# The EMC effect within the light-front Hamiltonian dynamics for few-nucleon bound systems

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# Based on

F.F, E.Pace, M.Rinaldi, G.Salmè, S.Scopetta and M.Viviani,
"The EMC effect for few-nucleon bound systems in Light-Front Hamiltonian Dynamics",
***Phys.Lett.B* 851 (2024) 138587**

E.Proietti, F.F, E.Pace, M.Rinaldi, G.Salmè and S.Scopetta,
"3He spin-dependent structure functions within the relativistic light-front Hamiltonian dynamics",
***Phys.Rev.C* 110 (2024) 3, L031303**

E.Pace, M.Rinaldi, G.Salmè and S.Scopetta
"The European Muon Collaboration effect in Light-Front Hamiltonian Dynamics",
**Phys. Lett. B 839(2023) 127810**

E.Pace, M.Rinaldi, G.Salmè and S.Scopetta
"EMC effect, few-nucleon system and Poincaré covariance",
**Phys. Scr. 95, 064008 (2020)**

R.Alessandro, A.Del Dotto, E.Pace, G.Perna, G.Salmè and S.Scopetta
"Light-Front Transverse Momentum Distributions for $J = 1/2$ Hadronic Systems in Valence Approximation",
**Phys.Rev.C 104(2021) 6,065204**

A. Del Dotto, E.Pace, G. Salmè and S.Scopetta
"Light-Front spin-dependent Spectral Function and Nucleon Momentum Distributions for a Three-Body system",
**Phys. Rev. C 95,014001 (2017)**

# Outline

- Overview
- The EMC effect
- The Light-Front Poincaré covariant approach
- Nuclear structure functions with Relativistic Hamiltonian Dynamics
- Numerical results for the EMC effect
- Numerical results for the $^3He$ SSFs
- Conclusions

# Overview

- Developed a new and rigorous **light-front formalism** for the unpolarized **Deep Inelastic Scattering** (**DIS**) and **applied to the** $^3He$**:** sizable **European Muon Collaboration** (**EMC**) **effect** predicted **[1]**

- The formalism is **extended to any nucleus** and applied to the $^4He$ and $^3H$ **[2]**

  - $^4He$ is a tightly bound nucleus ⇒ **Challenging** test to our approach

- The formalism is generalized for the **polarized DIS [3]** for the $^3He$

  - $^3He$ can be considered as an **effective polarized neutron target** ⇒ Extraction of the **neutron spin structure** is possible only through nuclear data

  - Experiments involving **polarized beams of** $^3He$ planned at **future facilities** such as EICs. Proposal for **positron beams** at JLab [A.Accardi et al., **Eur.Phys.J.A 57 (2021) 8, 261**]

**[1]** E.Pace, M.Rinaldi, G.Salmè and S.Scopetta**, Phys. Lett. B 839 (2023) 127810**

**[2]** F.F, E.Pace, M.Rinaldi, G.Salmè, S.Scopetta and M.Viviani**, *Phys.Lett.B* 851 (2024) 138587**

**[3]** E.Proietti, F.F, E.Pace, M.Rinaldi, G.Salmè and S.Scopetta, ***Phys.Rev.C* 110 (2024) 3, L031303**

# The EMC effect

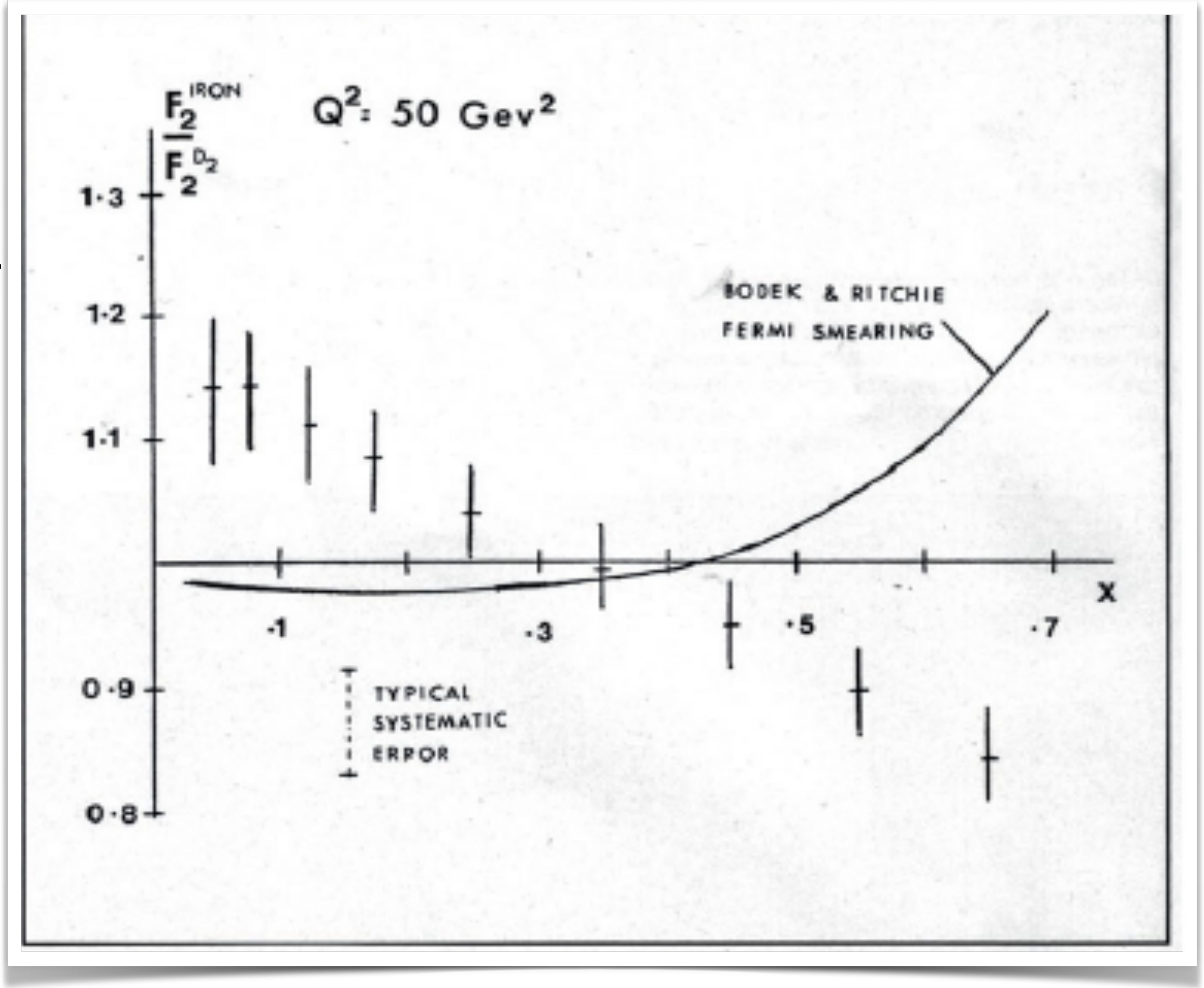
More than **40 years ago**, the **European Muon Collaboration (EMC)** measured (in **DIS** processes)

$$R(x) = F_2^{^{56}Fe}(x)/F_2^{^2H}(x)$$

Expected result: $R(x) = 1$

Result:

Aubert et al. **Phys.Lett. B123 (1983) 275**



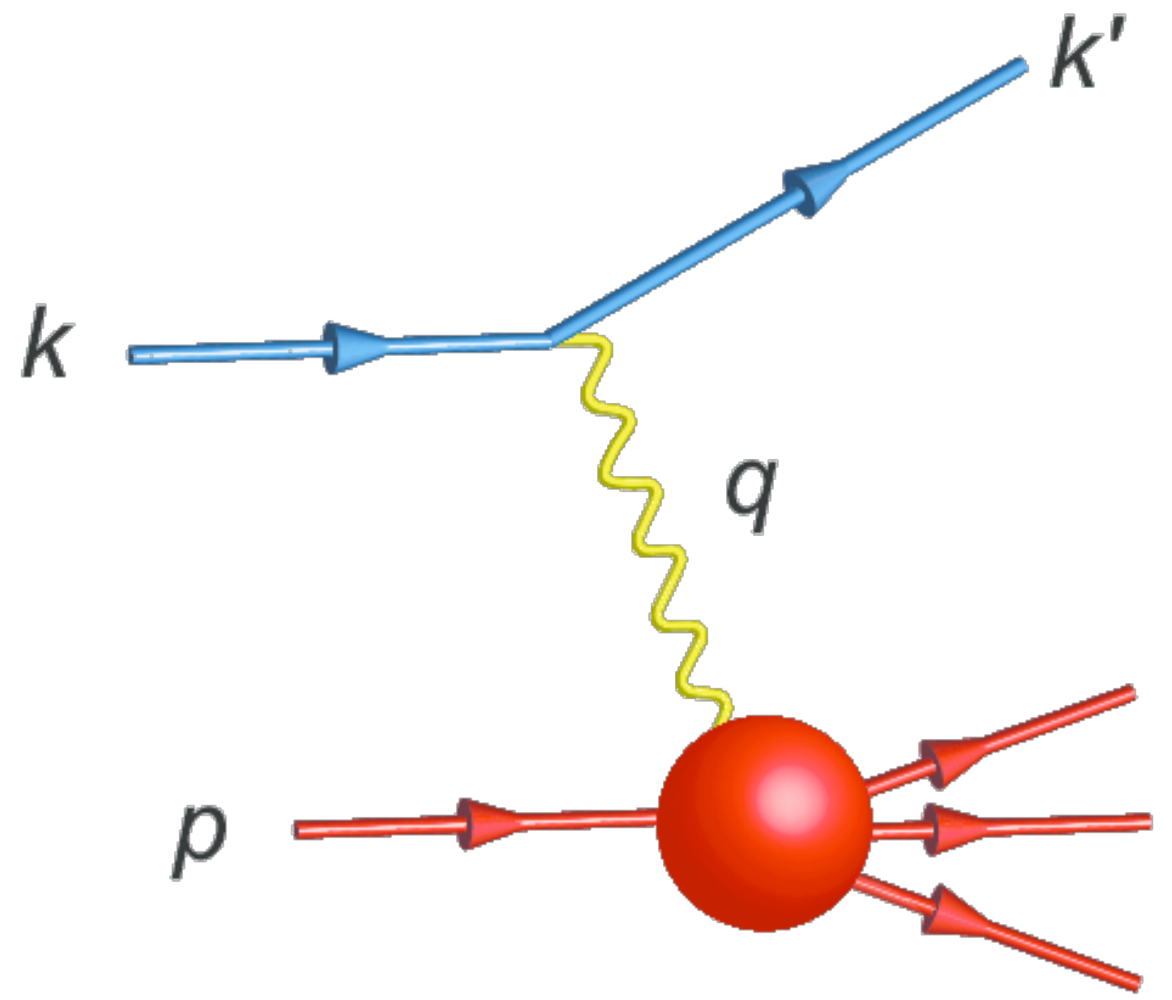

**Naive parton model interpretation:**

"Valence quarks, in the bound nucleon, are in average slower that in the free nucleon"

Is the bound proton bigger than the free one??

# The EMC effect

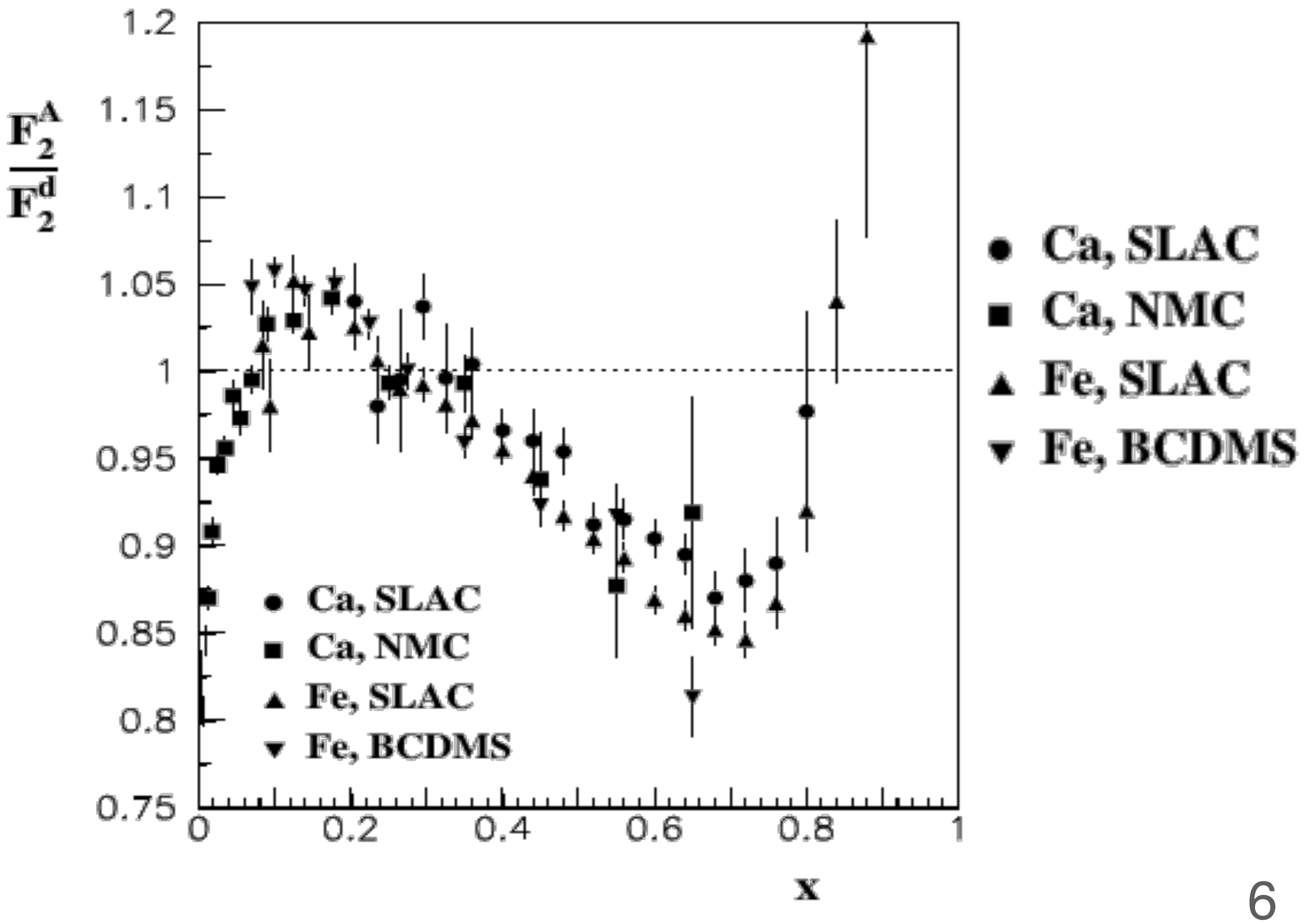
We remind that for DIS off nuclei: $0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$




- **x ≤ 0.3 "Shadowing region"**: coherence effects, the photon interacts with partons belonging to different nucleons
- **0.2 ≤ x ≤ 0.8 "EMC (binding) region"**: mainly valence quarks involved
- **0.8 ≤ x ≤ 1 "Fermi motion region"**

**main features:** universal behavior independent on $Q^2$; weakly dependent on A;
Scales with the density ρ → global property?
Or due to SRC → local property?

