lee-Suzuki变换软化核力

三体系统



超球谐基展开下束缚态收敛性

$$|\psi\rangle = \sum_K^{K_{max}} c_K \text{HH}(K)$$



广义Lanczos求和规则

- $\delta_{\text{pol}} \implies$ 光核响应函数求和规则利用直接对角化求和计算收敛很慢

$$I_{\hat{O}} = \int_0^{\infty} d\omega S_{\hat{O}}(\omega) g(\omega) \approx \sum_{n \neq 0}^{N_{\text{max}}} |\langle 0 | \hat{O} | n \rangle|^2 g(\omega_n)$$

- 利用Lanczos方法将 $I_{\hat{O}}$ 计算投影到有效子空间求和，同时避免直接计算响应函数 $S_{\hat{O}}(\omega)$
- 哈密顿量映射至Krylov子空间

$$V = \{|\phi_0\rangle, |\phi_1\rangle, \dots, |\phi_M\rangle\}$$

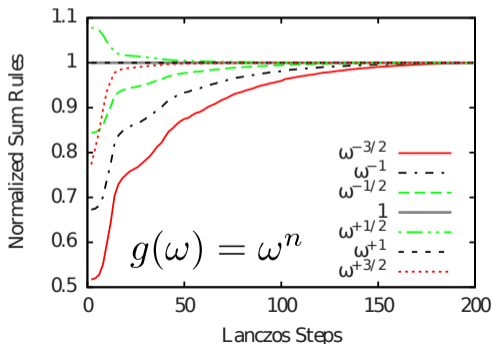
$$b_{i+1}|\phi_{i+1}\rangle = \hat{H}|\phi_i\rangle - a_i|\phi_i\rangle - b_i|\phi_{i-1}\rangle$$

$$|\phi_{-1}\rangle = 0; \quad |\phi_0\rangle = \frac{\hat{O}|0\rangle}{\sqrt{\langle 0|\hat{O}^\dagger\hat{O}|0\rangle}}; \quad \langle \phi_i|\phi_j\rangle = \delta_{ij}$$

$$\tilde{H}_M = QDQ^\dagger; \quad D = \text{diag}\{E_0, E_1, \dots, E_\nu, \dots, E_M\}$$

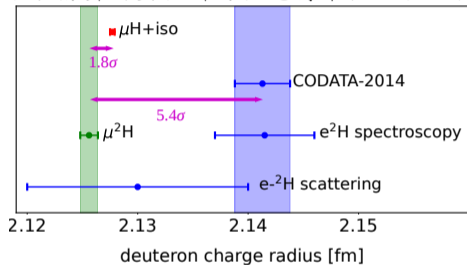
- $I_{\hat{O}}$ 随Lanczos迭代步数增加逐渐收敛

$$I_{\hat{O}} \approx \langle 0|\hat{O}^\dagger\hat{O}|0\rangle \sum_{\nu \neq 0}^M |Q_{\nu 0}|^2 g(\omega_\nu); \quad M \ll N_{\text{max}}$$



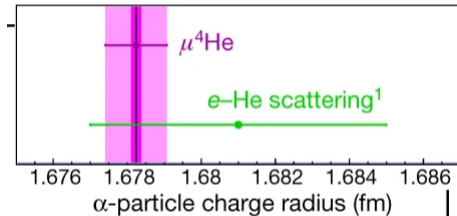
由 μ 原子兰姆位移测量核电荷半径

- 核极化理论预测被PSI实验用于提取核半径
- 原子精密谱实验所得核电荷半径误差由核极化效应理论误差主导



$$r_d = 2.12562(13)_{\text{exp}}(77)_{\text{theo}} \text{ fm}$$

Pohl, *et al.*, *Science* (2016)



$$r_\alpha = 1.67824(13)_{\text{exp}}(82)_{\text{theo}} \text{ fm}$$

Krauth *et al.*, *Nature* (2021)

核极化理论:

Hernandez, CJ, Bacca, Nevo-Dinur, Barnea, *PLB* 736 (2014) 344; *PRC* 100 (2019) 064315 ($\mu^2\text{H}$)

