Neutrinos for hadron structure

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Collaborators: X. Chen (IMP), R. Kunitomo (JWU), R. Petti (South Carolina), S. Wu (IMP), Y.-P. Xie (IMP)

References: SK, R. Petti, PoS (NuFact2021) 092;
X. Chen, SK, R. Kunitomo, S. Wu, Y.-P. Xie, arXiv:2401.11440.

第二届惠州大科学装置高精度物理研讨会
一暨基于HIAF加速器集群的缪子科学与技术研讨会

The 2nd Huizhou Large Scientific Facility High-Precision Physics Symposium
and Muon Science and Technology Symposium Based on HIAF Accelerator Cluster,
Sun Yat-sen University, Guangzhou, Guangdong, China, August 23-26, 2024
https://indico.impcas.ac.cn/event/63/

August 24, 2024

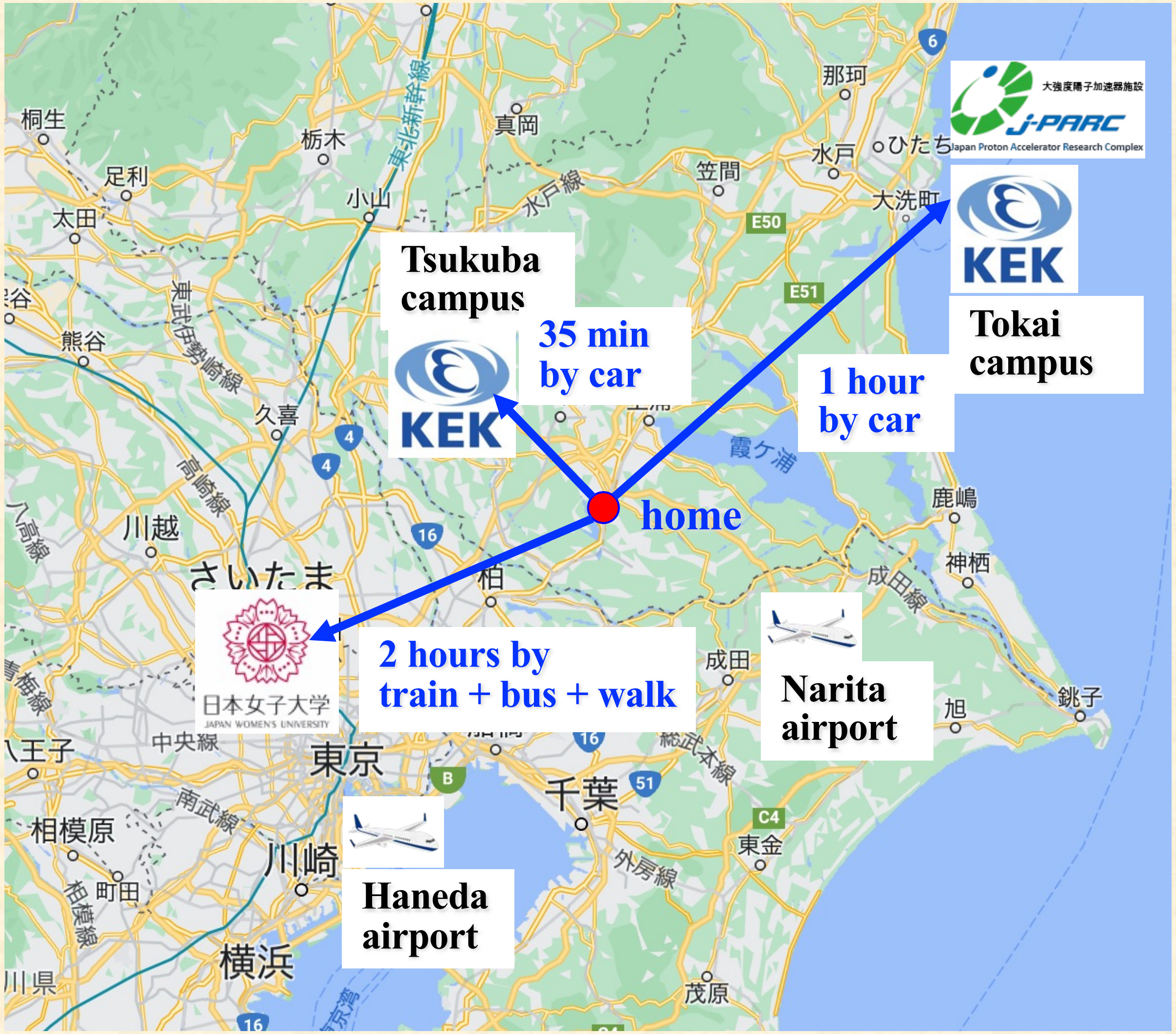
Self introduction

https://research.kek.jp/people/kumanos/

1985: Ph.D, MIT theory center
2022: Retired from KEK theory center
Now: Specially Appointed Professor, Japan Women's University
Professor Emeritus, Diamond Fellow, KEK

- 3-4 days / week at the Japan Women's University
- 1-2 days / week at the KEK Tsukuba campus

View of Ikebukuro downtown from my JWU office



Contents

HIAF (High Intensity heavy-ion Accelerator Facility)
Proton beam energy 9.3 GeV → 25 GeV → 100 GeV? in future.
High-energy neutrino nucleon interactions could be investigated!

1. Introduction
   - Nucleon structure functions in neutrino reactions
   - Motivations for studying GPDs (Generalized Parton Distributions)

slightly long introduction

2. Possible GPD studies at neutrino facilities (Fermilab, CERN, …)

3. GPD studies at other facilities
   - Spacelike GPDs: charged-lepton scattering (JLab, AMBER, EICs , …)
   - Timelike GPDs: e^+e^- scattering (KEKB, BES, …)
   - GPDs at hadron accelerator facilities (J-PARC, NICA, GSI, …)

may skip?

HIAF?

4. Future prospects on GPD projects

Appendix: Comments on GPDs for exotic hadrons

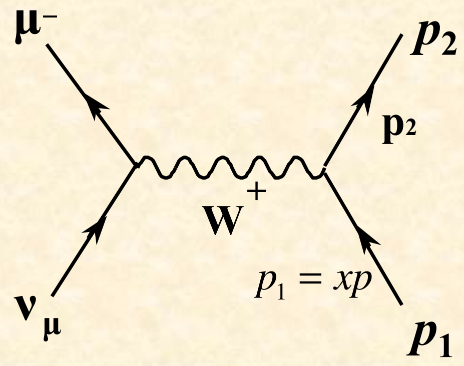
Nucleon structure functions in neutrino reactions

Neutrino deep inelastic scattering (CC: Charged Current)

Structure functions in parton model for neutrino-nucleon scattering (CC)

$$\frac{d\sigma^{CC}_{\nu,\bar{\nu}}}{dx\,dy} = \frac{G_F^2\,(s-M^2)}{2\pi\,(1+Q^2/M_W^2)^2}\left[x\,y^2 F_1^{CC} + \left(1-y-\frac{M\,x\,y}{2E}\right)F_2^{CC} \pm x\,y\left(1-\frac{y}{2}\right)F_3^{CC}\right]$$

$$J_\mu^{CC} = \bar{u}(p_2,\lambda_2)\,\gamma_\mu\,(1-\gamma_5)\,[\,d(p_1,\lambda_1)\cos\theta_c + s(p_1,\lambda_1)\sin\theta_c]$$
$$+\bar{c}(p_2,\lambda_2)\,\gamma_\mu\,(1-\gamma_5)\,[\,s(p_1,\lambda_1)\cos\theta_c - d(p_1,\lambda_1)\sin\theta_c]$$



In parton model

$F_2 = 2xF_1$ **(Callan-Gross relation)**

$$F_2^{\nu p(CC)} = 2x\left(d+s+\bar{u}+\bar{c}\right) \qquad F_2^{\nu n(CC)} = 2x\left(u+s+\bar{d}+\bar{c}\right)$$

$$xF_3^{\nu p(CC)} = 2x\left(d+s-\bar{u}-\bar{c}\right) \qquad xF_3^{\nu n(CC)} = 2x\left(u+s-\bar{d}-\bar{c}\right)$$

$$F_2^{\bar{\nu} p(CC)} = 2x\left(u+c+\bar{d}+\bar{s}\right) \qquad F_2^{\bar{\nu} n(CC)} = 2x\left(d+c+\bar{u}+\bar{s}\right)$$

$$xF_3^{\bar{\nu} p(CC)} = 2x\left(u+c-\bar{d}-\bar{s}\right) \qquad xF_3^{\bar{\nu} n(CC)} = 2x\left(d+c-\bar{u}-\bar{s}\right)$$

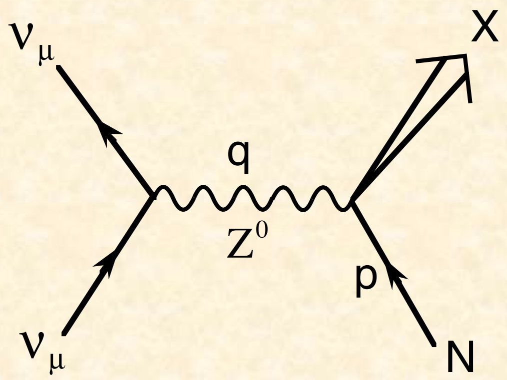
Determination of valence-quark distributions

$$\longrightarrow \frac{1}{2}\left[F_3^{\nu p} + F_3^{\bar{\nu} p}\right]_{CC} = \underline{u_v + d_v} + s - \bar{s} + c - \bar{c}$$

valence-quark distributions

Neutrino deep inelastic scattering (NC: Neutral Current)

$$M = \frac{\rho}{1+Q^2/M_Z^2}\frac{G_F}{\sqrt{2}}\,\bar{u}(k',\lambda')\,\gamma^\mu\,(1-\gamma_5)\,u(k,\lambda) < X\,|\,J_\mu^{NC}\,|\,p,\lambda_p >$$



$$\frac{d\sigma}{dE'd\Omega} = \frac{\rho^2 G_F^2}{(1+Q^2/M_Z^2)^2}\,\frac{k'}{32\pi^2 E}\,L^{\mu\nu}\,W_{\mu\nu}$$

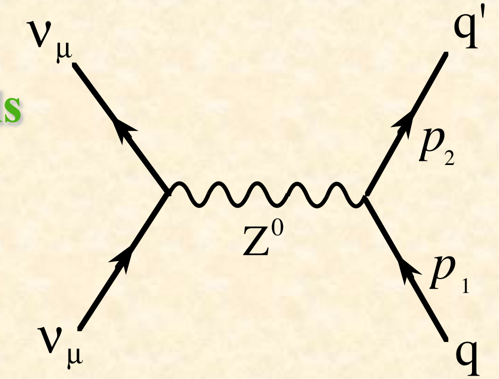
ρ is the strength of NC relative to CC

$$\frac{d\sigma^{NC}}{dx\,dy} = \frac{\rho^2 G_F^2\,(s-M^2)}{2\pi\,(1+Q^2/M_Z^2)^2}\left[x\,y^2F_1^{NC} + \left(1-y-\frac{M\,x\,y}{2E}\right)F_2^{NC} + x\,y\left(1-\frac{y}{2}\right)F_3^{NC}\right]$$

$\nu_\mu + p \to \nu_\mu + X$

Neutrino-quark scattering (NC)

see next pages for details



$$J_\mu^{NC} = \sum_q \bar{q}(p_2,\lambda_2)\,\gamma_\mu\,[\,g_L^q\,(1-\gamma_5) + g_R^q\,(1+\gamma_5)\,]\,q(p_1,\lambda_1)$$

$$g_L^{u,c} \equiv u_L = +\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\ ,\quad g_R^{u,c} \equiv u_R = -\frac{2}{3}\sin^2\theta_W$$

$$g_L^{d,s} \equiv d_L = -\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\ ,\quad g_R^{d,s} \equiv d_R = +\frac{1}{3}\sin^2\theta_W$$

$\sin^2\theta_W$: weak mixing angle

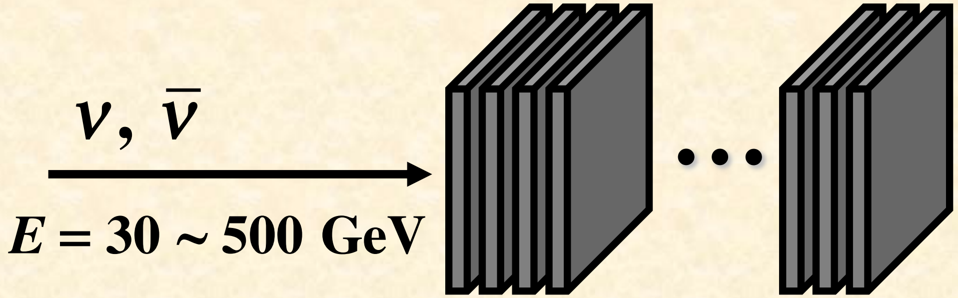
$$F_1^{\nu p(NC)} = F_2^{\nu p(NC)}/2x$$

$$F_2^{\nu p(NC)} = 2x\,[\,(u_L^2+u_R^2)\,\{u^+(x)+c^+(x)\} + (d_L^2+d_R^2)\,\{d^+(x)+s^+(x)\}\,]$$

$$xF_3^{\nu p(NC)} = 2x\,[\,(u_L^2-u_R^2)\,\{u^-(x)+c^-(x)\} + (d_L^2-d_R^2)\,\{d^-(x)+s^-(x)\}\,]$$

$$q^\pm(x) = q(x) \pm \bar{q}(x)$$

Neutrino DIS experiments

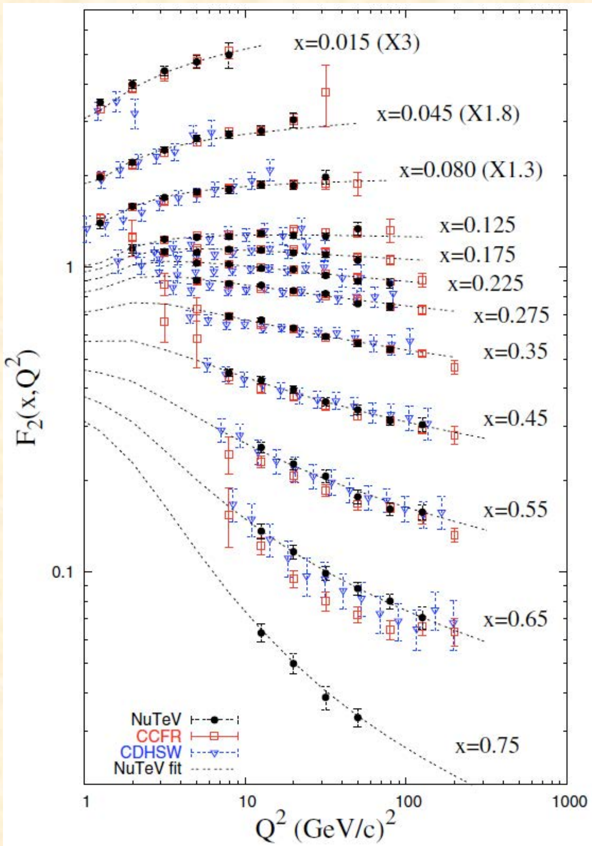


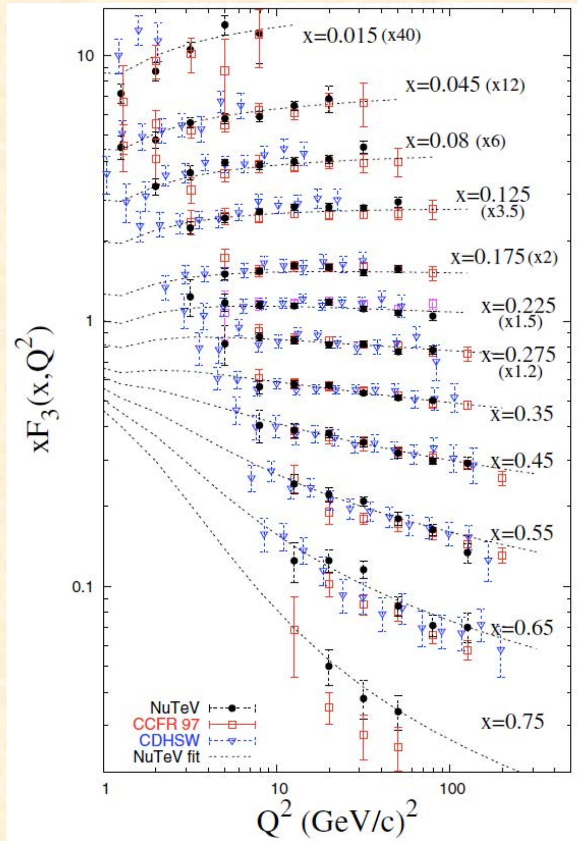
Huge Fe target (690 ton)

Experiment	Target	ν energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

MINERνA (He, C, Fe, Pb), …

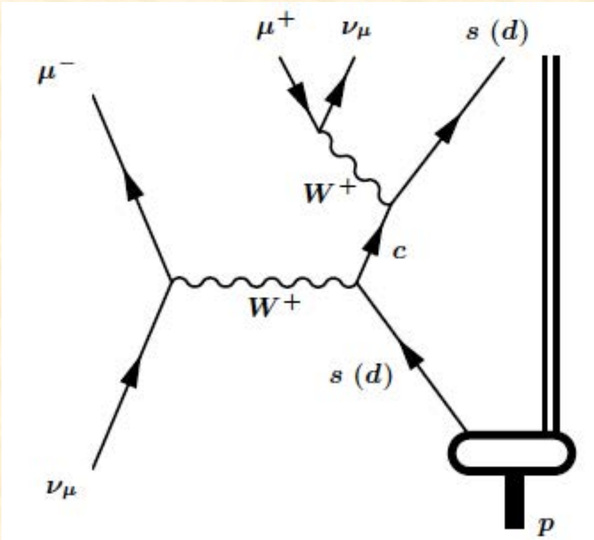
M. Tzanov *et al.* (NuTeV), PRD74 (2006) 012008.





Strangeness in the nucleon

Neutrino-induced opposite-sign dimuon events



A. Kayis-Topaksu *et al.*, NPB7 98 (2008) 1.
U. Dore, arXiv: 1103.4572 [hep-ex].

$$\kappa = \frac{\int dx\, x\,[s(x,Q^2) + \bar{s}(x,Q^2)]}{\int dx\, x\,[\bar{u}(x,Q^2) + \bar{d}(x,Q^2)]}$$

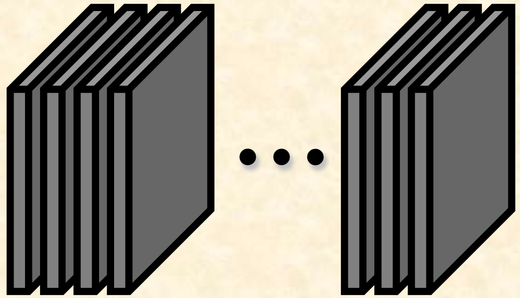
$$Q^2 = 20\ \text{GeV}^2$$

Experiment	κ
This analysis	0.33 ± 0.07
CDHS [1]	0.47 ± 0.09
CCFR [2]	0.44 ± 0.09
CHARM II [3]	0.39 ± 0.09
NOMAD [4]	0.48 ± 0.17
NuTeV [5]	0.38 ± 0.08

CCFR, NuTeV

$\nu,\ \bar{\nu}$

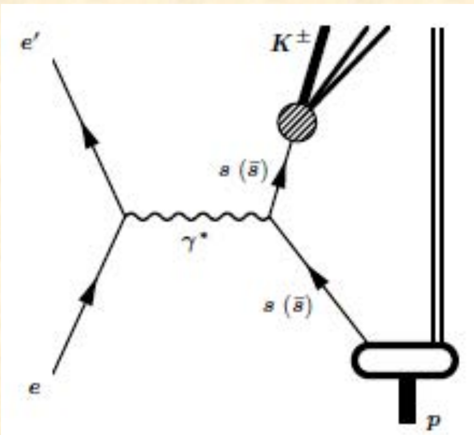
$E = 30 \sim 500$ GeV

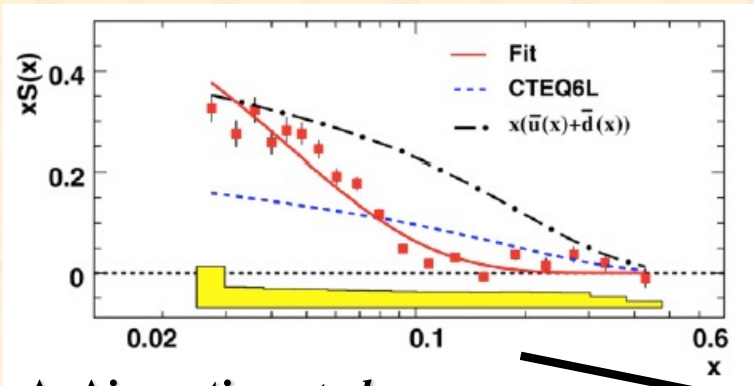


Huge Fe target (690 ton)

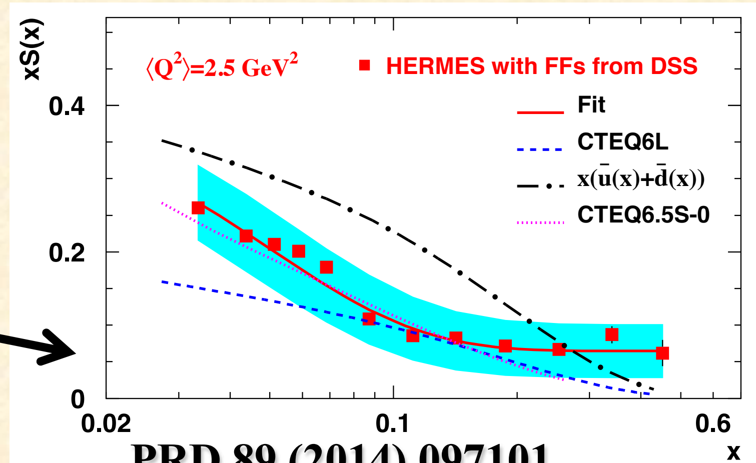
Issue: nuclear corrections

HERMES semi-inclusive measurement



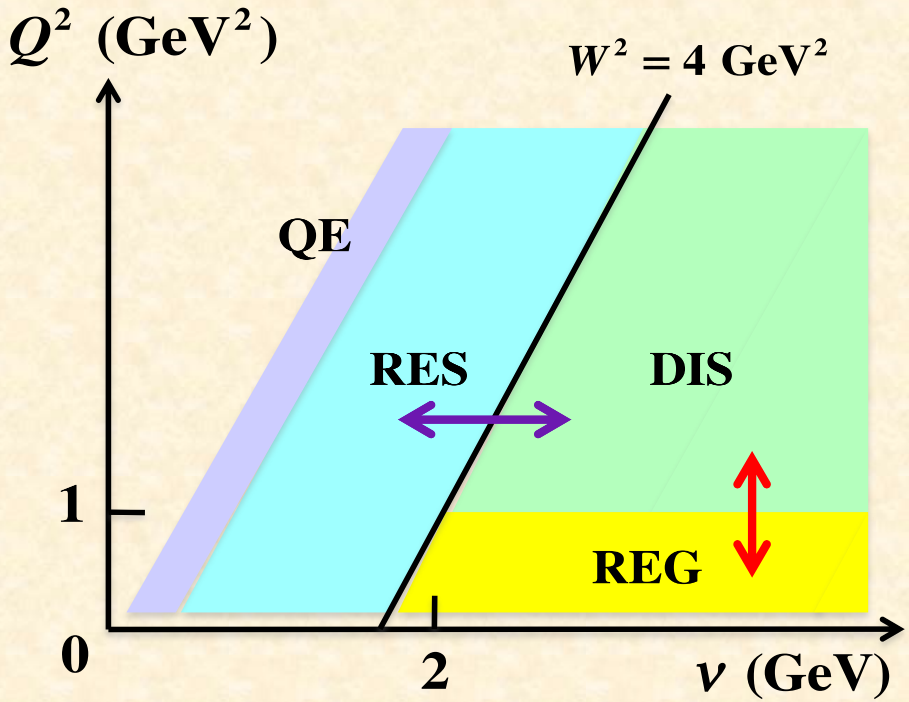


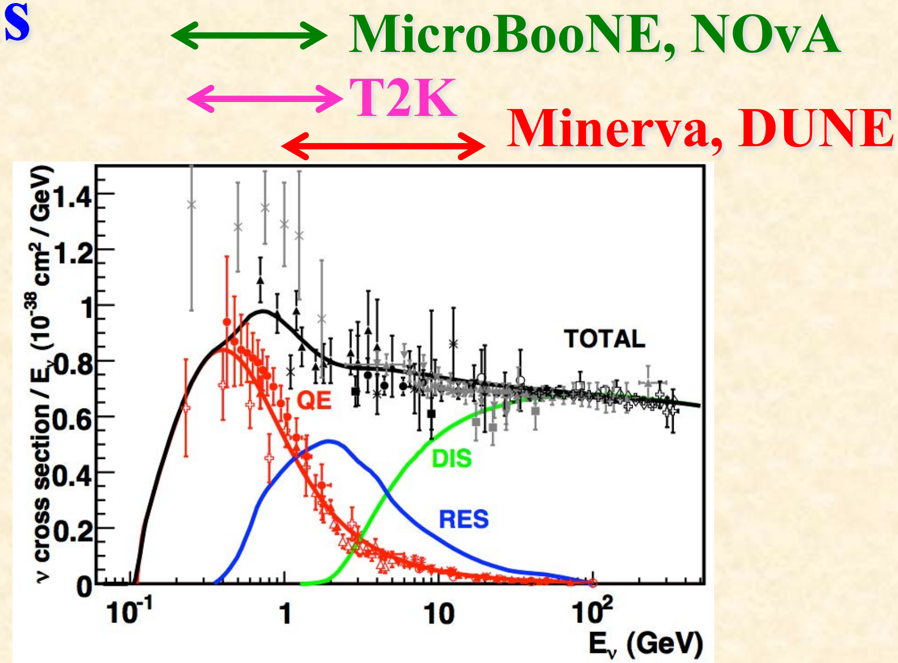
A. Airapetian *et al.*, PLB 666 (2008) 446.



