第三届惠州大装置高精度物理研讨会

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中科院近代物理所惠州研究部

Physics of $\eta^{(\prime)}$ rare decays

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Based on the excellent review article (materials taken from this paper unless otherwise mentioned, check it for original references):

L. Gan, B. Kubis, E. Passemar, and S. Tulin, *Precision tests of fundamental physics with η and η' mesons*, Phys.Rept. 945 (2022) 1 [arXiv:2007.00664]

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FKG, B. Kubis, A. Wirzba, *Anomalous decays of η' and η into four pions*, Phys.Rev. D 85 (2012) 014014;

A. Guevara, FKG, H.-J. Jing, *Novel method for determining the light quark mass ratio using $\eta'\to\eta\pi\pi$ decays*, arXiv:2502.02837

η, η'

- Symmetries of QCD:
  - Gauge, Lorentz, C
  - P, CP if there is no θ term
  - Chiral symmetry:

$$U(N)_L \times U(N)_R = SU(N)_L \times SU(N)_R \times U(1)_V \times U(1)_A$$
$$\rightarrow SU(N)_V \times U(1)_V$$

  Spontaneous symmetry breaking, chiral anomaly

- Goldstone bosons of $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$
  - $\pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0, \eta$
  - η' is not a Goldstone boson because of the $U(1)_A$ anomaly
  - U(1) anomaly suppressed in the large N_c limit, 9 Goldstone bosons

- Theory frameworks well defined:
  - (Large N_c) chiral perturbation theory (ChPT)
  - Dispersion relations: $\pi\pi$ scattering phase shifts well-known
  - Sometimes with vector-meson dominance model

η, η'

- Quantum numbers: $I^G J^{PC} = 0^+ 0^{-+}$
- C, P, CP eigenstates: can be used to test C and CP invariance, search for class-II interaction

Class	Violated	Conserved	Interaction
0		C, P, T, CP, CT, PT, CPT	strong, electromagnetic
I	C, P, CT, PT	T, CP, CPT	(weak, with no KM phase or flavor-mixing)
II	P, T, CP, CT	C, PT, CPT	
III	C, T, PT, CP	P, CT, CPT	
IV	C, P, T, CP, CT, PT	CPT	weak

- For η, $M_\eta = 547.862 \pm 0.017$ MeV, all its strong decays are forbidden at the lowest order by angular momentum conservation, C, P and CP invariance, and G-parity (isospin) conservation: tiny width $\Gamma_\eta = 1.31 \pm 0.05$ keV
- For η', $M_{\eta'} = 957.78 \pm 0.06$ MeV, has isospin-conserving decays into $\eta\pi\pi$, $\Gamma_{\eta'} = 196 \pm 9$ keV
- All additive quantum numbers are zero: flavor-conserving lab for new-physics search, little SM background

Physics of η, η'

- Precision tests of the Standard Model
  - CHPT, chiral anomaly
  - Fundamental parameters: quark mass ratio related to $m_d - m_u$
  - Contribution of η, η' to the hadronic light-by-light contribution to the muon $g-2$
  - Dynamics in the scalar sector: $f_0(500), a_0(980)$
- Beyond the Standard Model
  - New source of C/CP violation
  - Search for new light particles
    - dark photon, protophobic X boson, leptophobic $U(1)_B$ B boson
    - light Higgs-like scalars
    - axion-like particles (ALPs)
  - Weak decays BSM
  - Input for precision calculation of the QCD axion $a \to \gamma\gamma$

Physics of η decays

Channel	Expt. branching ratio	Discussion	Sect.
$\eta \to 2\gamma$	39.41(20)%	chiral anomaly, η–η' mixing	6.1
$\eta \to 3\pi^0$	32.68(23)%	$m_u - m_d$	5.1
$\eta \to \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χPT at $O(p^6)$, leptophobic B boson, light Higgs scalars	7, 10.1, 10.2
$\eta \to \pi^0\pi^0\gamma\gamma$	$< 1.2 \times 10^{-3}$	χPT, axion-like particles (ALPs)	10.3
$\eta \to 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$ [52]	
$\eta \to \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars	5.1, 9.2, 10.2
$\eta \to \pi^+\pi^-\gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation	6.3, 9.1
$\eta \to \pi^+\pi^-\gamma\gamma$	$< 2.1 \times 10^{-3}$	χPT, ALPs	10.3
$\eta \to e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g-2)_\mu$, dark photon, protophobic X boson	6.4, 10.1
$\eta \to \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon	6.4, 10.1
$\eta \to e^+e^-$	$< 7 \times 10^{-7}$	theory input for $(g-2)_\mu$, BSM weak decays	6.9, 8
$\eta \to \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation	6.9, 8, 9.1
$\eta \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs	9.2, 10.3
$\eta \to \pi^+\pi^-e^+e^-$	$2.68(11) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs	6.6, 9.1, 10.3
$\eta \to \pi^+\pi^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs	6.6, 9.1, 10.3
$\eta \to e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	theory input for $(g-2)_\mu$	6.7
$\eta \to e^+e^-\mu^+\mu^-$	$< 1.6 \times 10^{-4}$	theory input for $(g-2)_\mu$	6.7
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for $(g-2)_\mu$	6.7
$\eta \to \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	direct emission only	6.8
$\eta \to \pi^\pm e^\mp \nu_e$	$< 1.7 \times 10^{-4}$	second-class current	8
$\eta \to \pi^+\pi^-$	$< 4.4 \times 10^{-6}$ [53]	P/CP violation	9.1
$\eta \to 2\pi^0$	$< 3.5 \times 10^{-4}$	P/CP violation	9.1
$\eta \to 4\pi^0$	$< 6.9 \times 10^{-7}$	P/CP violation	6.5, 9.1

