STRUCTURE OF LIGHTEST NUCLEI IN THE VISIBLE UNIVERSE ON THE LIGHT FRONT

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(BLFQ Collaboration)

November 27, 2024

Overview

Motivation

Basis Light-Front Quantization (BLFQ)

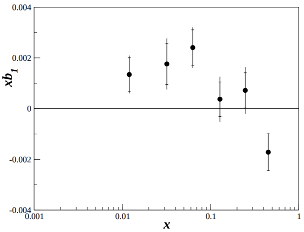
Extended Light-Front Holographic QCD approach

EM FFs and Structure Functions

Conclusion

Motivation

- Universe's lightest nuclei
- Deuteron possesses tensor structure:
  - Absent for spin-0 or 1/2 systems
  - Gluon Transversity
  - Proposals to study the structure of deuteron: JLab (approved), Fermilab (proposal in 2022), EICs...
  - Largely unexplored field yet : can open a new field of spin physics



PRL 95, 242001 (2005)

PR12-13-011

PR12-13-011

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-40
(Update to PR12-11-110)

FERMILAB-PUB-22-381-V

FERMILAB-PUB-22-381-V

The Transverse Structure of the Deuteron with Drell-Yan

The SpinQuest Collaboration[a]

A Letter of Intent to Jefferson Lab PAC 42

Search for Exotic Gluonic States in the Nucleus

ELSEVIER

Progress in Particle and Nuclear Physics
Volume 119, July 2021, 103858

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

Richness of Deuteron's Spin Structure

Hadron \ Quark	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5$ / σ^{i+})	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					[$h_1^\perp$]
L			g_{1L}		[$h_{1L}^\perp$]	
T		$f_{1T}^\perp$	g_{1T}		[h_1], [$h_{1T}^\perp$]	
LL	f_{1LL}					[$h_{1LL}^\perp$]
LT	f_{1LT}			g_{1LT}		[h_{1LT}], [$h_{1LT}^\perp$]
TT	f_{1TT}			g_{1TT}		[h_{1TT}], [$h_{1TT}^\perp$]

Twist-2 TMDs.

Kumano et al. 2406.01180

Hadron \ Quark	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5$ / σ^{i+})	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			g_{1L} (g_1)			
T					[h_1]	
LL	f_{1LL} (b_1)					
LT						*1 [h_{1LT}]
TT						

Twist-2 PDFs.

Recent Interest for Deuteron's GFFs

Deuteron gravitational form factors: Exchange currents

Fangcheng He and Ismail Zahed

Show more

Phys. Rev. C **110**, 014312 – **Published 8 July, 2024**

Gravitational form factors of light nuclei: Impulse approximation

Fangcheng He and Ismail Zahed

Show more

Phys. Rev. C **109**, 045209 – **Published 22 April, 2024**

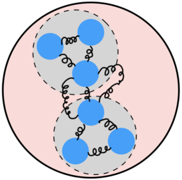
Gravitational form factors of nuclei in the Skyrme model

Alberto García Martín-Caro, Miguel Huidobro, and Yoshitaka Hatta

Show more

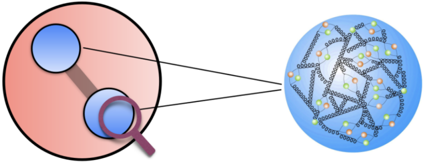
Phys. Rev. D **108**, 034014 – **Published 14 August, 2023**

How do we visualize the deuteron?



- A six-quark system
- Direct access to the Parton level
- Ability to achieve both quark and gluon distributions
- Hidden color

BLFQ Approach

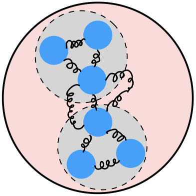


- A two-nucleon system
- Two-step approach
- Simple modeling

Light Front Holographic QCD

Basis Light-Front Quantization (BLFQ)

- Non-perturbative approach based on the Hamiltonian formalism : $P^+P^- |\Psi\rangle = M^2 |\Psi\rangle$
  - To solve relativistic many-body bound state problems.
  - Successfully implemented to investigate the structures of various baryons and mesons.
  - Motivation: To extend the approach to investigate light nuclei.