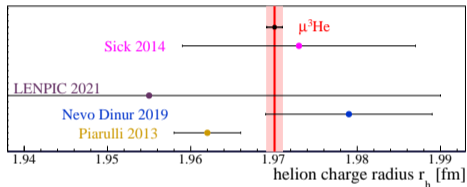
Hernandez, Ekström, Nevo Dinur, CJ, Bacca, Barnea, *PLB* 788 (2018) 377 ($\mu^2\text{H}$)

CJ, Nevo-Dinur, Bacca, Barnea, *PRL* 111 (2013) 143402 ($\mu^4\text{He}^+$)

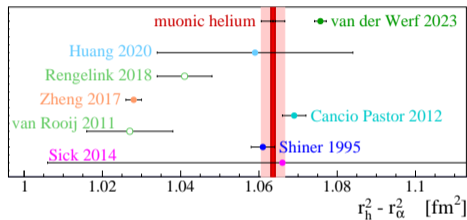
CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, *JPG* 45 (2018) 093002 ($\mu^{2,3}\text{H}$, $\mu^{3,4}\text{He}^+$)

由 μ 原子兰姆位移测量核电荷半径

- 核极化理论预测被PSI实验用于提取核半径
- 原子精密谱实验所得核电荷半径误差由核极化效应理论误差主导



$$r_h = 1.97007(12)_{\text{exp}}(93)_{\text{theo}} \text{ fm}$$



$$r_h^2 - r_\alpha^2 = 1.0636(6)_{\text{exp}}(30)_{\text{theo}} \text{ fm}^2$$

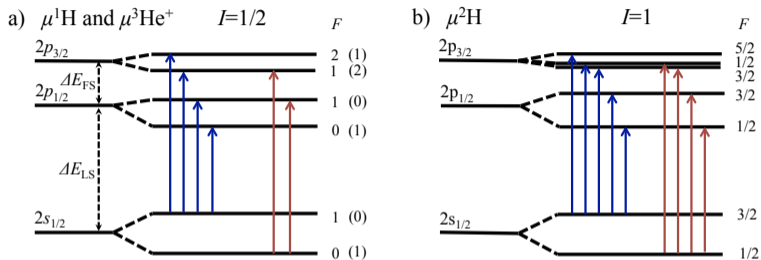
Schuhmann et al. (CREMA) arXiv:2305.11679

核极化理论:

Nevo Dinur, CJ, Bacca, Barnea, *PLB* 755 (2016) 380 ($\mu^3\text{H}$, $\mu^3\text{He}^+$)

CJ, Bacca, Barnea, Hernandez, Nevo-Dinur, *JPG* 45 (2018) 093002 ($\mu^{2,3}\text{H}$, $\mu^{3,4}\text{He}^+$)

μ 原子超精细劈裂谱测量核Zemach半径



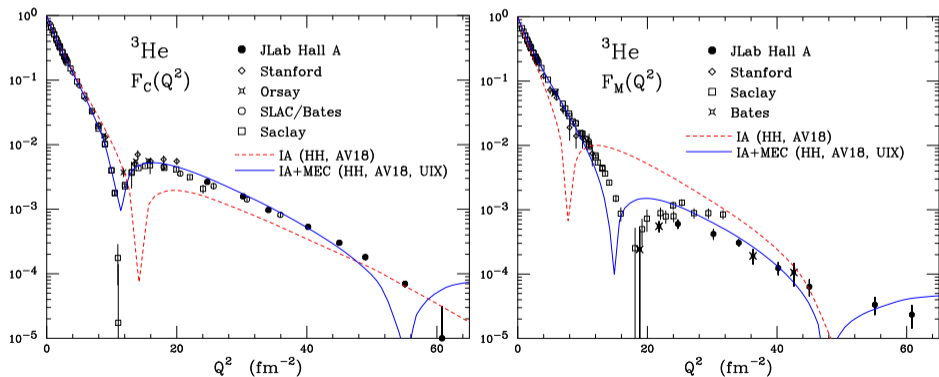
- Zemach半径 R_Z 由核电荷分布与磁矩分布共同决定

$$R_Z = \iint d\mathbf{r}d\mathbf{r}' \rho_E(\mathbf{r}) \rho_M(\mathbf{r}') |\mathbf{r} - \mathbf{r}'|$$