L. Gan, B. Kubis, E. Passemar, S. Tulin, arXiv:2007.00664

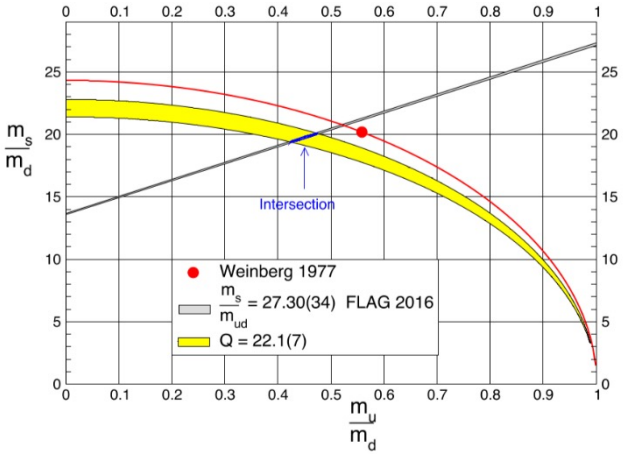
Physics of η' decays

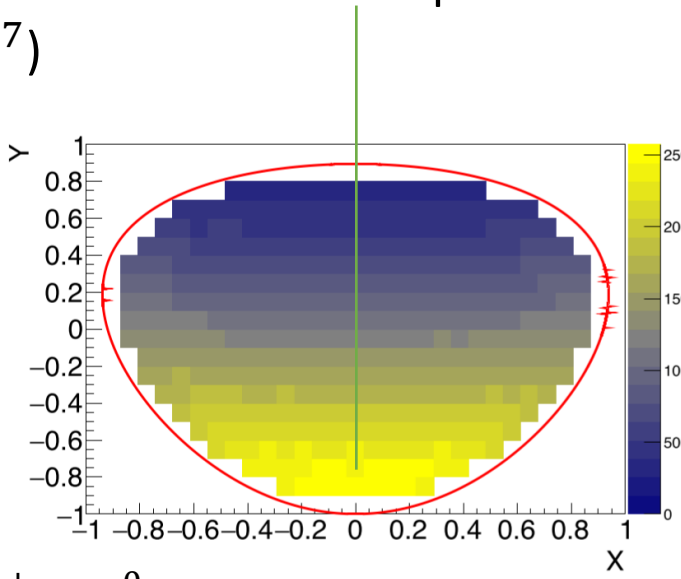
Channel	Expt. branching ratio	Discussion	Sect.
$\eta' \to \eta\pi^+\pi^-$	42.6(7)%	large-N_c χPT, light Higgs scalars	5.2, 10.2
$\eta' \to \pi^+\pi^-\gamma$	28.9(5)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation	6.3, 9.1
$\eta' \to \eta\pi^0\pi^0$	22.8(8)%	large-N_c χPT	5.2
$\eta' \to \omega\gamma$	2.489(76)% [55]	theory input for singly-virtual TFF and $(g-2)_\mu$	6.8
$\eta' \to \omega e^+e^-$	$2.0(4)\times10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$	6.8
$\eta' \to 2\gamma$	2.331(37)% [55]	chiral anomaly, η–η' mixing	6.1
$\eta' \to 3\pi^0$	2.54(18)% $^{(\star)}$	$m_u - m_d$	5.3
$\eta' \to \mu^+\mu^-\gamma$	$1.09(27)\times10^{-4}$	theory input for $(g-2)_\mu$, dark photon	6.4, 10.1
$\eta' \to e^+e^-\gamma$	$4.73(30)\times10^{-4}$	theory input for $(g-2)_\mu$, dark photon	6.4, 10.1
$\eta' \to \pi^+\pi^-\mu^+\mu^-$	$< 2.9\times10^{-5}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, dark photon, ALPs	6.6, 9.1, 10.1, 10.3
$\eta' \to \pi^+\pi^-e^+e^-$	$2.4(^{+1.3}_{-1.0})\times10^{-3}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, dark photon, ALPs	6.6, 9.1, 10.1, 10.3
$\eta' \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs	9.2, 10.3
$\eta' \to \pi^+\pi^-\pi^0$	$3.61(17)\times10^{-3}$	$m_u - m_d$, C/CP violation, light Higgs scalars	5.3, 9.2, 10.2
$\eta' \to 2(\pi^+\pi^-)$	$8.4(9)\times10^{-5}$	theory input for doubly-virtual TFF and $(g-2)_\mu$	6.5
$\eta' \to \pi^+\pi^-2\pi^0$	$1.8(4)\times10^{-4}$		6.5
$\eta' \to 2(\pi^+\pi^-)\pi^0$	$< 1.8\times10^{-3}$	ALPs	10.3
$\eta' \to K^\pm\pi^\mp$	$< 4\times10^{-5}$	weak interactions	8
$\eta' \to \pi^\pm e^\mp\nu_e$	$< 2.1\times10^{-4}$	second-class current	8
$\eta' \to \pi^0\gamma\gamma$	$3.20(24)\times10^{-3}$	vector and scalar dynamics, B boson, light Higgs scalars	7.4, 10.1, 10.2
$\eta' \to \eta\gamma\gamma$	$8.3(3.5)\times10^{-5}$ [56]	vector and scalar dynamics, B boson, light Higgs scalars	7.4, 10.1, 10.2
$\eta' \to 4\pi^0$	$< 4.94\times10^{-5}$ [57]	(S-wave) P/CP violation	6.5
$\eta' \to e^+e^-$	$< 5.6\times10^{-9}$	theory input for $(g-2)_\mu$, BSM weak decays	6.9, 8
$\eta' \to \mu^+\mu^-$		theory input for $(g-2)_\mu$, BSM weak decays	6.9, 8
$\eta' \to \ell^+\ell^-\ell^+\ell^-$		theory input for $(g-2)_\mu$	6.7
$\eta' \to \pi^+\pi^-\pi^0\gamma$		B boson	10.1
$\eta' \to \pi^+\pi^-$	$< 1.8\times10^{-5}$	P/CP violation	9.1
$\eta' \to 2\pi^0$	$< 4\times10^{-4}$	P/CP violation	9.1

L. Gan, B. Kubis, E. Passemar, S. Tulin, arXiv:2007.00664

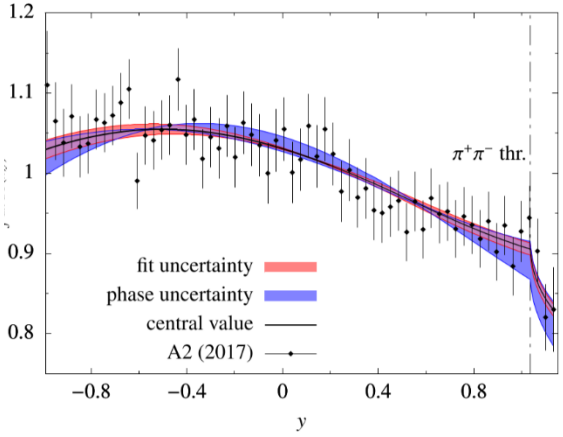
η, η' decays

- $\eta \to 3\pi$:
  - Isospin breaking, $\propto (m_d - m_u)$, em correction negligible
  - quark mass ratios, together with lattice results of m_s/m_{ud}; $m_u \neq 0$
  - C violation from the $\pi^+\pi^-$ asymmetry in the Dalitz distribution
  - $\pi^+\pi^-$ threshold cusps in $\eta \to 3\pi^0$ and $\eta' \to \eta\pi^0\pi^0$, probe the $\pi\pi$ scattering length, but the current data cannot compete with the $K^+ \to \pi^+\pi^0\pi^0$ decay (NA48/2: 6.031×10^7)