**Explanation (exotic) advocated:** confinement radius bigger for bound nucleons, quarks in bags with 6, 9,..., 3A
**quark, pion cloud effects... Alone or mixed with conventional ones...**



# The EMC effect

**Status of "conventional" calculations for light nuclei:**

**NR Calculations:** qualitative agreement but no fulfillment of both **particle and momentum sum rules**… Not under control

Our approach is aimed to include **only nucleonic dof** through **conventional nuclear physics** in a **Poincaré-covariant** approach that preserves the **macroscopic locality**. The only way to **fulfill sum rules** while using **realistic NR nuclear potentials** is to **embed relativistic effects**

The lack of the **Poincarè covariance** and **macroscopic locality** generates **biases** for the study of **genuine QCD effects** (nucleon swelling, exotic quark configurations …). We provide a **reliable baseline** for the calculation of the **nuclear SFs** where only the **well known nuclear part** is considered

# Need for a relativistic treatment

**Why do we need a relativistic treatment ?**

**General answer:** to develop an advanced scheme, appropriate for the kinematics of **JLAB12** and **EIC**

- The **Standard Model of Few-Nucleon Systems**, with nucleon and meson degrees of freedom within a non relativistic **(NR)** framework, has achieved **high sophistication** (e.g. the NR $^3$He and $^3$H Spectral Functions * )

- Covariance wrt the **Poincaré Group,** needed for nucleons at **large 4-momenta** and pointing to **high precision** measurements

Our definitely preferred framework for embedding the successful **NR phenomenology:**

**Light-front Relativistic Hamiltonian Dynamics** (LFRHD, fixed dof) + **Bakamjian-Thomas** (BT) construction of the Poincaré generators for an interacting theory.

* Kievsky, Pace, Salmè and Viviani **PRC 56, 64 (1997)**

# The relativistic Hamiltonian dynamics framework

In **RHD+BT**, one can address both Poincaré **covariance** and **macroscopic locality**, general principles to be implemented in presence of interaction:

**Poincaré covariance** → The 10 generators, $P^\mu$ → $4D$ displacements and $M^{\nu\mu}$ → Lorentz transformation, have to fulfill:

$$[P^\mu, P^\nu] = 0; [M^{\mu\nu}, P^\rho] = -i(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu)$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\sigma}M^{\mu\sigma})$$

**Macroscopic locality** (= cluster separability (relevant in nuclear physics)): i.e. observables associated to different space-time regions must commute in the limit of **large space like separation** (i.e. causally disconnected). In this way, when a system is separated into disjoint subsystems by a sufficiently large space like separation, then the **subsystems behave as independent systems***

This requires a **careful choice** of the i**ntrinsic relativistic coordinates**

*B.D.Keister and W.N.Polyzou, **Adv.Nucl.Phys. 20 (1991), 225-479**

# Advantages of the Light-Front framework

**The Light-Front framework** has several advantages:

- **7 Kinematical generators**: i) **3 LF boosts** (in instant form they are dynamical!) ;
  ii) $\tilde{P} = (P^+ = P^0 + P^3, \mathbf{P}_\perp)$ ; iii) Rotation around the z-axis

- The **LF boosts** have a subgroup structure: trivial separation of intrinsic and global motion, as in the NR case

- $P^+ \geq 0 \rightarrow$ **meaningful Fock expansion**, once massless constituents are absent

- The **infinite-monentum frame (IMF)** description of DIS is easily included

**Drawback: the transverse LF-rotations are dynamical!**

**But** within the **Bakamjian-Thomas (BT)** construction of the generators in an interacting theory, one can construct an intrinsic angular momentum fully kinematical

# The Bakamjian-Thomas construction

Bakamjian and Thomas **(PR 92 (1953) 1300)** proposed an **explicit construction** of 10 Poincaré generators in presence of **interactions.** The key ingredient is the **mass operator**:

i) Only the **mass operator** $M$ contains the interaction

ii) It generates the dependence of the **3 dynamical generators** ($P^-$ and LF transverse rotations) upon the interaction

iii) The eigenvalue equation $M^2|\psi> = s|\psi>$ is formally equivalent to the Schrödinger equation

The **mass operator** is given by the sum of $M_0$ with an interaction V: $M_0 + V$. The interaction V **must commute** with all the **kinematical generators** and with the **non-interacting angular momentum**, as in the NR case

Light-Cone coordinates: $a = (a^-, \tilde{a}), a^{\pm} = a^0 \pm a^z, \tilde{a} = (a^+, \mathbf{a}_\perp)$

# The BT construction for a nuclear system

For a generic nucleus A, the mass operator is

$$M_{BT}[1,2,3,\ldots,A] = M_0[1,2,3,\ldots,A] + V(\mathbf{k}^2; \mathbf{k}\cdot\mathbf{k}_i; \mathbf{k}_j\cdot\mathbf{k}_i)$$

2 and 3 body forces operator

Where the free mass operator of the system is: $M_0[1,2,3,\ldots,A] = \sum_i^A \sqrt{m^2 + \mathbf{k}_i^2}$

Momenta in the intrinsic reference frame $\sum_{i=1}^{A} \mathbf{k}_i = 0$

The **commutation rules** impose to $V$ **invariance for translations and rotations** as well as **independence on the total momentum**, as it occurs for $V^{NR}$

**One can assume** $M_{BT}[1,2,\ldots,A] \sim M^{NR}$

Therefore what has been learned till now about **the nuclear interaction**, within a **non-relativistic framework**, can be re-used in a **Poincaré covariant framework**.

# Reference frames

For a correct description of the **structure functions**, so that the **macro-locality** is implemented, it is crucial to distinguish between **different frames**, moving with respect to each other:

- The **Lab frame**, where $\tilde{P} = (M_{BT}, \mathbf{0}_\perp)$
- The **intrinsic LF frame** of the whole system, $[1,2,\ldots,A]$, where $\tilde{P} = (M_0[1,2,\ldots,A], \mathbf{0}_\perp)$ with $k_i^+ = \xi_i M_0[1,2,\ldots,A]$ and $M_0[1,2,\ldots,A] = \sum_{i=1}^{A} \sqrt{m^2 + \mathbf{k}_i^2}$
- The **intrinsic LF frame** of the cluster $[1;2,3,\ldots,(A-1)]$ where $\tilde{P} = (\mathcal{M}_0[1;2,3,\ldots,A-1]), \mathbf{0}_\perp)$ with $k^+ = \xi \mathcal{M}_0[1;2,3,\ldots,A-1]$ and $\mathcal{M}_0[1;2,3,\ldots,A-1] = \sqrt{m^2+\kappa^2} + \sqrt{M_s^2+\kappa^2}$

While $\mathbf{p}_\perp^{LAB} = \mathbf{k}_{1\perp} = \kappa_\perp$

$M_s = (A-1)m + \epsilon$ is the mass of the fully interacting spectator system

# LF spectral function

Since we use an **impulse approximation** assumption, we have to define the **spin-dependent LF spectral function** $P^{\tau}_{\sigma'\sigma}(\tilde{\kappa}, \epsilon, \mathbf{S}, M)$

$$P^{N}_{\sigma'\sigma}(\tilde{\kappa}, \epsilon, \mathbf{S}, M) = \sum_{JJ_z} \sum_{TT_z} \rho(\epsilon)_{LF} < tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} \,|\, \Psi_{JM}; \mathbf{S}, T_A T_{Az} > < \Psi_{JM}; \mathbf{S}, T_A T_{Az} \,|_{LF} tT; \alpha, \epsilon; JJ_z; \tau\sigma, \tilde{\kappa} >_{LF}$$

$|\, tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} >_{LF}$ is the **tensor product** of the plane wave of the struck nucleon $[1]$ and the state of the fully interacting spectator system $[2,\ldots,A-1]$ in the intrinsic reference frame of the cluster $[1;2,3,\ldots,A-1]$ when the spectator system has energy $\epsilon$. It fulfills the **macrolocality***

$|\,\Psi_{JM}; \mathbf{S}, T_A T_{Az} >_{LF}$ is the **eigenstate** of $M_{BT}[1,\ldots,A] \sim M^{NR}$ in the **intrinsic frame** of the system $[1,2,\ldots,A]$

The **LF spectral function** contains the **determinant of the Jacobian** of the **transformation between the intrinsic frames** $[1;2,3,\ldots,A-1]$ and $[1,2,\ldots,A]$, connected each other by a **LF boost**

*B.D.Keister and W.N.Polyzou, **Adv.Nucl.Phys. 20 (1991), 225-479**

# LF spectral function

We can express the **LF overlap** in terms of the **IF overlap** using **Melosh rotations:**

$$< tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} \,|\, \Psi_{JM}; \mathbf{S}, T_A T_{Az} >_{LF} \rightarrow \; < tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa \,|\, \Psi_{JM}; \mathbf{S}, T_A T_{Az} >_{IF}$$

Then we can approximate the **IF overlap** into a **NR overlap** by using the NR wave function for the nucleus, thanks to the **BT construction:**

$$< tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa \,|\, \Psi_{JM}; \mathbf{S}, T_A T_{Az} >_{IF} \; \sim \; < tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa \,|\, \Psi_{JM}; \mathbf{S}, T_A T_{Az} >_{NR}$$

**Poincarè covariance** preserved but using the **successful NR phenomenology**

We used wave functions of $^2H, ^3H, ^3He, ^4He$ calculated through 3 different potentials: **Av18+UIX*** and 2 versions of the **Norfolk $\chi EFT$ interactions NVIa+3N**** and **NVIb+3N****

*R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, **Phys. Rev. C 51 (1995) 38–51;** R. B. Wiringa et al., **Phys. Rev. Lett. 74 (1995) 4396–4399**

**M.Viviani et al., **Phys. Rev. C 107 (1) (2023) 014314;** M. Piarulli et al.,**Phys. Rev. Lett. 120 (5) (2018) 052503;** M. Piarulli, S. Pastore, R. B. Wiringa, S. Brusilow, R. Lim,**Phys. Rev. C 107 (1) (2023) 014314**

# Hadronic tensor

In our approach the **symmetric part** of the **hadronic tensor** is found to be *

$$W_A^{s,\mu\nu} = \sum_N \sum_\sigma \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{2(2\pi)^3 \kappa^+} \frac{1}{\xi} P^N(\tilde{\kappa}, \epsilon)\, w_{N,\sigma}^{s,\mu\nu}(p,q)$$

hadronic tensor of the nucleon

Unpolarized LF spectral function:

$$P^N(\tilde{\kappa}, \epsilon) = \frac{1}{2j+1} \sum_{\mathcal{M}} P^N_{\sigma\sigma}(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M})$$

$W_A^{s,\mu\nu}$ is parametrized by the SFs $F_2^A(x)$ and $F_1^A(x)$:

$$F_2^A(x) = -\frac{1}{2} x g_{\mu\nu} W_A^{s,\mu\nu} = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{d\kappa^+}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) F_2^N(z)$$

Free nucleon SF

Where $x = \dfrac{Q^2}{2P_A \cdot q}$ and $\xi = \dfrac{\kappa^+}{\mathcal{M}_0[1;2,3,\ldots,A-1]}$ with $z = \dfrac{Q^2}{2p \cdot q} = \dfrac{p}{P_A^+}\dfrac{x}{\xi}$

* E.Pace, M.Rinaldi, G.Salmè and S. Scopetta, **Phys. Scr. 95, 064008 (2020)**

# LC momentum distribution

$$F_2^A(x) = -\frac{1}{2} x g_{\mu\nu} W_A^{s,\mu\nu} = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{d\kappa^+}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) F_2^N(z)$$

In the Bjorken limit $\int d\epsilon \int d\kappa^+ = \int d\kappa^+ \int d\epsilon$ so we can use the **light-cone momentum distribution** (LCMD) instead of the **LF spectral function** *

$$\textbf{LCMD}: f_1^N(\xi) = \sum \int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{1}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) \frac{E_s}{1-\xi} = \int d\mathbf{k}_\perp n^n(\xi, \mathbf{k}_\perp)$$

**LF momentum distribution:**

$$n^N(\xi, \mathbf{k}_\perp) = \frac{1}{2\pi} \int \prod_{i=2}^{A-1} [d\mathbf{k}_i] \left| \frac{\partial k_z}{\partial \xi} \right| \mathcal{N}^N(\mathbf{k}, \mathbf{k}_2, \ldots, \mathbf{k}_{A-1})$$

Squared nuclear wave function. **Thanks to the BT construction, one is allowed to use the NR one**

Determinant of the Jacobian matrix. **LF boost: effect of a Poincaré covariance approach**

* A. Del Dotto, E.Pace, G. Salmè and S.Scopetta, **Phys. Rev. C 95,014001 (2017)**

# LC momentum distribution

LCMD: $f_1^N(\xi) = \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{1}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) \frac{E_s}{1-\xi} = \int d\mathbf{k}_\perp n^n(\xi, \mathbf{k}_\perp)$

Since our approach fulfill both **macro-locality** and **Poincaré covariance** the LC momentum distribution must satisfies **2 essential sum rules:**

$A = \int_0^1 d\xi [Z f_1^p(\xi) + (A-Z) f^n(\xi)]$: Baryon number SR;

