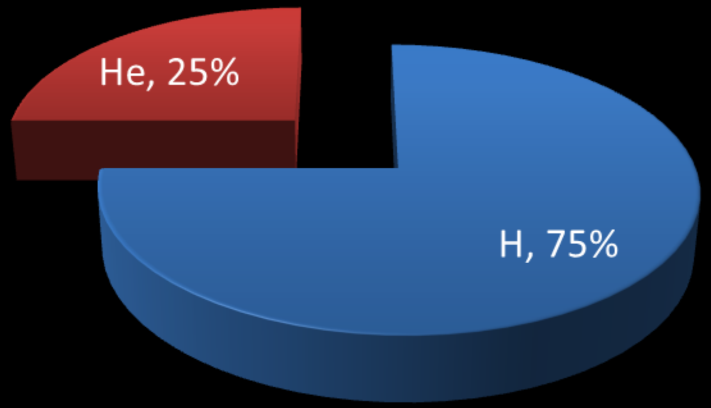
Nuclear Reactions in stars

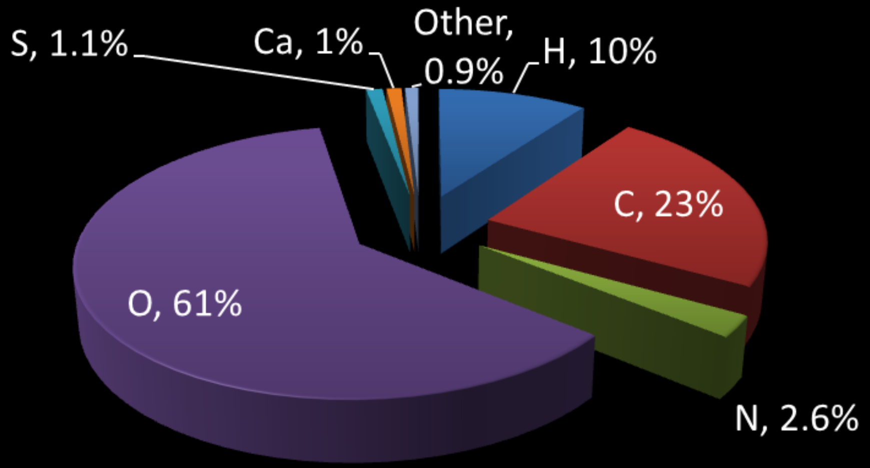
Xiaodong Tang
Institute of Modern Physics, CAS
Joint Department for Nuclear Physics,
Lanzhou University and IMP, CAS

Origin of Elements

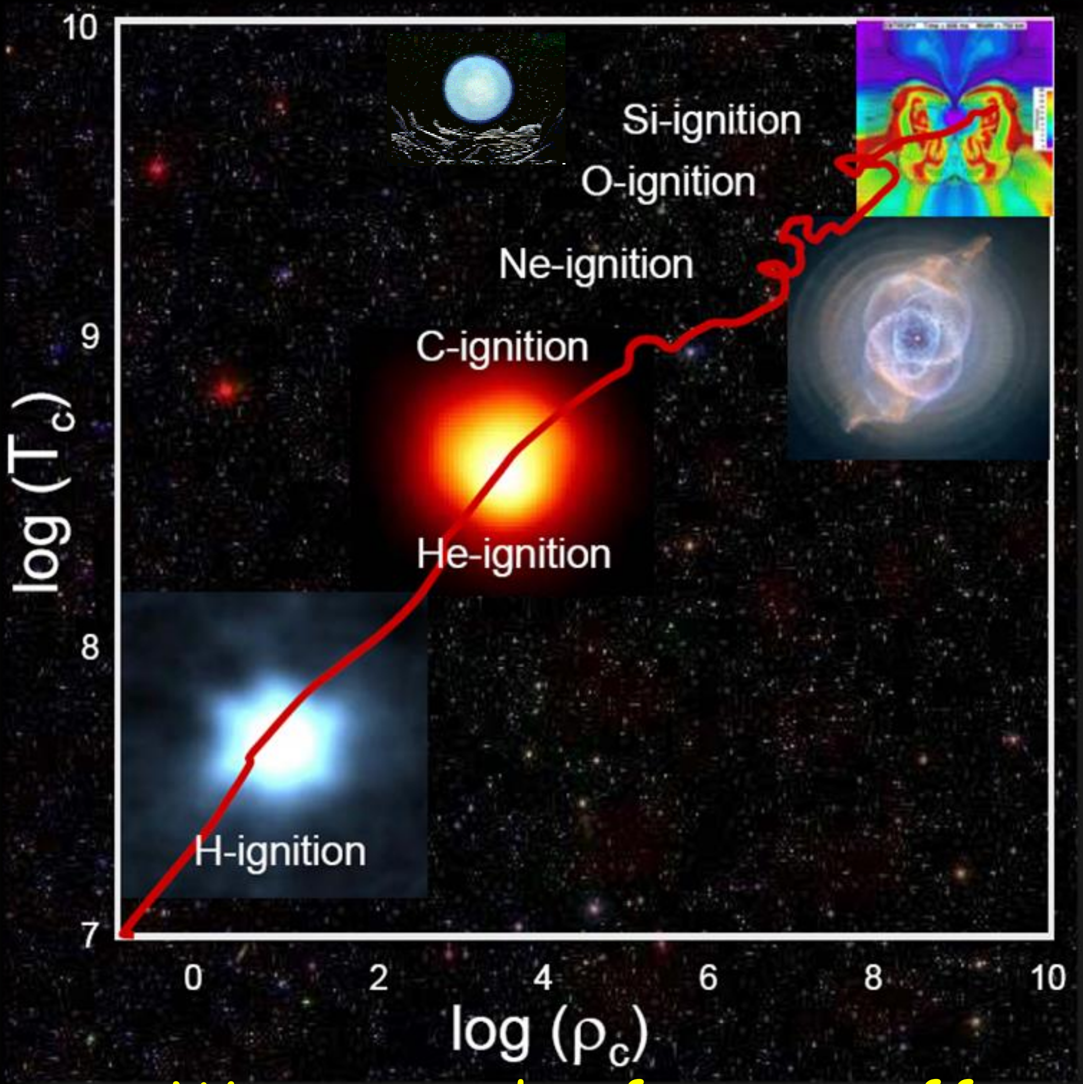
Other isotopes (^{2}H,^{3}He,^{6}Li,^{7}Li)<0.00001



3 mins



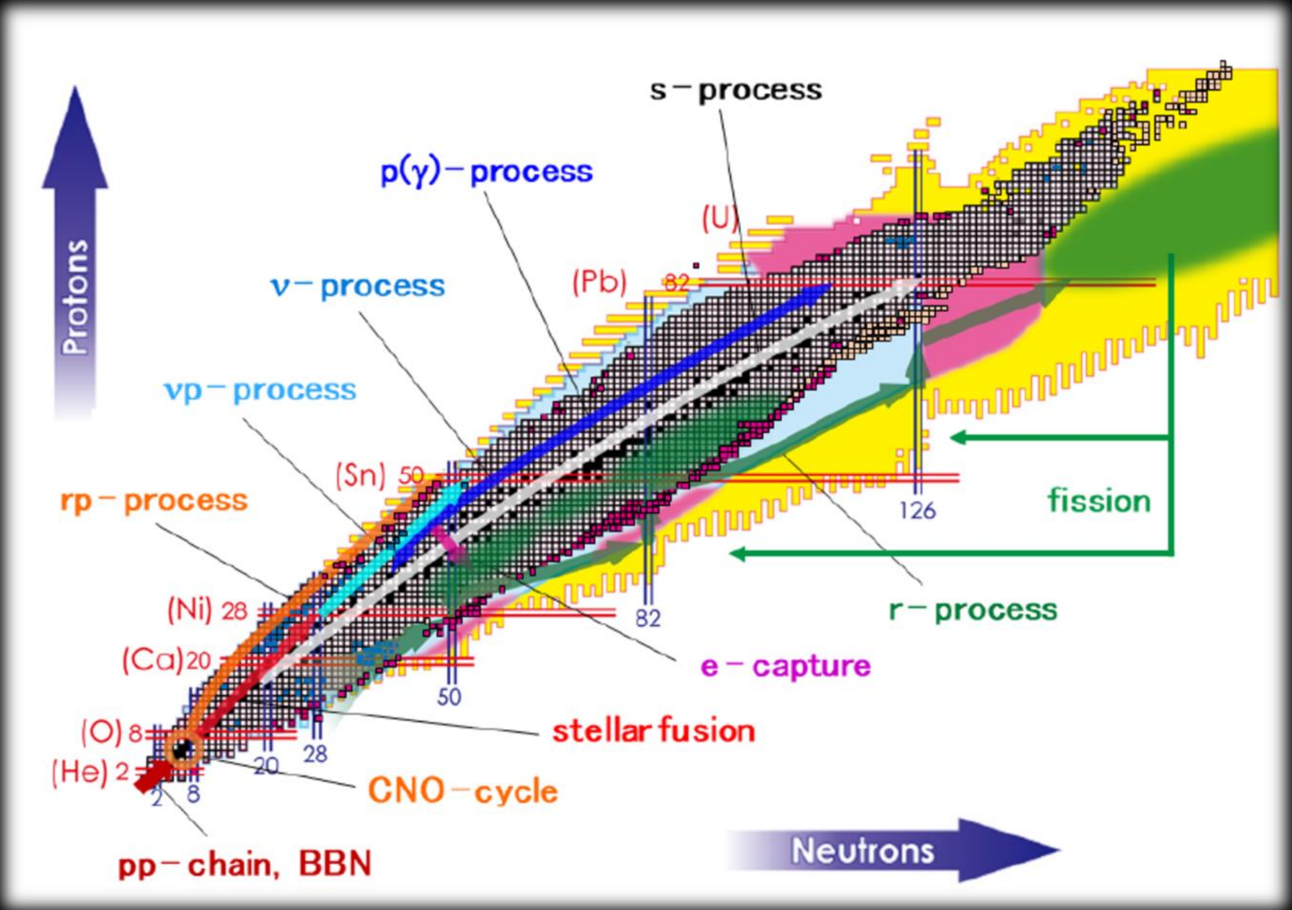
13.8 years



We are made of star-stuff.

Carl Sagan

Nucleosynthesis in stars



1) Big bang

2) Stellar quiescent burning

(pp-chain, stellar fusion, s-process)

3) Stellar explosive burning

(r-, νp-, ν-, p-, rp-processes)

4) Cosmic Ray spallation

- **What is the origin of the elements in the cosmos?**
- **What are the nuclear reactions that drive the evolution of stars and stellar explosions?**

"Progress in Nuclear Astrophysics of East and Southeast Asia", AAPPS bulletin(2021) 31:18

OUTLINE

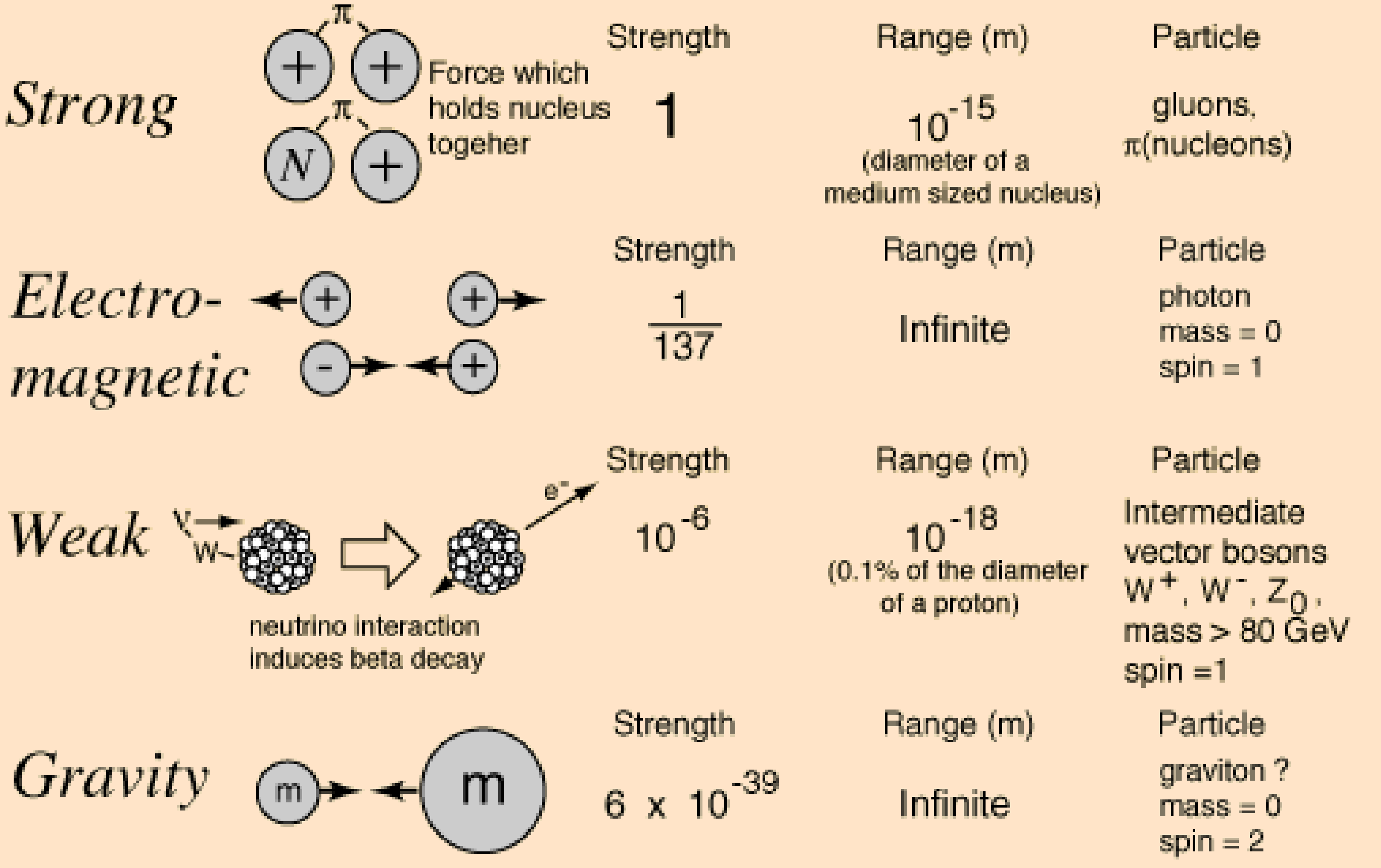
- Reaction Cross Section
- Reaction Rate
- pp-chain: p+p reaction
- Production of Carbon and Oxygen: triple alpha reaction and $^{12}C(\alpha,\gamma)^{16}O$
- Carbon burning: $^{12}C+^{12}C$

References

- C. Rolfs, Cauldron in Cosmos
- C. Iliadis, Nuclear Physics of Stars
- D. Clayton, Principles of Stellar Evolution and Nucleosynthesis
- M. Wiescher+M. Aliotta, lecture note of Nuclear Astrophysics
- E. Vogt, R-matrix theory, https://archive.jinaweb.org/events/matrix/05-R-matrix.pdf
- Descouvemont and Baye, R-matrix theory
- AZURE, https://azure.nd.edu/
- JINA Reaclib, https://reaclib.jinaweb.org/
- Nuclear data compilation: https://nucldata.tunl.duke.edu/, https://www.nndc.bnl.gov/

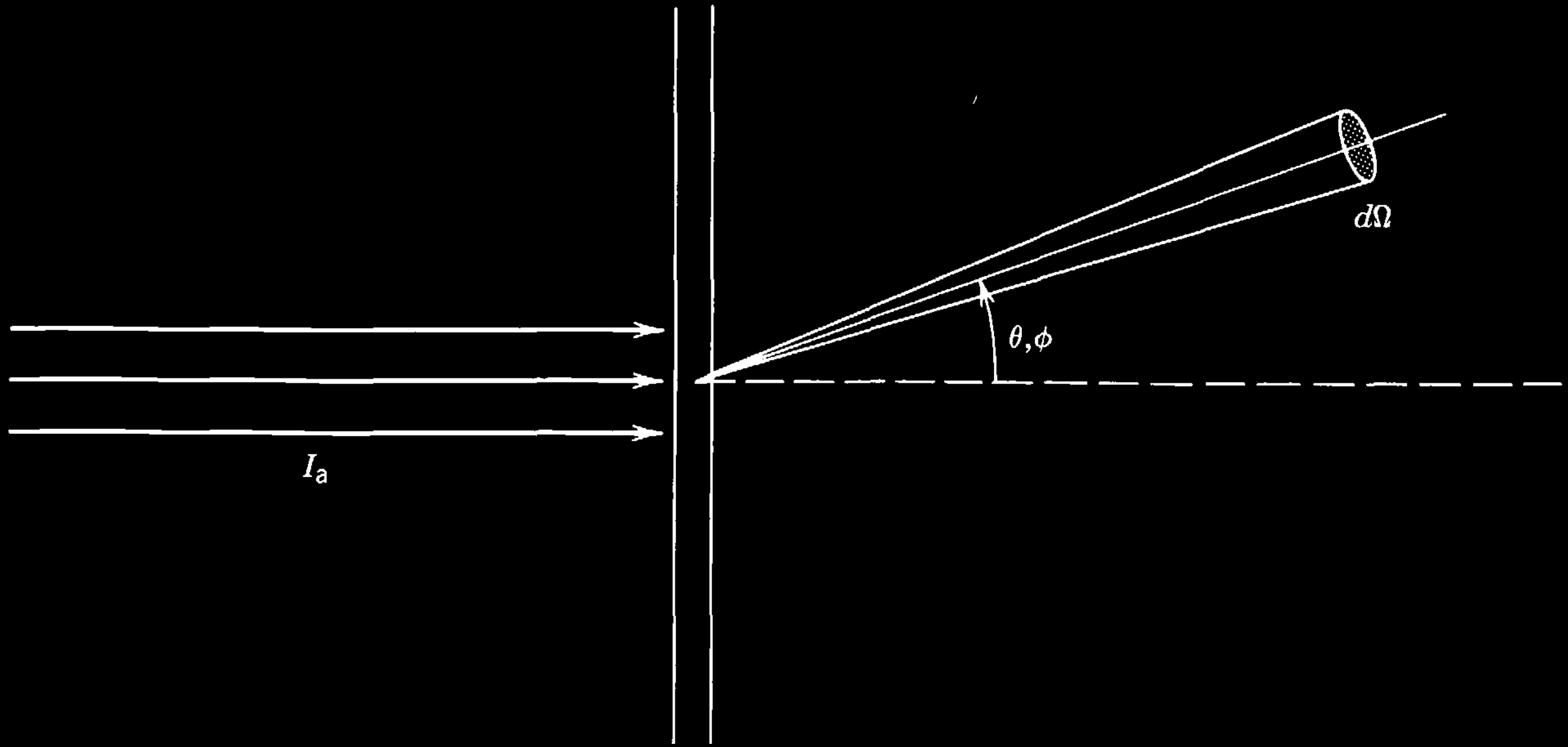
Fundamental Forces

http://hyperphysics.phy-astr.gsu.edu/hbase/Forces/funfor.html



		Strength	Range (m)	Particle
Strong	Force which holds nucleus togeher	1	10^{-15} (diameter of a medium sized nucleus)	gluons, π(nucleons)
Electro-magnetic		$\frac{1}{137}$	Infinite	photon mass = 0 spin = 1
Weak	neutrino interaction induces beta decay	10^{-6}	10^{-18} (0.1% of the diameter of a proton)	Intermediate vector bosons W^+, W^-, Z_0, mass > 80 GeV spin =1
Gravity		6×10^{-39}	Infinite	graviton ? mass = 0 spin = 2

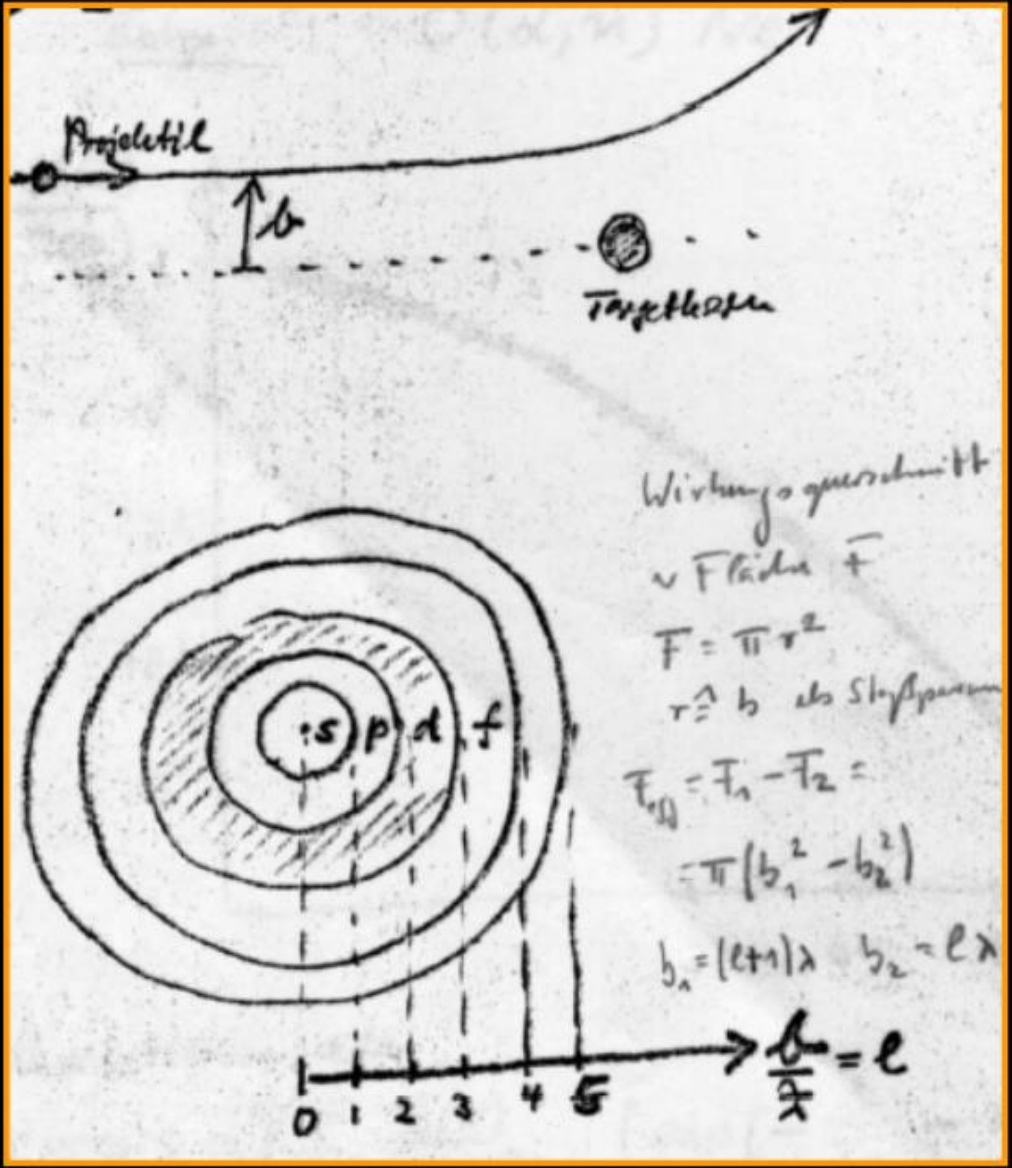
Cross section



$$\sigma \equiv \frac{\left(\text{number of interactions per time}\right)}{\left(\text{number of incident particles per area per time}\right)\left(\text{number of target nuclei within the beam}\right)}$$

1 barn = 10^{-24} cm^2 = 100 fm^2

Cross Section Estimates



cross section = area!

$$\sigma \cong F_{nucleus} = \pi \cdot r_{nucleus}^2 \approx (1.26)^2 \cdot \pi \cdot A^{2/3} \left[fm^2\right]$$

projectile orbital momentum ℓ

$$p = \hbar k = \frac{\hbar}{\bar{\lambda}}; \quad L = \vec{b} \times \vec{p} \cong |b| \cdot |p| = b \cdot \hbar k = b \cdot \frac{\hbar}{\bar{\lambda}}$$

$$L = \hbar \ell \quad \Rightarrow \quad b = \bar{\lambda} \cdot \ell$$

orbital momentum ℓ in σ

$$\sigma_\ell \cong \quad F_\ell = \pi \cdot \left(r_{\ell+1}^2 - r_\ell^2\right) = \pi \cdot \left(b_{\ell+1}^2 - b_\ell^2\right)$$

$$\sigma_\ell \cong \pi \cdot \left((\ell+1)^2 - \ell^2\right) \cdot \bar{\lambda}^2 = \pi \cdot (2\ell + 1) \cdot \bar{\lambda}^2$$

$$\sigma_\ell(E) \propto \frac{2\ell + 1}{E}$$

Cross section

cross section depends sensitively on:

- **type of interaction**
- **properties of the nuclei involved**
- **reaction mechanism**

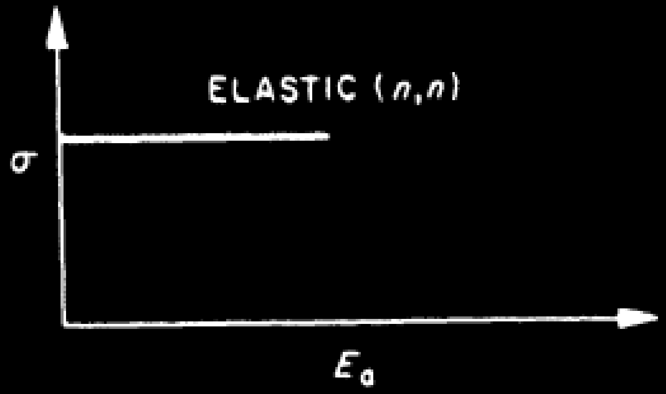
and can vary by orders of magnitude, depending on the interaction

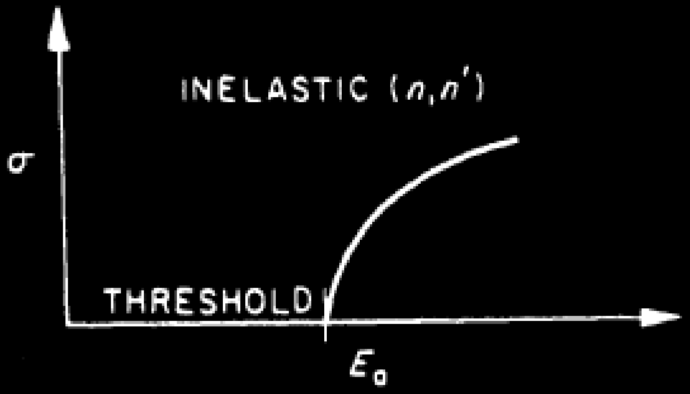
examples:

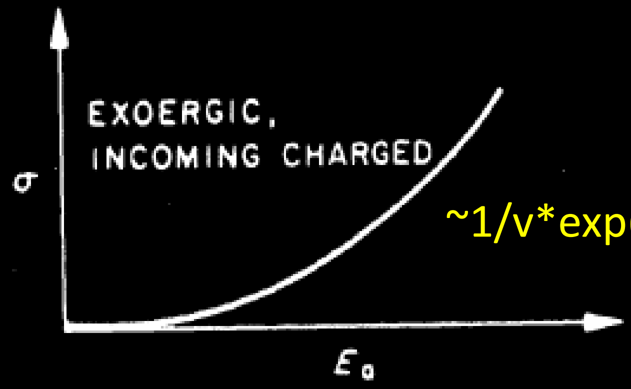
Reaction	Force	σ (barn)	E_{proj} (MeV)
$^{15}N(p,\alpha)^{12}C$	strong	0.5	2.0
$^{3}He(\alpha,\gamma)^{7}Be$	electromagnetic	10^{-6}	2.0
$p(p,e^{+}\nu)d$	weak	10^{-20}	2.0

1 barn = 10^{-24} cm^2 = 100 fm^2

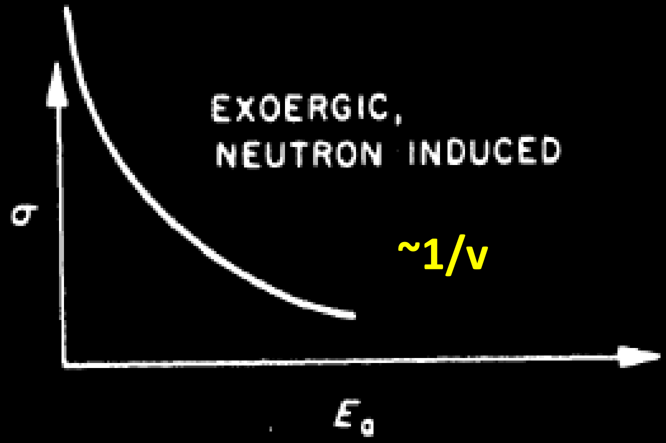
Behavior of cross section near threshold



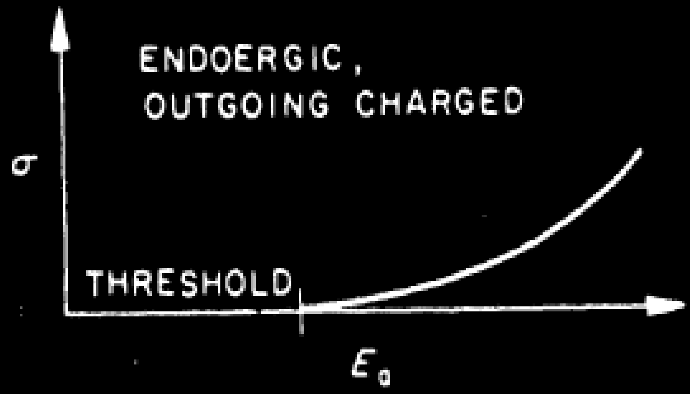




~1/v*exp(-2πη)



~1/v

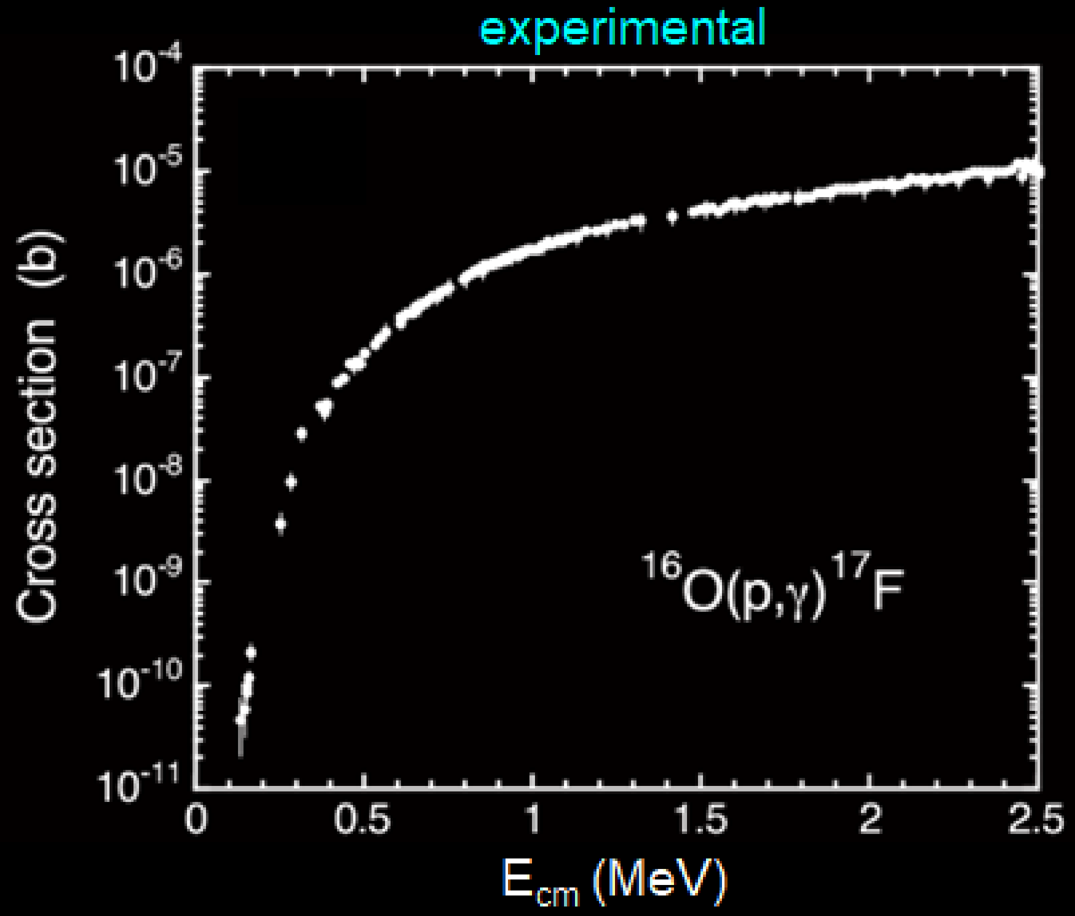


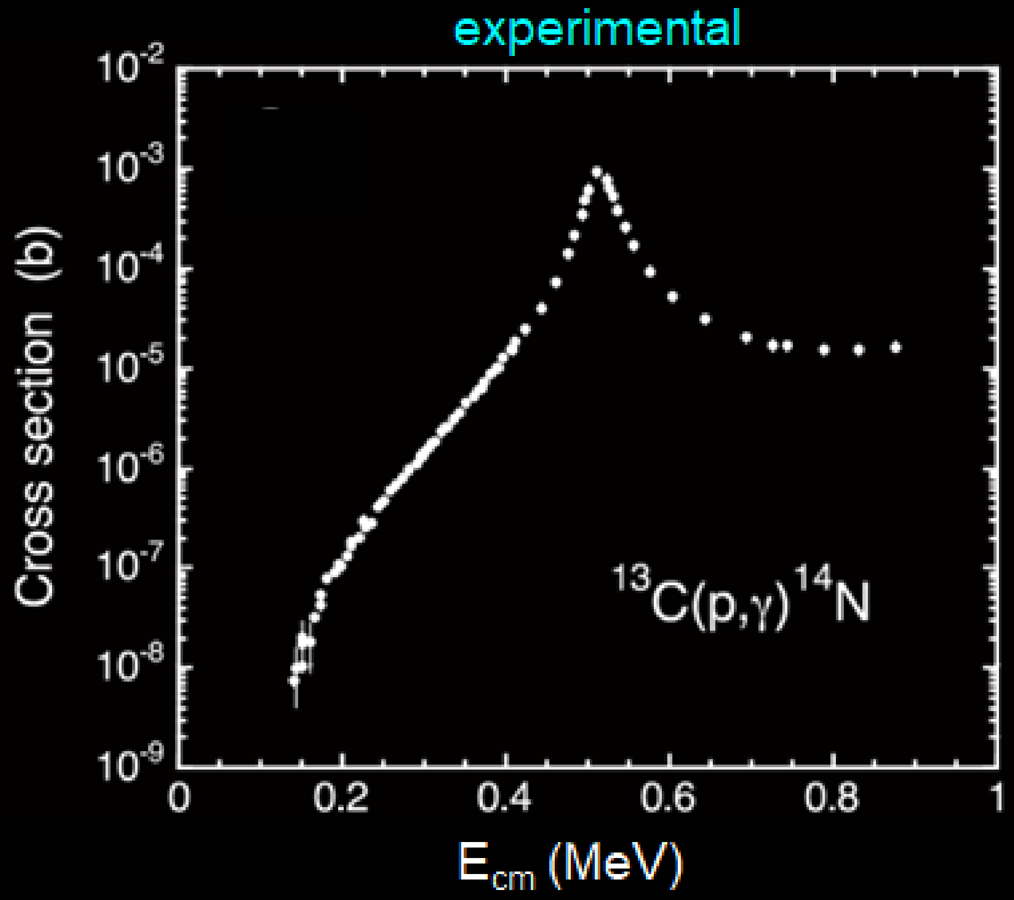
$$2\pi\eta = 0.989534 Z_0 Z_1 \sqrt{\frac{1}{E}\frac{M_0 M_1}{M_0 + M_1}}$$

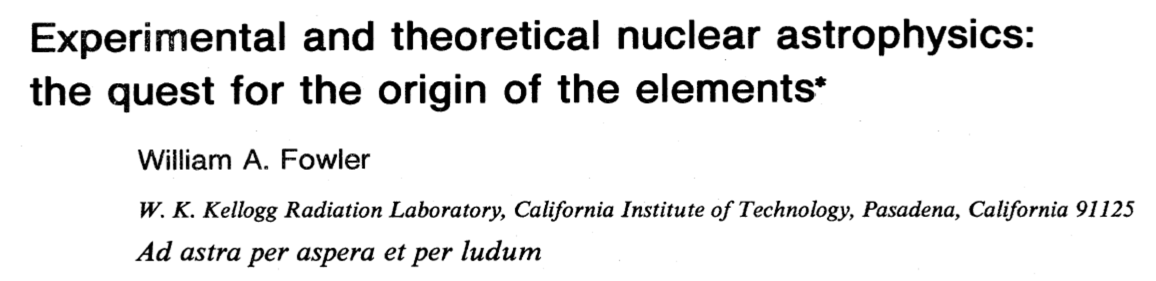
η is the Sommerfeld parameter
E is the energy in MeV
M

G.R. Satchler, Introduction to nuclear reactions
E. Segre, Nuclei and particles

Cross section (Charged particle induced Reaction)







Experimental and theoretical nuclear astrophysics: the quest for the origin of the elements*

William A. Fowler

W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125

Ad astra per aspera et per ludum

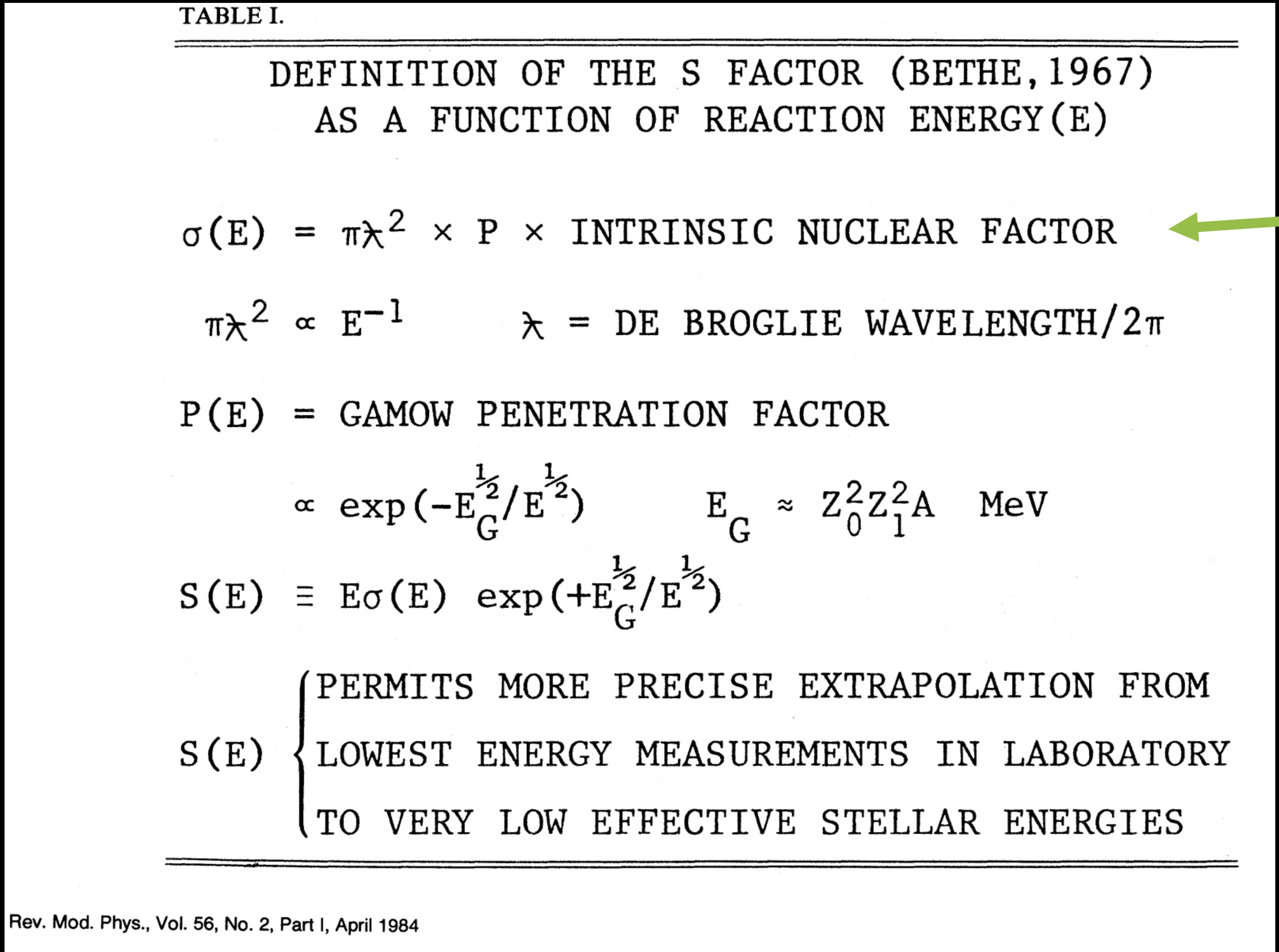


TABLE I.

DEFINITION OF THE S FACTOR (BETHE, 1967)
AS A FUNCTION OF REACTION ENERGY (E)

$$\sigma(E) = \pi\bar{\lambda}^2 \times P \times \text{INTRINSIC NUCLEAR FACTOR}$$

s-wave !

$$\pi\bar{\lambda}^2 \propto E^{-1} \qquad \bar{\lambda} = \text{DE BROGLIE WAVELENGTH}/2\pi$$

$$P(E) = \text{GAMOW PENETRATION FACTOR}$$

$$\propto \exp(-E_G^{1/2}/E^{1/2}) \qquad E_G \approx Z_0^2 Z_1^2 A \ \text{MeV}$$

$$S(E) \equiv E\sigma(E) \ \exp(+E_G^{1/2}/E^{1/2})$$

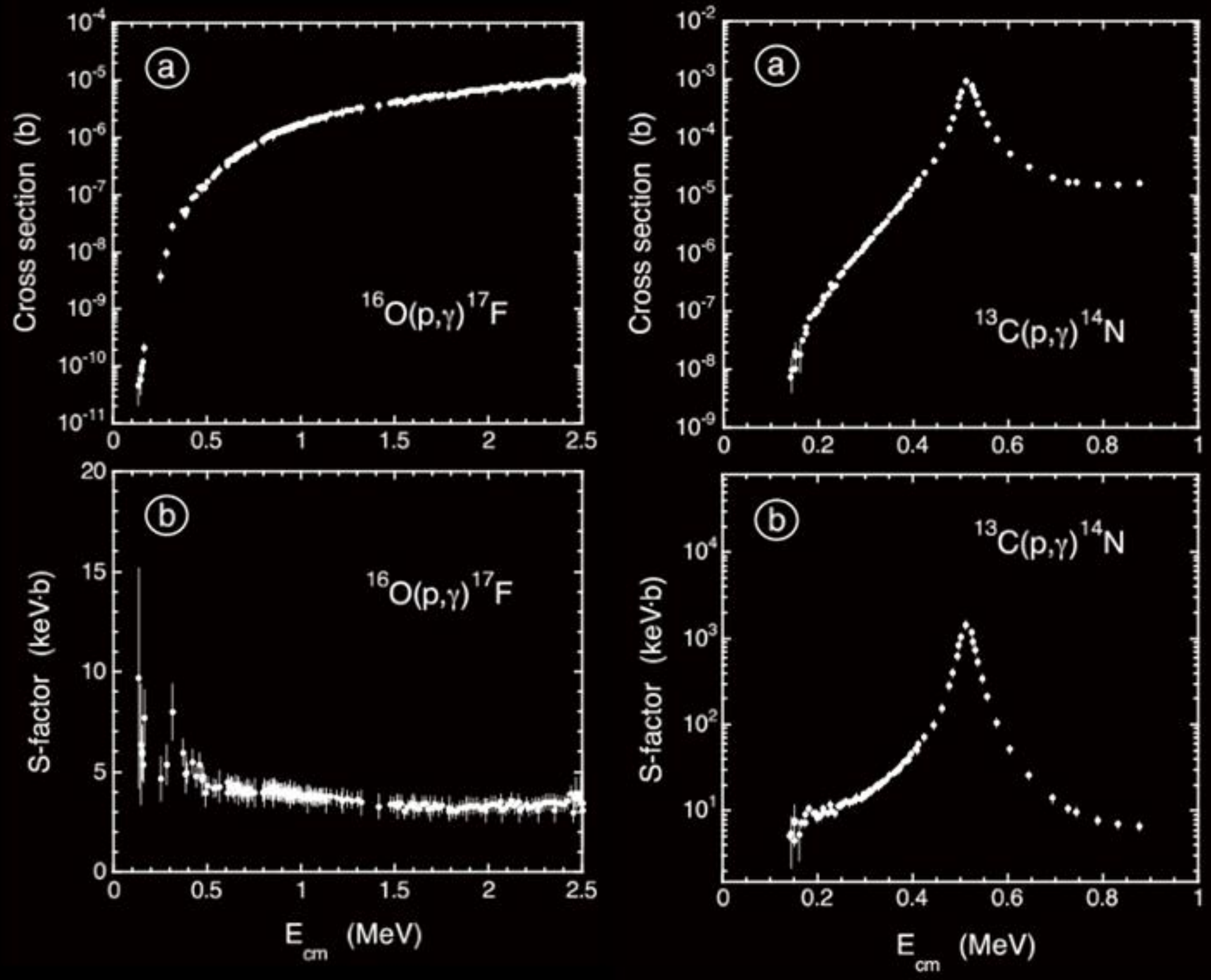
$$S(E) \begin{cases} \text{PERMITS MORE PRECISE EXTRAPOLATION FROM} \\ \text{LOWEST ENERGY MEASUREMENTS IN LABORATORY} \\ \text{TO VERY LOW EFFECTIVE STELLAR ENERGIES} \end{cases}$$