$\eta \to \pi^+\pi^-\pi^0$



$\eta' \to \eta\pi^0\pi^0$

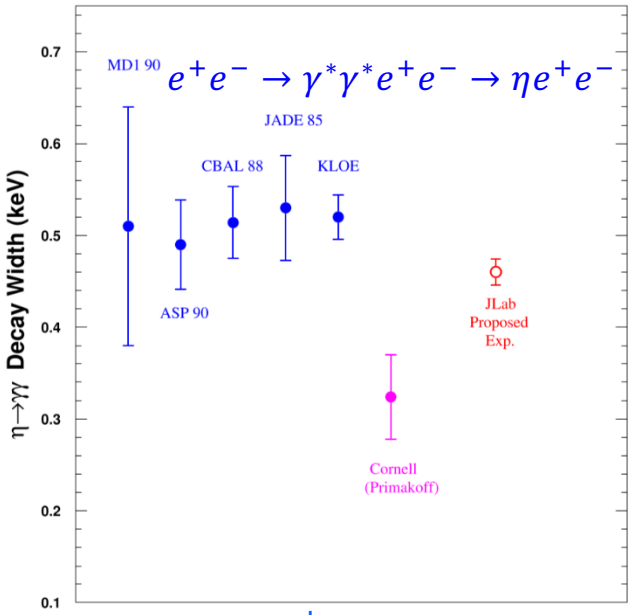
Two-photon decays

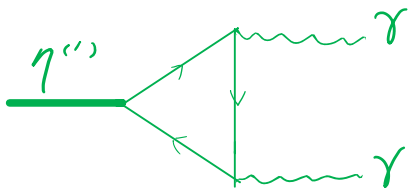
- $\eta^{(\prime)} \to \gamma\gamma$: chiral anomaly

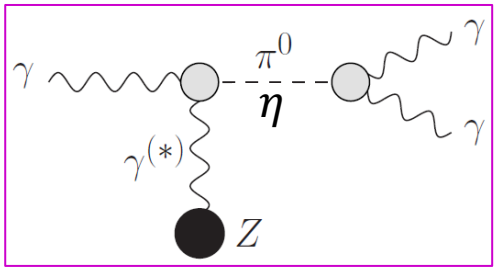
$$\Gamma(P \to \gamma\gamma) = \frac{\pi \alpha_{\rm em}^2 M_P^3}{4} \left| F_{P\gamma\gamma} \right|^2$$

$$F_{\eta\gamma\gamma} = \frac{1}{12\pi^2}\left[\frac{5\cos\phi}{F_q} - \frac{\sqrt{2}\sin\phi}{F_s}\right],$$

$$F_{\eta'\gamma\gamma} = \frac{1}{12\pi^2}\left[\frac{5\sin\phi}{F_q} + \frac{\sqrt{2}\cos\phi}{F_s}\right]$$







- Discrepancy between measurements from e^+e^- and Primakoff measurements
- Together with the branching fraction, affect the values of partial decay widths of the other channels
- Measuring the $\eta - \eta'$ mixing
- Determination of $O(p^6)$ anomalous LECs C_7^W and C_8^W, input of precision calculation of QCD axion decay $a \to \gamma\gamma$

$$g_{a\gamma\gamma} = \frac{\alpha_{\rm em}}{2\pi f_a}\left(\frac{\mathcal{E}}{\mathcal{C}}\boxed{-2.05(3)}\right)$$ using only $\eta \to \gamma\gamma$ assuming $|C_7^W| \ll |C_8^W|$ (Kampf, Mussallam, PRD79,076005)

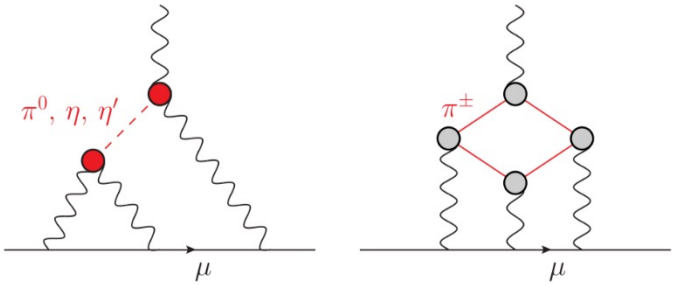
Z.-Y. Lu, M.-L. Du, FKG, U.-G. Meißner, T. Vonk, JHEP05(2020)001

$\boxed{-1.92(4)}$ using both $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$, leading to $|C_7^W| \gg |C_8^W|$

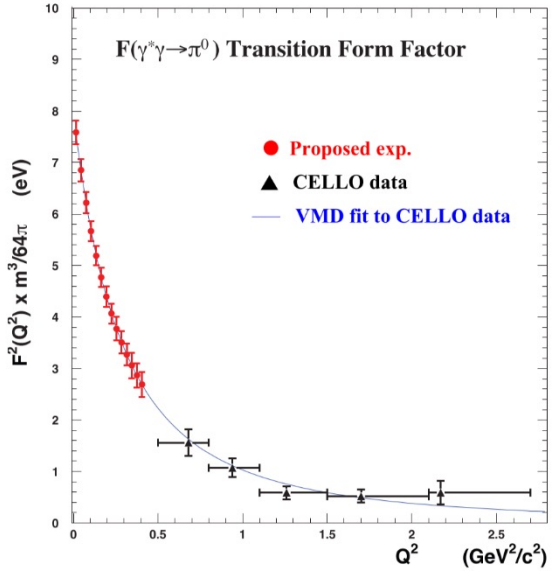
G.G. di Cortona et al., JHEP01(2016)034

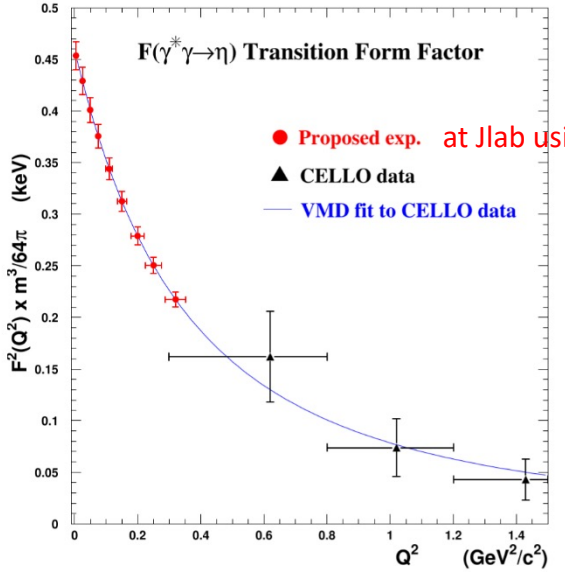
Transition form factors (TFFs)

- TFF: $F_{\eta\gamma\gamma}(Q_1^2, Q_2^2)$, $F_{\eta\gamma\gamma} = F_{\eta\gamma\gamma}(0,0)$
- Input to the dispersive analysis of the hadronic light-by-light contribution to $(g-2)_\mu$



- Singly or doubly-virtual TFFs can be measured with Primakoff effect and $\eta^{(\prime)} \to l^+l^-\gamma, \pi^+\pi^-\gamma, l^+l^-l^+l^-, \pi^+\pi^-l^+l^-$





at Jlab using Primakoff eff.