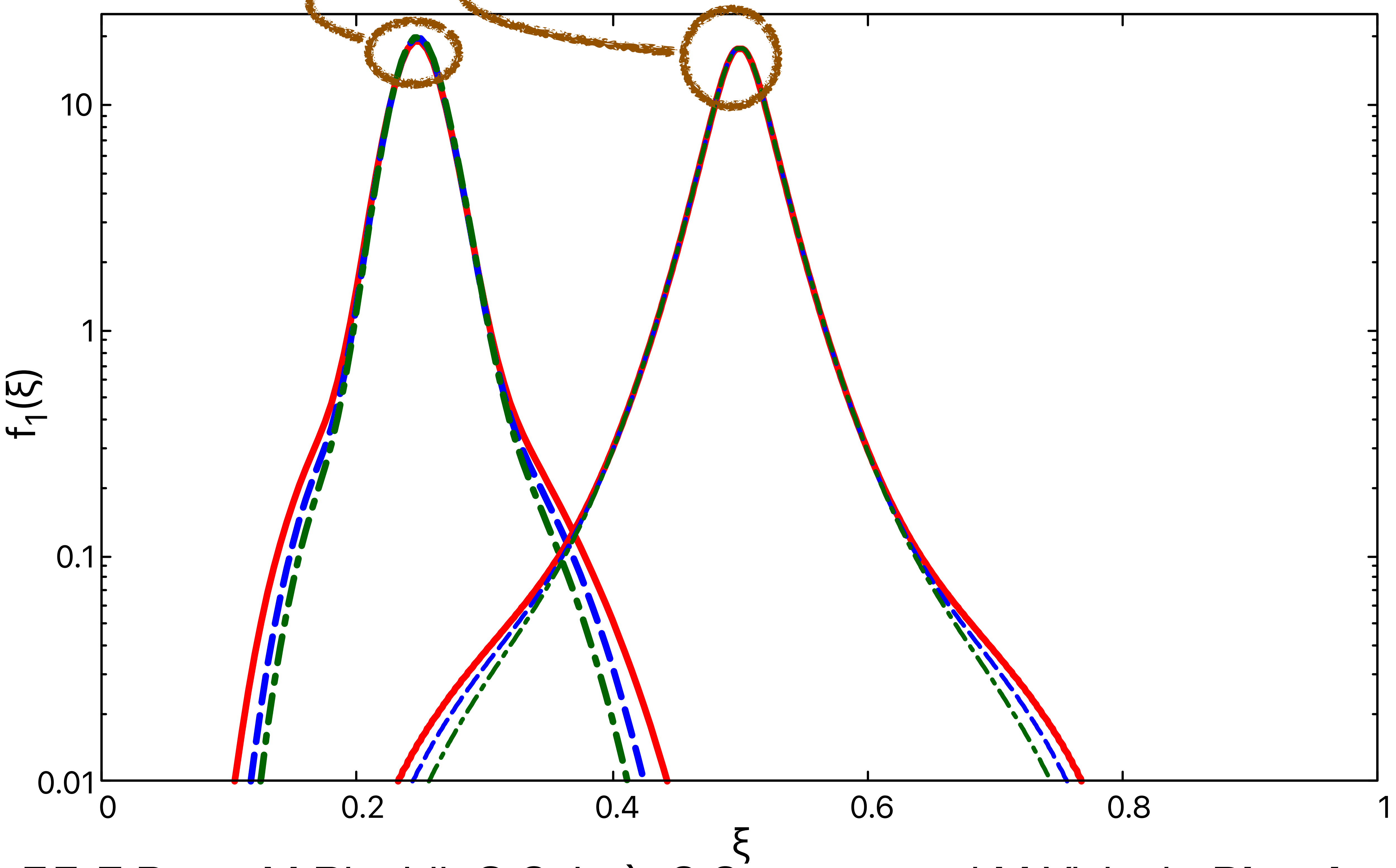
$1 = Z < \xi >_p + (Z-N) < \xi >_n \, ; < \xi >_N = \int_0^1 d\xi \, \xi \, f_1^N(\xi)$: Momentum SR (MSR)

Within the LFHD we are able to fulfill **both sum rules at the same time!**

Not possible with **IF approach** (Frankfurt & Strikman; Miller;….80's)

# LC momentum distribution: numerical results for $^4He$

The distributions are peaked at 1/A with an accuracy of 1/1000:
**MSR and Number of baryon sum rules are numerically satisfied**




- The tails of the distributions are generated by the **short range correlations (SRC)** induced by the potentials (i.e the high-momentum content of the 1-body momentum distribution)
- The tails of the LC momentum distribution calculated by the Av18/UIX potential is larger than the ones obtained by the $\chi$EFT potentials for both $^4He$ and deuteron
- This difference will partially cancel out on the **EMC ratio**

F.F, E.Pace, M.Rinaldi, G.Salmè, S.Scopetta and M.Viviani, ***Phys.Lett.B*** **851 (2024) 138587**

**LC momentum distribution for $^4He$ (peaked at 0.25) and deuteron (peaked at 0.5)**

**Solid lines: Av18/UIX. Dashed lines:NVIb+3N. Dot-dashed lines: NVIa+3N**

# Convolution formula for the nuclear structure function

To calculate the EMC ratio $R^A_{EMC}(x) = \frac{F_2^A(x)}{F_2^d(x)}$ for any nucleus A, we need a **NR realistic wave function** and a **parametrization for the free-nucleon structure functions**

$$F_2^A(x) = \sum_N \int_{\xi_{min}}^1 d\xi \; F_2^N\left(\frac{mx}{\xi M_A}\right) \; f_1^N(\xi)$$

We need both $F_2^n$ and $F_2^p$

One can choice a **parametrization** for $F_2^p$ and a **parametrization for the ratio** $\frac{F_2^n}{F_2^p}$ because $F_2^n$ could be only extracted by **nuclear DIS data**

We used a parametrization for the ratio $\frac{F_2^n}{F_2^p}$ extracted by **MARATHON data*** in Ref. [1]

*MARATHON Coll., **Phys. Rev. Lett 128 (2022) 13,132003**

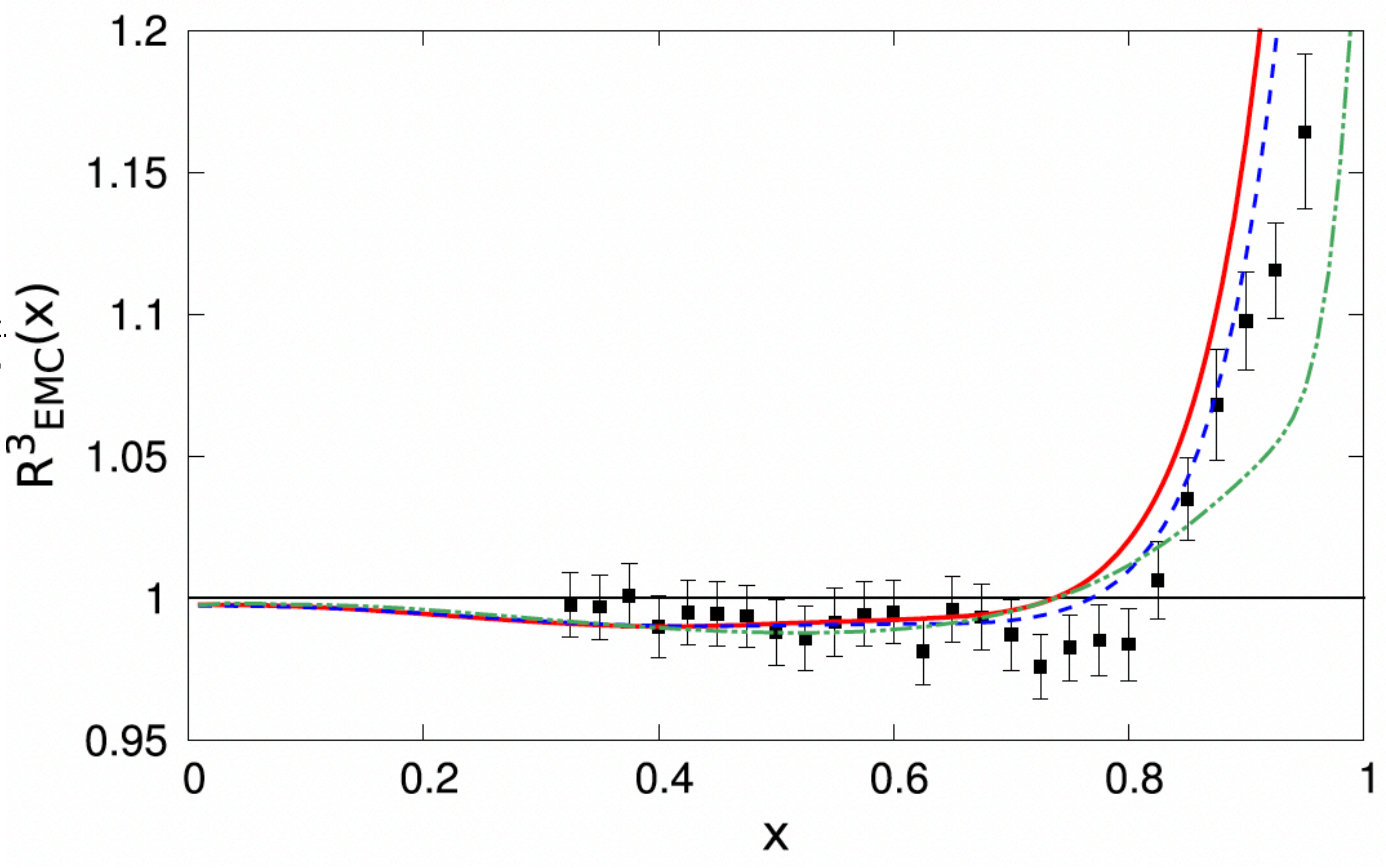
[1] E.Pace, M.Rinaldi, G.Salmè and S. Scopetta, **Phys. Scr. 95, 064008 (2020)**

# The EMC effect:results for $^3He$

E.Pace, M.Rinaldi, G.Salmè and S.Scopetta, **Phys. Lett. B 839(2023) 127810**

**[1]** J. Arrington, et al, **Phys. Rev. C 104 (6) (2021) 065203**

**[2]** S. A. Kulagin and R. Petti, **Phys. Rev. C 82, 054614 (2010)**




**Solid line: Av18/UIX + SMC***
**Dashed line:Av18 + SMC***
**Dotted-dashed: Av18/UIX +CJ15****

**Full squares: JLab data from experiment E03103** [1] **as reanalyzed in** [2]

****[B. Adeva, et al., Phys. Lett. B 412 (1997) 414–424.]***

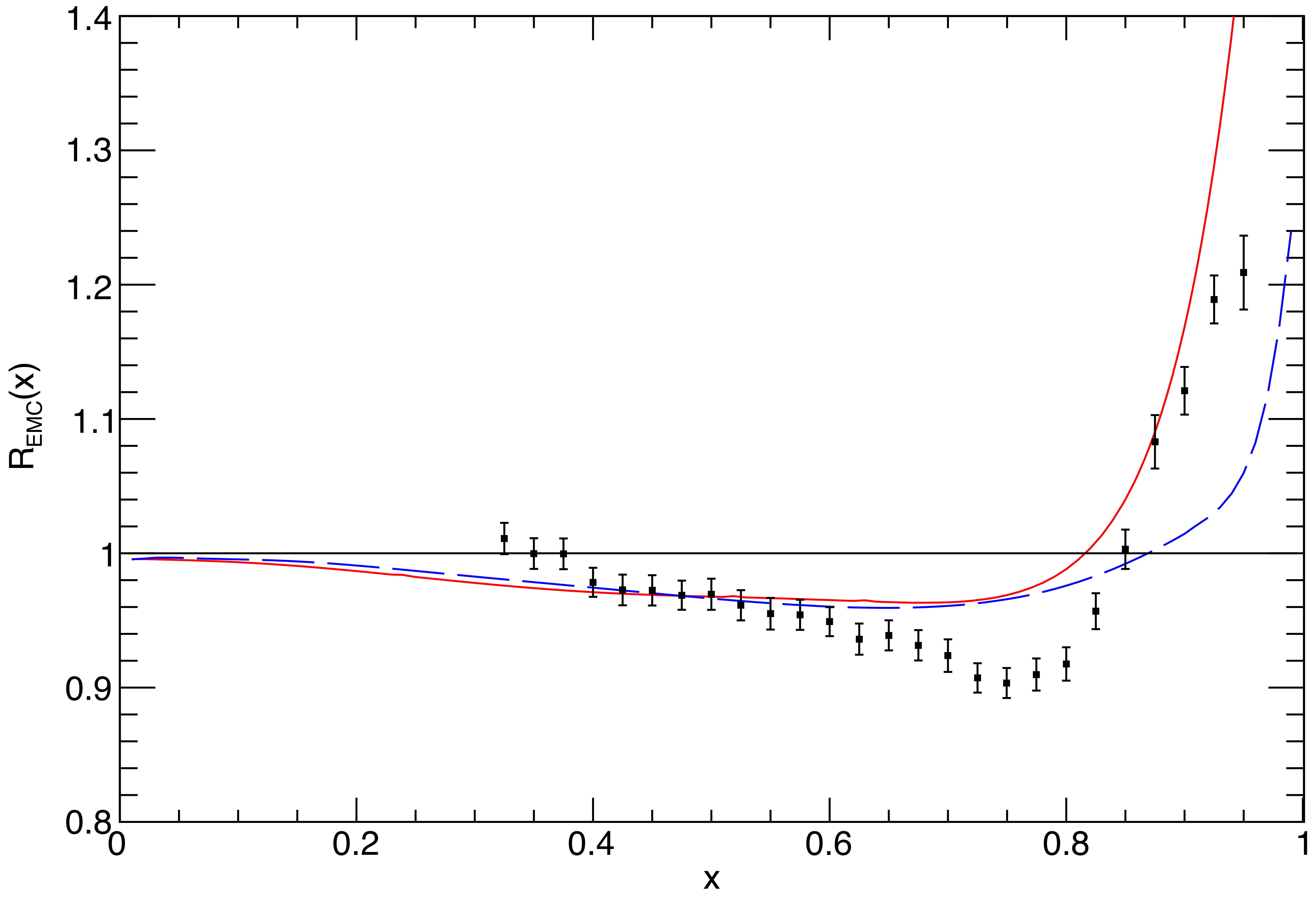
****[A. Accardi, L. T. Brady, W. Melnitchouk, J. F. Owens, N. Sato, Phys. Rev. D 93 (11) (2016) 114017]**