Rev. Mod. Phys., Vol. 56, No. 2, Part I, April 1984

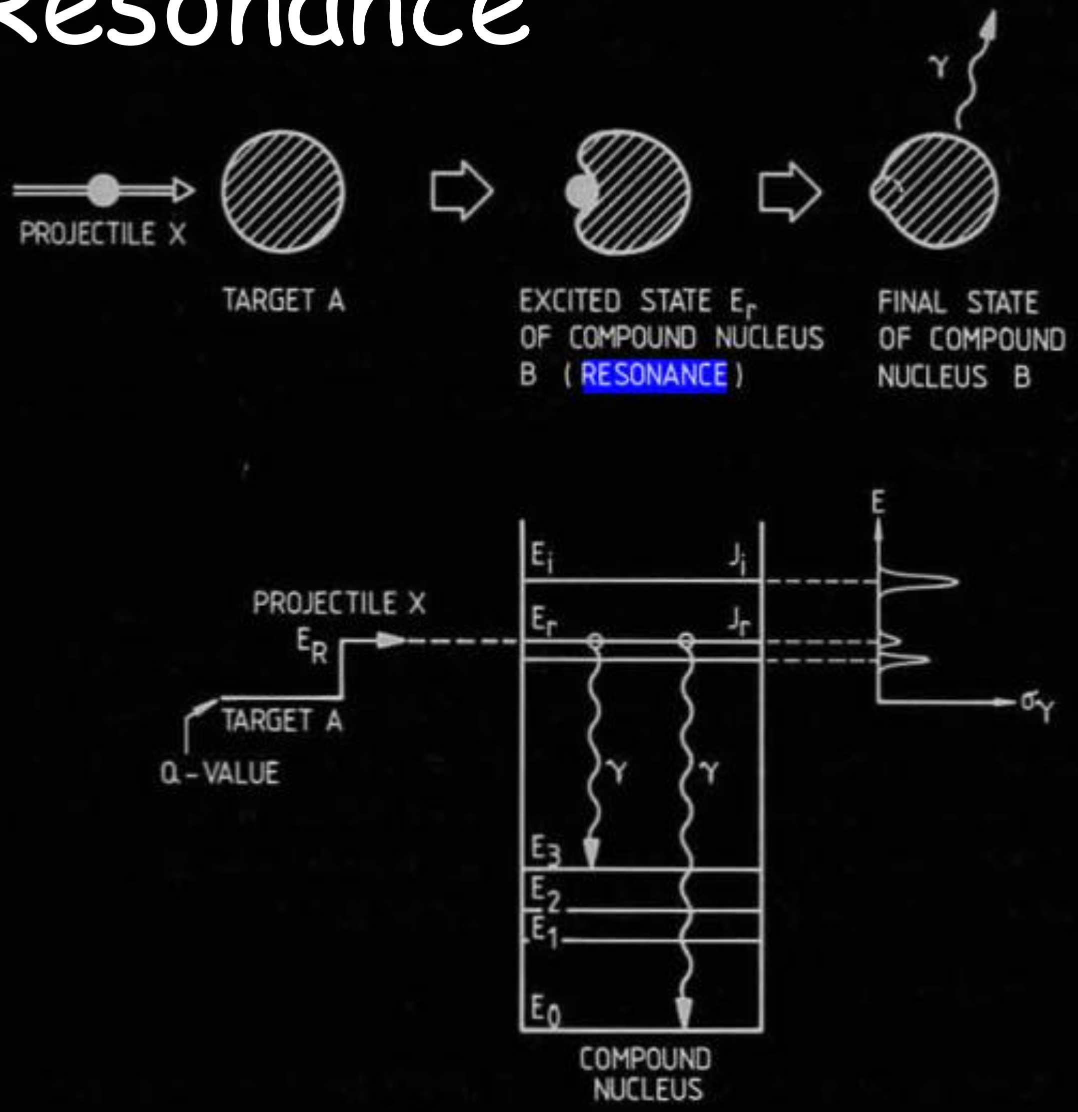
S factor: complication of nuclear structure

$$\sigma(E) \equiv \frac{1}{E} e^{-2\pi\eta} \boxed{S(E)}$$

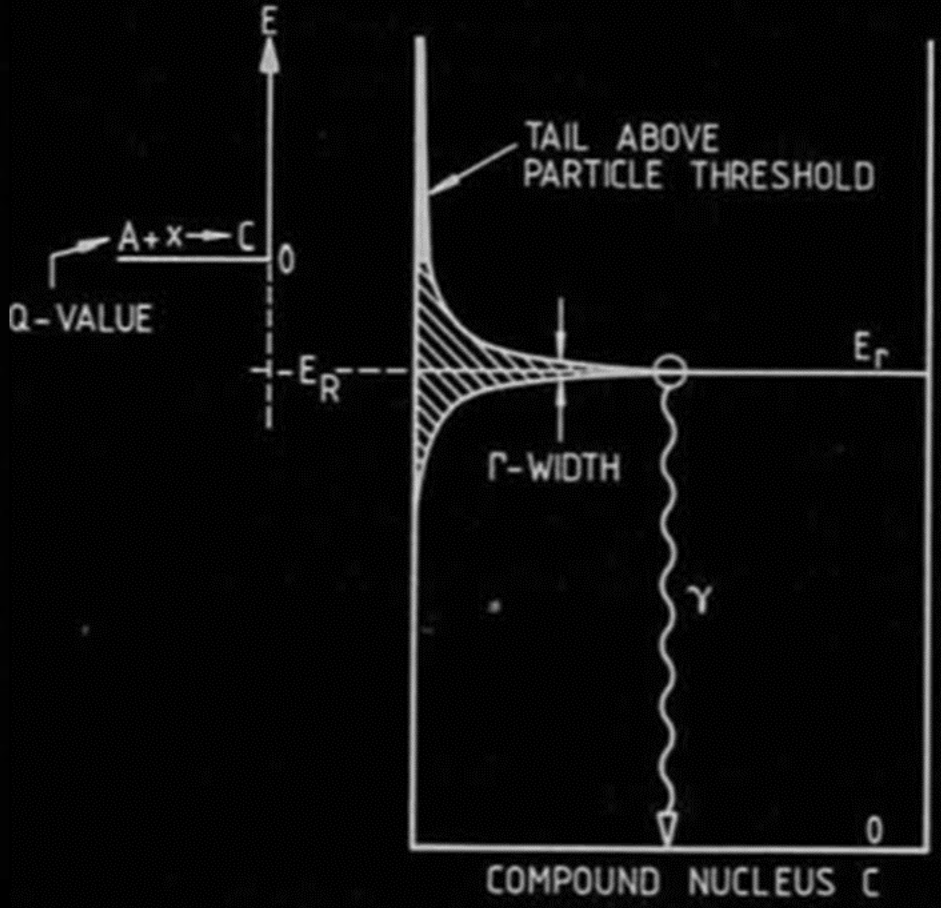
“astrophysical S-factor”



Resonance



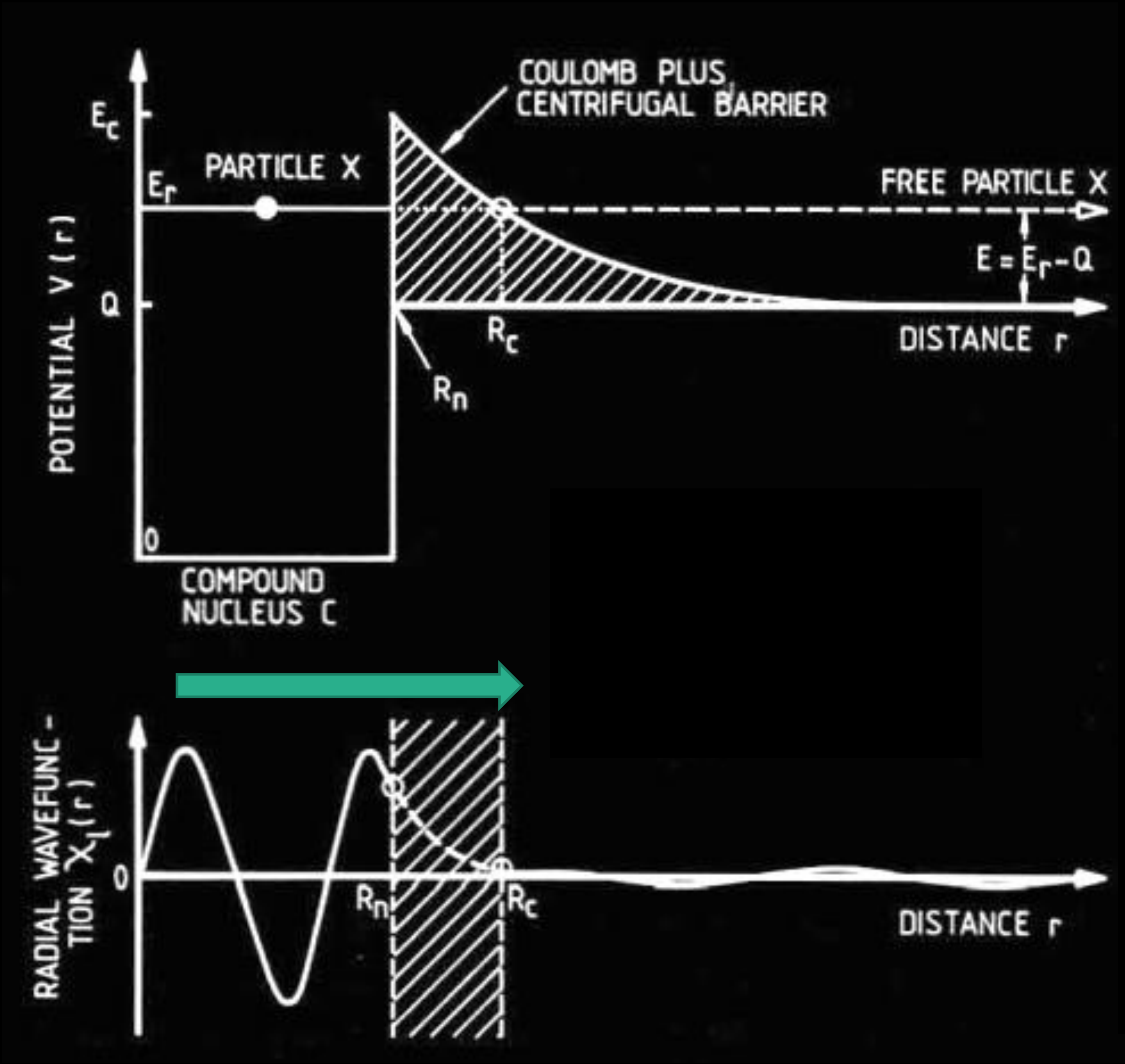
Resonance above the threshold

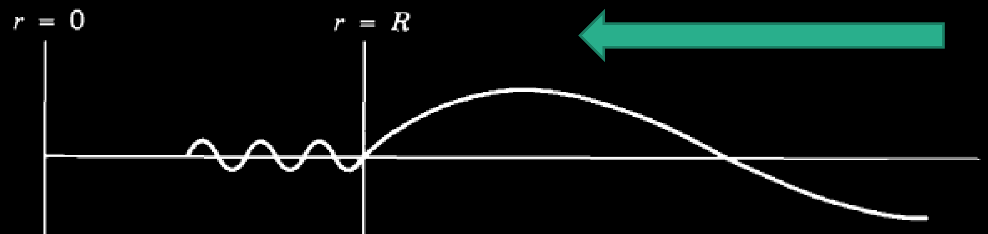


Sub-threshold Resonance

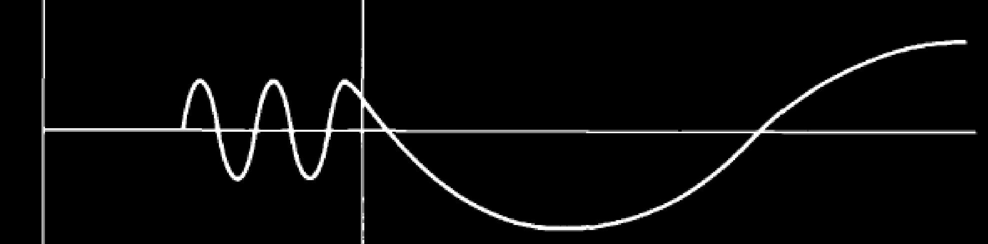
Decay of an excited state

Resonance of reaction

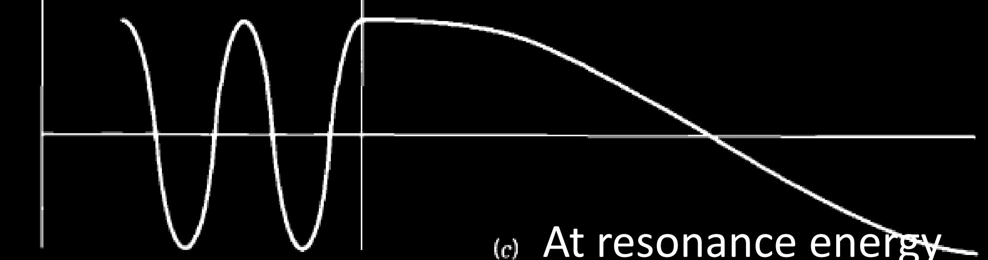




(a) Far from resonance



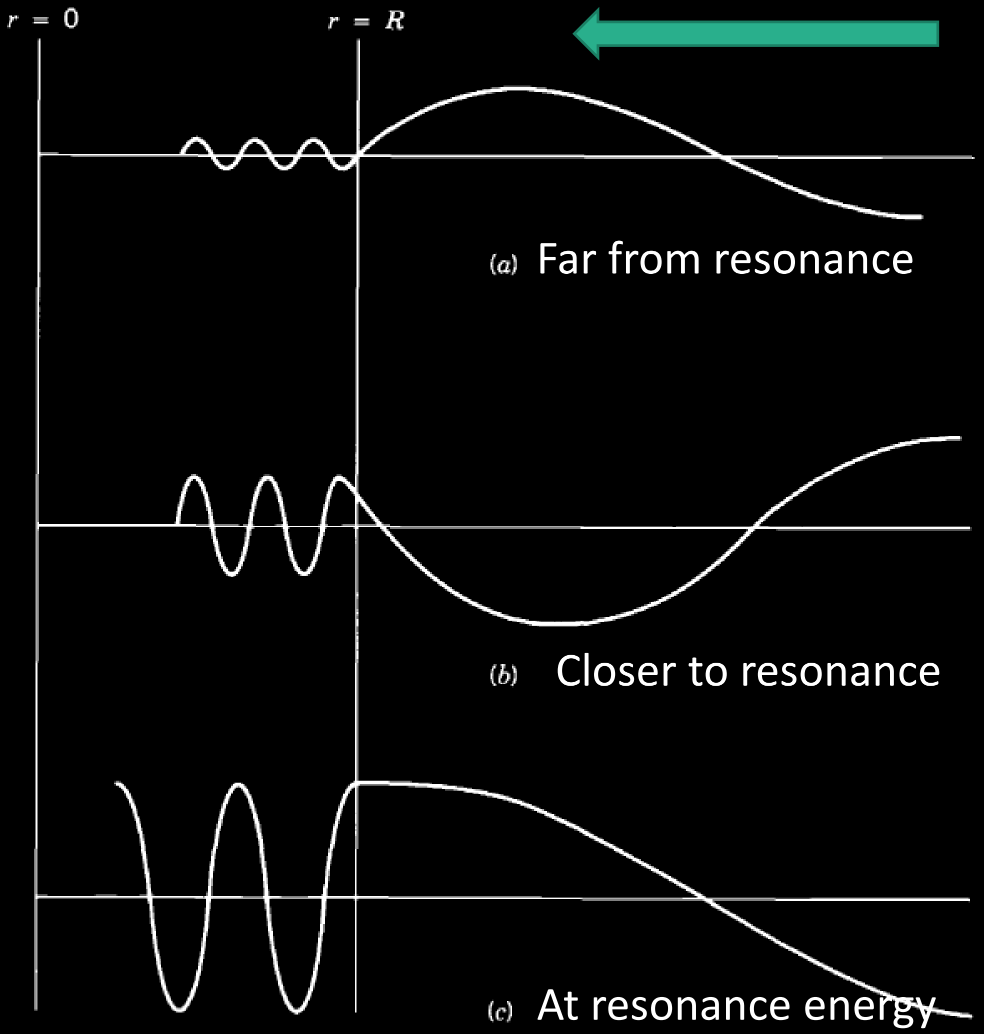
(b) Closer to resonance



(c) At resonance energy

Resonance

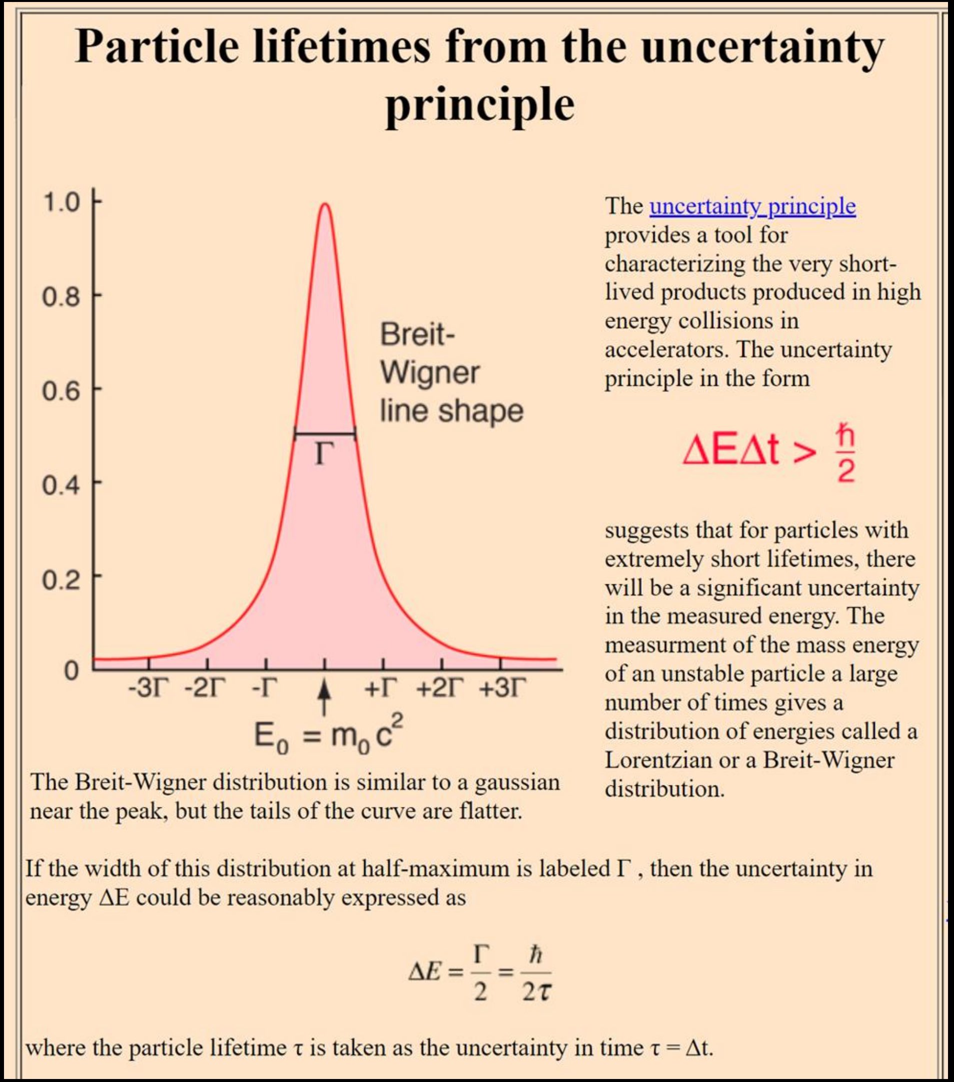
$$\boxed{\Gamma = \lambda\hbar}$$



(a) Far from resonance

(b) Closer to resonance

(c) At resonance energy



Particle lifetimes from the uncertainty principle

The uncertainty principle provides a tool for characterizing the very short-lived products produced in high energy collisions in accelerators. The uncertainty principle in the form

$$\Delta E \Delta t > \frac{\hbar}{2}$$

suggests that for particles with extremely short lifetimes, there will be a significant uncertainty in the measured energy. The measurment of the mass energy of an unstable particle a large number of times gives a distribution of energies called a Lorentzian or a Breit-Wigner distribution.

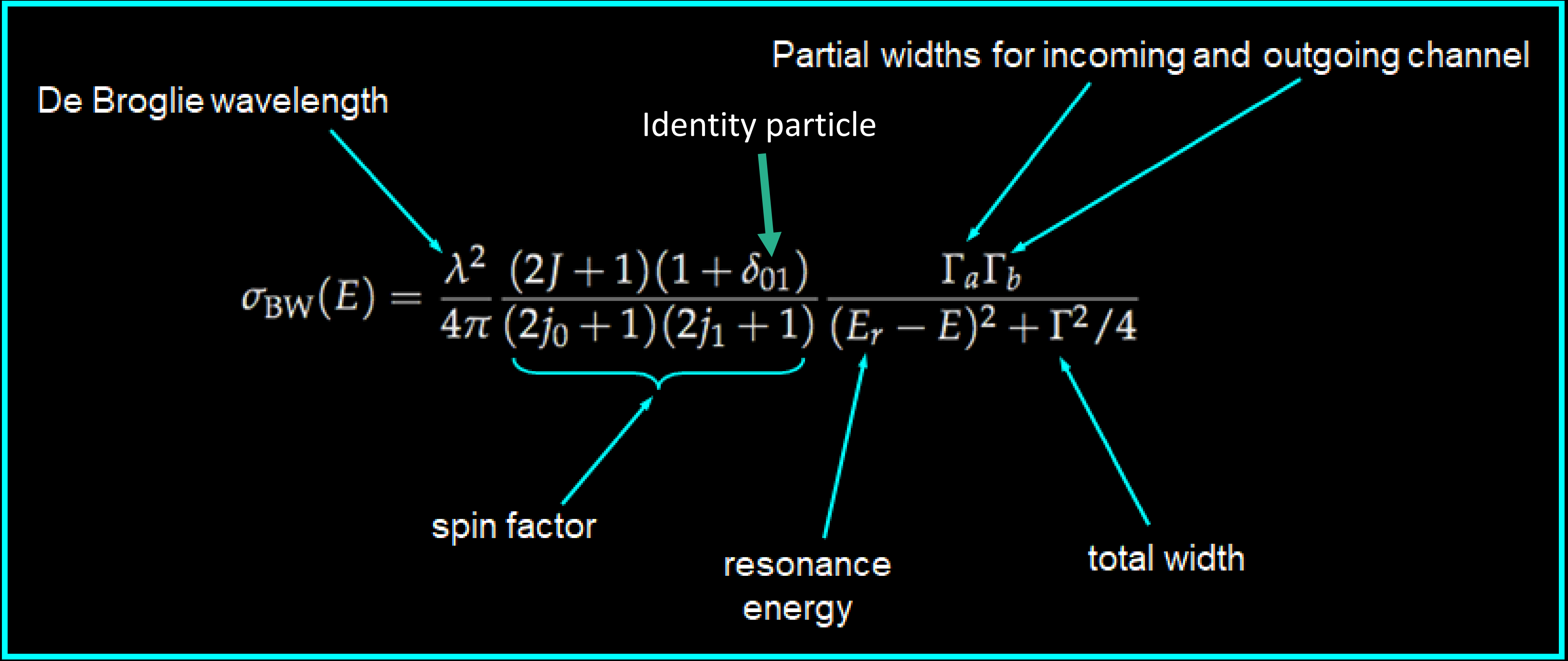
The Breit-Wigner distribution is similar to a gaussian near the peak, but the tails of the curve are flatter.

If the width of this distribution at half-maximum is labeled Γ , then the uncertainty in energy ΔE could be reasonably expressed as

$$\Delta E = \frac{\Gamma}{2} = \frac{\hbar}{2\tau}$$

where the particle lifetime τ is taken as the uncertainty in time τ = Δt.

Breit-Wigner formula



Partial widths for incoming and outgoing channel

De Broglie wavelength

Identity particle

$$\sigma_{\mathrm{BW}}(E) = \frac{\lambda^2}{4\pi} \underbrace{\frac{(2J+1)(1+\delta_{01})}{(2j_0+1)(2j_1+1)}}_{\text{spin factor}} \frac{\Gamma_a \Gamma_b}{(E_r - E)^2 + \Gamma^2/4}$$

spin factor

resonance energy

total width

Resonance strength: $\omega\gamma = \frac{(2J+1)}{(2J_p+1)(2J_t+1)} \frac{\Gamma_a \Gamma_b}{\Gamma}$ **Total width** $\Gamma = \sum_i \Gamma_i$

Partial widths

a constant if there is only 1 level

For proton, neutron, alpha…: $\Gamma_{\lambda c} = 2P_c\gamma^2_{\lambda c}$

Reduced width: $\gamma^2_{\lambda c} = \theta^2_{\lambda c}\frac{3\hbar^2}{2mR^2}$

Dimensionless reduced width<1

Wigner limit

Descouvemont and Baye, R matrix theory, arXiv:1001.0678

$$P_\ell = R\left(\frac{k}{F_\ell^2+G_\ell^2}\right)_{r=R}$$

where F_l is the regular Coulomb function and G_l is the irregular Coulomb function

$$2\pi\eta = 0.989534 Z_0 Z_1 \sqrt{\frac{1}{E}\frac{M_0 M_1}{M_0+M_1}}$$

η is the Sommerfeld parameter
E is the energy in MeV
M_i is in the unit of amu

For s—wave charged particle,**(l=0) and $E<<E_c$,**

$P_0 \sim kR\exp(-2\pi\eta)$

For neutron,

$F_l(\rho)=\rho*jl(\rho)$

$G_l(\rho)=\rho*nl(\rho)$

Eg. s-wave neutron (l=0),

$P_0=kR$

Penetration factors

$$P_\ell = R\left(\frac{k}{F_\ell^2 + G_\ell^2}\right)_{r=R}$$

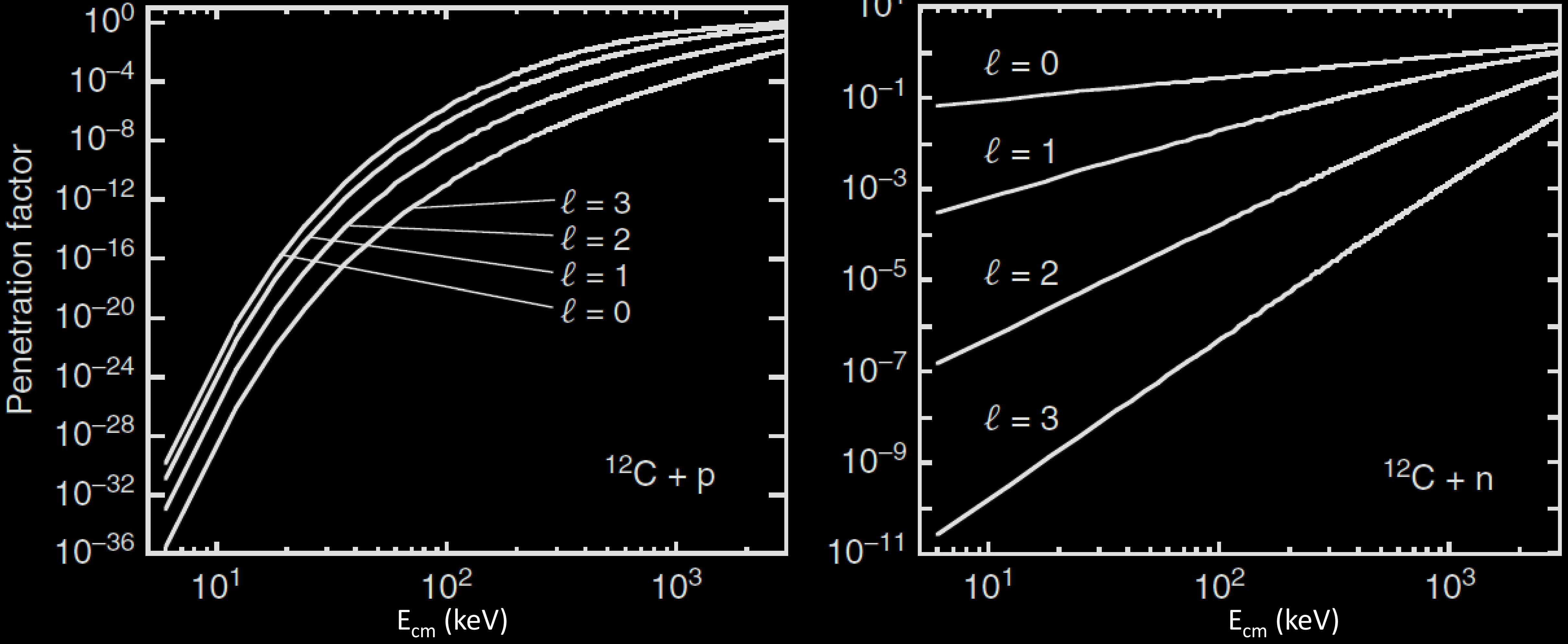


Fig. 2.21 from Iliadis's book

Particle width

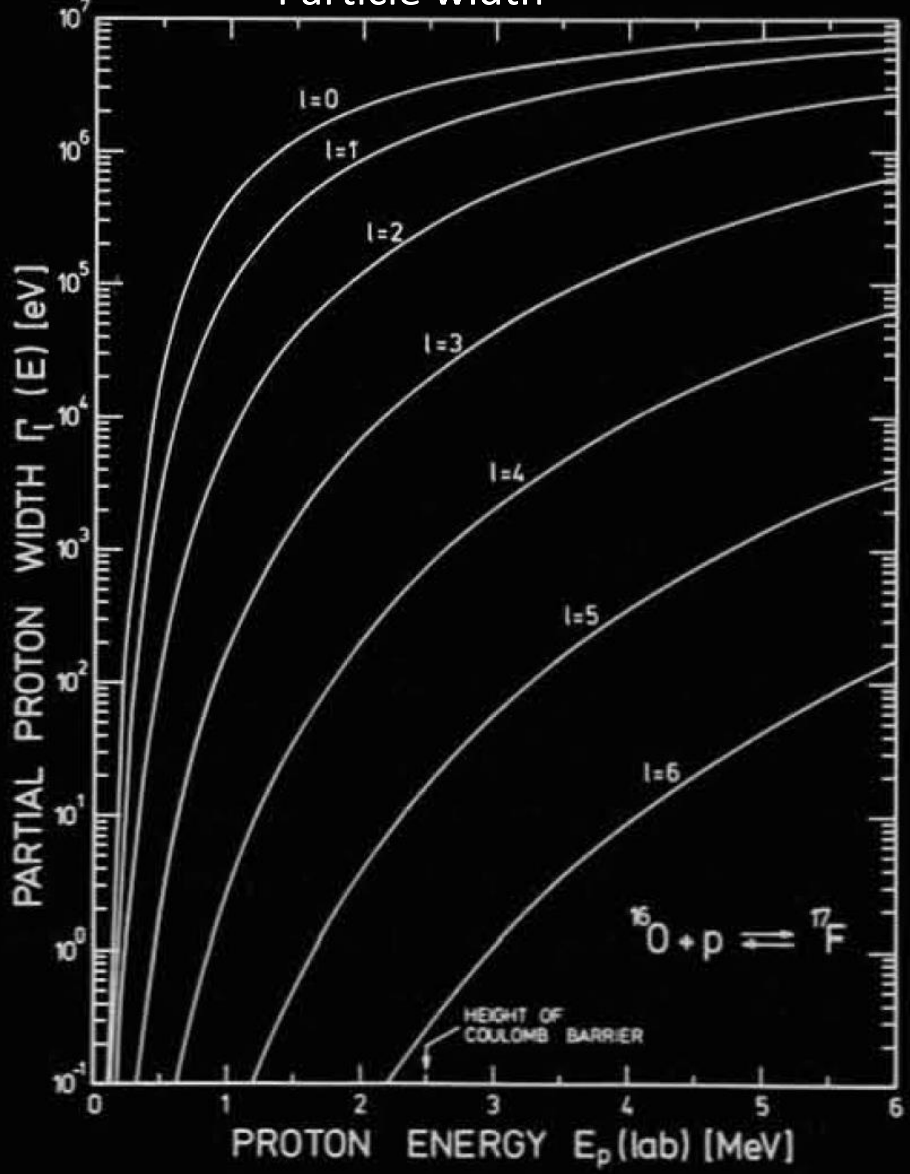


FIGURE 4.14. The maximum partial width $\Gamma_l(E)$ of the reaction channel $^{16}O + p \rightleftarrows {}^{17}F$ for increasing values of the orbital angular momentum l ($R_n = 4.6$ fm). The energetic location of the Coulomb barrier is also indicated. Note the logarithmic scale of the ordinate.

Width reported in NNDC

$$\Gamma_l(E) = \Gamma_l(E_r)\frac{P_l(E)}{P_l(E_r)}$$

$$P_\ell = R\left(\frac{k}{F_\ell^2 + G_\ell^2}\right)_{r=R}$$

$$\rho = 0.218735 \cdot r\sqrt{\frac{M_pM_t}{M_p + M_t}E}$$

$$\eta = 0.157489 \cdot Z_pZ_t\sqrt{\frac{M_pM_t}{M_p + M_t}\frac{1}{E}}$$

where M_i, E and r are in units of u, MeV and fm, respectively.

Partial widths

Decay constant of electromagnetic transition

$$\lambda(\varpi L) = \frac{8\pi}{L[(2L+1)!!]^2}\frac{1}{\hbar}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(\varpi L)$$

reduced transition probability

multipolarity

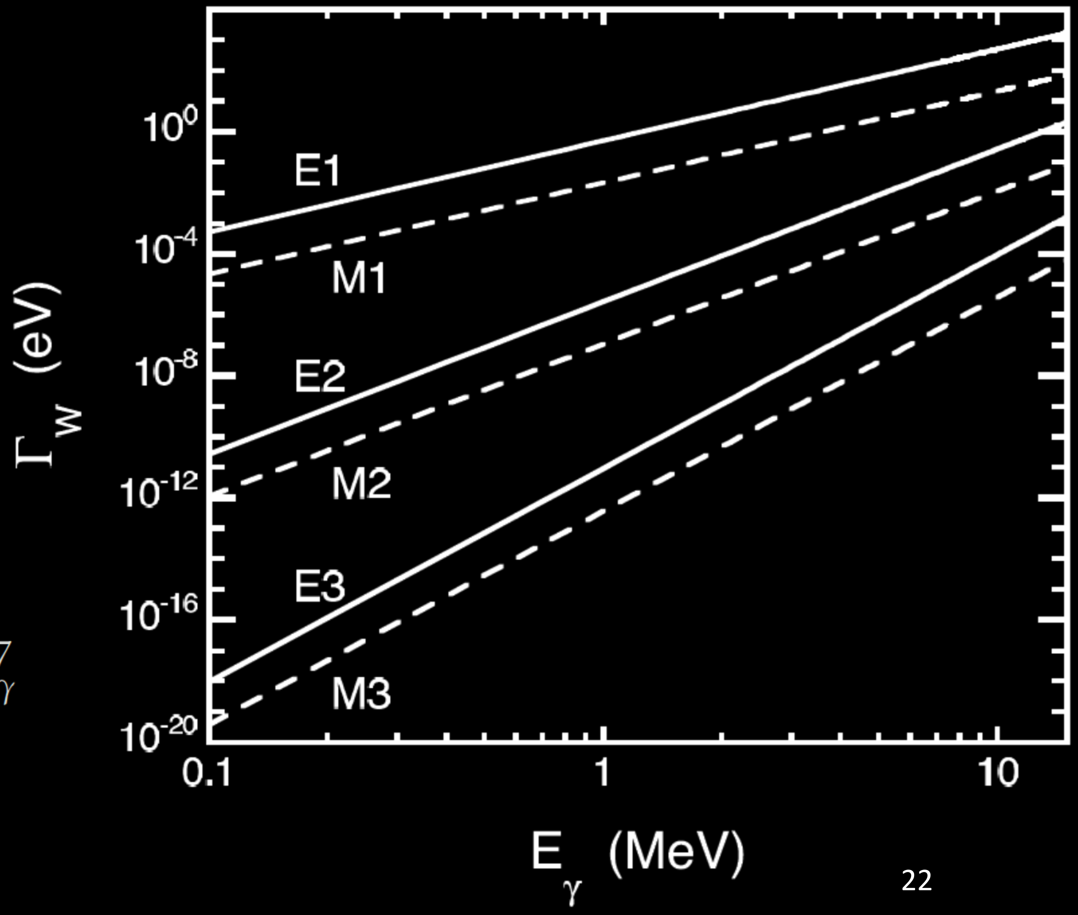
$$\boxed{\Gamma = \lambda\hbar \sim E_\gamma^{2L+1}}$$

Weisskopf estimates in units of eV, E_γ in MeV

$$\lambda_W(\mathrm{E1})\hbar = 6.8\times10^{-2}A^{2/3}E_\gamma^3, \qquad \lambda_W(\mathrm{M1})\hbar = 2.1\times10^{-2}E_\gamma^3$$

$$\lambda_W(\mathrm{E2})\hbar = 4.9\times10^{-8}A^{4/3}E_\gamma^5, \qquad \lambda_W(\mathrm{M2})\hbar = 1.5\times10^{-8}A^{2/3}E_\gamma^5$$

$$\lambda_W(\mathrm{E3})\hbar = 2.3\times10^{-14}A^{2}E_\gamma^7, \qquad \lambda_W(\mathrm{M3})\hbar = 6.8\times10^{-15}A^{4/3}E_\gamma^7$$



Transitions with changed parity : E1, M2, E3
Transitions with non changed parity: M1,E2, M3

Compare the energy dependence of the particle width (s-wave) with that of the gamma width at E_{cm}=0.01, 0.1, 1 MeV. Assume that the gamma ray decay to the ground state of ^{17}F.

Particle width

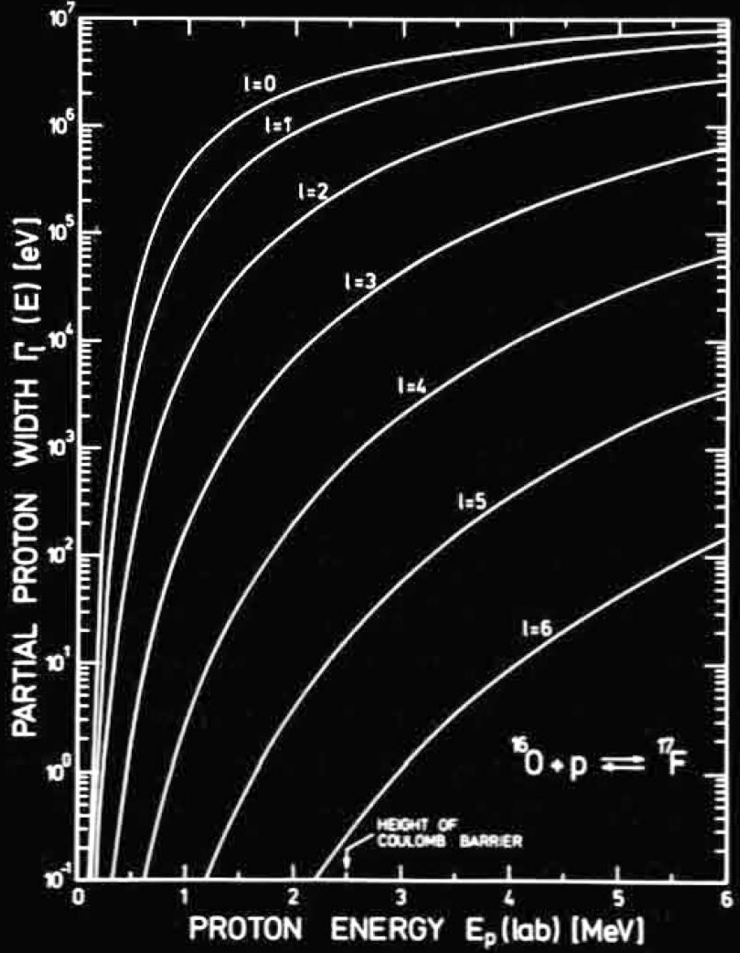
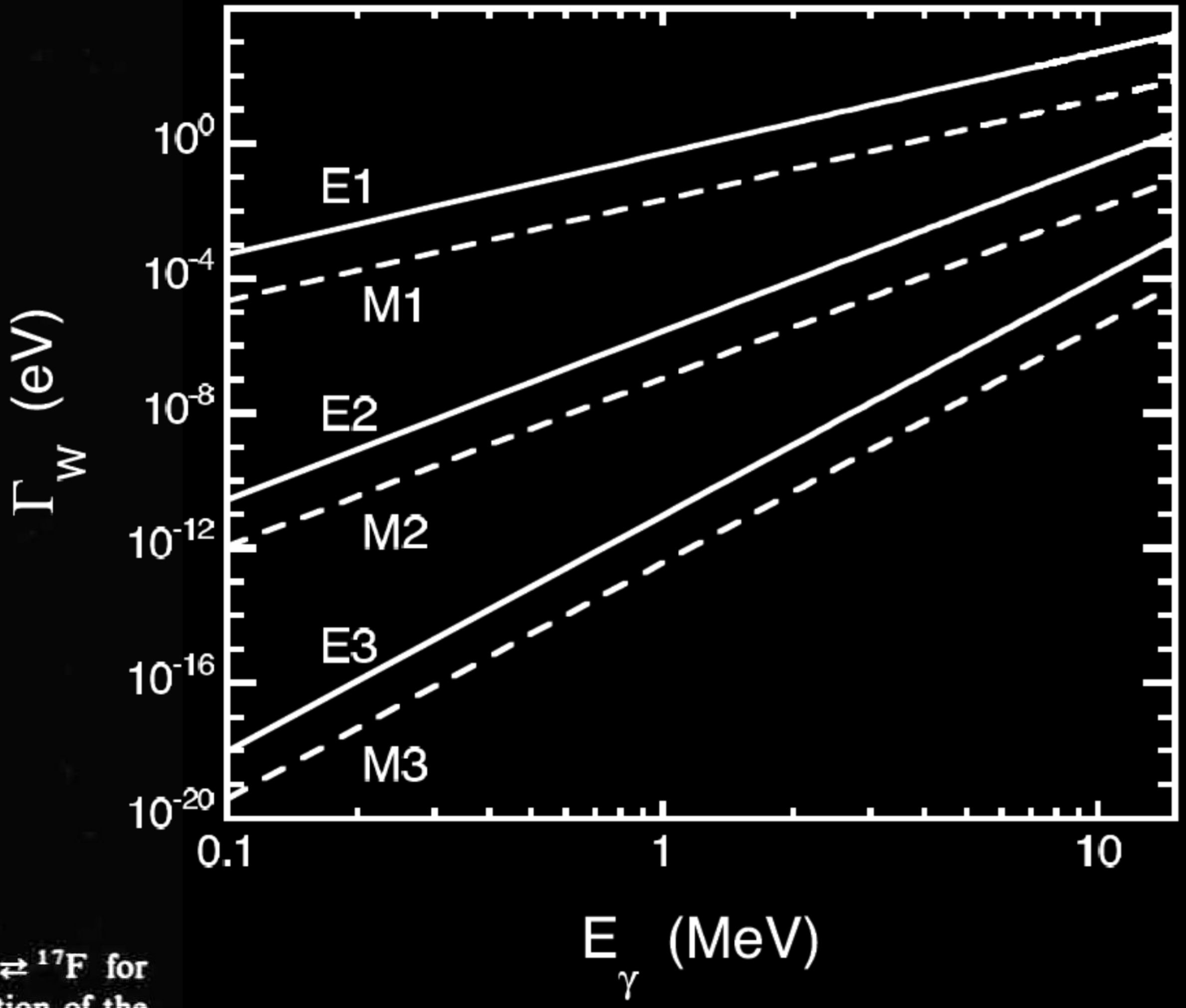
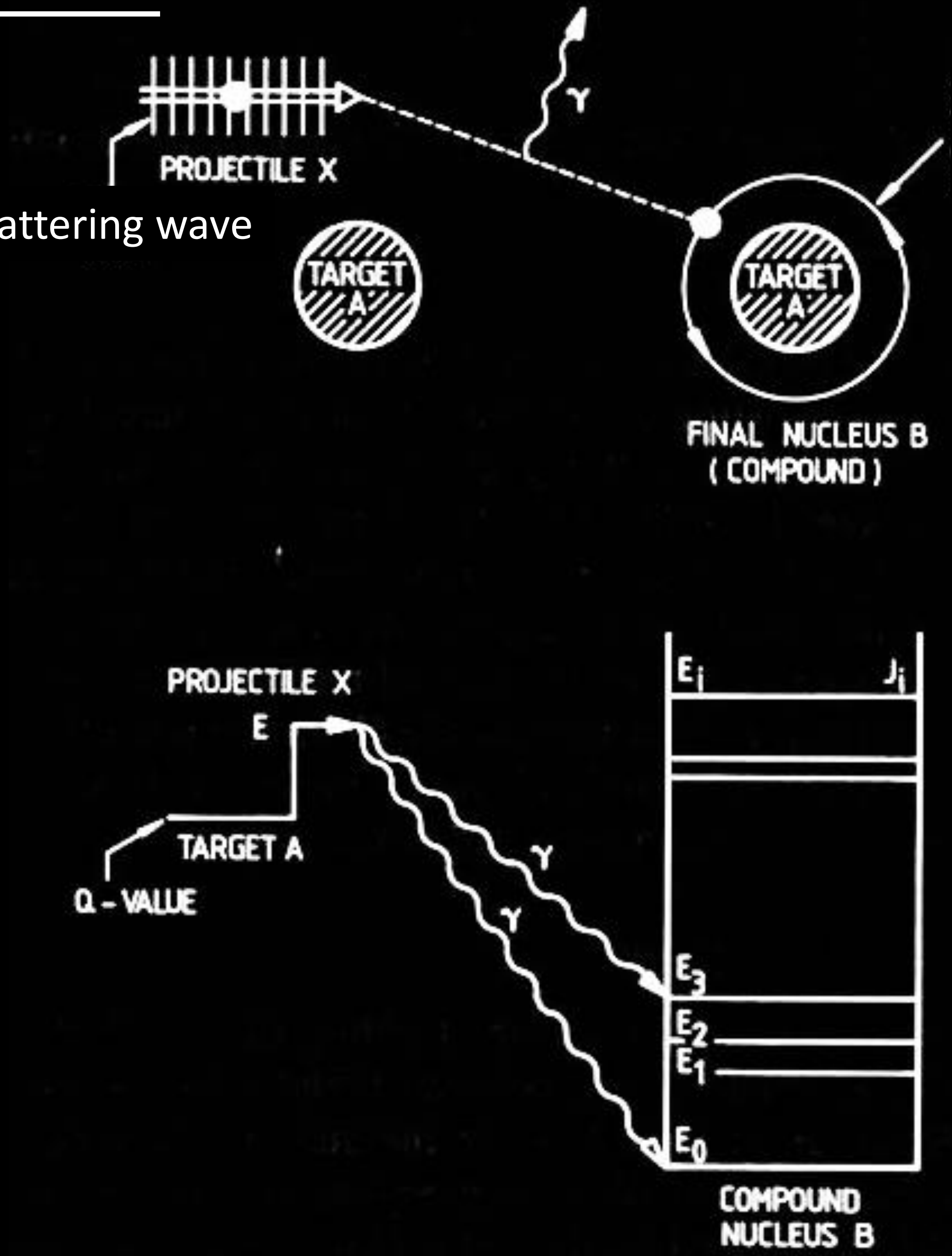


FIGURE 4.14. The maximum partial width $\Gamma_l(E)$ of the reaction channel $^{16}O + p \rightleftarrows {}^{17}F$ for increasing values of the orbital angular momentum l ($R_n = 4.6$ fm). The energetic location of the Coulomb barrier is also indicated. Note the logarithmic scale of the ordinate.