- μH 超精细结构: CREMA、J-PARC、RIKEN-RAL
- $\mu^3\text{He}^+$ 超精细结构: CREMA

核磁矩分布研究存在分歧

- ^3He 电荷形状因子 F_C : 实验与理论一致
- ^3He 磁形状因子 F_M : 实验与理论在高动量处存在分歧
- 缪原子谱学为研究原子核磁矩分布提供更精确的测量



超精细劈裂谱中的核结构效应

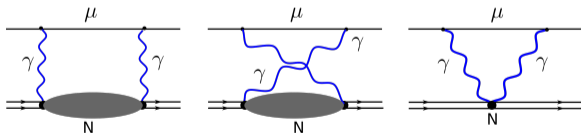
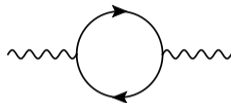
- 从 e/μ 原子超精细劈裂提取核Zemach半径

$$E_{\text{HFS}}(nS) = E_F(1 + \delta_{\text{QED}} + \delta_{\text{TPE}})$$

- Fermi接触项
 - 核-轻子自旋耦合

$$E_F = \frac{2\pi\alpha g_m}{3m_\ell m_N} \phi_n^2(0) \langle \vec{\sigma}^{(\ell)} \cdot \vec{I} \rangle$$

- 量子电动力学修正
 - 真空极化效应
 - 轻子自能修正
 - 相对论反冲修正
- 双光子交换 (核结构效应)
 - 核极化
 - 弹性项 (Zemach 半径)



氘原子超精细劈裂谱中核结构效应

- 双光子交换项主导了超精细劈裂实验测量与QED理论预测的偏差
- $e^2\text{H}$ 超精细谱中的核结构效应：理论与实验吻合（争议性）
- $\mu^2\text{H}$ 超精细谱中的核结构效应：理论与实验有很大分歧

$e^2\text{H } 1\text{S } E_{HFS}(2\gamma)$ [kHz]

$\nu_{\text{exp}} - \nu_{\text{qed}}$	45 [1]
Khriplovich, Milstein 2004	43 (model dependent)
Friar 2005	46 (+18)
	(1N pol/recoil)

$\mu^2\text{H } 2\text{S } E_{HFS}(2\gamma)$ [meV]

$\nu_{\text{exp}} - \nu_{\text{qed}}$	0.0966(73) [2]
Kalinowski, Pachucki 2018	0.0383

[1] Wineland, Ramsey, PRA (1972)

[2] Pohl et al., Science (2016)

氘原子超精细劈裂谱中核结构效应

- 双光子交换项主导了超精细劈裂实验测量与QED理论预测的偏差
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$e^2\text{H } 1\text{S } E_{HFS}(2\gamma) \text{ [kHz]}$

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	(1N pol/recoil)

$\mu^2\text{H } 2\text{S } E_{HFS}(2\gamma) \text{ [meV]}$

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[1] Wineland, Ramsey, PRA (1972)

[2] Pohl et al., Science (2016)

- 原有理论未能严格处理核激发（极化项）

PHYSICAL REVIEW LETTERS **133**, 042502 (2024)

Nuclear Structure Effects on Hyperfine Splittings in Ordinary and Muonic Deuterium

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²Southern Center for Nuclear-Science Theory, Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, China

³Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA

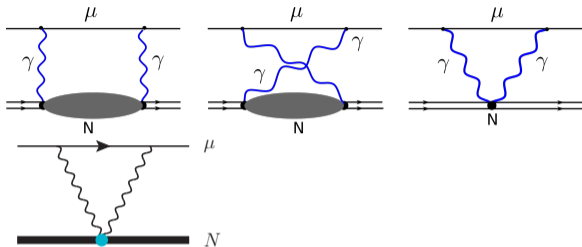
⁴Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