**Small** but **solid** effect, comparable to the experimental data

# The EMC effect: results for $^4He$

F.F, E.Pace, M.Rinaldi, G.Salmè, S.Scopetta and M.Viviani, ***Phys.Lett.B*** **851 (2024) 138587**

**Full squares: JLab data from experiment E03103**




**Both lines calculated with Av18/UIX**

**Solid line: SMC parametrization of $F_2^p$ ***

**Dashed line: CJ15 +TMC Parametrization of $F_2^p$****

**$F_2^n$ extracted from MARATHON data**

***[B. Adeva, et al., Phys. Lett. B 412 (1997) 414–424.]**

****[A. Accardi, L. T. Brady, W. Melnitchouk, J. F. Owens, N. Sato, Phys. Rev. D 93 (11) (2016) 114017]**

The dependence on the choice of the **free nucleon SFs** is largely under control in the **properly EMC region**

# The EMC effect: results for $^4He$

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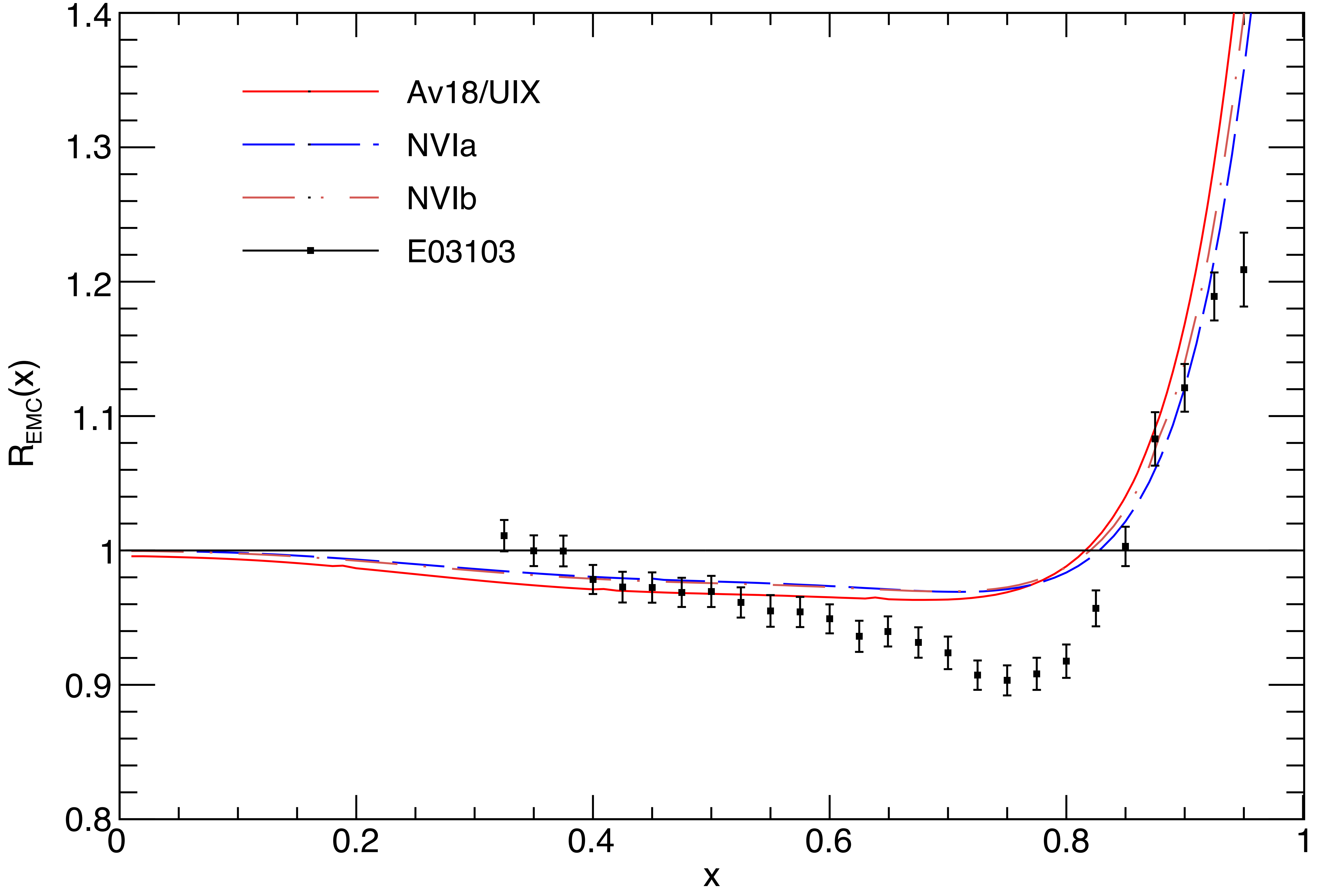



**Full squares: JLab data from experiment E03103**

The differences between the calculations from different potentials are of the **same order for both nuclei.** Definitely **smaller than the difference between data and theoretical prediction**

# Hadronic tensor II

For the **polarized DIS** we need to calculate the **antysimmetric** part of the **hadronic tensor**:

$$W_A^{a,\mu\nu} = \sum_N \sum_\sigma \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\kappa d\kappa^+}{2(2\pi)^3\kappa^+} \frac{1}{\xi} P_\sigma^N(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M}) \, w_{N,\sigma}^{a,\mu\nu}(p, q)$$

hadronic tensor of the nucleon

Spin-dependent LF spectral function

$W_A^{a,\mu\nu}$ is parametrized by the the **spin-dependent SFs (SSFs)** $g_1^A(x, Q^2)$ and $g_2^A(x, Q^2)$

As for the unpolarized case, in the **Bjorken limit** we can write a **convolution formula** for the **SSFs**:

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi \left[ g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi) \right], j = 1,2$$

# Spin-dependent SFs

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi \left[g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi)\right], j = 1,2$$

The spin-dependent **LCMD** $l_j^N(\xi)$ and $h_j^N(\xi)$ are related to the **transverse momentum-dependent distributions (TMDs)** of the nucleons $\Delta f^N, g_{1T}^N, \Delta'_T f^N, h_{1L}^N, h_{1T}^N$

We used the **TMDs** for $^3He$ calculated with the **Av18** potential in Ref. **[1]**

**GRSV** parametrization [2] for the $g_1^N(x)$ SSF

$g_2^N(x)$ extracted by $g_1^N(x)$ with the **Wandzura-Wilczek** formula [3]:

$$g_2^N(x) = -g_1^N(x) + \int_x^1 dy \frac{g_1^N(y)}{y}$$

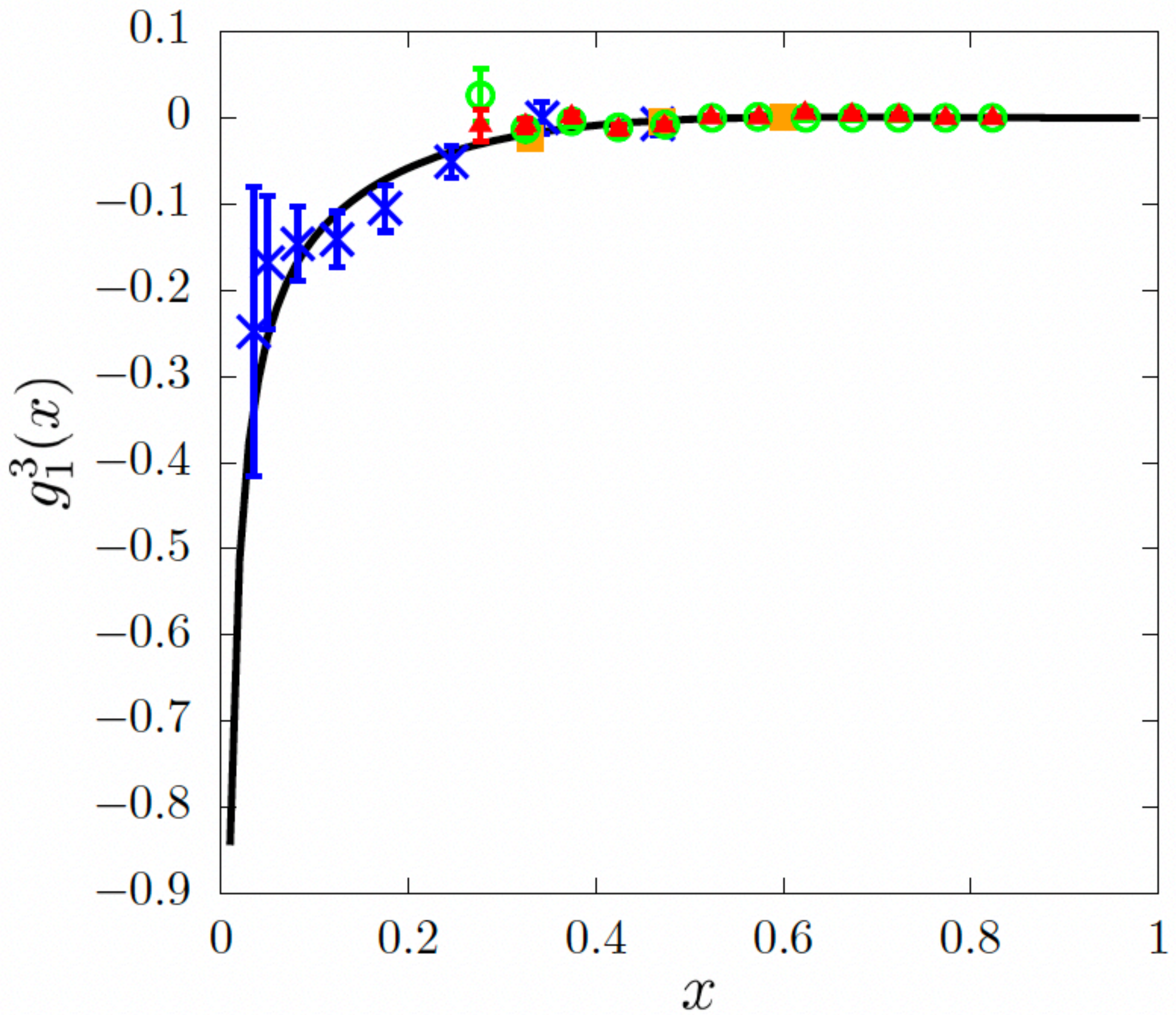

**[1]** R.Alessandro, A.Del Dotto, E.Pace, G.Perna, G.Salmè and S.Scopetta, **Phys.Rev.C 104(2021) 6,065204**

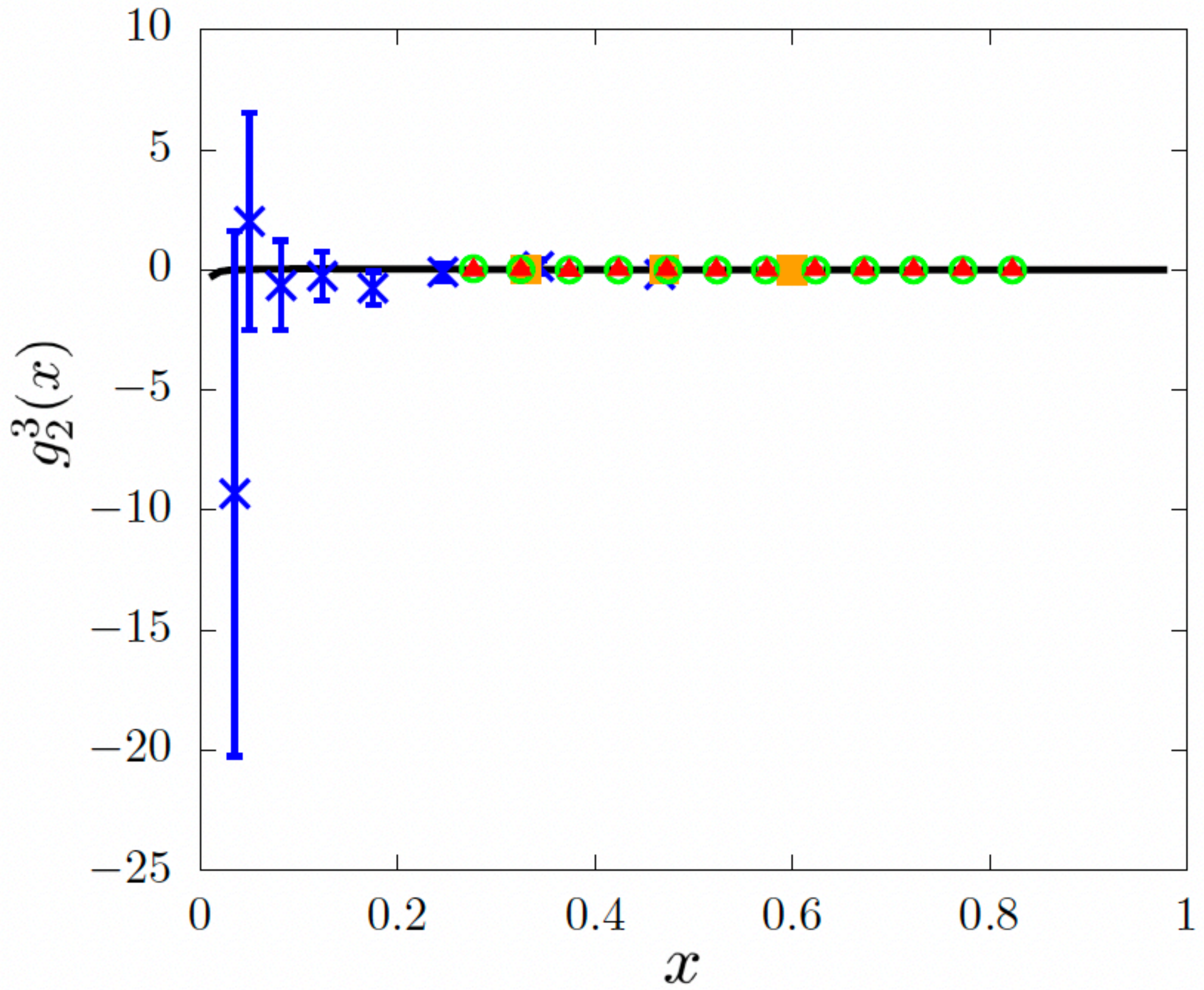
**[2]** M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, **Phys. Rev. D 63, 094005 (2001)**