Transition form factors

- Theory predictions of $\pi^0, \eta^{(\prime)} \to l^+l^-l^+l^-$, accessible at the HIAF eta factory (most of them only have upper bounds to date)

$\pi^0 \to 2(e^+e^-)$
3.47×10^{-5} [449]
$3.36689(5) \times 10^{-5}$ [450]
$3.40(1) \times 10^{-5}$ [452]
$3.3919(13) \times 10^{-5}$ [447]

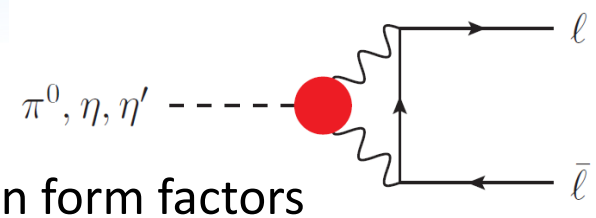
$\eta \to 2(e^+e^-)$	$\eta' \to 2(e^+e^-)$
$2.71(2) \times 10^{-5}$ [450]	$2.10(45) \times 10^{-6}$ [450]
$2.701(14) \times 10^{-5}$ [447]	

$\eta \to e^+e^-\mu^+\mu^-$	$\eta' \to e^+e^-\mu^+\mu^-$
$2.39(7) \times 10^{-6}$ [450]	$6.39(91) \times 10^{-7}$ [450]
$2.335(12) \times 10^{-6}$ [447]	

$\eta \to 2(\mu^+\mu^-)$	$\eta' \to 2(\mu^+\mu^-)$
$3.98(15) \times 10^{-9}$ [450]	$1.69(36) \times 10^{-8}$ [450]
$3.878(20) \times 10^{-9}$ [447]	

实验	η 个数	η’ 个数	备注
KLOE	10^8	0.5 x 10^6	
KLOE - II	3 x 10^8	1.5 x 10^6	
COSY	5 x 10^8	No	
MAMI	2.5 x 10^8	6 x 10^6	
JLab 12	~10^9	~10^9	
BESIII	10^6		arXiv:1709.04627
REDTOP	2 x10^12	?	一年
REDTOP II	~ 10^13		一年
HIAF	10^{12}	10^{11}	一年

Transition form factors



- $\pi^0, \eta^{(\prime)} \to l^+ l^-$
  - Related to the doubly-virtual transition form factors
  - Probe weak decays beyond the SM (Z contributes at permille level, precision measurements)
- Current experimental results on the branching fractions

$\pi^0 \to e^+e^-$	$\eta \to e^+e^-$	$\eta \to \mu^+\mu^-$	$\eta' \to e^+e^-$	$\eta' \to \mu^+\mu^-$
$7.48(38) \times 10^{-8}$ [460]	$\leq 7 \times 10^{-7}$ [461]	$5.8(8) \times 10^{-6}$ [54]	$\leq 5.6 \times 10^{-9}$ [462, 463]	—

- Theory predictions, accessible at eta factory

$\pi^0 \to e^+e^-$	$\eta \to e^+e^-$	$\eta' \to e^+e^-$
$6.23(5) \times 10^{-8}$ [483] $6.22(3) \times 10^{-8}$ [452]	$5.31(^{+0.14}_{-0.04}) \times 10^{-9}$ [472]	$1.81(18) \times 10^{-10}$ [472]
	$\eta \to \mu^+\mu^-$	$\eta' \to \mu^+\mu^-$
	$4.71(^{+0.05}_{-0.21}) \times 10^{-6}$ [472]	$1.36(^{+0.29}_{-0.26}) \times 10^{-7}$ [472]

$\eta^{(\prime)} \to 4\pi$

FKG, B. Kubis, A. Wirzba, PRD85(2012)014014

- $\eta' \to \pi^+\pi^-\pi^+\pi^-, \pi^+\pi^-\pi^0\pi^0$:
  - $O(p^6)$ (NNLO) in CHPT
  - Kaon loop + VMD for $O(p^6)$ counterterm lead to predictions

$$\mathcal{B}(\eta' \to 2(\pi^+\pi^-)) = (1.0 \pm 0.3) \times 10^{-4},$$
$$\mathcal{B}(\eta' \to \pi^+\pi^- 2\pi^0) = (2.4 \pm 0.7) \times 10^{-4}.$$

  - BESIII measurements verified the predictions

BESIII, PRL112(2014)251801

$$\mathcal{B}(\eta' \to 2(\pi^+\pi^-)) = 8.4(9)(3) \times 10^{-5}, \qquad \mathcal{B}(\eta' \to \pi^+\pi^- 2\pi^0) = 1.8(4)(1) \times 10^{-4}$$