^2H and $\mu^2\text{H}$ 超精细劈裂谱中的双光子修正

- 双光子修正

$$E_{\text{TPE}} = E_{\text{el}} + E_{\text{pol}} + E_{1\text{N}}$$

- 弹性部分 $F_c(q)$, $F_m(q)$, $F_Q(q)$: $\sim r_Z$
- 非弹性矢量极化
- $E_{1\text{N}}$: 轻子-单核子双光子交换 (核子结构)



$$\delta_{\text{pol}}^{(0,1)} \propto \int d\omega \int dq h^{(0,1)}(\omega, q) S^{(0,1)}(\omega, q)$$

$$\text{电荷-磁流耦合: } S^{(0)}(\omega, q) = -\frac{1}{q^2} \text{Im} \sum_{N \neq N_0} \int \frac{d\hat{q}}{4\pi} \langle N_0 II | [\vec{q} \times \vec{J}_m^\dagger(\vec{q})]_3 | N \rangle \langle N | \rho(\vec{q}) | N_0 II \rangle \delta(\omega - \frac{q^2}{2m_A} - \omega_N)$$

$$\text{运流电流-磁流耦合: } S^{(1)}(\omega, q) = -\text{Im} \sum_{N \neq N_0} \int \frac{d\hat{q}}{4\pi} \epsilon^{3jk} \langle N_0 II | \vec{J}_{m,j}^\dagger(\vec{q}) | N \rangle \langle N | \vec{J}_{c,k}(\vec{q}) | N_0 II \rangle \delta(\omega - \frac{q^2}{2m_A} - \omega_N)$$

- 无 π 介子有效场论计算 [CJ*](#), Zhang, Platter, Phys. Rev. Lett. 133, 042502 (2024)

无 π 介子有效场理论计算核极化效应

- 无 π 介子有效场论通过接触势构建核子间低能相互作用
- 理论预测仅需要极少的输入参数: a_t, r_t, Q_d at NNLO (5% 精度)

$$\begin{aligned} \mathcal{L} = & N^\dagger \left[i\partial_0 + \frac{\nabla^2}{2M} \right] N - C_0 \left(N^T P_i N \right)^\dagger \left(N^T P_i N \right) \\ & + \frac{1}{8} C_2 \left[\left(N^T P_i N \right)^\dagger \left(N^T \nabla^2 P_i N \right) + h.c. \right] - \frac{1}{16} C_4 \left(N^T \nabla^2 P_i N \right)^\dagger \left(N^T \nabla^2 P_i N \right) \\ & + \frac{1}{4} C_0^{(sd)} \left\{ \left(N^T P^i N \right)^\dagger \left[N^T P^j \left(\nabla_i \nabla_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) N \right] + h.c. \right\} \end{aligned}$$

- np散射t-矩阵 \mathcal{A}_n 逐阶展开:

$$\mathcal{A}_0 = \text{[Contact diagram]} + \text{[Bubble diagram]} + \dots$$

$$\mathcal{A}_1 = \text{[Two contact diagrams]}$$

$$\mathcal{A}_2 = \text{[Two diagrams with two contact vertices]} + \text{[Diagram with one contact vertex and a bubble]}$$

$$\text{[Contact diagram]} = \text{[Bubble diagram]} + \text{[Crossing diagram]} + \text{[Bubble diagram]} + \dots$$

- on-shell t-matrix

$$\mathcal{A}_t(p, p; E) = -\frac{4\pi}{m_N} \frac{1}{\gamma + ip} \left[1 + \rho(\gamma - ip)/2 + \rho^2(\gamma - ip)^2/4 \right]$$

- off-shell t-matrix

$$\mathcal{A}_t^{(0)}(k, p; E) = -\frac{4\pi}{m_N} \frac{1}{\gamma + ip}$$

$$\mathcal{A}_t^{(1)}(k, p; E) = -\frac{2\pi}{m_N} \frac{\rho}{\gamma + ip} \left[\gamma - ip + \frac{1}{2(\gamma - \mu)} (k^2 - p^2) \right]$$