**[3]** S. Wandzura and F. Wilczek, **Phys. Lett. B 72, 195 (1977)**

# $^3He$ SSFs

E.Proietti, F.F, E.Pace, M.Rinaldi, G.Salmè and S.Scopetta, ***Phys.Rev.C* 110 (2024) 3, L031303**




Experimental data from [1] (crosses), **[2] (squares)** and [3] (triangles)


[1] P. L. Anthony et al., **Phys. Rev. D 54, 6620 (1996)** [2] X. Zheng et al.,**Phys. Rev. Lett. 92, 012004 (2004)**

[3] D. Flay et al., **Phys. Rev. D 94, 052003 (2016)**

# Neutron SSFs

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi \left[ g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi) \right]$$

One can **approximate** this equation using that $l_j^N(\xi), h_j^N(\xi)$ are **peaked** around $\xi \simeq 1/A$ and so extract the **neutron SSFs**:

$$g_j^{\bar{n}}(x) = \frac{1}{p_j^n} \left[ g_j^{^3He}(x) - 2 p_j^p g_j^p(x) \right]$$

Where the **effective polarization** $p_j^N$ are **integral** of the **TMDs** $\Delta f(\xi, k_\perp)$ **and** $\Delta_T' f(\xi, k_\perp)$*

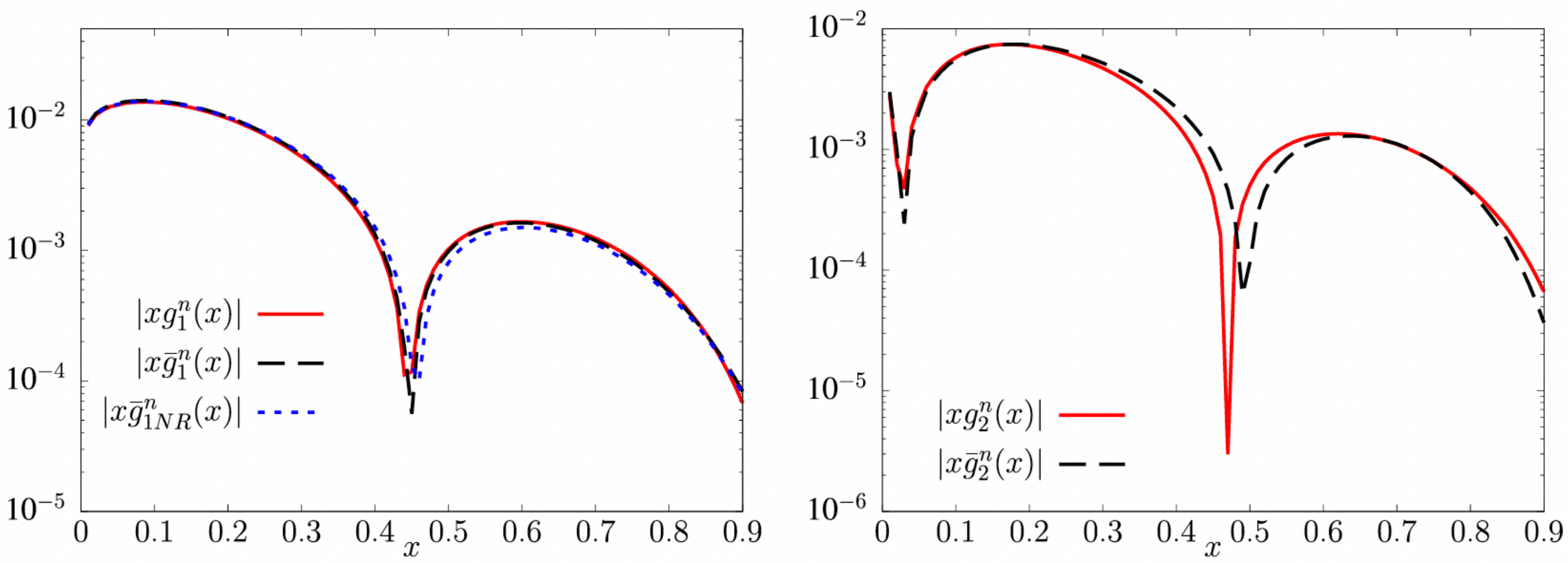
$$p_1^N = \int_0^1 d\xi \int d\mathbf{k}_\perp \Delta f(\xi, k_\perp) \text{ and } p_2^N = \int_0^1 d\xi \int d\mathbf{k}_\perp \Delta_T' f(\xi, k_\perp)$$

We compared our extraction of the **neutron SSFS** with the one of the **GRSV parametrization** and with the **NR extraction**, obtained through the effective polarizations calculated from a NR spectral function*

* R.Alessandro, A.Del Dotto, E.Pace, G.Perna, G.Salmè and S.Scopetta, **Phys.Rev.C 104(2021) 6,065204**

# Neutron SSFs

E.Proietti, F.F, E.Pace, M.Rinaldi, G.Salmè and S.Scopetta, ***Phys.Rev.C* 110 (2024) 3, L031303**




**Solide lines: GRSV parametrization** of the free neutron SSFs

**Dashed lines**: extraction of the free neutron SSFs from **relativistic** effective polarizations

**Dotted line**: extraction of the free neutron SSFs from **non-relativistic** effective polarizations

# Conclusions

- We proposed a **rigorous formalism** for the calculation of **nuclear SFs and SSFs** involving only **nucleonic DOF** with the conventional nuclear physics

- For $^3He$ we obtain results in **agreement** with **experimental data** for both **EMC effect** and **SSFs.** Useful analysis for **planned experiments** in future facilities

- For $^4He$ the deviations from experimental data could be ascribed to **genuine QCD effects: our results provide** a **reliable baseline to study exotic phenomena**

- In every case the **dependence** on the choice of the **nuclear potential** and the **free nucleon SFs** is largely **under control**

## To do next:

- Include **off-shell** corrections to our calculations

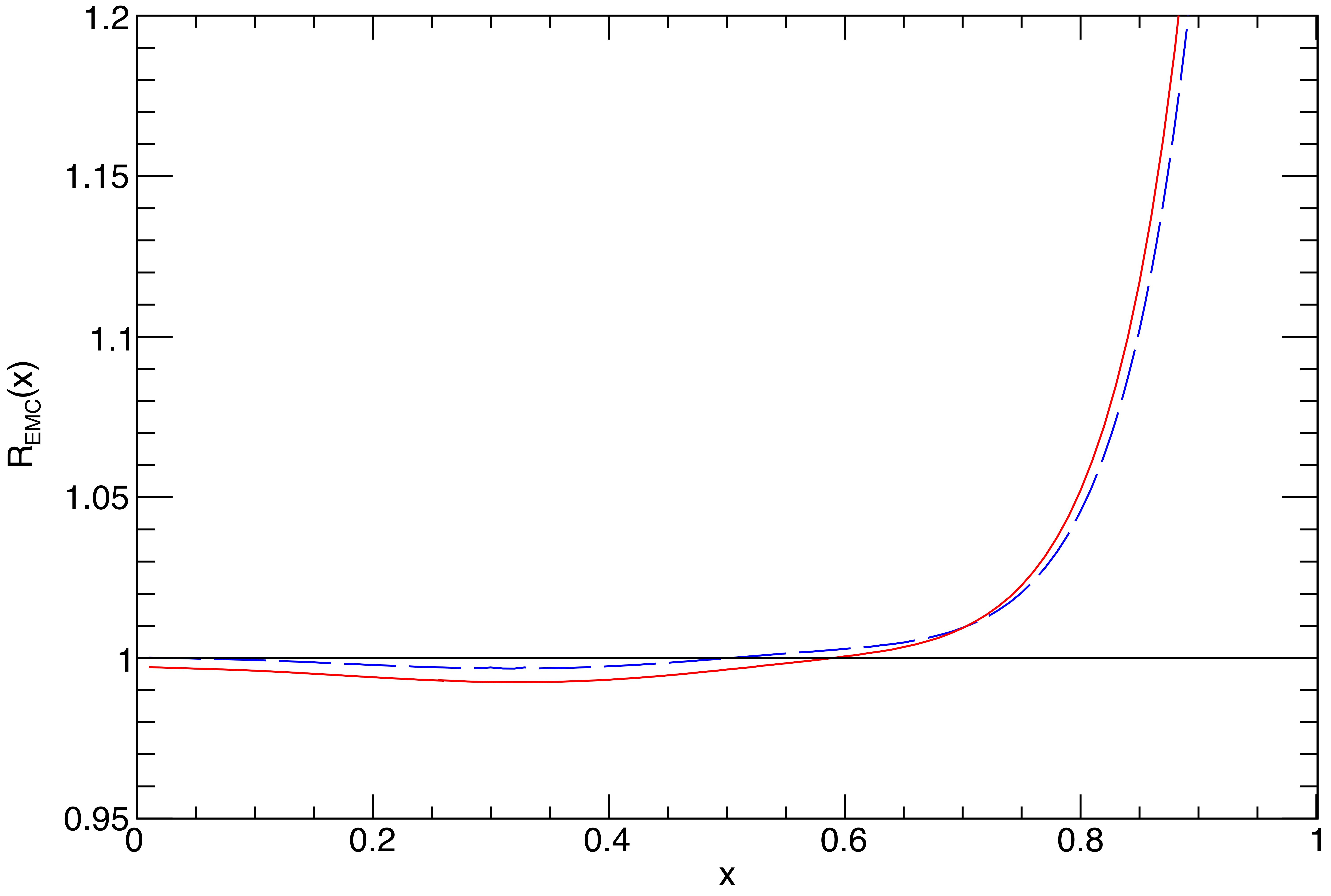
- Calculate the EMC effect for **heavier nuclei.** We are **working on** $^6Li$

## In preparation:

- With the s**ame approach** we are developing a new formalism for the calculation of the **nuclear Double Parton Distributions (DPDs)** for **light nuclei***

* A.Ceccopieri, F.F., N.Iles, E.Pace, M.Rinaldi and G. Salmè, **in preparation**

# Tritium EMC effect




Results similar to $^3He$ and $^4He$

No experimental data

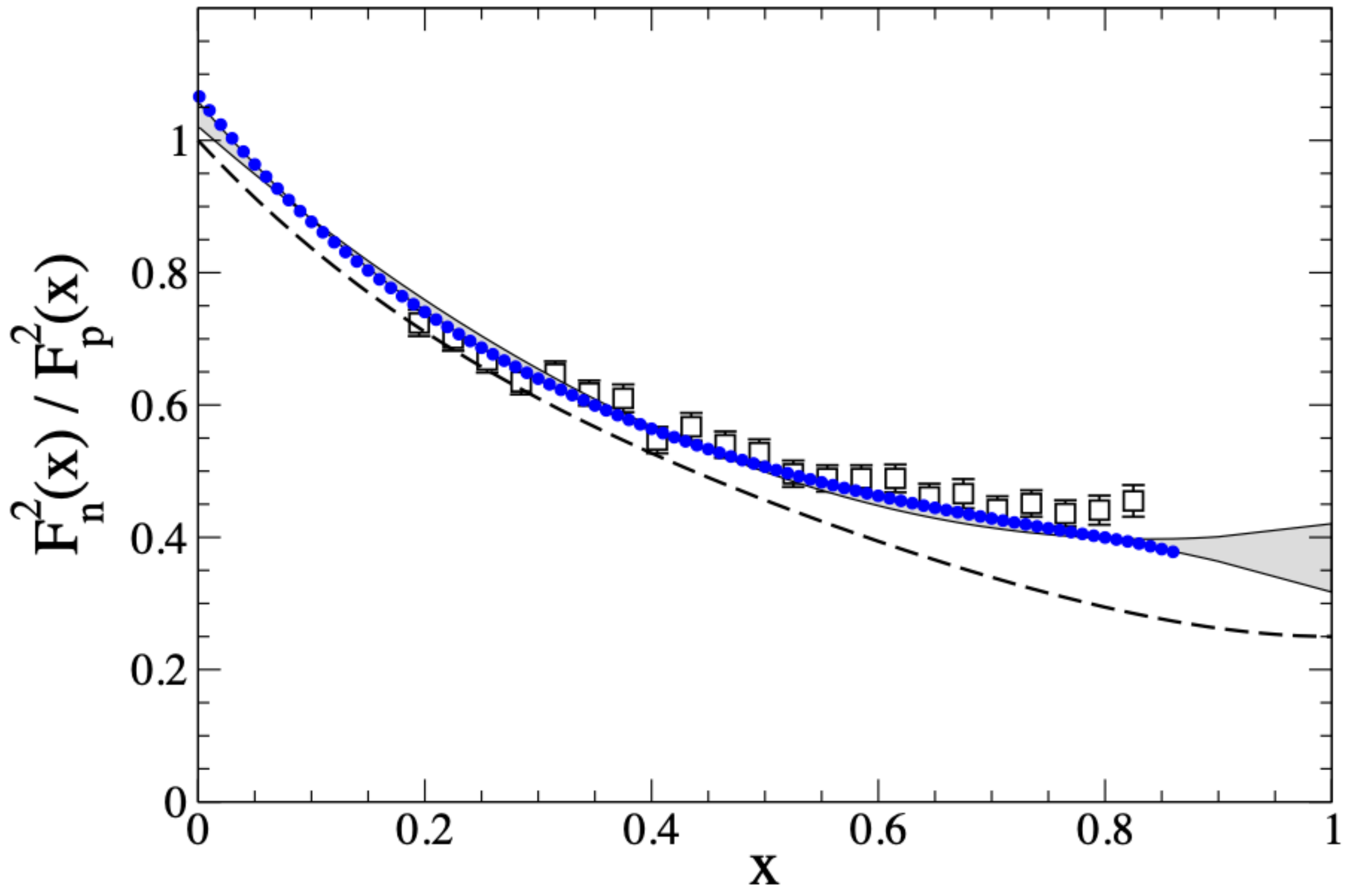
Solid line: Av18/UIX; Dashed-line: NVIb/UIX

# Extraction of $F_2^n/F_2^p$ via MARATHON data

MARATHON coll. : experimental data of the super-ratio $R^{ht}(x) = F_2^{^3He}(x)/F_2^{3H}(x)$

$^3He$: 2p + n; $^3H$: n + 2p

Is possible to extract the ratio $F_2^n(x)/F_2^p(x)$ through the super-ratio




**E.Pace, M.Rinaldi, G.Salmè and S.Scopetta Phys. Lett. B 839(2023) 127810**

Dashed line: ratio from SMC collaboration
Empty squares: MARATHON extraction
Solid line: cubic and conic extractions from $F_2^p$ SMC parametrization, fitted to MARATHON data

# Canonical and LF spin

- In Instant form (initial hyperplane t=0), one can couple spins and orbital angular momenta via Clebsch-Gordan (CG) coefficients. In this form the three rotation generators are independent of the the interaction.