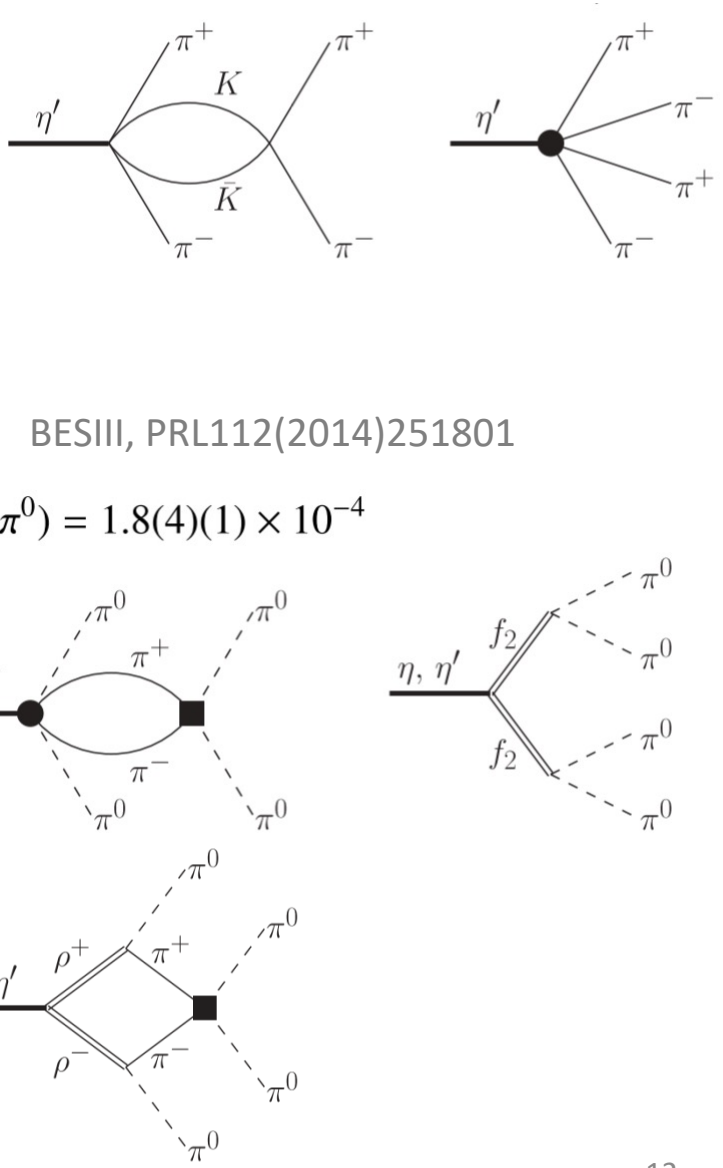
- $\eta' \to 4\pi^0$:
  - $O(p^{10})$ in CHPT, D-wave between pions

$$\mathcal{B}(\eta' \to 4\pi^0) \sim 4 \times 10^{-8}$$

- $\eta \to 4\pi^0$:
  - S-wave CP forbidden
  - Tiny phase space for D-wave leads to

$$\mathcal{B}(\eta \to 4\pi^0) \sim 3 \times 10^{-30}$$

  - If found, signal for CP violation



C-violating processes

- C-violating processes:
  - strictly violating
  - violating via single-photon exchange

Channel	Branching ratio	Note	Ref.
$\eta \to 3\gamma$	$< 1.6 \times 10^{-5}$	sensitive to C,CP-violating as well as C,P-violating but CP-conserving BSM (safe from EDM constraints)	[598]
$\eta \to \pi^0\gamma$	$< 9 \times 10^{-5}$	Violates angular momentum conservation or gauge invariance	[599]
$\eta \to \pi^0 e^+ e^-$	$< 7.5 \times 10^{-6}$	C, CP-violating as single-γ process	[508]
$\eta \to \pi^0 \mu^+ \mu^-$	$< 5 \times 10^{-6}$	C, CP-violating as single-γ process	[600]
$\eta \to 2\pi^0\gamma$	$< 5 \times 10^{-4}$		[601]
$\eta \to 3\pi^0\gamma$	$< 6 \times 10^{-5}$		[601]
$\eta' \to 3\gamma$	$< 1.0 \times 10^{-4}$		[515]
$\eta' \to \pi^0 e^+ e^-$	$< 1.4 \times 10^{-3}$	C, CP-violating as single-γ process	[602]
$\eta' \to \pi^0 \mu^+ \mu^-$	$< 6.0 \times 10^{-5}$	C, CP-violating as single-γ process	[600]
$\eta' \to \eta e^+ e^-$	$< 2.4 \times 10^{-3}$	C, CP-violating as single-γ process	[602]
$\eta' \to \eta \mu^+ \mu^-$	$< 1.5 \times 10^{-5}$	C, CP-violating as single-γ process	[600]

SM predictions: $\mathcal{B}(\eta \to 3\gamma) \sim 10^{-25}$–$10^{-17}$, $\mathcal{B}(\eta' \to 3\gamma) \sim 10^{-23}$–$10^{-15}$

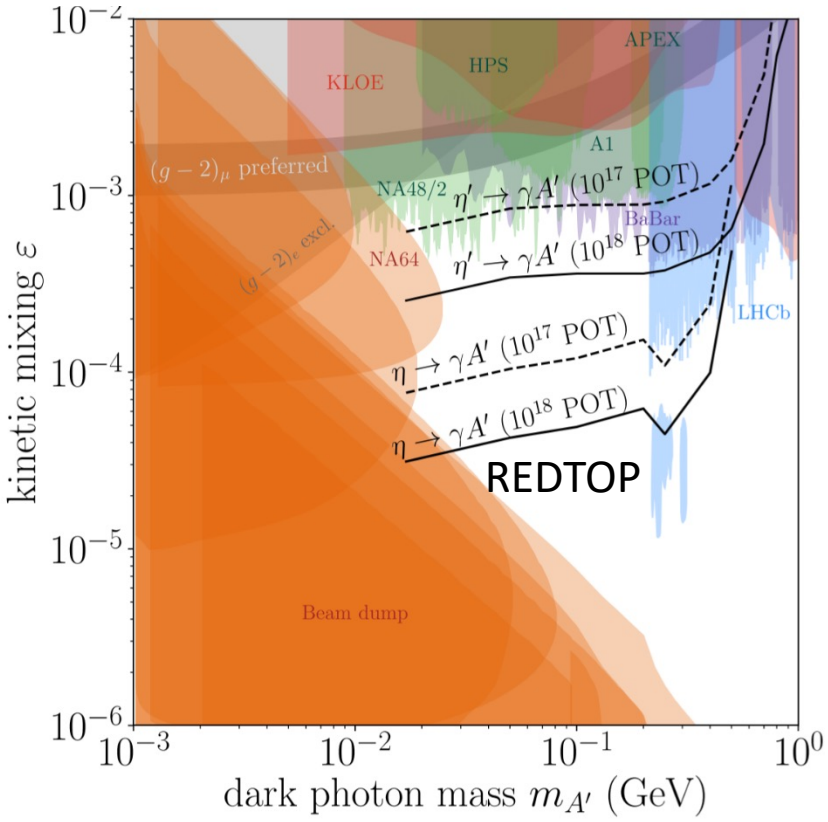
Three or more orders of magnitude below the current sensitivity for $\eta \to \pi^0 l^+ l^-$

- Charge asymmetry of $\pi^\pm$ distributions in $\eta \to \pi^+\pi^-\pi^0$

Search for new vector bosons BSM

- Dark photon: the region preferred for explaining $(g-2)_\mu$ within 2σ is excluded