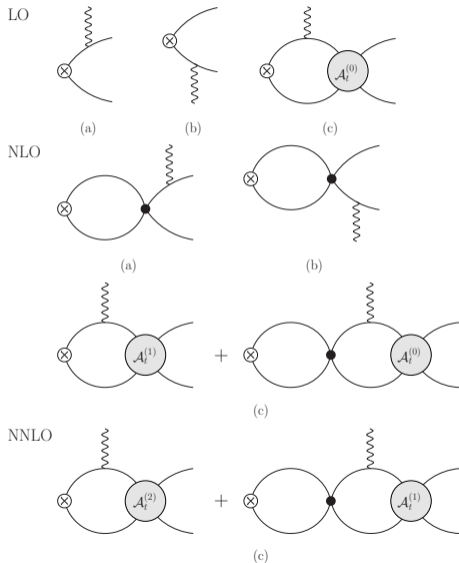
$$\mathcal{A}_t^{(2)}(k, p; E) = -\frac{\pi}{m_N} \frac{\rho^2}{\gamma + ip} \left[(\gamma - ip)^2 + \frac{\gamma - ip}{\gamma - \mu} \left(1 + \frac{\gamma + ip}{\gamma - \mu} \right) \frac{k^2 - p^2}{2} \right]$$

^2H and $^2\mu\text{H}$ 超精细劈裂谱中的双光子修正

- 核激发模式中的单核子电磁流贡献：

核电荷密度 ρ_E ，对流电流 \vec{J}_C ，磁流 \vec{J}_m 均对超精细谱产生核修正

$$\begin{aligned} \mathcal{L}_{\text{EM},1b} = & -eN^\dagger \frac{1+\tau_3}{2} N A_0 \\ & - \frac{ie}{2m_N} \left[N^\dagger \overleftrightarrow{\nabla} \frac{1+\tau_3}{2} N \right] \cdot \vec{A} \\ & + \frac{e}{2m_N} N^\dagger (\kappa_0 + \kappa_1 \tau_3) \vec{\sigma} \cdot \vec{B} N \end{aligned}$$

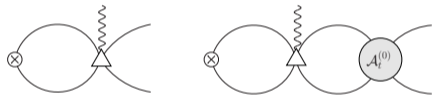


^2H and $^2\mu\text{H}$ 超精细劈裂谱中的双光子修正

- 双核子对流电流、磁流（介子交换流）修正

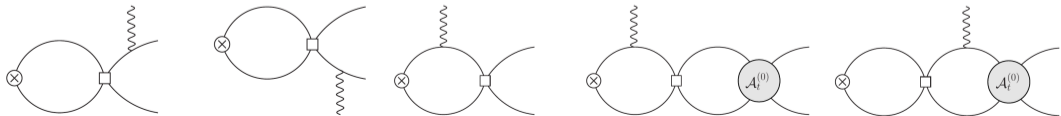
$$\mathcal{L}_{2,C} = ie \frac{C_2}{4} \left[(N^T P_i N)^\dagger (N^T \overleftrightarrow{\nabla} P_i \tau_3 N) + \text{h.c.} \right] \cdot \vec{A}$$

$$\mathcal{L}_{2,B} = -ie L_2 \epsilon_{ijk} (N^T P_i N)^\dagger (N^T P_j N) B_k + \text{h.c.}$$



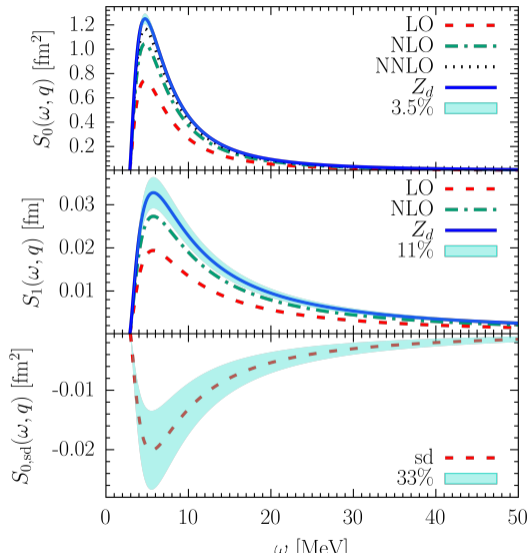
- SD耦合修正

$$\mathcal{L}_{2,Q} = -e L_Q (N^T P_i N)^\dagger (N^T P_j N) \left(\nabla^i \nabla^j - \frac{1}{3} \nabla^2 \delta_{ij} \right) A_0$$



超精细劈裂双光子修正中的核极化响应函数

- $S^{(0)}(\omega, q)$: 电荷密度-磁流跃迁 (LO)
- $S^{(1)}(\omega, q)$: 对流电流-磁流跃迁 (NLO)
- $S_{sd}^{(0)}(\omega, q)$: SD耦合修正 (NNLO)
- 核响应函数计算结果逐阶收敛