Gamma width (Weisskopf Estimation)

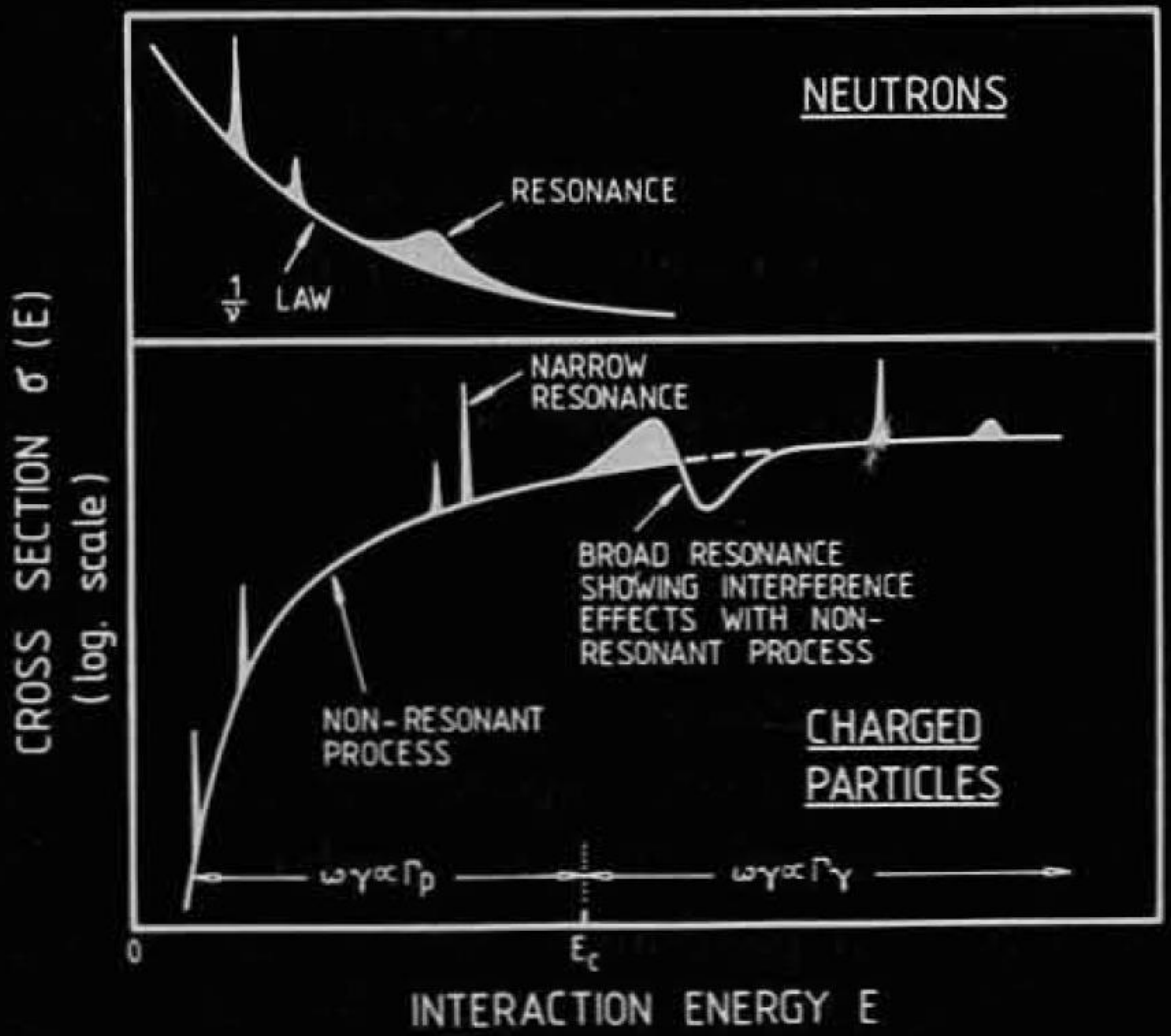


Direct capture



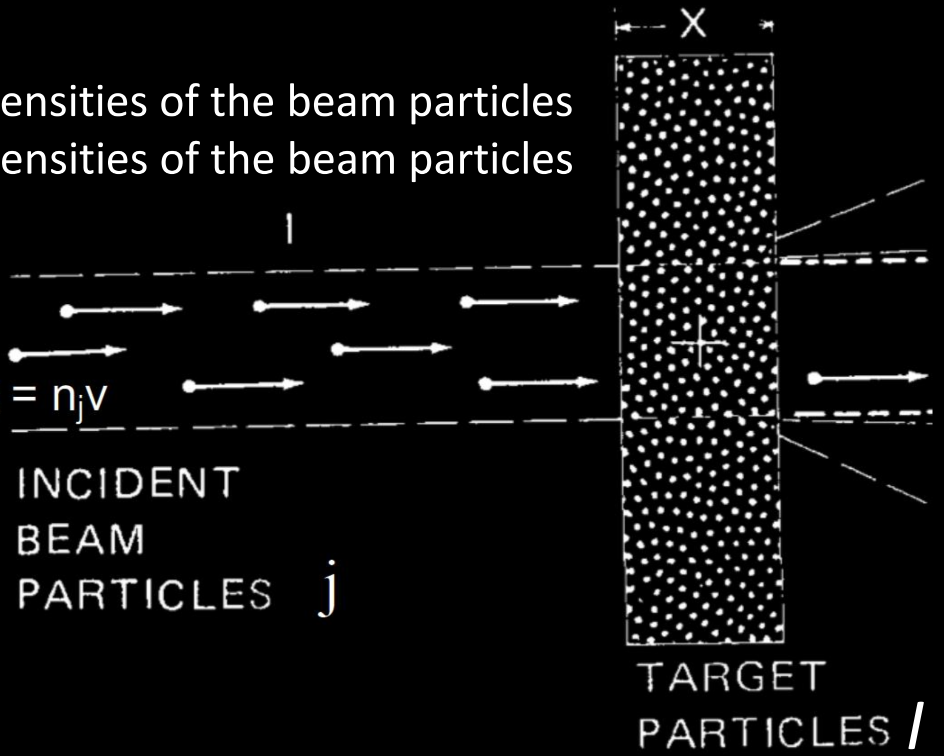
Bound state wave

Scattering wave

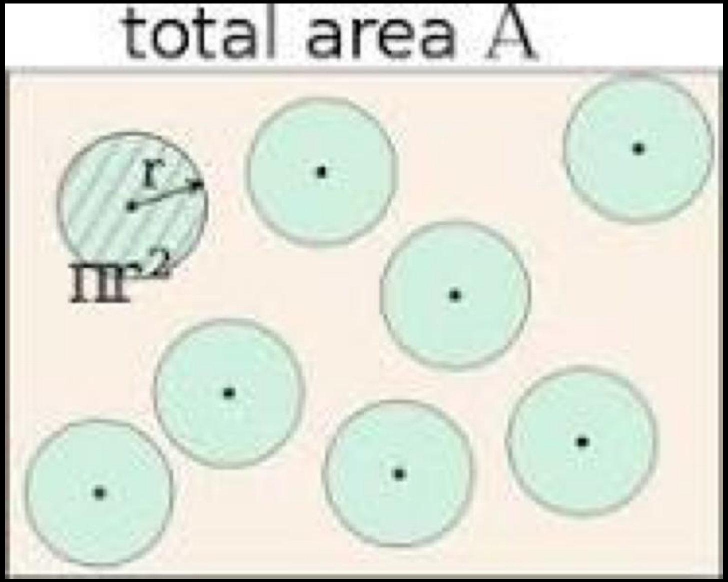

NEUTRONS
RESONANCE
$\frac{1}{v}$ LAW
NARROW RESONANCE
BROAD RESONANCE SHOWING INTERFERENCE EFFECTS WITH NON-RESONANT PROCESS
NON-RESONANT PROCESS
CHARGED PARTICLES
$\omega\gamma \propto \Gamma_p$
$\omega\gamma \propto \Gamma_\gamma$
0
E_c
INTERACTION ENERGY E
CROSS SECTION σ (E) (log. scale)

Reaction Rate

n_j: number densities of the beam particles
n_l: number densities of the beam particles



flux = $n_j v$



Total area of target nuclei per cm^3 = $n_l \sigma_l$

Reaction rate per cm^3 per sec = $n_j v\, n_l\, \sigma_l$

The reaction rate for the two reactants, *I* and *j* as in e.g., I (j,k) L is:

$$n_I n_j \sigma_{Ij} \mathrm{v}$$

which has units "reactions cm^{-3} s^{-1}"

It is often more convenient to write abundances in terms of the mole fractions,

$$Y_I = \frac{X_I}{A_I} \qquad n_I = \rho N_A Y_I$$

$$\left(\frac{gm}{cm^3}\right)\left(\frac{atoms}{Mole}\right)\left(\frac{Mole}{gm}\right)$$

so that the rate becomes

$$(\rho N_A)^2 Y_I Y_j \sigma_{Ij} \mathrm{v}$$

and a term in a rate equation decribing the destruction of I might be

$$\boxed{\frac{dY_I}{dt} = -\rho Y_I Y_j N_A \langle \sigma_{Ij} \mathrm{v} \rangle + \ldots.}$$

Equivalent to

$$\frac{dn_I}{dt} = -n_I n_j \langle \sigma_{Ij} v \rangle + \ldots$$

Stellar reaction rates

$$r = \frac{1}{1+\delta_{pT}} Y_T Y_p \rho^2 \, \mathrm{N}_\mathrm{A}^2 < \sigma v >$$

reactions per s and cm^3

$$\lambda = \frac{1}{1+\delta_{pT}} Y_p \rho \, \mathrm{N}_\mathrm{A} < \sigma v >$$

reactions per s &
Target nucleus

this is usually referred to
as the **stellar reaction rate**

units of stellar reaction rate $\mathrm{N_A}$<sv>: usually cm^3/s/mole

$$n_T = \rho \cdot N_A \cdot \frac{X_T}{A_T} = \rho \cdot N_A \cdot Y_T$$

X_T; mass fraction
Y_T: abundance

Change in Abundance

$$A + a \Rightarrow B$$

A reaction is a random process with a reaction probability (reaction rate) and follows the laws of radioactive decay:

Depletion of isotope A

$$\frac{dn_A}{dt} = -n_A \lambda = -n_A \underline{Y_a \rho \, \mathrm{N_A} < \sigma v >}$$

Formation of isotope B

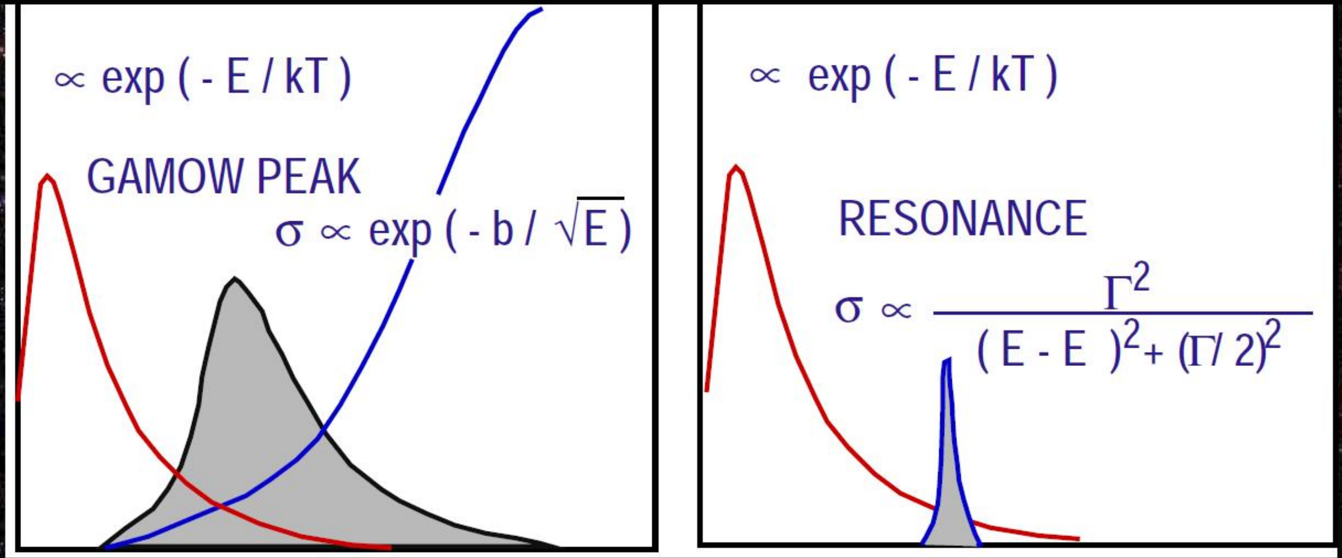
$$\frac{dn_B}{dt} = +n_A \lambda$$

consequently:

$$n_A(t) = n_{0A} \, \mathrm{e}^{-\lambda \mathrm{t}}$$

$$n_B(t) = n_{0A} (1 - \mathrm{e}^{-\lambda \mathrm{t}})$$

Gamow peak & Reaction rate



Nonresonant Reaction Contributions

$$N_A < \sigma v > \propto T^{-3/2} \int \sigma E \exp (- E / kT) \quad d E$$

Resonant Reaction Rate

$$N_A < \sigma v > \propto T^{-3/2} \omega\gamma \exp (- E_R / kT)$$

σ: cross section

ωγ: res. strength

E_R: res. energy

The Gamow Range of Stellar Burning

The **Gamow window** or the range of relevant cross section for "non-resonant" processes is calculated:

Check derivation in book

$$E_0 = \left(\frac{bkT}{2}\right)^{3/2} = 0.122 \cdot \left(Z_1^2 Z_2^2 A\right)^{1/3} T_9^{2/3} \text{ MeV}$$

$$\Delta E = \frac{4}{\sqrt{3}}\sqrt{E_0 kT} = 0.2368 \cdot \left(Z_1^2 Z_2^2 A\right)^{1/6} T_9^{5/6} \text{ MeV}$$

with A "reduced mass number" and T_9 the temperature in GK

Reaction rate from S-factor

If S-factor ~ constant over the Gamow range
the rate is calculated in terms of the S-factor

$$S(E)=S(E_0)$$

$$N_A < \sigma v >= 7.83 \cdot 10^9 \left(\frac{Z_1 Z_2}{\mu T_9} \right)^{1/3} S(E_0)[\text{MeV barn}]\ \mathrm{e}^{-4.2487\left(\frac{Z_1^2 Z_2^2 \mu}{T_9} \right)^{1/3}}$$

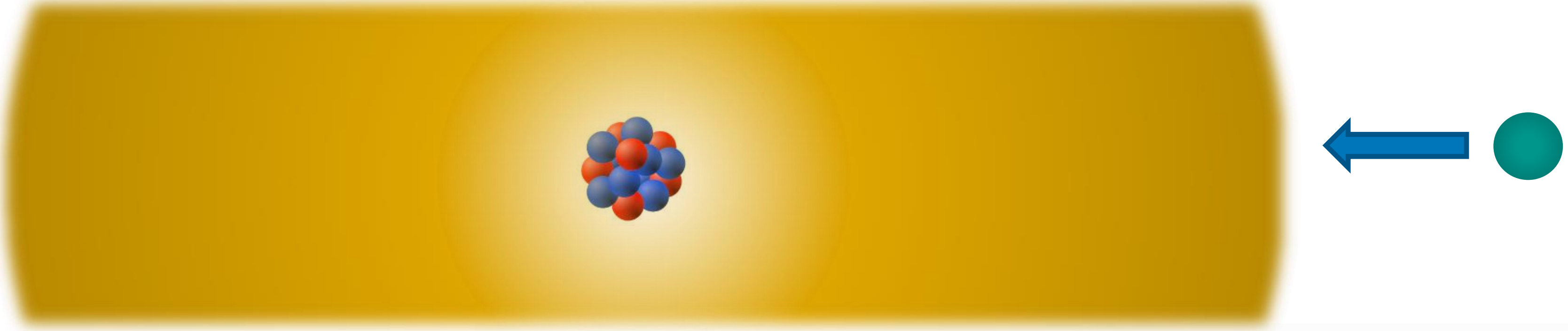
Otherwise energy dependence needs to be approximated!

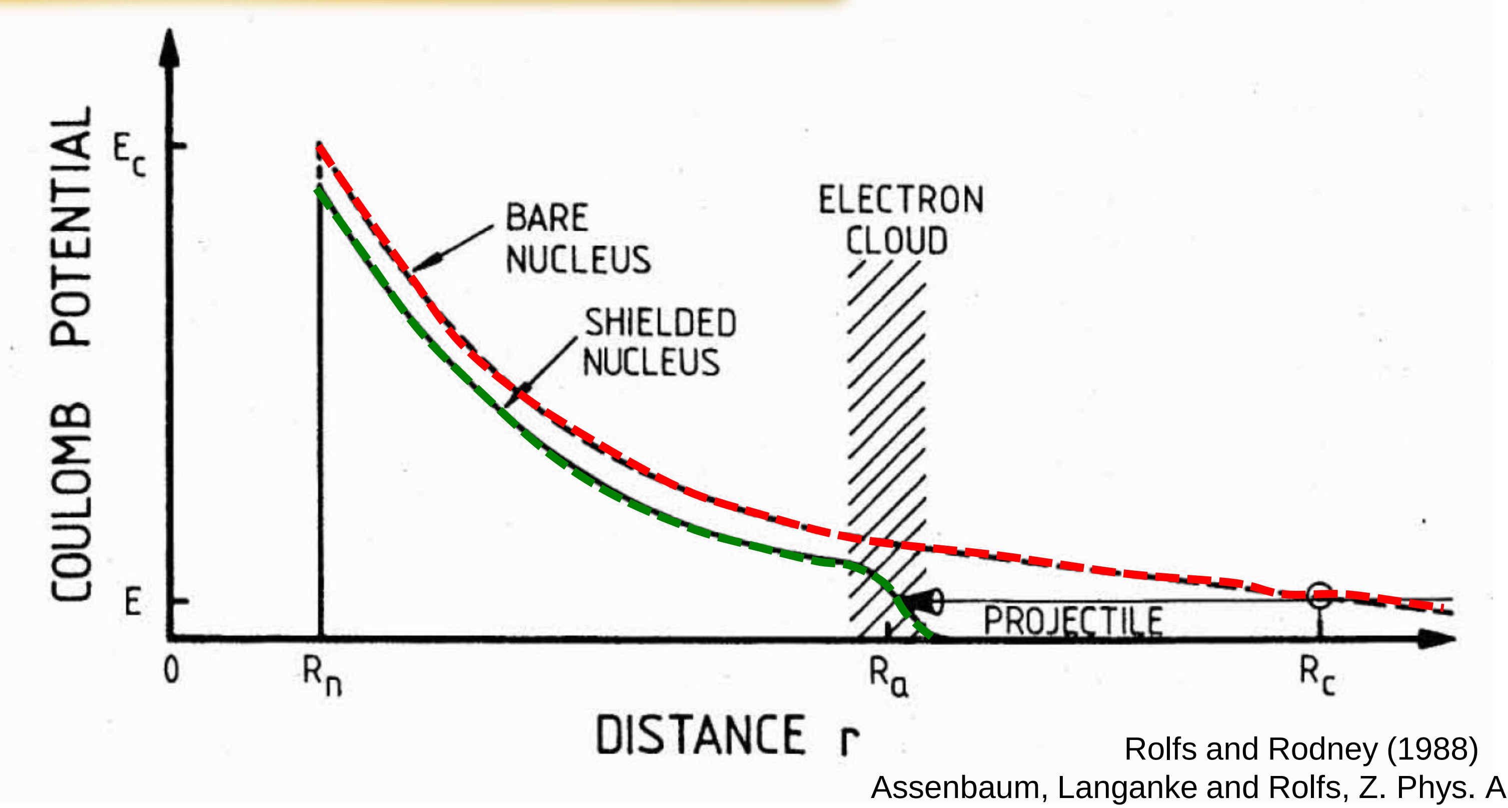
Screening effect

U_e: Screening potential

JUNA

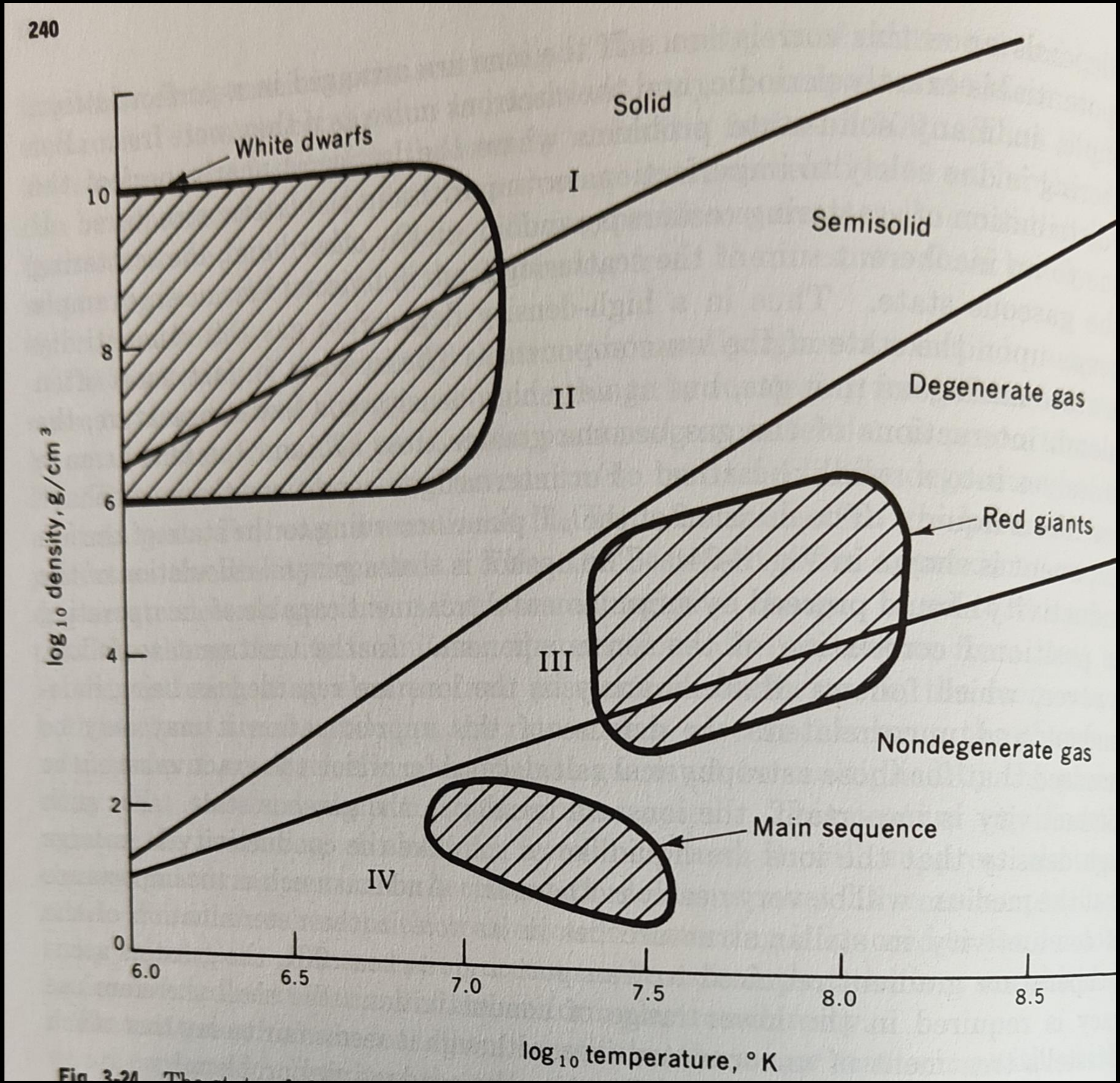
$$f_{\text{lab}}(E) = \sigma_s(E)/\sigma_b(E) \approx \exp(\pi\eta U_e/E) \geq 1$$



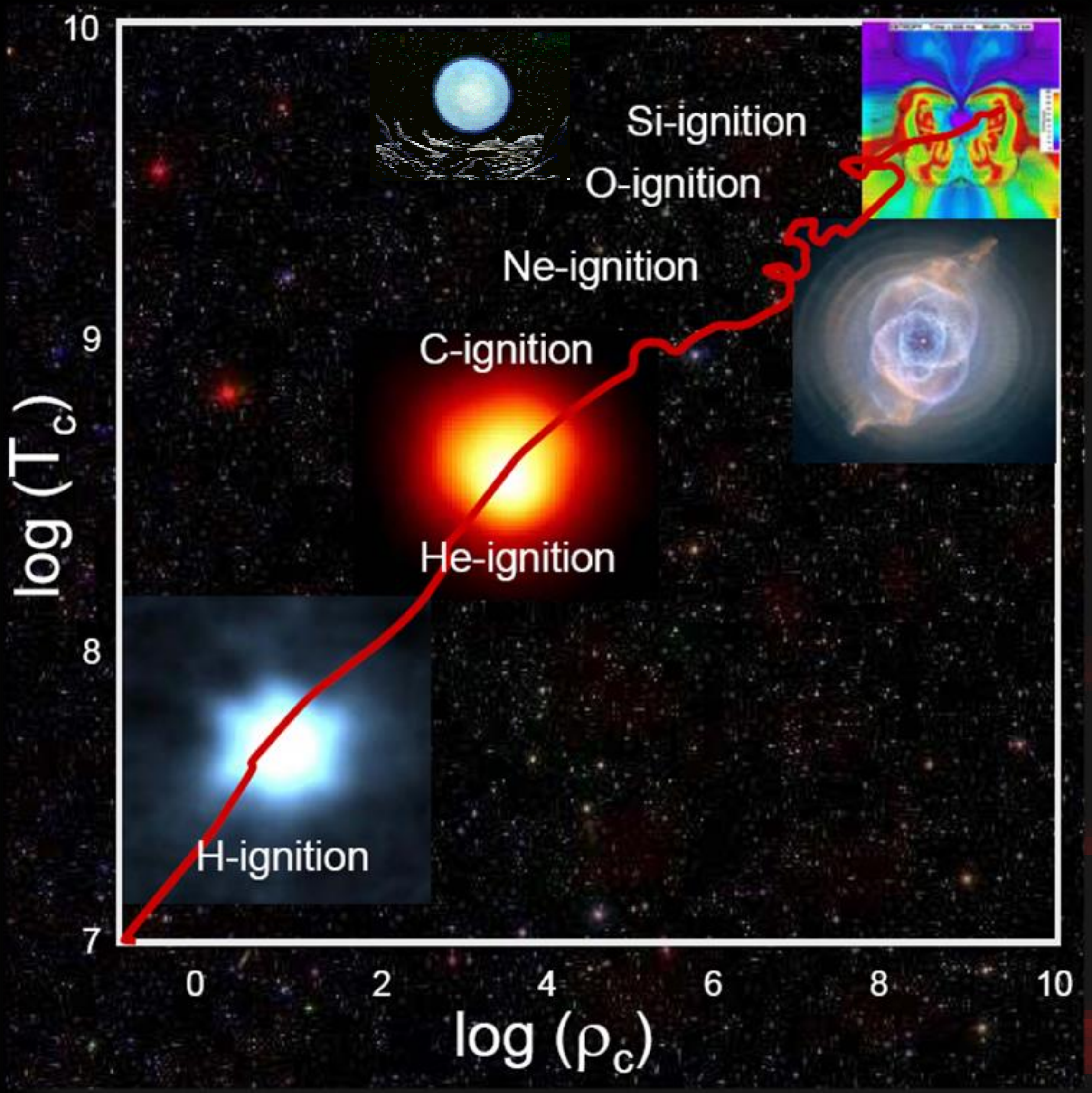


Rolfs and Rodney (1988)

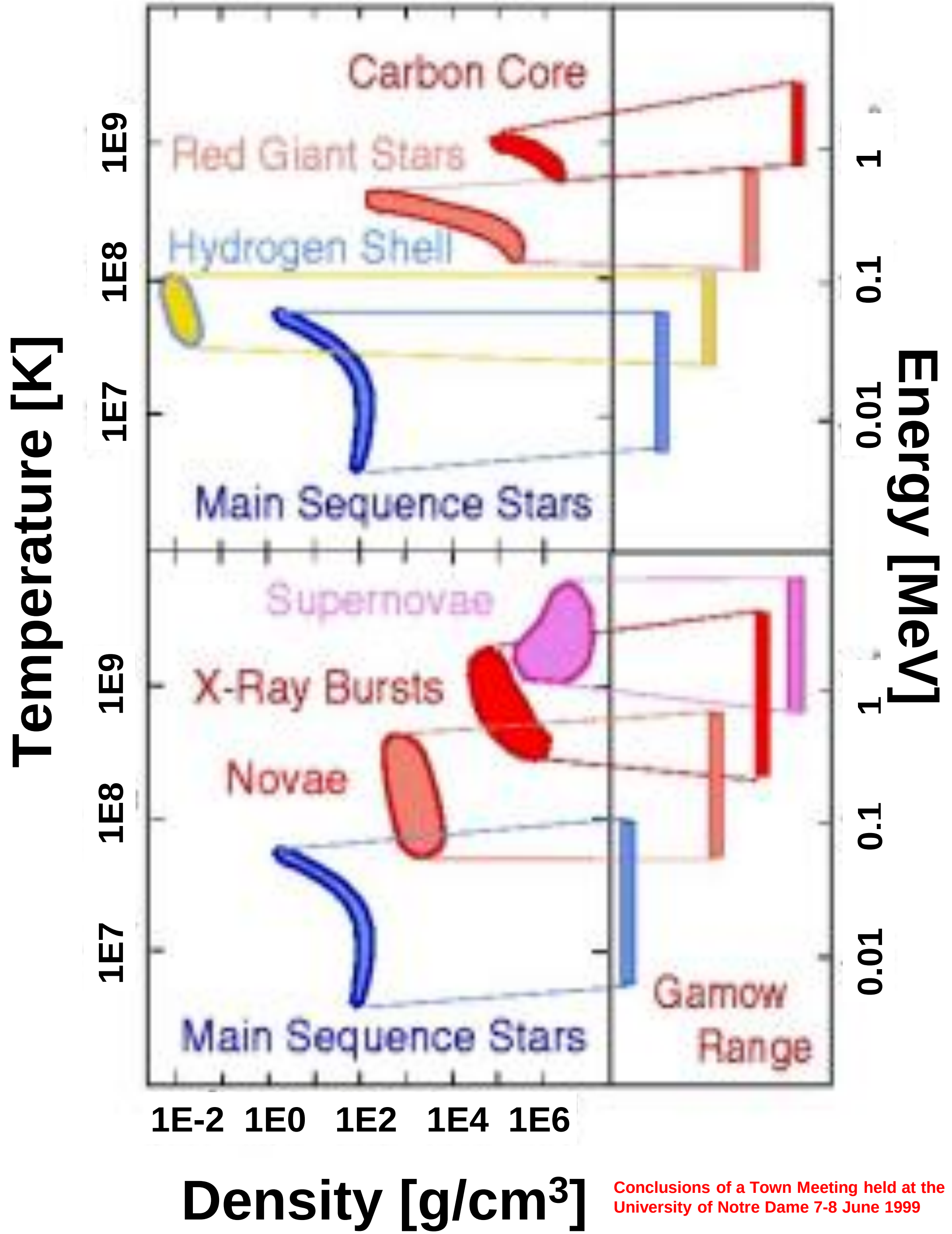
Assenbaum, Langanke and Rolfs, Z. Phys. A327, 461 (1987)



The assumption of ideal gas is not always valid!

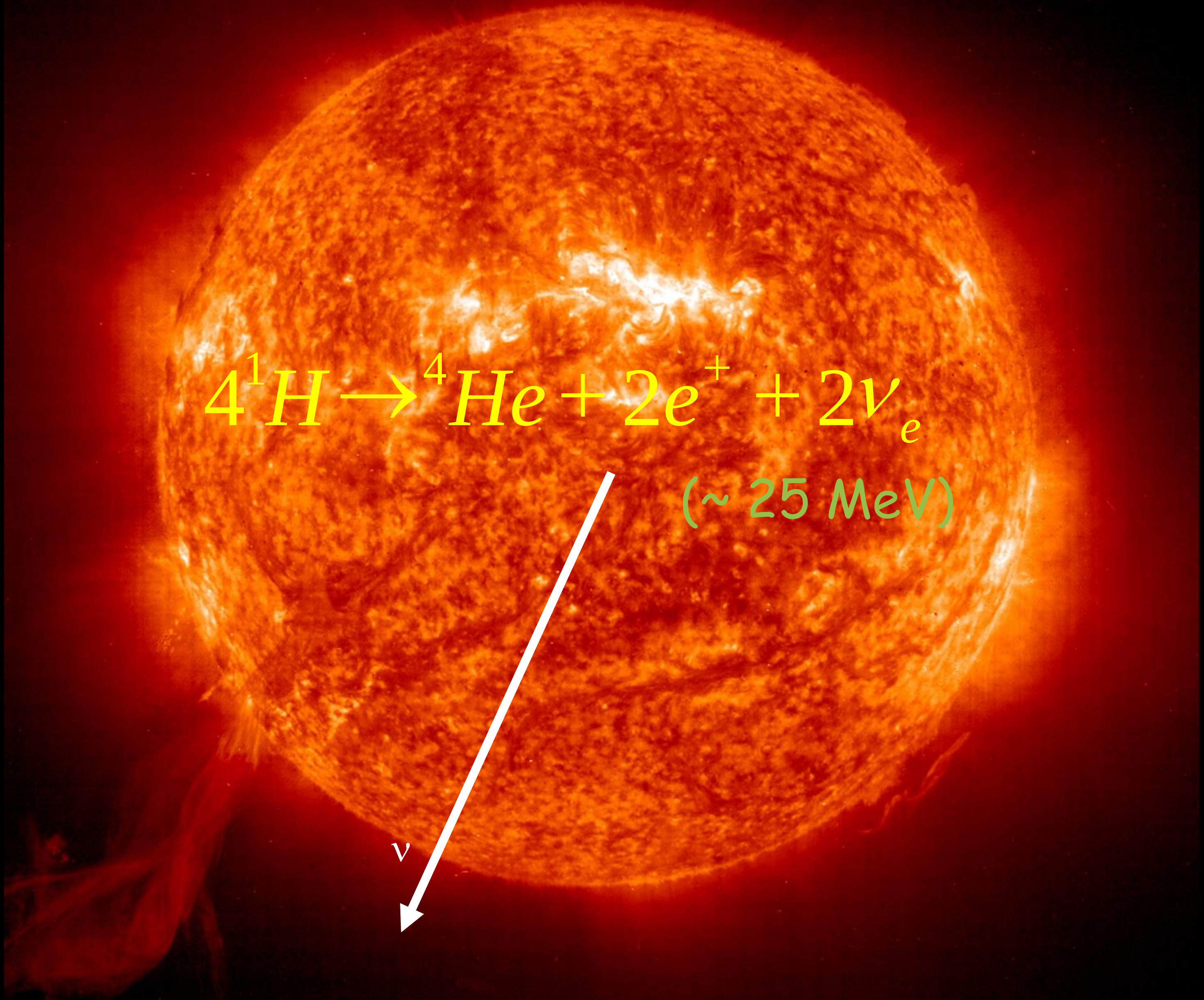


Woosley, Heger, and Weaver, Evolution and explosion of massive stars, Rev. Mod. Phys. (2002)



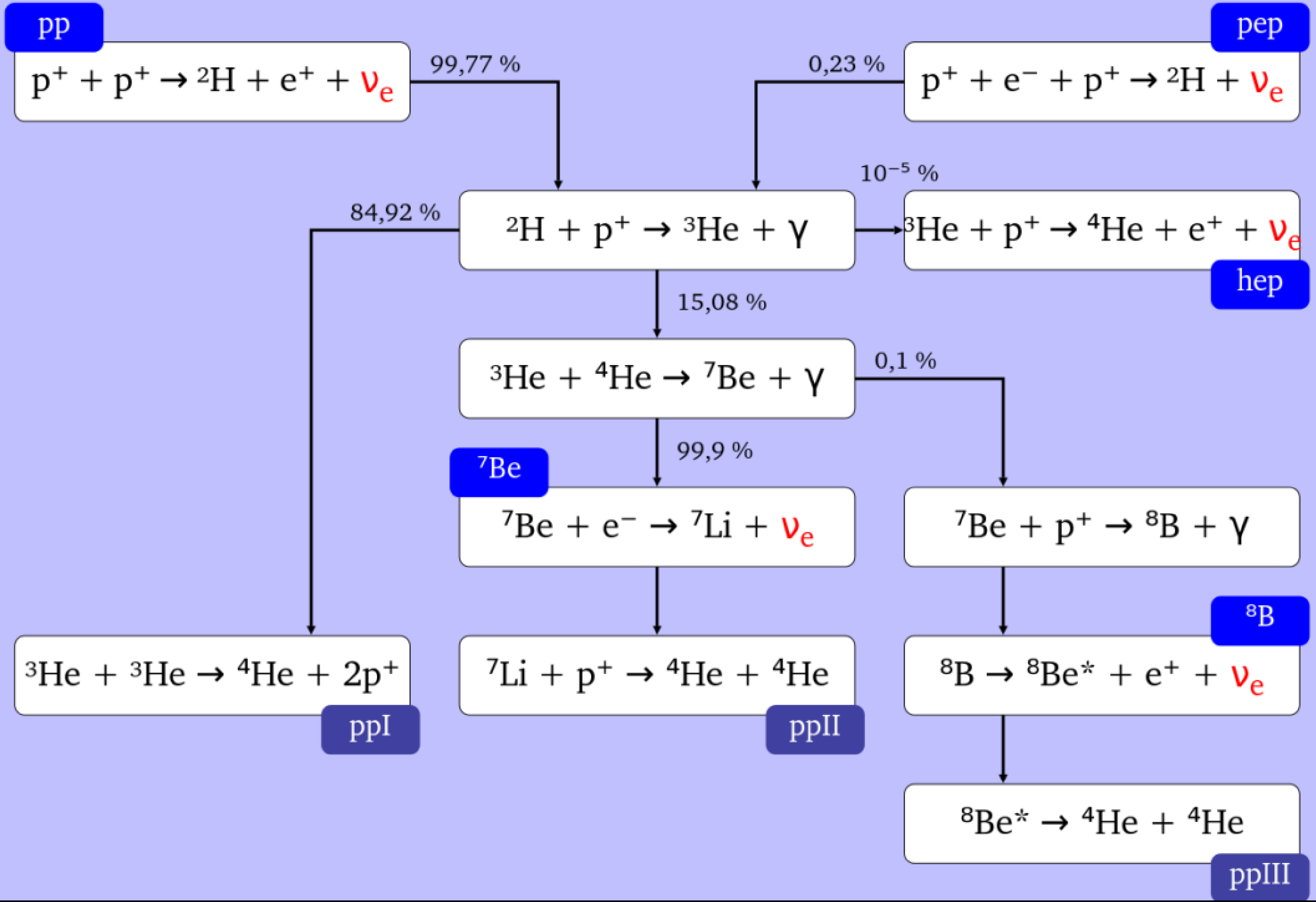
Conclusions of a Town Meeting held at the University of Notre Dame 7-8 June 1999

The first success of multi-messenger astronomy



$$4\,{}^{1}H \rightarrow {}^{4}He + 2e^{+} + 2\nu_{e}$$

(~ 25 MeV)



pp

$p^+ + p^+ \rightarrow {}^2H + e^+ + \nu_e$

99,77 %

pep

$p^+ + e^- + p^+ \rightarrow {}^2H + \nu_e$

0,23 %

$^2H + p^+ \rightarrow {}^3He + \gamma$

84,92 %

10⁻⁵ %

$^3He + p^+ \rightarrow {}^4He + e^+ + \nu_e$

hep

15,08 %

$^3He + {}^4He \rightarrow {}^7Be + \gamma$

99,9 %

0,1 %

^{7}Be

$^7Be + e^- \rightarrow {}^7Li + \nu_e$

$^7Be + p^+ \rightarrow {}^8B + \gamma$

$^3He + {}^3He \rightarrow {}^4He + 2p^+$

ppI

$^7Li + p^+ \rightarrow {}^4He + {}^4He$

ppII

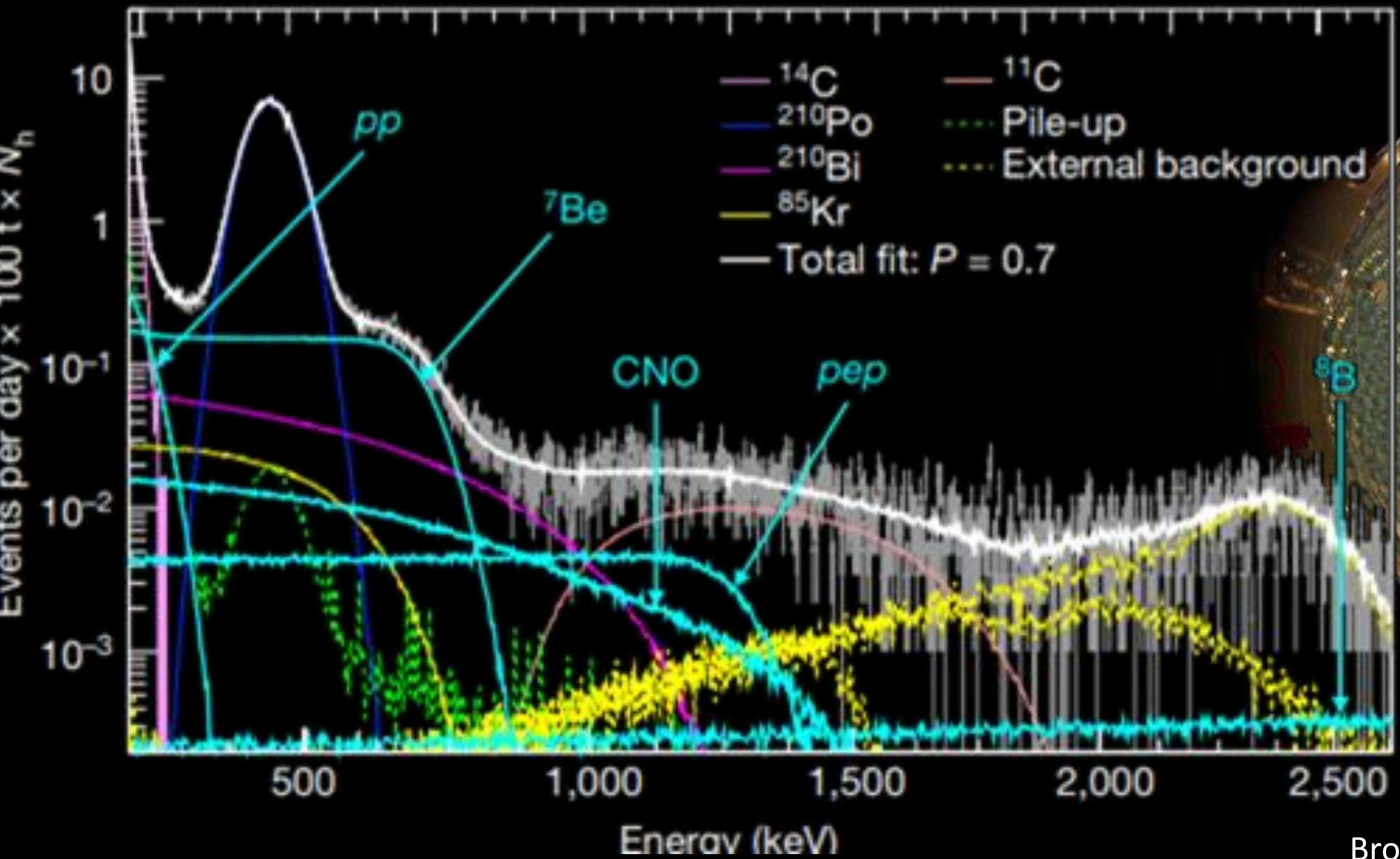
^{8}B

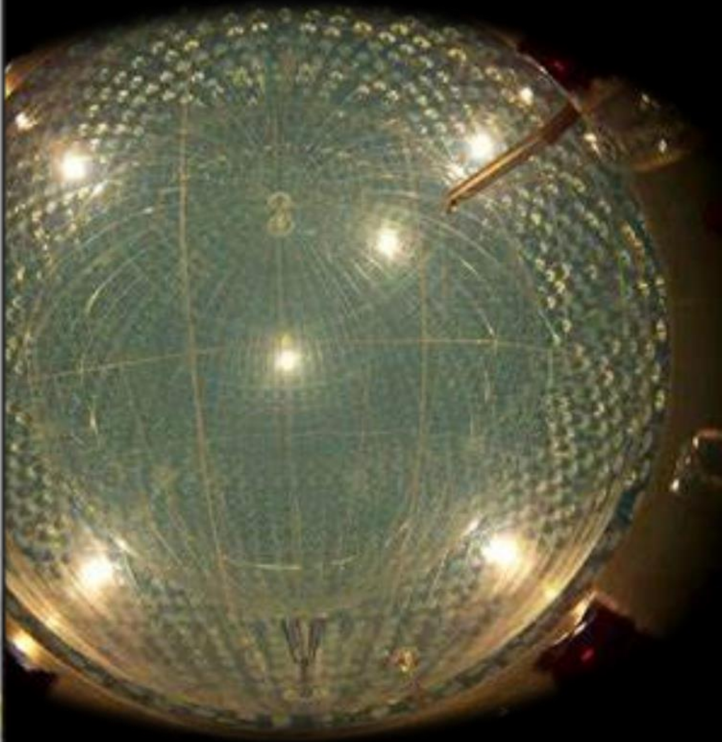
$^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

$^8Be^* \rightarrow {}^4He + {}^4He$

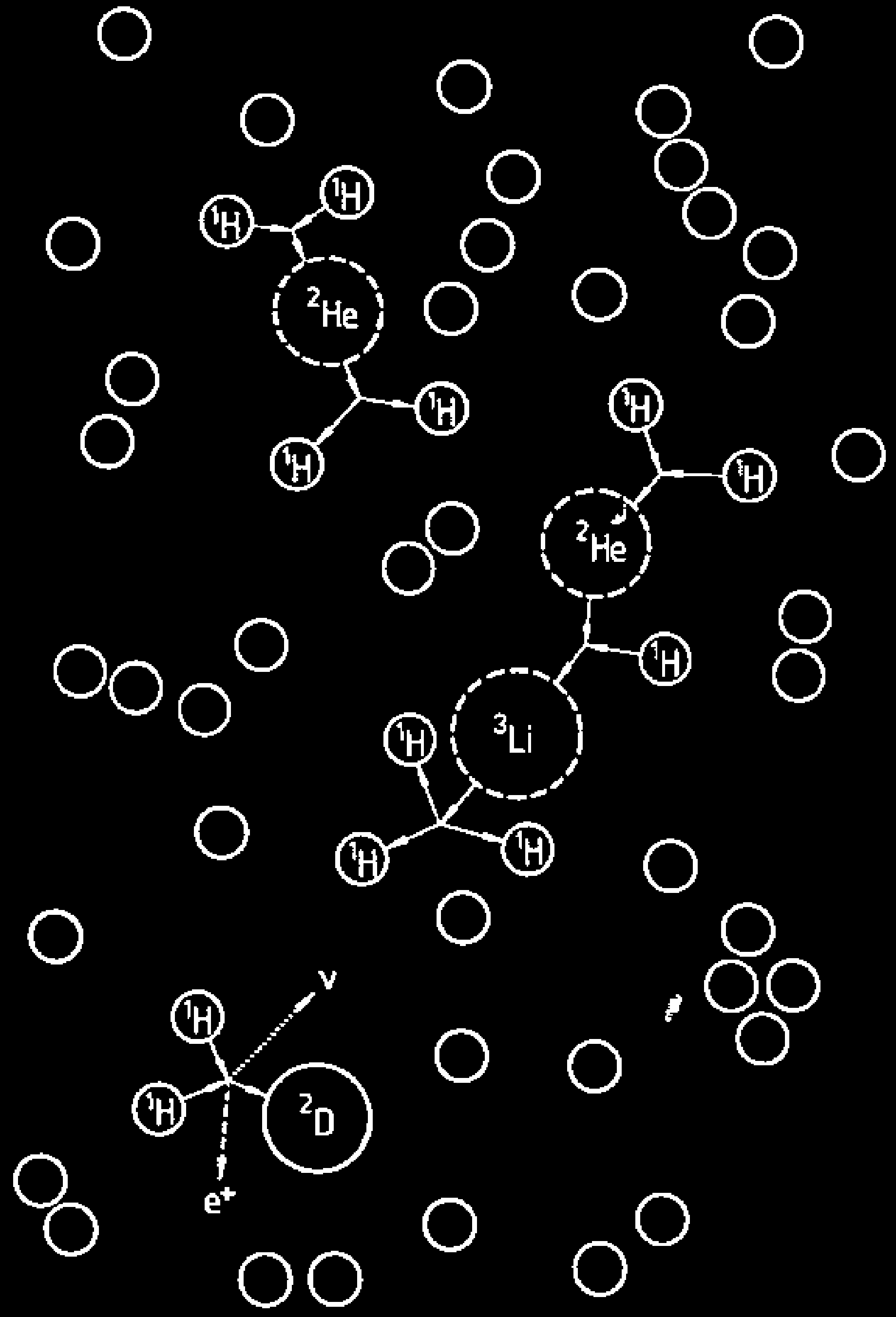
ppIII

First measurement of pp-chain solar neutrinos





Broxino collaboration, Nature (2018)



The p-p chain

Strong interaction:
$p+p \rightarrow {}^2He \rightarrow p+p$
${}^2He+p \rightarrow {}^3Li \rightarrow p+p+p$

Weak interaction:
$p+p \rightarrow d+e^++\nu_e$ (Q=0.42MeV)

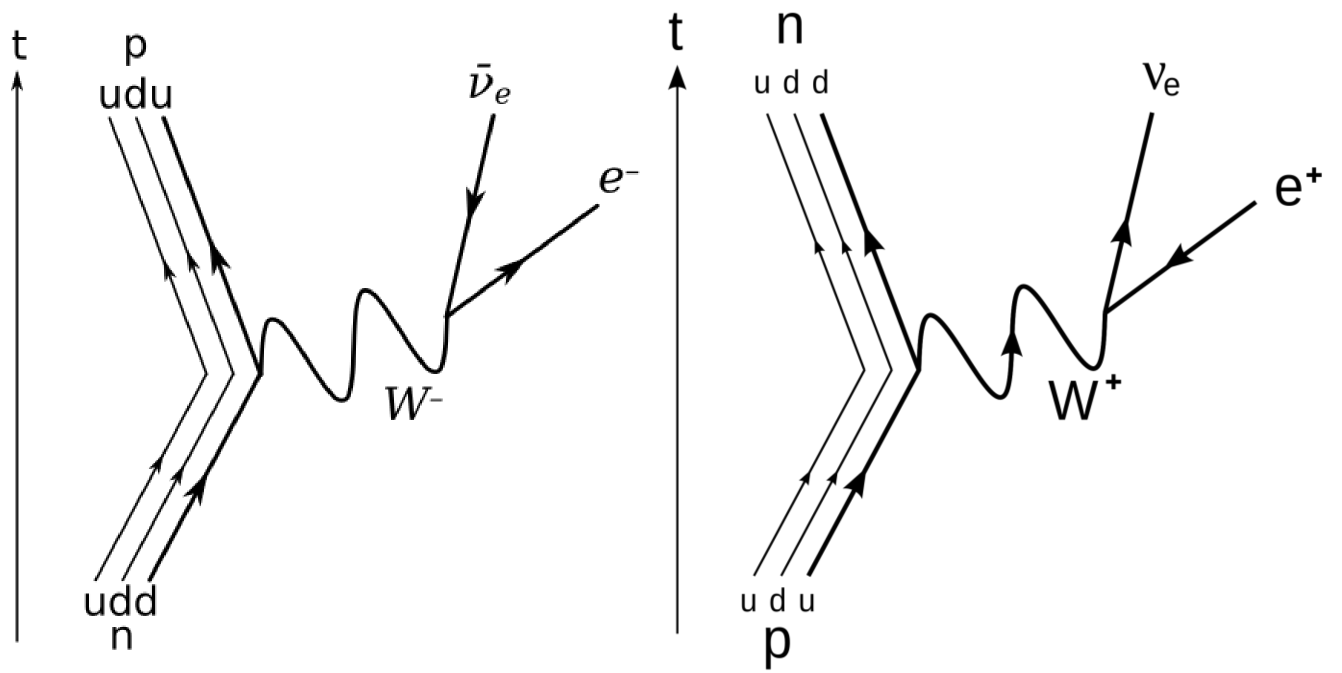
The Fermi decays ($\Delta I = 0$) are often referred to as the "superallowed" decays while Gamow–Teller ($\Delta I = 1$) decays are simple "allowed" decays.