- P^+ : longitudinal momentum of the targeted nuclei
  P^- : LF Hamiltonian

Fock state expansion of the deuteron state

$$|\Psi\rangle_D = \psi_{6q} |qqq\ qqq\rangle + \psi_{6q+1g} |qqq\ qqq\ g\rangle \quad + \psi_{6q+q\bar{q}} |qqq\ qqq\ q\bar{q}\rangle + \ ...$$

- $\psi_{...}$: LFWFs associated with the Fock components $|...\rangle$.

[1] J.P.Vary, H. Honkanen, J. Li, P. Maris, S.J.Brodsky, A. Harindranath, G.F. de Teramond, PRC 81, 035205 (2010).

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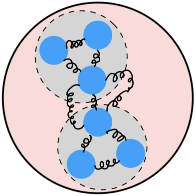

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Basis function

- 2D-HO basis in transverse direction, $\Phi_{nm}(\mathbf{p}_\perp)$
- Plane wave in longitudinal direction

- Parton's basis state is identified by $|\alpha_i\rangle = |k_i, n_i, m_i, \lambda_i\rangle$
- Many-body basis states are identified as the direct product of the Fock-particle basis states $|\alpha\rangle = \otimes\,|\alpha_i\rangle$.

Fock space truncation:
$|\Psi\rangle_D = \psi_{6q}\,|qqq\ qqq\rangle + \psi_{6q1g}\,|qqq\ qqq\ g\rangle$

Basis truncation:
$\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$
$\sum_i k_i = K$, $x = \frac{k_i}{K}$

Hamiltonian P^-_{QCD}

- K.E. terms of quarks and gluons
- quark-gluon vertex interaction
- instantaneous gluon interaction

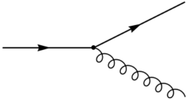
[1] J.P.Vary, H. Honkanen, J. Li, P. Maris, S.J.Brodsky, A. Harindranath, G.F. de Téramond, PRC 81, 035205 (2010).

Light-front QCD Hamiltonian

[Brodsky et al, 1998]

$$P_{-,LFQCD} = \frac{1}{2}\int d^3x\, \bar{\psi}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\,\psi - \frac{1}{2}\int d^3x\, A^i_a (i\partial^\perp)^2 A^i_a$$

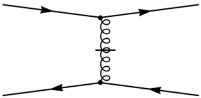
$$+g\int d^3x\, \bar{\psi}\gamma_\mu A^\mu\, \psi$$



$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+}\gamma_\nu A^\nu \psi$$

$$-ig^2\int d^3x\, f^{abc}\bar{\psi}\gamma^+ T^c \psi \frac{1}{(i\partial^+)^2}(i\partial^+ A^\mu_a A_{\mu b})$$

$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi$$



$$+ig\int d^3x\, f^{abc} i\partial^\mu A^{\nu a} A^b_\mu A^c_\nu$$

$$-\frac{1}{2}g^2\int d^3x\, f^{abc}\, f^{ade}\; i\partial^+ A^\mu_b A_{\mu c} \frac{1}{(i\partial^+)^2}(i\partial^+ A^+_d A_{\nu e})$$

$$+\frac{1}{4}g^2\int d^3x\, f^{abc}\, f^{ade}\; A^\mu_b A^\nu_c A_{\mu d} A_{\nu e}.$$

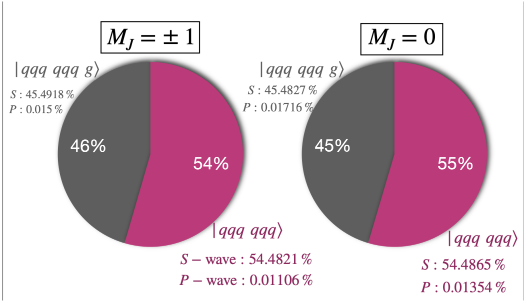
6

[1] S.J. Brodsky, H.C. Pauli, S.S. Pinsky, Phys. Rep. 301, 299-486 (1998)