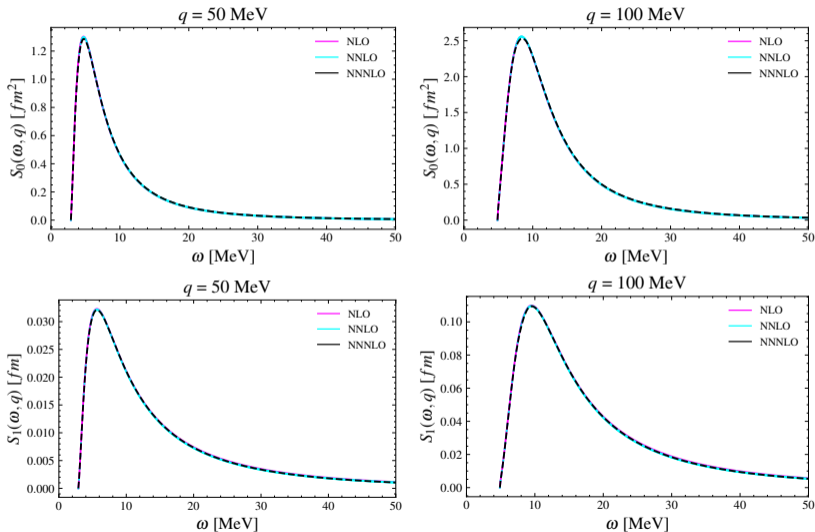
^2H and $\mu^2\text{H}$ 超精细劈裂中的双光子修正 (无 π 介子有效场核力)

	^2H (1S)	$\mu^2\text{H}$ (1S)	$\mu^2\text{H}$ (2S)
E_{1p} (Antognini 2022)	-35.54(8)	-1.018(2)	-0.1272(2)
E_{1n} (Tomalak 2019)	9.6(1.0)	0.08(3)	0.010(4)
E_{el}	-42.1(2.1)	-0.984(46)	-0.123(6)
E_{pol}	109.8(4.5)	2.86(12)	0.358(14)
E_{TPE}	kHz	meV	meV
This work	41.7(4.4)	0.94(11)	0.117(13)
Khriplovich, Milstein 2004	43		
Friar, Payne 2005 _{mod}	64.5		
Kalinowskim, Pauckci 2018		0.304(68)	0.0383(86)
$\nu_{exp} - \nu_{qed}$	45.2		0.0966(73)

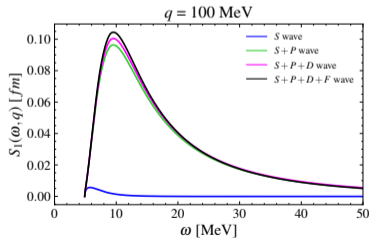
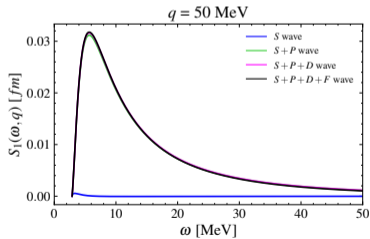
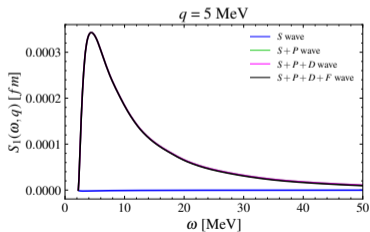
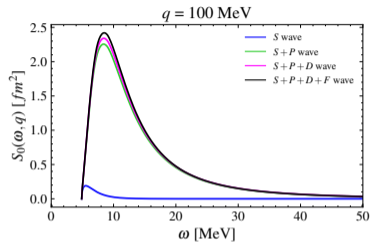
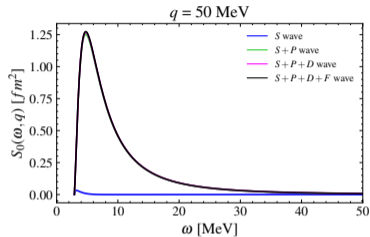
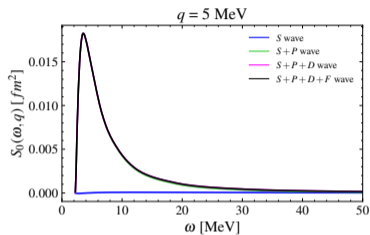
- 计算结果较好地解释实验-QED理论偏差 ($0.8 - 1.3\sigma$)
- 单核子修正的理论误差可能被低估 (手征微扰论与色散关系计算相差10倍)