Forbidden decays are those which are substantially more improbable, due to parity violation, and as a result have long decay times.

Now the angular momentum (*L*) of the $\beta + \nu$ systems can be non-zero (in the center-of-mass frame of the system).

Below are the Observed Selection Rules for Nuclear Beta-Decay:[5]

Transition	*L*	Δ*I*	Δ*π*
Fermi	0	0	0
Gamow–Teller	0	0, 1	0
first-forbidden (parity change)	1	0, 1, 2	1
second-forbidden (no parity change)	2	1, 2, 3	0
third-forbidden (parity change)	3	2, 3, 4	1
fourth-forbidden (no parity change)	4	3, 4, 5	0



Each of the above have Fermi ($S = 0$) and Gamow–Teller ($S = 1$) decays.

So for the "first-forbidden" transitions you have

$$\vec{I} = \vec{L} + \vec{S} = \vec{1} + \vec{0} \Rightarrow \Delta I = 0, 1 \text{ Fermi}$$

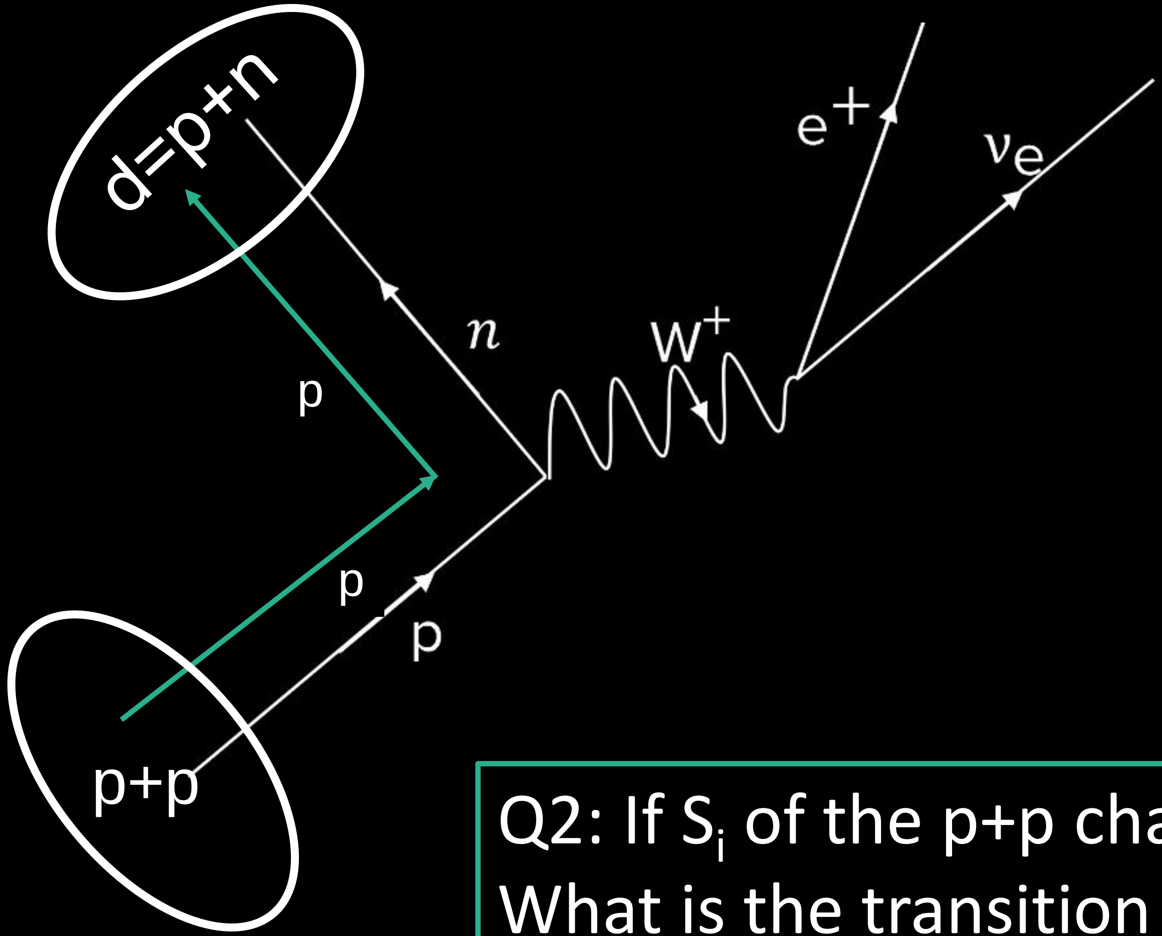
and

$$\vec{I} = \vec{L} + \vec{S} = \vec{1} + \vec{1} \Rightarrow \Delta I = 0, 1, 2 \text{ Gamow–Teller}$$

https://en.wikipedia.org/wiki/Beta_decay_transition

Samuel S.M. Wong (2004). Introductory Nuclear Physics (2nd ed.). Wiley-VCH. p. 200

Weak Interaction



Initial state: p+p $(S_i=0, l_i=0, J_i=0, +)$

Q1: What is the transition mode?

Final state: d=p+n $(S_f=1, l_f=0, J_f=1,+)$

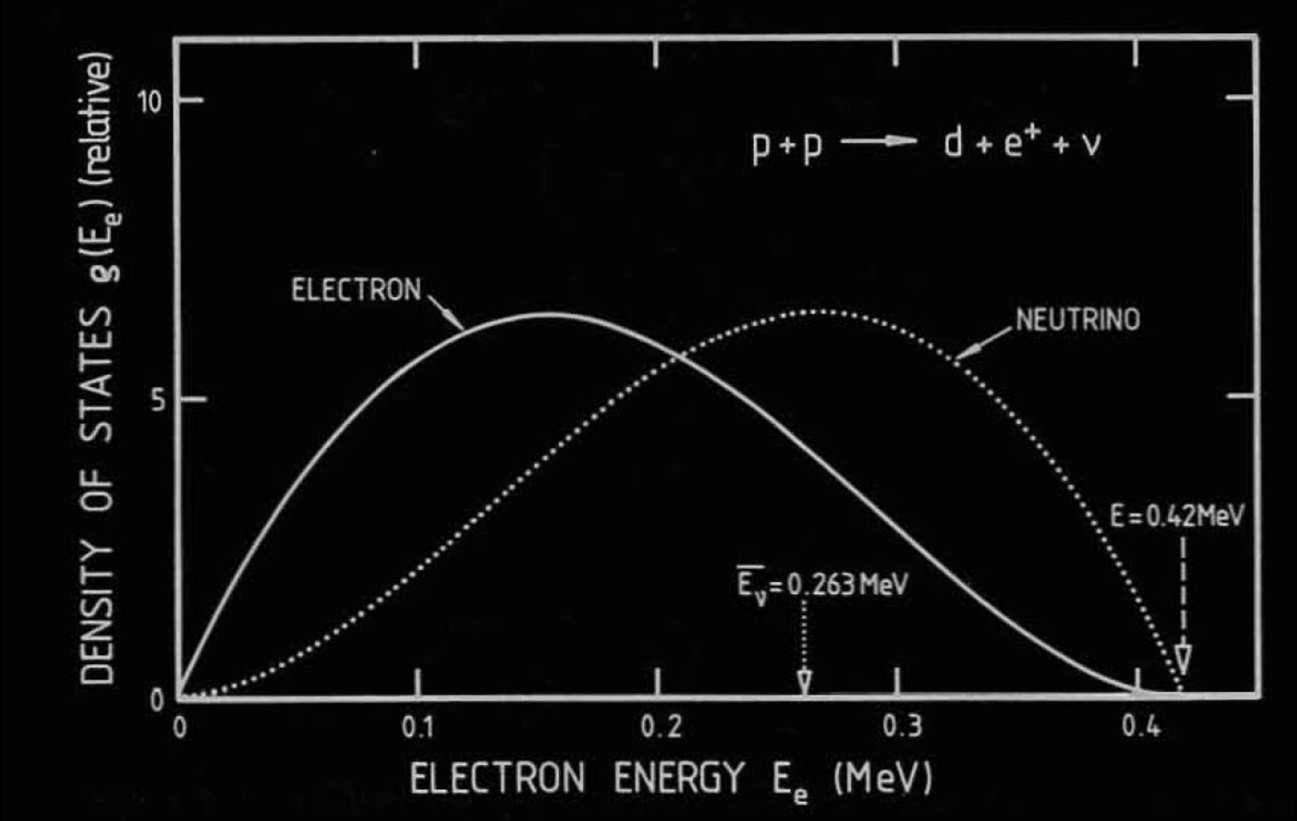
Q2: If S_i of the p+p channel is 1, what are the choices for l_f ? What is the transition mode of the weak interaction ?

Fermi's Golden Rule

$$d\sigma = \frac{2\pi}{\hbar}\frac{\rho(E)}{v_i}\left|\langle f\middle|H_\beta\middle|\mathrm{i}\rangle\right|^2$$

- $\rho(E)$: statistical factor representing the density of final states
- v_i: relative velocity in the incident channel
- $<f|H_\beta|i>$: transition matrix element between the initial (p+p) and final state ($d+e^+ + \nu_e$) resulting from the weak interaction represented by H_β

Statistical factor: $\rho(E)$



Assuming a zero rest mass for the neutrino and neglecting the recoil energy of the deuterium, the total energy E is shared between the electron and neutrino,

i.e., $E = E_e + E_\nu = E_e + cp_\nu$

$$\rho(E) = \frac{dN}{dE} = dn_e \frac{dn_\nu}{dE} = \frac{16\pi^2 V^2}{c^3 h^6} p_e^2 (E - E_e)^2 dp_e = \rho(E_e) dp_e$$

The differential cross section can now be written as

$$d\sigma = \frac{2\pi}{\hbar}\frac{1}{v_i}\frac{16\pi^2}{c^3h^6}\, g^2 M_{\text{spin}}^2 M_{\text{space}}^2 p_e^2 (E - E_e)^2 dp_e\ ,$$

and the total cross section is obtained by integration over the total range of electron momenta:

$$\sigma = \frac{2\pi}{\hbar}\frac{1}{v_i}\frac{16\pi^2}{c^3h^6}\, g^2 M_{\text{spin}}^2 M_{\text{space}}^2 \int_0^E p_e^2 (E - E_e)^2 dp_e\ .$$

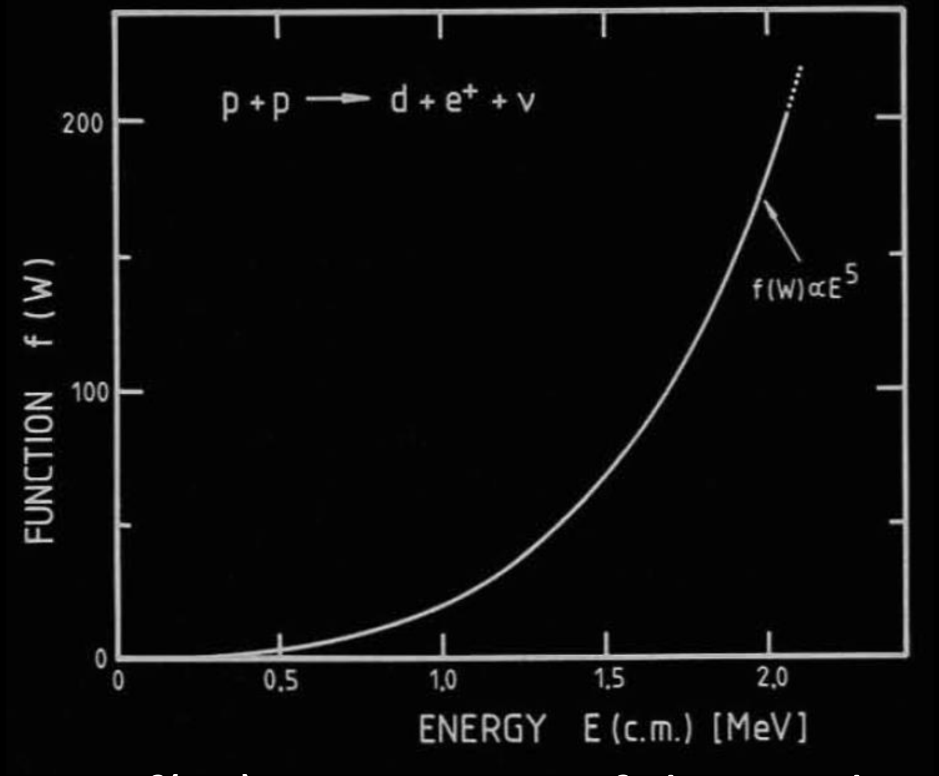
Introducing the new variable

$$W = \frac{E + m_e c^2}{m^e c^2}\ ,$$

one arrives at an integral transformation:

$$\int_0^E p_e^2 (E - E_e)^2 dp_e = \frac{(m_e c^2)^5}{c^3} \int_1^W (W_e^2 - 1)^{1/2} (W - W_e)^2 W_e\, dW_e$$

$$\sigma = \frac{m_e^5 c^4}{2\pi^3 \hbar^7} f(w) g^2 M_{spin}^2 M_{space}^2$$



f(W): a measure of the total phase space available for the reaction (W=E/$m_e c^2$+1)

Sum over the final states

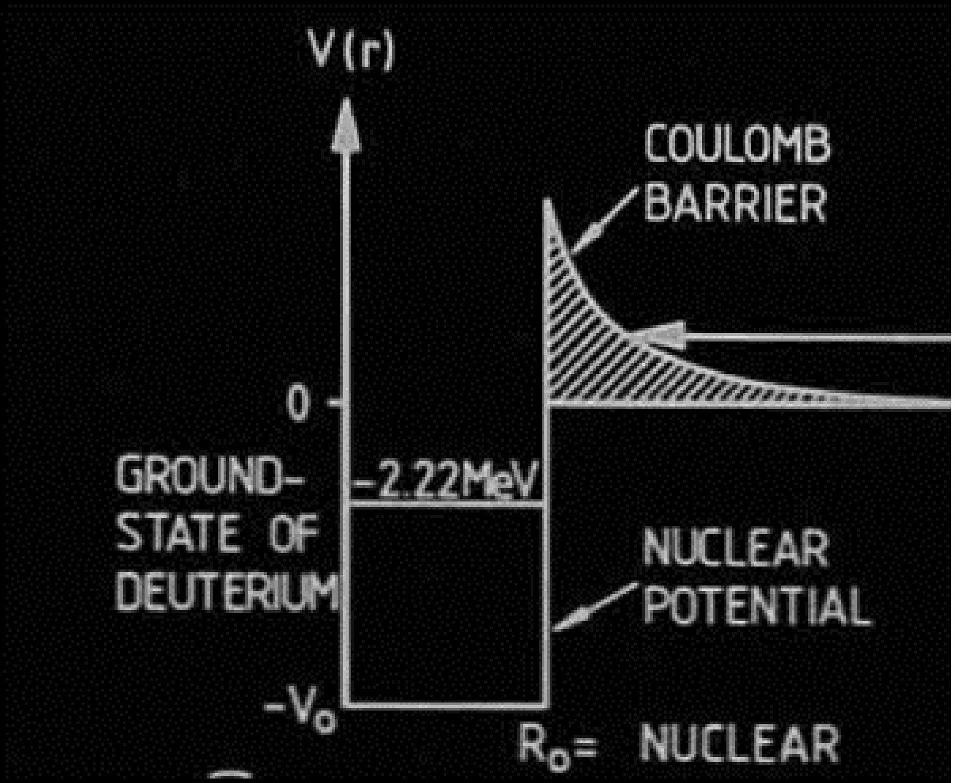
$$M_{spin}^2 = \frac{(2J+1)}{(2J_1+1)(2J_2+1)} = 3$$

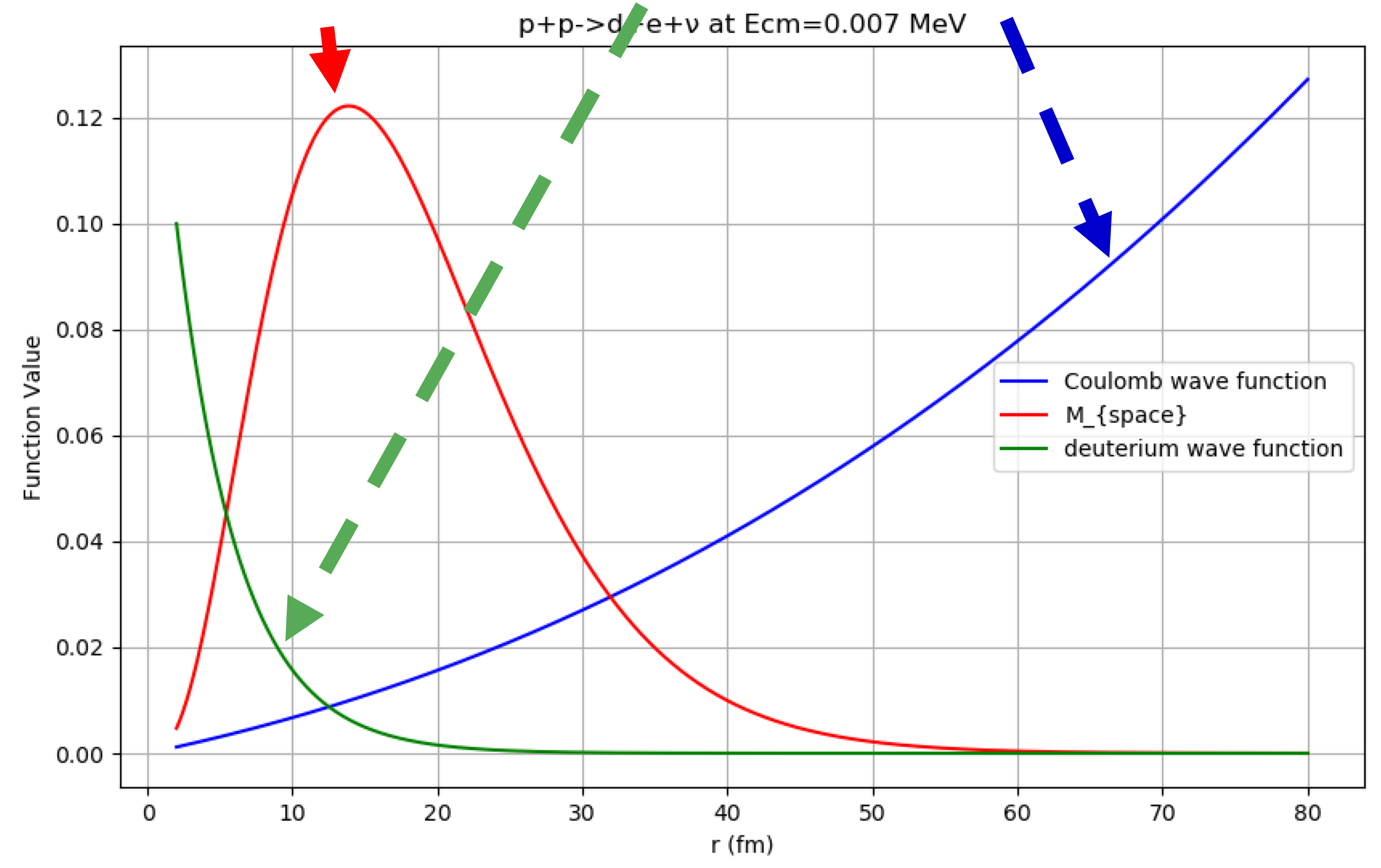
Average over the initial states

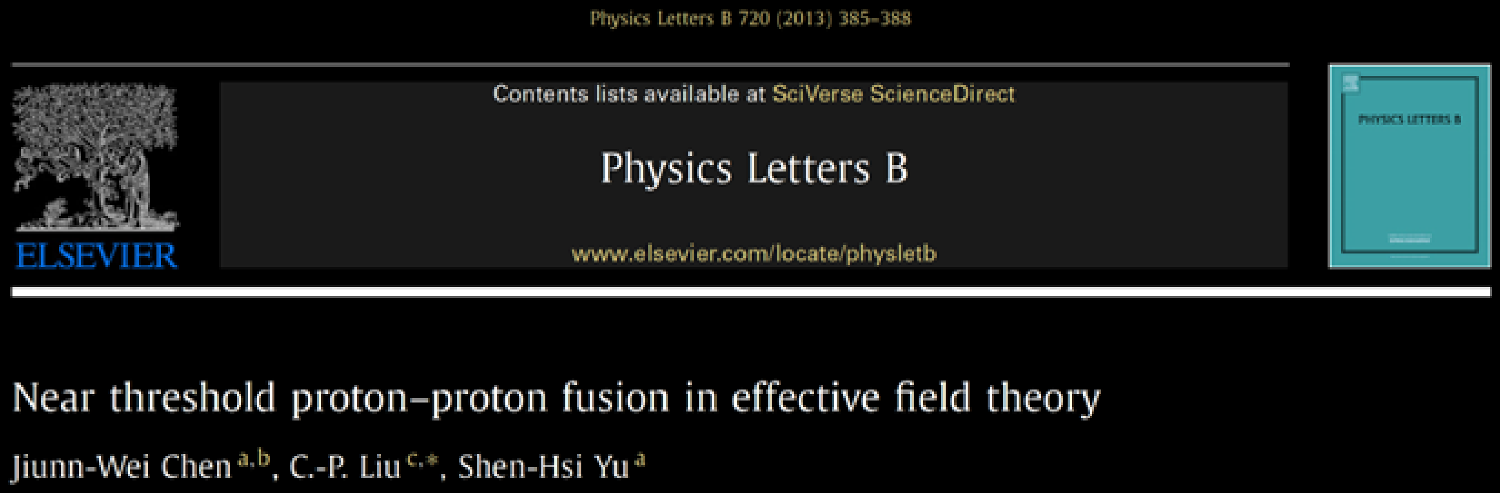
g: Strength of weak interaction can be determined by allowed GT transition, eg. ^{6}He decay ($0^+ \rightarrow 1^+$)

Space matrix

$$M_{space} = \int_0^\infty \chi_f(r)\chi_i(r)r^2dr \;\; cm^{3/2}$$







Physics Letters B 720 (2013) 385–388

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Near threshold proton–proton fusion in effective field theory

Jiunn-Wei Chen [a,b], C.-P. Liu [c,*], Shen-Hsi Yu [a]

$$S_{11}(0) = (3.99 \pm 0.14) \times 10^{-25}\ \mathrm{MeV\,b},$$

$$S'_{11}(0) = S_{11}(0)(11.3 \pm 0.1)\ \mathrm{MeV}^{-1},$$

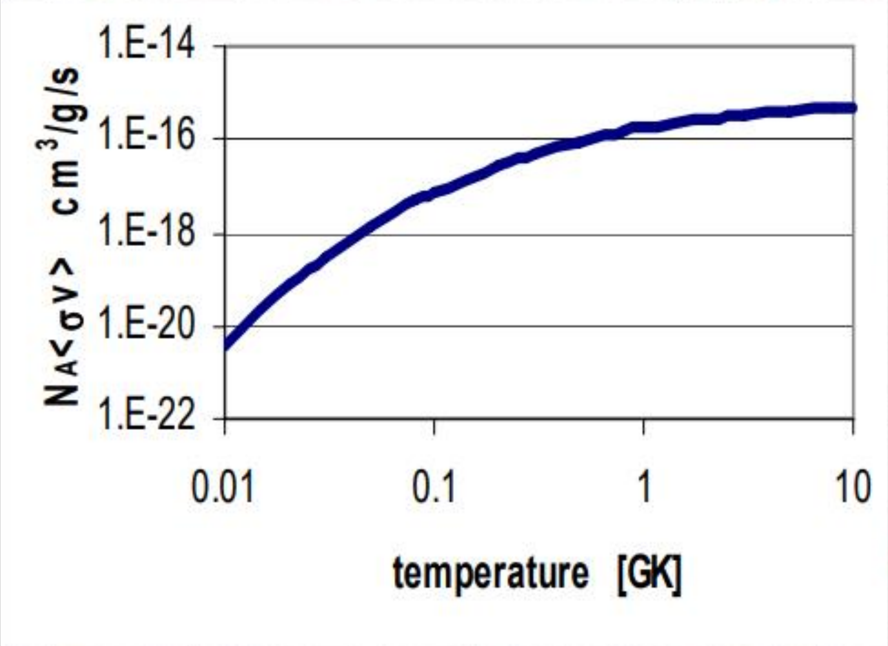
$$S''_{11}(0) = S_{11}(0)(170 \pm 2)\ \mathrm{MeV}^{-2}.$$

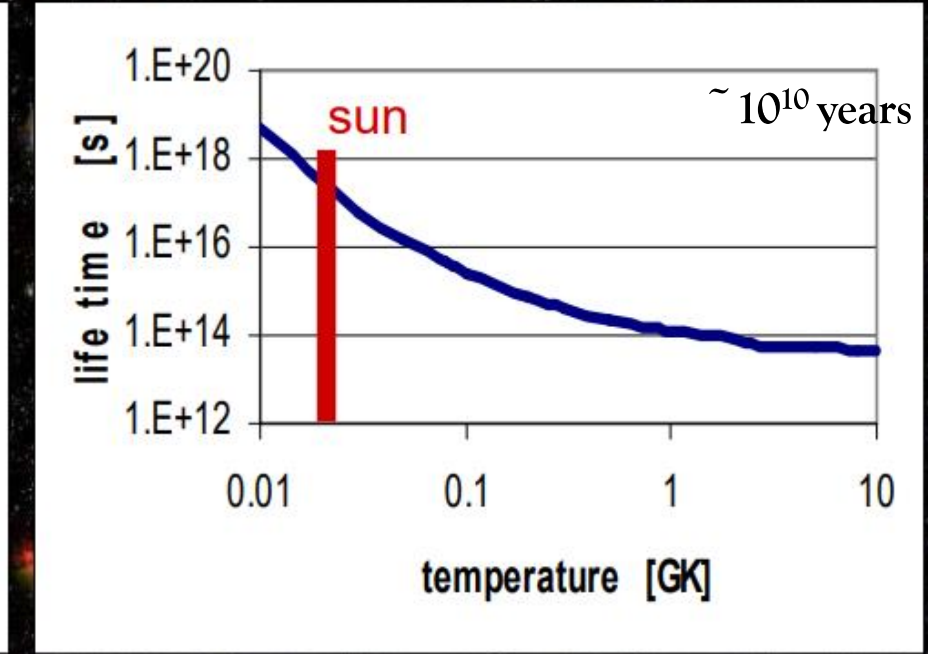
$$S(E) \simeq S(0) + S'(0)\ E + \frac{S''(0)}{2}\ E^2 + \cdots$$

The p+p reaction

$^1H(p,e^+\nu)^2H$ is a reaction based on weak interaction mechanism

the S-factor is calculated: $S=5\cdot10^{-25}$ MeV-barn

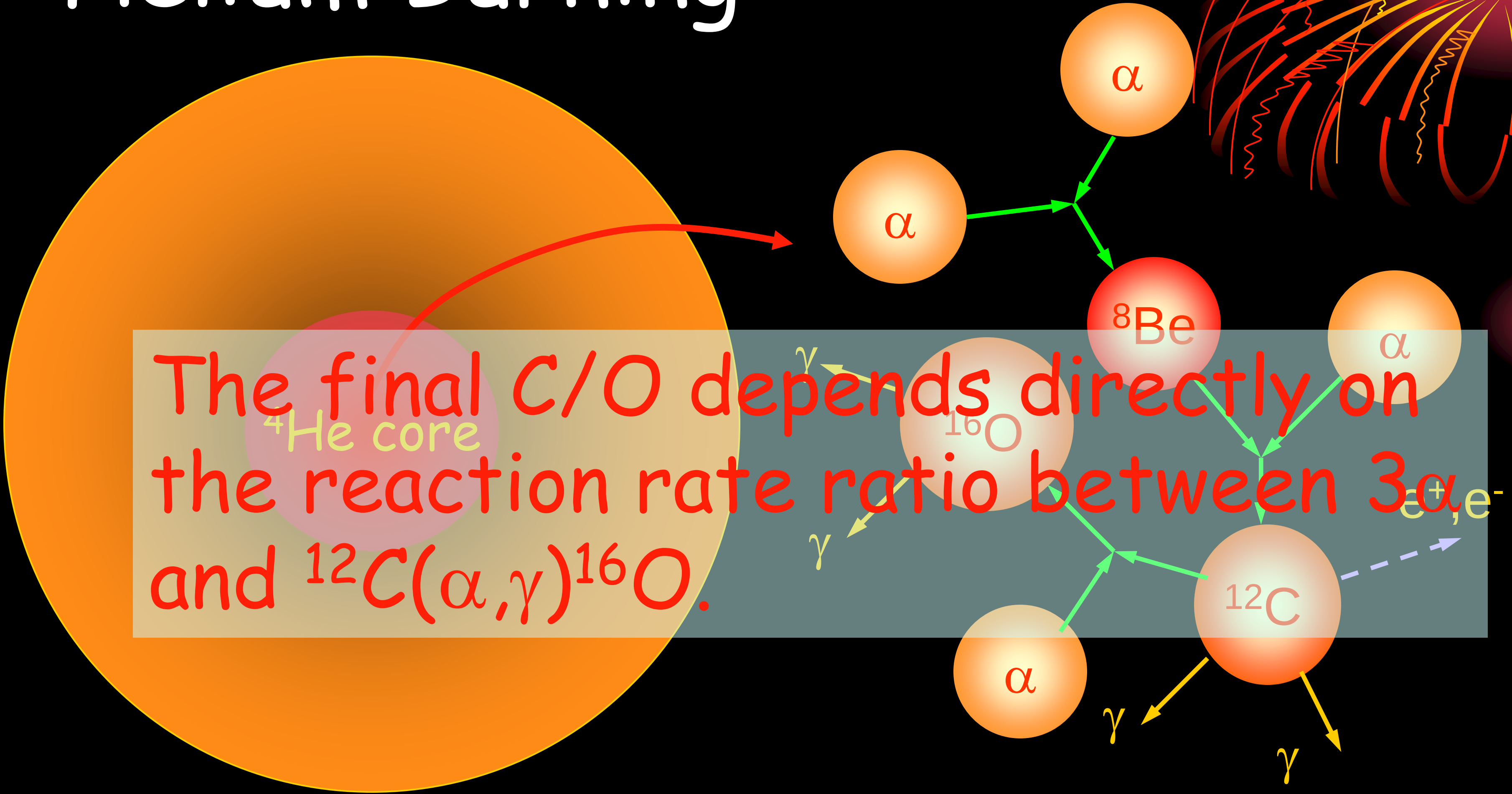




What would be the life time of hydrogen with strong interaction $S=5\cdot10^{-5}$ MeV-barn?

- Find the reaction rate from JINA REACLIB and obtain the two plots on the left side.
- Estimate the new life time of the Sun
- Will the Sun be stable with such a change in the p-p reaction rate?

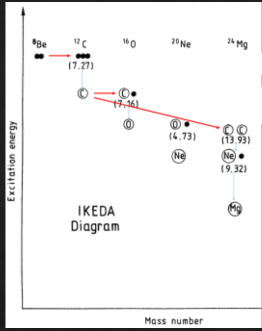
Helium Burning

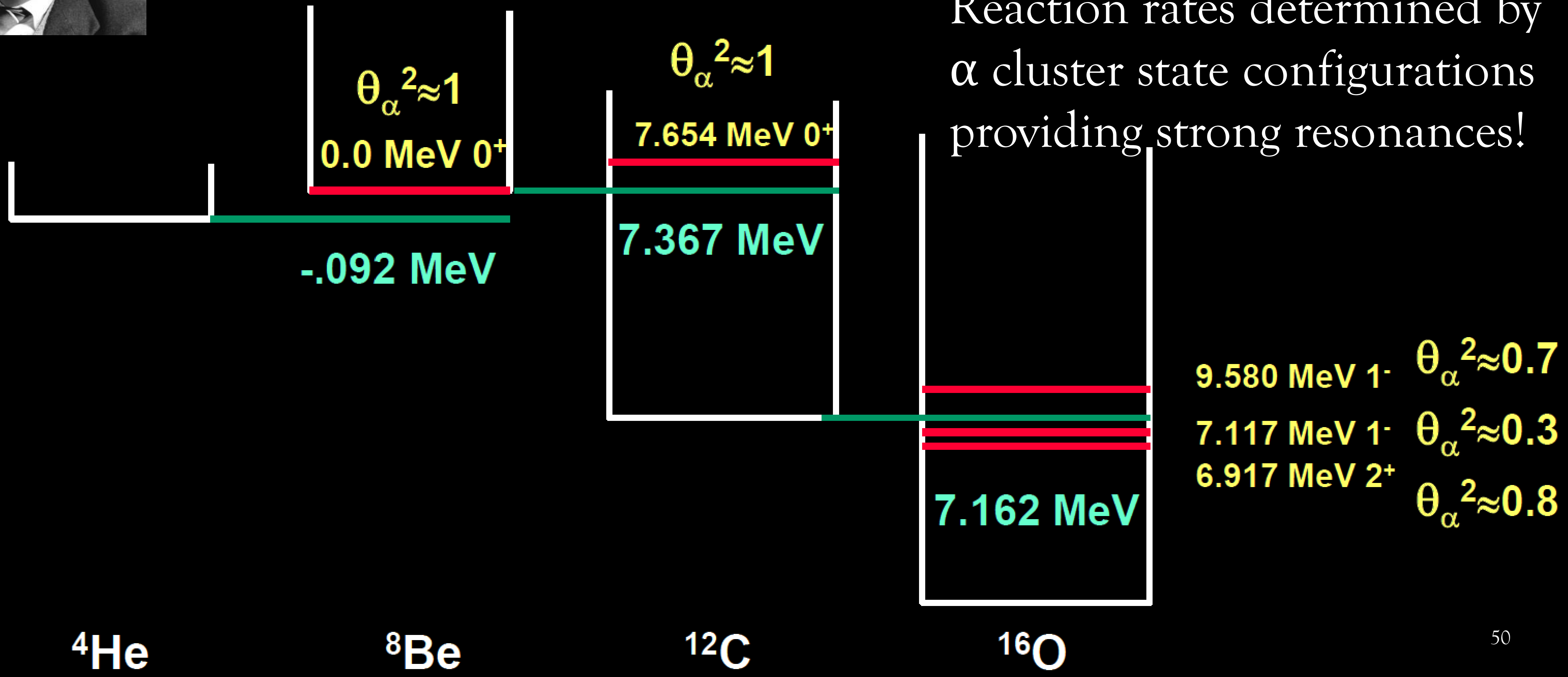


The final C/O depends directly on the reaction rate ratio between 3α and $^{12}C(\alpha,\gamma)^{16}O$.

T ~ 0.2 billion Kelvin

Helium burning :3α and $^{12}C(\alpha,\gamma)^{16}O$

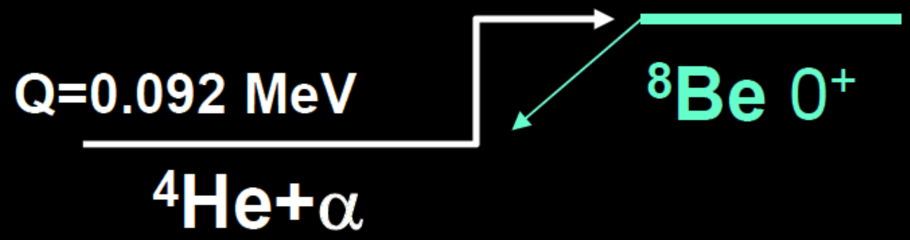




Reaction rates determined by α cluster state configurations providing strong resonances!

The (ααα) reaction as a two-step process

first step!



$T_{1/2}(^8Be) = 9.7\cdot10^{-17}$ s
Γ_α=6.8 eV
pure α cluster configuration

fast capture ⇒ equilibrium between capture and decay

Interaction time: $t \approx \frac{2R_\alpha}{v_\alpha} = \frac{2\cdot1.3\cdot A^{1/3}}{\sqrt{\frac{2E_\alpha^{cm}}{\mu}}} \approx \frac{4.17\ fm}{3.8\cdot10^{24}\ \ fm/s} \approx 10^{-24}\ s << \tau(^8Be)$

Application of Saha Equation
For calculating ^{8}Be equilibrium: $N(^8Be) = N_\alpha^2 \cdot \hbar^3 \cdot \left(\frac{2\pi}{\mu\cdot kT}\right)^{3/2} \cdot e^{\left(-\frac{Q}{kT}\right)}$

Case of typical He-burning: T=0.1GK ⇒ T_9=0.1; ρ=10^5 g/cm^3

$$N\left({}^{8}Be\right)=6\cdot10^{-35}\cdot N_{\alpha}^{2}\cdot T_{9}^{-3/2}\cdot e^{\left(-\frac{1.068}{T_9}\right)}$$

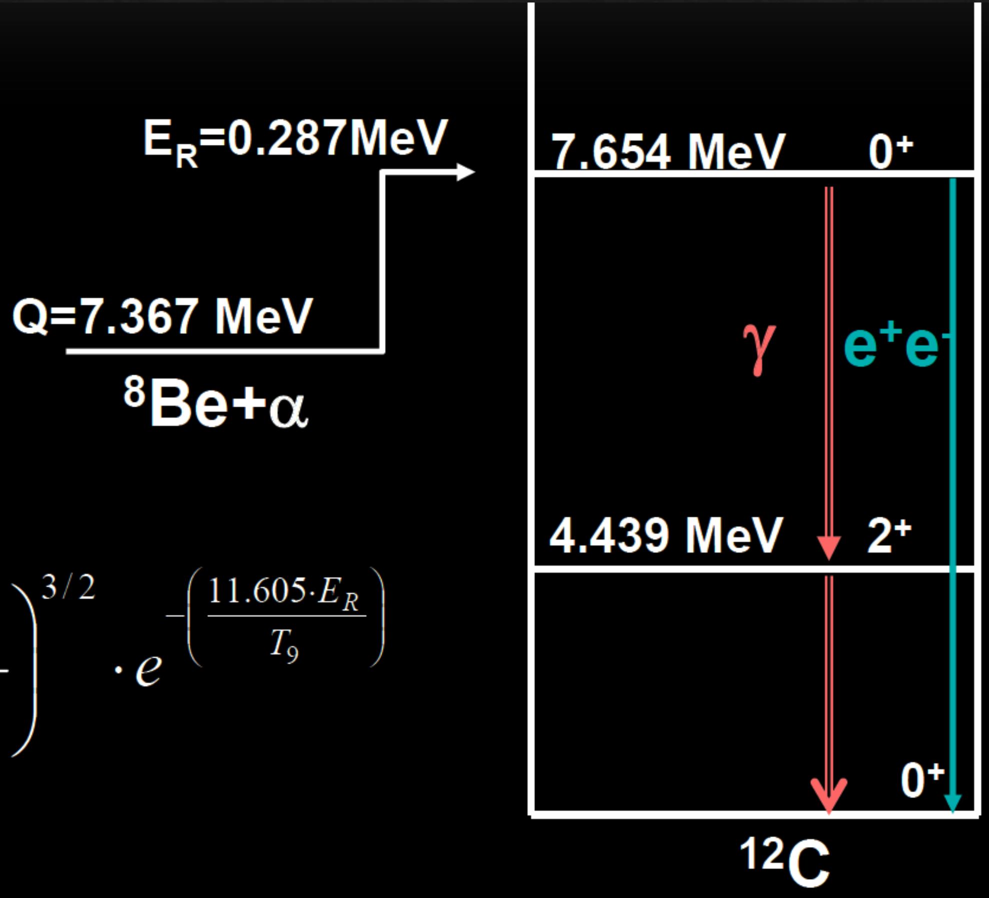
$$N\left({}^{8}Be\right)\approx 4.4\cdot10^{-38}\cdot N_{\alpha}^{2}$$

$$N=\rho\cdot N_{A}\cdot\frac{X_i}{A_i}\quad\Rightarrow\quad\frac{X\left({}^{8}Be\right)}{X_{\alpha}^{2}}\approx1.3\cdot10^{-9}$$

- Calculate the ^{8}Be equilibrium abundance in stellar helium burning as a function of stellar temperature (0.1 Gk—10 Gk)

Second step: Resonant capture by ^{8}Be

The Hoyle resonance!



$$N_A\langle\sigma v\rangle = 1.54\cdot10^{11}\cdot\omega\gamma\cdot\left(\frac{1}{\mu\cdot T_9}\right)^{3/2}\cdot e^{-\left(\frac{11.605\cdot E_R}{T_9}\right)}$$

$$\omega\gamma = (2J+1)\cdot\frac{\Gamma_{in}\cdot\Gamma_{out}}{\Gamma_{tot}}$$

Decay by sequential E2 γ transitions or internal e^+ e^- pair conversion

Γ_γ/Γ=4.16(4)E-4

Γ_π/Γ=6.7(6)E-6

Γ=9.3(9) eV

ωγ=3.87(39) meV

Kelly, Purcell and Sheu, NPA(2017)

$$r_{\alpha\alpha\alpha} = N_{^8Be} \cdot \rho \cdot \frac{X_\alpha}{A_\alpha} \cdot N_A \langle {}^8Be(\alpha,\gamma)^{12}C \rangle$$

Step 1

Number density of alpha particle

Step 2

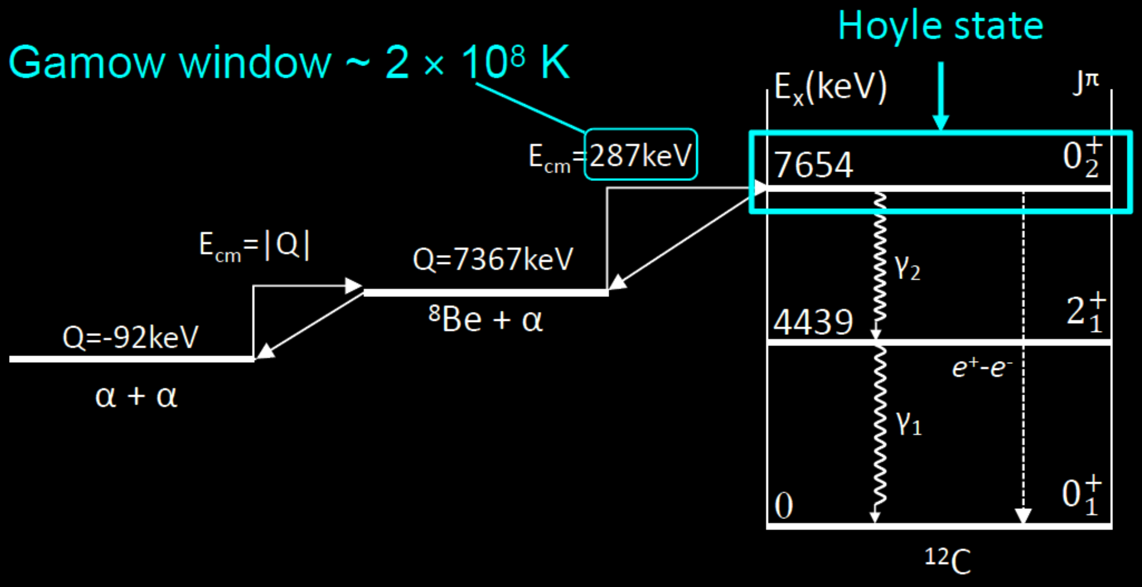
$$N({}^8Be) = 6 \cdot 10^{-35} \cdot N_\alpha^2 \cdot T_9^{-3/2} \cdot e^{\left(-\frac{1.068}{T_9}\right)}$$

$$N_A \langle {}^8Be(\alpha,\gamma)^{12}C \rangle = 126.4 \cdot (T_9)^{-3/2} \cdot e^{-\left(\frac{3.331}{T_9}\right)}$$

$$r_{\alpha\alpha\alpha} = \frac{1.26 \cdot 10^{-56}}{1+\delta_{\alpha\alpha}} \cdot N_\alpha^3 \cdot T_9^{-3} \cdot e^{\left(-\frac{11.605 \cdot (0.092+0.278)}{T_9}\right)}$$

$$r_{\alpha\alpha\alpha} = 1.38 \cdot 10^{15} \cdot \rho^3 \cdot \left(\frac{X_\alpha}{4}\right)^3 \cdot T_9^{-3} \cdot e^{\left(-\frac{4.294}{T_9}\right)} \quad [cm^{-3} s^{-1}]$$

Important resonance parameters



The 3α reaction proceeds via 3α resonanc

3α-process rate: $\langle\sigma v\rangle \propto \omega\gamma \exp(-\frac{E_R}{hT})$

Most 3α reaction resonances decay back to 3α.

Only tiny fraction of them radiatively decay to g.s. (~ 1 in 2500)

$$\Gamma_\alpha \gg \Gamma_{\text{rad}}$$

$$\omega\gamma = \frac{\Gamma_\alpha\Gamma_{\text{rad}}}{\Gamma} = \frac{\Gamma_\alpha\Gamma_{\text{rad}}}{\Gamma_\alpha+\Gamma_{\text{rad}}} = \frac{\Gamma_{\text{rad}}}{1+\Gamma_{\text{rad}}/\Gamma_\alpha}$$

$$\approx \Gamma_{\text{rad}}$$

^{12}C(e,e')^{12}C*(7.654) → Γ_π=62.3±2 μeV

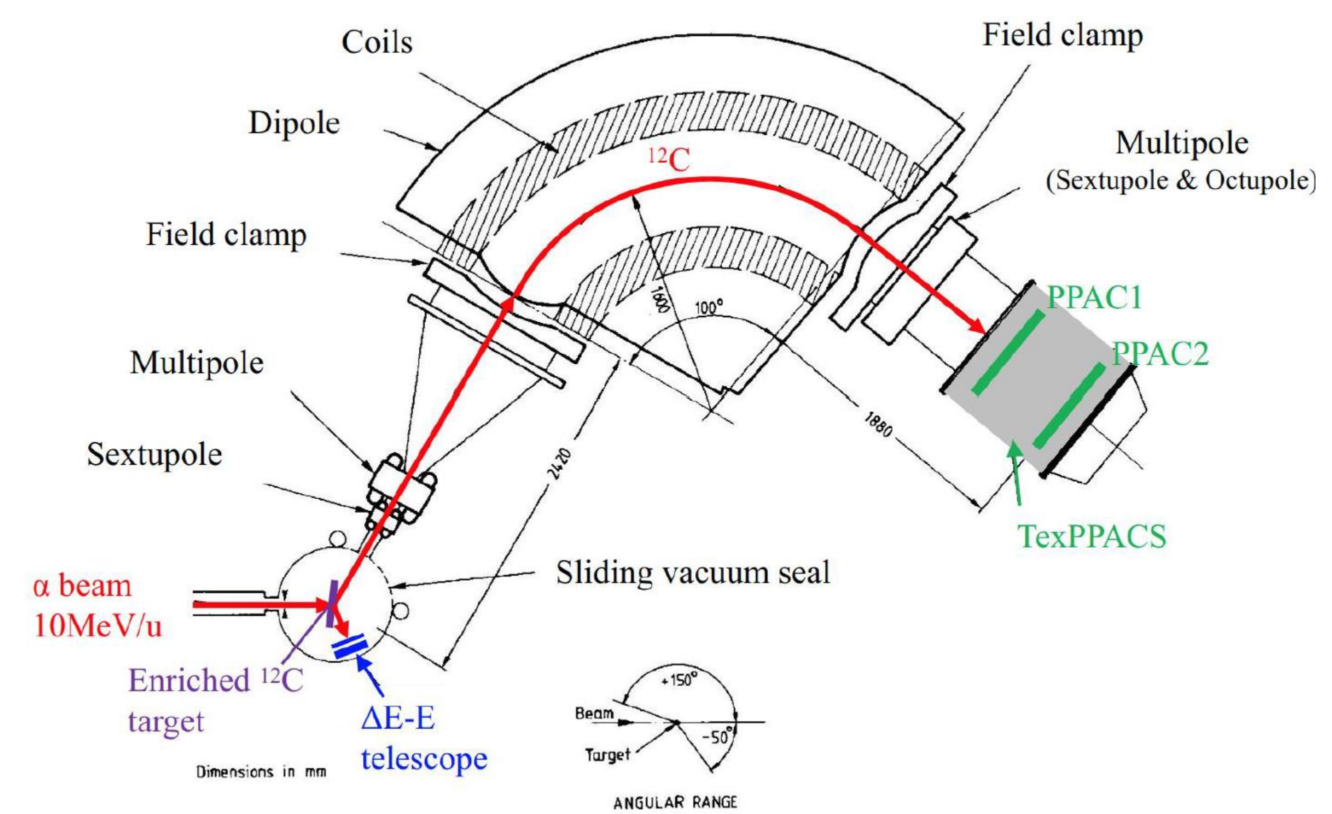
PRL 105, 022501 (2010)

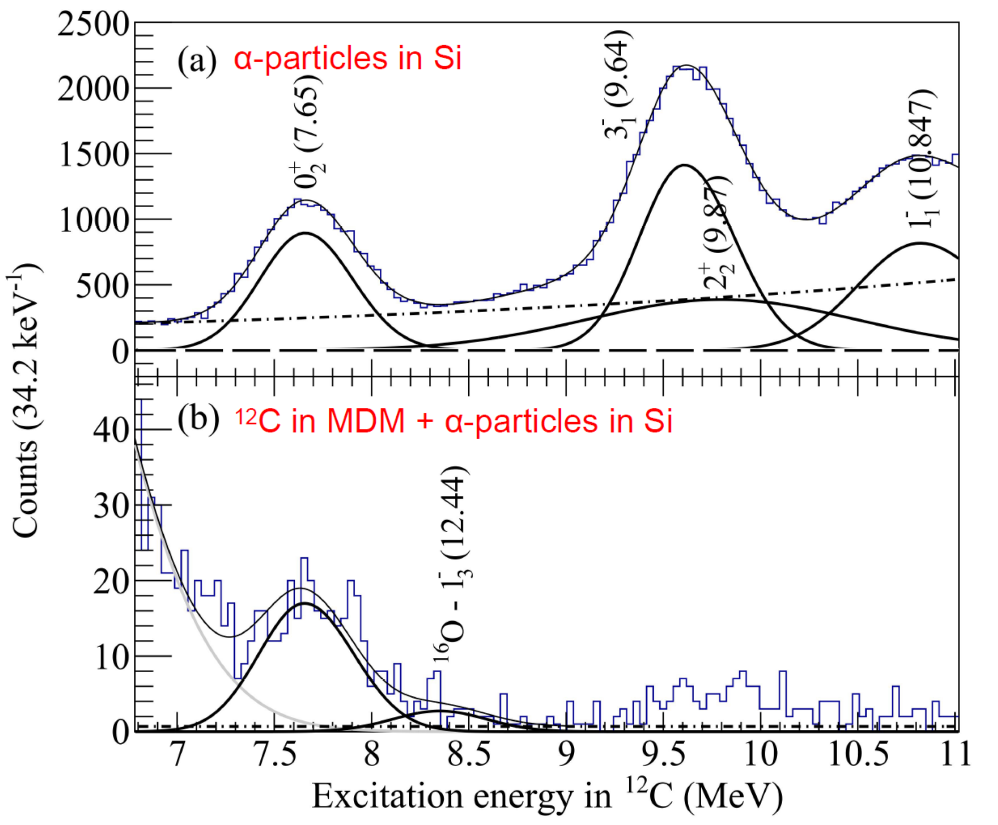
$$\Gamma_{\text{rad}} = \frac{\Gamma_{\text{rad}}}{\Gamma} \times \frac{\Gamma}{\Gamma_\pi(\text{E0})} \times \Gamma_\pi(\text{E0})$$

$\Gamma_{\text{rad}}/\Gamma$ is a key parameter to precisely determine the 3α rate.