PRD 89 (2014) 097101.

Kinematical regions in ν reactions





J.L. Hewett *et al.*, arXiv:1205.2671,
Proceedings of the 2011 workshop
on Fundamental Physics at the Intensity Frontier

ν interaction part is the major source
of experimental errors
in ν oscillation measurements.

Depending on the neutrino beam energy, different physics mechanisms contribute to the cross section.

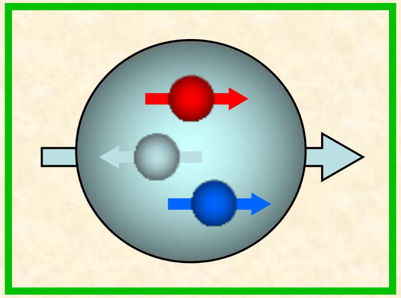
- QE (Quasi elastic)
- RES (Resonance)
- DIS (Deep inelastic scattering)
- REG (Regge)

Our work:
Towards a unified model of neutrino-nucleus reactions for neutrino oscillation experiments,
S. X. Nakamura *et al.*, Rep. Prog. Phys. 80 (2017) 056301.

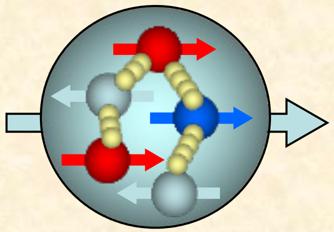
Motivations for studying gravitational form factors and GPDs

Origin of nucleon spin

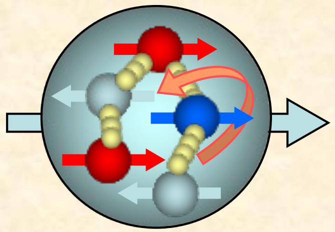
"old" standard model



gluon spin



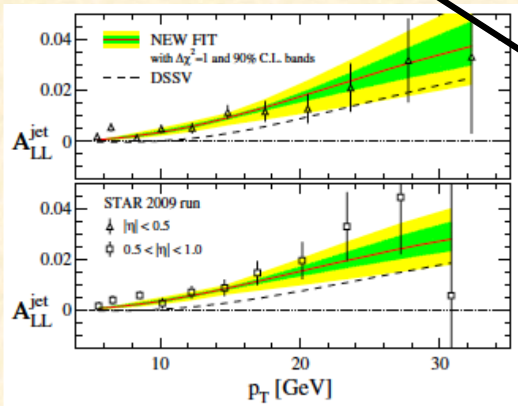
angular momentum

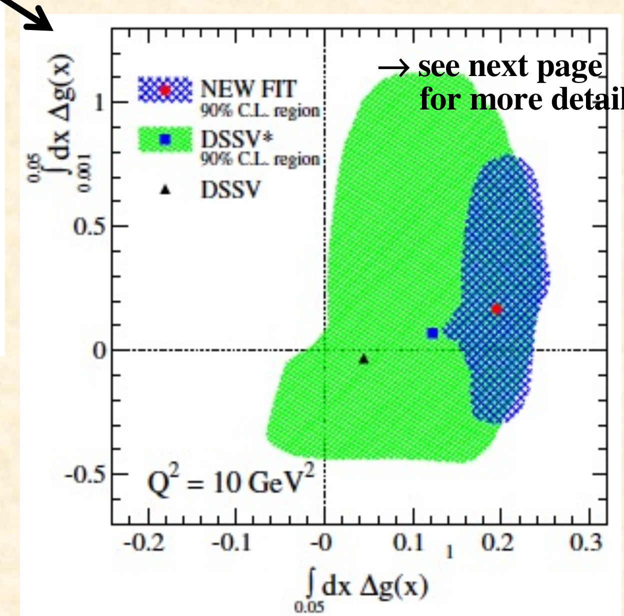


$$p_{\uparrow} = \frac{1}{3\sqrt{2}}\left(uud\left[2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right] + \text{permutations} \right)$$

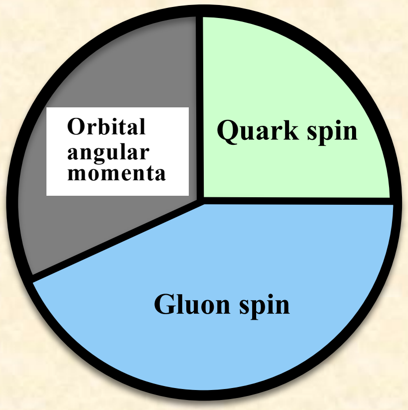
$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

$$\Delta\Sigma = \sum_i \int dx \left[\Delta q_i(x) + \Delta\bar{q}_i(x) \right] \rightarrow 1\ (100\%)$$

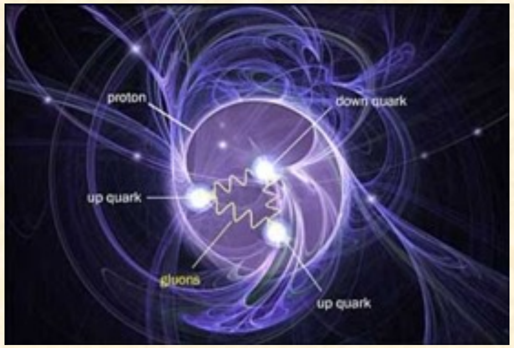




→ see next page for more details



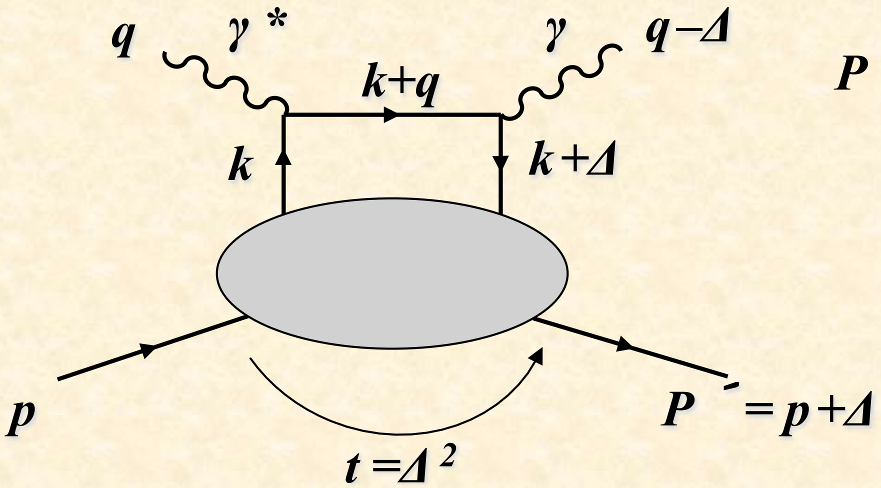
"A possible" spin decmposion



Scientific American (2014)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_{q,g}$$

Generalized Parton Distributions (GPDs)



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $\quad x = \frac{Q^2}{2p\cdot q}$

Momentum transfer squared $\quad t = \Delta^2$

Skewdness parameter $\quad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z/2)\gamma^+\psi(z/2)|p\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2)|p\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

Forward limit: PDFs $\quad H(x,\xi,t)\big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\quad \int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$

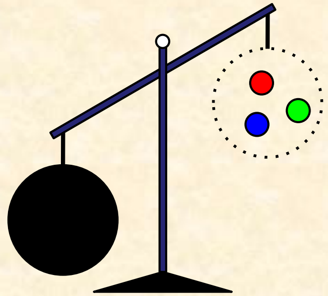
Axial and Pseudoscalar form factors G_A, G_P $\quad \int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$

Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right], \quad J_q = \frac{1}{2}\Delta q + L_q$

$\Rightarrow$ probe L_q, key quantity to solve the spin puzzle!

Origin of hadron masses

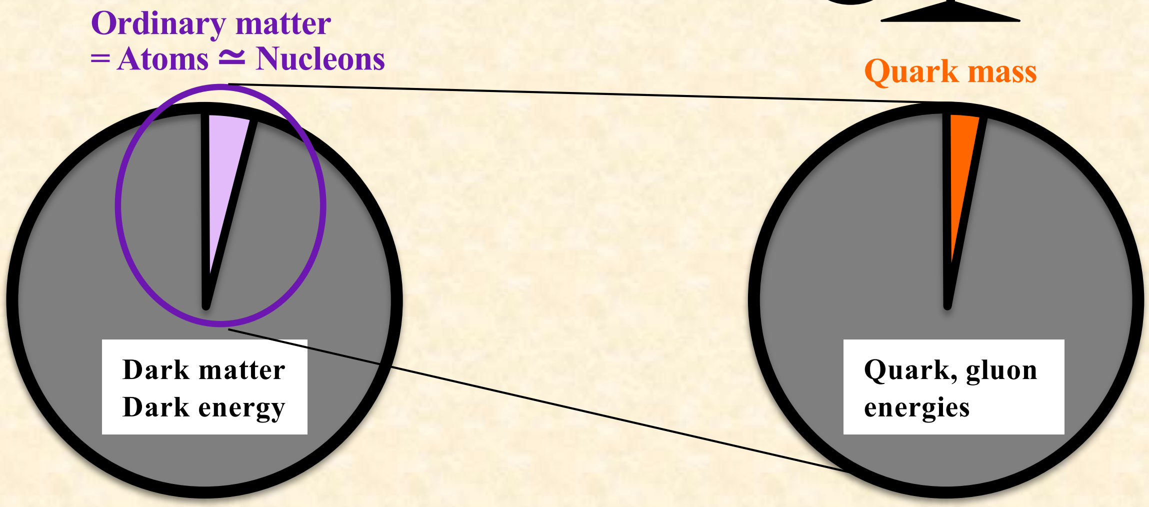


Mass and spin of the nucleon are two of fundamental quantities in physics.

Nucleon mass: $M = \left\langle p \left| \int d^3x \, T^{00}(x) \right| p \right\rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\ \alpha}(x)$$



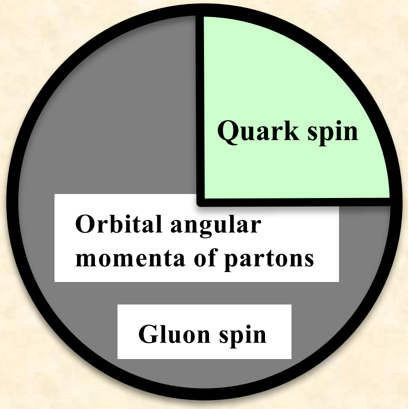
Dark matter

Origin of nucleon mass

Nucleon spin: $\frac{1}{2} = \left\langle p \left| J^3 \right| p \right\rangle$

3rd component of total angular momentum: $J^3 = \frac{1}{2}\varepsilon^{3jk} \int d^3x \, M^{3jk}(x)$

Angular-momentum density: $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x) x^{\mu} - T^{\alpha\mu}(x) x^{\nu}$



Origin of nucleon spin

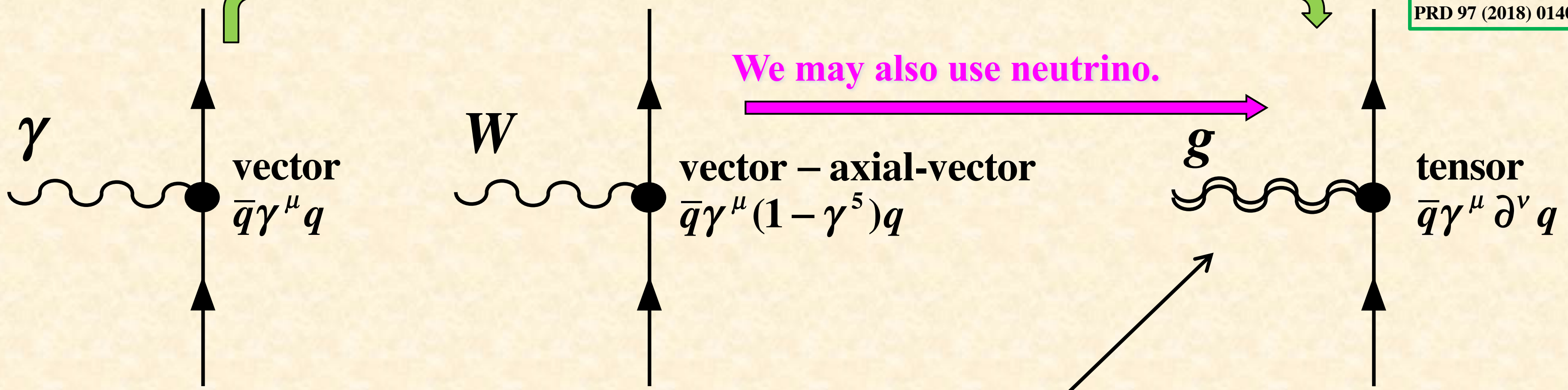
("Dark spin")

Why "gravitational" interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev, PRD 97 (2018) 014020.

We may also use neutrino.



It is possible to probe gravitational sources in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2)\gamma^+ q(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$$

Non-local operator of GPDs/GDAs:

$$(P^+)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z^+=0,\vec{z}_\perp=0}$$

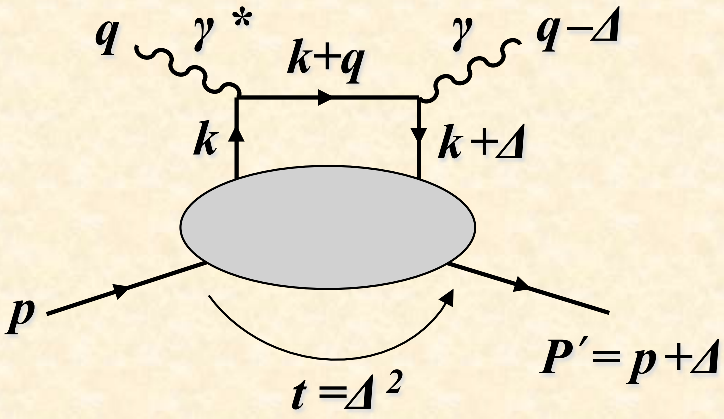
$$= \left(i\frac{\partial}{\partial z^-} \right)^{n-1} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z=0}$$

$$= \bar{q}(0)\gamma^+ \left(i\overleftrightarrow{\partial}^+ \right)^{n-1} q(0)$$

= energy-momentum tensor of a quark for $n=2$
(electromagnetic for $n=1$)

= source of gravitiy

Virtual Compton
or (timelike) two-photon process



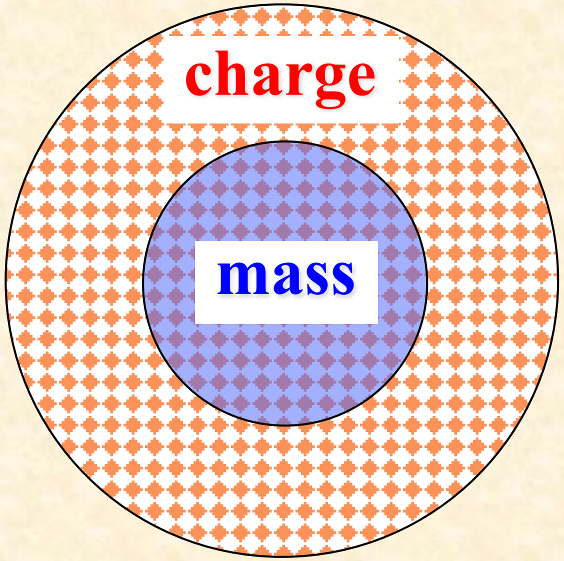
Gravitational form factors and radii for pion

This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

$$\Leftrightarrow \quad \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

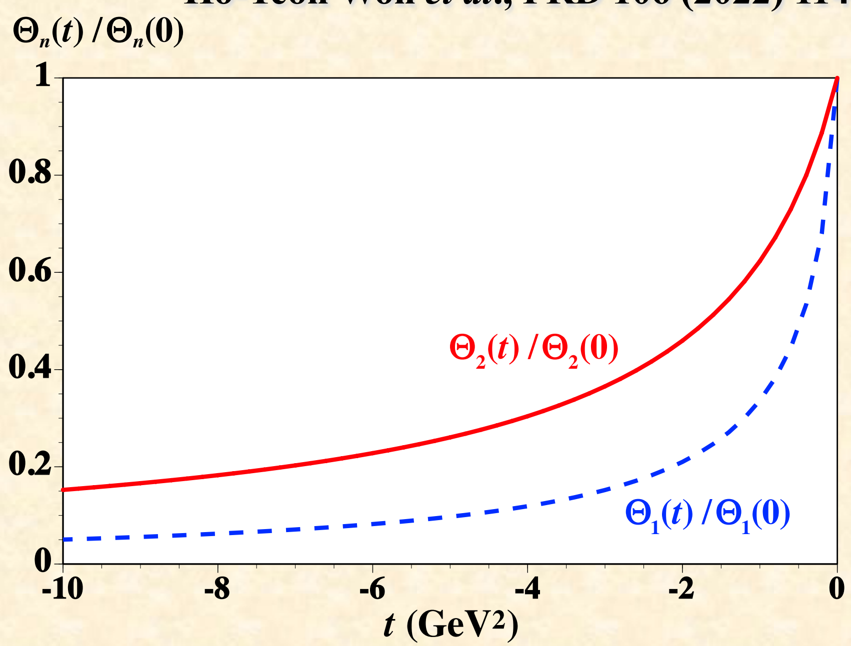
SK, Q.-T. Song, O. Teryaev PRD 97 (2018) 014020.

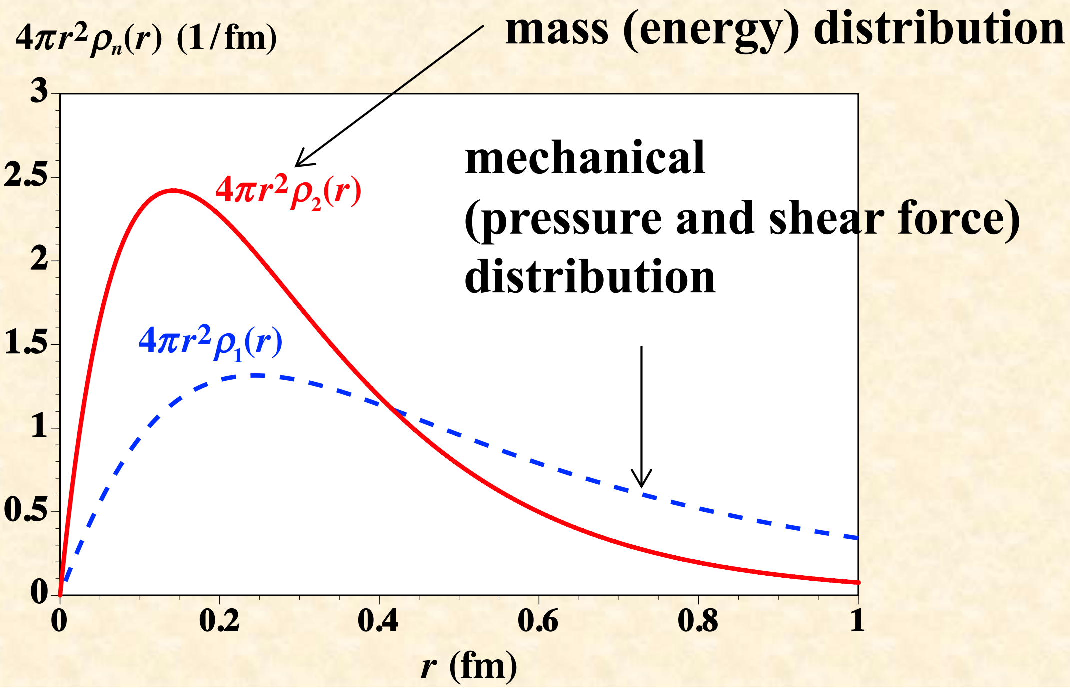


Related theoretical studies:

A. Freeseand I. C. Cloet, PRC 100 (2019) 015201;
P. E. Shanahan and W. Detmold, PRD 99 (2019) 014511;
C. D. Roberts *et al.*, Prog. Part. Nucl. Phys.120 (2021) 103883;
J.-L. Zhang et al. PLB 815 (2021) 136158;
June-Young Kim and Hyun-Chul Kim, PRD 104 (2021) 074019;
Ho-Yeon Won *et al.*, PRD 106 (2022) 114009.

Proton mass radius:
R. Wang, W. Kou, Y.-P. Xie, X. Chen, PRD 103 (2021) L091501.





Nucleon pressure

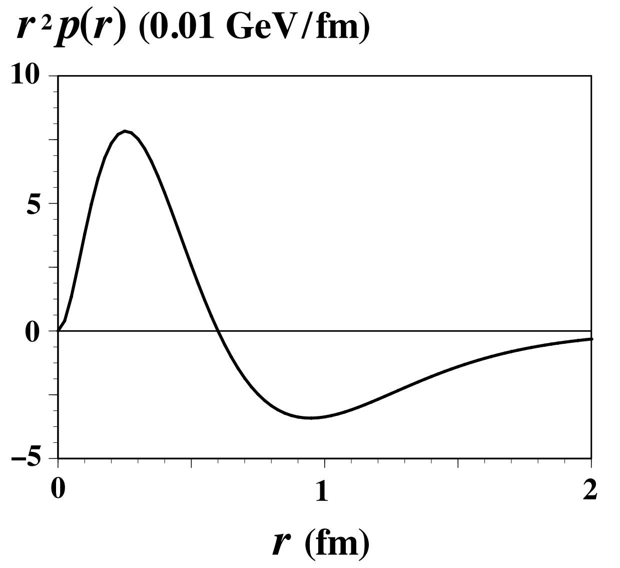
$$\langle N(p')|T_q^{\mu\nu}(0)|N(p)\rangle = \bar{u}(p')\left[A\gamma^{(\mu}\bar{P}^{\nu)} + B\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}Mg^{\mu\nu}\right]u(p)$$

Recent progress

V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557 (2018) 396;

M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) 1830025;

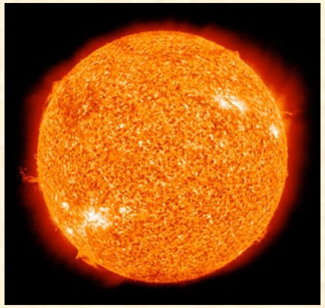
C. Lorce, H. Moutarde, and A. P. Tranwinski, Eur. Phys. J. C 79 (2019) 89.