[CJ*](#), Zhang, Platter, Phys. Rev. Lett. 133, 042502 (2024)

手征核力计算核响应函数（各阶贡献）



手征核力计算核响应函数 (各分波贡献)



^2H and $\mu^2\text{H}$ 超精细劈裂中的核极化贡献 (手征核力)

Hyperfine Energy Shift E_{pol}			
	$^2\text{H}(1S)$	$\mu^2\text{H}(1S)$	$\mu^2\text{H}(2S)$
	kHz	meV	meV
NLO _{RS450}	109.5	2.819	0.352
N2LO _{RS450}	109.4	2.816	0.352
N3LO _{Idaho}	110.2	2.834	0.354
Pionless EFT	109.8(4.5)	2.86(12)	0.358(14)

- 初步完成手征核力对HFS中核极化贡献的计算
- 与无 π 介子有效场计算结果一致

A>3 原子

- 双光子交换对原子超精细劈裂修正理论存在空缺
 - 核极化效应随 Z 和 A 变化的趋势不明
 - 高阶电磁作用算符的贡献可能更重要
- 结合HFS实验—QED—电子散射数据:
 - 提取核极化贡献: ${}^3\text{He} \ll {}^2\text{H}$; ${}^7\text{Li} \ll {}^6\text{Li}$
 - 核极化作用机制不清

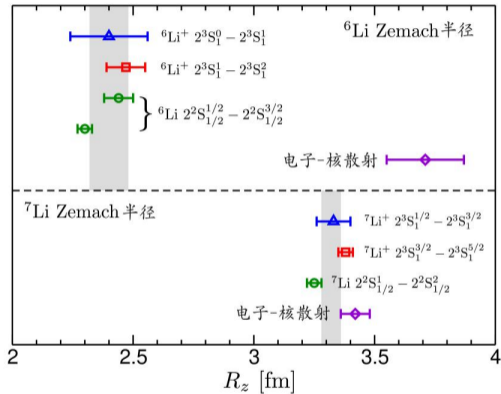
PHYSICAL REVIEW A **107**, 052802 (2023)

3He超精细劈裂

Nuclear polarizability effects in ${}^3\text{He}^+$ hyperfine splitting

Vojtěch Patkóš¹, Vladimir A. Yerokhin² and Krzysztof Pachucki³

We obtain a surprising result that the nuclear polarizability of the helion yields just 3% of the total nuclear correction, which is smaller than for the proton.



6Li与7Li超精细劈裂

Puchalski, Pachucki, PRL 111, 243001 (2013)
Qi et al., PRL 125, 183002 (2020)
Li et al., PRL 124, 063002 (2020)

惠州大科学装置缪原子精密谱设想

● 物理设想：

- 高强度缪子源，支持高精度缪原子谱学研究
- 研究核电荷/磁矩分布
- 研究核极化效应，对比康普顿散射实验，验证手征微扰理论
- 验证束缚态QED
- 探索对称性破缺

● 技术设想：

- 借鉴PSI-CREMA激光谱学与QUARTET X射线技术。
- 借鉴 μ X实验，开发微克级靶材制备，适配稀有同位素研究。

总结

- 质子半径难题与原子精密谱
 - 向轻子普适性与高阶QED理论提出挑战
 - 核极化效应连接光核反应与原子谱
 - 由低能核结构理论探究精密物理
- 通过第一性原理计算原子谱中核效应
 - 双光子交换对 μ 原子兰姆位移谱的修正
 - 双光子交换对 $e^2\text{H}$ 、 $\mu^2\text{H}$ 超精细劈裂谱的修正
 - 将理论输入提高到百分比的精度，比由光核反应数据提取的核极化结果更加精确
- 惠州大科学装置为通过缪原子精密谱研究原子核结构提供新的机遇