Parameters and Decomposition of Spin States

m_u	m_d	b	b_inst	g
1.0 GeV	0.95 GeV	0.32 GeV	5 GeV	3.9

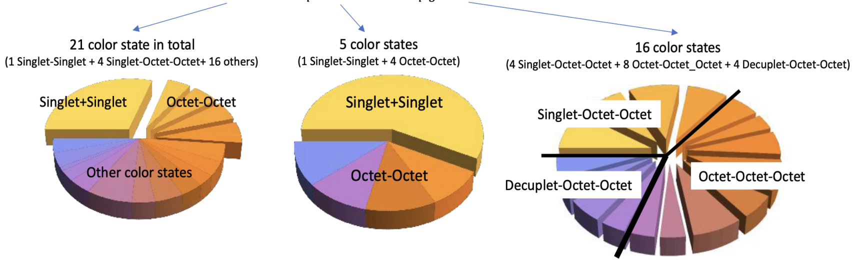
$$N_{max} = 8; K = 7$$



- Number of Color singlet states in $|qqq\ qqq\rangle$: 5
- Number of Color singlet states in $|qqq\ qqq\ g\rangle$: 16

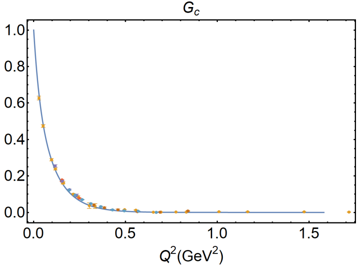
Decomposition of Color States

Fock sector expansion: $|\text{Deuteron}\rangle = \psi_{6q}|qqq\ qqq\rangle + \psi_{6q1g}|qqq\ qqq\ g\rangle$

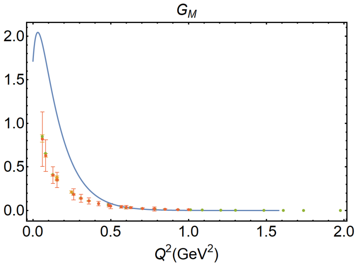


EM Form Factors : Very preliminary results from BLFQ

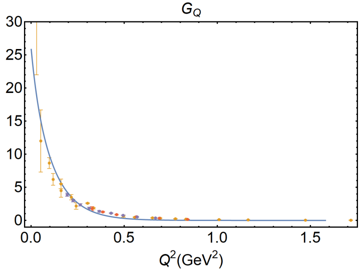
Charge FF



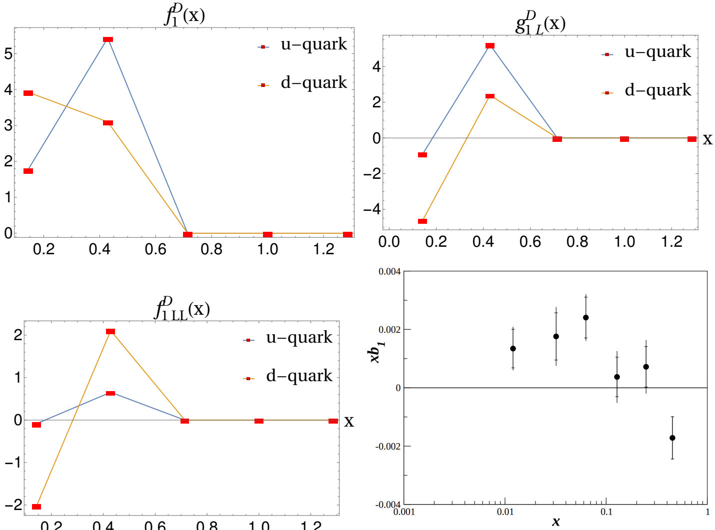
Magnetic FF



Quadrupole FF

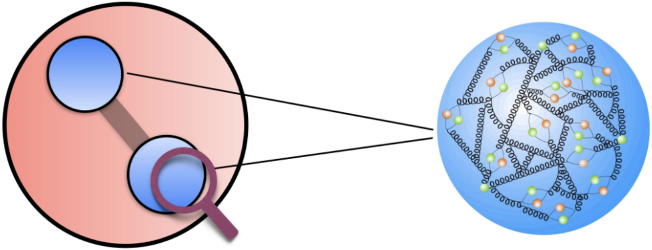


PDFs : Very preliminary results from BLFQ



$\mu^2 = 5 \text{ GeV}^2$

- Capability to achieve the gluon transversity.