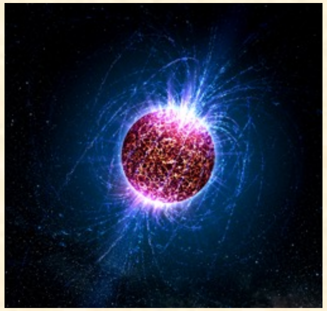
Highest pressure in nature   1 Pa (Pascal) = 1 N/m^2

Earth atmosphere
10^5 Pa =1000 hPa

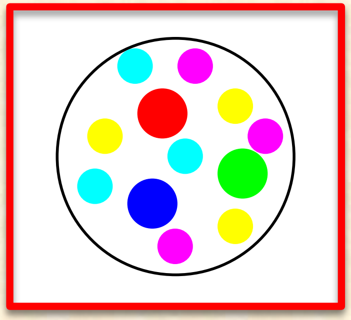
Center of earth
10^{11} Pa = 100GPa



Center of Sun
10^{16} Pa = 10 PPa



Neutron star
10^{34} Pa

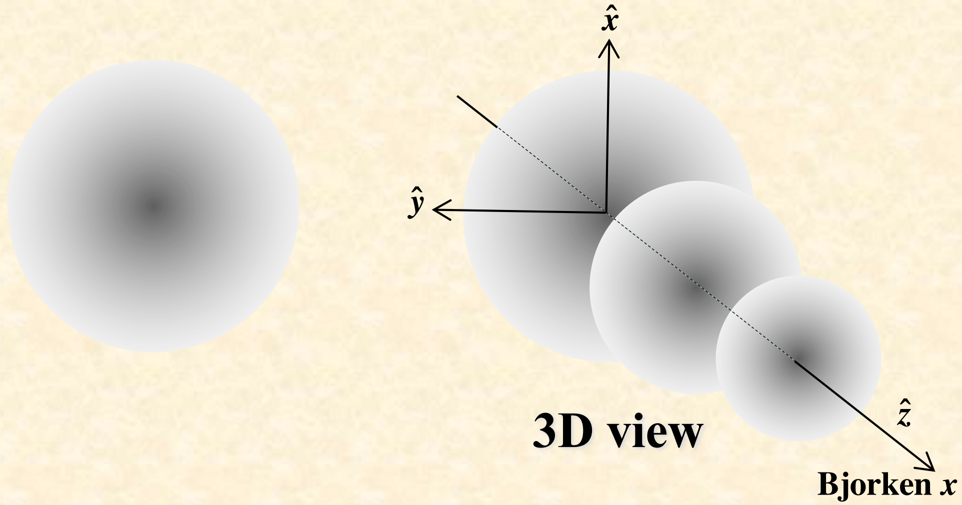


Hadron
10^{35} Pa

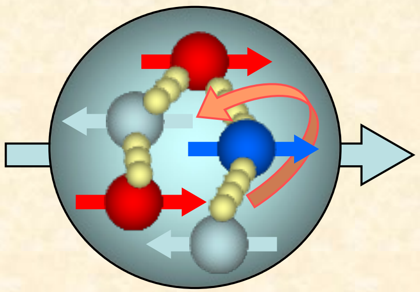
Proton (hadrons) puzzle studies by hadron tomography

Hadron tomography

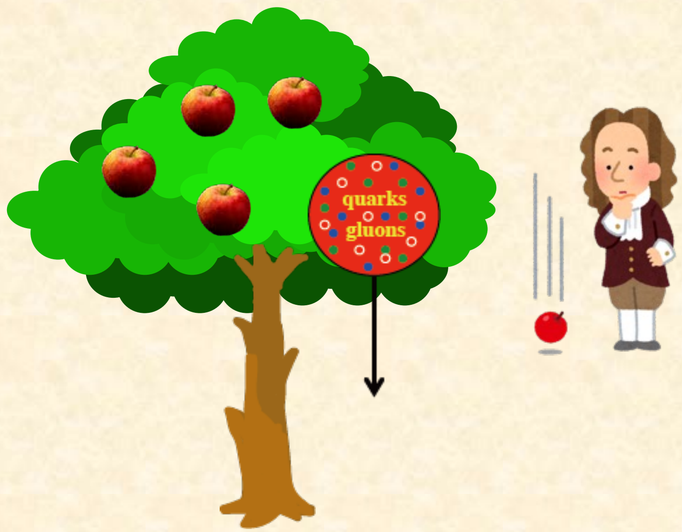
Proton radius puzzle



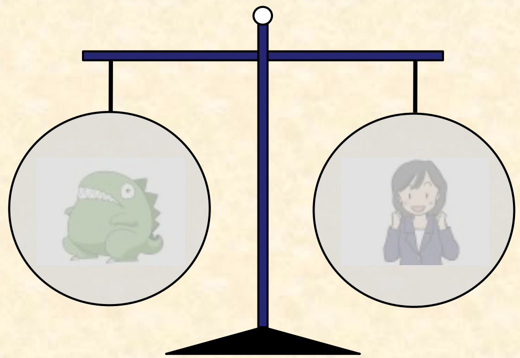
Origin of nucleon spin



Source of gravity (mass)



Exotic hadrons



Possible GPD studies at neutrino facilities

X. Chen, SK, R. Kunitomo, S. Wu, Y.-P. Xie, arXiv:2401.11440

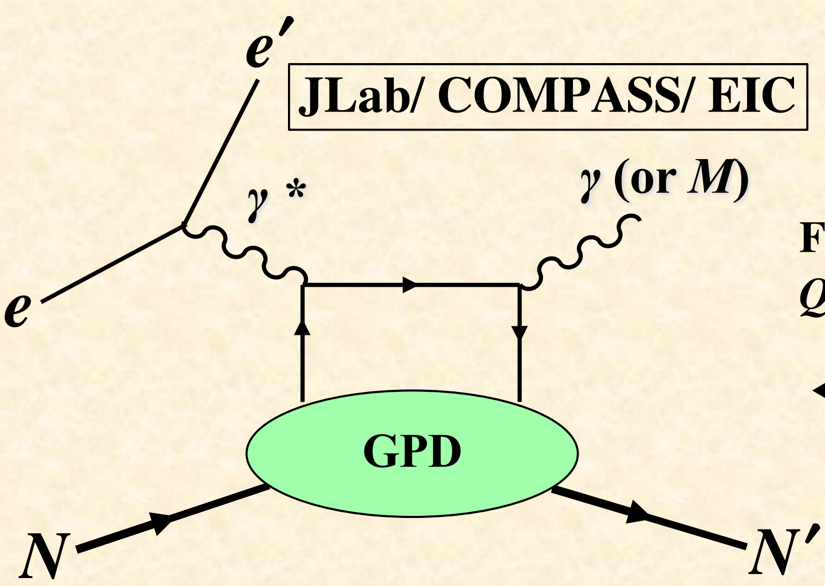
See also

SK, EPJ Web Conf. 208 (2019) 07003.

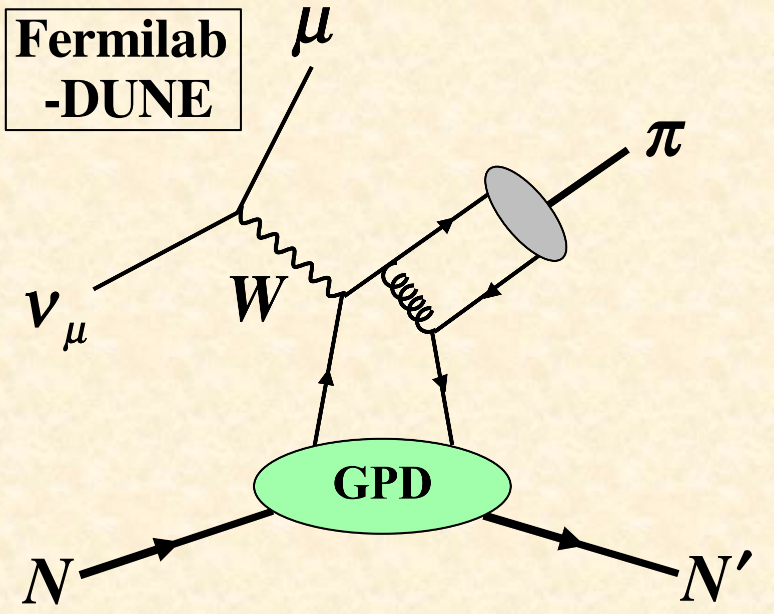
EIC yellow report, R. Abdul Khalek *et al*., arXiv:2103.05419, Sec. 7.5.2, Neutrino physics by SK and R. Petti.

SK and R. Petti, PoS (NuFact2021) 092.

Neutrino reactions for gravitational form factors @Fermilab-DUNE (Origins of hadron masses and pressures)

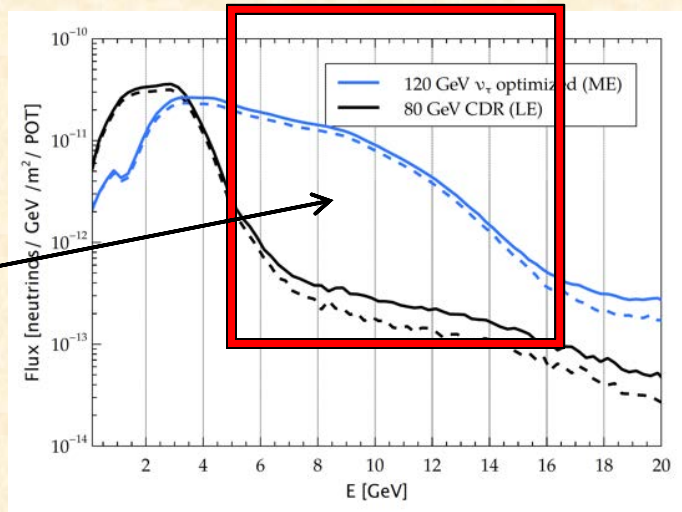


Factorization condition:
$Q^2 \gg |t|, \Lambda^2_{QCD}$



Deep Underground Neutrino Experiment (DUNE)
at Long-Baseline Neutrino Facility (LBNF)

High-energy part of the LBNF ν beam can be used for the GPD studies.



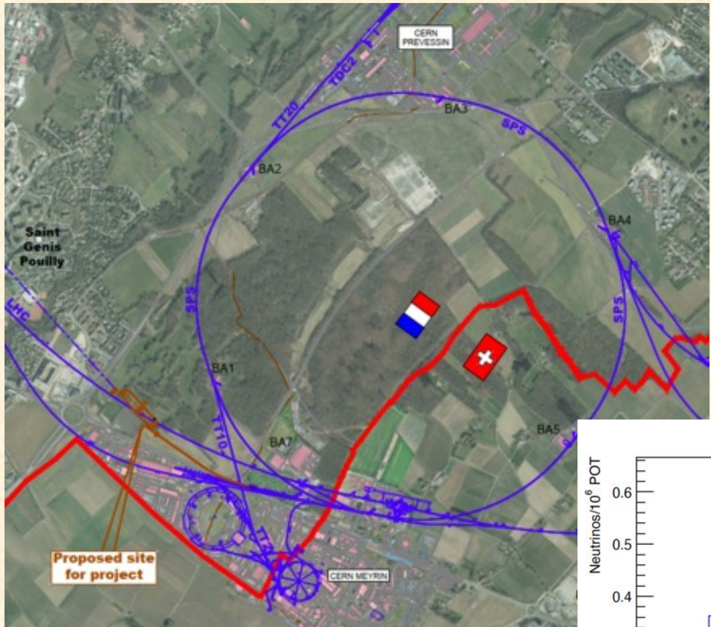
J. Rout *et al.*, PRD 102 (2020) 116018

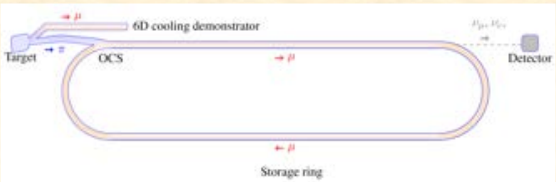
nuSTORM (Neutrinos from Stored Muons)

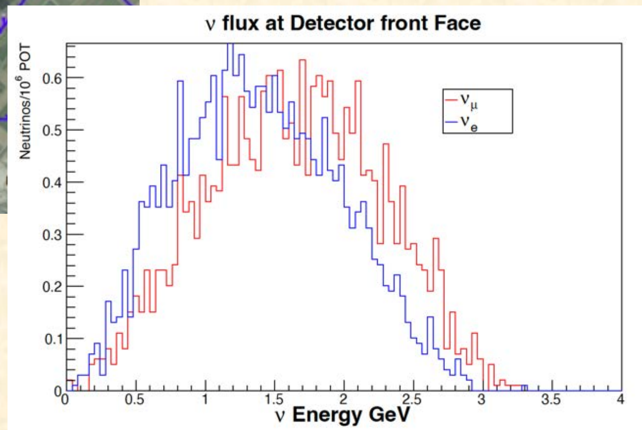
Fermilab

Feasibility Study, C. C. Ahdida *et al*., (2020);
L. A. Ruso *et al*., arXiv:2203.07545.

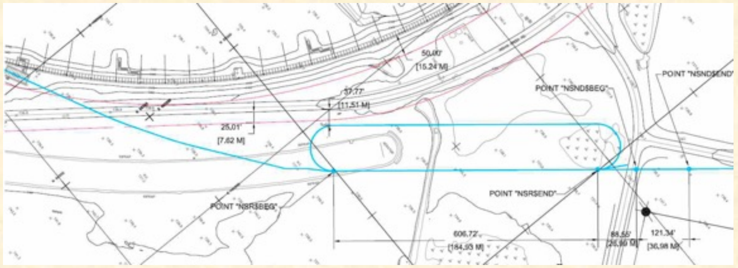
CERN

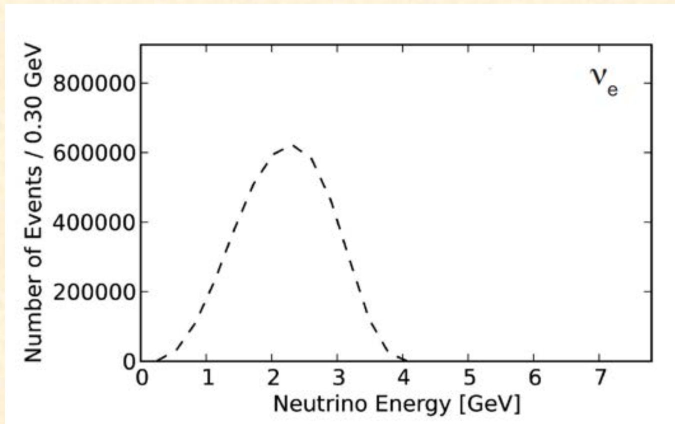






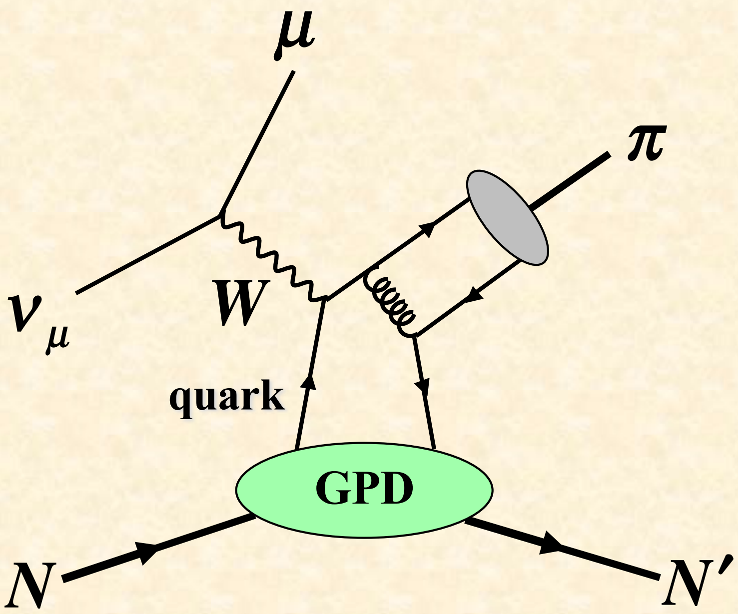
Letter of Intent, arXiv:1206.0294,
P. Kyberd *et al*. (2012);
Proposal, D. Adey *et al*., arXiv:1308.6822.
No recent update.





At this stage, the considered beam energy
is not high enough for structure-function studies;
however, high-energy option could be possible.
(personal communications: Xianguo Lu)
→ SK's talk at the nuSTORM-collaboration meeting
on July, 15, 2024
They could be interested in the higher-energy possibility.

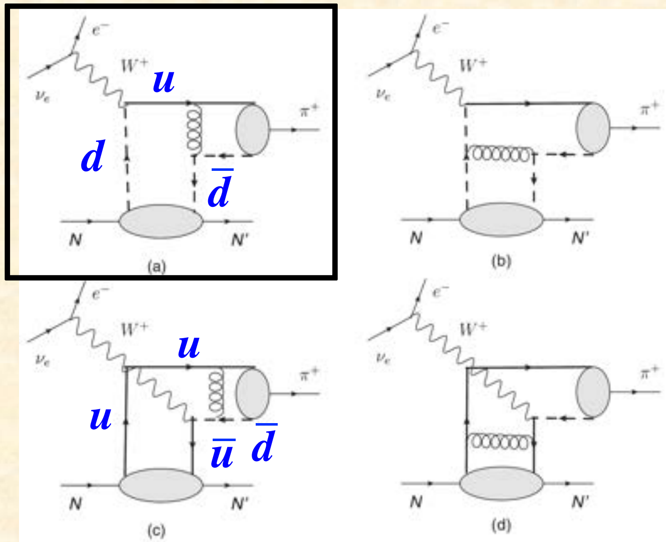
Recent work on pion production in neutrino reaction for GPD studies



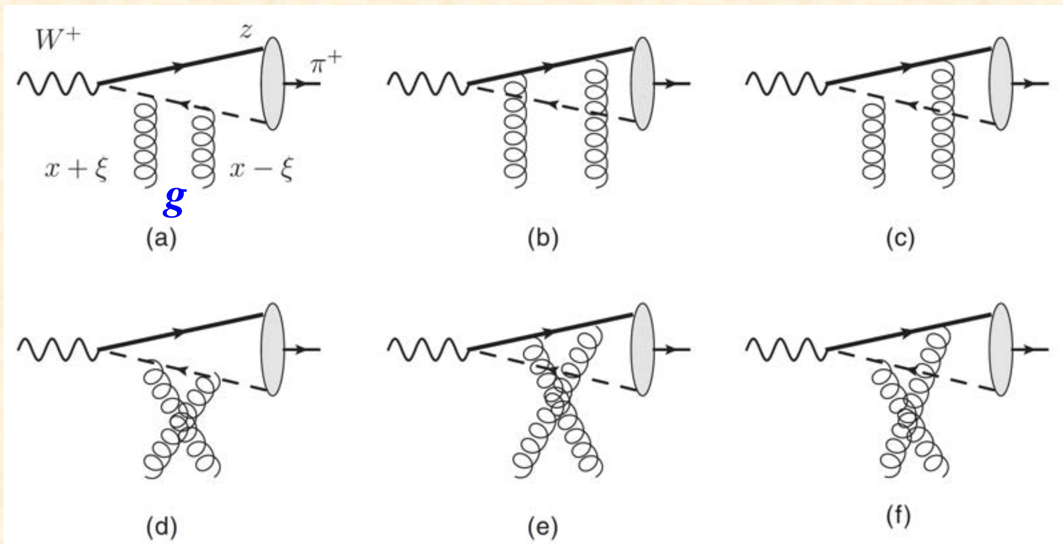
B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D 95, 114029 (2017).

There are several processes to contribute to the pion-production cross section, including the gluon GPD terms.

Quark GPDs



Gluon GPDs



Cross section formalism

B. Pire, L. Szymanowski, J. Wagner, Phys. Rev. D 95, 114029 (2017).

Cross section

$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} = \Gamma\,\varepsilon\,\sigma_L, \quad \varepsilon \simeq \frac{1-y}{1-y+y^2/2}, \quad \Gamma = \frac{G_F^2 Q^2}{32\,(2\pi)^4 (s-m_N^2)^2 y\,(1-\varepsilon)\sqrt{1+4x^2 m_N^2/Q^2}}$$

$$\sigma_L = \varepsilon_L^{*\mu} W_{\mu\nu} \varepsilon_L^\nu = \frac{1}{Q^2}\Big[(1-\xi^2)\Big\{\left|C_q\mathcal{H}_q + C_g\mathcal{H}_g\right|^2 + \left|C_q\tilde{\mathcal{H}}_q\right|^2\Big\} + \frac{\xi^4}{1-\xi^2}\Big\{\left|C_q\mathcal{E}_q + C_g\mathcal{E}_g\right|^2 + \left|C_q\tilde{\mathcal{E}}_q\right|^2\Big\}$$
$$-2\xi^2\,\mathrm{Re}\Big\{(C_q\mathcal{H}_q + C_g\mathcal{H}_g)(C_q\mathcal{E}_q + C_g\mathcal{E}_g)^*\Big\} - 2\xi^2\,\mathrm{Re}\Big\{C_q\tilde{\mathcal{H}}_q (C_q\tilde{\mathcal{E}}_q)^*\Big\}\Big]$$

Quark contributions

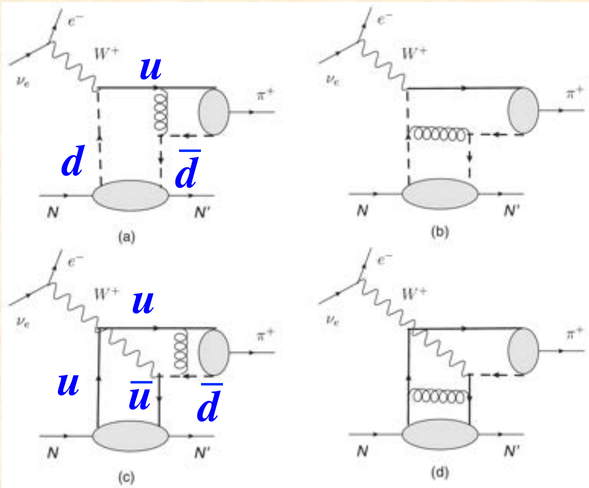
$$T_q = -i\frac{C_q}{2Q} N(p')\left[\mathcal{H}_q \hat{n} + \mathcal{E}_q \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} - \tilde{\mathcal{H}}_q \hat{n}\gamma_5 - \tilde{\mathcal{E}}_q \frac{\gamma_5 n\cdot\Delta}{2m_N}\right] N(p)$$

$$\mathcal{F}_q = 2f_\pi \int \frac{dz\;\phi_\pi(z)}{1-z} \int dx\, \frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

= (pion distribution amplitude) · (quark GPD)

$$F_q(x,\xi,t) \equiv F_d(x,\xi,t) - F_u(-x,\xi,t)$$

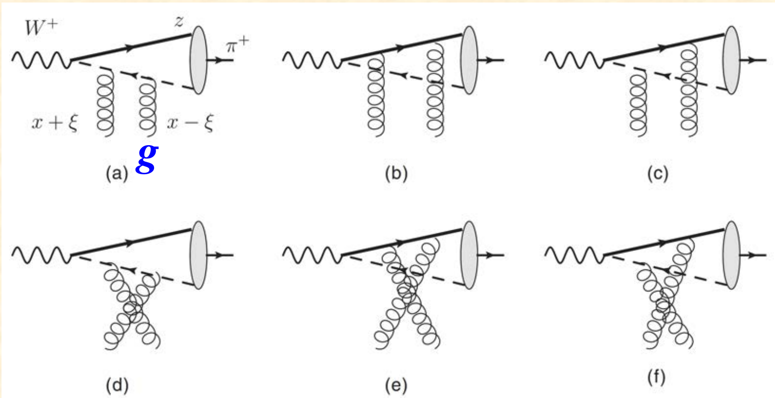
$$F = H,\ E,\ \tilde{H},\ \tilde{E}$$



Gluon contributions

$$T_g = -i\frac{C_g}{2Q} N(p')\left[\mathcal{H}^g \hat{n} + \mathcal{E}^g \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N}\right] N(p)$$

$$\mathcal{F}_g = \frac{8f_\pi}{\xi} \int \frac{dz\;\phi_\pi(z)}{z(1-z)} \int dx\, \frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$



GK (Goloskokov-Kroll) - 2013 parametrization

P. Kroll, H. Moutarde, F. Sabatie, Eur. Pjys. J. C 73 (2013) 2278.

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0}$$

$$= \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0}$$

$$= \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

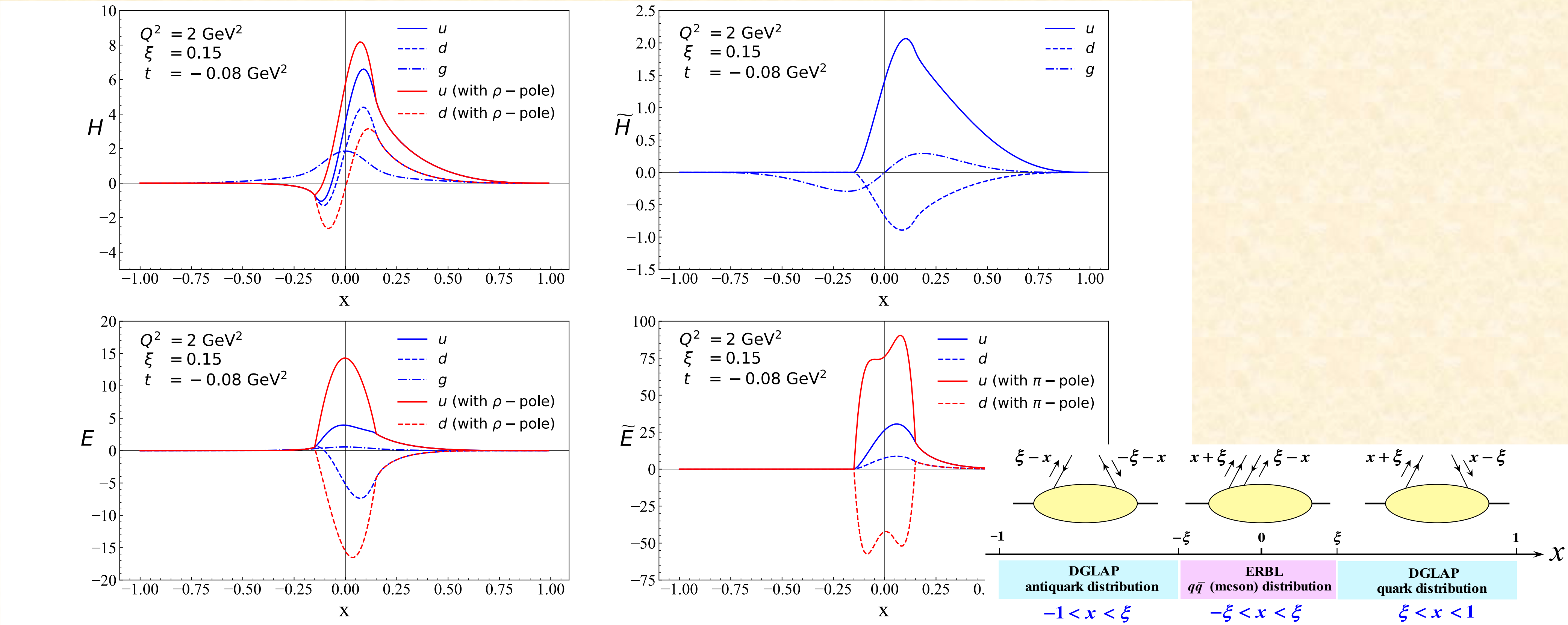
$$F_i(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-x) f_i(\beta,\alpha,t) + D_i(x',t)\Theta(\xi^2-x^2)$$

$$f_i(\beta,\alpha,t) = F_i(\beta, \xi=0, t=0) e^{tp_{h_i}(\beta)} \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)} \frac{\left[(1-|\beta|)^2-\alpha^2\right]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$\Theta(\xi^2-x^2) = \begin{cases} 1 & \xi^2 > x^2 \\ 0 & \xi^2 < x^2 \end{cases}, \quad p_{h_i}(\beta) = -\alpha'_{h_i}\ln\beta + b_{h_i}$$

$$F_i(\beta, \xi=0, t=0) = \beta^{-\delta_i}(1-\beta)^{2n_i+1}\sum_{j=0}^{3} c_{f_j}\beta^{j/2},$$

parameters determined by global analysis



Cross sections

π^+ production: $\nu p \to \ell^- \pi^+ p$

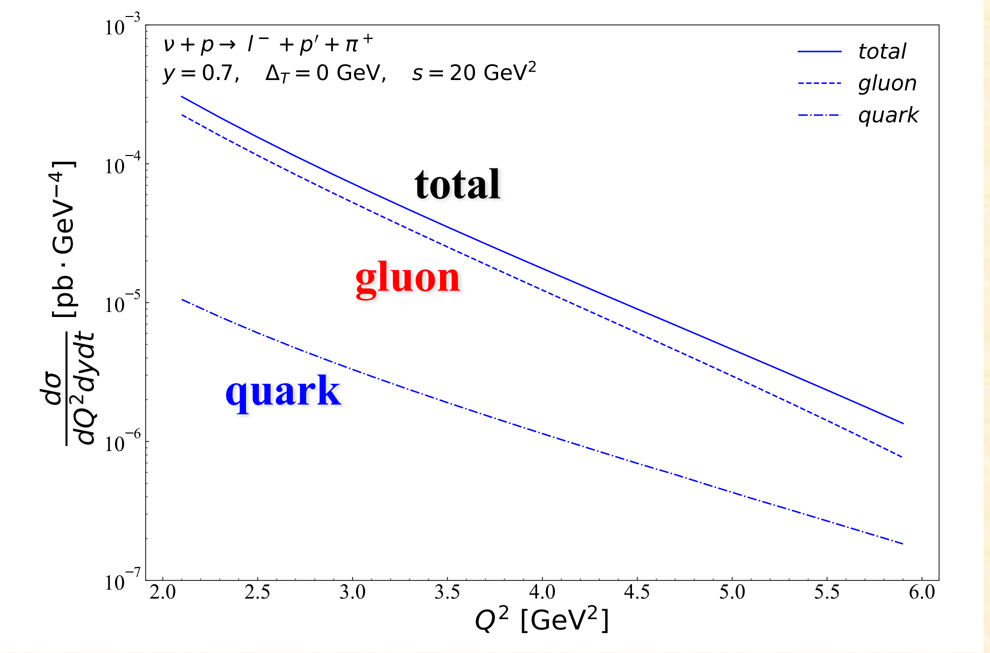
$$\mathcal{F}_q = 2 f_\pi \int \frac{dz\, \phi_\pi(z)}{1-z} \int dx \frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

$$\mathcal{F}_g = \frac{8 f_\pi}{\xi} \int \frac{dz\, \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$