Courtesy of Z.F. Luo

Measurement of $\Gamma_{rad}/\Gamma_{tot}$



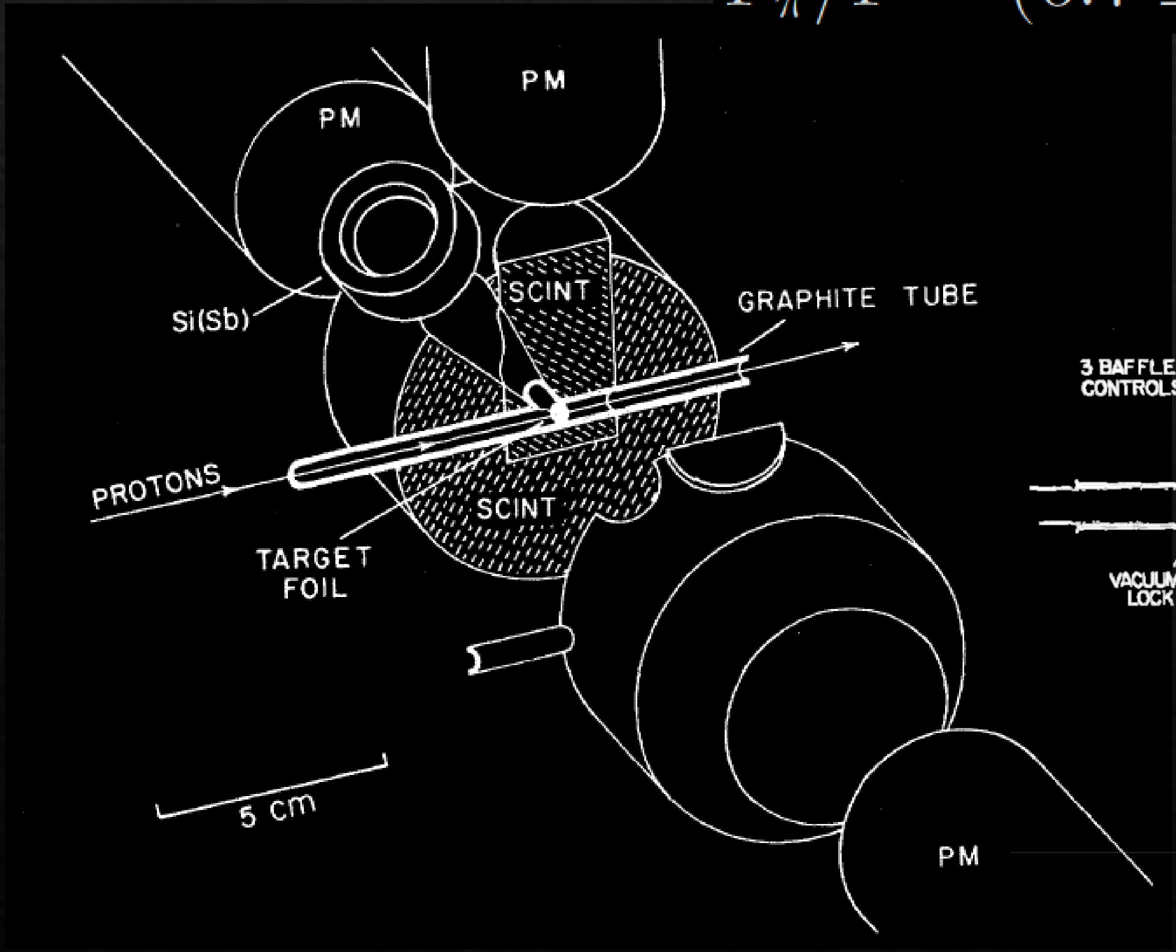


$$\Gamma_{rad}/\Gamma \times 10^4 = 4.0 \pm 0.3\,(\text{stat.}) \pm 0.16\,(\text{syst.})$$

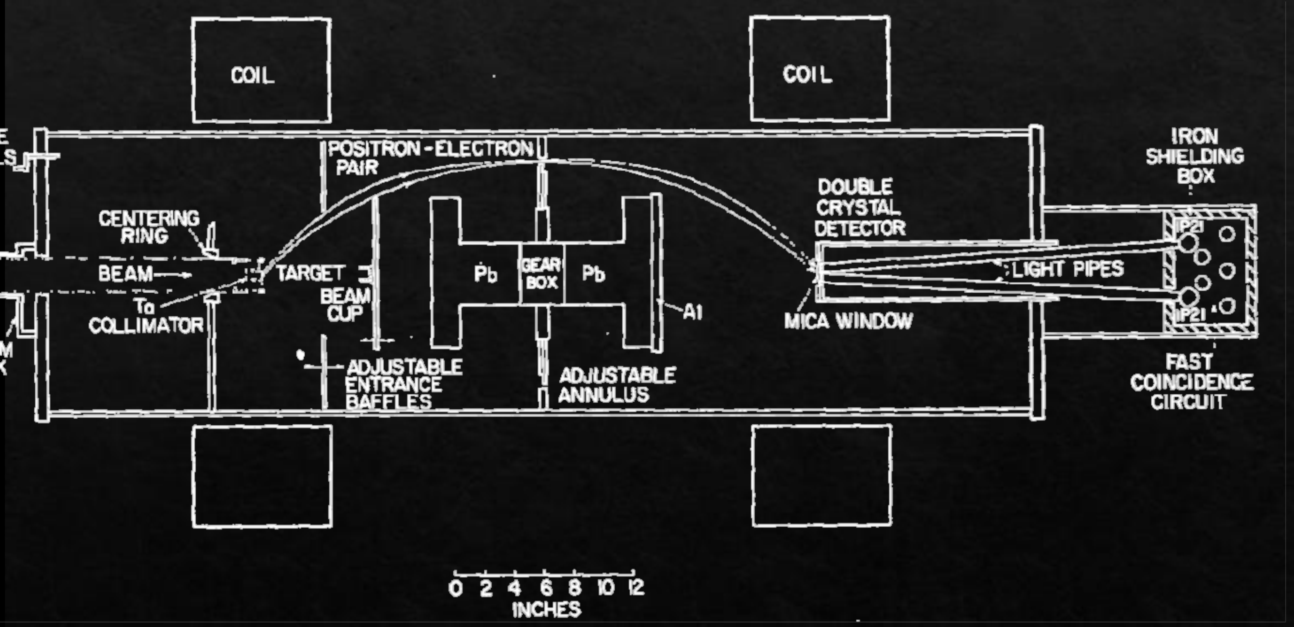
Z.F. Luo et al., Phys. Rev. C 109, 025801 (2024)

Measurement of $\Gamma_{pair}/\Gamma_{tot}$

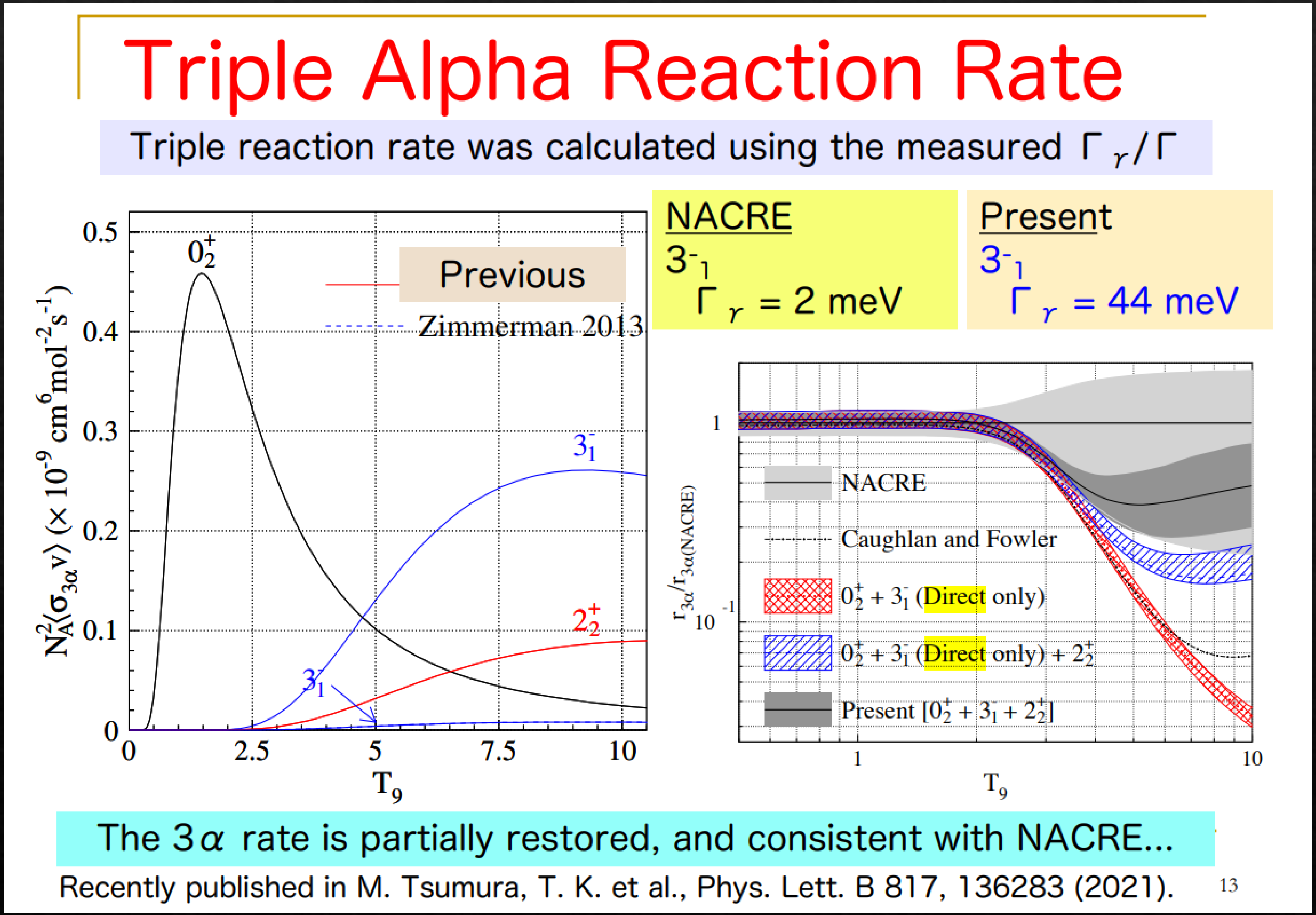
$$\Gamma_\pi/\Gamma = (6.7 \pm 0.6) \times 10^{-6}$$



Roberson et al., PRC15, 1072 (1977)



Alburger, PRC16, 1394 (1977)



Triple Alpha Reaction Rate

Triple reaction rate was calculated using the measured Γ_γ/Γ

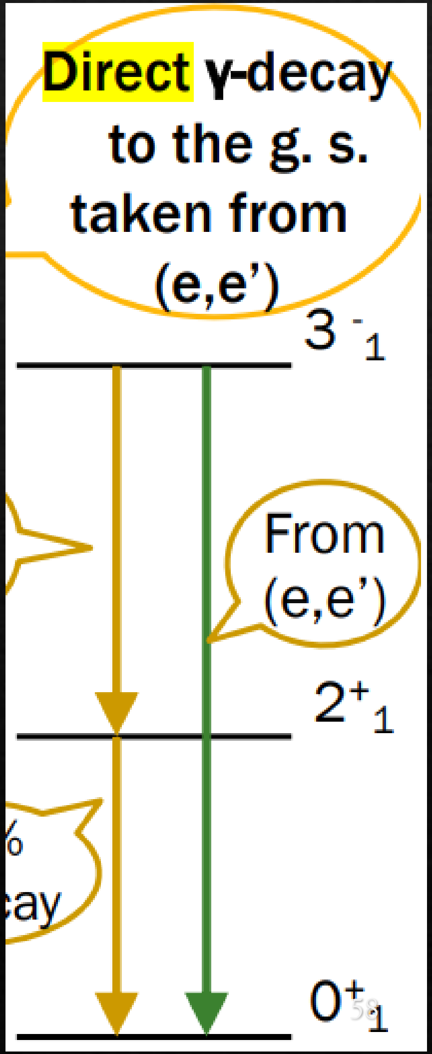
NACRE
3^-_1
Γ_γ = 2 meV

Present
3^-_1
Γ_γ = 44 meV

The 3α rate is partially restored, and consistent with NACRE...

Recently published in M. Tsumura, T. K. et al., Phys. Lett. B 817, 136283 (2021).

Higher temperature

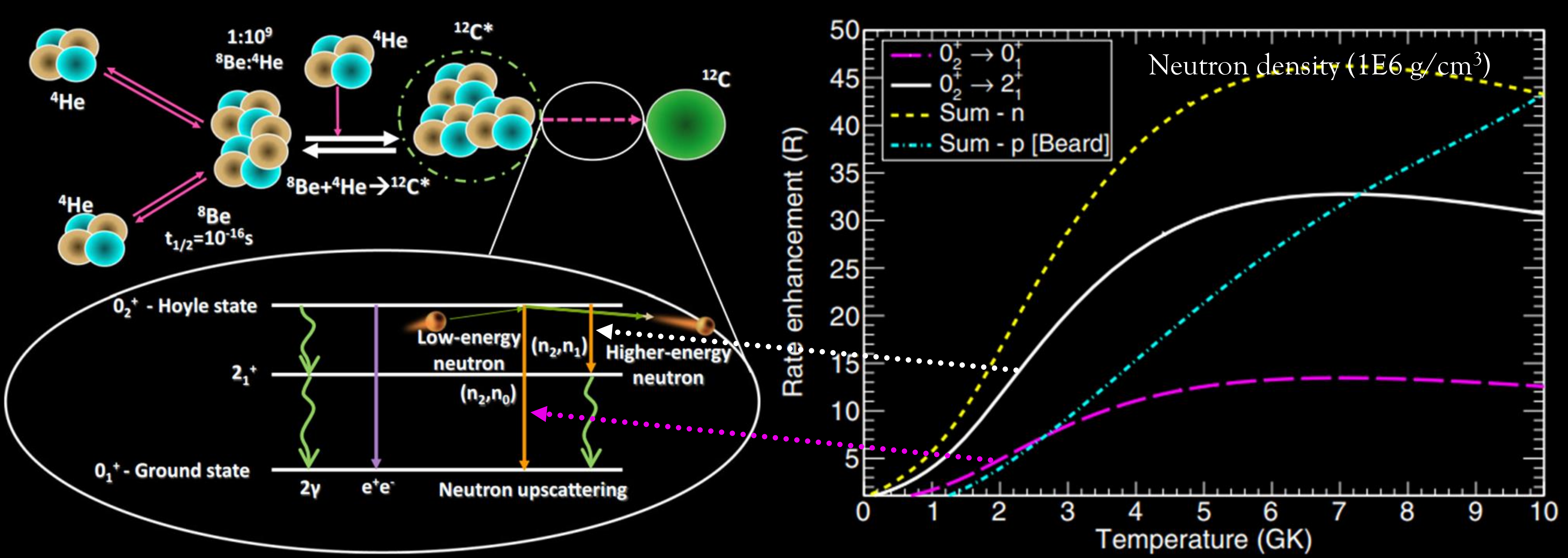


https://www2.yukawa.kyoto-u.ac.jp/~nuc2021/slides/kawabata_t.pdf

Higher density

Upscattering enhances the Hoyle state's decay rate to the bound states of ^{12}C

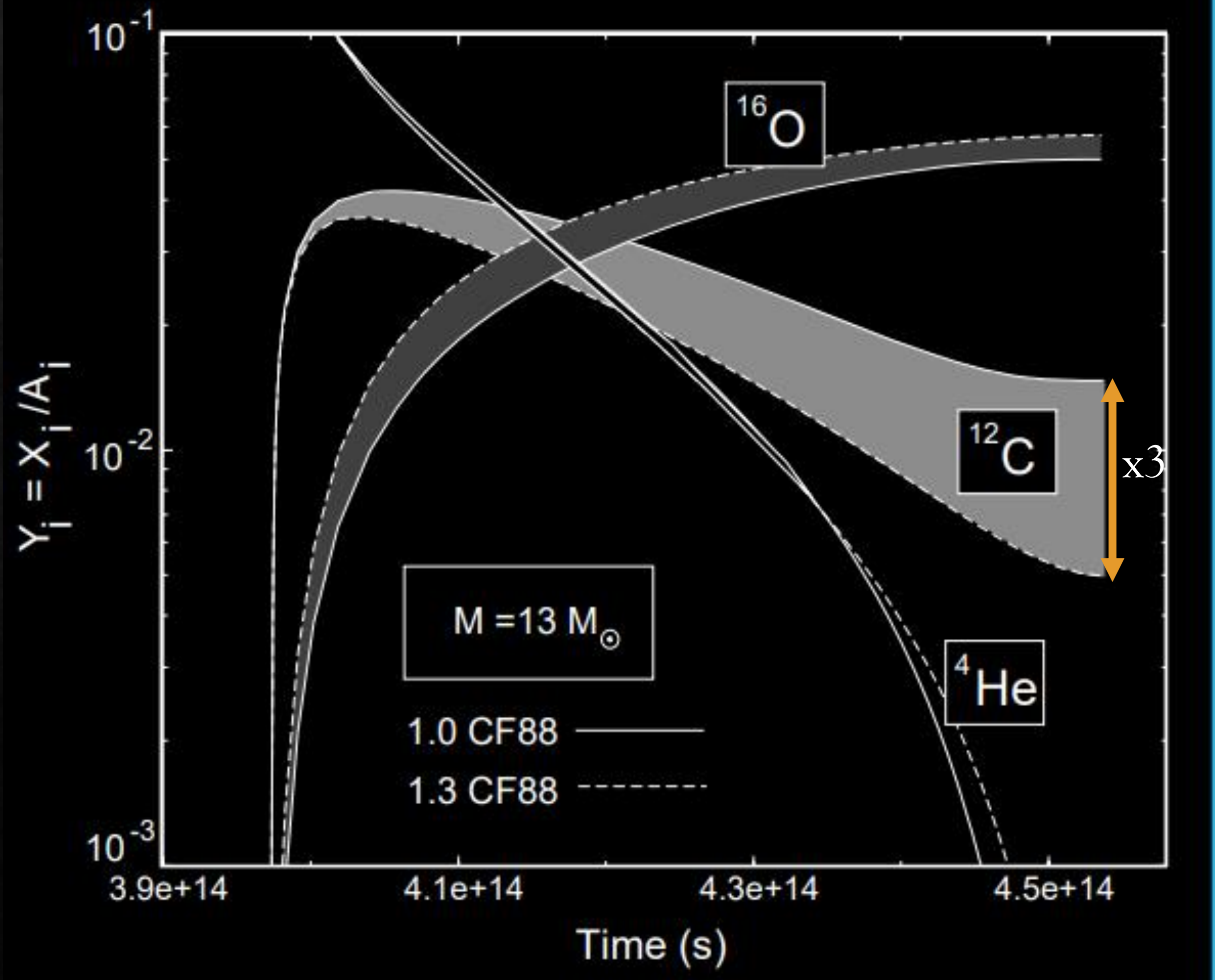
Upscattering: Low energy proton/neutron + ^{12}C(7.654 MeV)→Higher energy neutron + ^{12}C(g.s.)/^{12}C(4.44 MeV)



Neutron density (1E6 g/cm^3)

Bishop et al., Nat Commun **13**, 2151 (2022); Beard et al., Phys. Rev. Lett. 119, 112701 (2017); Jin et al., Nature 588, 57–60 (2020)

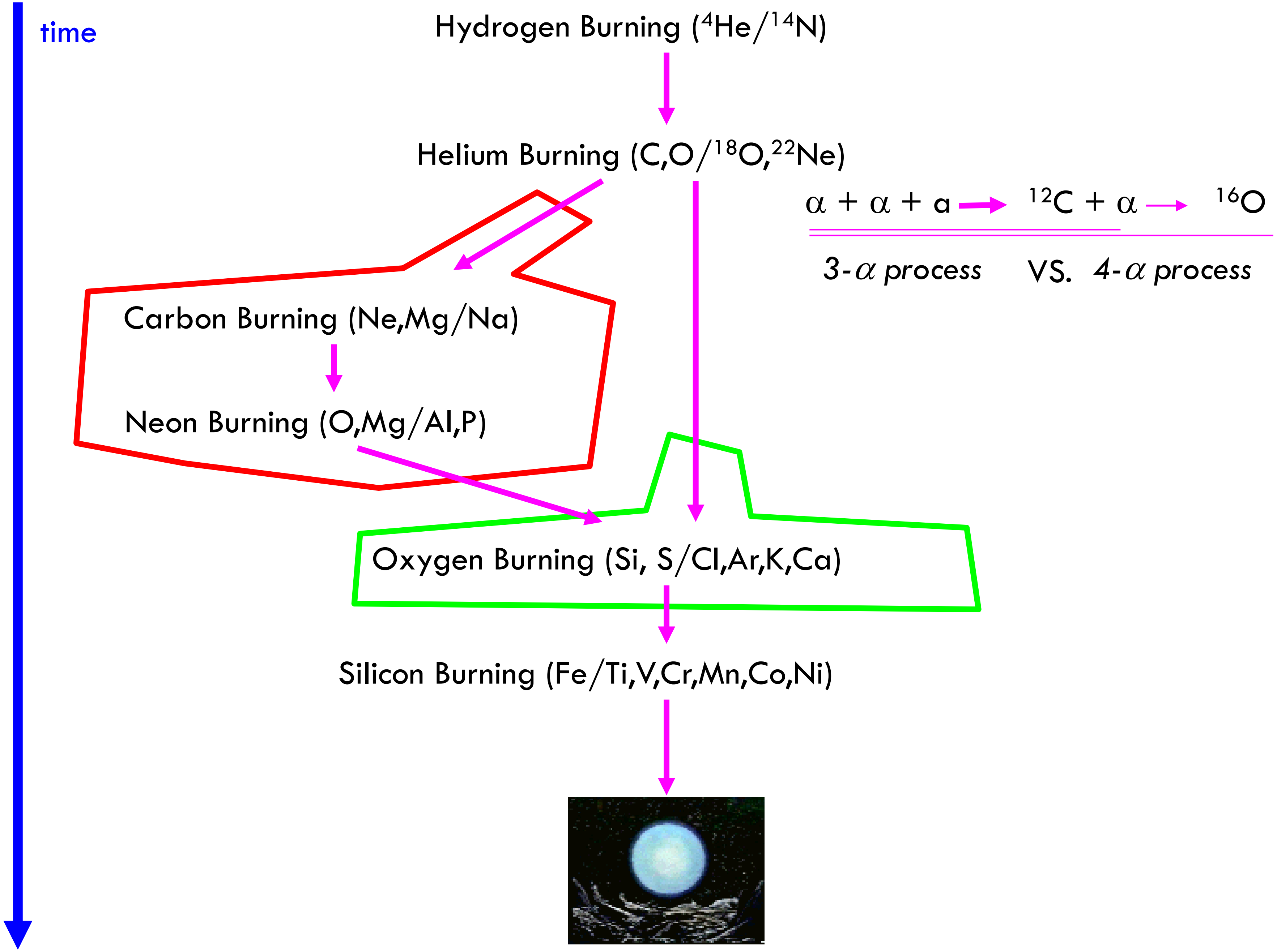
Impact of the uncertain $^{12}C(\alpha,\gamma)^{16}O$ rate



- **Late Stellar Evolution**
  determines Carbon and/or Oxygen phase

- **Type Ia Supernova**
  central carbon burning of C/O white dwarf

- **Type II Supernova**
  shock-front nucleosynthesis in C and He shells of pre-supernova star



Hydrogen Burning (^{4}He/^{14}N)

Helium Burning (C,O/^{18}O,^{22}Ne)

$\alpha + \alpha + a \rightarrow {}^{12}C + \alpha \rightarrow {}^{16}O$

3-α process VS. 4-α process

Carbon Burning (Ne,Mg/Na)

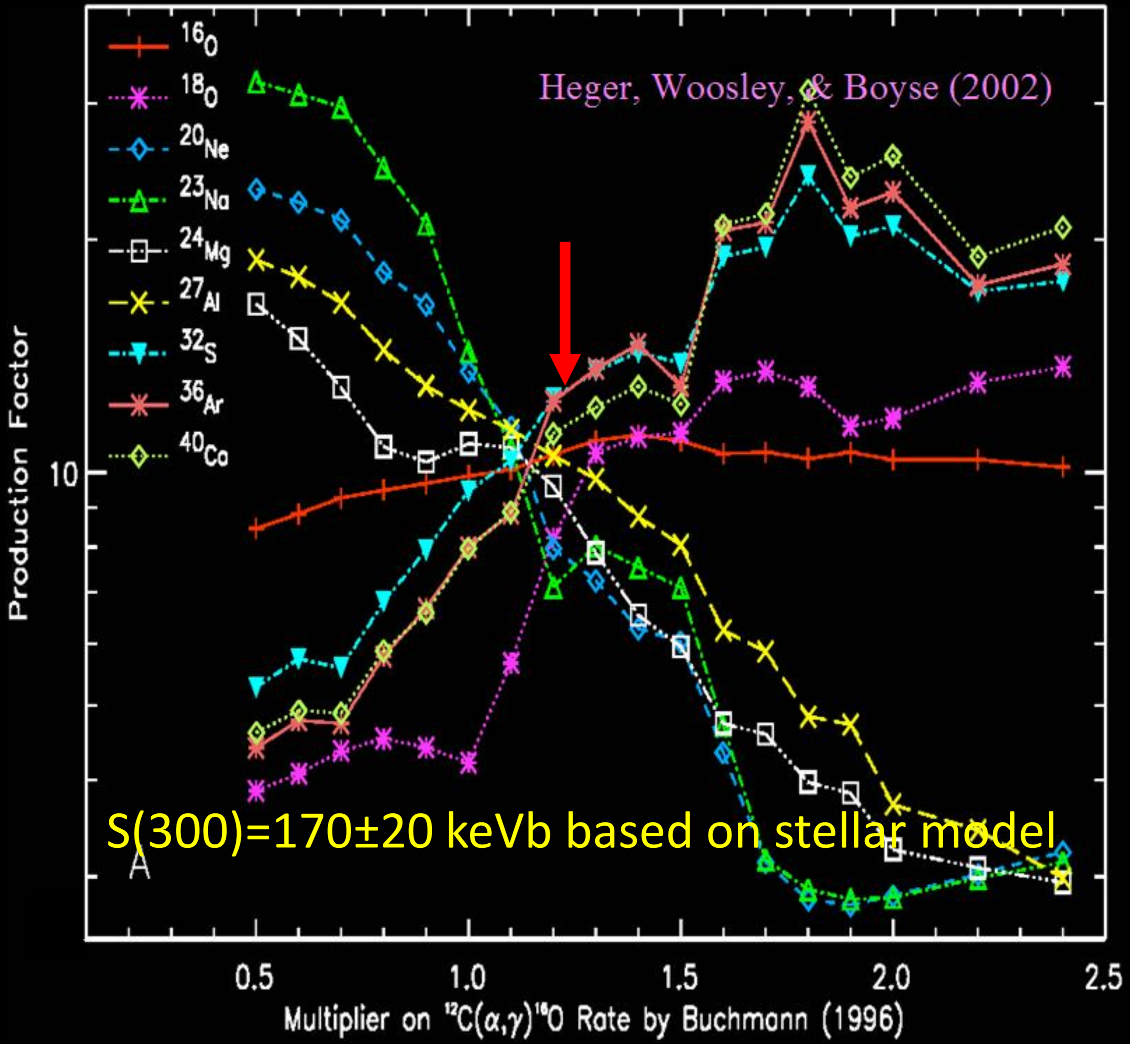
Neon Burning (O,Mg/Al,P)

Oxygen Burning (Si, S/Cl,Ar,K,Ca)

Silicon Burning (Fe/Ti,V,Cr,Mn,Co,Ni)

Influences of the uncertainty in the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate in Nucleosynthesis

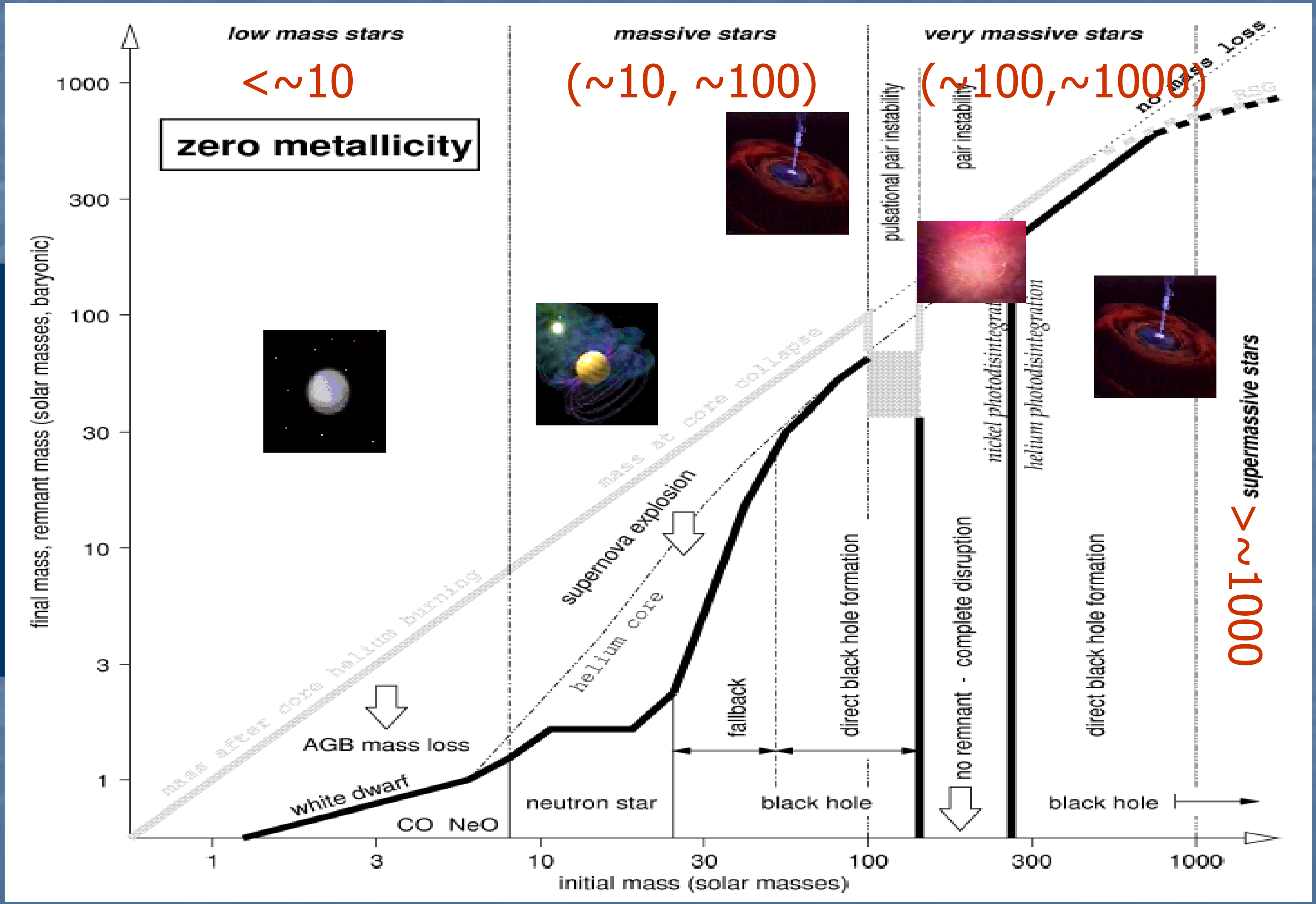
A uncertainty of ~10% is needed!



S(300)=170±20 keVb based on stellar model

Black hole mass gap

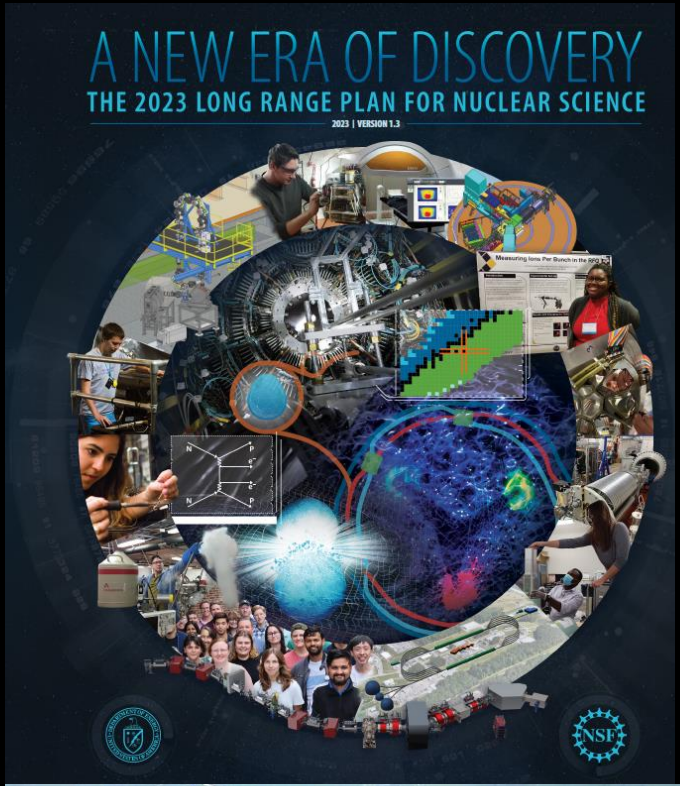
A **pair-instability supernova** is a type of supernova predicted to occur when pair production. The explosion is trigged by the $^{16}O+^{16}O$ fusion reaction.



A. Heger & S. Woosley, ApJ.. 567(2002)532,
Woosley, Heger and Weaver, Rev. Mod. Phys. 74, 1015

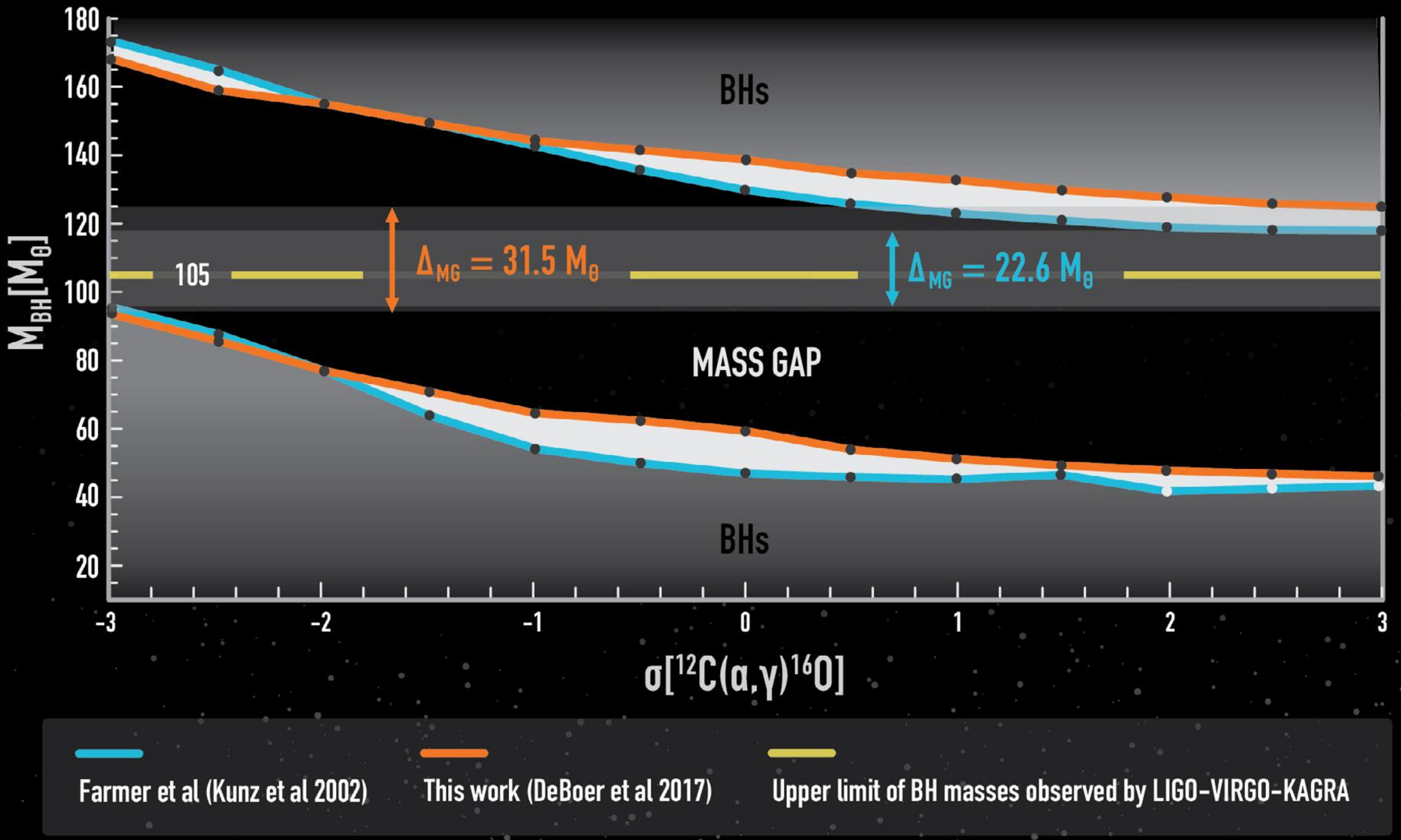
A slide from my Ph.D. defense

Impact on Multi-Messenger Astronomy



A NEW ERA OF DISCOVERY
THE 2023 LONG RANGE PLAN FOR NUCLEAR SCIENCE
2023 | VERSION 1.3

LIGO



Farmer et al., ApJ 902:L36(2020)
NSAC LONG RANGE PLAN (2023)

Holy grail for nuclear astrophysicists

Uncertainty in the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate affects not only the nucleosynthesis but also the explosion itself.

The determination of the ratio C/O produced in helium burning is a problem of paramount importance in Nuclear Astrophysics.
W. Fowler, Nobel lecture, 1983

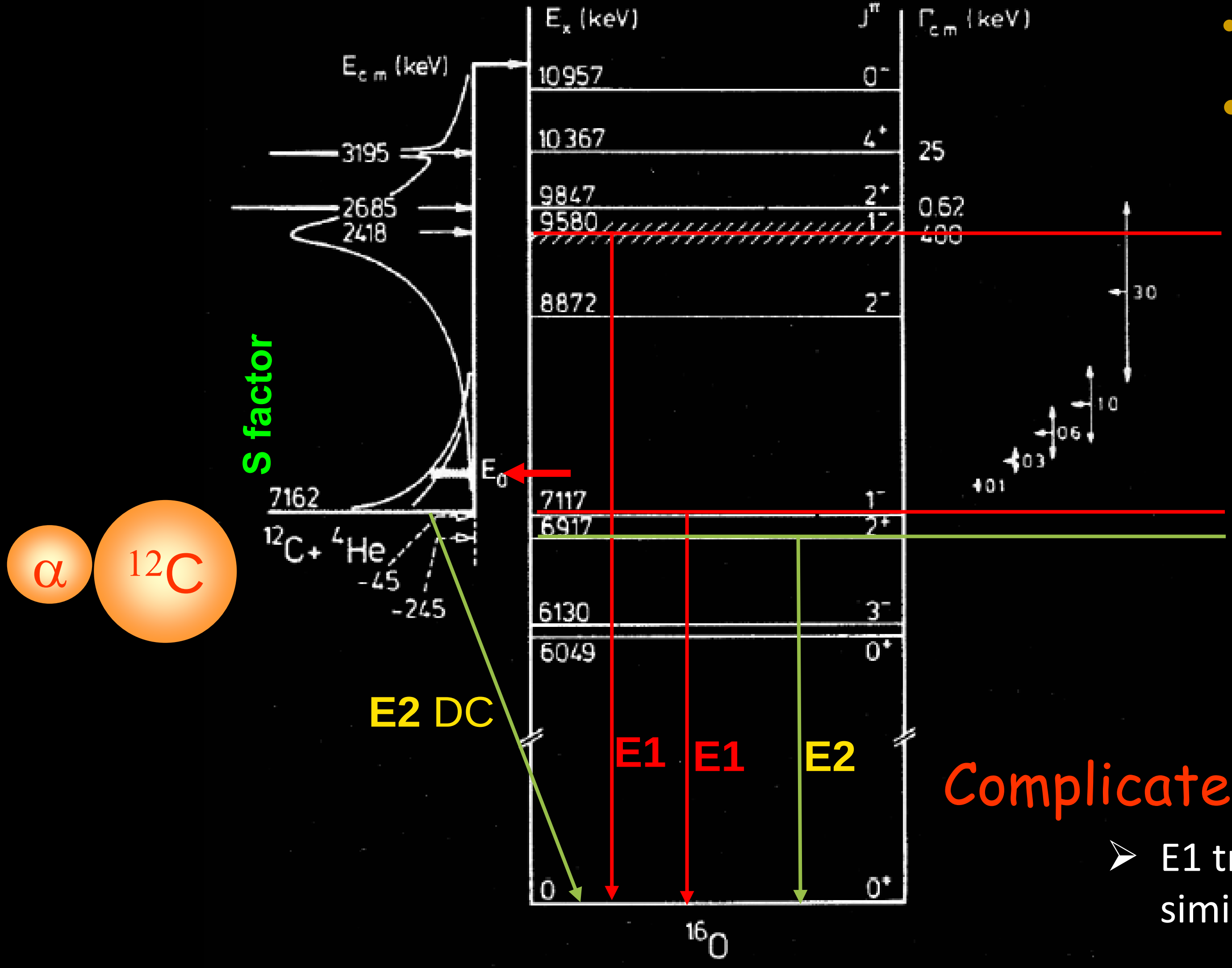
We hope that…will keenly motivate experimentalists to undertake the difficult task of accurately measuring this rate.

Weaver & Woosley, Phys. Rep. 227 (1993) 65

The fusion of ^{4}He and ^{12}C nuclei to ^{16}O is the most important nuclear reaction in the development of massive stars.
NuPECC Long Range Plan

Level Scheme of ^{16}O

- **3 resonances**
- **1 direct capture**



resonance
(high lying)

resonance
(sub threshold)

resonance
(sub threshold)

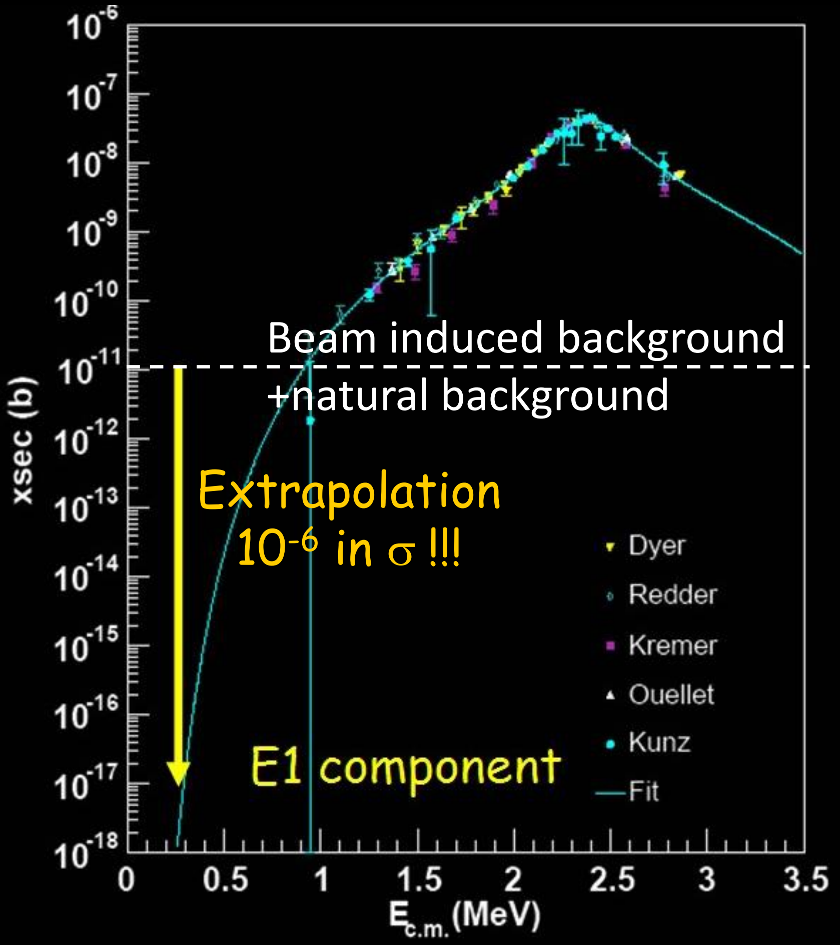
Complicated reaction mechanism

- ➢ E1 transition requires isospin mixing leads to similar strength o to E2

A fundamental challenges for nuclear astrophysics :
Measure reaction rate at extremely low energies

NSAC Long Range Plan (2023)

$^{12}C(\alpha,\gamma)^{16}O$

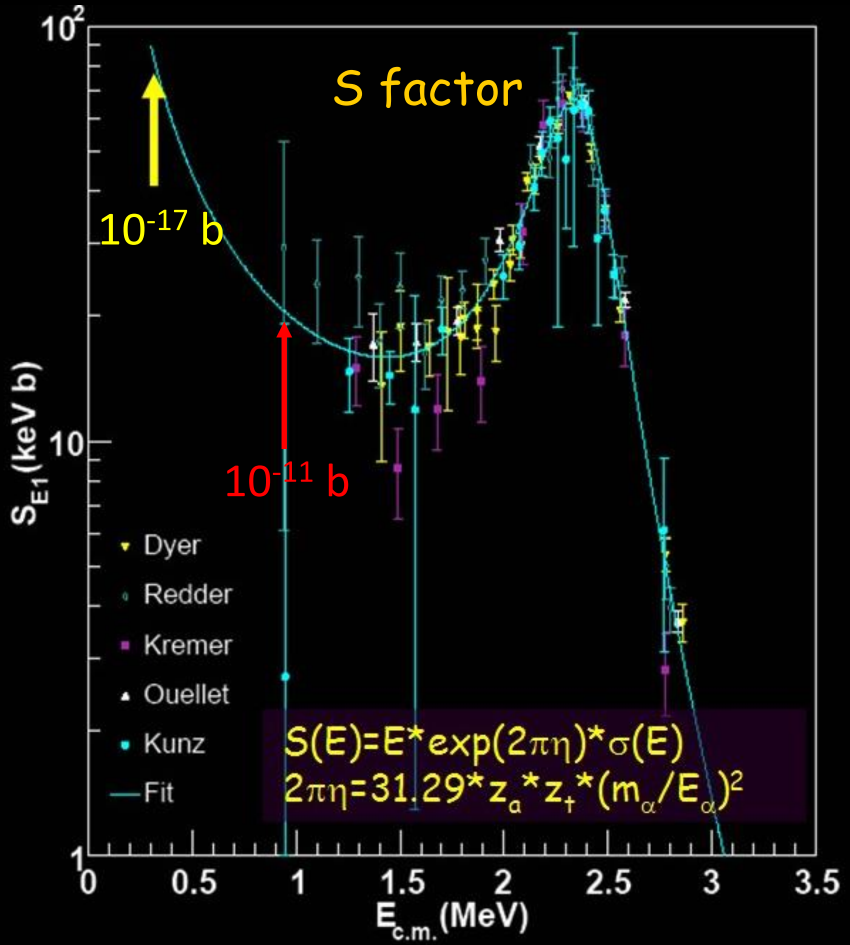


Beam induced background
+natural background

Extrapolation
10^{-6} in σ !!!