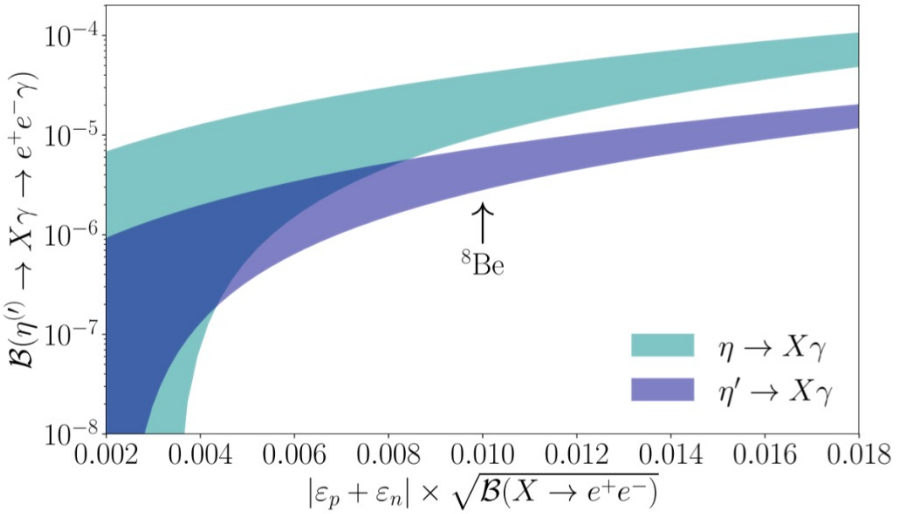
$$\mathcal{L}_{\rm int} = -e\varepsilon j^\mu_{\rm em} A'_\mu$$



POT: protons on target

- Protophobic vector boson (X): candidate to explain the 16.7 MeV anomaly in $^8\mathrm{Be}^* \to {}^8\mathrm{Be}\, e^+e^-$

$$^8\mathrm{Be}^* \to {}^8\mathrm{Be}\, X \to {}^8\mathrm{Be}\, e^+e^-$$



allowed ranges from $\pi^0 \to X\gamma \to e^+e^-\gamma$

$\varepsilon_{p,n}$: the X changes of p, n in units of e

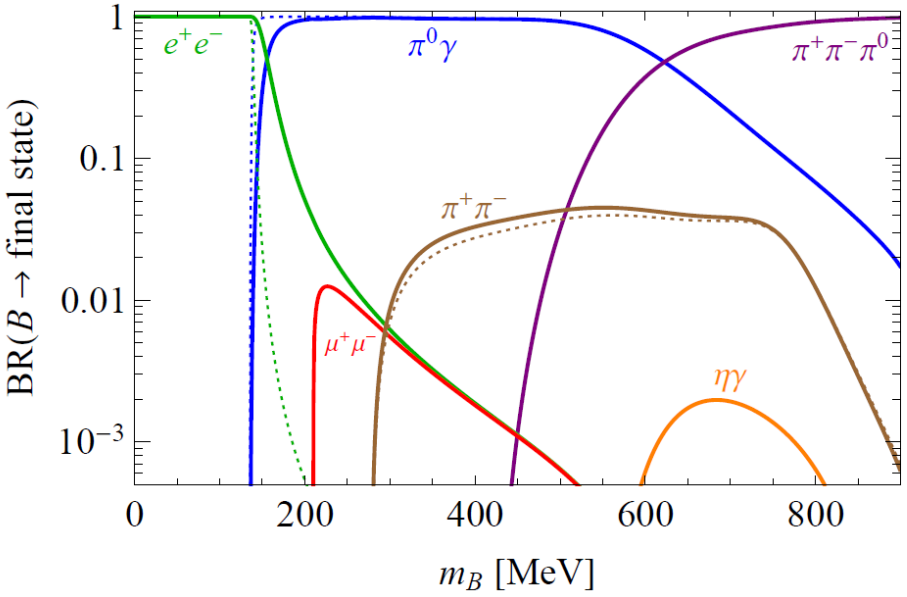
➢ The same channel also probes the Z' boson from $U(1)_{B-L}$

Search for new vector bosons BSM

- Leptophobic $U(1)_B$ boson: gauge boson if the $U(1)_B$ is a gauge symmetry

$$\mathcal{L}_{\text{int}} = \left(\tfrac{1}{3}g_B + \varepsilon e Q_q\right)\bar{q}\gamma^\mu q B_\mu - \varepsilon e \bar{\ell}\gamma^\mu \ell B_\mu$$