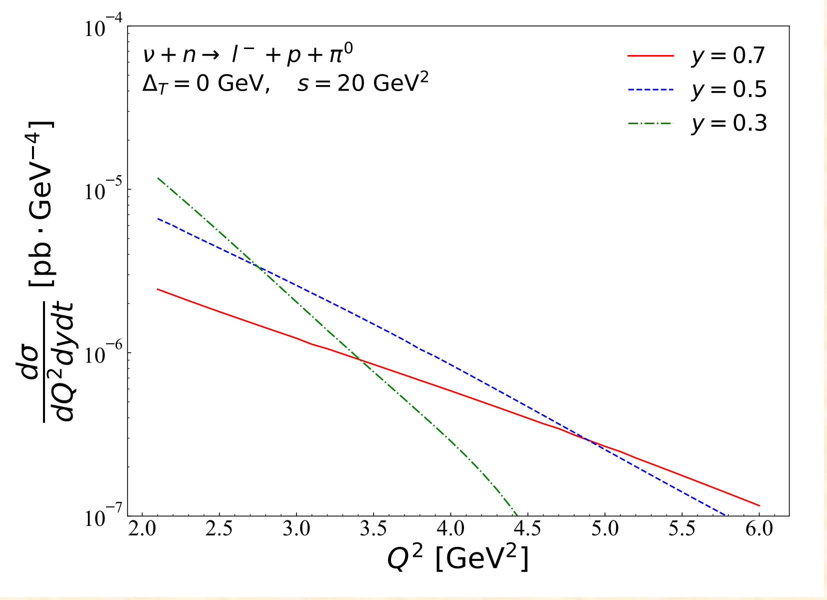
$$\frac{\mathcal{F}_q}{\mathcal{F}_g} \sim \frac{\xi}{8} = \frac{0.1 \sim 0.3}{8} = 0.01 \sim 0.04$$

$= \text{a few } \% \ll 1$

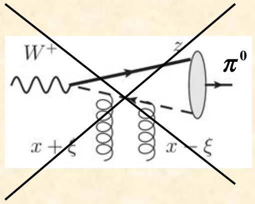
gluon ≫ quark



π^0 production: $\nu n \to \ell^- \pi^0 p$



no gluon



no gluon for π^0

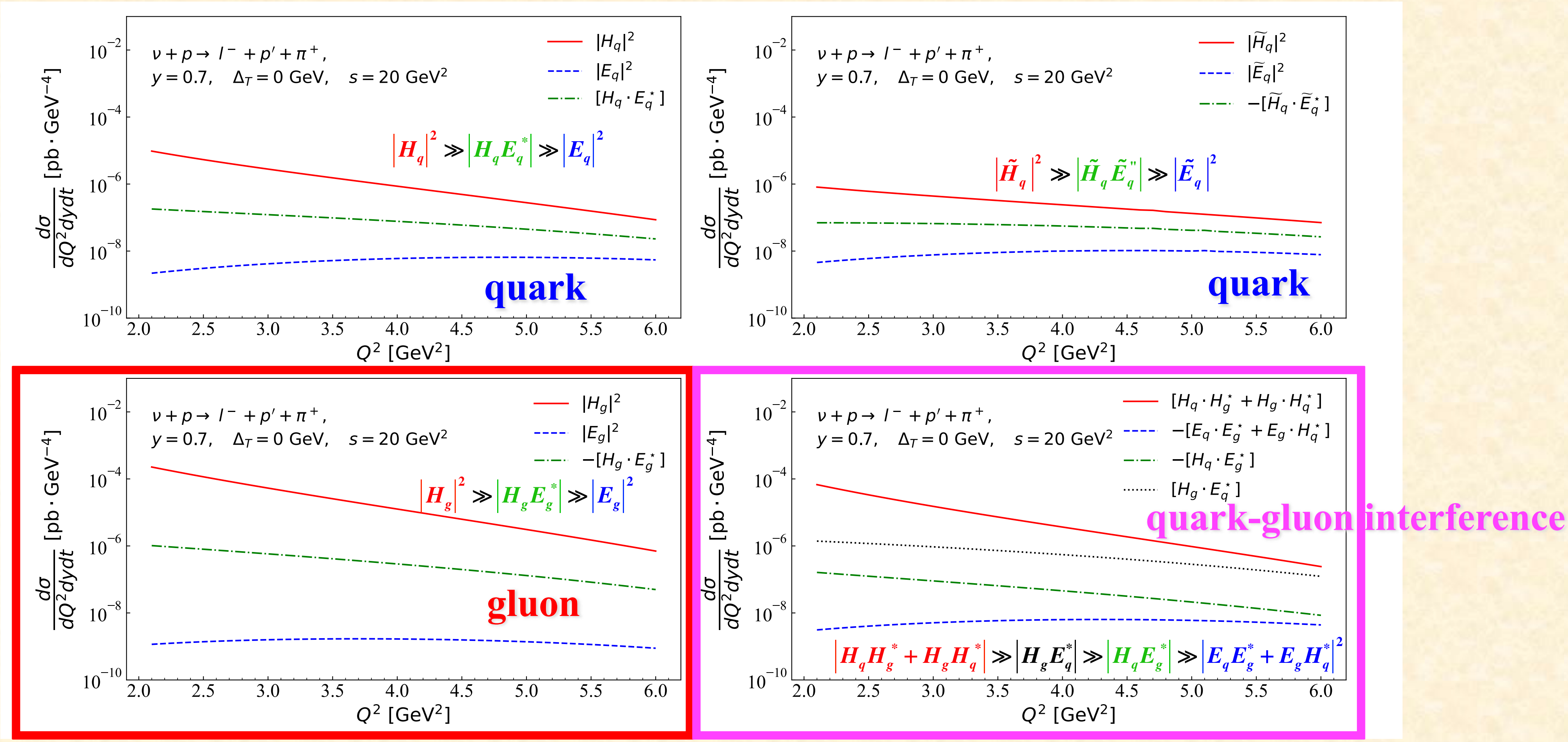
Neutrino GPD studies are complementary to the charged-lepton projects.

- **Gluon GPDs could be probed in charged-pion production.**
- **Quark GPDs could be probed in π^0 production.**
- **Flavor dependece of quark GPDs could be investigated.**

Contribution of each term to the π^+-production cross section

$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} \propto \frac{1}{Q^2}\Big[(1-\xi^2)\Big\{\left|C_q\mathcal{H}_q + C_g\mathcal{H}_g\right|^2 + \left|C_q\tilde{\mathcal{H}}_q\right|^2\Big\} + \frac{\xi^4}{1-\xi^2}\Big\{\left|C_q\mathcal{E}_q + C_g\mathcal{E}_g\right|^2 + \left|C_q\tilde{\mathcal{E}}_q\right|^2\Big\}$$

$$-2\xi^2\,\mathrm{Re}\Big\{(C_q\mathcal{H}_q + C_g\mathcal{H}_g)(C_q\mathcal{E}_q + C_g\mathcal{E}_g)^*\Big\} - 2\xi^2\,\mathrm{Re}\Big\{C_q\tilde{\mathcal{H}}_q(C_q\tilde{\mathcal{E}}_q)^*\Big\}\Big]$$

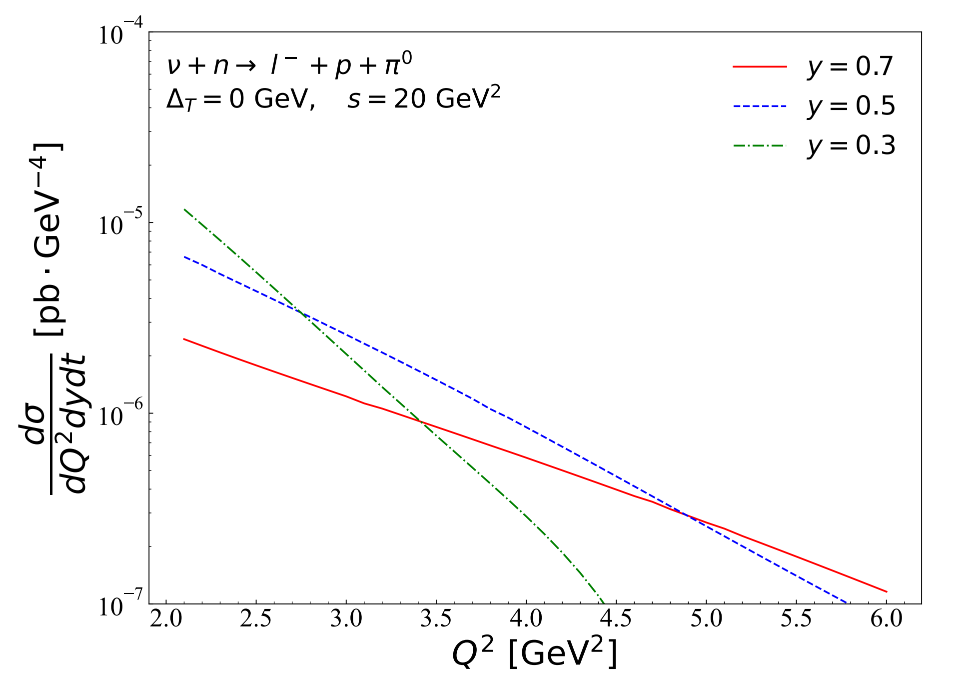


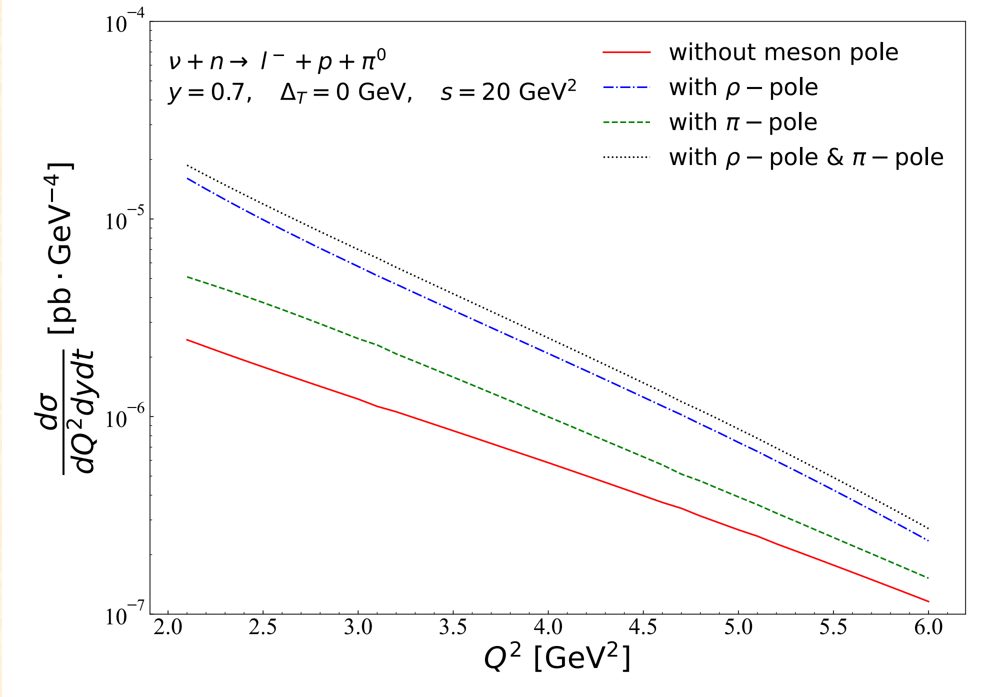
$$H_g > H_q > \tilde{H}_q > E_q,\ \tilde{E}_q,\ E_g$$

- π^+ production is senstive to gluon $\mathcal{H}_g$.
- Sizable quark-gluon interference $\mathcal{H}_q\mathcal{H}_g$.

Contribution of each term to the π^0-production cross section

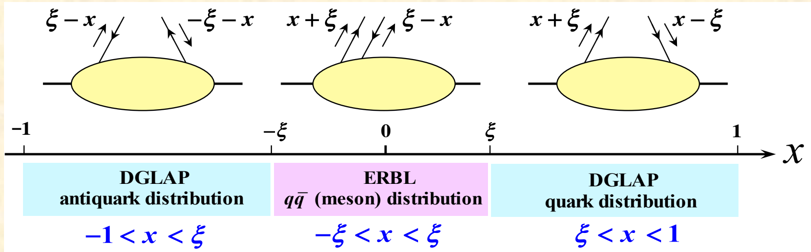
$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} \propto \frac{1}{Q^2}\Big[(1-\xi^2)\Big\{\left|C_q \mathcal{H}_q + \cancel{C_g \mathcal{H}_g}\right|^2 + \left|C_q \tilde{\mathcal{H}}_q\right|^2\Big\} + \frac{\xi^4}{1-\xi^2}\Big\{\left|C_q \mathcal{E}_q + \cancel{C_g \mathcal{E}_g}\right|^2 + \left|C_q \tilde{\mathcal{E}}_q\right|^2\Big\}$$
$$-2\xi^2\,\mathrm{Re}\Big\{(C_q \mathcal{H}_q + \cancel{C_g \mathcal{H}_g})(C_q \mathcal{E}_q + \cancel{C_g \mathcal{E}_g})^*\Big\} - 2\xi^2\,\mathrm{Re}\Big\{C_q \tilde{\mathcal{H}}_q (C_q \mathcal{E}_q)^*\Big\}\Big]$$





$\cancel{H_g} > H_q > \tilde{H}_q > E_q,\ \tilde{E}_q,\ \cancel{E_g}$

- π^0 production is senstive to quark $\mathcal{H}_q$.
- GPDs in the ERBL (Efremov-Radyushkin-Brodsky-Lepage) region could be probed.



Gravitational form factors, Prospects on neutrino GPD project

Nucleon mass: $M = \left\langle N(p) \right| \int d^3x\, T^{00}(x) \left| N(p) \right\rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F_\alpha^{\ \nu}(x) = T_q^{\mu\nu}(x) + T_g^{\mu\nu}(x)$$

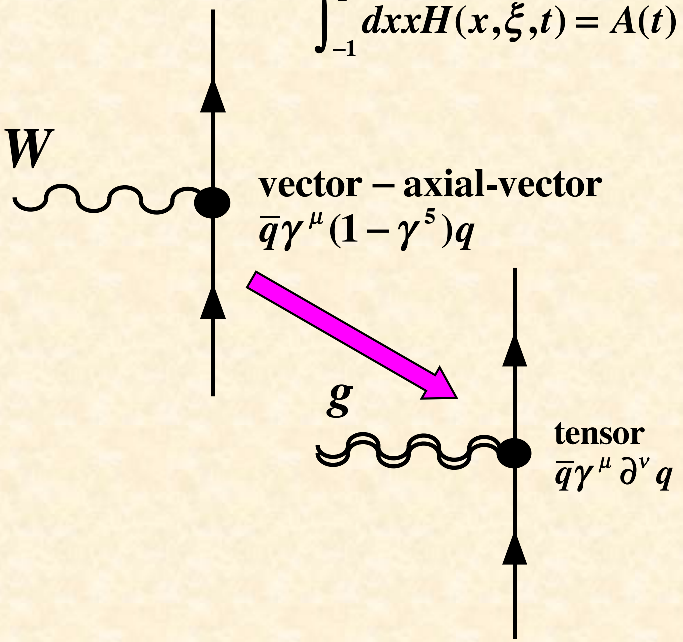
Gravitational form factors: A, B, C, D

$$\left\langle N(p') \right| \int d^3x\, T^{\mu\nu}(x) \left| N(p) \right\rangle = u(p') \left[A\gamma^{\{\mu}\bar{P}^{\nu\}} + B\frac{\bar{P}^{\{\nu} i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M} + CMg^{\nu\nu} + D\frac{\Delta^\mu\Delta^\nu - g^{\nu\nu}\Delta^2}{M} \right] u(p)$$

T^{00}: $$\left\langle N(p') \right| \int d^3x\, T^{00}(x) \left| N(p) \right\rangle = 2ME\left[A(t) - \frac{t}{4M^2}\{A(t) - 2B(t) + D(t)\} \right]$$

GPDs and gravitational form factors:

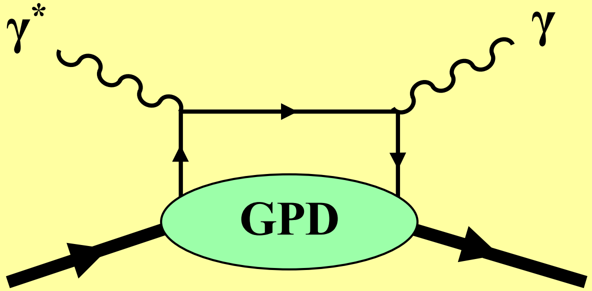
$$\int_{-1}^{1} dx\, x H(x,\xi,t) = A(t) + \xi^2 D(t), \quad \int_{-1}^{1} dx\, x E(x,\xi,t) = B(t) - \xi^2 D(t)$$

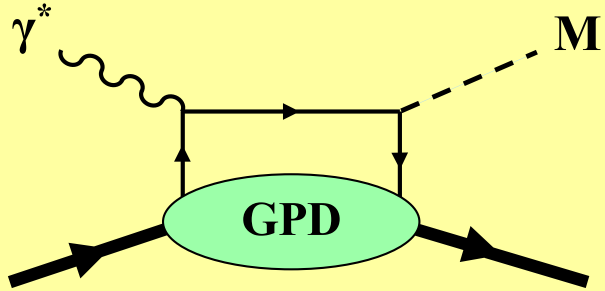


- Neutrino-scattering experiments (LBNF, nuSTORM) are valuable and complementary to JLab, AMBER, KEK-B, and the other facility projects in the sense that the cross sections are sensitive to quark flavor.
- This project is already in progress.
  The new detector, which was the basis of various GPD measurements, was selected by the DUNE collaboration to be part of the near detector complex (R. Petti, 2021).

  nuSTORM at CERN?

Charged-lepton scattering on spacelike GPDs

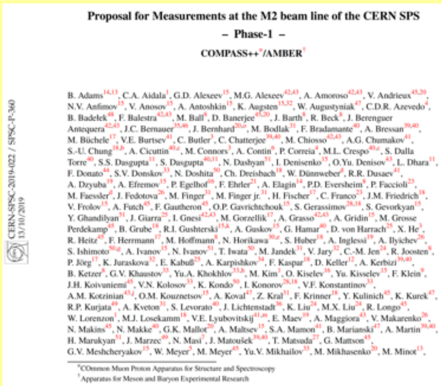




DVCS (Deeply Virtual Compton Scattering) **DVMP (Deeply Virtual Meson Production)**

Jefferson Lab

CERN-AMBER



Proposal for Measurements at the M2 beam line of the CERN SPS

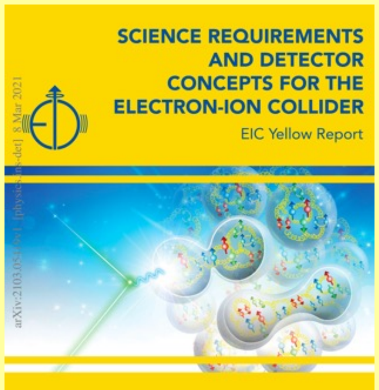
– Phase-1 –

COMPASS++*/AMBER†

CERN-SPSC-2019-022 / SPSC-P-360

13/10/2019

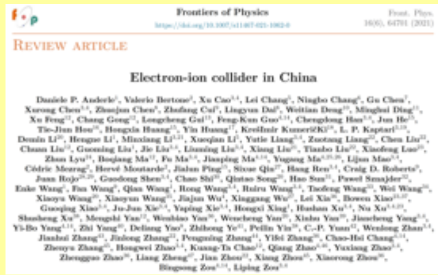
EIC-US



SCIENCE REQUIREMENTS AND DETECTOR CONCEPTS FOR THE ELECTRON-ION COLLIDER

EIC Yellow Report

EicC



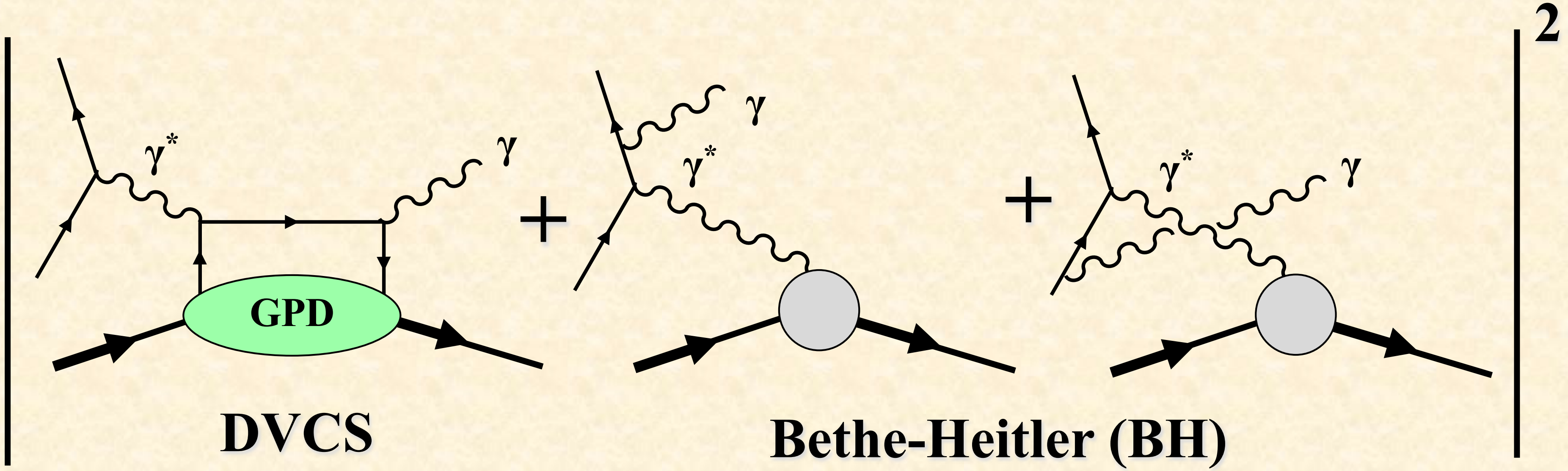
Frontiers of Physics

Review Article

Electron-ion collider in China

Deeply Virtual Compton Scattering (DVCS)

$d\sigma =$



$$\frac{d\sigma(eN \to e'N'\gamma)}{dQ^2\, dx\, dt\, d\phi} \propto \left|T_{DVCS} + T_{BH}\right|^2$$

HERMES, JLab, COMPASS/AMBER, EIC, EicC, ⋯

e.g. **Polarized beam:** $d\sigma(e^{\uparrow}) - d\sigma(e^{\downarrow}) \propto T_{BH} * \mathrm{Im}\left(T_{DVCS}\right)$

$$\mathrm{Re}\,\mathcal{H}_q = e_q^2 \mathcal{P}\int_0^1 dx \left[H^q(x,\xi,t) - H^q(-x,\xi,t)\right]\left(\frac{1}{\xi - x} + \frac{1}{\xi + x}\right)$$

$$\mathrm{Im}\,\mathcal{H}_q = \pi e_q^2 \left[H^q(\xi,\xi,t) - H^q(-\xi,\xi,t)\right]$$

- **Polarized beam, unpolarized target:** $\mathrm{Im}\left\{\mathcal{H}, \tilde{\mathcal{H}}, \mathcal{E}\right\}$
- **Unpolarized beam, longigudinally-polarized target:** $\mathrm{Im}\left\{\mathcal{H}, \tilde{\mathcal{H}}\right\}$
- **Polarized beam, longigudinally-polarized target:** $\mathrm{Re}\left\{\mathcal{H}, \tilde{\mathcal{H}}\right\}$
- **Unpolarized beam, transversely-polarized target:** $\mathrm{Im}\left\{\mathcal{H}, \mathcal{E}\right\}$

Recent measurement at JLab

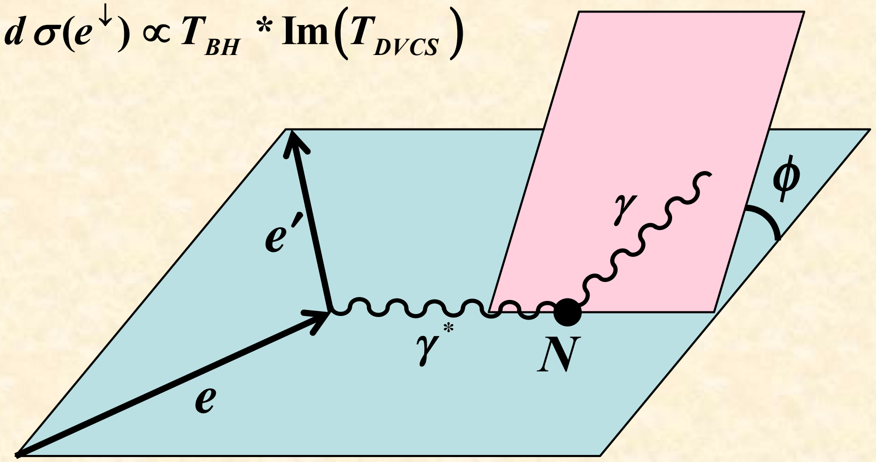
F. Georges et al., PRL 128 (2022) 252002.