E1 component

1 barn=10^{-24} cm^2



S factor

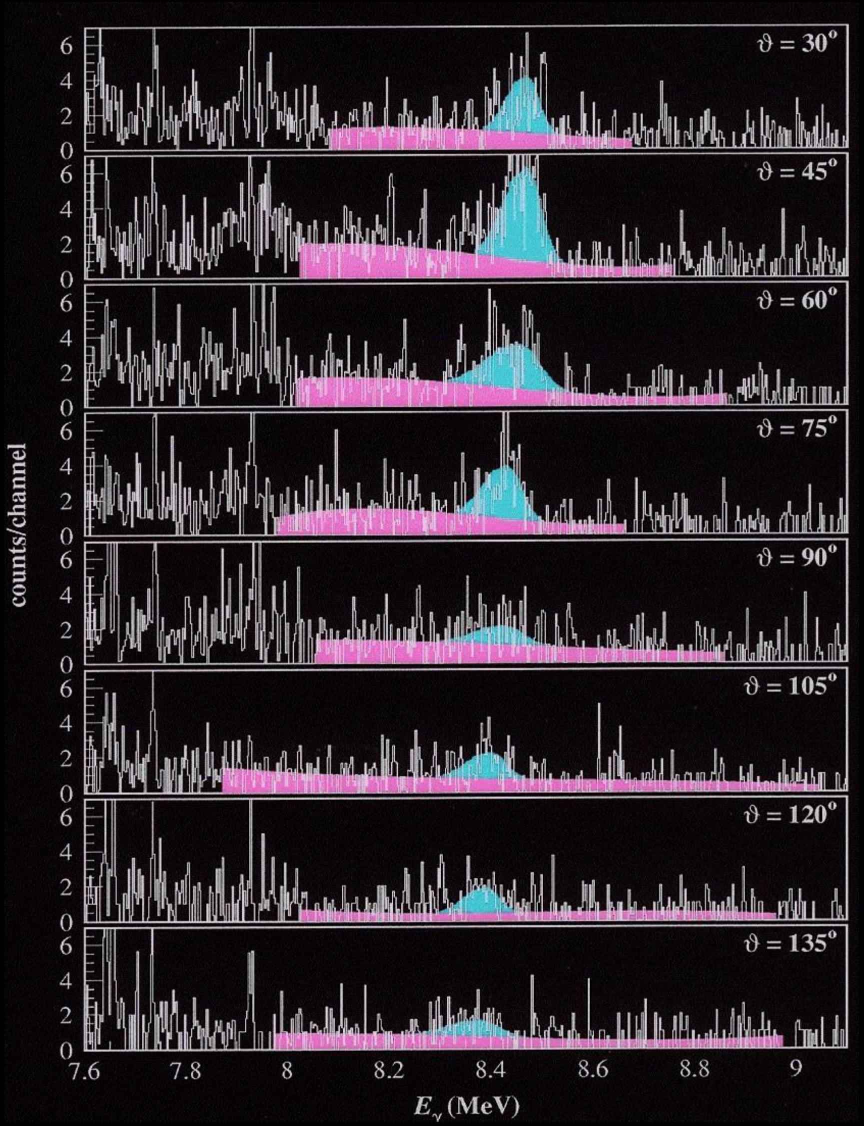
10^{-17} b

10^{-11} b

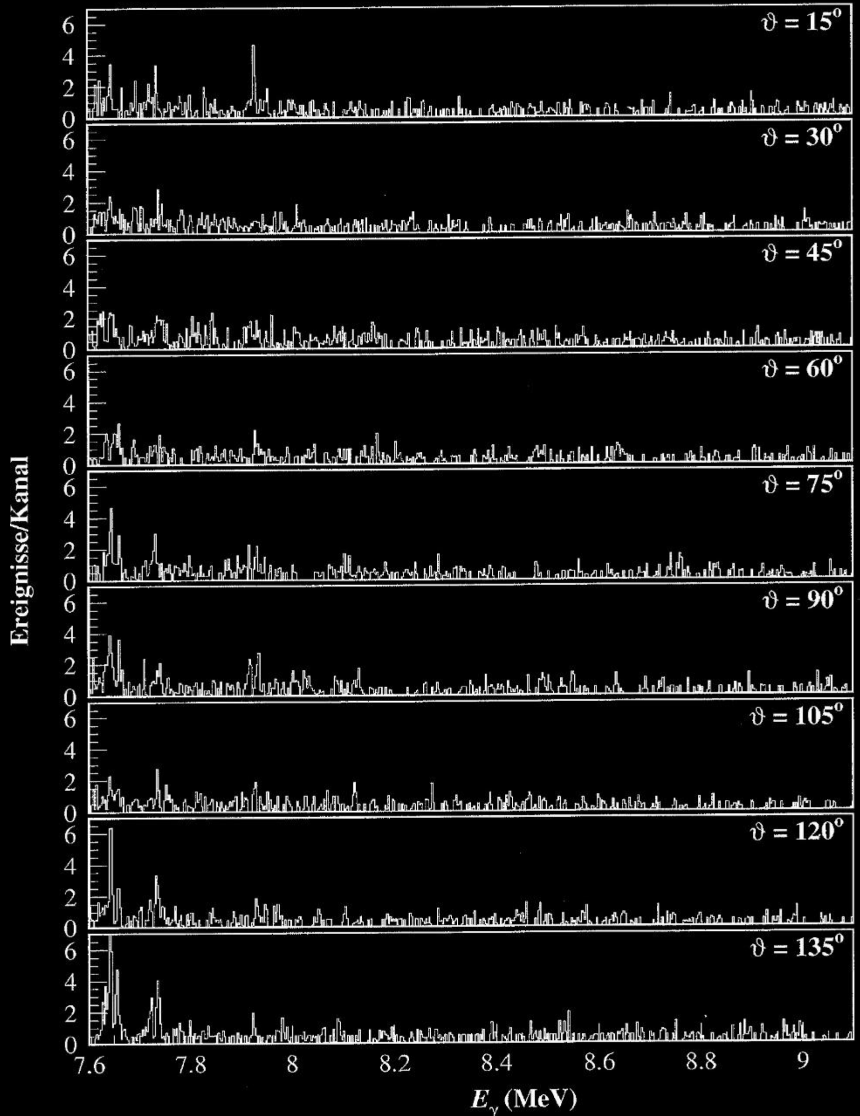
$S(E)=E^{*}\exp(2\pi\eta)^{*}\sigma(E)$

$2\pi\eta=31.29^{*}z_a{}^{*}z_t{}^{*}(m_\alpha/E_\alpha)^2$

Difficulties in direct measurement: $^{12}C(\alpha,\gamma)^{16}O$ (1974-)



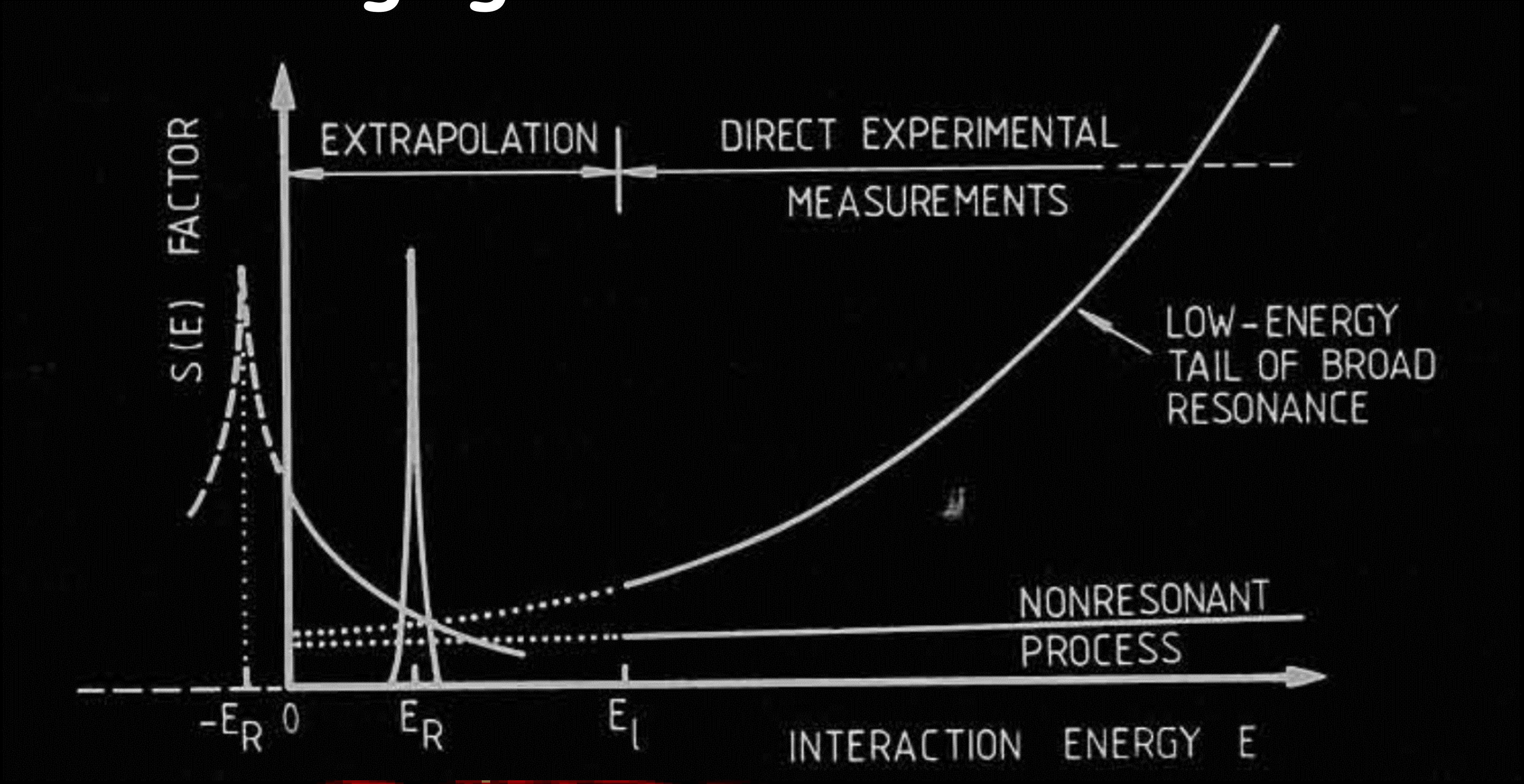
E_{cm}=1.254 MeV



E_{cm}=0.945 MeV

Kunz et al. PRL 86(2001)3244

A challenging task

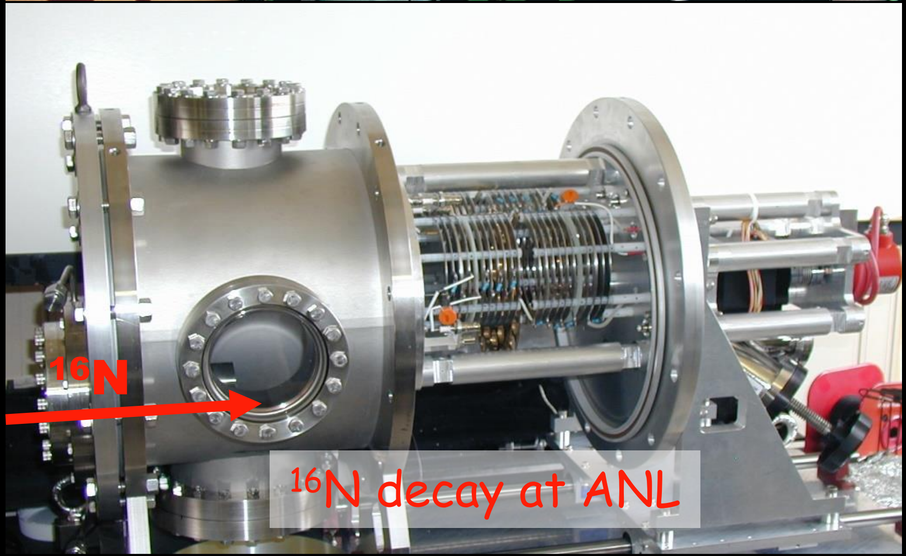


Rolfs and Rodney, Cauldrons in the cosmos, (1988)

0.5 mA α

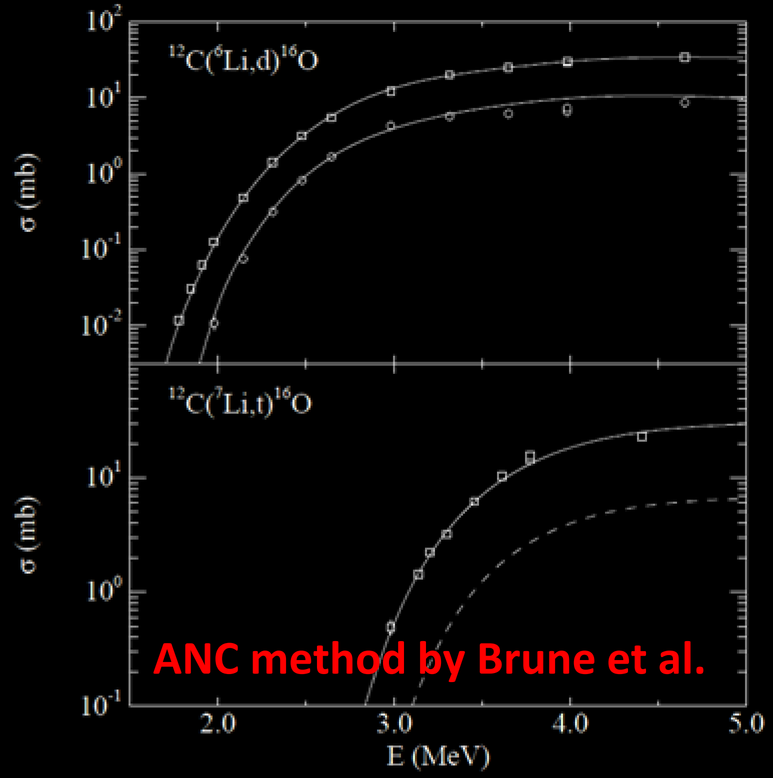
$^{12}C(\alpha,\gamma)^{16}O$ at Stuggart

$^{12}C(\alpha,\alpha)^{12}C$ at Notre Dame



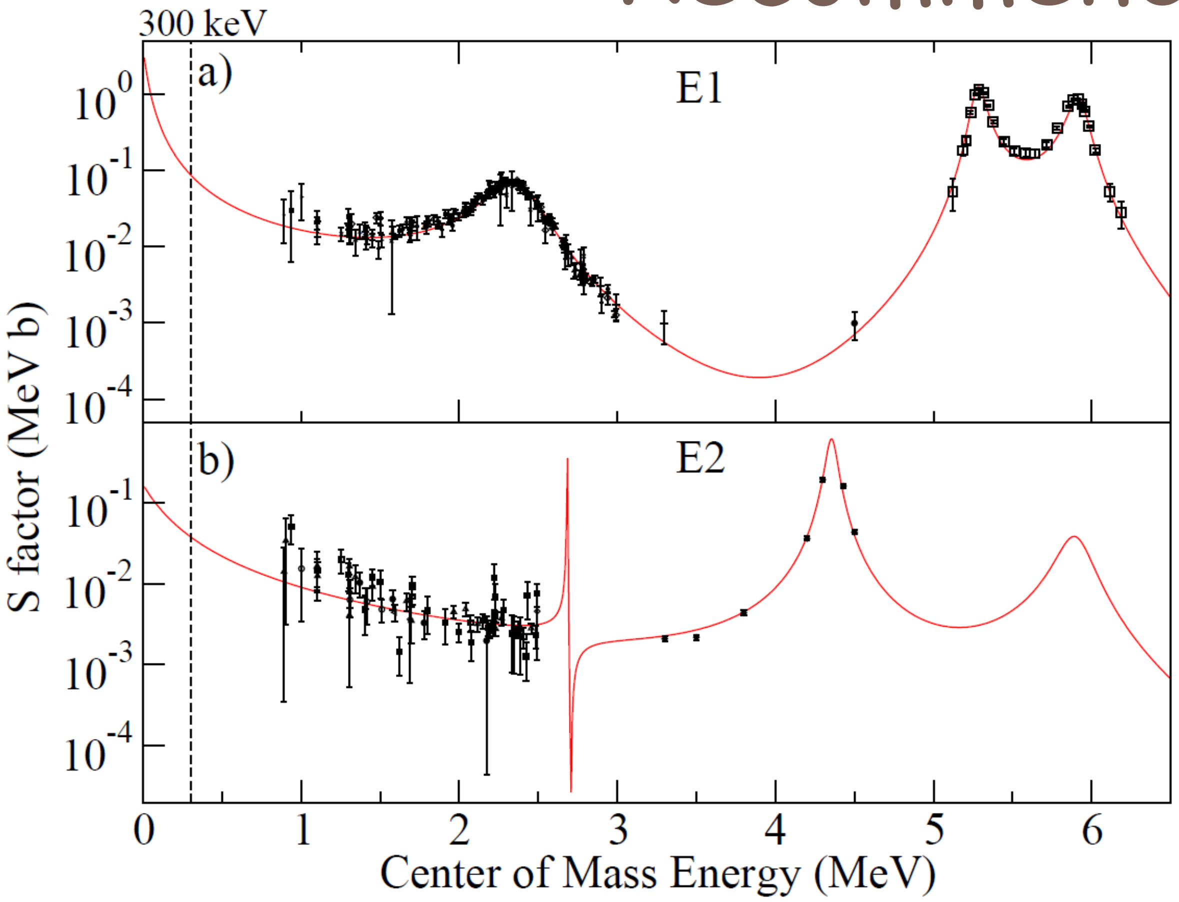
^{16}N

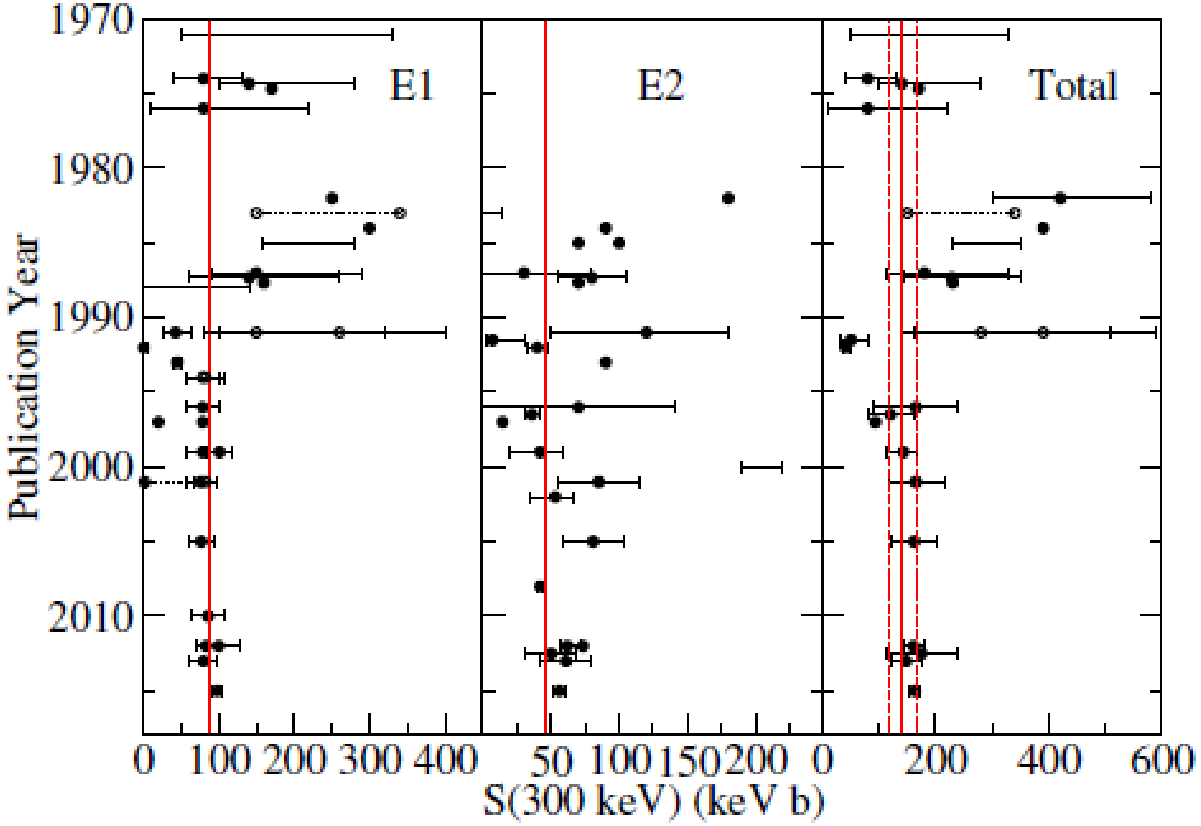
^{16}N decay at ANL



ANC method by Brune et al.

Recommended S factor





S(E1)=86.3 keVb; S(E2)=45.3 keVb; S(cascade)=7 keVb

Total S factor = $140 \pm 21_{(\mathrm{MC})}\,{}^{+18}_{-11}{}_{(\mathrm{model})}$ keV b.

ANC plays the key role to fix the strengths of the subthreshold states and direct capture

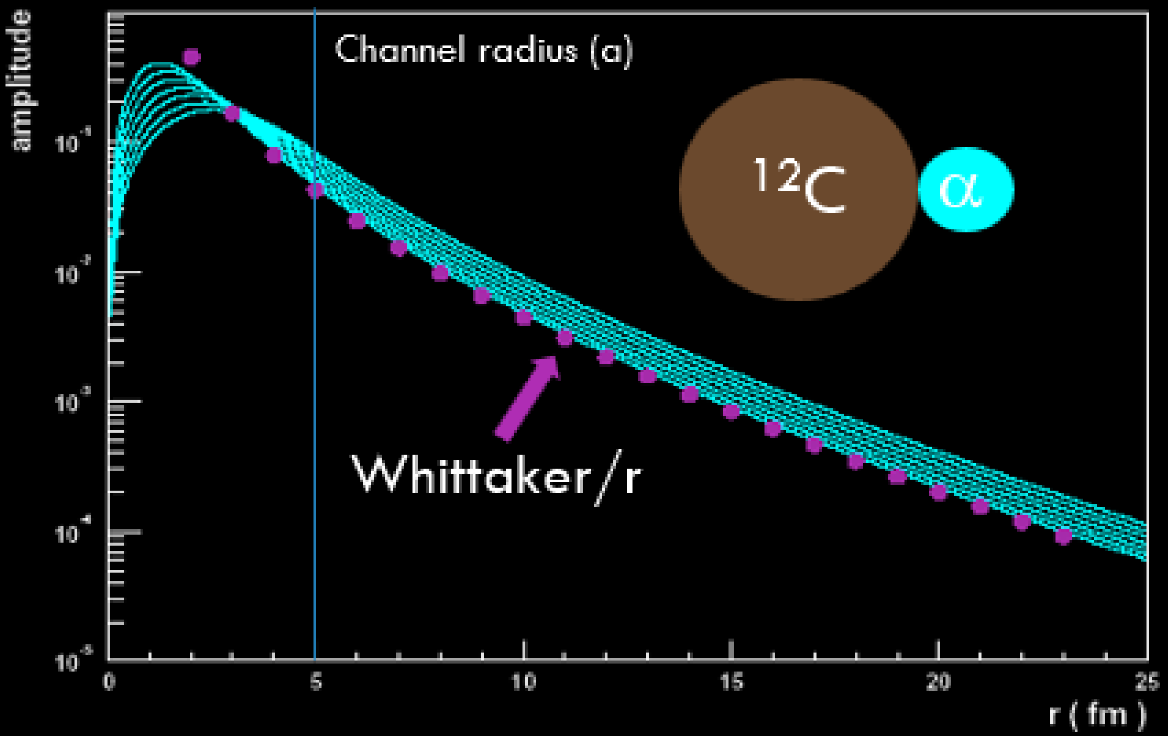
deBoer et al., RMP(2017)

Bound state wave function and Whittaker function

$$I^A_{Bplj}(r) \rightarrow C^A_{Bplj}\frac{W_{-\eta,l+1/2}(2k_B r)}{r} \qquad (r > R_N)$$

$$u_{lj}(r) \rightarrow b_{lj}\frac{W_{-\eta,l+1/2}(2k_B r)}{r} \qquad (r > R_N)$$

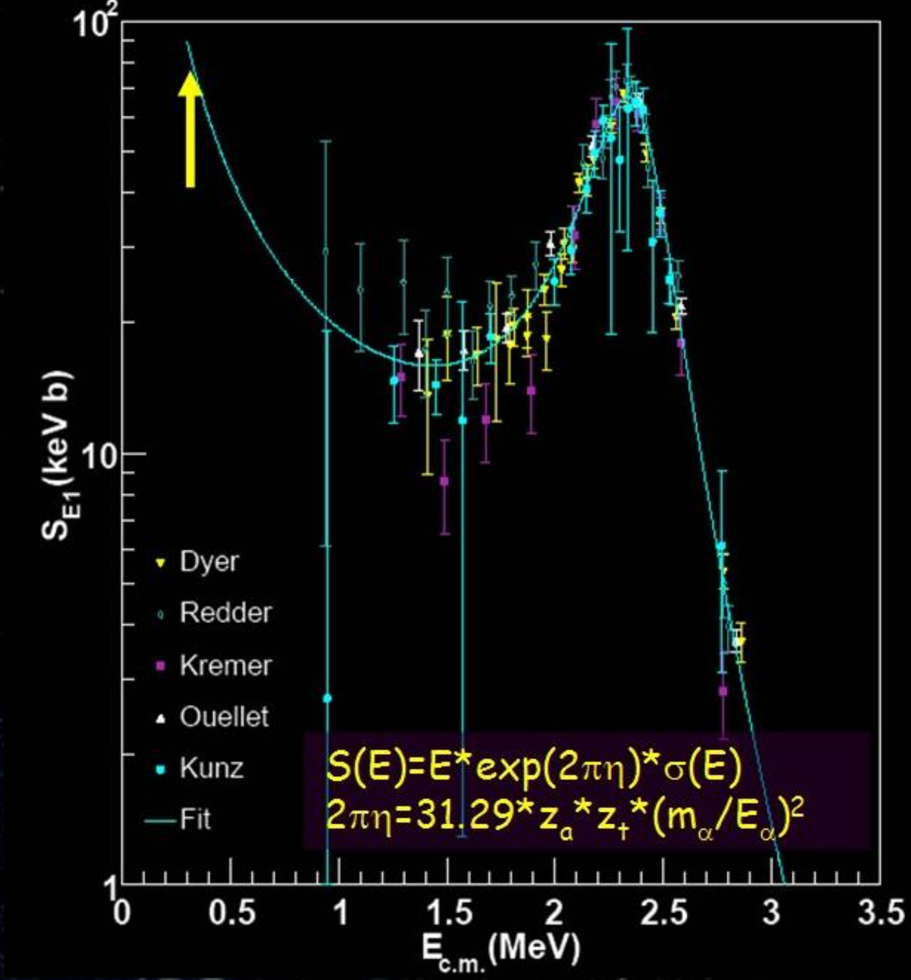
$$I^A_{Bplj}(r) \rightarrow C^A_{Bplj}\frac{u_{lj}(r)}{b_{lj}} \qquad (r > R_N)$$



- Both tails dominated by Coulomb interaction->same solution
- ANC describes the absolute magnitude of the tail of the overlap function and is determined by the complicated internal n-n interaction
- b_{lj}, obtained with potential model, strongly depends on geometric parameters

Subthreshold resonance:

From $^{12}C(^{6}Li,d)^{16}O$ to $^{12}C(\alpha,\gamma)^{16}O$



C. R. Brune *et al.*, Phys. Rev. Lett. **83**, 4025 (1999).

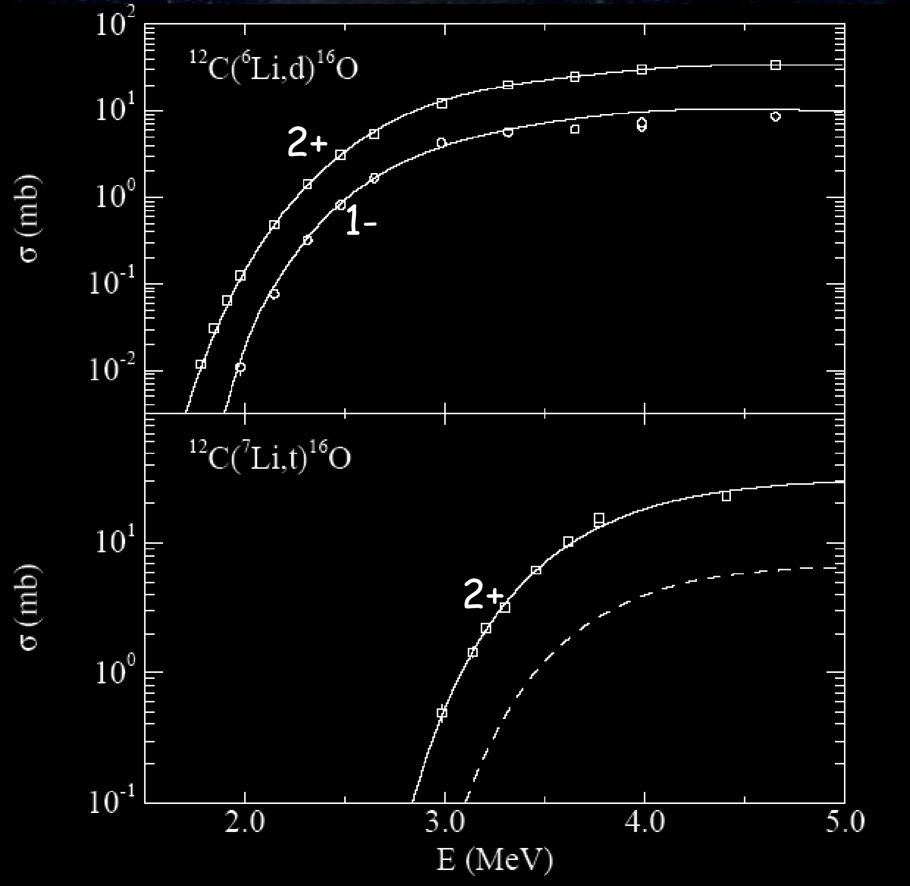
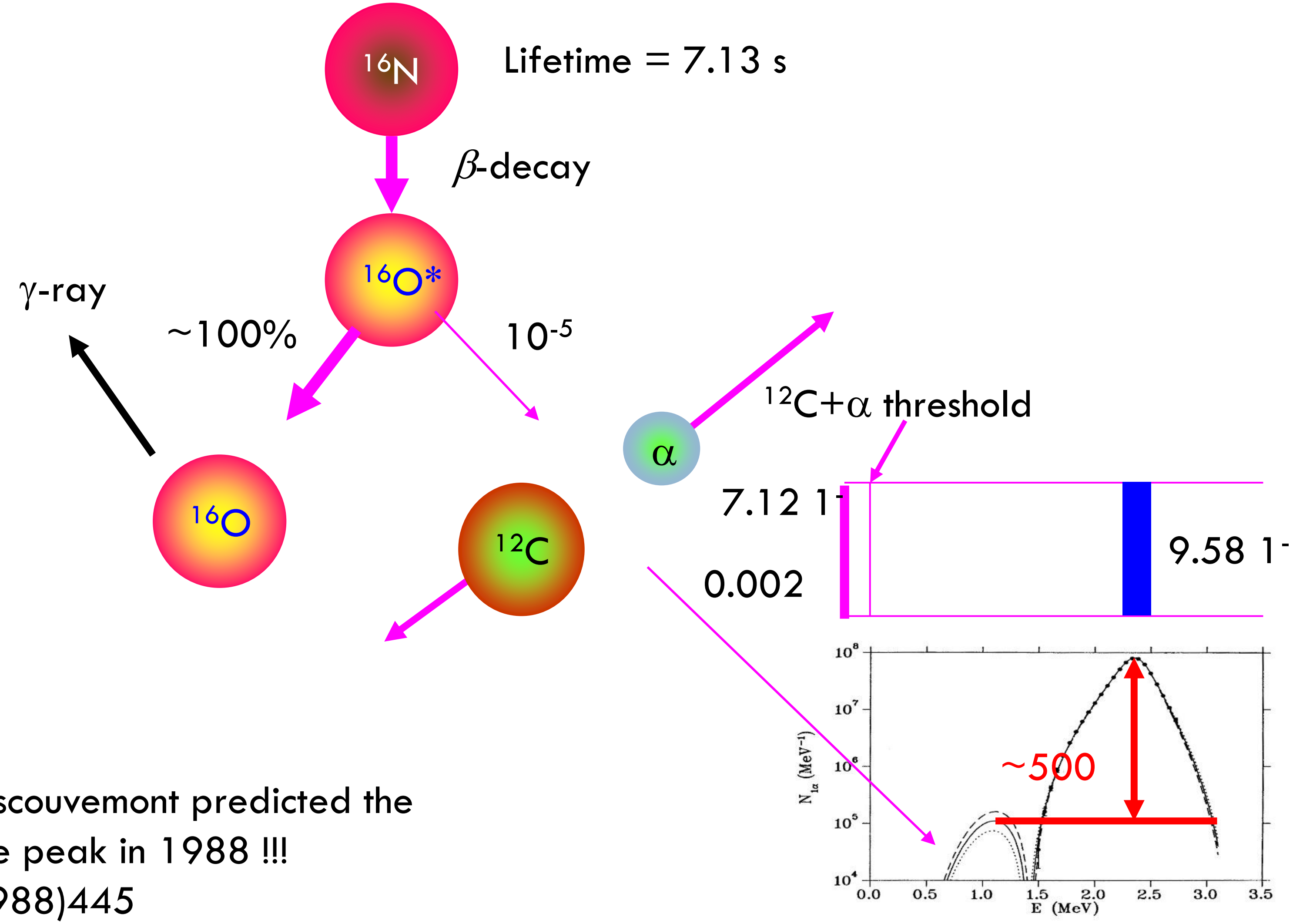


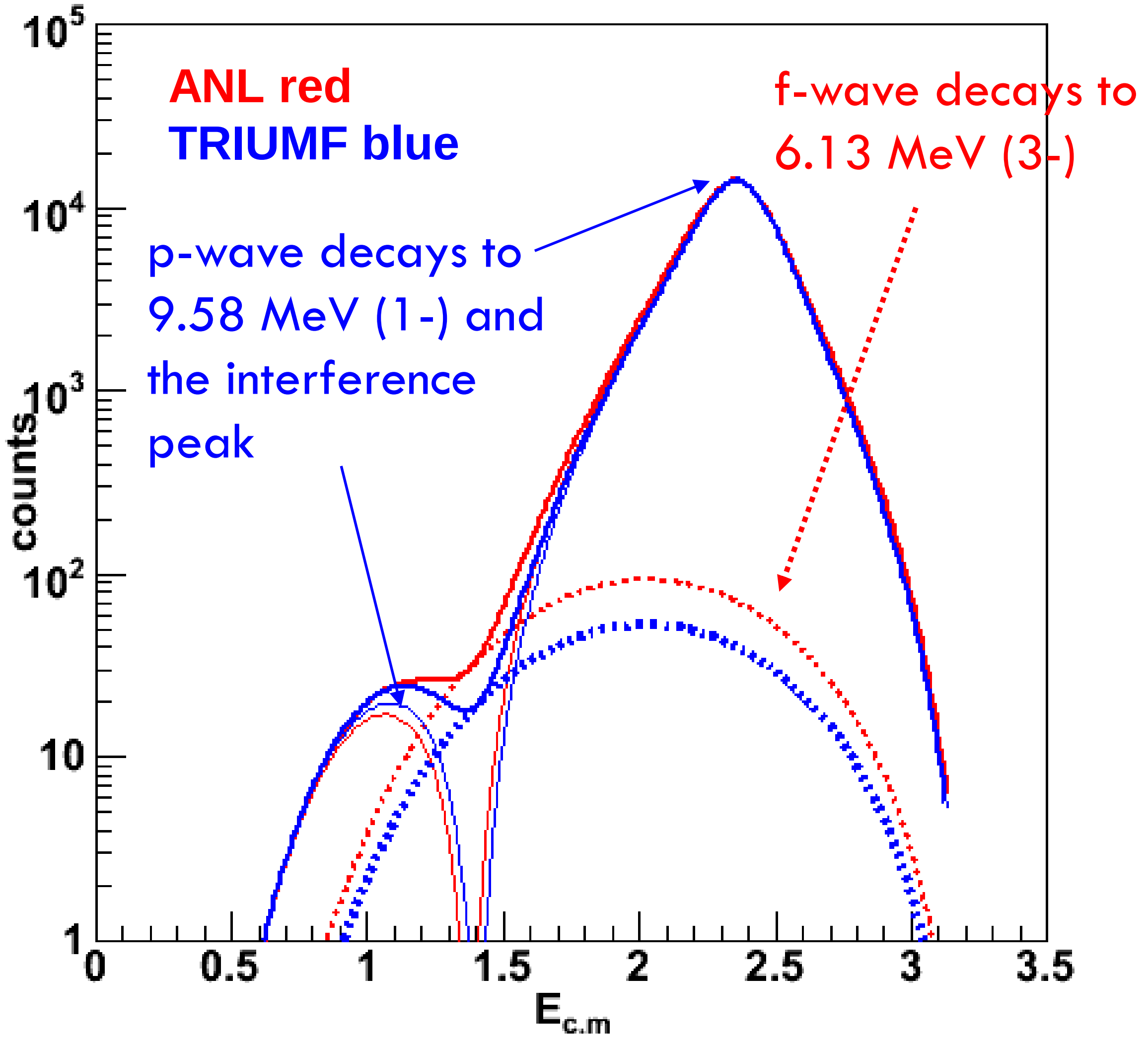
FIG. 2. Total cross sections measured using ^{6}Li (upper panel) and ^{7}Li (lower panel) beams for the 6.92-MeV 2^+ state of ^{16}O (□) and the 7.12-MeV 1^- state (○, ^{6}Li beam only). The solid curves are DWBA calculations normalized to the data; the dashed curve is described in the text.

^{16}N β-delayed α decay



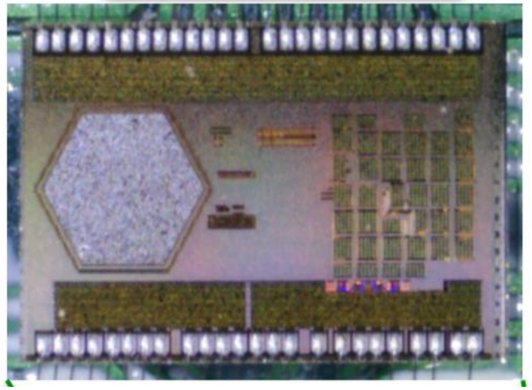
Baye & Descouvemont predicted the interference peak in 1988 !!!
NPA458(1988)445

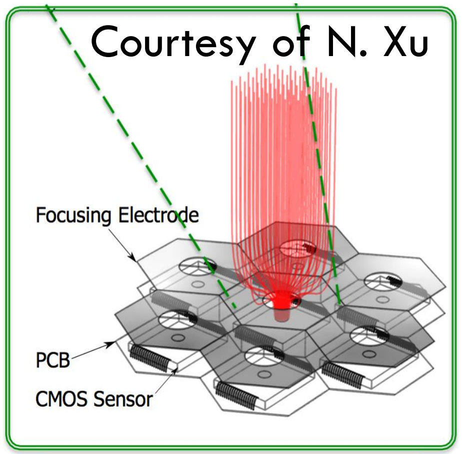
Another ^{16}N decay experiment is needed to resolve the tension!



Azuma et al., PRC(1994); Tang et al., PRL(2007), PRC(2010)

Topmetal

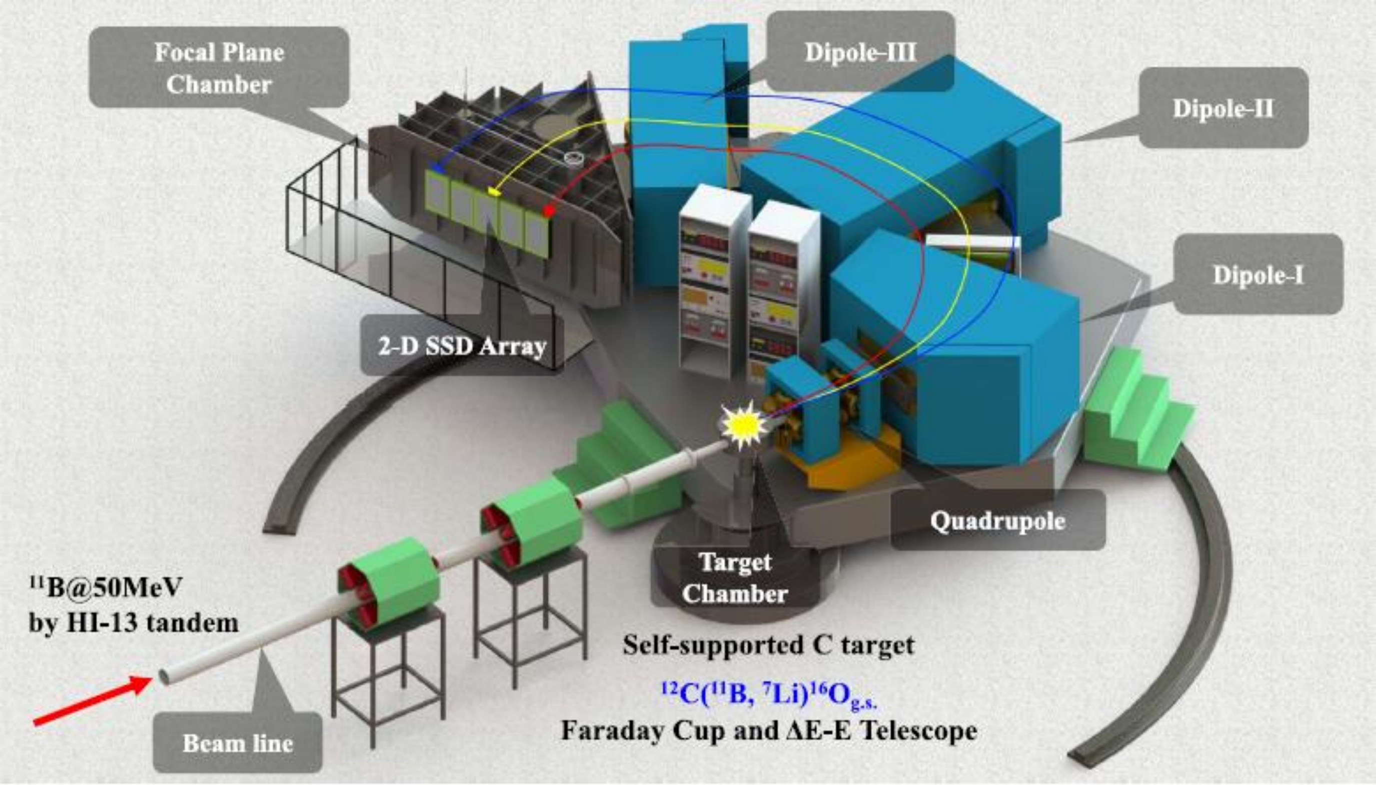


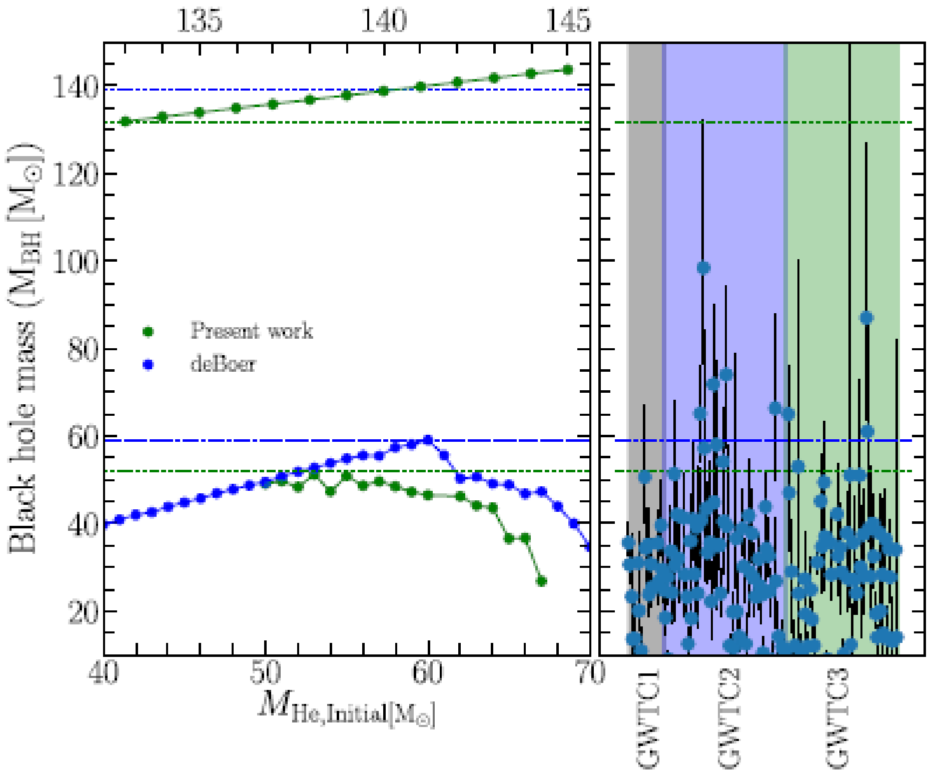


Courtesy of N. Xu

Topmetal CMOS Array

New measurement of ANC of ^{16}O(g.s.) leads to larger S(E2)

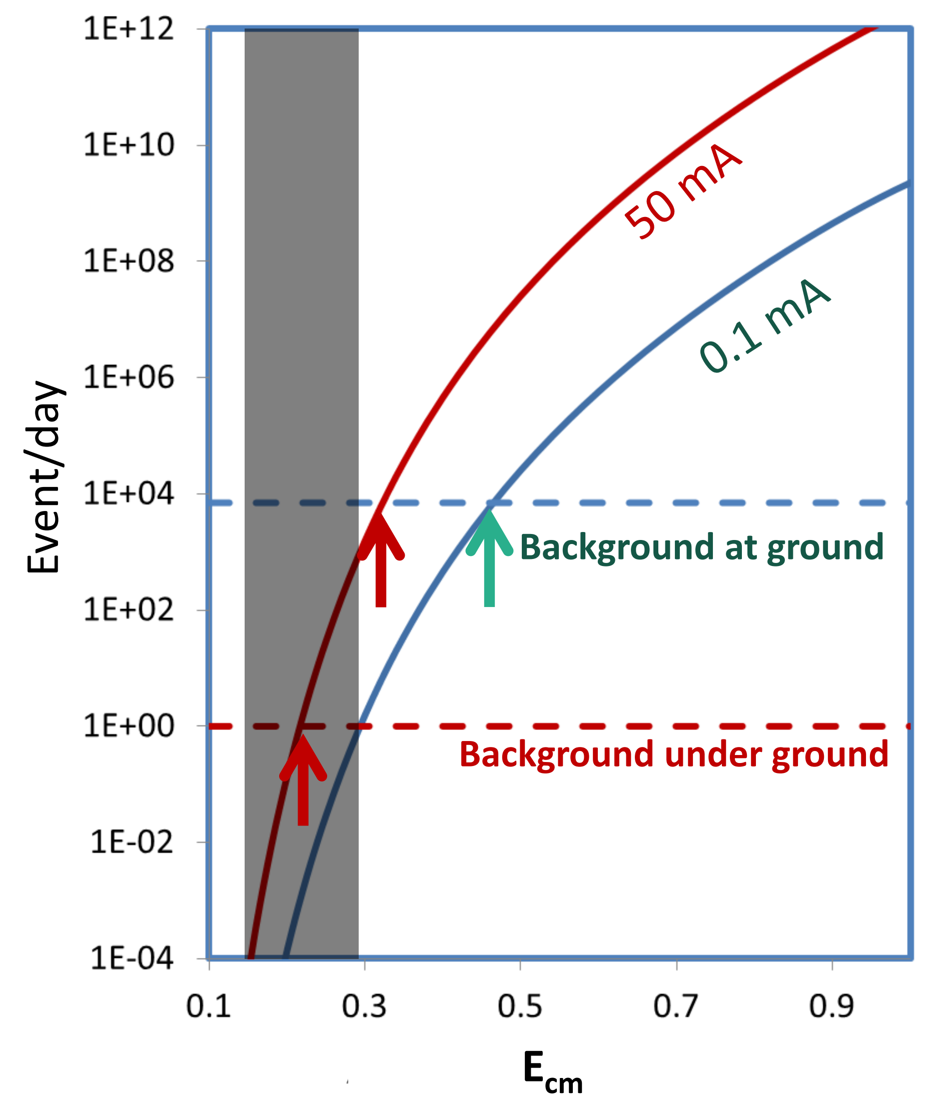


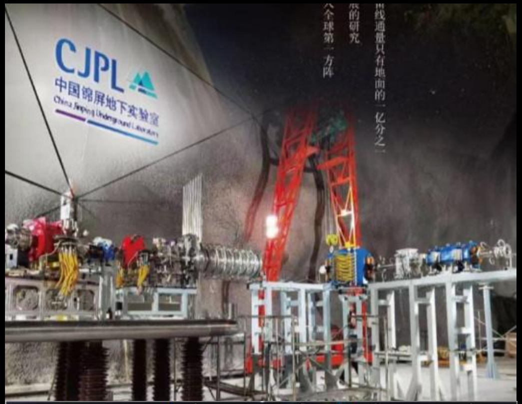


- S(E2) increases from 45 keVb to 70±7 keVb → Total S factor = 162 keVb (err TBD)
- The updated $^{12}C(\alpha, \gamma)^{16}O$ reaction rate decreases the lower and upper edges of the black hole mass gap about 12% and 5%, respectively.