Light-Front Schrödinger Wave Equation

- Light-front wave equation:

$$\left(\frac{m_p^2}{z} + \frac{m_n^2}{1-z} - \frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\text{eff}}\right)\Psi(z,\zeta) = M_D^2\Psi(z,\zeta)$$

- Light-front wavefunction:

$$\Psi(z,\zeta,\varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}e^{iL\varphi}X(z)$$

- $\zeta = \sqrt{z(1-z)}\mathbf{b}_\perp$, $X(z) = \sqrt{z(1-z)}\chi(z)$

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_\perp(\zeta)\right)\phi(\zeta) = M_{\perp D}^2\phi(\zeta)$$

$$\left(\frac{m_p^2}{z} + \frac{m_n^2}{1-z} + U_\parallel(z)\right)\chi(z) = M_{\parallel D}^2\chi(z)$$

- Assumption: $U_{\text{eff}} = U_\perp(\zeta) + U_\parallel(z)$; Mass: $M_D^2 = M_{\perp D}^2 + M_{\parallel D}^2$
- LFWF: $\Psi(z,\zeta) = \sqrt{z(1-z)}\chi(z)\,\phi(\zeta)$

[1] S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and J. Erlich, Phys. Rept. 584, 1 (2015)

Light-Front Holographic QCD: contains transverse dynamics

- Unique confining potential [1]: $U_\perp^{\rm LFH}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- Meson mass spectra:

$$M^2_{\perp D}(n_\perp, J, L) = 4\kappa^2\left(n_\perp + \frac{J+L}{2}\right) \quad ; \quad J = L + S$$

- Transverse part of the wave function:

$$\phi_{n_\perp L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n_\perp!}{(n_\perp+L)!}}\zeta^{1/2+L}\exp\left(\frac{-\kappa^2\zeta^2}{2}\right) L^L_{n_\perp}(\kappa^2\zeta^2)$$

- Transverse part of the deuteron LFWF in momentum space:

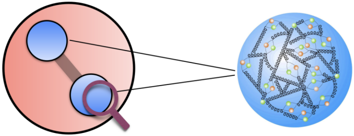
$$\Psi(z, k_\perp^2) = \frac{1}{\sqrt{z(1-z)}}\exp\left(-\frac{k_\perp^2}{2\kappa^2 z(1-z)}\right)$$

[1] S.J. Brodsky, G.F. de Téramond, H.G. Dosh, J. Erlich, Physics Reports 584, 1 (2015)

The 't Hooft Equation: contains longitudinal dynamics

- Derived from the $(1+1)$-dim QCD Lagrangian in the large N_c limit [1] :

$$\left(\frac{m_p^2}{z}+\frac{m_n^2}{1-z}\right)\chi(z)+\frac{g^2}{\pi}\mathcal{P}\int \mathrm{d}y\frac{\chi(z)-\chi(Z)}{(z-Z)^2}=M_{\parallel D}^2\chi(z)$$



- Extend to light nuclei

$$\mathrm{m}_n = m_p = 0.80 \pm 0.08 \text{ GeV};\ \kappa = 0.13 \pm 0.013 \text{ GeV};\ g = 0.5 \pm 0.05 \text{ GeV}$$

$$M_D = \sqrt{M_{\perp D}^2 + M_{\parallel D}^2} = 1.80 \pm 0.18 \text{ GeV}$$

[1] G. 't Hooft, Nucl. Phys. B 75 (1974) 461–470

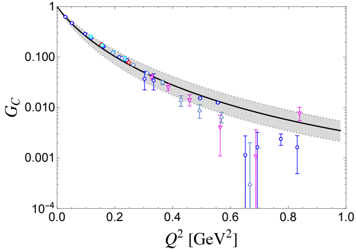
[2] Phys. Lett. B 823, 136754(2021); Phys. Rev. D 104, 074013 (2021)