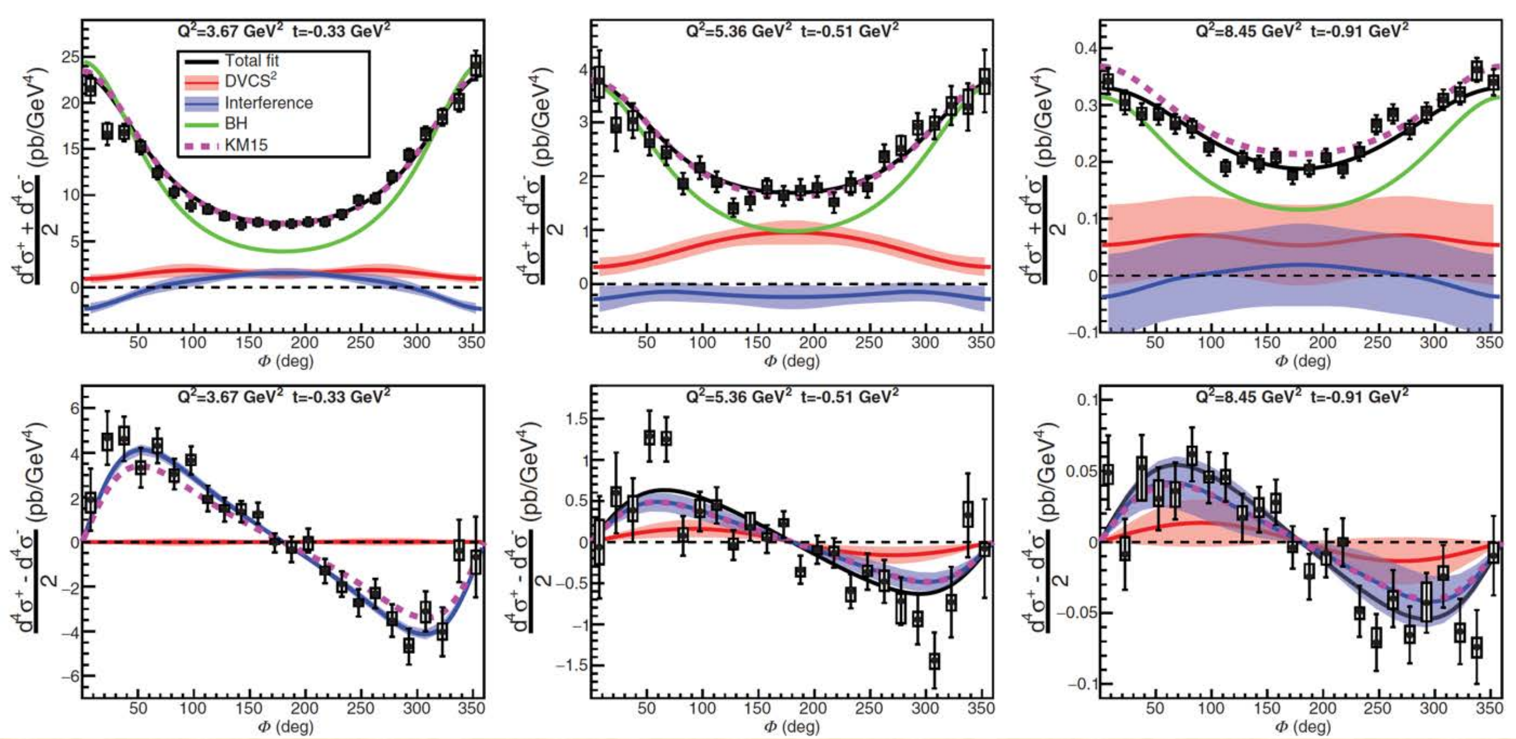
$$\frac{d\sigma(eN \to e'N'\gamma)}{dQ^2\,dx\,dt\,d\phi} \propto \left|T_{DVCS} + T_{BH}\right|^2,$$ e.g. Polarized beam: $d\sigma(e^{\uparrow}) - d\sigma(e^{\downarrow}) \propto T_{BH} * \mathrm{Im}\left(T_{DVCS}\right)$

$$\mathrm{Re}\,\mathcal{H}_q = e_q^2 \mathcal{P}\int_0^1 dx \left[H^q(x,\xi,t) - H^q(-x,\xi,t)\right]\left(\frac{1}{\xi - x} + \frac{1}{\xi + x}\right)$$

$$\mathrm{Im}\,\mathcal{H}_q = \pi e_q^2 \left[H^q(\xi,\xi,t) - H^q(-\xi,\xi,t)\right]$$

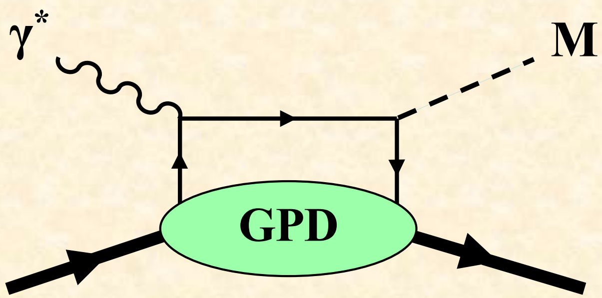
- Polarized beam, unpolarized target: $\mathrm{Im}\left\{\mathcal{H}, \tilde{\mathcal{H}}, \mathcal{E}\right\}$



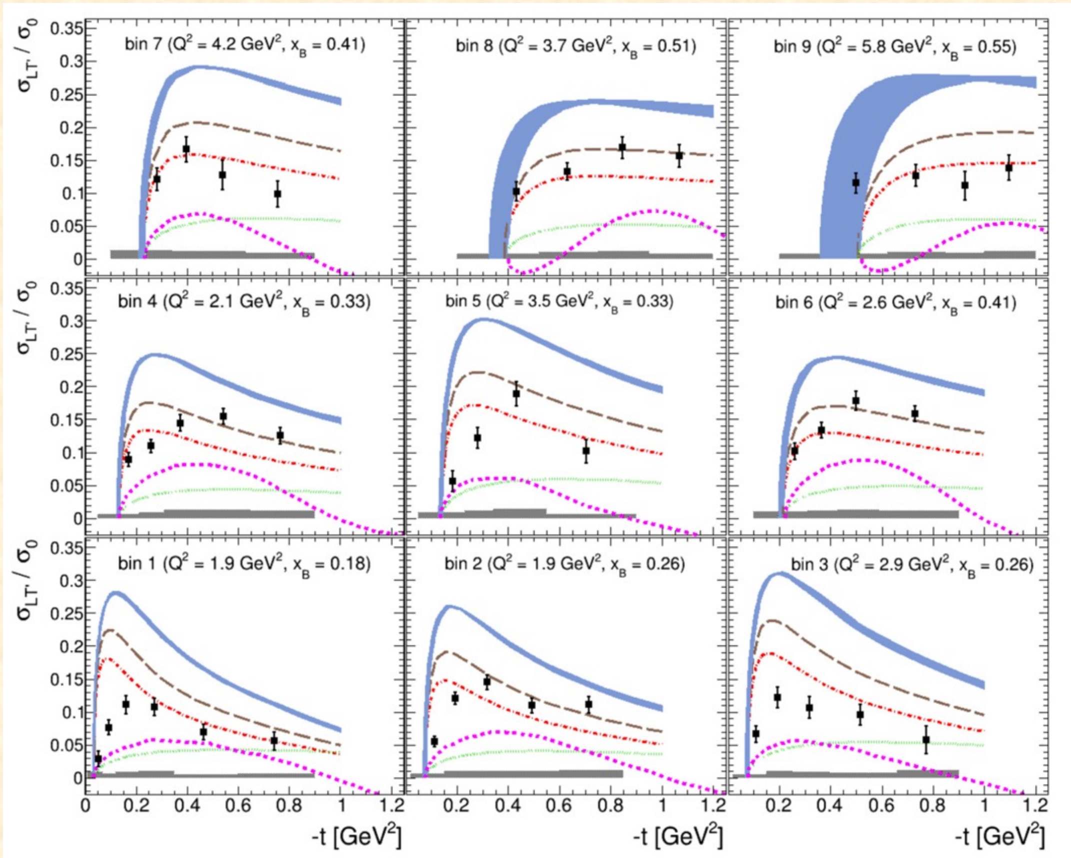


ϕ dependent cross sections

Deeply Virtual Meson Production (DVMP)



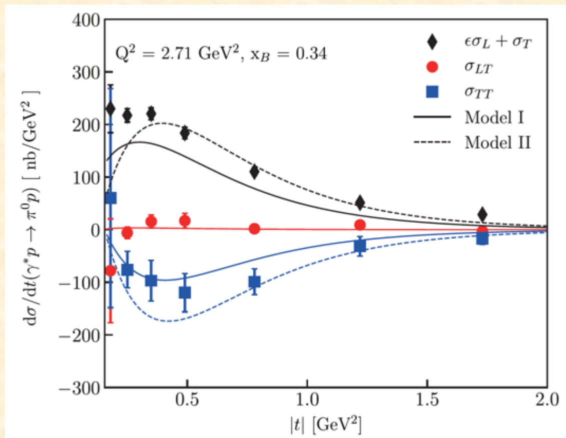
$$\frac{d\sigma(eN \to e'N'\gamma)}{dQ^2\,dx\,dt\,d\phi} \propto \sigma_T + \varepsilon\sigma_L + \varepsilon\sigma_{TT}\cos(2\phi) + \sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT}\cos(\phi) + P_b\sqrt{2\varepsilon(1-\varepsilon)}\sigma_{LT'}\sin(\phi)$$



S. Diehl *et al.*, PLB 839 (2023) 137761.

	Meson	Flavor
$\mathcal{H}_T, \mathcal{E}_T$	π^+	$\Delta u - \Delta d$
	π^0	$2\Delta u + \Delta d$
	η	$2\Delta u - \Delta d + 2\Delta s$
$\mathcal{H}, \mathcal{E}$	ρ^+	$u - d$
	ρ^0	$2u + d$
	ω	$2u - d$
	ϕ	g

K. Joo, EIC Asia workshop (2024).

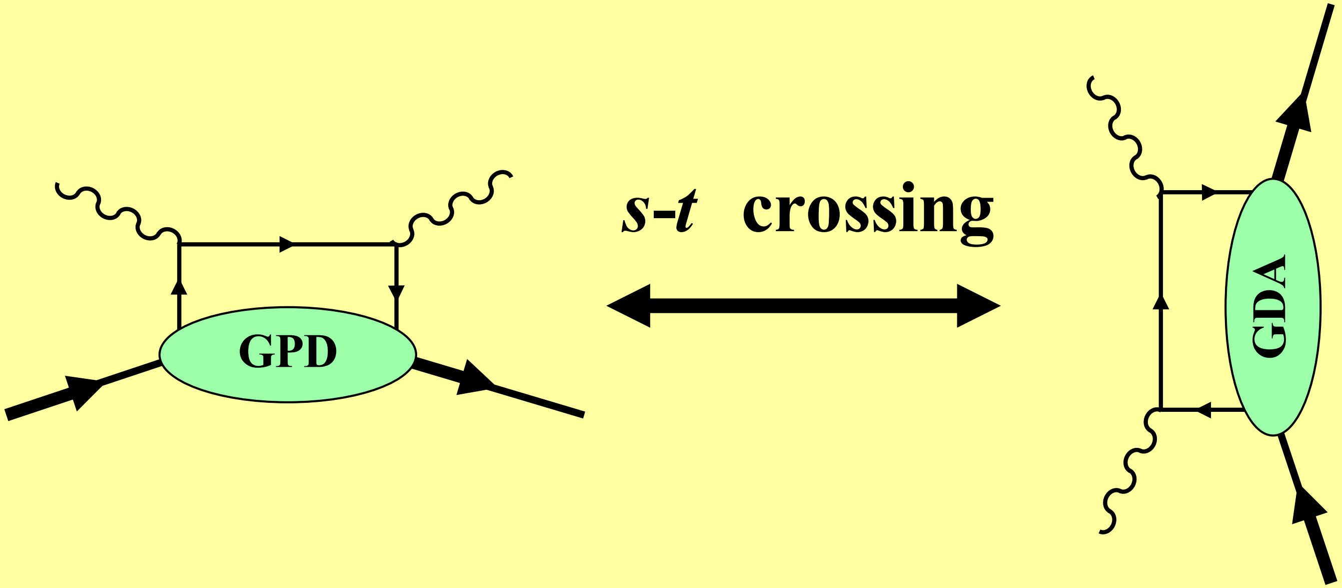


S. V. Goloskokov, Ya-Ping Xie, Xurong Chen, Chin. Phys. C 46 (2022) 123101.

e^+e^- facilities on timelike GPDs

Generalized Distribution Amplitudes
(GDAs = timelike GPDs)

Spacelike GPDs **GDA = Timelike GPDs**



Extraction of GDAs and gravitational form factors from KEKB data.

SK, Q.-T. Song, O. Teryaev, Phys. Rev. D 97 (2018) 014020.

GPD $H_q^h(x,\xi,t)$ and GDA(= timelike GPD) $\Phi_q^{hh}(z,\zeta,W^2)$

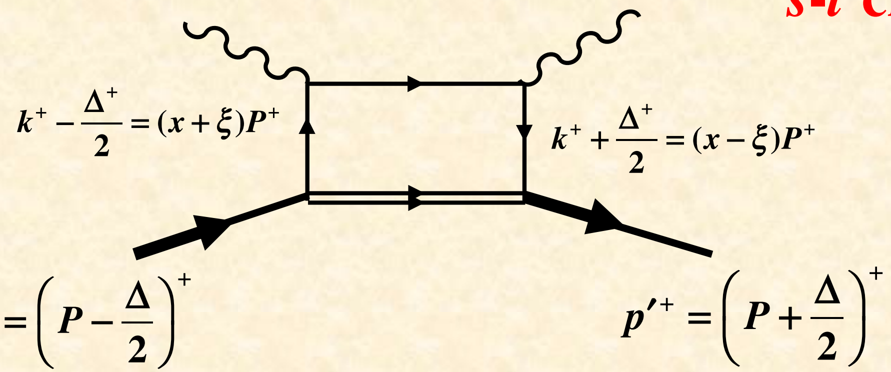
GPD: $H_q(x,\xi,t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | h(p) \rangle \Big|_{y^+=0,\vec{y}_\perp=0}$, $\quad P^+ = \frac{(p+p')^+}{2}$

GDA: $\Phi_q(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p)\bar{h}(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | 0 \rangle \Big|_{y^+=0,\vec{y}_\perp=0}$

DA: $\Phi_q^\pi(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2)\gamma^+\gamma_5\psi(y/2) | 0 \rangle \Big|_{y^+=0,\vec{y}_\perp=0}$

$H_q^h(x,\xi,t)$ ⟷ $\Phi_q^{hh}(z,\zeta,W^2)$

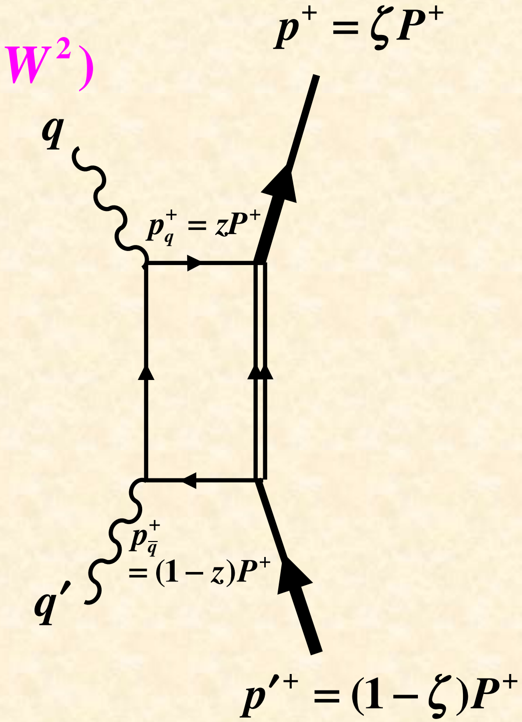
***s*-*t* crossing**



$z \Leftrightarrow \frac{1-x/\xi}{2}$

$\zeta \Leftrightarrow \frac{1-1/\xi}{2}$

$W^2 \Leftrightarrow t$



JLab / COMPASS | **KEKB**

$P = \frac{p+p'}{2}$, $\Delta = p' - p$

Bjorken variable: $x = \frac{Q^2}{2p\cdot q}$

Momentum transfer squared: $t = \Delta^2$

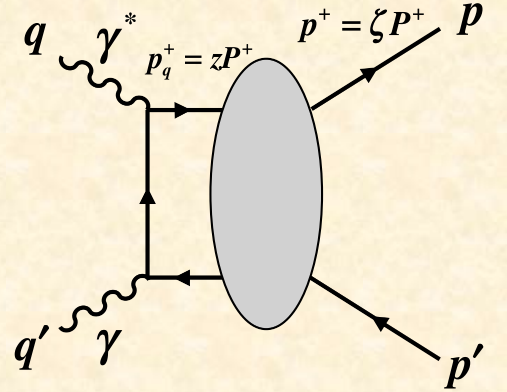
Skewdness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

Bjorken variable for $\gamma\gamma^*$: $z = \frac{Q^2}{2q\cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{P^+} = \frac{1+\beta\cos\theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

Cross section for $\gamma^* \gamma \to \pi^0 \pi^0$



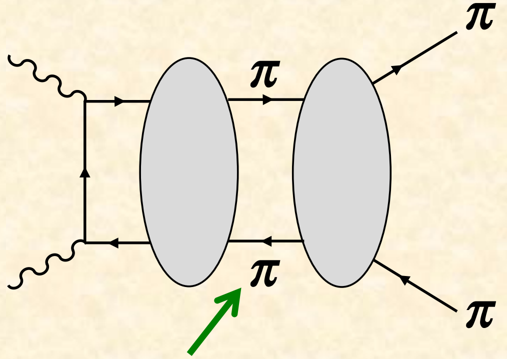
$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}\overline{\sum_{\lambda,\lambda'}}|\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i\int d^4\xi e^{-i\xi\cdot q}\left\langle \pi(p)\pi(p') \middle| TJ_{em}^\mu(\xi)J_{em}^\nu(0) \middle| 0 \right\rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2}\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = -\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')g_T^{\mu\nu}\sum_q \frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

GDA (timelike GPD): $\Phi_q^{\pi\pi}(z,\zeta,s) = \int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle \pi(p)\pi(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|0\rangle\Big|_{y^+=0,\vec{y}_\perp=0}$

$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}|A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$



Including intermediate resonance contributions

- **Continuum: GDAs without intermediate-resonance contribution**

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N_\pi z^\alpha(1-z)^\alpha(2z-1)\zeta(1-\zeta)F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{\left[1+(s-4m_\pi^2)/\Lambda^2\right]^{n-1}}, \quad n = 2 \text{ according to constituent counting rule}$$

- **Resonances: There exist resonance contributions to the cross section.**

$$\sum_q \Phi_q^{\pi\pi}(z,\zeta,W^2) = 18N_f z^\alpha(1-z)^\alpha(2z-1)\left[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)\right]$$

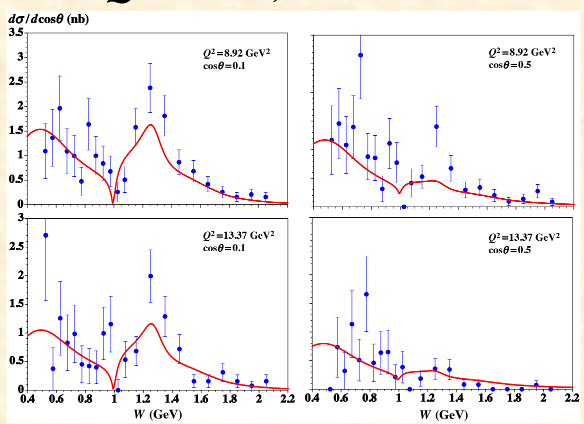
$$P_2(x) = \frac{1}{2}(3x^2-1)$$

$\tilde{B}_{10}(W) =$ resonance $[f_0(500), f_0(980)]$ + continuum

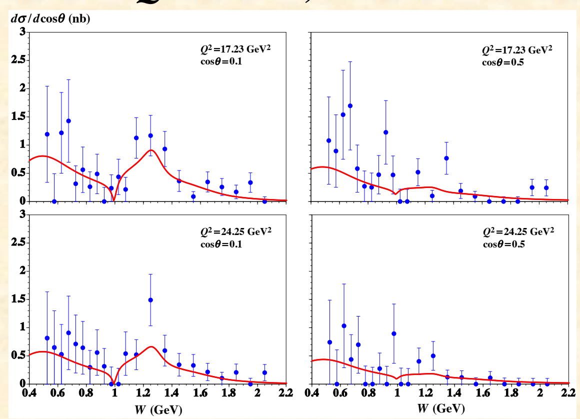
$\tilde{B}_{12}(W) =$ resonance$[f_2(1270)]$ + continuum

Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.

$Q^2 = 8.92,\ 13.37\ \text{GeV}^2$



$Q^2 = 17.23,\ 24.25\ \text{GeV}^2$



Gravitational form factors and radii for pion

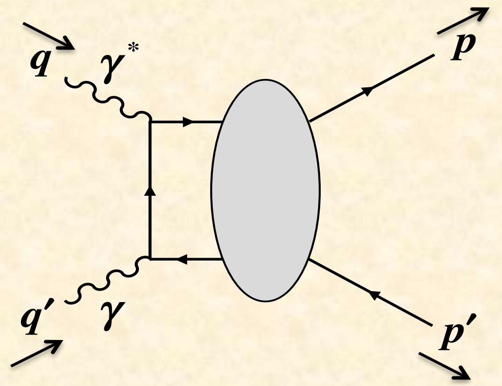
$$\int_0^1 dz(2z-1)\Phi_q^{\pi^0\pi^0}(z,\zeta,s) = \frac{2}{(P^+)^2}\langle \pi^0(p)\pi^0(p')|T_q^{++}(0)|0\rangle$$

$$\langle \pi^0(p)\pi^0(p')|T_q^{\mu\nu}(0)|0\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu} - P^\mu P^\nu\right)\Theta_{1,q}(s) + \Delta^\mu\Delta^\nu\Theta_{2,q}(s)\right]$$

$$P = \frac{p+p'}{2}, \qquad \Delta = p' - p$$

$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}$, $\Theta_{2,q}$: gravitational form factos for pion



See also Hyeon-Dong Son, Hyun-Chul Kim, PRD90 (2014) 111901.