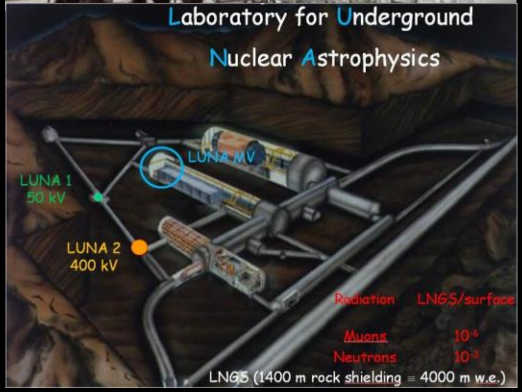
Shen et al, PRL(2022), ApJ(2023)

Challenging the tiny cross-sections

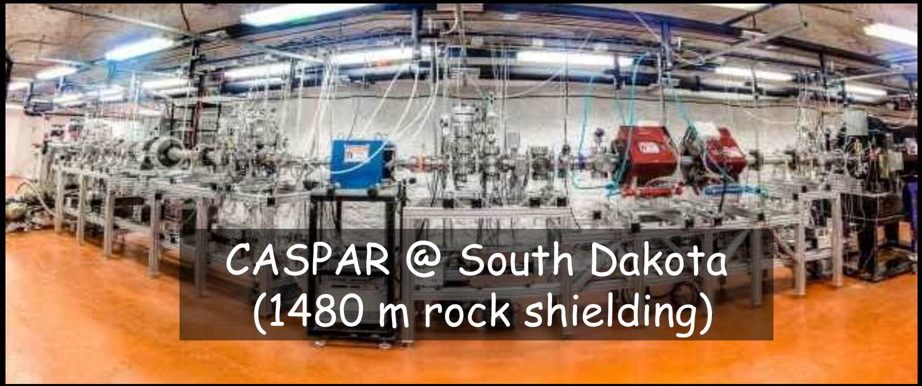




JUNA@China
Jinping
Underground
Laboratory
(2400 m rock shielding)

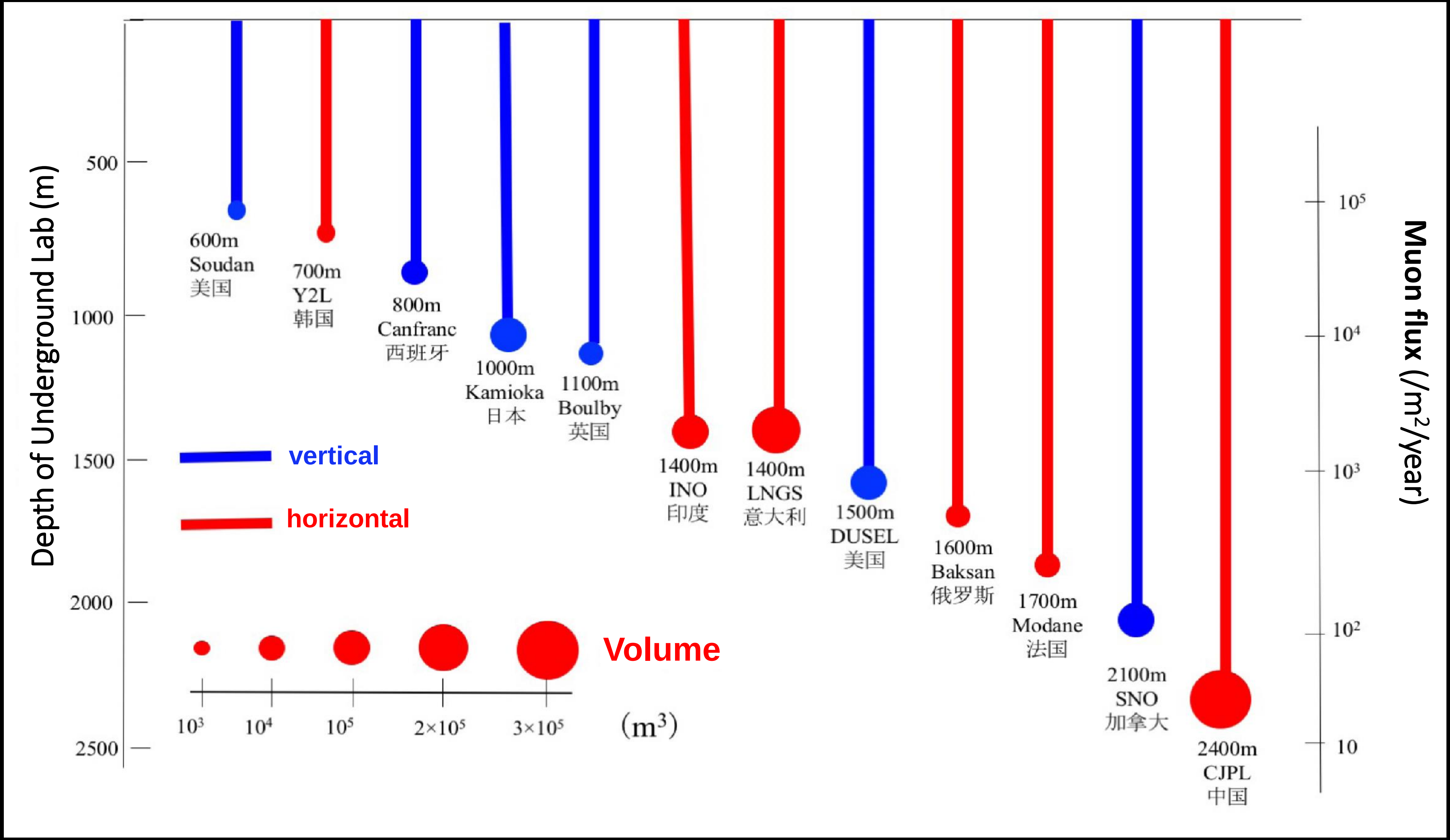


LUNA @
Gran Sasso
Italy
1400m rock shielding

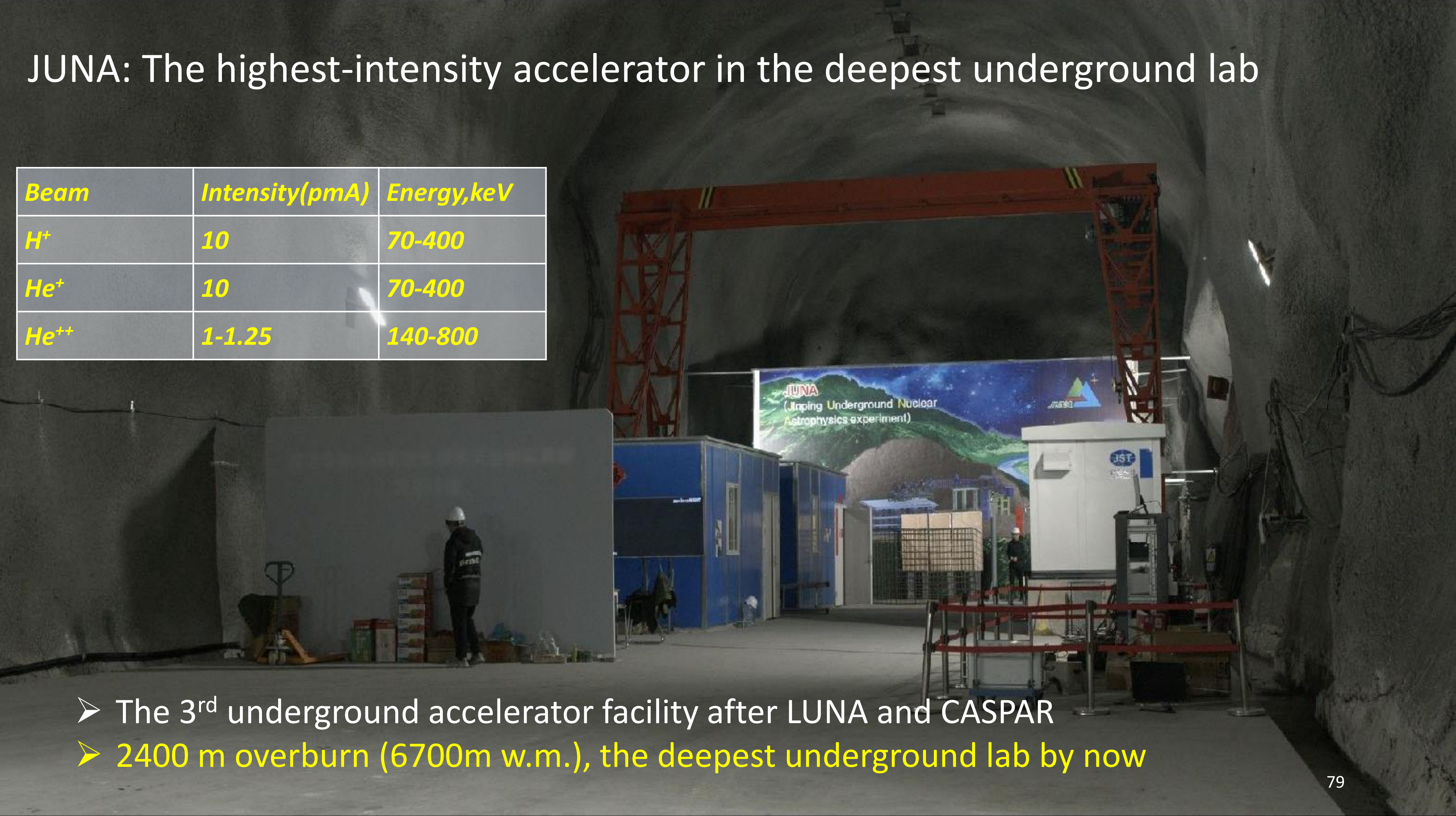


CASPAR @ South Dakota
(1480 m rock shielding)

Comparison of underground laboratories



JUNA: The highest-intensity accelerator in the deepest underground lab



Beam	*Intensity(pmA)*	*Energy,keV*
H^+	*10*	*70-400*
He^+	*10*	*70-400*
He^{++}	*1-1.25*	*140-800*

- ➢ The 3rd underground accelerator facility after LUNA and CASPAR
- ➢ 2400 m overburn (6700m w.m.), the deepest underground lab by now

Jinping Underground Nuclear Astrophysics(JUNA) projects (2015-2021)

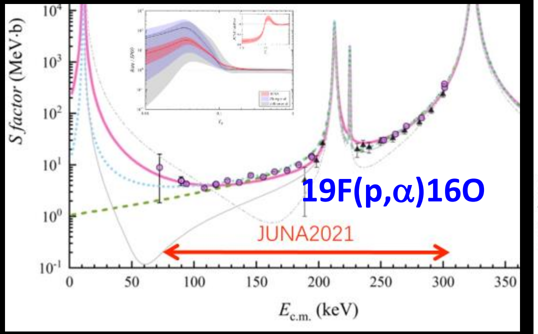
CIAE， W.P. Liu
$^{12}C(\alpha,\gamma)^{16}O$

BNU, J.J. He
$^{19}F(p,\alpha)^{16}O$
$^{19}F(p,\gamma)^{20}Ne$

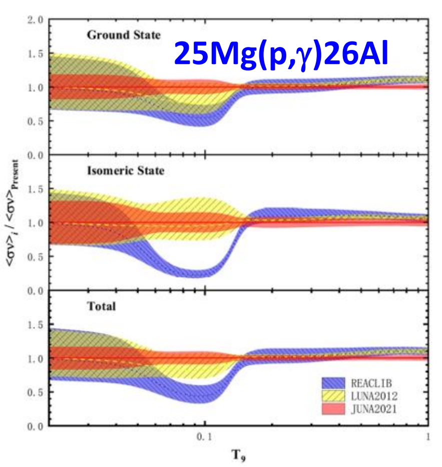
CIAE, Z.H. Li
$^{25}Mg(p,\gamma)^{26}Al$

IMP, X.D. Tang
$^{13}C(\alpha,n)^{16}O$

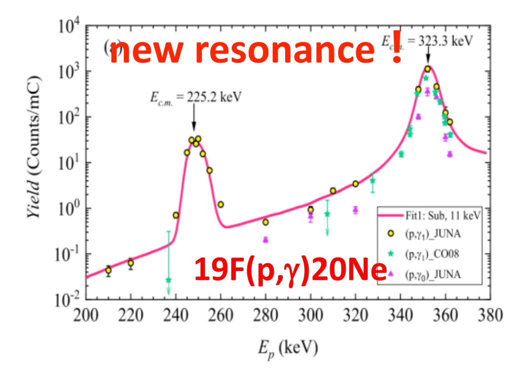
CIAE, G. Lian
Accelerator and Infrastructure



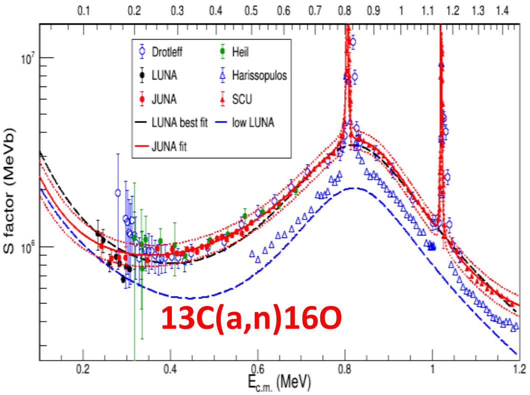
Zhang et al., PRL(2021)



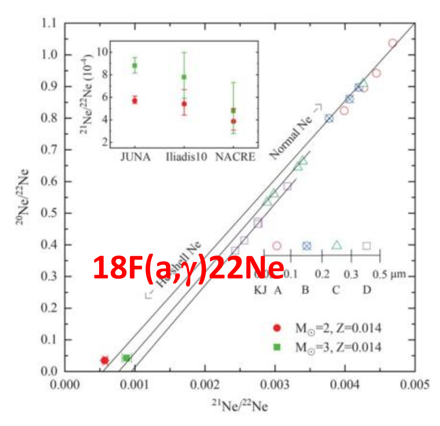
Su et al., Science Bulletin(2022)



Zhang et al,
Nature (2022)

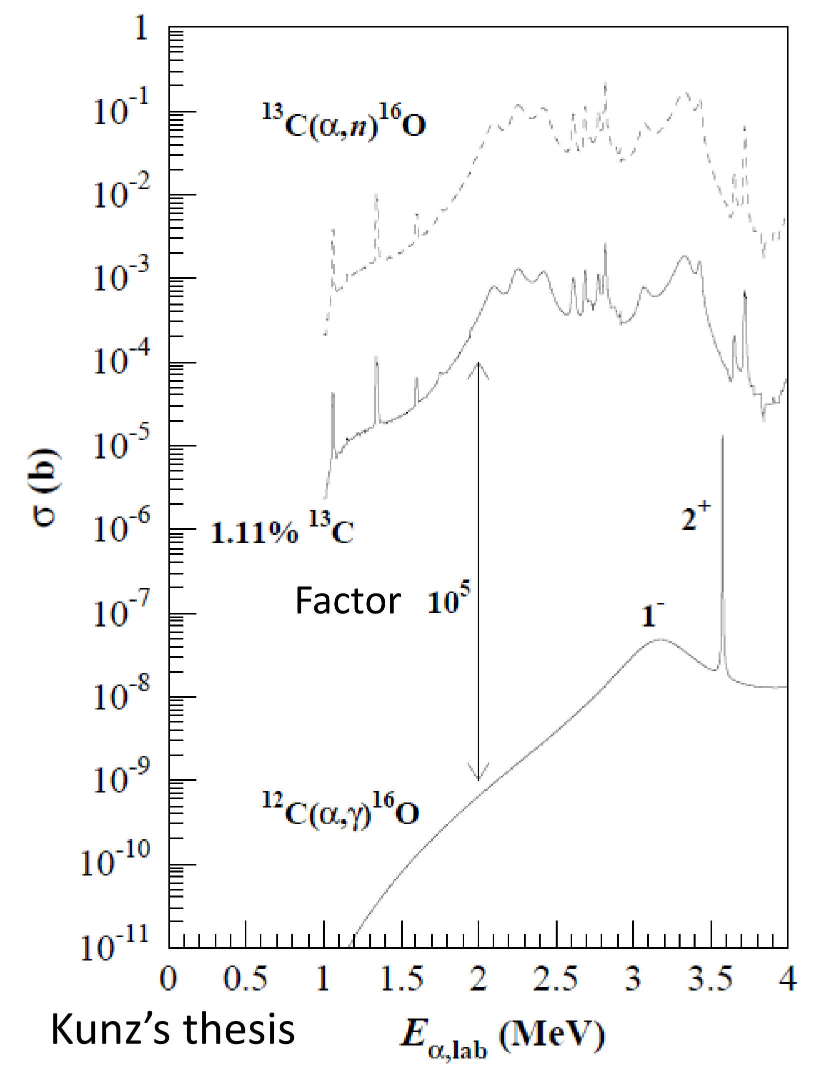


B.S. Gao et al,
PRL (2022)

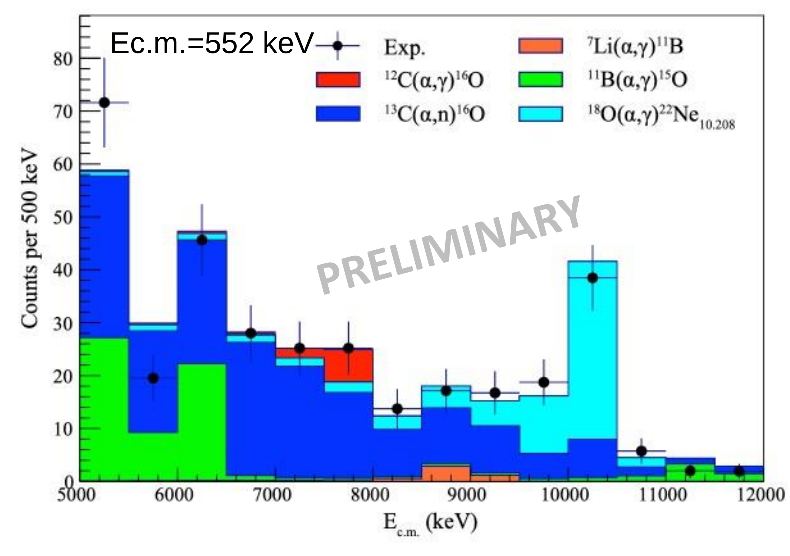


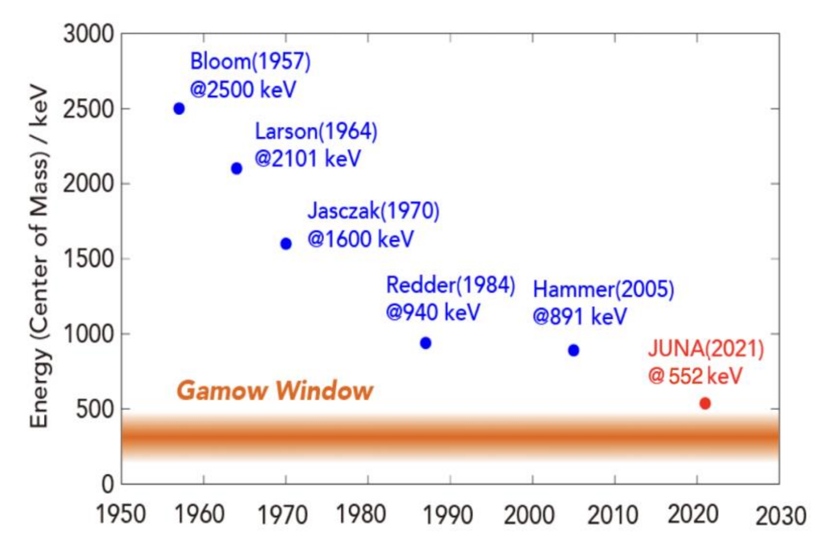
Wang et al,
PRL (2023)

$^{12}C(\alpha,\gamma)^{16}O$: better sensitivity



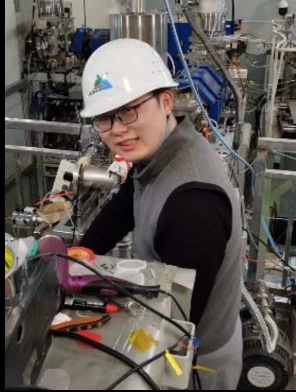
Kunz's thesis

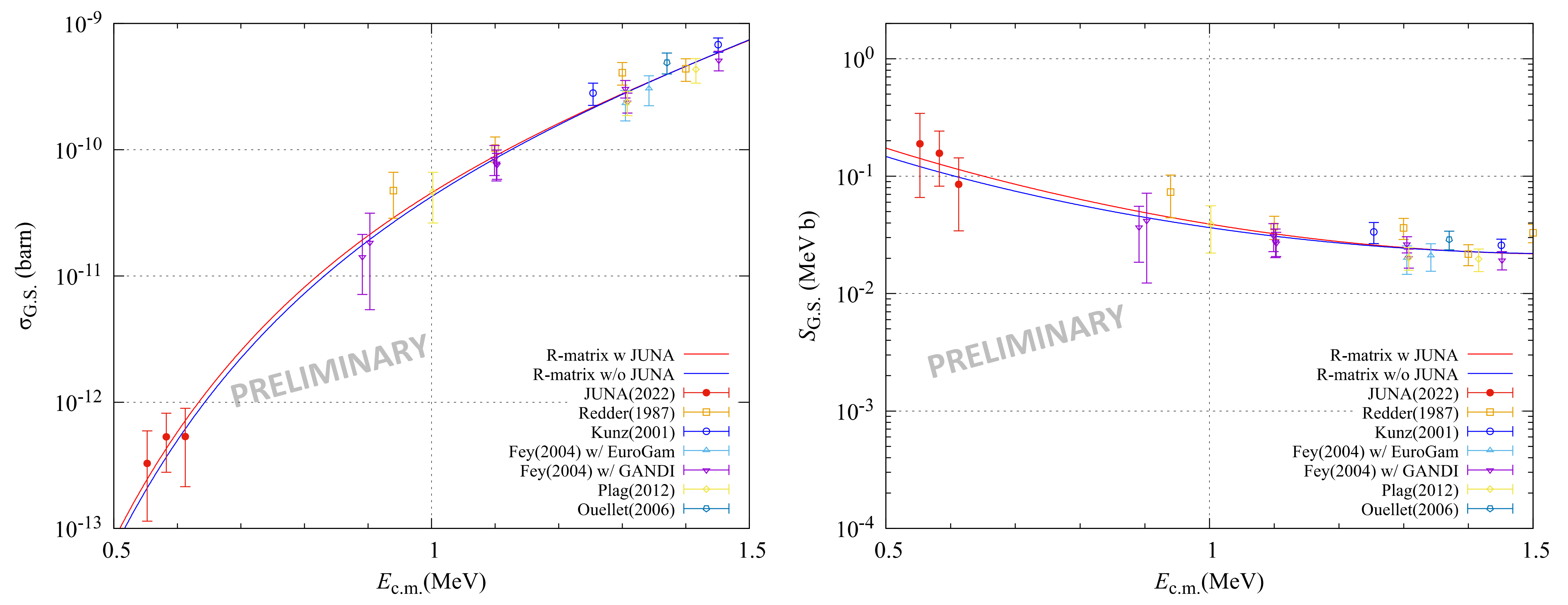




- ➢ FCVA implantation CTi thick targets with enriched ^{12}C sample
- ➢ BGO+LaBr3 (Lanthanum bromide) veto
- ➢ Background is dominated by $^{18}O(\alpha,\gamma)^{22}Ne$ contaminations
- ➢ Sensitivity: 10^{-11}b→ 10^{-12} b @$E_{c.m.}$ = 552 keV

A great progress towards stellar energies





Dr. Yangpin Shen, ypshen@ciae.ac.cn, fermi09@me.com

$^{12}C(\alpha,\gamma)^{16}O$ reaction rate

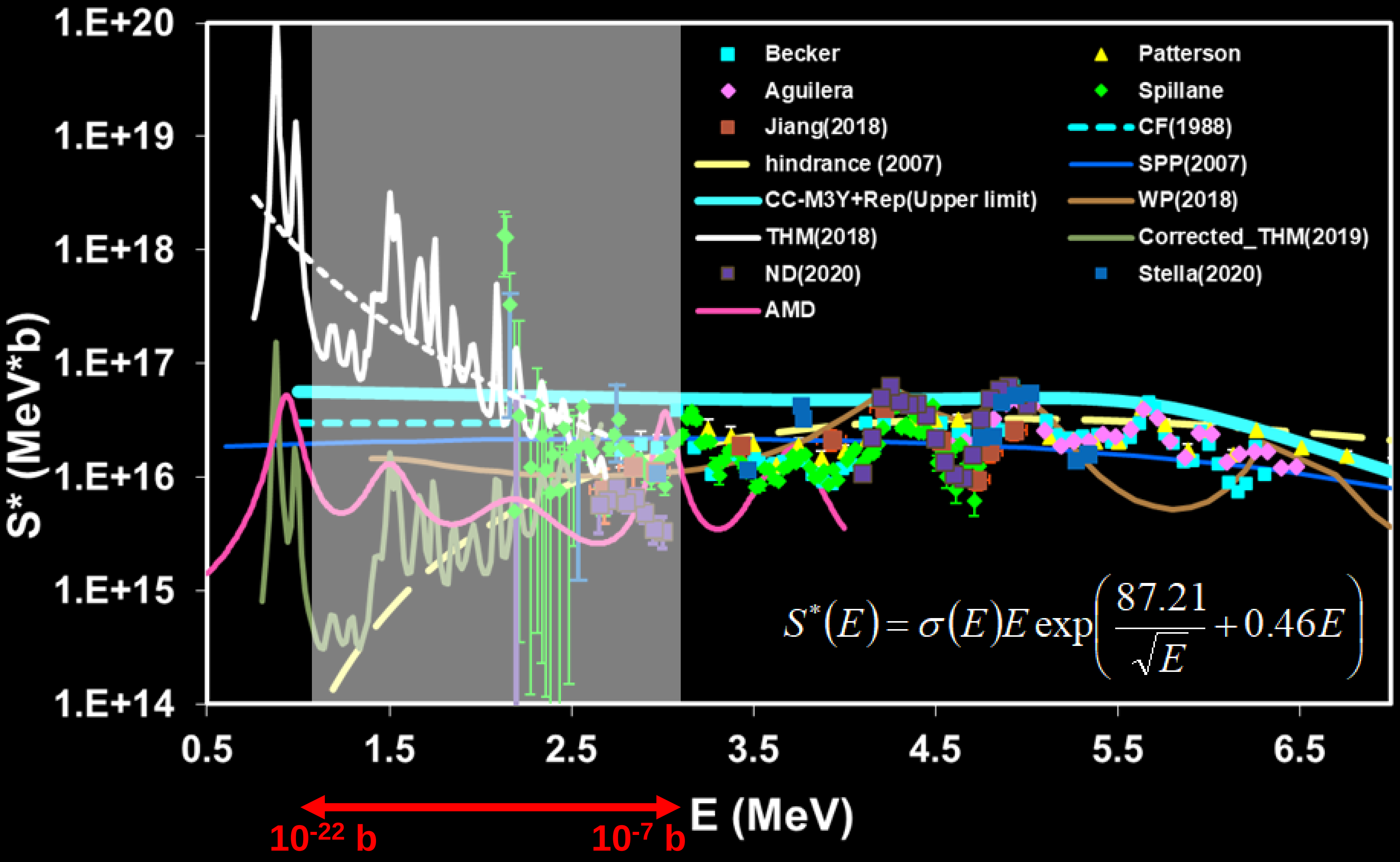
$$N_A\langle\sigma\upsilon\rangle = 6.9\cdot 10^8 \cdot T_9^{-2/3}\cdot S_{eff}\left[MeV-b\right]\cdot e^{-\frac{32.11}{T_9^{1/3}}} \quad \left[\frac{cm^3}{s}\right]$$

$$S_{eff}\approx 0.17\left[MeV-b\right]$$

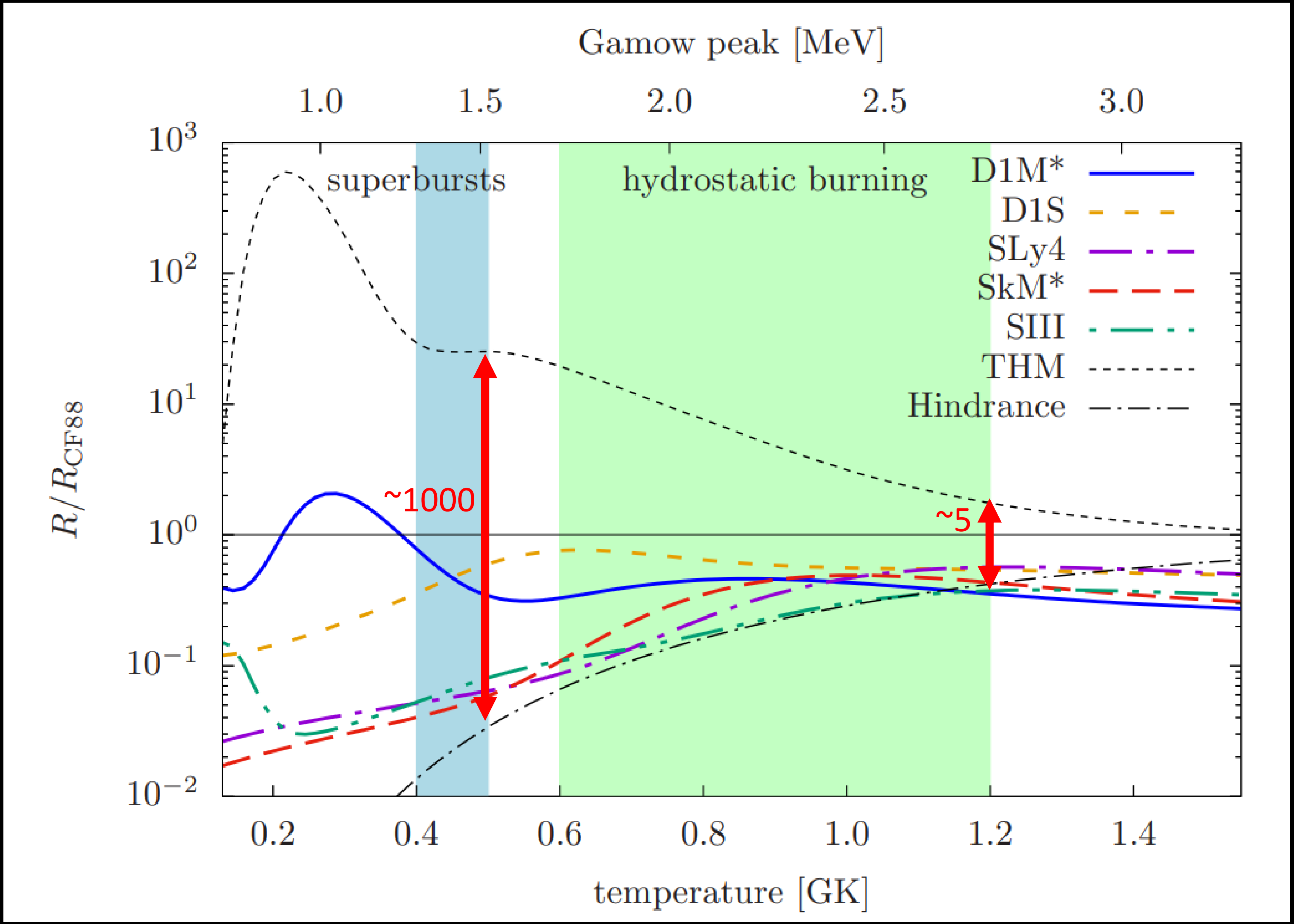
$$N_A\langle\sigma\upsilon\rangle \approx 1.2\cdot 10^8 \cdot T_9^{-2/3}\cdot e^{-\frac{32.11}{T_9^{1/3}}} \quad \left[\frac{cm^3}{s}\right]$$

Only very crude estimate!
E-T dependency needs to be considered!

^{12}C+^{12}C Fusion Reaction (1960-)

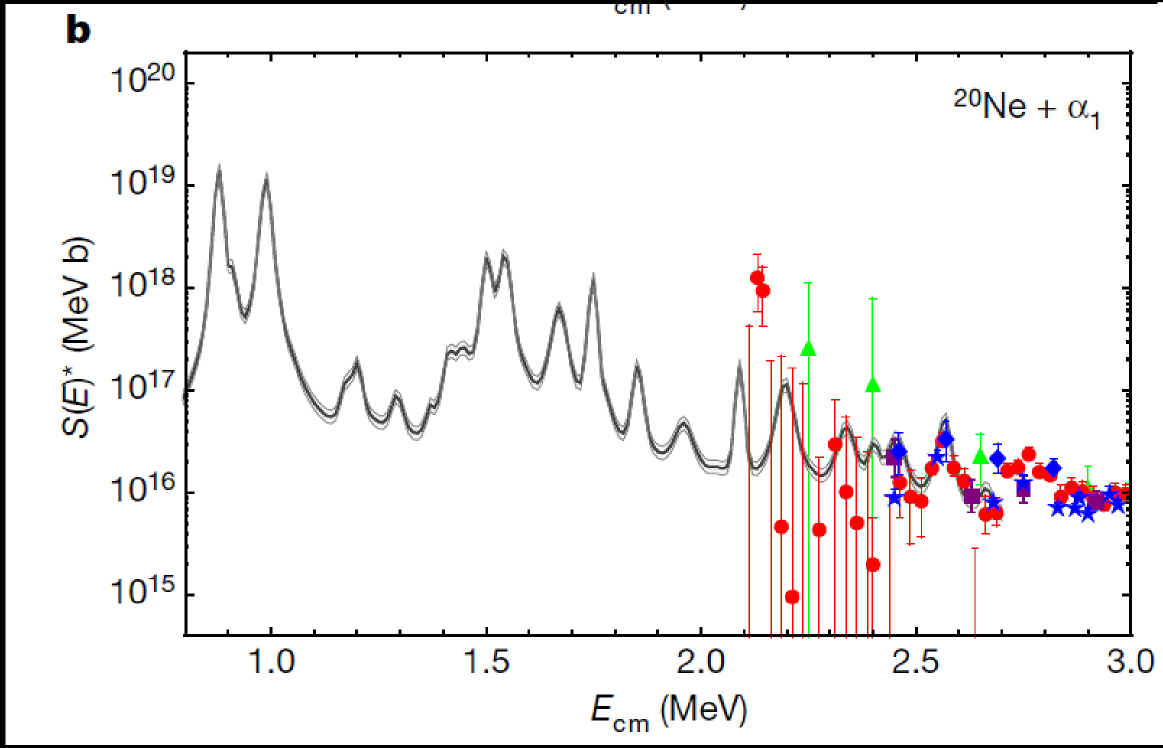


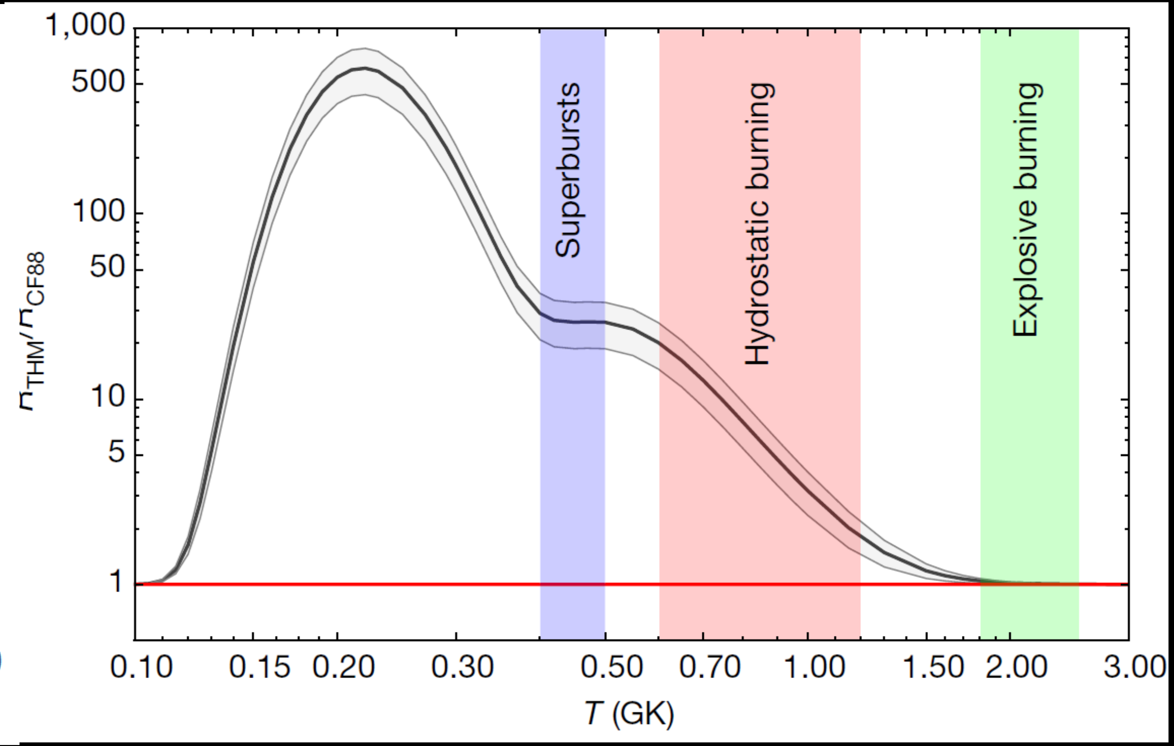
$$S^{*}(E)=\sigma(E)E\exp\left(\frac{87.21}{\sqrt{E}}+0.46E\right)$$



Taniguchi & Kimura, Phys. Lett. B 849 (2024) 138434

THM: Carbon burning can trigger superbursts

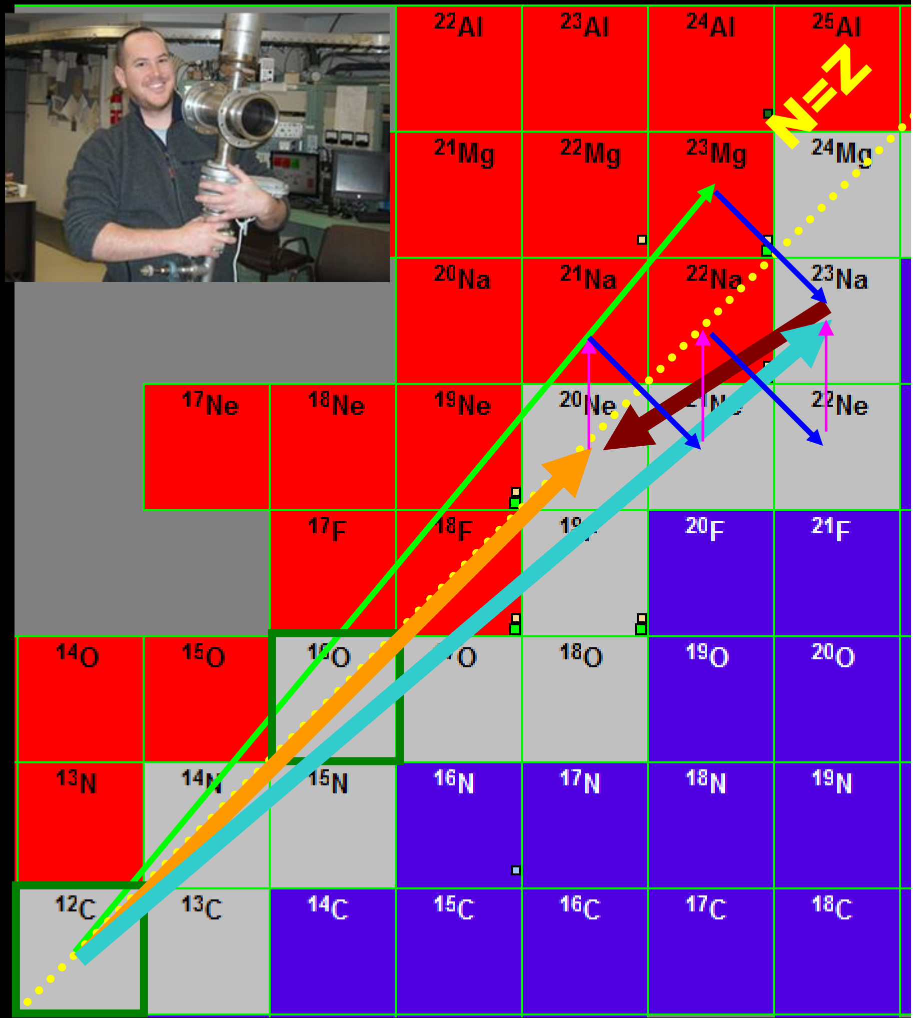


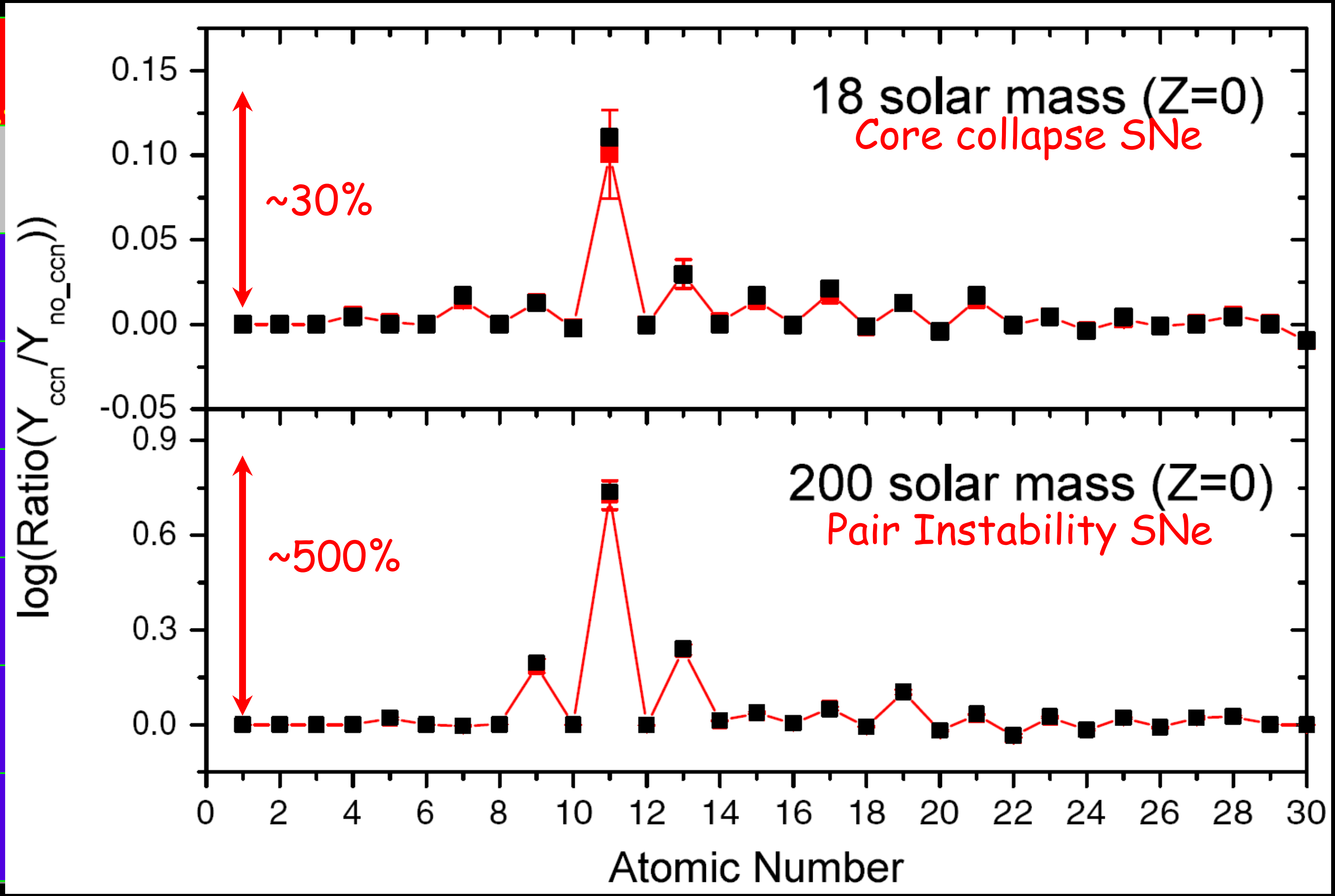


- Increase in the ^{12}C + ^{12}C fusion rate from resonances at astrophysical energies
- This change matches the observationally inferred ignition depths and can be translated into an ignition temperature below 0.5 GK, compatible with the calculated crust temperature

Tumino et al., Nature (2018)

$^{12}C(^{12}C,n)^{23}Mg$: neutron source in Pop-III stars





- ➢ Decay of ^{23}Mg changes the electron fraction and the even-odd pattern in the production yield
- ➢ Direct measurement was performed within Gamow window