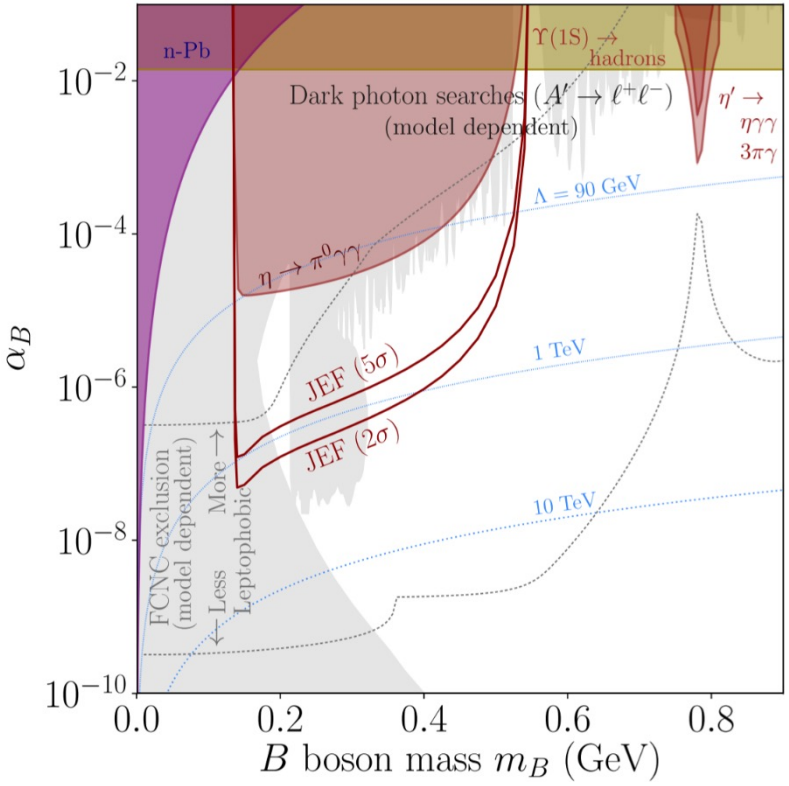
- Branching fractions



solid: $\varepsilon = g_B e/(4\pi)^2$

dashed: $\varepsilon = 0.1 g_B e/(4\pi)^2$

- Exclusion regions



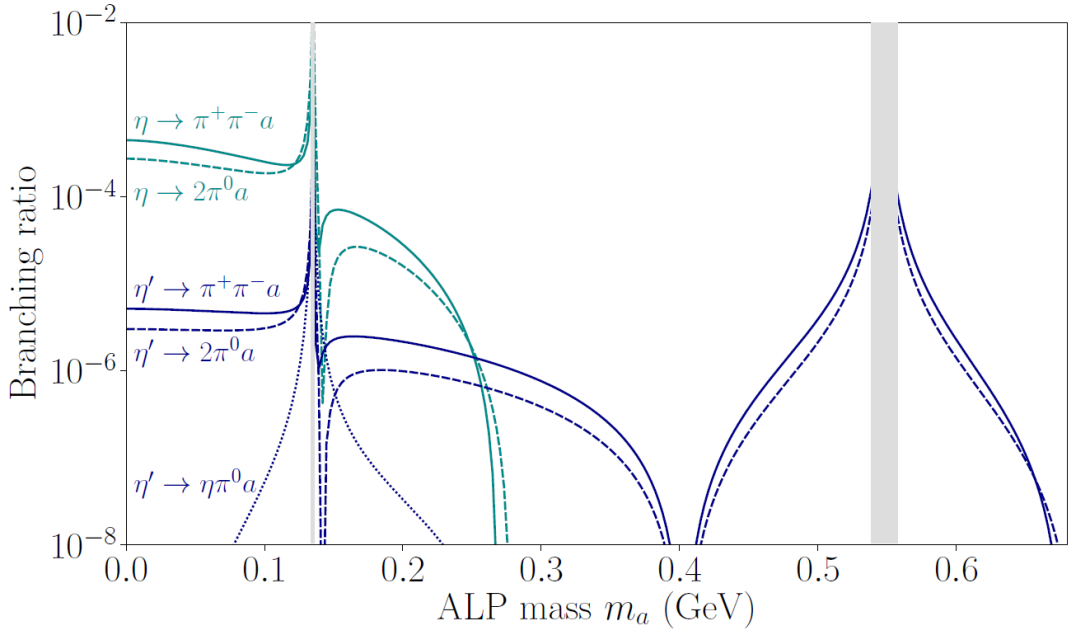
- Channels

$\eta \to B\gamma \to \pi^0\gamma\gamma, \quad \eta' \to B\gamma \to \pi^0\gamma\gamma, \quad \eta' \to B\gamma \to \pi^+\pi^-\pi^0\gamma, \quad \eta' \to B\gamma \to \eta\gamma\gamma.$

Search for ALPs

- Axion-like particles (ALPs): mediating interactions between dark matter and SM
- Branching ratios of decay modes involving an ALP, assuming $f_a = 10$ GeV and no direct quark-ALP coupling ($c_a = 0$; through gluons)

$$\mathcal{L}_{\rm ALP} = \mathcal{L}_{\rm QCD} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{1}{2}m_0^2 a^2 - \frac{\alpha_s}{8\pi f_a} a\, G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \frac{\alpha_{\rm em} c_\gamma}{8\pi f_a} a\, F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{\partial^\mu a}{2f_a}\bar{q}c_q\gamma_\mu\gamma_5 q - \frac{\partial^\mu a}{2f_a}\bar{\ell}c_\ell\gamma_\mu\gamma_5\ell$$



- Channels:
  - $\eta, \eta' \to 2\pi a \to 2\pi\gamma\gamma$
  - $\eta, \eta' \to \pi^+\pi^- a \to \pi^+\pi^- l^+l^-$; $\eta, \eta' \to \pi^0\pi^0 a \to \pi^0\pi^0 l^+l^-$
  - More η' decay channels

Search for new scalars

- New scalar bosons BSM
  - Low-mass scalars (below about 10 MeV) excluded:
    - Big bang nucleosynthesis arguments for $S \to \gamma\gamma$ exclude $m_S < 20\ \mathrm{MeV}$
    - Beam dump experiments and $(g-2)_\mu$ exclude $m_S \lesssim 10\ \mathrm{MeV}$
  - Intermediate mass region ($10\ \mathrm{MeV} \lesssim m_S < 2M_\pi$), can be searched for in

$$\eta \to \pi^0 S \to \pi^0\gamma\gamma, \pi^0 e^+e^-, \pi^0\mu^+\mu^-$$

  - High mass ($m_S > 2M_\pi$), can be searched for in

$$\eta, \eta' \to \pi^0 S \to 3\pi, \qquad \eta' \to \eta S \to \eta\pi\pi$$

- Allowed parameter space needs to be worked out

Extraction of Q value from $\eta' \to \eta\pi\pi$

- Q: fundamental parameter, isospin breaking quark mass ratio ($\hat{m} = \frac{m_u + m_d}{2}$)

$$Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

Leutwyler's ellipse: $\left(\frac{m_u}{m_d}\right)^2 + \frac{1}{Q^2}\left(\frac{m_s}{m_d}\right)^2 = 1$ H. Leutwyler (1996)

- Extracted from isospin breaking quantities
  - Using Dashen's theorem: $(\Delta M_K^2)_{\mathrm{EM}} = (\Delta M_\pi^2)_{\mathrm{EM}} + \mathcal{O}(e^2 m_q)$; rel. corrections $\mathcal{O}(m_q^2, \delta, e^2)$

$$Q_D^2 = \frac{(M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2 + M_{\pi^0}^2)(M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2 - M_{\pi^0}^2)}{4M_{\pi^0}^2 (M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2 - M_{\pi^0}^2)} = (24.3)^2$$

  - $\eta \to \pi^+\pi^-\pi^0, 3\pi^0$:

Q	Refs.
24.3	from Dashen's theorem [44]
22.7 ± 0.8	A. V. Anisovich & H. Leutwyler [17]
23.1 ± 0.7	K. Kampf *et al.* [38]
22.1 ± 0.7	G. Colangelo *et al.* [41]
21.50 ± 0.97	M. Albaladejo & B. Moussallam [42]
22.4 ± 0.3	D. Stamen *et al.* [45]
23.3 ± 0.5	FLAG ($N_f = 2 + 1$) [43]
22.5 ± 0.5	FLAG ($N_f = 2 + 1 + 1$) [43]