[3] Phys. Lett. B 836, 137628 (2023)

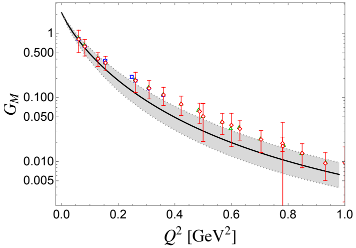
[4] Phys. Rev. D 109, 094017 (2024)

Electromagnetic Form Factors

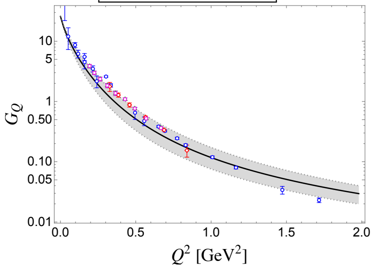
Charge FF



Magnetic FF



Quadrupole FF



$$G_C(0) = 1,\ G_M(0) = M_D^2 \mathcal{Q}_D = 25.83,$$
$$G_Q(0) = \frac{M_D}{M_N} \mu_D = 1.714$$

$$\sqrt{\langle r_C^2 \rangle} = 2.10 \pm 0.20\ (2.13 \pm 0.003 \pm 0.009)\ \text{fm}$$

$$\sqrt{\langle r_M^2 \rangle} = 2.27 \pm 0.21\ (1.90 \pm 0.14)\ \text{fm}$$

Nucleon's Longitudinal Momentum-dependent Distribution Functions

- Nucleon longitudinal momentum distribution functions:

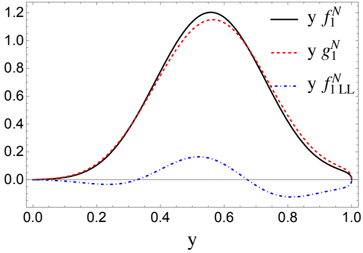
$$f_1^N(z) = \frac{1}{6}\left[P_\uparrow^1(z) + P_\uparrow^{-1}(z) + P_\uparrow^0(z)\right]$$

$$g_{1L}^N(z) = \frac{1}{4}\left[P_\uparrow^1(z) - P_\downarrow^1(z)\right]$$

$$f_{1LL}^N(z) = \frac{1}{4}\left[2P_\uparrow^0(z) - \left(P_\uparrow^1(z) + P_\uparrow^{-1}(z)\right)\right]$$

where $P_\uparrow^\Lambda(z) = \int \mathrm{d}^2\mathbf{k}_\perp \sum_{\bar{h}} \left|\Psi_{\uparrow\bar{h}}^\Lambda(z,\mathbf{k}_\perp)\right|^2$;

$P_\downarrow^\Lambda(z) = \int \mathrm{d}^2\mathbf{k}_\perp \sum_{\bar{h}} \left|\Psi_{\downarrow\bar{h}}^\Lambda(z,\mathbf{k}_\perp)\right|^2$



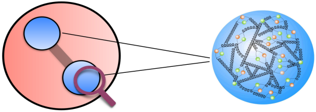
- Qualitative consistency with other spin-1 systems (like ρ-meson).

Structure Functions

- Structure functions: $x\sum_f e_f^2\ \{\text{PDF}\}^D(x,Q^2)$
  - PDF of deuteron at the level of its valence quarks

$$\{\text{PDF}\}^D(x,Q^2)=\frac{1}{2}\sum_{\text{nucleon}}\int_x^1\frac{\mathrm{d}y}{y}\mathcal{F}^{\text{nucleon}}(y)\otimes\{\text{PDF}\}^f\left(\frac{x}{y},Q^2\right)$$

  where $\mathcal{F}^{\text{nucleon}}$: nucleon longitudinal momentum distribution
- $\{\text{PDF}\}^f$ @ $Q^2=5\ \text{GeV}^2$ are obtained from NNPDF global fits.