Bucher et al., PRL (2015)

nature › articles › article

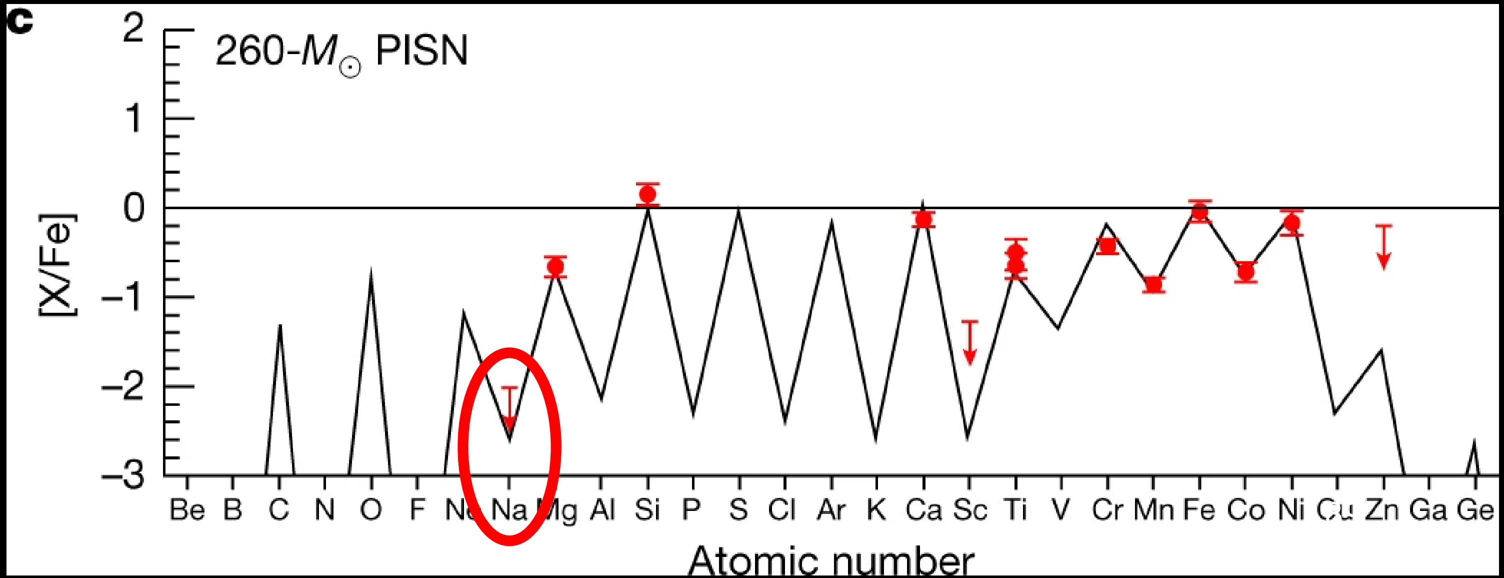
Article | Open access | Published: 07 June 2023

A metal-poor star with abundances from a pair-instability supernova

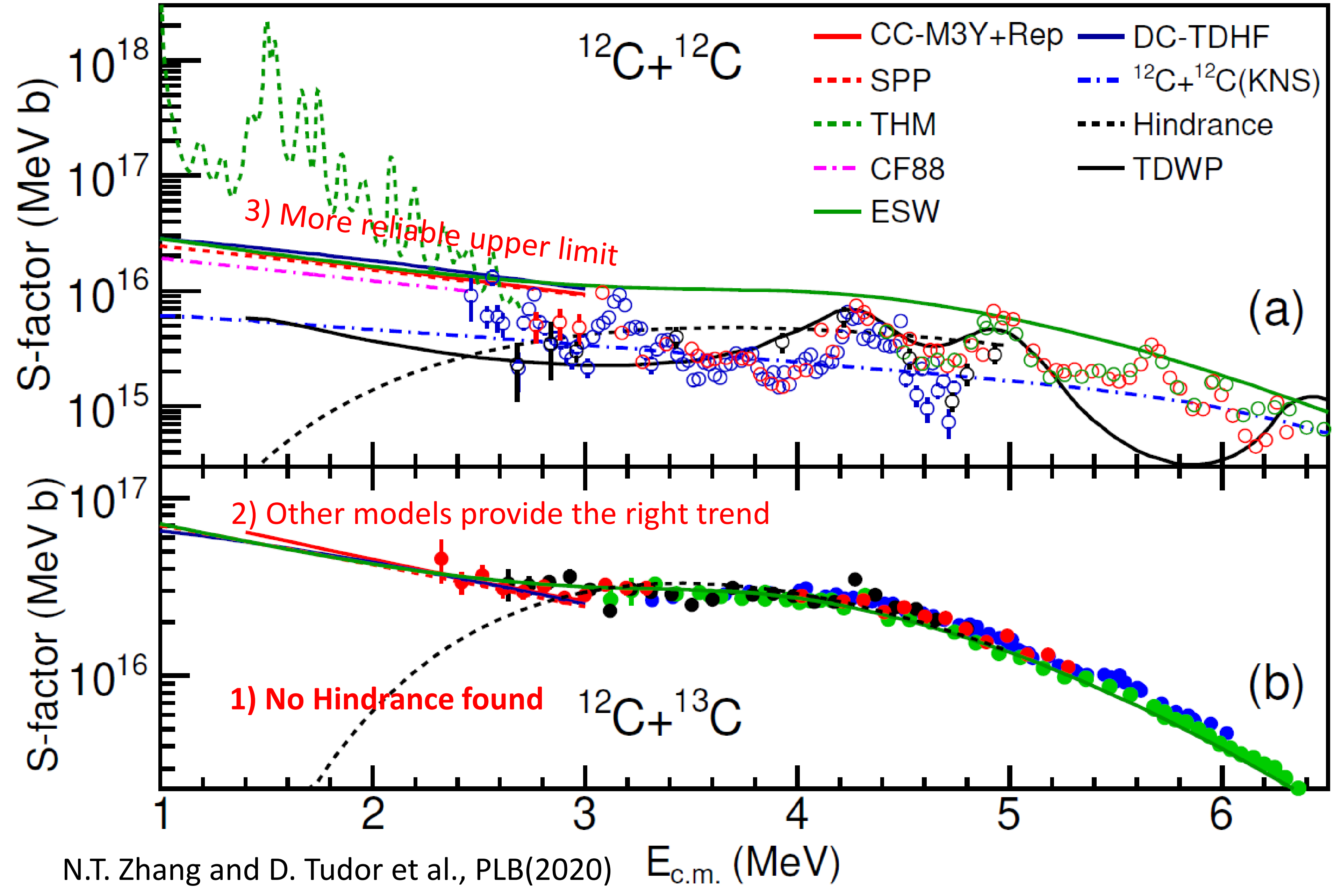
Qian-Fan Xing, Gang Zhao, Zheng-Wei Liu, Alexander Heger, Zhan-Wen Han, Wako Aoki, Yu-Qin Chen, Miho N. Ishigaki, Hai-Ning Li & Jing-Kun Zhao

Nature **618**, 712–715 (2023) | Cite this article

Our rate has been used in KEPLER to predict the production of Na in PISN



Test of hindrance and upper limit of $^{12}C+^{12}C$ based on systematics



3) More reliable upper limit

2) Other models provide the right trend

1) No Hindrance found

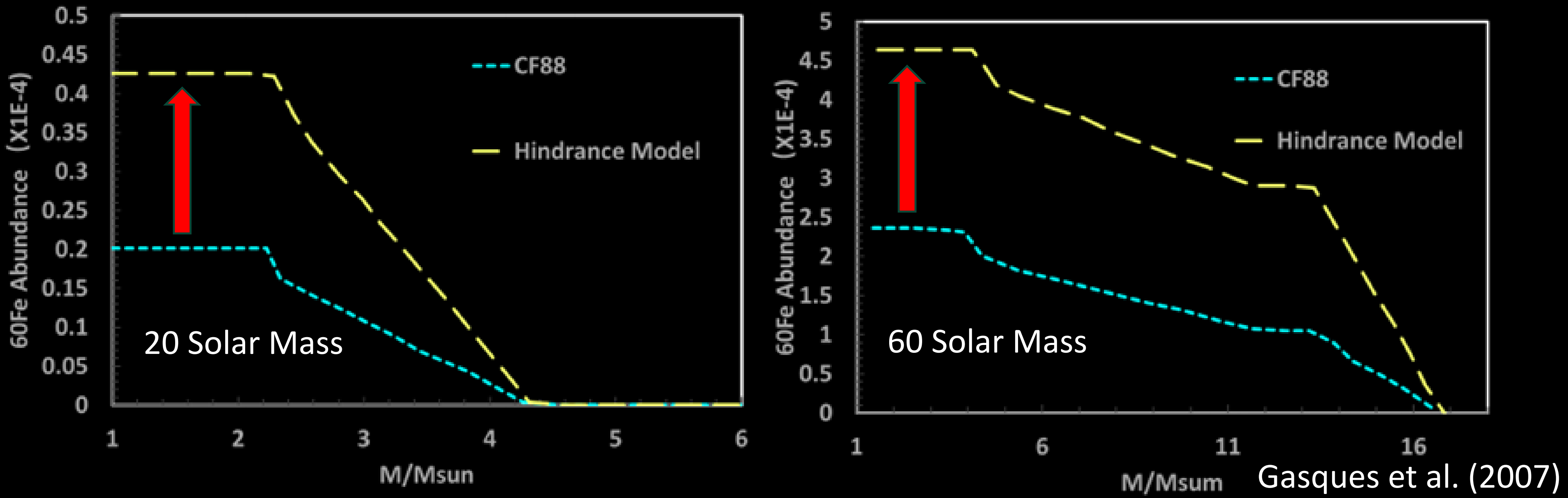
N.T. Zhang and D. Tudor et al., PLB(2020)

N.T.Zhang(IMP)

D. Tudor (IFIN-HH)

L. Trache (IFIN-HH)

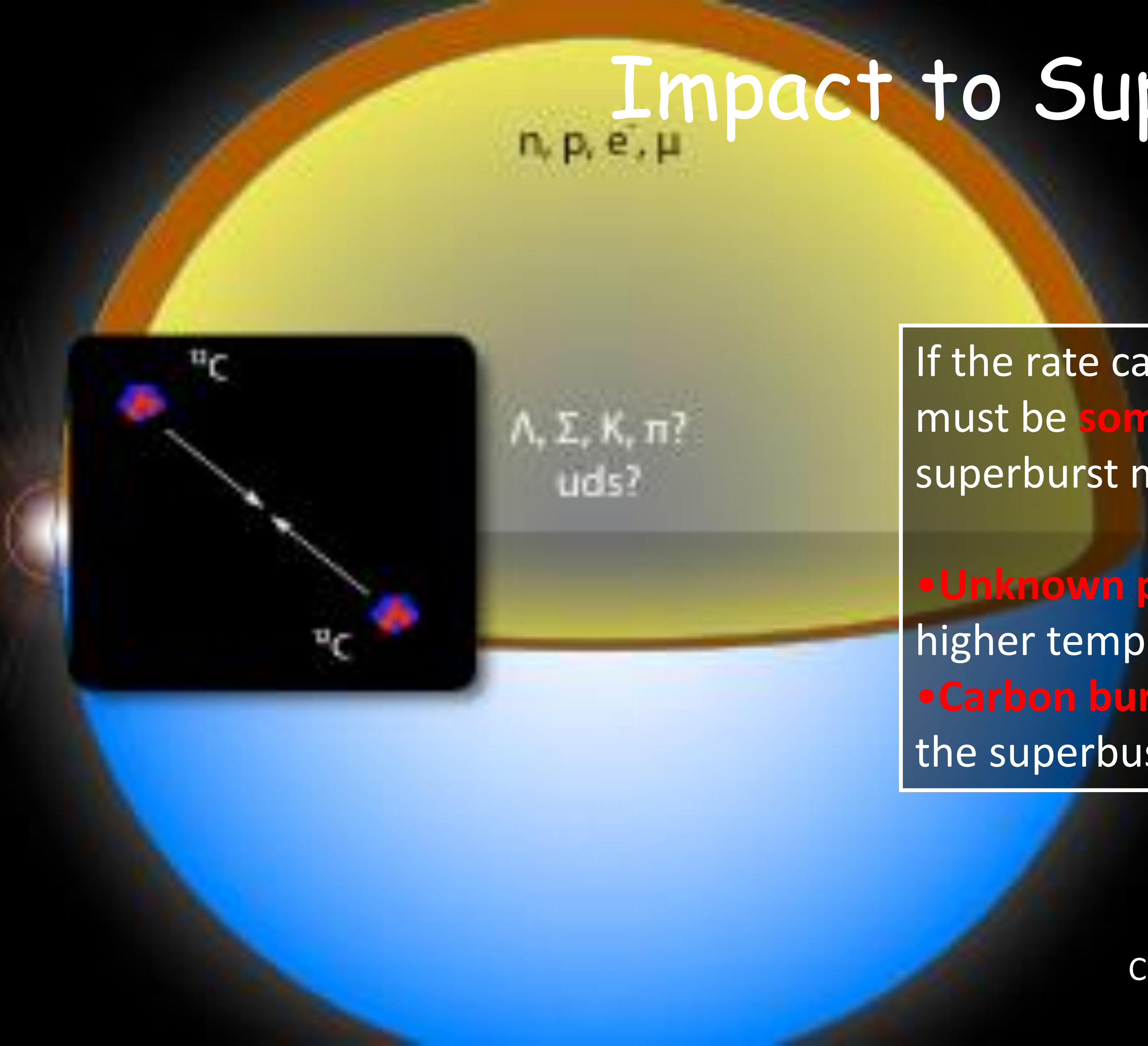
Impact on ^{60}Fe in massive stars



Gasques et al. (2007)

- ➢Enhanced ^{60}Fe production provided by the hindrance fusion rates would further enhance the already overpredicted ^{60}Fe abundance in the galaxy
- ➢Enlarge the discrepancy: [Perdition: 0.45 vs. Observation: 0.15± 0.04]
- ➢Our result rules out such a scenario

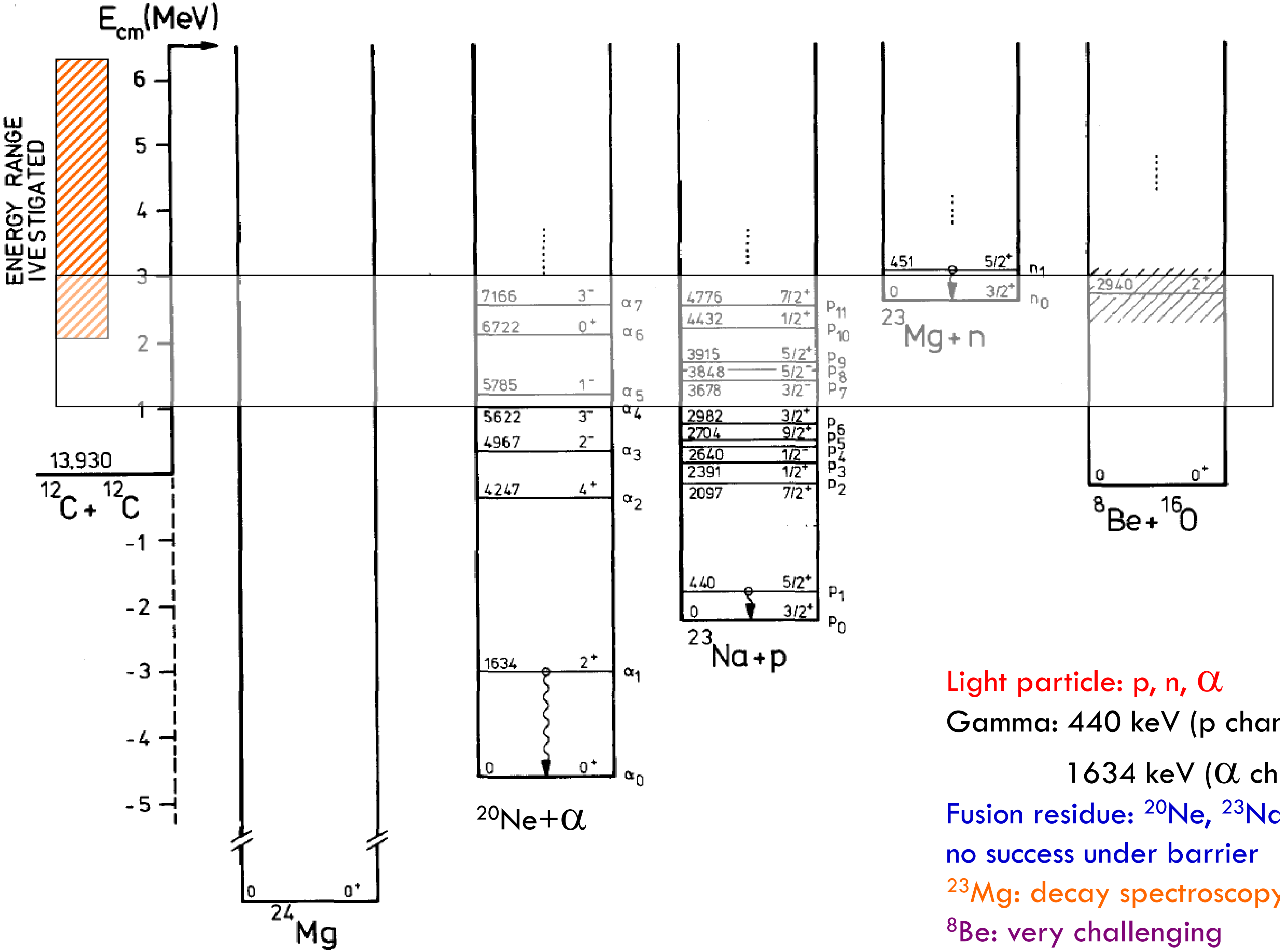
Impact to Superburst model



If the rate can not be as that high, there must be **some physics missing** in the superburst model.

- **Unknown process to heat** up the crust to higher temperature.
- **Carbon burning is not the one triggered** the superbust!

Communication with Ed. Brown(MSU)



Light particle: p, n, α

Gamma: 440 keV (p channel)

1634 keV (α channel)

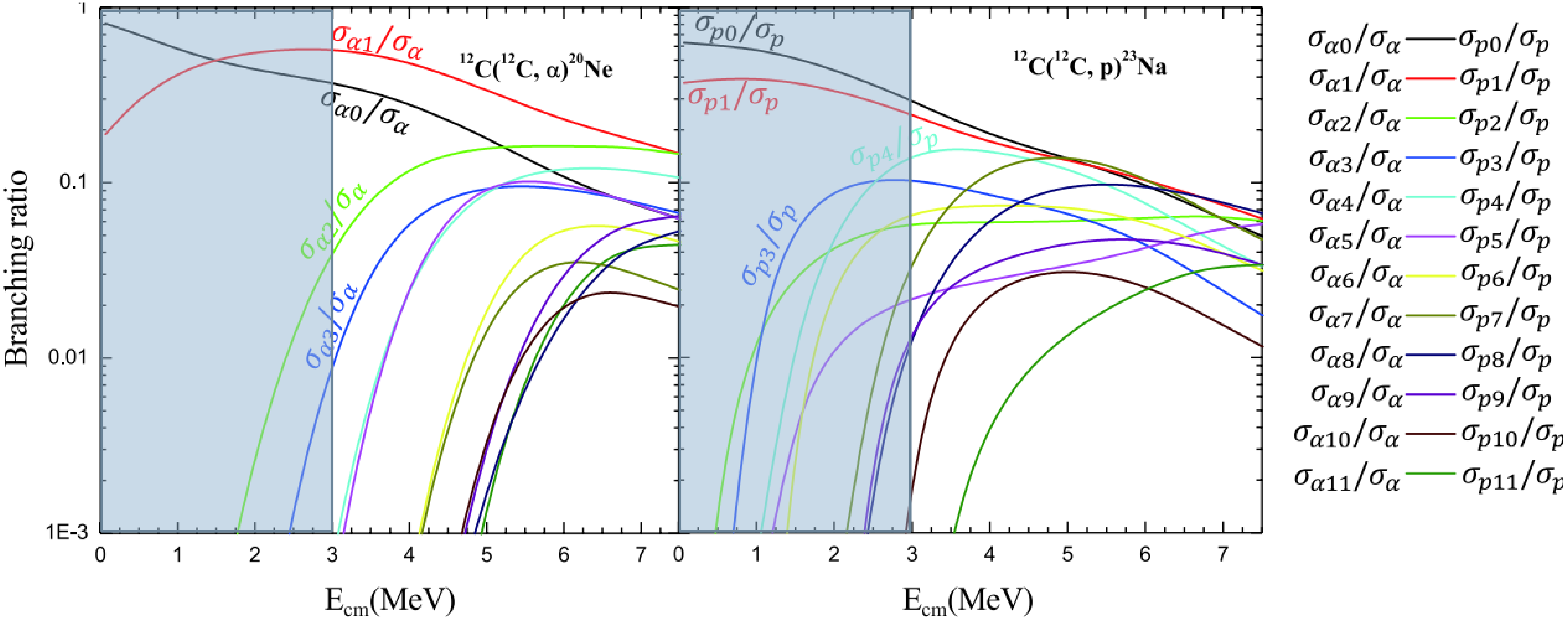
Fusion residue: ^{20}Ne, ^{23}Na …

no success under barrier

^{23}Mg: decay spectroscopy

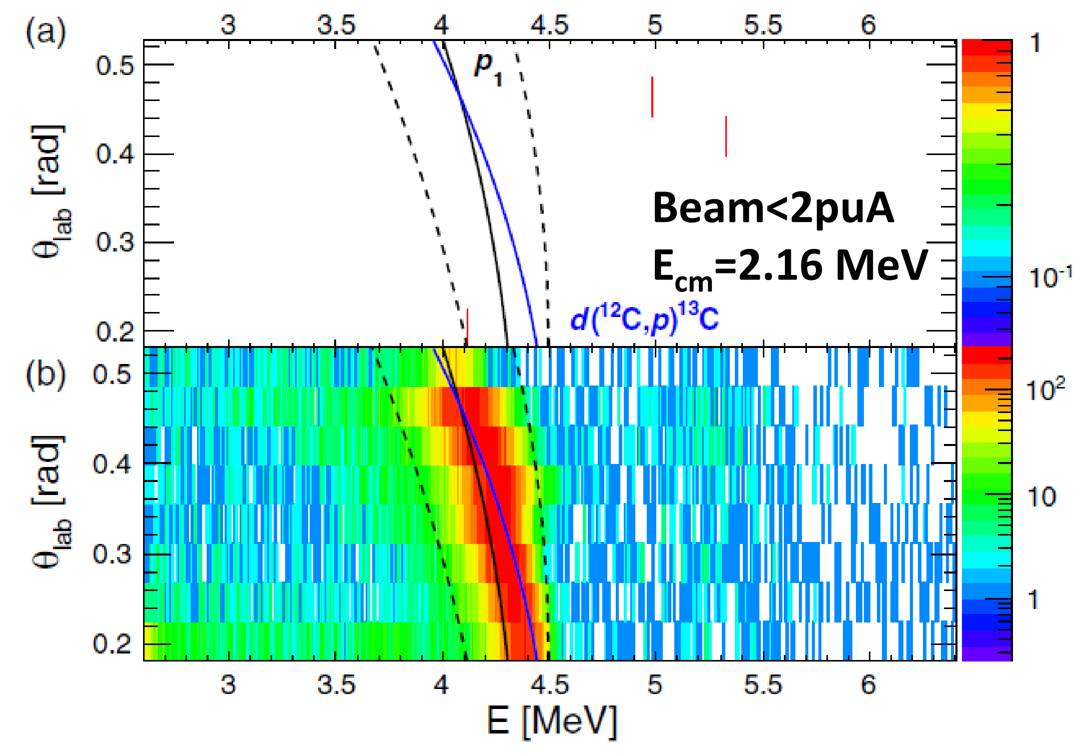
^{8}Be: very challenging

Why do some channels vanish at lower energies?

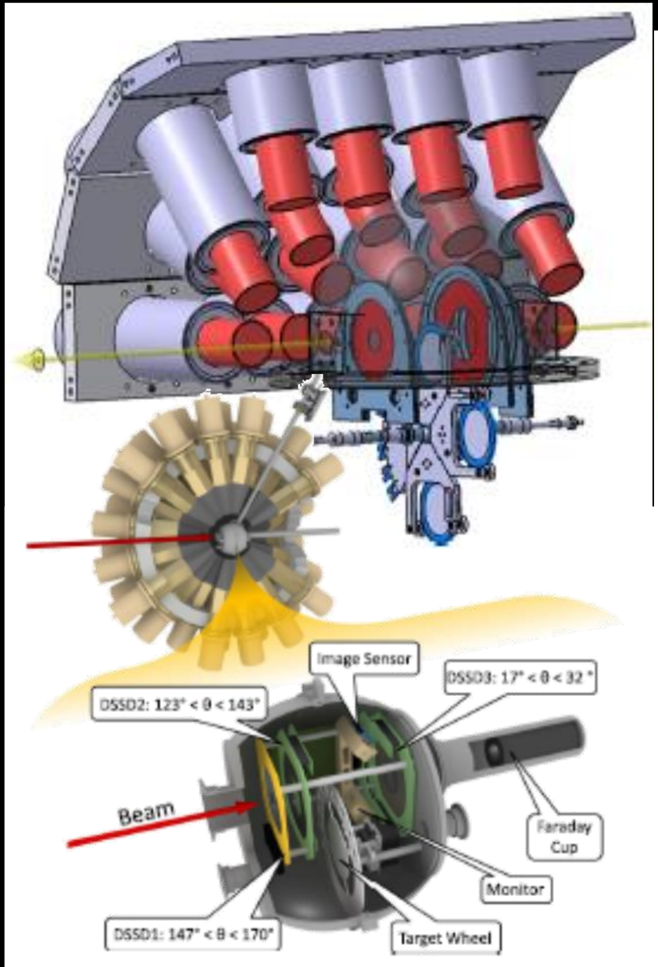


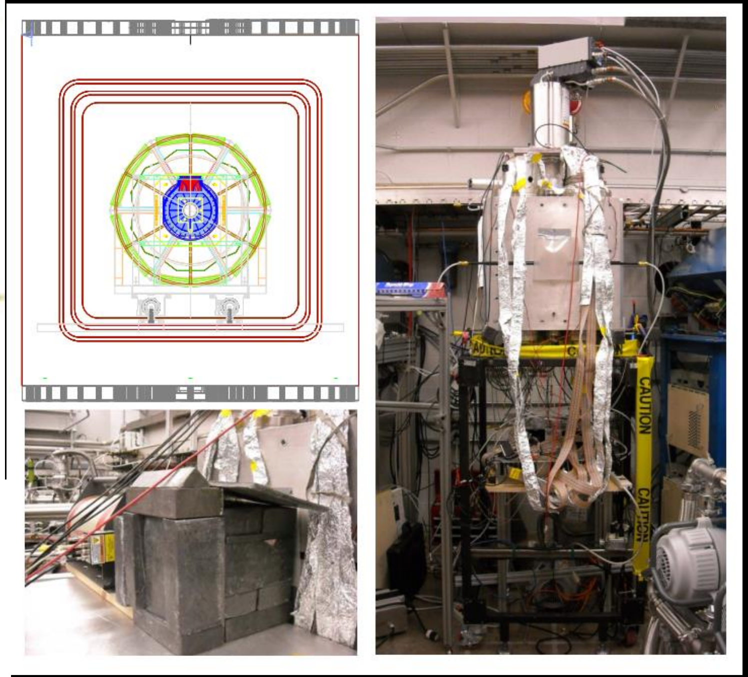
Y.J.Li, X.Fang+, Chin. Phys. C(2020), DOI: 10.1088/1674-1137/abae56

Particle-γ coincidence at lower stellar energies



Fruet+ PRL(2020)





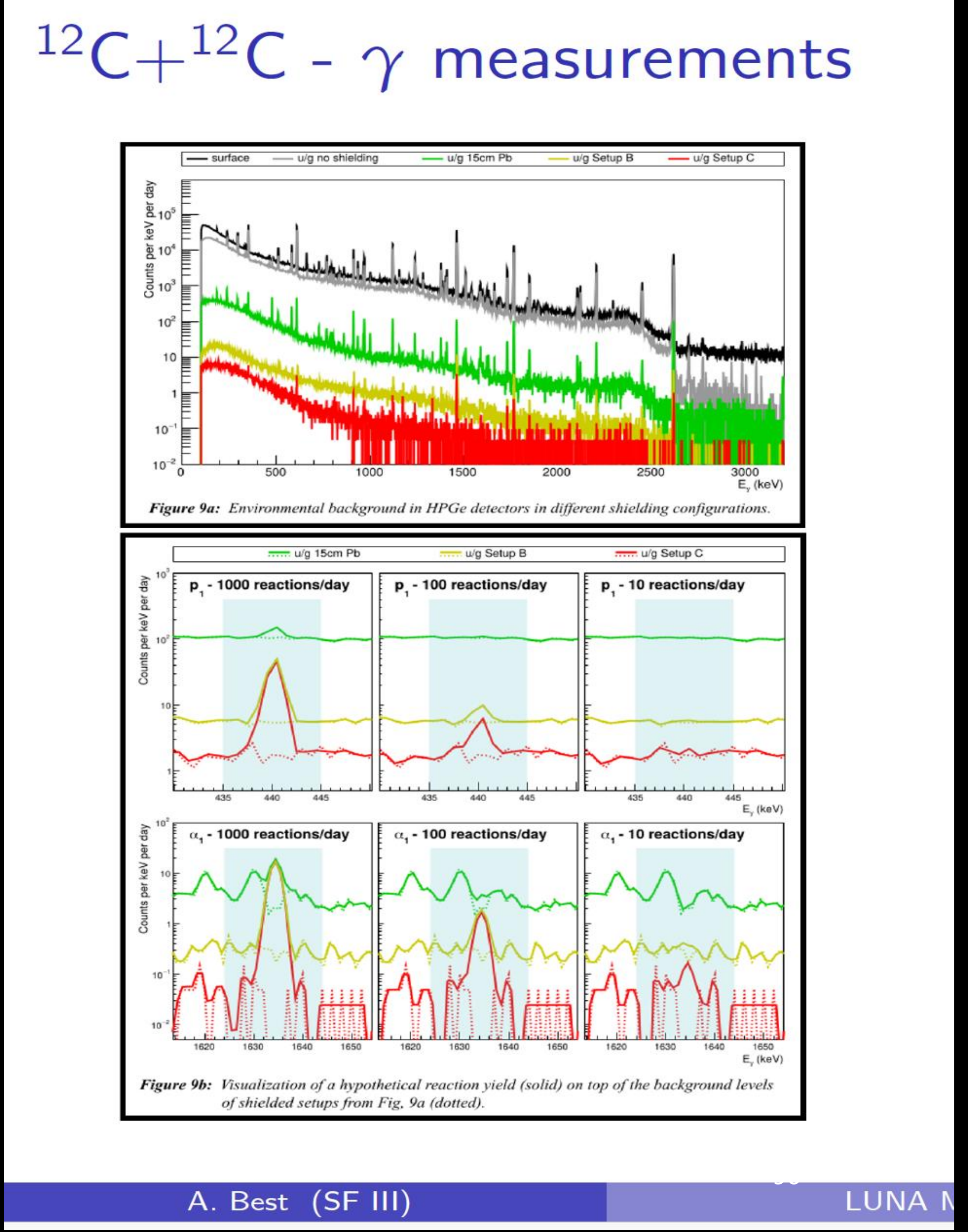
Beam<15puA

Jiang et al. (2012), Jiang et al. (2018)
Heine et al. (2018), Tan et al. (2021),
Fruet et al. (2021)

- ➢ Particle-γ coincidence technique pushed the measurement down to sub-nb level
- ➢ Only detect p_1 and α_1 channels

Carbon fusion project at LUNA-MV

Massive lead shield and radon flushing → push sensitivity to better than 100 reactions/day



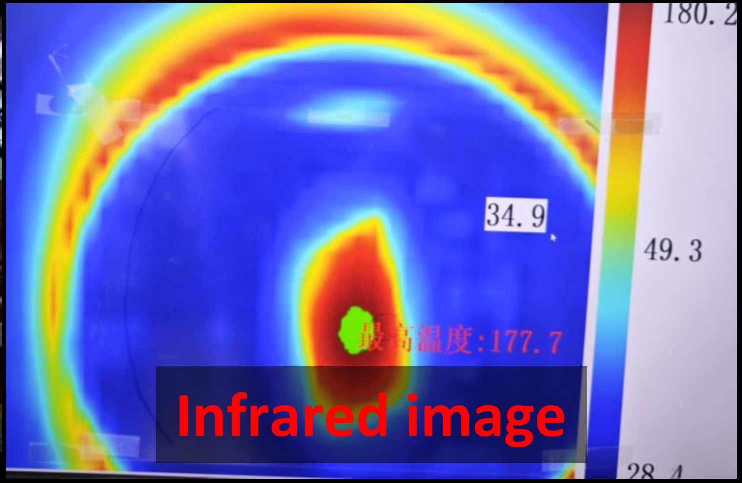
$^{12}C+^{12}C$ - γ measurements

Figure 9a: *Environmental background in HPGe detectors in different shielding configurations.*

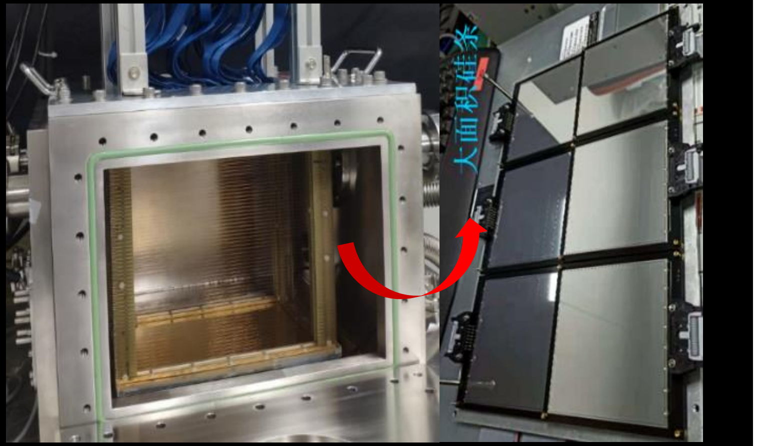
Figure 9b: *Visualization of a hypothetical reaction yield (solid) on top of the background levels of shielded setups from Fig. 9a (dotted).*

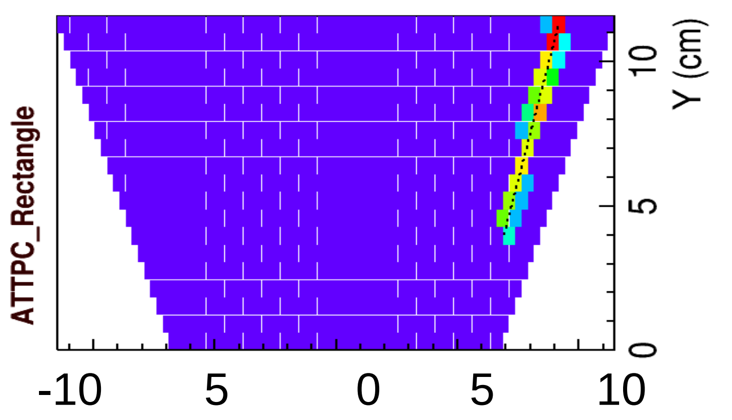
High Intensity+Time Projection Chamber


Low Energy Accelerator Facility (LEAF)



Infrared image





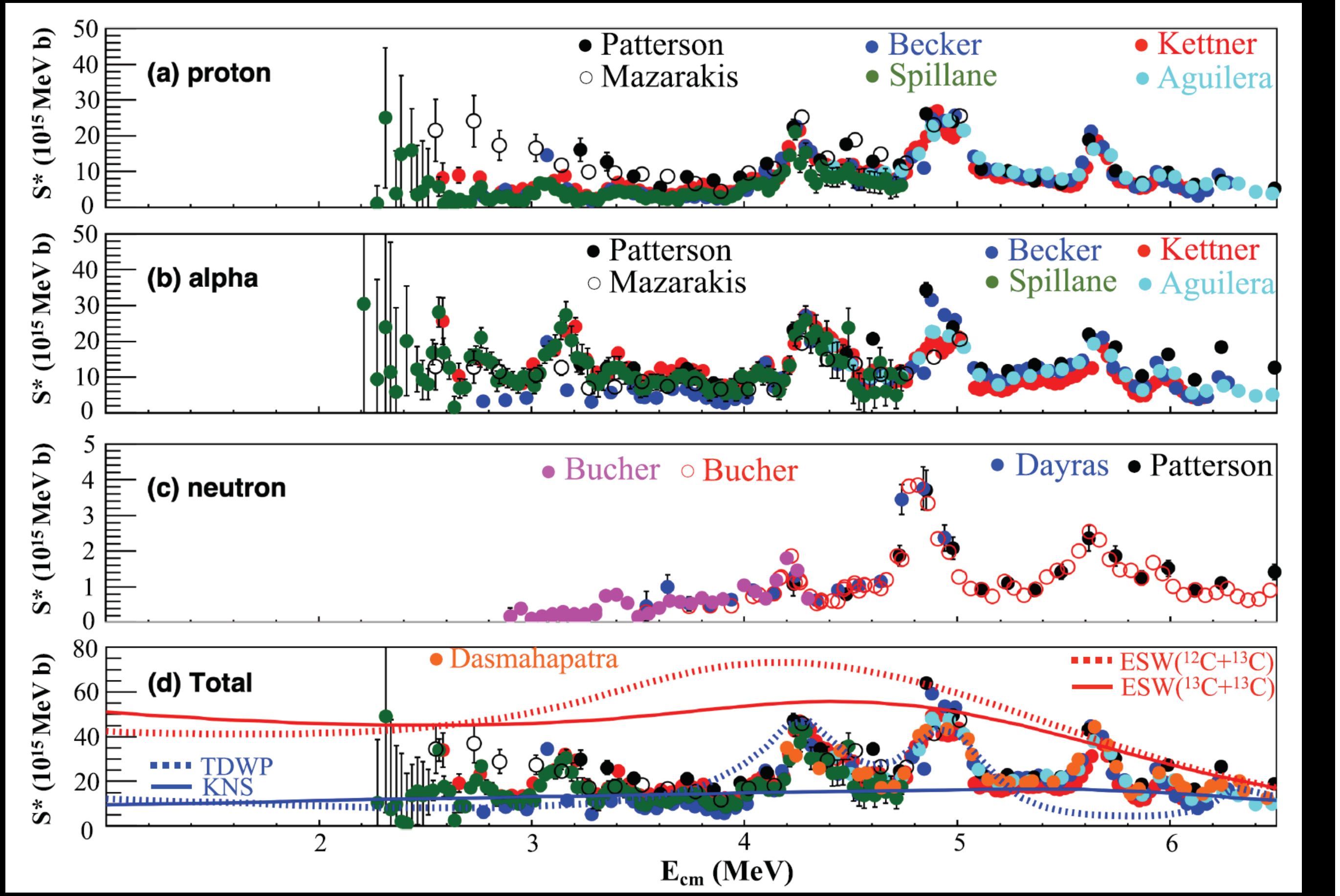

Max. 70puA on target

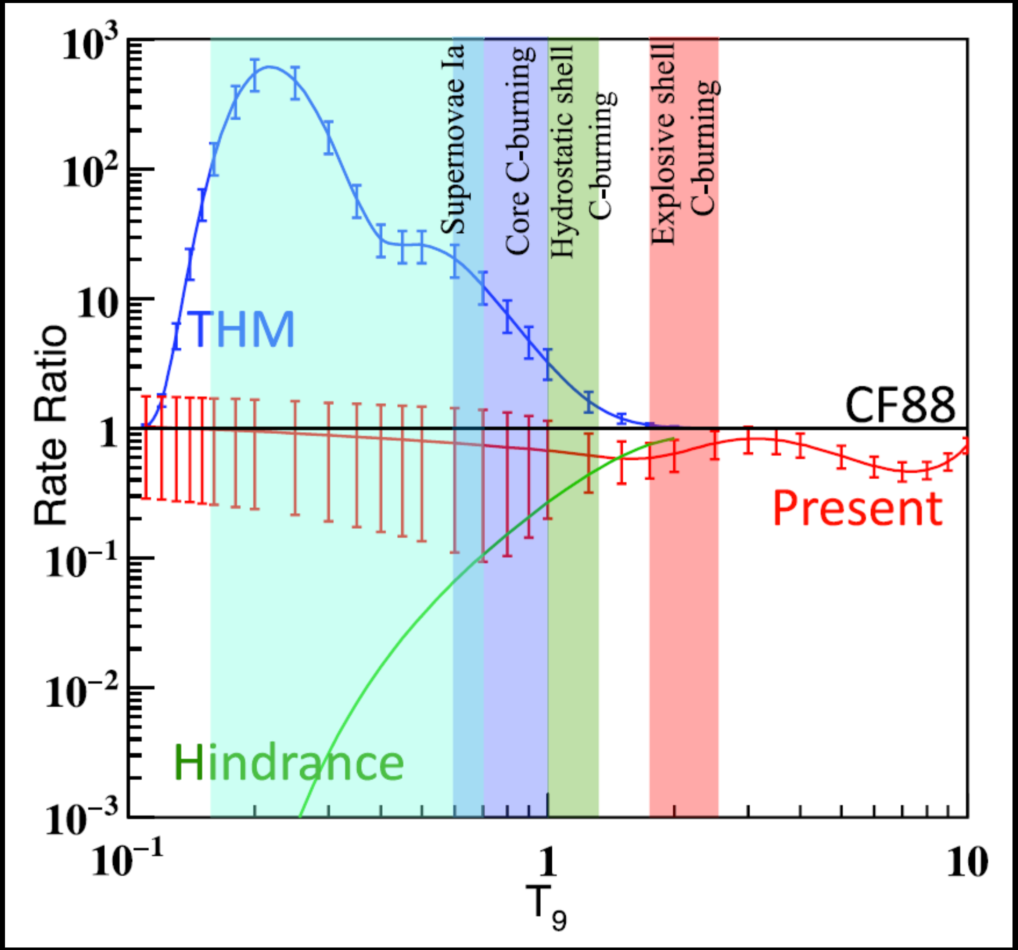
- LINAC: High Intensity beam up to 200 puA
- Time Projection Chamber: Ultra sensitive tracking detector
- Complementary to LUNA-MV and other experiments

Y.Z. Li's talk in OMEG pre-Symposium

New rate

Y.J.Li, X.Fang+ (2020), DOI: 10.1088/1674-1137/abae56





- Combining **the new upper limits with the empirical lower limit and the prediction of TDWP**, the 12C+12C S* factors are better constrained despite the unknown resonances within the unmeasured energy range.
- Revision is needed if there are currently unknown relatively strong resonances

EXPERIMENTAL AND THEORETICAL NUCLEAR ASTROPHYSICS; THE QUEST FOR THE ORIGIN OF THE ELEMENTS

Nobel lecture, 8 December, 1983

by

WILLIAM A. FOWLER

W. K. Kellogg Radiation Laboratory
California Institute of Technology, Pasadena, California 91125

Ad astra per aspera et per ludum

Nuclear astrophysics: the unfinished quest for the origin of the elements

Jordi José

Departament de Física i Enginyeria Nuclear, EUETIB, Universitat Politècnica de Catalunya, E-08036 Barcelona, Spain; Institut d'Estudis Espacials de Catalunya, E-08034 Barcelona, Spain

E-mail: `jordi.jose@upc.edu`

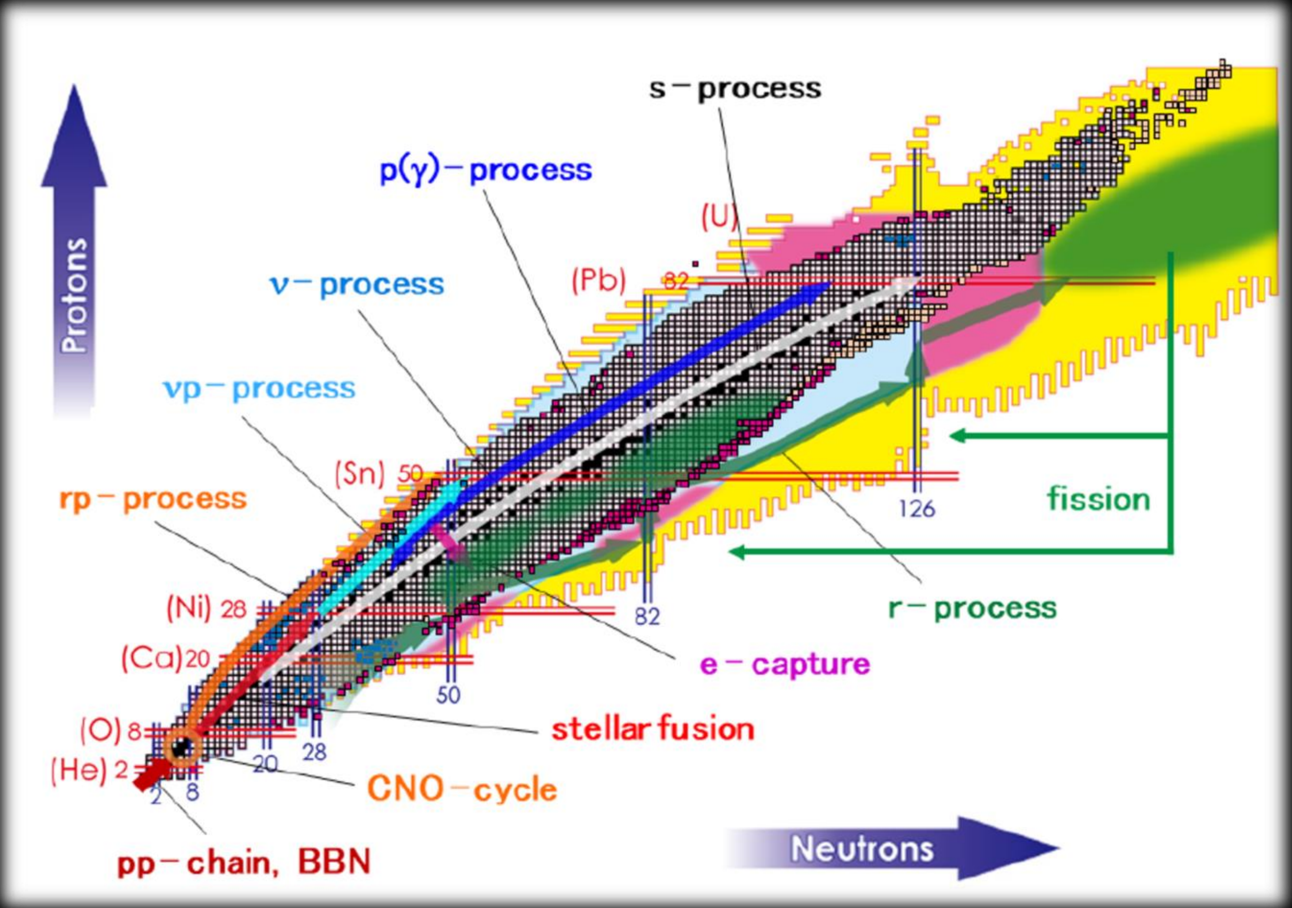
Christian Iliadis

Department of Physics & Astronomy, University of North Carolina, Chapel Hill, North Carolina, 27599, USA; Triangle Universities Nuclear Laboratory, Durham, North Carolina 27708, USA

E-mail: `iliadis@unc.edu`

(i) Why do predictions of helioseismology disagree with those of the standard solar model?
(ii) What is the solution to the lithium problem in Big Bang nucleosynthesis?
(iii) What do the observed light-nuclide and s-process abundances tell us about convection and dredge-up in massive stars and AGB stars?
(iv) What are the production sites of the γ-ray emitting radioisotopes ^{26}Al, ^{44}Ti and ^{60}Fe?
(v) What is the origin of about 30 rare and neutron deficient nuclides beyond the iron peak (p-nuclides)?
(vi) What causes core-collapse supernovae to explode?
(vii) What is the extend of neutrino-induced nucleosynthesis (ν-process)?
(viii) What is the extend of the nucleosynthesis in proton-rich outflows in the early ejecta of core-collapse supernovae (νp-process)?
(ix) What are the sites of the r-process?
(x) What causes the discrepancy between models and observations regarding the mass ejected during classical nova outbursts?
(xi) Which are the physical mechanisms driving convective mixing in novae?
(xii) What are the progenitors of type Ia supernovae?
(xiii) What is the nucleosynthesis endpoint in type I X-ray bursts? Is there any matter ejected from those systems?
(xiv) What is the impact of stellar mergers on Galactic chemical abundances?
(xv) What are the production and acceleration sites of Galactic cosmic rays?

Summary



Interdisciplinary feature of nuclear astrophysics demands **the close collaborations among astronomers, astrophysicists, and nuclear physicists and among the facilities**. As we demonstrated in the paper, **NO single facility or model will answer all the quests in our field**. How to be successful in nuclear astrophysics? Here are the advises from Willy Fowler: **seek for truth, work hard, and help people**

Progress in nuclear astrophysics of east and southeast Asia
Aziz et al. AAPPS Bulletin (2021) 31:18, https://doi.org/10.1007/s43673-021-00018-z