Extraction of Q value from $\eta' \to \eta\pi\pi$

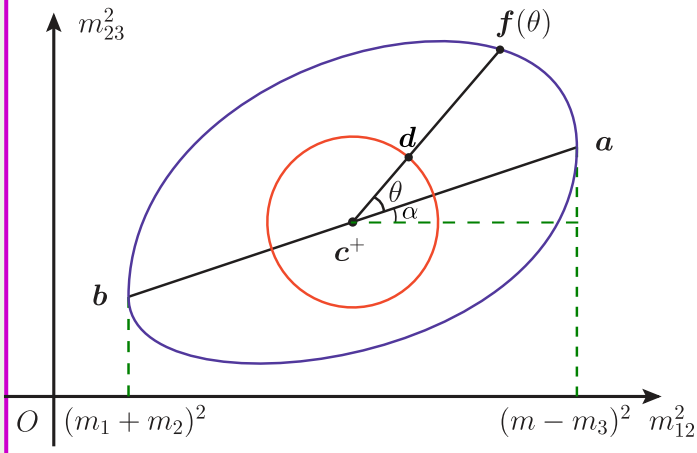
A. Guevara, FKG, H.-J. Jing, arXiv:2502.02837

- $\eta' \to \eta\pi^+\pi^-$ and $\eta' \to \eta\pi^0\pi^0$ are isospin conserving reactions
  - But their difference is isospin breaking, can we extract Q from η' decays?
- Unit disk mapping: Dalitz plot ↦ unit disk
  - Dalitz plot regions of $\eta' \to \eta\pi^+\pi^-$ and $\eta' \to \eta\pi^0\pi^0$ map to the same unit disk
  - Take difference inside the unit disk ⇒ purely isospin breaking !
  - Practical procedure:
    - Measure Dalitz plot distribution parameters
    - Discretize unit disk into bins
    - Map Dalitz dists. to unit disk and take difference
    - Parametrize decay amplitudes using UChPT or dispersion relation
    - Fit to the difference inside the unit disk

$$d\Gamma_{\rm diff}(r,\theta) \equiv \frac{d^2}{dm_{12}^2 dm_{23}^2}\left[\Gamma_{\pi^0}(r,\theta) - \Gamma_{\pi^\pm}(r,\theta)\right]$$

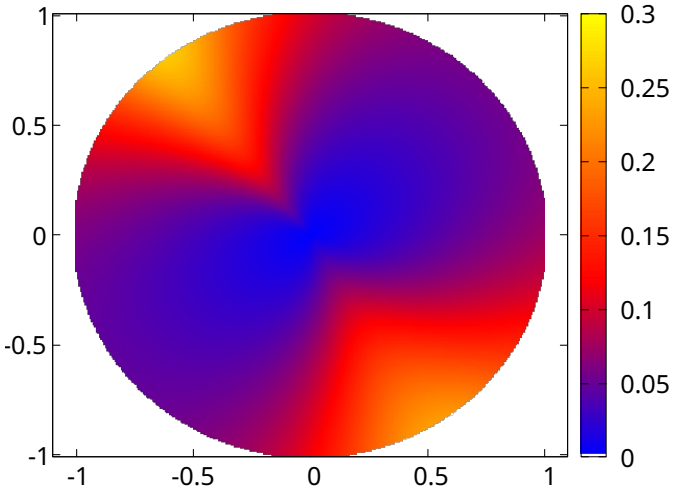
$$d\Gamma_{\rm diff}(r,\theta) = 2\,{\rm Re}\left(\mathcal{M}_{\rm IC}^{*}\mathcal{M}_{\rm IB}\right) + \mathcal{O}\left(Q^{-8}\right) \propto \frac{1}{Q^4}$$

    - Exact the Q value



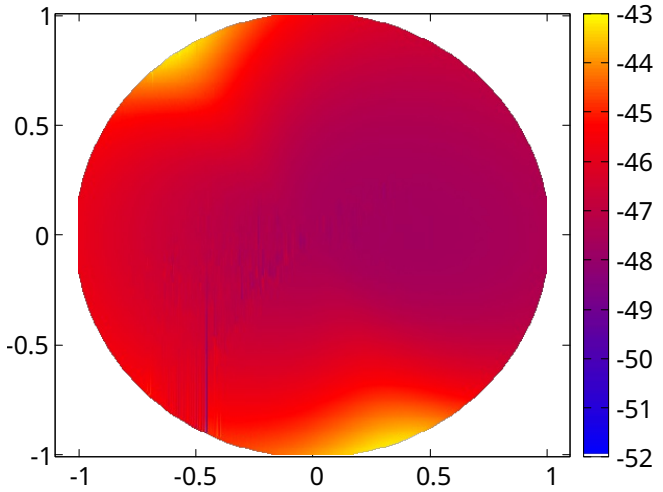
Extraction of Q value from $\eta' \to \eta\pi\pi$

A. Guevara, FKG, H.-J. Jing, arXiv:2502.02837



Unit disk degenerated from BESIII measurement of $\eta' \to \eta\pi^+\pi^-$ (3.5×10^5 events)

BESIII, PRD 97 (2018) 012003



Unit disk difference from isospin breaking

Take UChPT amplitude from

R. Escribano et al., JHEP 05 (2011) 094

Possible improvements:

- Include higher order terms in Dalitz plot distribution
- Use dispersive amplitudes

Q	Refs.
24.3	from Dashen's theorem [44]
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23.3 ± 0.5	FLAG ($N_f = 2+1$) [43]
22.5 ± 0.5	FLAG ($N_f = 2+1+1$) [43]
22.2 ± 1.1	this work

Summary

- Rich physics (both precision tests of SM and BSM) can be done at eta factories
- Most interesting processes suggested in L. Gan et al., arXiv:2007.00664

Decay channel	Standard Model	Discrete symmetries	Light BSM particles
$\eta \to \pi^+\pi^-\pi^0$	light quark masses	C/CP violation	scalar bosons (also η')
$\eta^{(\prime)} \to \gamma\gamma$	η–η' mixing, precision partial widths		
$\eta^{(\prime)} \to \ell^+\ell^-\gamma$	$(g-2)_\mu$		Z' bosons, dark photon
$\eta \to \pi^0\gamma\gamma$	higher-order χPT, scalar dynamics		$U(1)_B$ boson, scalar bosons
$\eta^{(\prime)} \to \mu^+\mu^-$	$(g-2)_\mu$, precision tests	CP violation	
$\eta \to \pi^0\ell^+\ell^-$		C violation	scalar bosons
$\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$	$(g-2)_\mu$		ALPs, dark photon
$\eta^{(\prime)} \to \pi^0\pi^0\ell^+\ell^-$		C violation	ALPs

- $\eta' \to \eta\pi\pi$ can also be used to extract the Q value using the unit disk mapping method

Thank you for your attention!