Analyiss of $\gamma^*\gamma \to \pi^0\pi^0$ cross section

$\Rightarrow$ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z,\zeta,s)$

$\Rightarrow$ Timelike gravitational form factors $\Theta_{1,q}(s)$, $\Theta_{2,q}(s)$

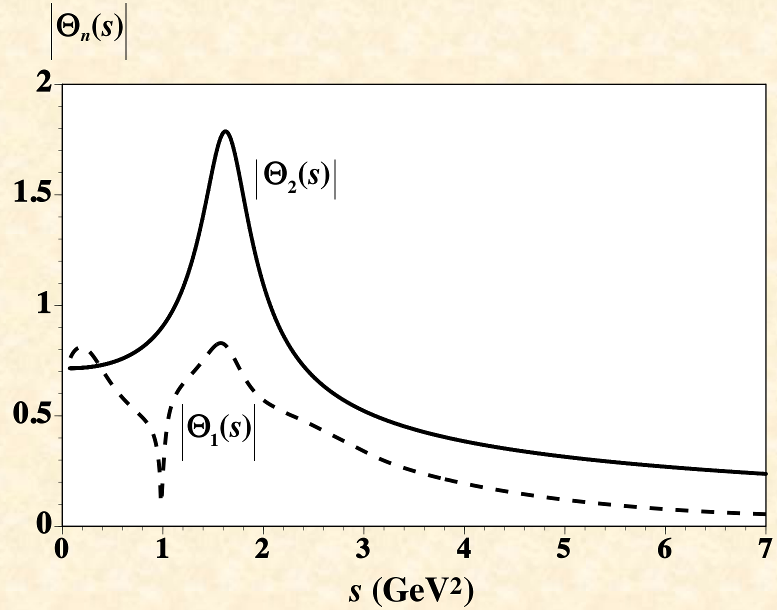
$\Rightarrow$ Spacelike gravitational form factors $\Theta_{1,q}(t)$, $\Theta_{2,q}(t)$

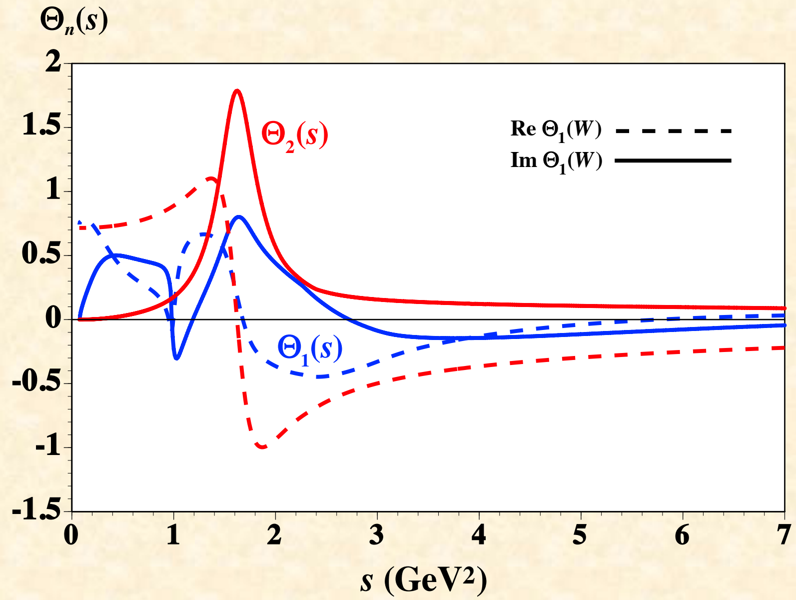
$\Rightarrow$ Gravitational radii of pion

Gravitational form factors:

Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.

Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009; Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12 (2018) 008; K. Tanaka, JHEP 01 (2019) 120.





Spacelike gravitational form factors and radii for pion

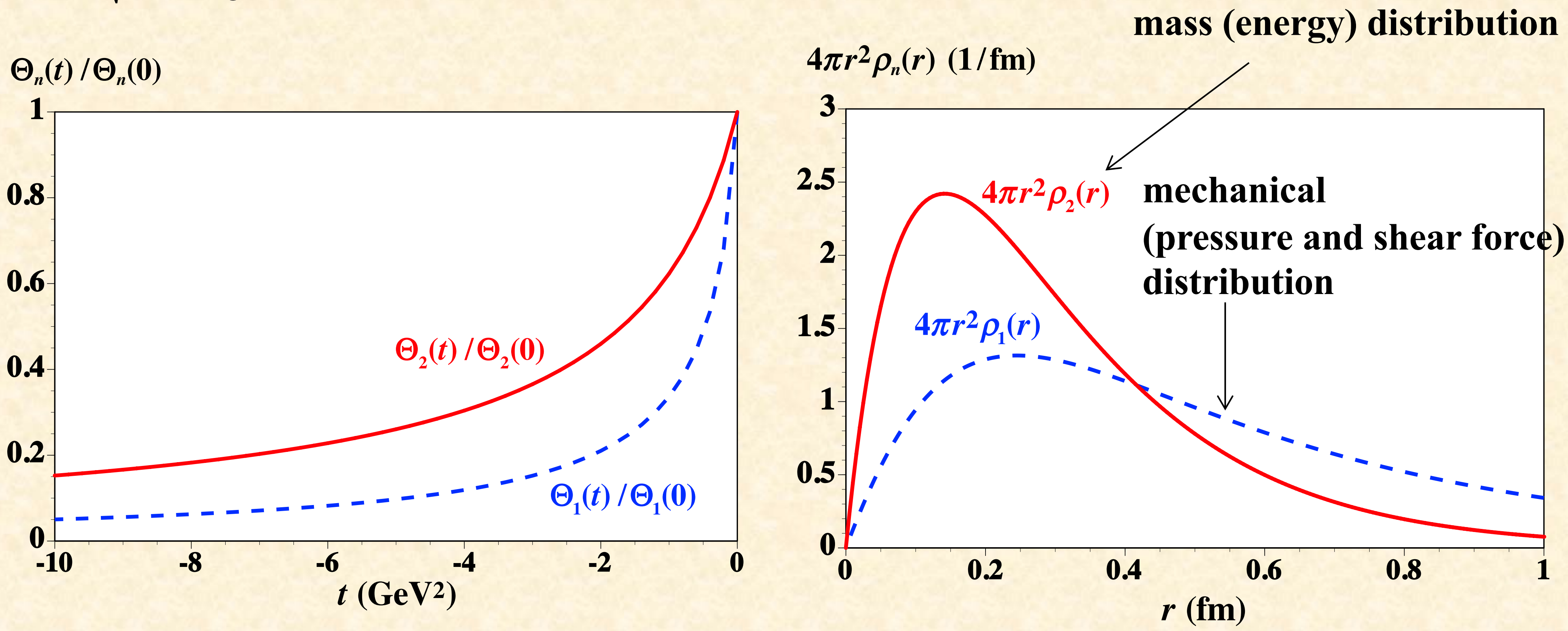
$$F(s)=\Theta_1(s),\ \Theta_1(s),\quad F(t)=\int_{4m_\pi^2}^{\infty} ds\,\frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r)=\frac{1}{(2\pi)^3}\int d^3q\,e^{-i\vec{q}\cdot\vec{r}}F(q)=\frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\,e^{-\sqrt{s}r}\,\mathrm{Im}\,F(s)$$

This is the first report on gravitational radii of hadrons from actual experimental measurements.

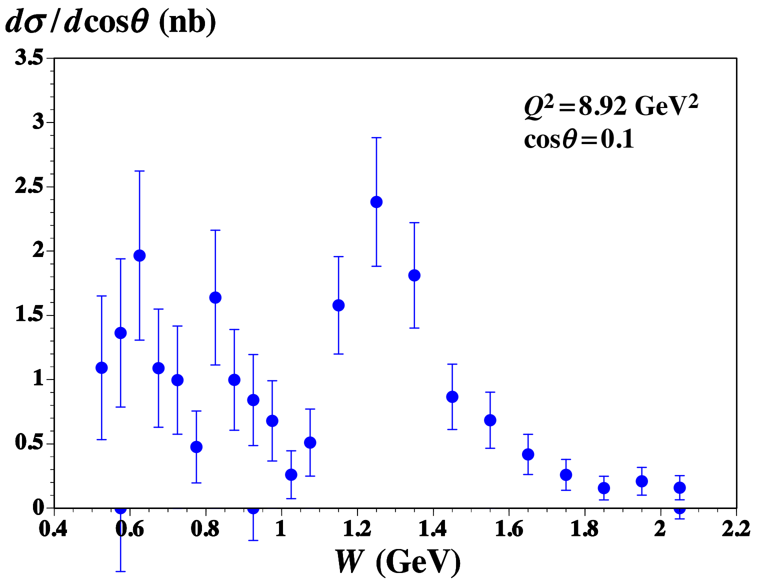
$$\sqrt{\langle r^2\rangle_{\text{mass}}}=0.32\sim0.39\ \text{fm},\quad \sqrt{\langle r^2\rangle_{\text{mech}}}=0.82\sim0.88\ \text{fm}$$

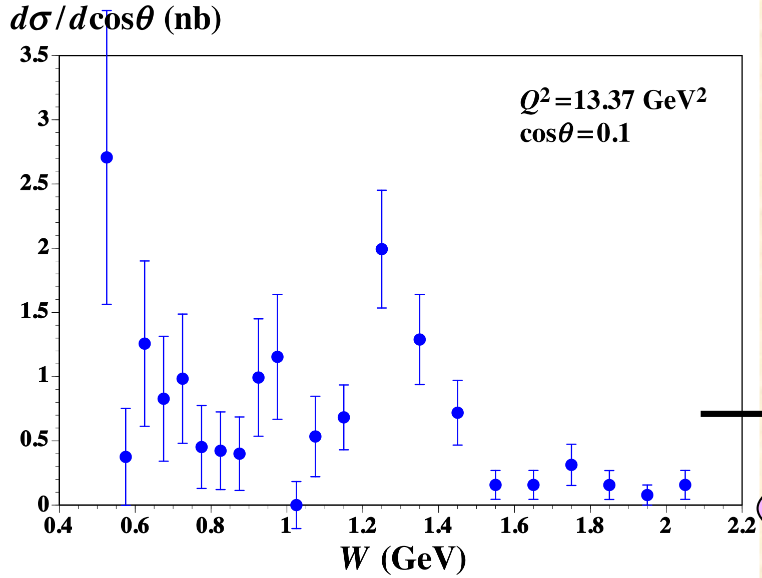
← First finding on gravitational radius from actual experimental measurements

$$\Leftrightarrow\ \sqrt{\langle r^2\rangle_{\text{charge}}}=0.672\pm0.008\ \text{fm}$$

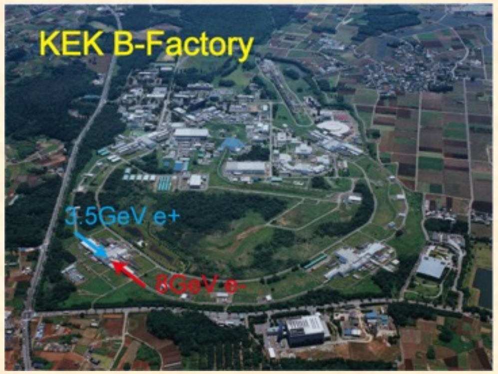


Super KEKB, ILC, FCC-ee,⋯





The errors are dominated by statistical errors, and they will be significantly reduced by super-KEKB.

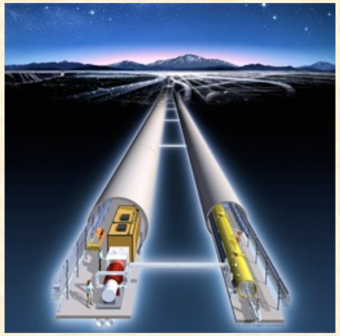


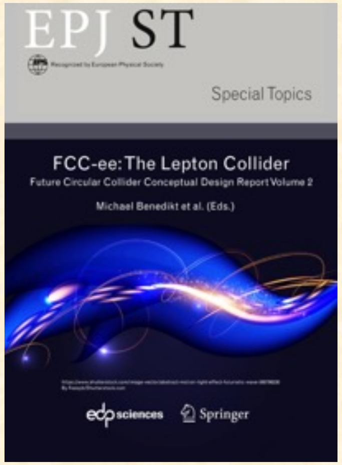
From KEKB to ILC, FCC-ee, ⋯

- Very Large Q^2
- Large W^2

for extracting GDAs

ILC, FCC-ee, ⋯

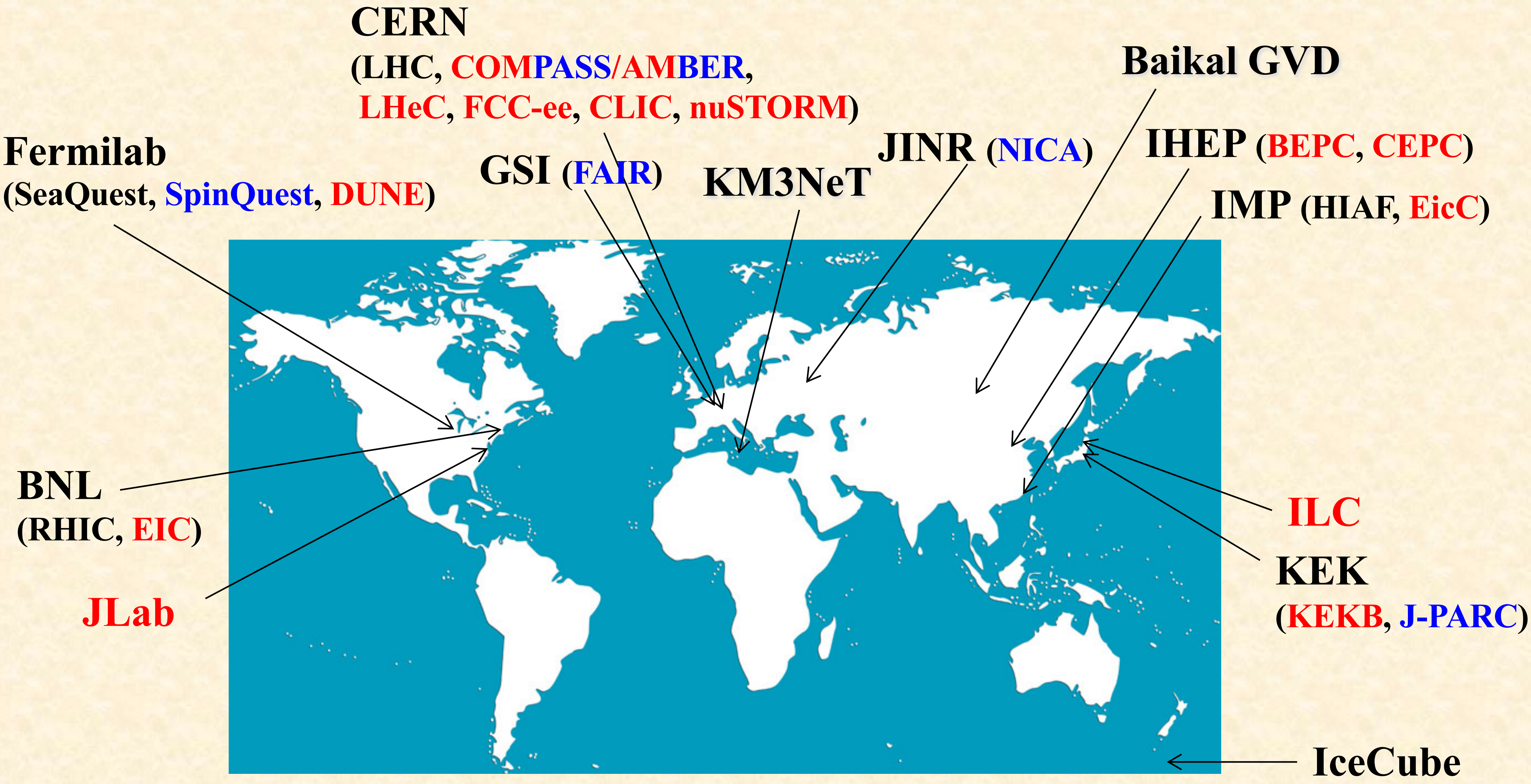




Hadron accelerator facilities on GPDs

(including future possibilities)

High-energy hadron physics experiments: hadron facilities

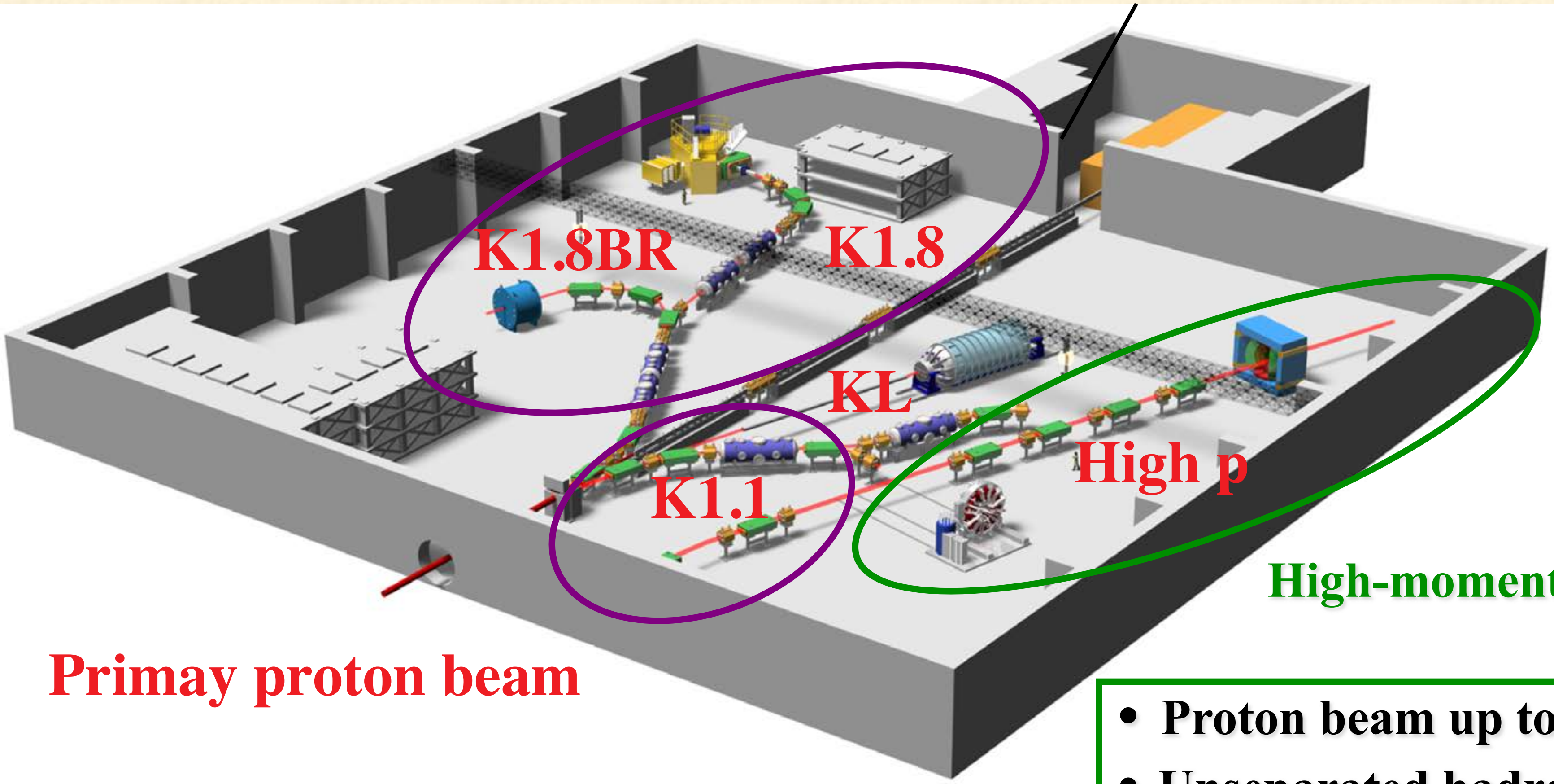


Facilities on hadron structure functions on GPDs including future possibilities.

Hadron accelerator facilities. Lepton accelerator facilities.

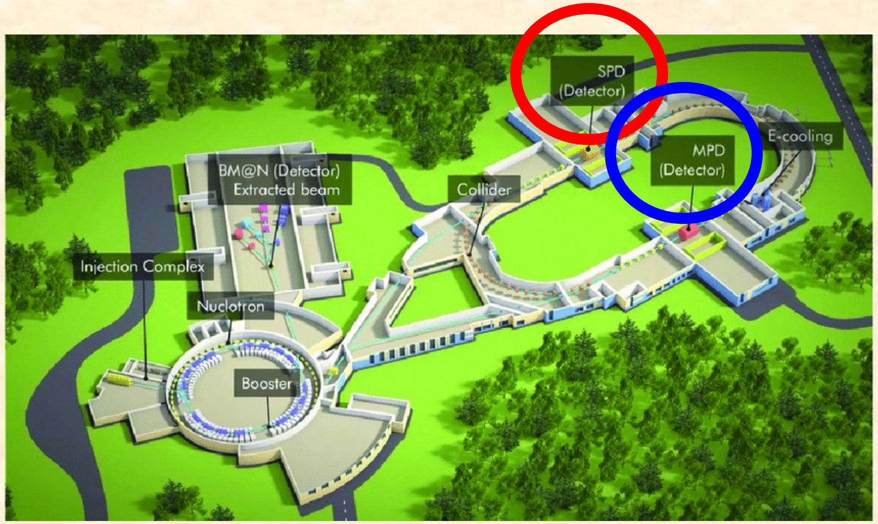
Hadron facility

(Low energy) Kaon and pion experiments are done at these beamlines.



- Proton beam up to 30 GeV
- Unseparated hadron (pion, …) beam up to 15~20 GeV

Nuclotron-based Ion Collider fAcility (NICA)



SPD (Spin Physics Detector for physics with polarized beams)
MPD (MultiPurpose Detector for heavy ion physics)

$\vec{p}+\vec{p}$: $\sqrt{s_{pp}} = 12 \sim 27$ GeV

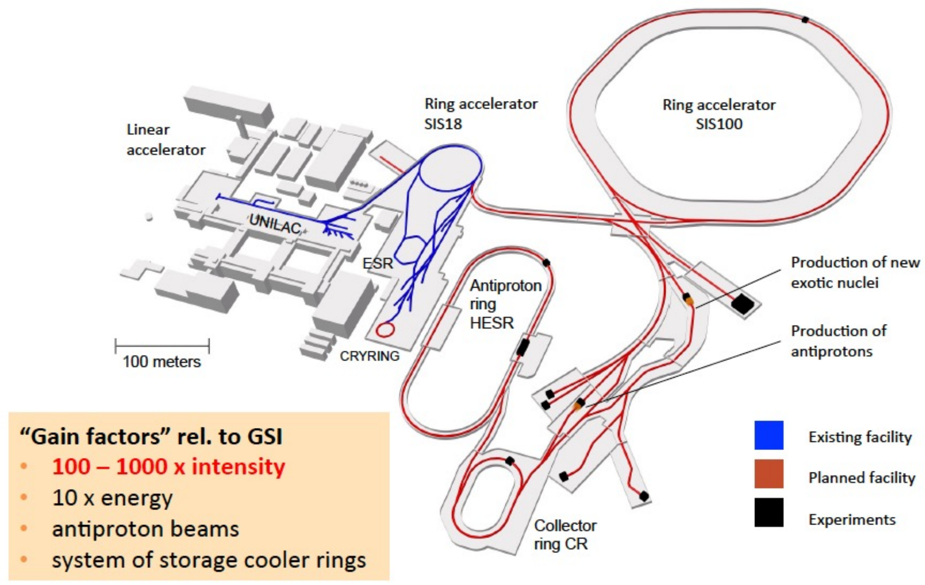
$\vec{d}+\vec{d}$: $\sqrt{s_{NN}} = 4 \sim 14$ GeV

$\vec{p}+\vec{d}$ is also possilbe.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al*. (NICA project), arXiv:2011.15005, Progress in Nuclear and Particle Physics in press.

Unique opportunity in high-energy spin physics, especially on the deuteron spin physics.

GSI-FAIR



“Gain factors” rel. to GSI

- **100 – 1000 x intensity**
- 10 x energy
- antiproton beams
- system of storage cooler rings

APPA
- Atomic Physics and Fundamental Symmetries,
- Plasma Physics,
- Materials Research,
- Radiation Biology,
- Cancer Therapy with Ion Beams / Space Research

CBM
- Dense and Hot Nuclear Matter

NUSTAR
- Nuclear Structure and Reaction Studies with nuclei far off stability,
- Physics of Explosive Nucleosynthesis (r-process)

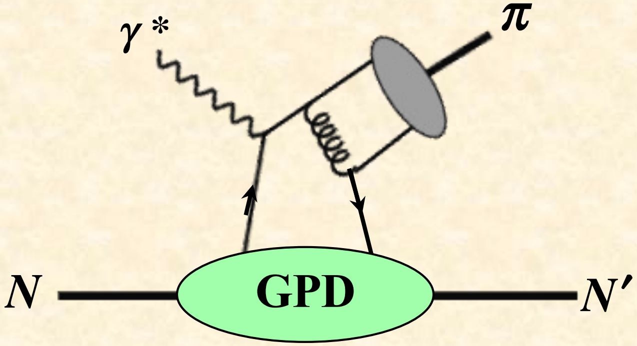
PANDA
- Hadron Structure & Dynamics with cooled antiproton beams

Possible studies on GPDs at hadron accelerator facilities

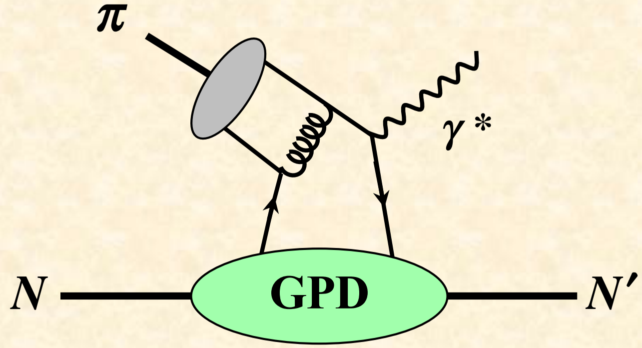
SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003;
T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka,
PRD 93 (2016) 114034.
J-PARC LoI 2019-07, J.-K. Ahn *et al*. (2019).
J-PARC proposal under preparation (2024),
Please get in touch with W.-C. Chang, N. Tomida
if you are interested in this project.