- To embed the CG machinery in the LFHD one needs unitary operators, the so-called Melosh rotations that relate the LF spin wave function and the canonical one. For a particle of spin (1/2) with LF momentum $\tilde{\mathbf{k}} \equiv \{k^+, \vec{k}_\perp\}$

$$|\mathbf{k}; \frac{1}{2}, \sigma\rangle_c = \sum_{\sigma'} \underbrace{D^{1/2}_{\sigma', \sigma}(R_M(\tilde{\mathbf{k}}))}_{\text{Wigner rotation for the J=1/2 case}} |\tilde{\mathbf{k}}; \frac{1}{2}, \sigma'\rangle_{LF}$$

**Wigner rotation for the J=1/2 case**

- $R_M(\tilde{\mathbf{k}})$ is the Melosh rotation connecting the intrinsic LF and canonical frames, reached through different boosts from a given frame where the particle is moving

$$D^{1/2}[R_M(\tilde{\mathbf{k}})]_{\sigma'\sigma} = \chi^\dagger_{\sigma'} \frac{m + k^+ - \imath \boldsymbol{\sigma} \cdot (\hat{z} \times \mathbf{k}_\perp)}{\sqrt{(m+k^+)^2 + |\mathbf{k}_\perp|^2}} \chi_\sigma = {}_{LF}\langle \tilde{\mathbf{k}}; s\sigma' | \mathbf{k}; s\sigma\rangle_c$$

$\chi_\sigma$ → **two-dimensional spinor**

**N.B. If** $|\mathbf{k}_\perp| << k^+, m \longrightarrow D_{\sigma'\sigma} \simeq I_{\sigma'\sigma}$

# LF spectral function and LC Correlator

The fermion correlator in terms of the LF coordinates is [e.g., Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002)]

p = fermion momentum

isospin

$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \frac{1}{2}\int d\xi^- d\xi^+ d\boldsymbol{\xi}_T\ e^{i\,p\xi}\ \langle P,S,A|\bar{\psi}^{\tau}_{\beta}(0)\psi^{\tau}_{\alpha}(\xi)|A,S,P\rangle$$

parent system (nucleus, nucleon..)

The particle contribution to the correlator in valence approximation, i.e. the result obtained if the antifermion contributions are disregarded, is related to the LF SF:

$$\Phi^{\tau p}(p,P,S) = \frac{(\not{p}_{on} + m)}{2m}\ \Phi^{\tau}(p,P,S)\ \frac{(\not{p}_{on} + m)}{2m} = \frac{2\pi\ (P^+)^2}{(p^+)^2\ 4m}\ \frac{E_S}{\mathcal{M}_0[1,(23)]}\sum_{\sigma\sigma'}\{\ u_{\alpha}(\tilde{\mathbf{p}},\sigma')\ \mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\ \bar{u}_{\beta}(\tilde{\mathbf{p}},\sigma)\ \}$$

In deriving this expression it naturally appears the momentum $\tilde{\boldsymbol{\kappa}}$ in the intrinsic reference frame of the cluster [1,(23)], where particle 1 is free and the (23) pair is fully interacting.

# TMDs and LF spectral function

14

TMDs → (obtained from) Tr LC correlator → (obtained from) Tr Spectral function

$$\mathrm{Tr}(\gamma^+\Phi^{\mathrm{P}}) = D\ Tr\left[\hat{\mathcal{P}}_{\mathcal{M}}(\tilde{\kappa},\epsilon,S)\right]$$

$$\mathrm{Tr}(\gamma^+\gamma_5\ \Phi^{\mathrm{P}}) = D\ Tr\left[\sigma_z\ \hat{\mathcal{P}}_{\mathcal{M}}(\tilde{\kappa},\epsilon,S)\right]$$

$$\mathrm{Tr}(\not{p}_\perp\ \gamma^+\ \gamma_5\ \Phi^{\mathrm{P}}) = D\ Tr\left[\boldsymbol{p}_\perp\cdot\boldsymbol{\sigma}\ \hat{\mathcal{P}}_{\mathcal{M}}(\tilde{\kappa},\epsilon,S)\right]$$

$$D = \frac{(P^+)^2}{p^+}\ \frac{\pi}{m}\ \frac{E_S}{\mathcal{M}_0[1,(23)]}$$

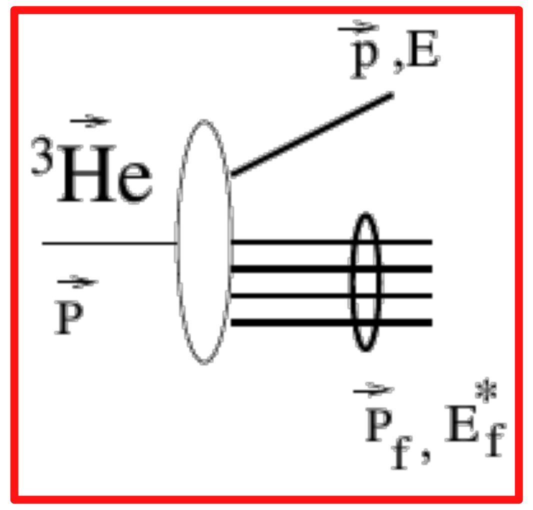
The integration $\frac{1}{2}\int\frac{dp^+dp^-}{(2\pi)^4}\ \delta[p^+-xP^+]\ P^+$ of Tr of SF

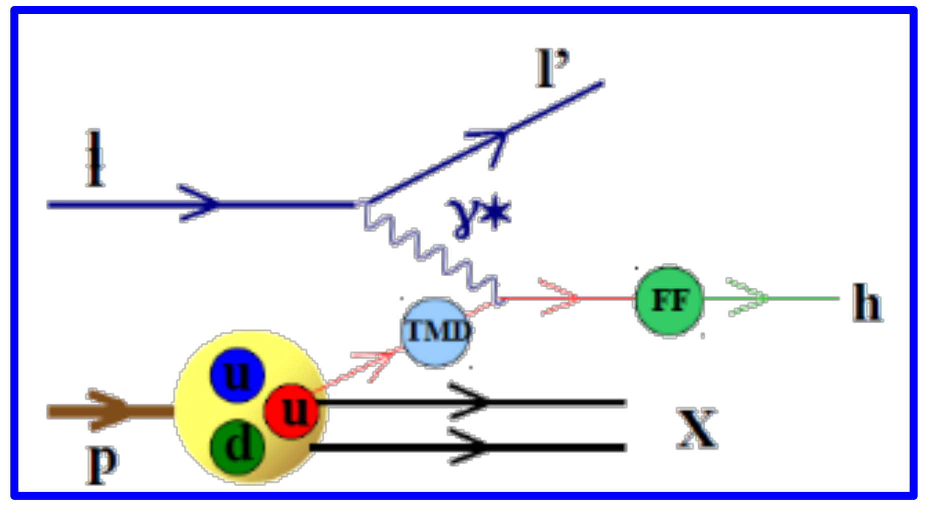
$$f(x,\mathbf{p}_\perp^2) = b_0 \quad \Delta f(x,|\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + b_{5,\mathcal{M}} \quad g_{1T}(x,|\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b_{4,\mathcal{M}}$$

$$\Delta'_T f(x,|\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + \frac{1}{2}\ b_{2,\mathcal{M}} \quad h_{1L}^\perp(x,|\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b_{3,\mathcal{M}} \quad h_{1T}^\perp(x,|\mathbf{p}_\perp|^2) = \frac{M^2}{|\mathbf{p}_\perp|^2}\ b_{2,\mathcal{M}}$$

# TMDs and $^3$He LF spectral function

The procedure works for any three-body J = 1/2 system (in valence approx!)




**$^3$He**

- $p$, $p$, $n$
- $(e, e'p)$ reactions
- $p$ detection
- PW Impulse Approximation
- spin-dep response functions
- light-cone momentum distributions
- norms, effective polarizations

CORRESPONDENCE

**Proton**

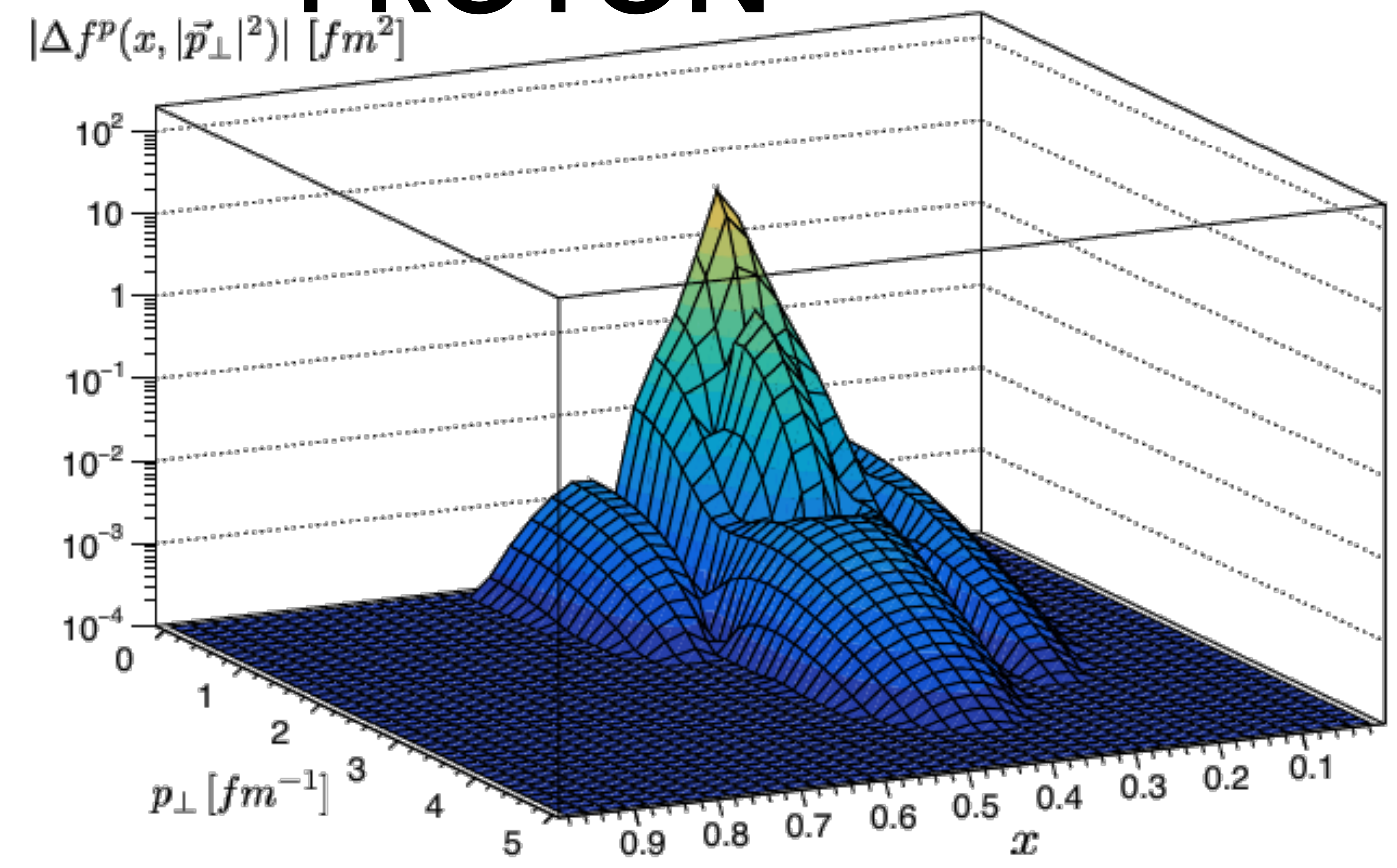
- $u_v$, $u_v$, $d_v$
- SIDIS
- no $q_v$ detection, fragmentation...
- leading twist
- TMDs
- PDFs
- charges (axial, tensor...)

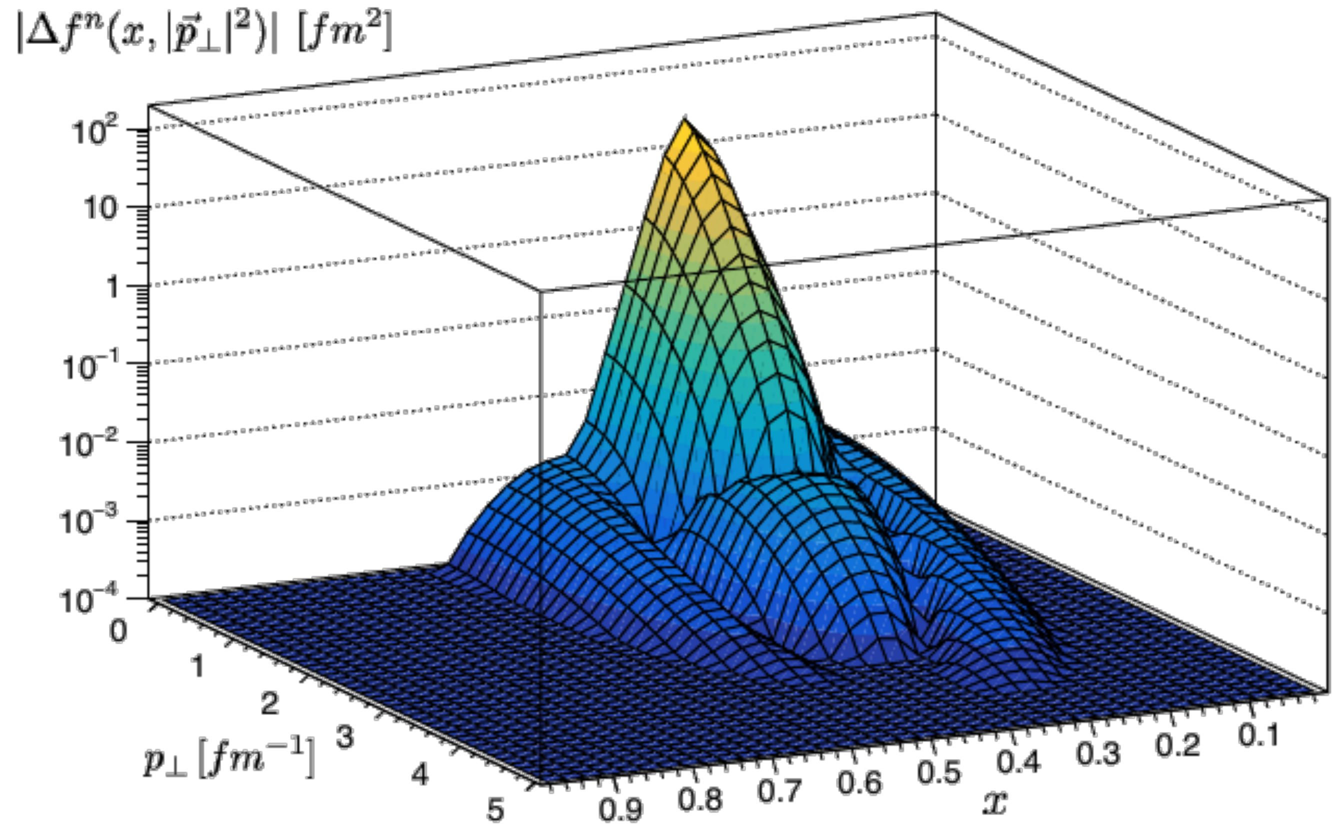
- the $^3$He TMDs could be obtained from spin asymmetries in $^3\vec{He}(\vec{e}, e'p)$ experiments: in progress!
- We show our calculation for the TMDs of He using Av18 + UIX wfs (A. Kievsky, M. Viviani et al.)
  Thus testing LFRHD and of the importance of Relativity in nuclear structure.