F_2	$\rightarrow$	f_1^N	$\otimes$	unpolarized NNPDF
g_1	$\rightarrow$	g_{1L}^N	$\otimes$	helicity NNPDF
b_1	$\rightarrow$	f_{1LL}^N	$\otimes$	unpolarized NNPDF

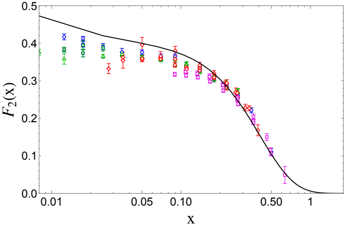
PHYSICAL REVIEW C VOLUME 44, NUMBER 3 SEPTEMBER 1991

Convenient parametrization for deep inelastic structure functions of the deuteron

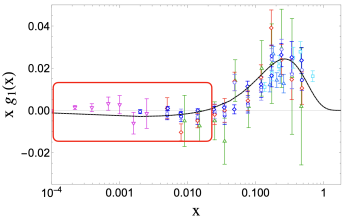
Hafsa Khan and Pervez Hoodbhoy
Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan
(Received 27 December 1990)

Structure Functions

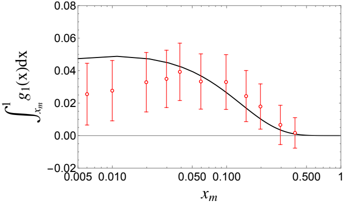
unpolarized



helicity

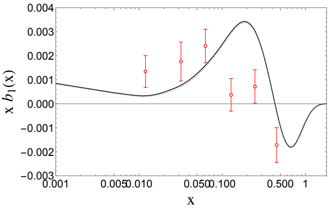


Moments of g_1

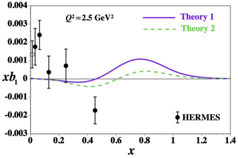


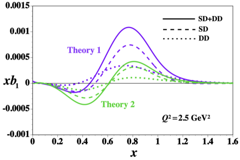
- Discrepancy at low-x: absence of the non-nucleonic contributions.
- Remarkable description of the data.

Tensor-polarized structure function



Cosyn et al. PRD 95, 074036 (2017); Kumano EPJ A 60, 205 (2024)





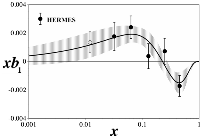


Fig. 4 HERMES data and parametrized b_1 structure function with its uncertainty band given by $\Delta\chi^2 = 1$. The data with $Q^2 < 1$ GeV^2 is shown by the open circle.

- Captures the overall qualitative behavior.
- $\int_{0.02}^{0.85} \mathrm{d}x \, b_1^D(x) = 0.41 \times 10^{-2}$; HERMES: $(0.35 \pm 0.10_{\text{stat}} \pm 0.18_{\text{sys}}) \times 10^{-2}$

  Kumano(2024): 0.0058 ± 0.0047 [EPJ A 60 (2024) 10, 205]

[1] A. Airapetian, et al. (HERMES Collaboration), PRL 95, 242001 (2005)

Conclusion

- Deuteron, a lightest nuclei with spin-1, contains enriched information at the level of its partons.
- Showed some very preliminary results on deuteron structure functions from LF Hamiltonian approach.
    - Qualitatively consistent results.
    - able to achieve gluon transversity distribution.
    - able to study the color structure of deuteron.
- Studied the deuteron structure functions using LF holographic QCD approach alongwith the 't Hooft Equation.
    - Good agreement with the experimental data.


THANK YOU
谢谢

Light-Front Holographic QCD : contains transverse dynamics

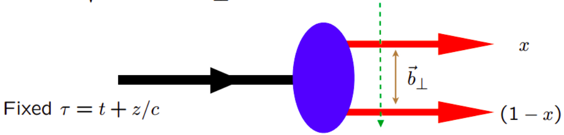
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$\psi(x, \vec{b}_\perp)$ ⟷ $\phi(z)$

$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$ ⟷ z



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

***Light-Front Holography**: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

[1] S.J. Brodsky, arXiv-hep:1611.07194