GPD projects at JLab /EIC and J-PARC

JLab / EIC

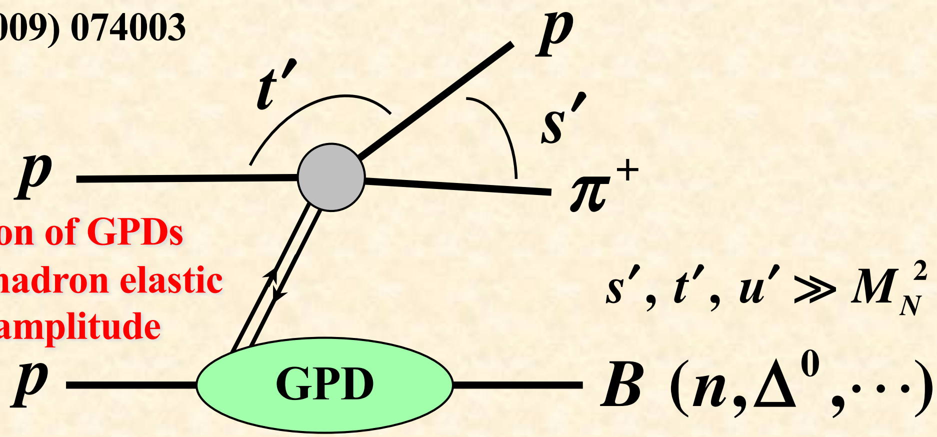


J-PARC



$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003



Investigation of GPDs
with 2→3 hadron elastic
scattering amplitude

$s', t', u' \gg M_N^2$

J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

$\pi + N \rightarrow \gamma + \gamma + N'$

$h + M_B \rightarrow h' + \gamma + M_D$

$h + M_B \rightarrow h' + M_C + M_D$

Physics of J-PARC high-momentum beamline

2019 — 2021 — 2023 — 2025

Approved (E16, E50) → **Hadron masses in nuclear medium** (E16)

. . .

E50: **Charmed baryons**

GPDs with E16 (proposal?)

e.g. $p + p \rightarrow p + \pi^{+} + n$

GPDs with E50 (LoI)

e.g. $\pi^{-} + p \rightarrow \mu^{+}\mu^{-} + n$

There is a possibility for high-energy hadron physics, including nucleon structure, ...

Proposal

Electron pair spectrometer at the J-PARC 50-GeV PS to explore the chiral symmetry in QCD

April 28, 2006
June 07, 2006 rev.1

S. Yokkaichi[1], H. En'yo, M. Naruki, R. Muto, T. Tabaru
RIKEN
K. Ozawa, H. Hamagaki
Center for Nuclear Study, Graduate School of Science, University of Tokyo
K. Shigaki
Graduate School of Science, Hiroshima University
S. Sawada, M. Sekimoto
High Energy Accelerator Research Organization (KEK)
F. Sakuma, K. Aoki
Department of Physics, Kyoto University

KEK/J-PARC-PAC 2012-19

Charmed Baryon Spectroscopy via the (π, D^{*-}) reaction

Y. Morino, T. Nakano,* H. Noumi†,* K. Shirotori, Y. Sugaya, and T. Yamaga
Research Center for Nuclear Physics (RCNP), Osaka University, 10-1, Mihogaoka, Ibaraki, Osaka, 567-0047, Japan
K. Ozawa‡
Institute of Particle and Nuclear Studies(IPNS), High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan
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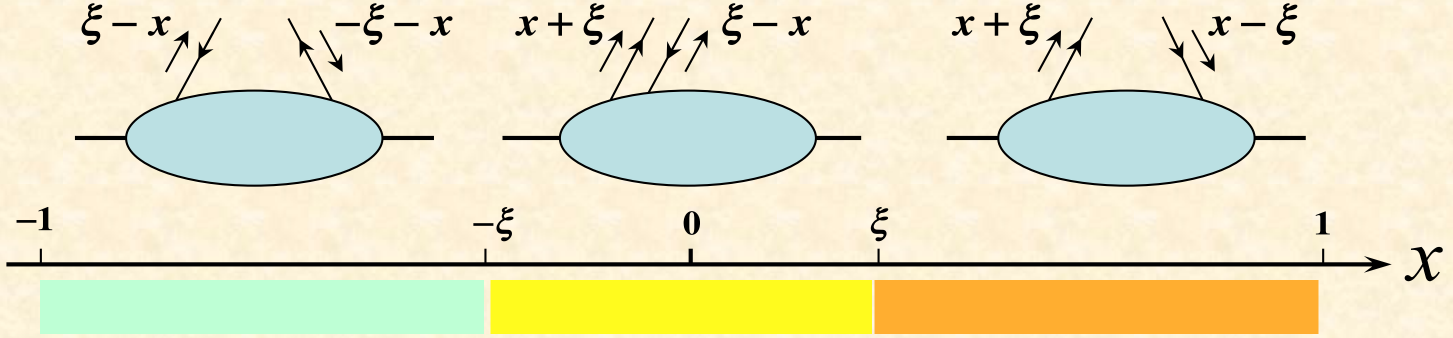
Department of Physics and Astronomy, Seoul National University, Seoul 151-747, Korea

LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J- PARC

JungKeun Ahn,[1] Sakiko Ashikag,[2] Wen-Chen Chang,[3,*] Seonho Choi,[4] Stefan Diehl,[5] Yuji Goto,[6] Kenneth Hicks,[7] Youichi Igarashi,[8] Kyungseon Joo,[5] Shunzo Kumano,[9,10] Yue Ma,[6] Kei Nagai,[3] Kenichi Nakano,[11] Masayuki Niiyama,[12] Hiroyuki Noumi,[13,8,†] Hiroaki Ohnishi,[14] Jen-Chieh Peng,[15] Hiroyuki Sako,[16] Shin'ya Sawada,[8,‡] Takahiro Sawada,[17] Kotaro Shirotori,[13] Kazuhiro Tanaka,[18,10] and Natsuki Tomida[13]

GPDs in different x regions and GPDs at hadron facilities



$-1 < x < \xi \quad (x + \xi < 0,\ x - \xi < 0)$

$\xi < x < 1 \quad (x + \xi > 0,\ x - \xi > 0)$

$-\xi < x < \xi \quad (x + \xi > 0,\ x - \xi < 0)$

Quark distribution

Emission of quark with momentum fraction $x+\xi$
Absorption of quark with momentum fraction $x-\xi$

$q\bar{q}$(meson)-like distribution amplitude

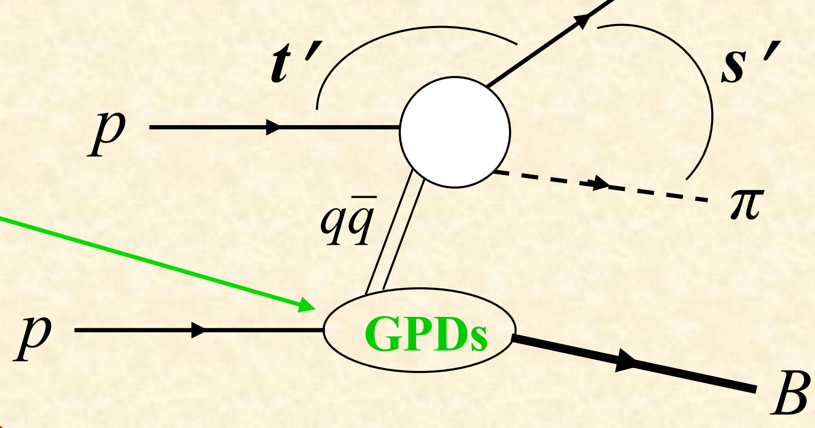
Emission of quark with momentum fraction $x+\xi$
Emission of antiquark with momentum fraction $\xi-x$

Antiquark distribution

Emission of antiquark with momentum fraction $\xi-x$
Absorption of antiquark with momentum fraction $-\xi-x$

Consider a hard reaction with

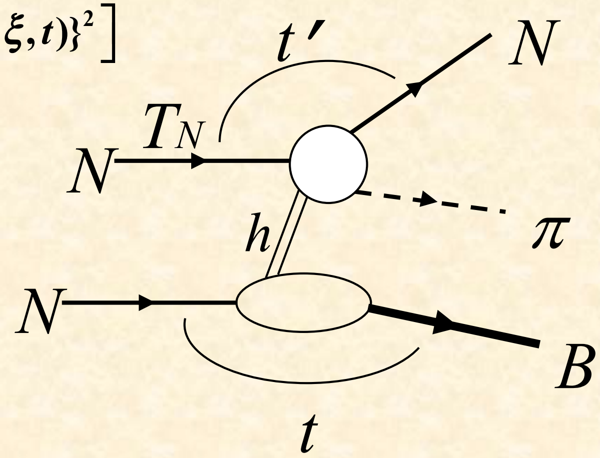
$|s'|,\ |t'|,\ |u'| \gg M_N^2,\quad |t| \ll M_N^2$



Efremov-Radyushkin -Brodsky-Lepage (ERBL) region

Cross section estimate (ξ dependence)

$$\frac{d\sigma_{NN\to N\pi B}}{d\xi\, dt\, dt'} \propto \frac{d\sigma_{MN\to\pi N}}{dt'}\Big[8(1-\xi^2)\{H(x,\xi,t)\}^2+16\xi^2 H(x,\xi,t)E(x,\xi,t)-\frac{t}{m_N^2}(1+\xi)^2\{E(x,\xi,t)\}^2$$
$$+8(1-\xi^2)\{\tilde{H}(x,\xi,t)\}^2+18\xi^2\tilde{H}(x,\xi,t)\tilde{E}(x,\xi,t)-\frac{2t\xi^2}{m_N^2}\{\tilde{E}(x,\xi,t)\}^2\Big]$$



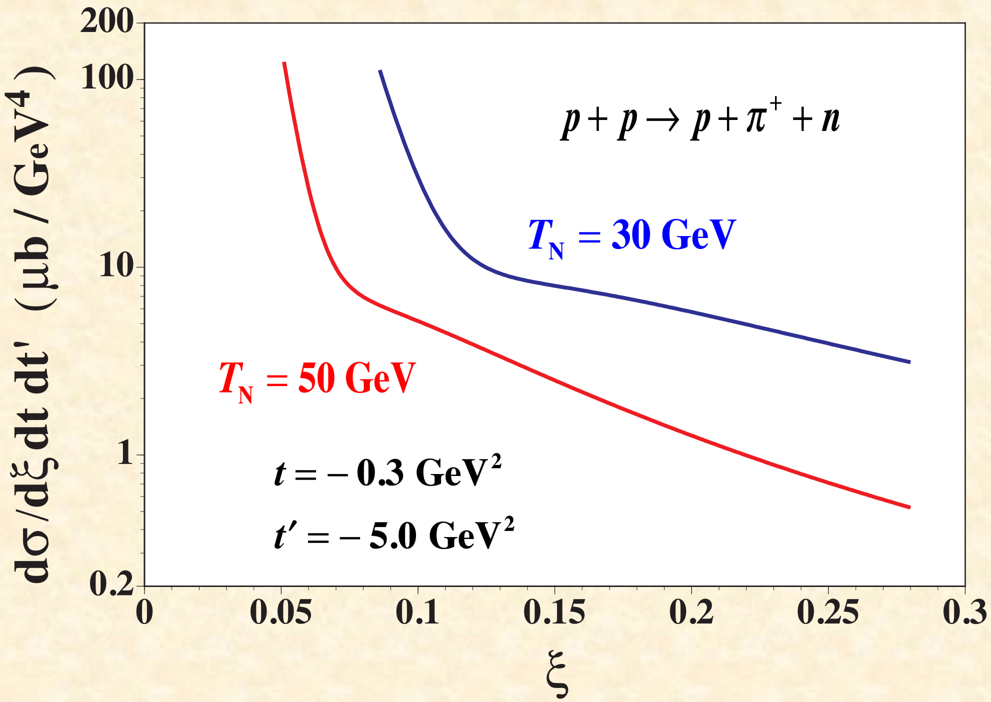
Skewdness parameter: $\xi = \dfrac{p_N^+ - p_B^+}{p_N^+ + p_B^+}$

$\dfrac{d\sigma}{d\xi dt dt'} \left(\dfrac{\mu\text{b}}{\text{GeV}^2}\right)$ as a function of ξ

at fixed $T_N = 30$ (50) GeV,

$t = -\,0.3$ GeV^2, $t' = -\,5$ GeV^2.

At this stage, our numerical results are for rough order of magnitude estimates on cross sections by assuming π- and ρ-like intermediate states.



For the details, please look at SK, M. Strikman, K. Sudoh, PRD 80 (2009) 074003.

Exclusive Drell-Yan $\pi^- + p \to \mu^+\mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde H^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \,\mathrm{Re}\left\{\tilde H^{du}(-\xi,\xi,t)^* \tilde E^{du}(-\xi,\xi,t)\right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde E^{du}(-\xi,\xi,t)\right|^2\right]$$

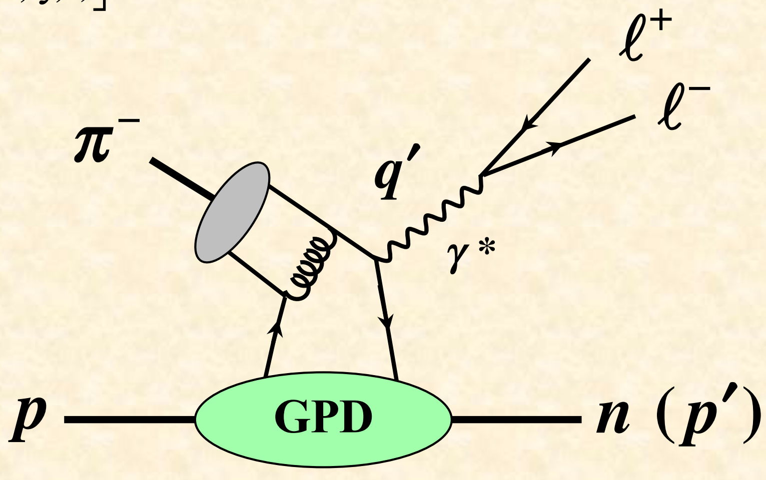
$$Q'^2 = q'^2,\ \ t=(p-p')^2,\ \ \tau = \frac{Q'^2}{2p\cdot q_\pi} \simeq \frac{Q'^2}{s-m_\pi^2}$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p(p')|\bar q(-z/2)\gamma^+\gamma_5 q(z/2)|p(p)\rangle\Big|_{z^+=0,\vec z_\perp=0} = \frac{1}{2P^+}\left[\tilde H_p^q(x,\xi,t)\bar u(p')\gamma^+\gamma_5 u(p) + \tilde E_p^q(x,\xi,t)\bar u(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

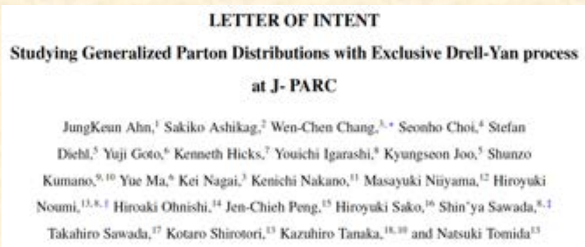
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle n(p')|\bar q_d(-z/2)\gamma^+\gamma_5 q_u(z/2)|p(p)\rangle\Big|_{z^+=0,\vec z_\perp=0} = \frac{1}{2P^+}\left[\tilde H_{p\to n}^{du}(x,\xi,t)\bar u(p')\gamma^+\gamma_5 u(p) + \tilde E_{p\to n}^{du}(x,\xi,t)\bar u(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

$$\tilde H^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon}\right]\left[\tilde H^d(x',\xi,t) - \tilde H^u(x',\xi,t)\right]$$

$$\tilde E^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon}\right]\left[\tilde E^d(x',\xi,t) - \tilde E^u(x',\xi,t)\right]$$



T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka, PRD93 (2016) 114034.


LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J-PARC

JungKeun Ahn,[1] Sakiko Ashikag,[2] Wen-Chen Chang,[3,*] Seonho Choi,[4] Stefan Diehl,[5] Yuji Goto,[6] Kenneth Hicks,[7] Youichi Igarashi,[8] Kyungseon Joo,[5] Shunzo Kumano,[9,10] Yue Ma,[6] Kei Nagai,[3] Kenichi Nakano,[11] Masayuki Niiyama,[12] Hiroyuki Noumi,[13,6,†] Hiroaki Ohnishi,[14] Jen-Chieh Peng,[15] Hiroyuki Sako,[16] Shin'ya Sawada,[8,‡] Takahiro Sawada,[17] Kotaro Shirotori,[13] Kazuhiro Tanaka,[18,10] and Natsuki Tomida[13]

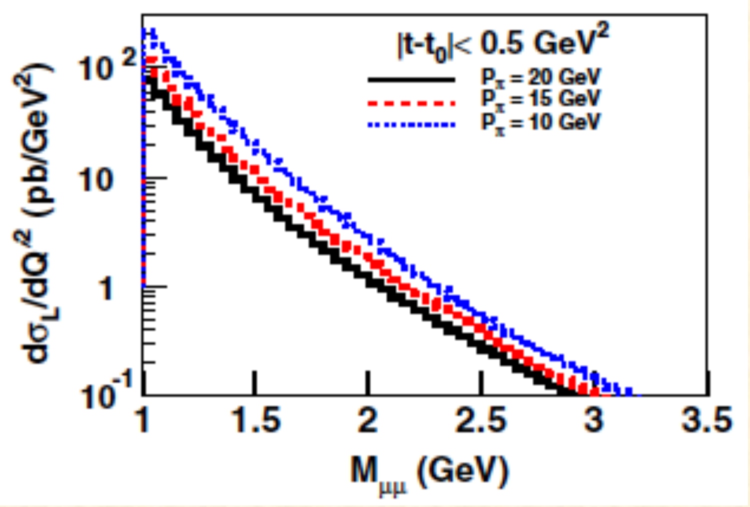
LoI for a J-PARC experiment

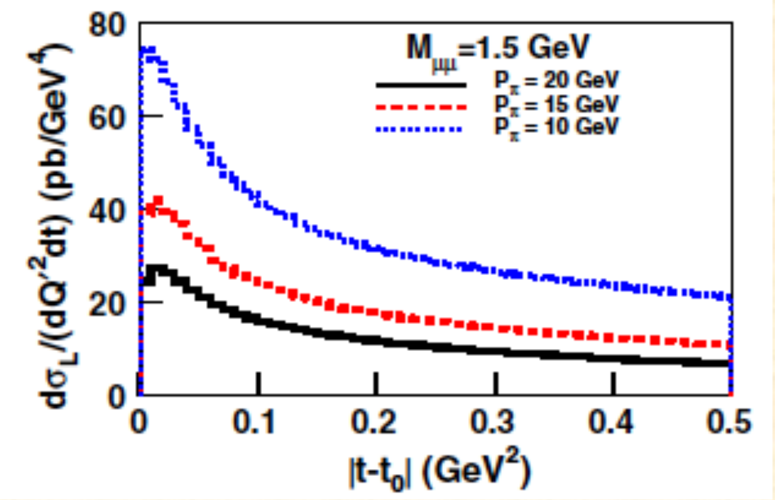
$$\pi^-(\bar u d) + p(uud) \to n(udd) + \gamma^*(\to \ell^+\ell^-)$$

Expected Drell-Yan events at J-PARC

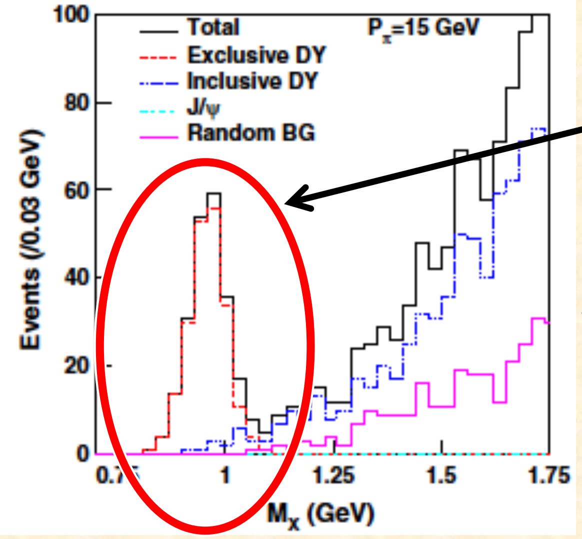
$$Q'^2 = q'^2,\ t = (p-p')^2,\ \tau = \frac{Q'^2}{2p\cdot q_\pi} \simeq \frac{Q'^2}{s-m_N^2}$$

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \mathrm{Re}\left\{\tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t)\right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]$$





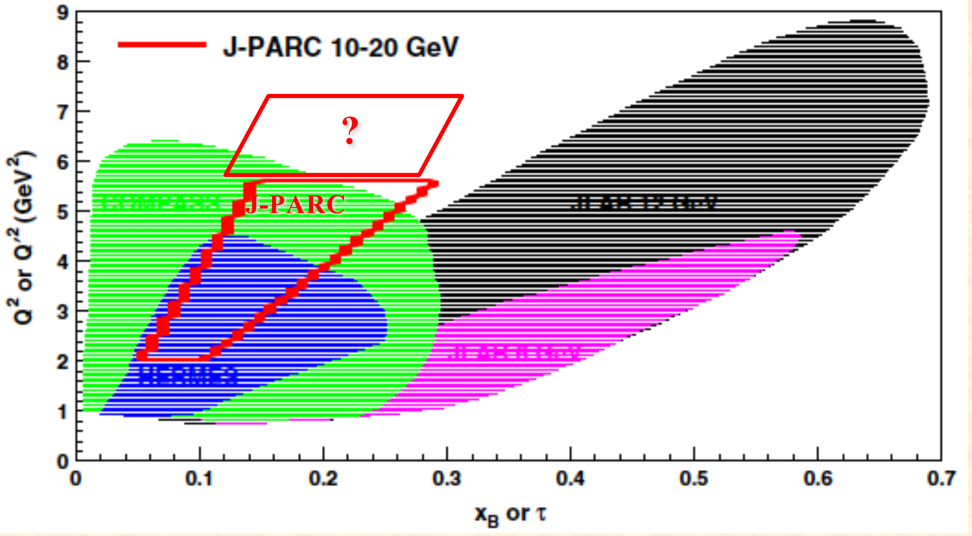
Missing mass



Exclusive Drell-Yan

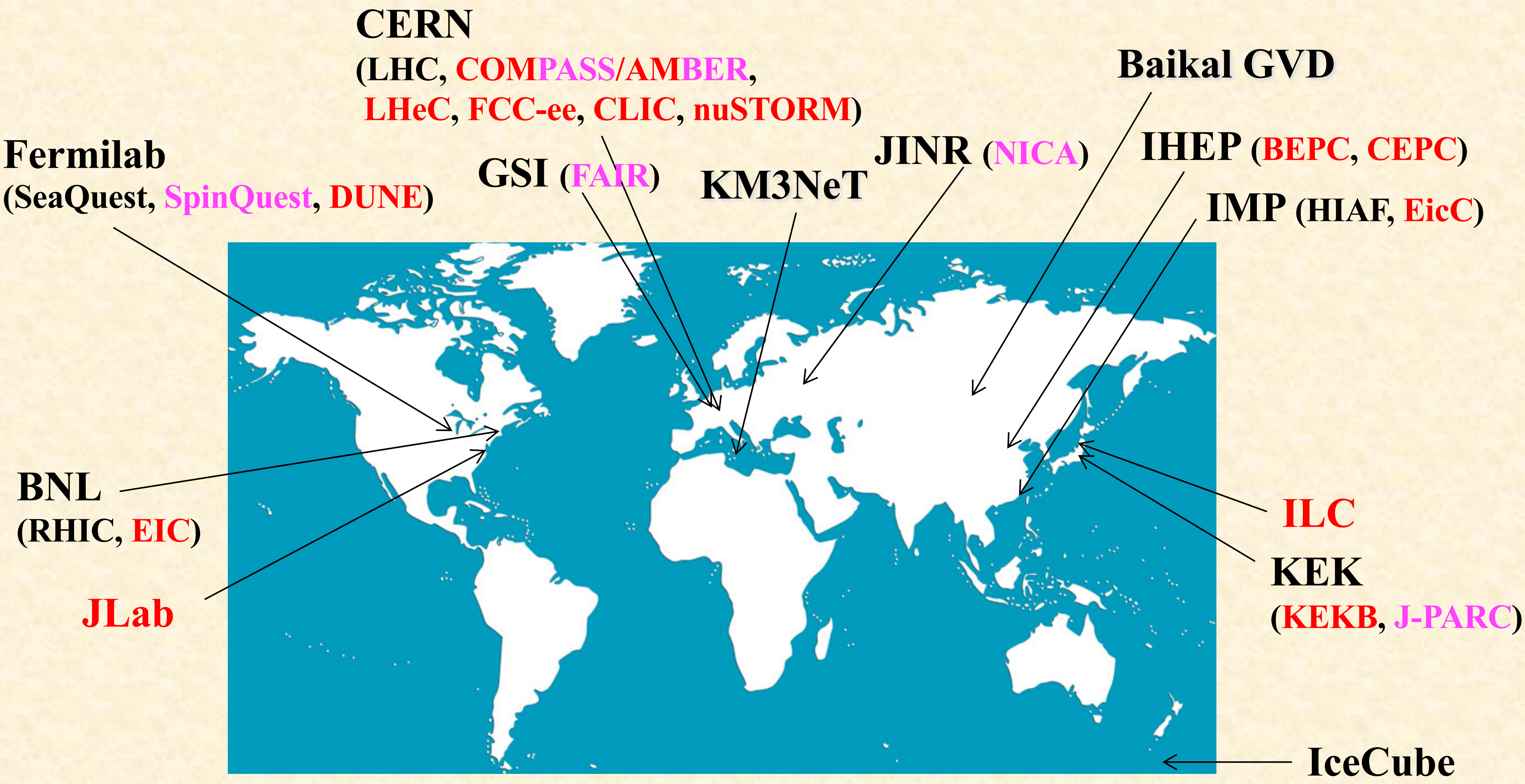
$$M_X^2 = (q+p-q')^2$$

$$q = p_\pi,\ p = p_p,\ q = p_{\mu^+\mu^-}$$



Future prospects
on GPD projects

High-energy hadron physics experiments

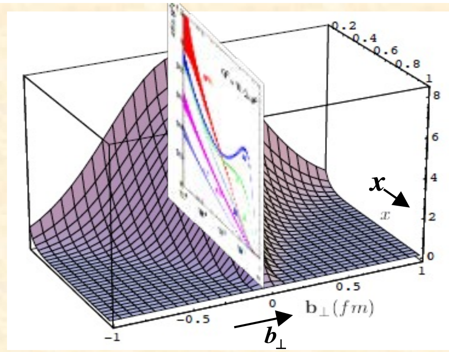


Facilities on hadron structure functions on GPDs including future possibilities.