# $^3$He TMDs

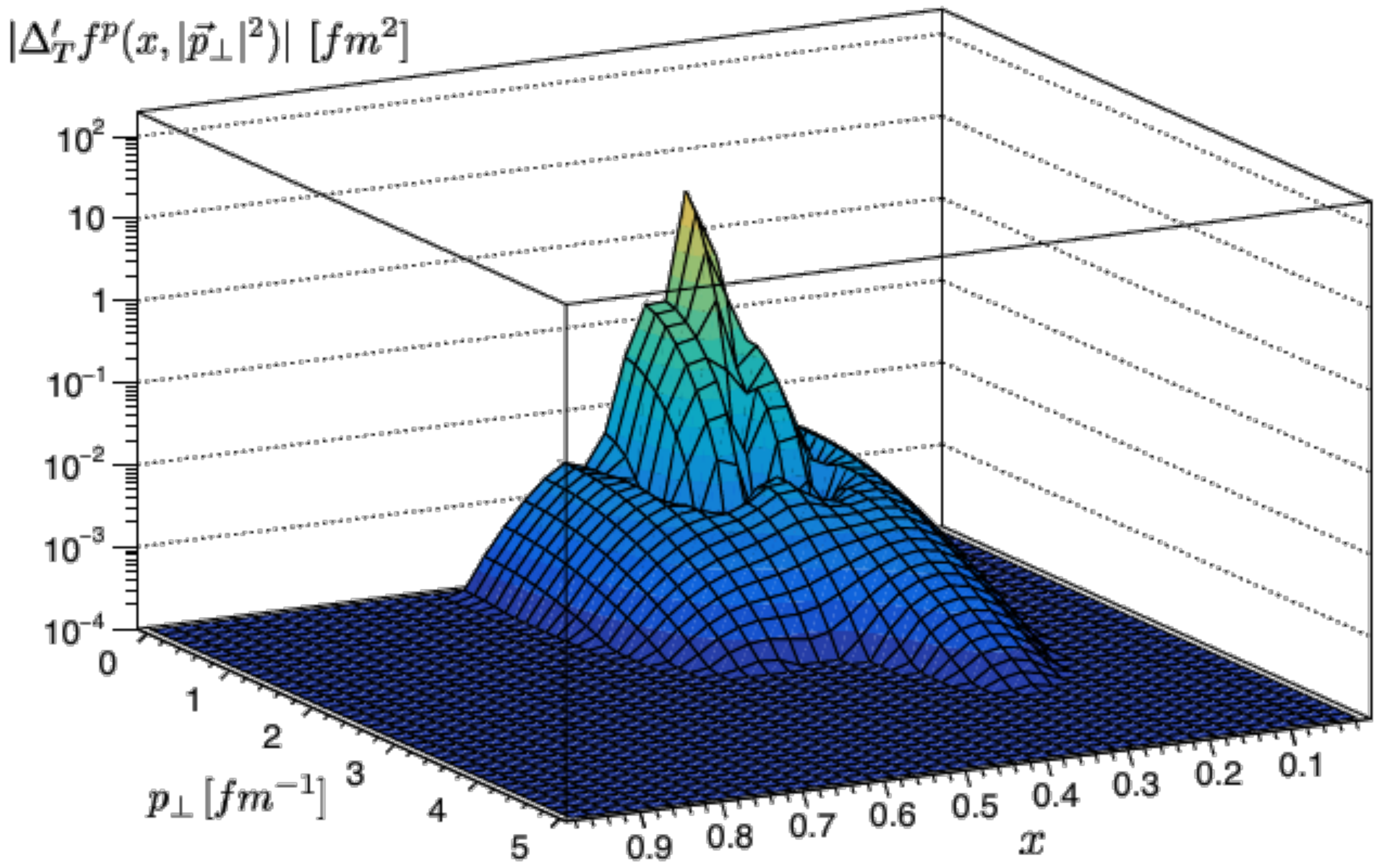
**Numerical results** A. Del Dotto, E. Pace, G. Perna, A. Rocco, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)