Hadron accelerator facilities. Lepton accelerator facilities.

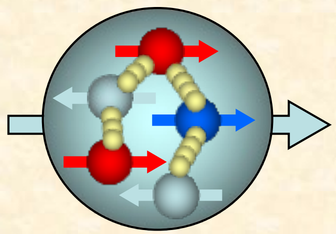
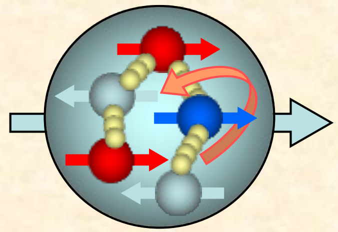
By hadron tomography

3D view of hadrons

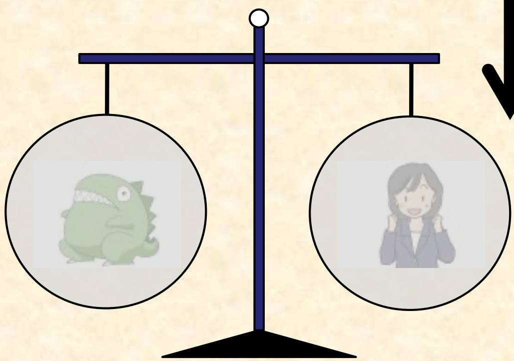


Origin of nucleon spin

By the tomography, we determine

 or  .

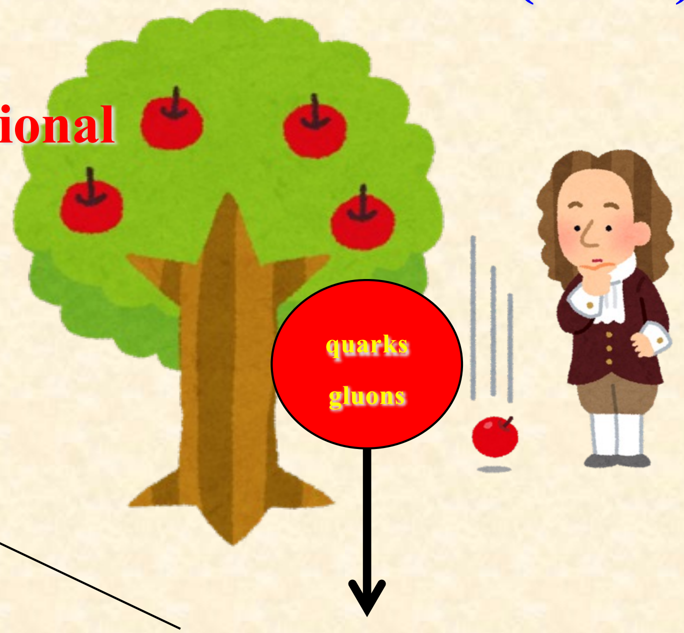
Exotic hadrons



By tomography, we determine  or  .

Origin of gravitational source (mass)

By tomography, we determine gravitational sources in terms of quarks and gluons.



Summary on the GPDs

Hadron-tomography and gravitational form factors

- **Puzzle to find the origin of hadron masses and pressures in terms of quark and gluon degrees of freedom**
- **Puzzle to find the origin of nucleon spin**
- **Exotic hadron candidates could be studied in the same tomography method.**
- **There are world-wide lepton and hadron accelerator facilities which has been used and could be used in future for the GPD studies.**

  In addition to the JLab/EIC type electron scattering projects, the GPD studies are possible by neutrino- and hadron-beam facilities and e^+e^- colliders.
- **If the HIAF will have a high-energy proton beam in future, a wider hadron physics project is possible, such as the GPDs by the neutrino reactions.**

Time has come to understand the gravitational sources in microscopic (instead of usual macroscopic/cosmic) world in terms of quark and gluon degrees of freedom.

The End

The End

Appendix

GPDs for exotic hadrons

(If transition GPDs could be studied, this exotic-hadron project becomes realistic.)

H. Kawamura and SK,
Phys. Rev. D 89 (2014) 054007.

Constituent counting rule for exotic hadrons:
H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006.

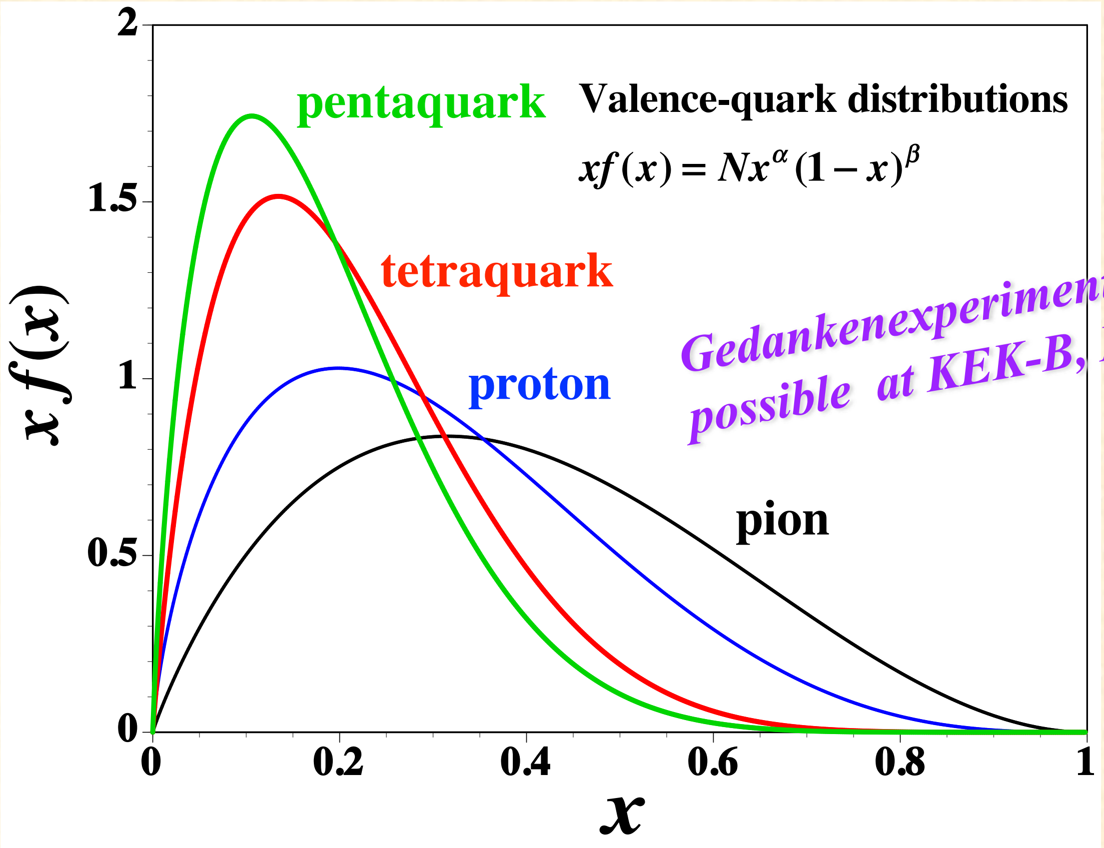
Simple function of GPDs

$$H_q^h(x,t) = f(x)F(t,x)$$

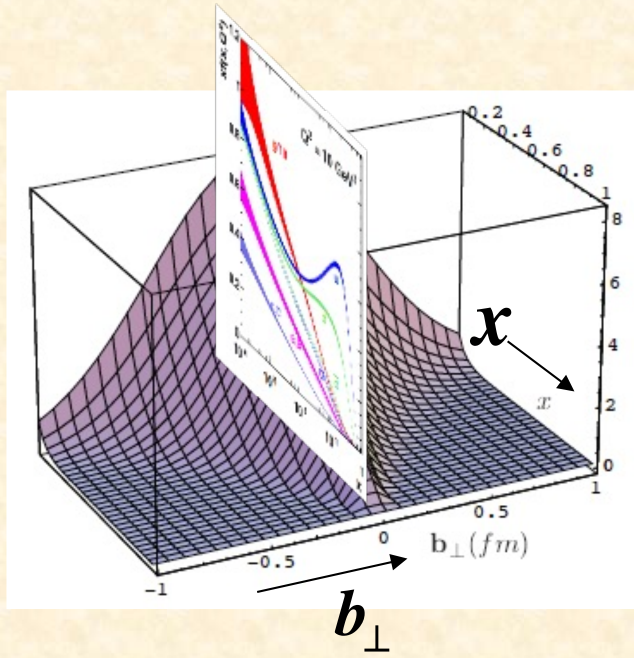
M. Guidal, M.V. Polyakov, A.V. Radyushkin, M. Vanderhaeghen, PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n}(1-x)^{\beta_n}$

- **Valence-quark number sum rule (charge and baryon numbers):** $\int_0^1 dx\, f(x) = n$
- **Constituent conting rule at** $x \to 1$: $\beta_n = 2n - 3 + 2\Delta S$ **(**n **= number of constituents)**
- **Momentum carried by quarks** $\langle x \rangle_q \simeq \int_0^1 dx\, x f(x)$



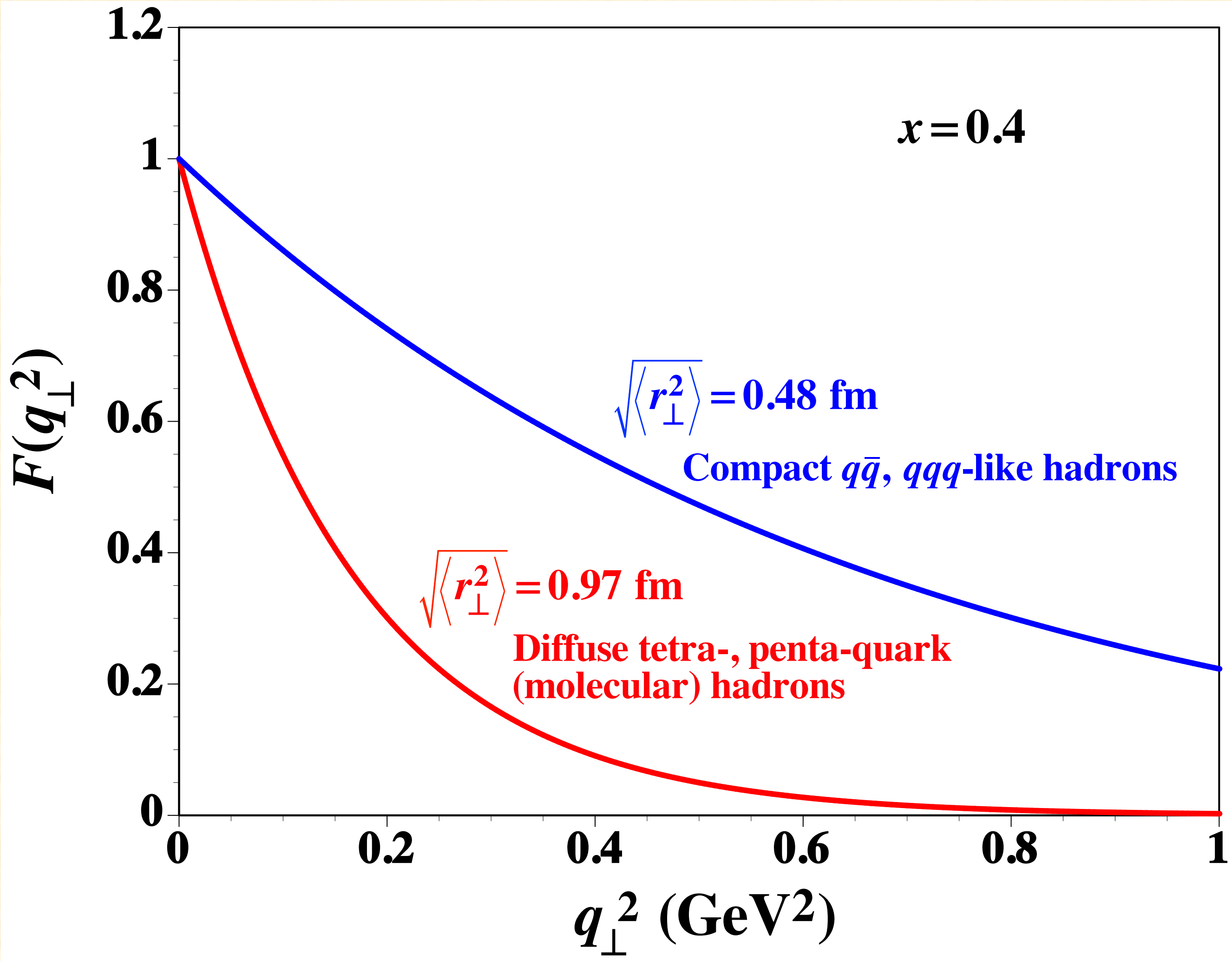
Gedankenexperiment, but possible at KEK-B, ILC, ...

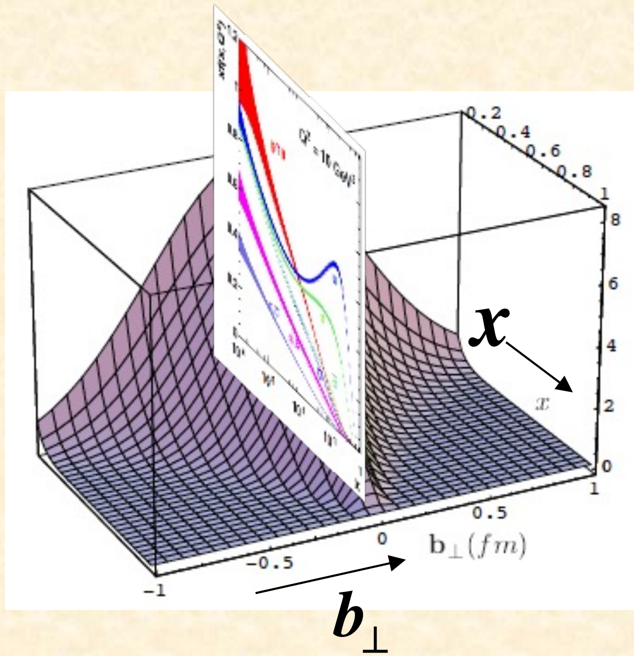


Recent study on Zc(3900),
C. Han, X. Wang, W. Kou, X. Chen, arXiv:2407.05923 (2024).

Two-dimensional form factor

$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$





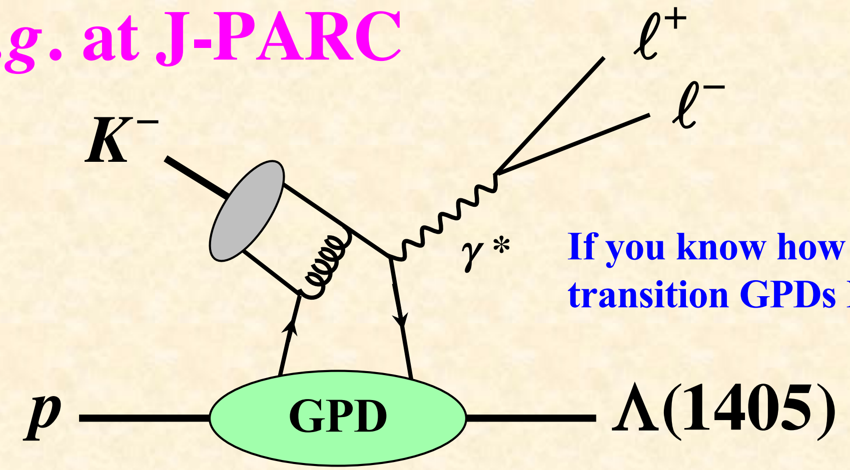
GPDs for exotic hadrons !?

Because stable targets do not exist for exotic hadrons, it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or → $s \leftrightarrow t$ crossed qunatity = GDAs at KEKB, Linear Collider

***e.g.* at J-PARC**

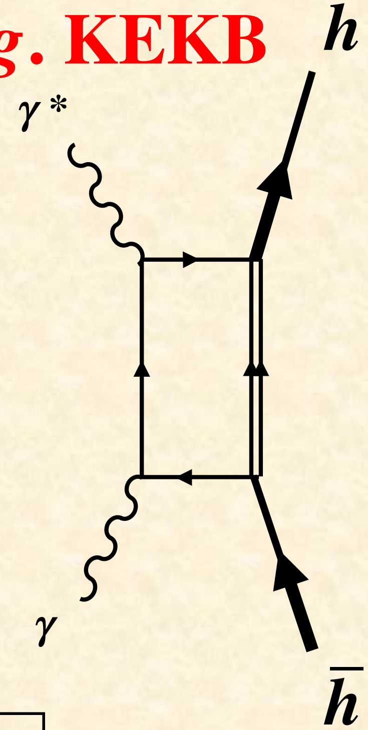


$K^-(\bar{u}s) + p(uud) \to \Lambda_{1405}(uud\bar{u}s) + \gamma^*$

Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate

If you know how to handle this kind of transition GPDs N→Λ, please inform me.

***e.g.* KEKB**



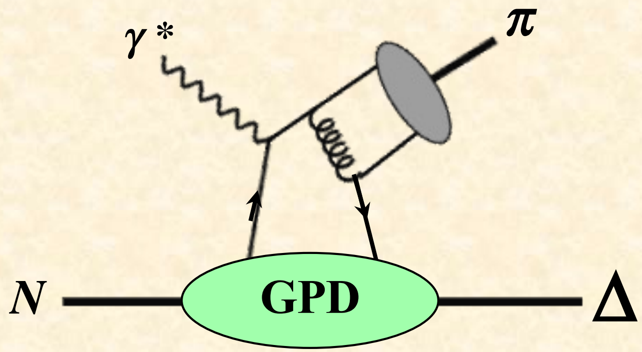
See H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006
for constituent-counting rule for exotic hadron candidates.

Transition GPDs for exotic hadrons

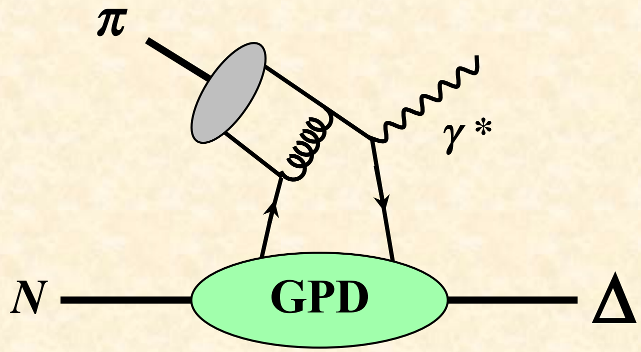
S. Diehl *et al*. (SK, 15th author),
arXiv:2405.15386, submitted for Eur. Phys. J. A

Transition GPDs from N to Δ

JLab / EIC



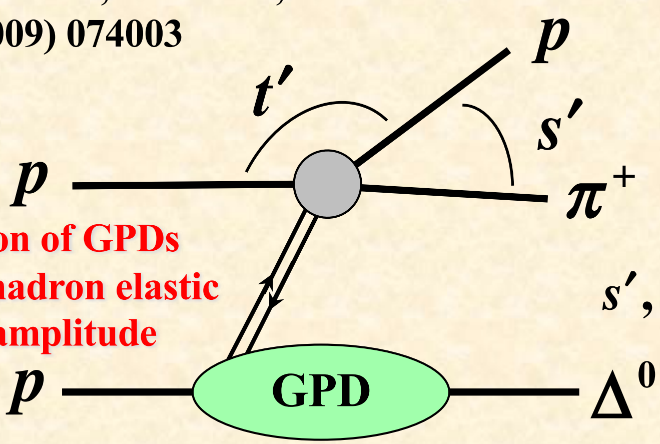
J-PARC



In future

$K^- + p \rightarrow \Lambda_{1405} + \gamma^*$?

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003



Investigation of GPDs with 2→3 hadron elastic scattering amplitude

$s', t', u' \gg M_N^2$

J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

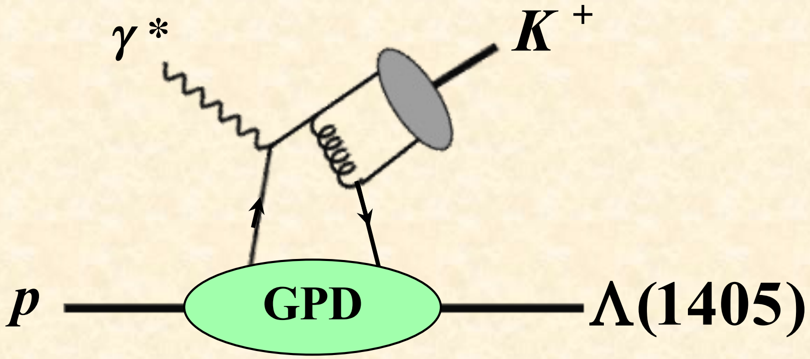
$\pi + N \rightarrow \gamma + \gamma + N'$

$h + M_B \rightarrow h' + \gamma + M_D$

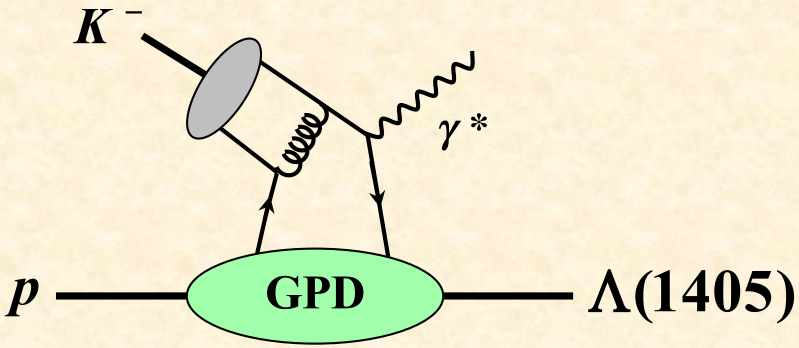
$h + M_B \rightarrow h' + M_C + M_D$

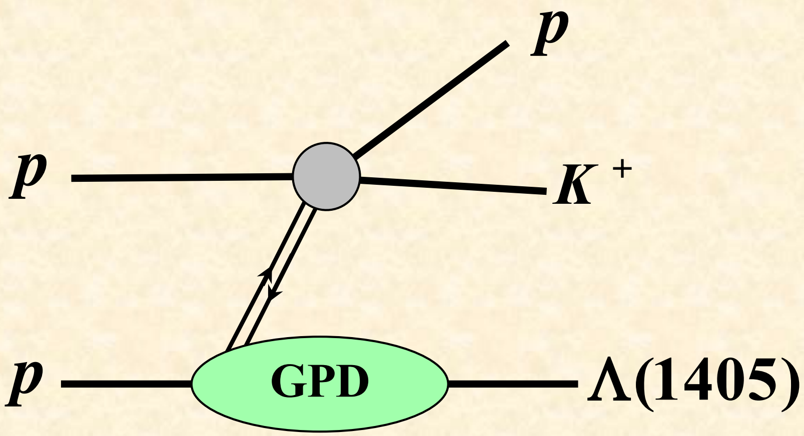
Transition GPDs for exotic hadrons

JLab / EIC



J-PARC





However, there is no theoretical study on the N→Λ(1405) transition GPDs at this stage.