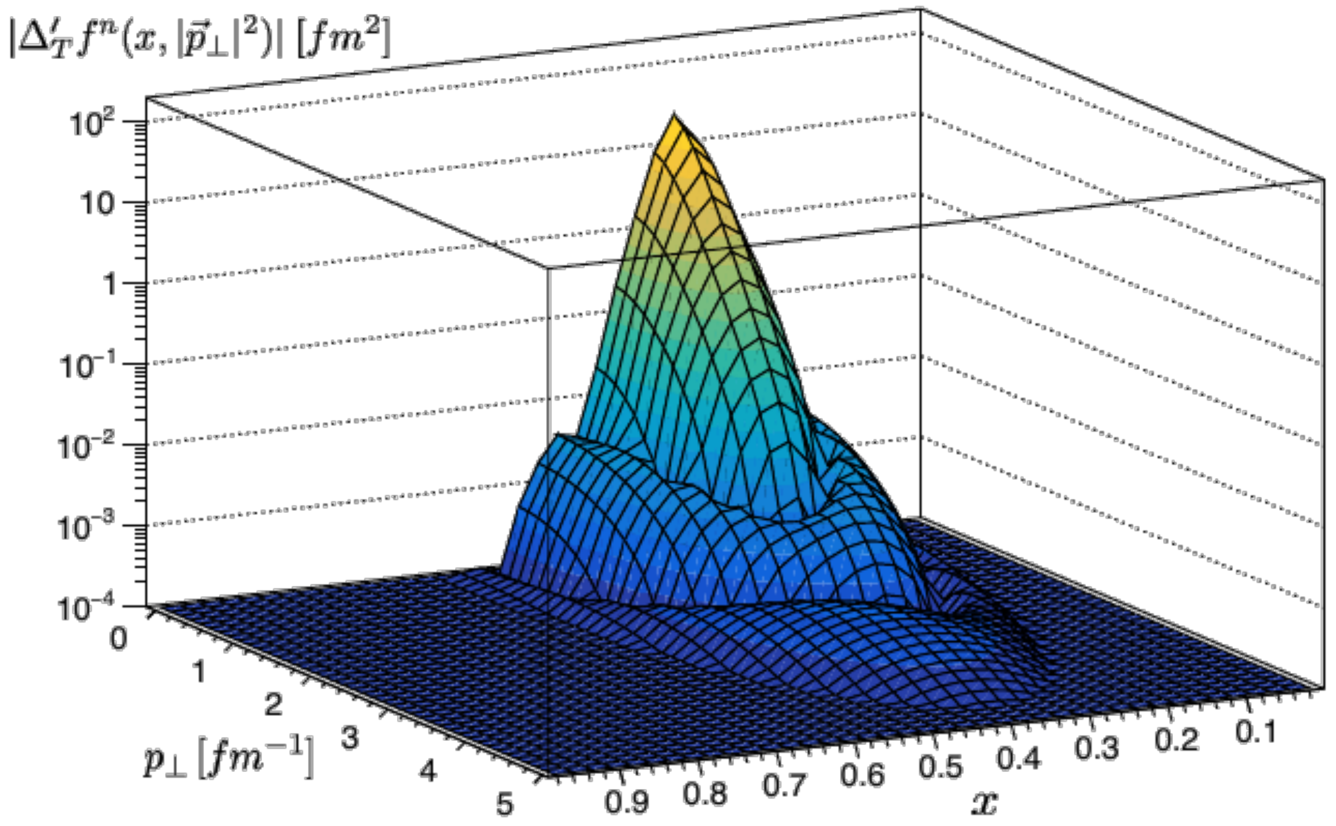
**PROTON** | **NEUTRON**




$\Delta f^{\tau}(x,|\mathbf{p}_{\perp}|^2)$



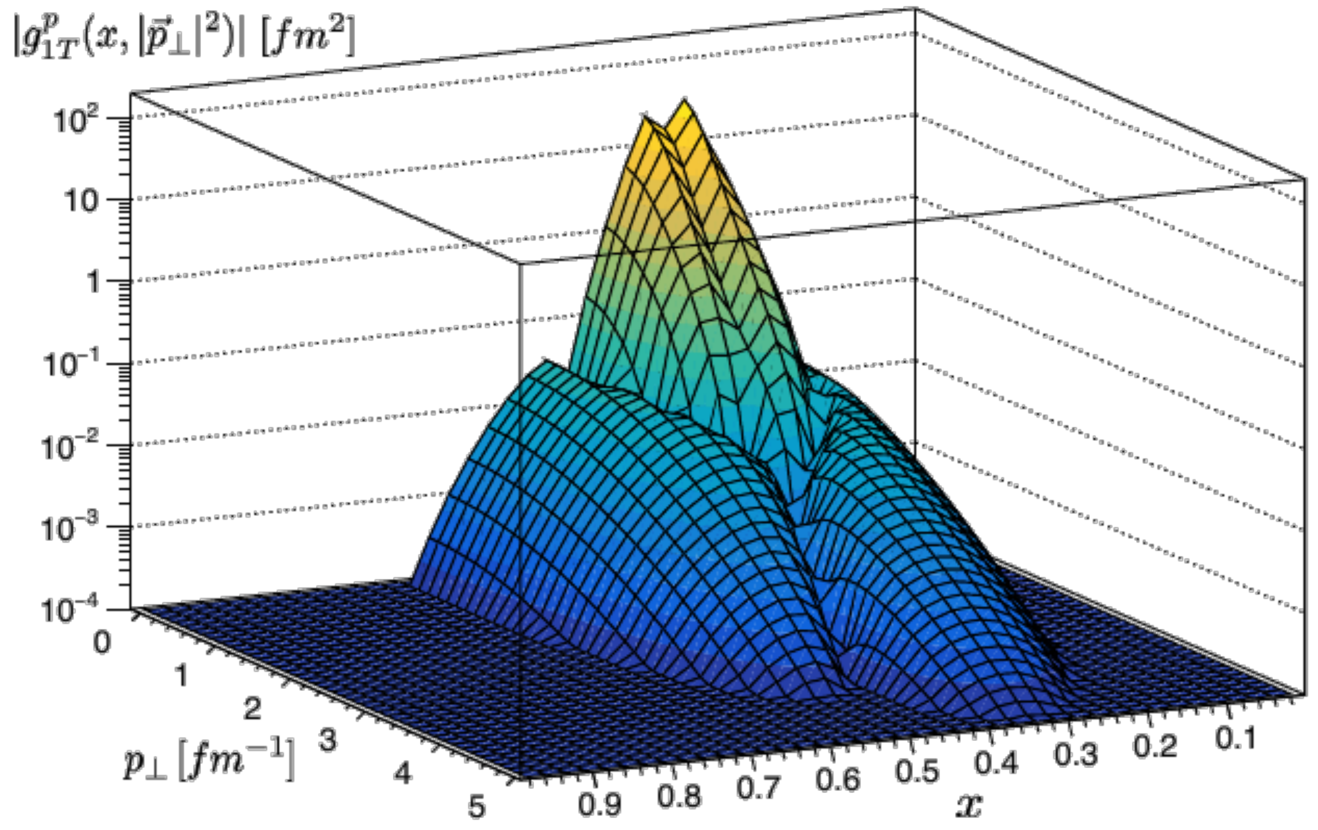




$\Delta'_T f^{\tau}(x,|\mathbf{p}_{\perp}|^2)$

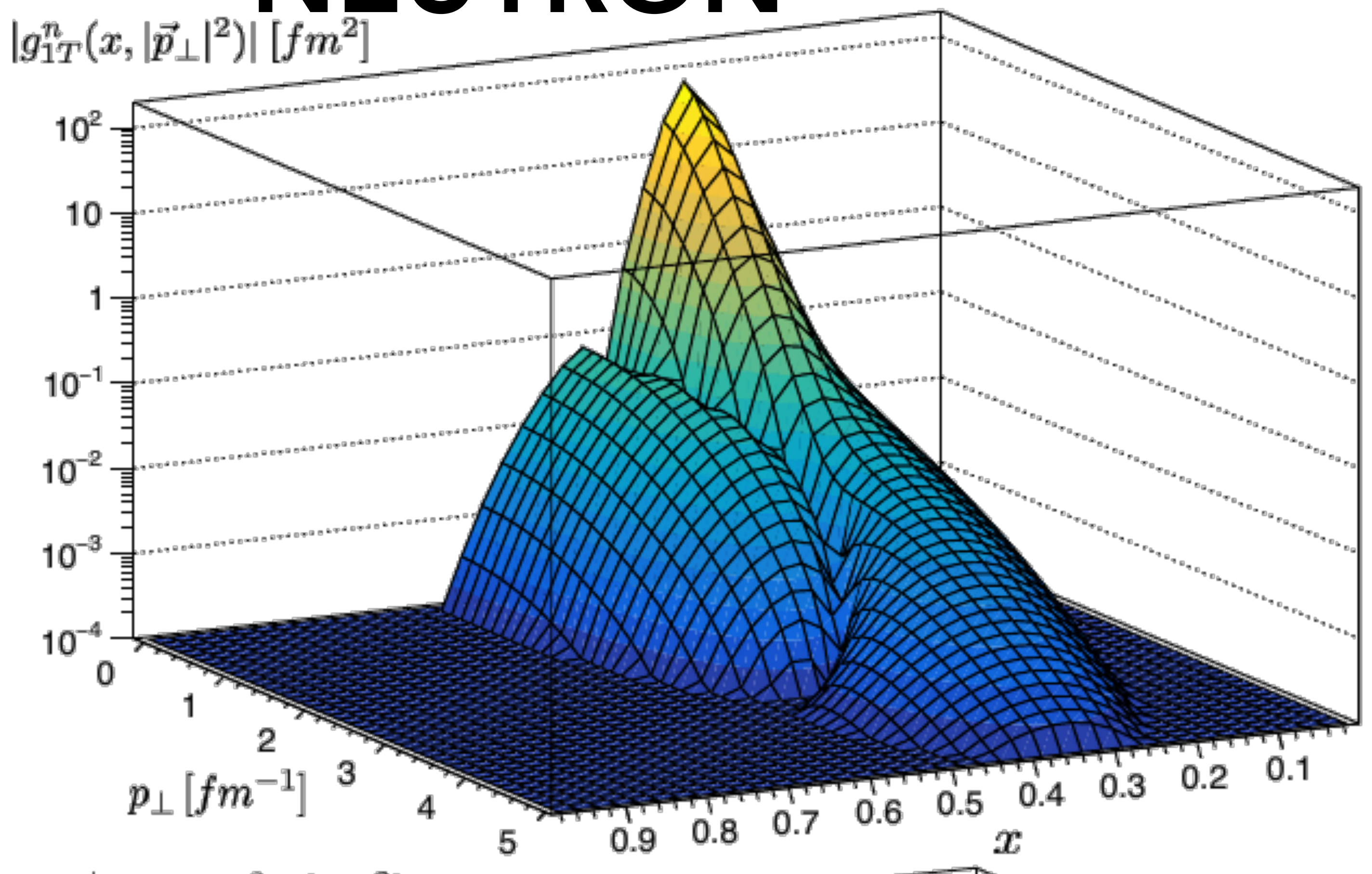
# $^3$He TMDs

**Numerical results** A. Del Dotto, E. Pace, G. Perna, A. Rocco, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)

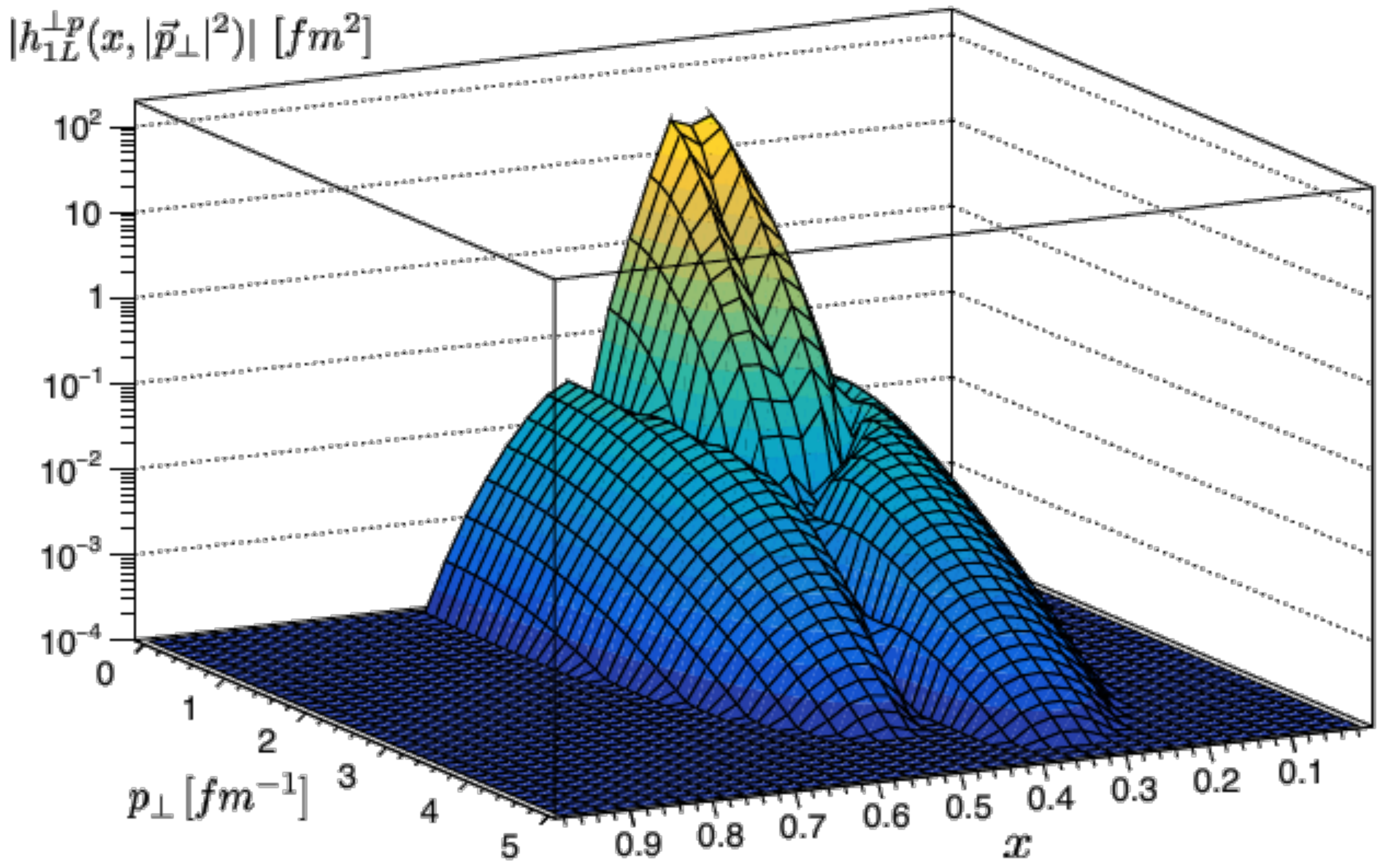
**PROTON**

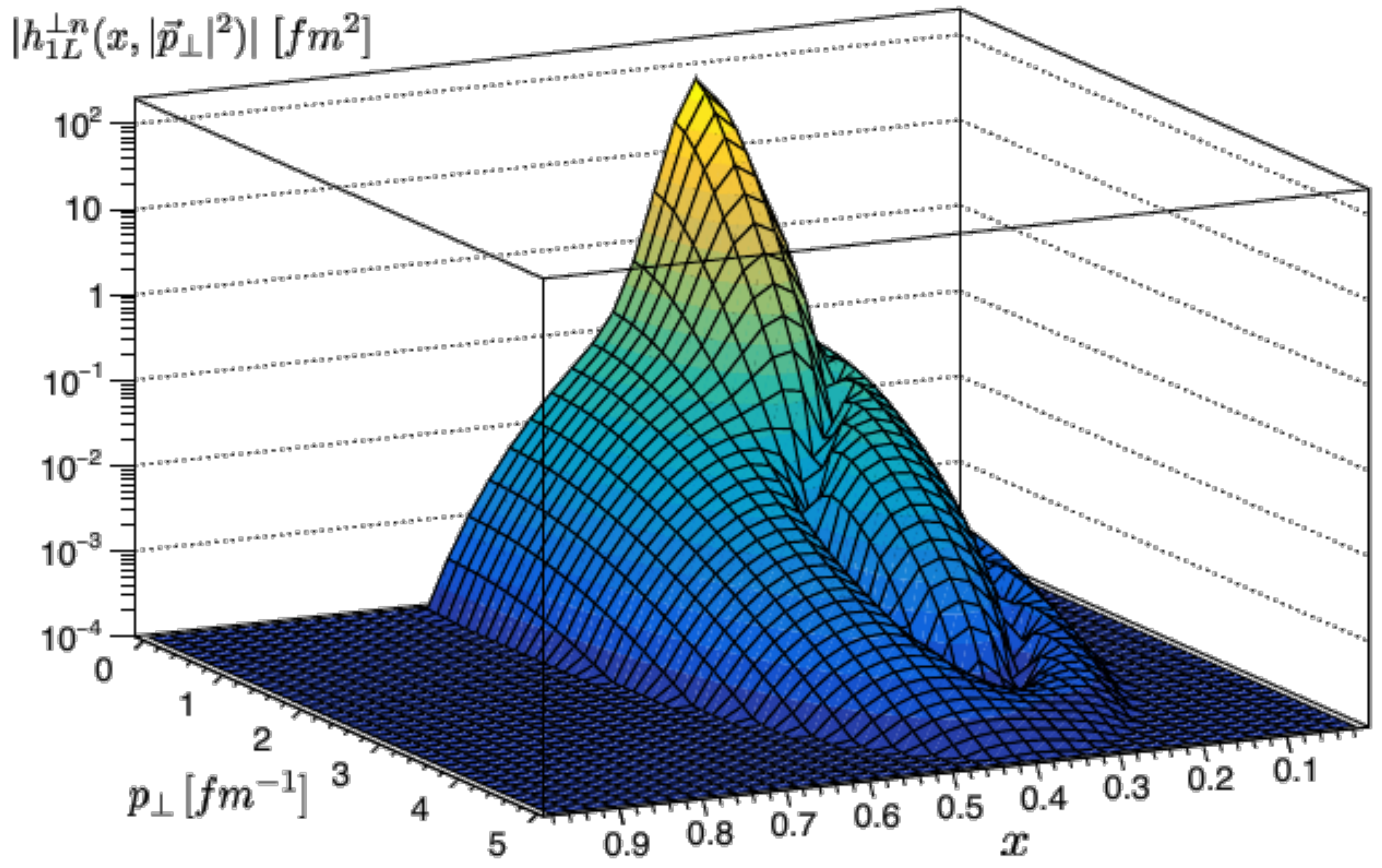



**NEUTRON**




Absolute value of the nucleon longitudinal-polarization distribution, $g_{1T}^{\tau}(x, |\mathbf{p}_\perp|^2)$, in a transversely polarized $^3$He.



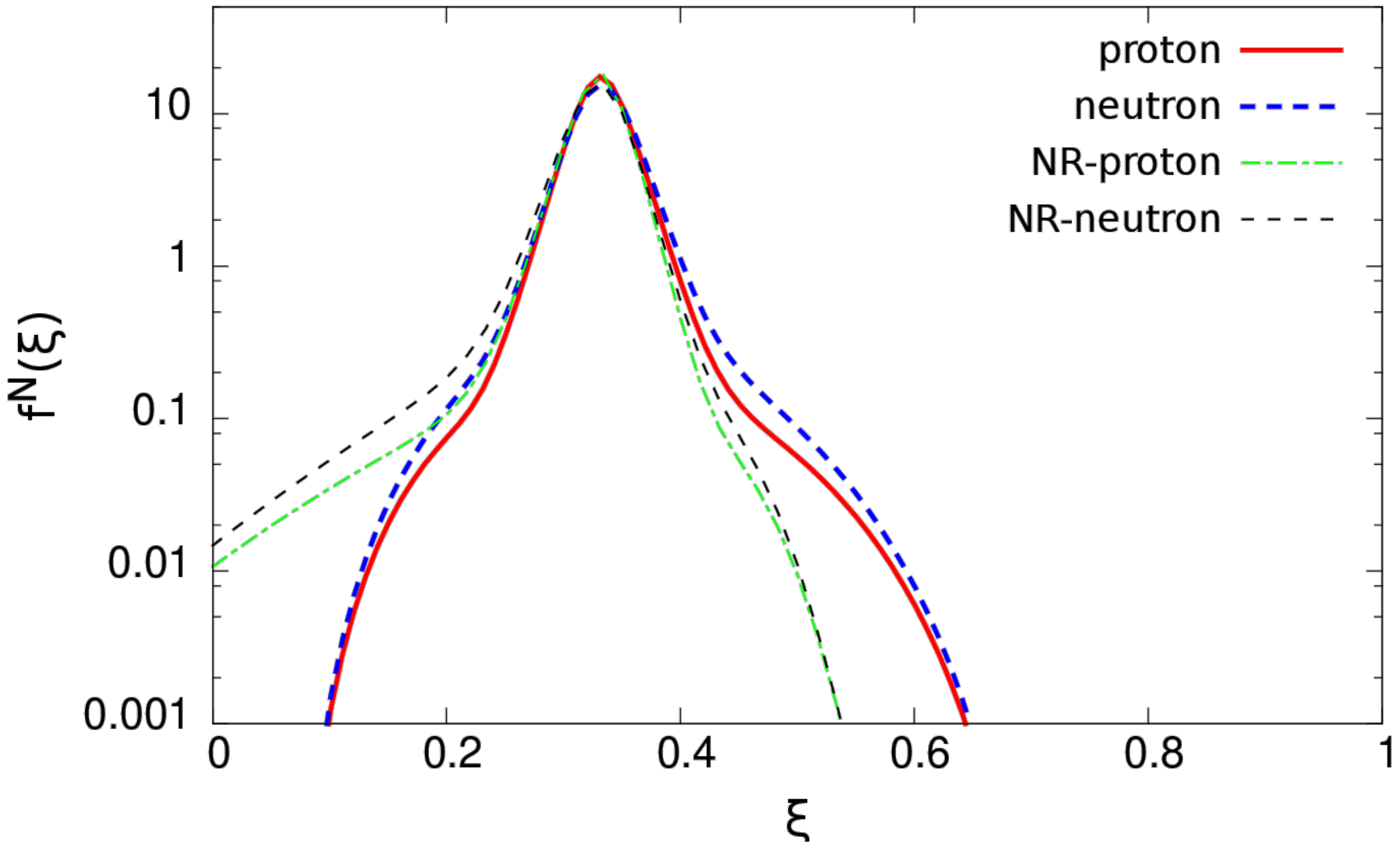




Absolute value of the nucleon transverse-polarization distribution, $h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2)$ in a longitudinally polarized $^3$He.

# LC momentum distributions

From the normalization of the Spectral Function one has

$$f_\tau^A(\xi) = \int d\mathbf{k}_\perp \; n^\tau(\xi, \mathbf{k}_\perp) \quad \Longrightarrow \quad \int_0^1 d\xi \; f_\tau^A(\xi) \; = \; 1$$

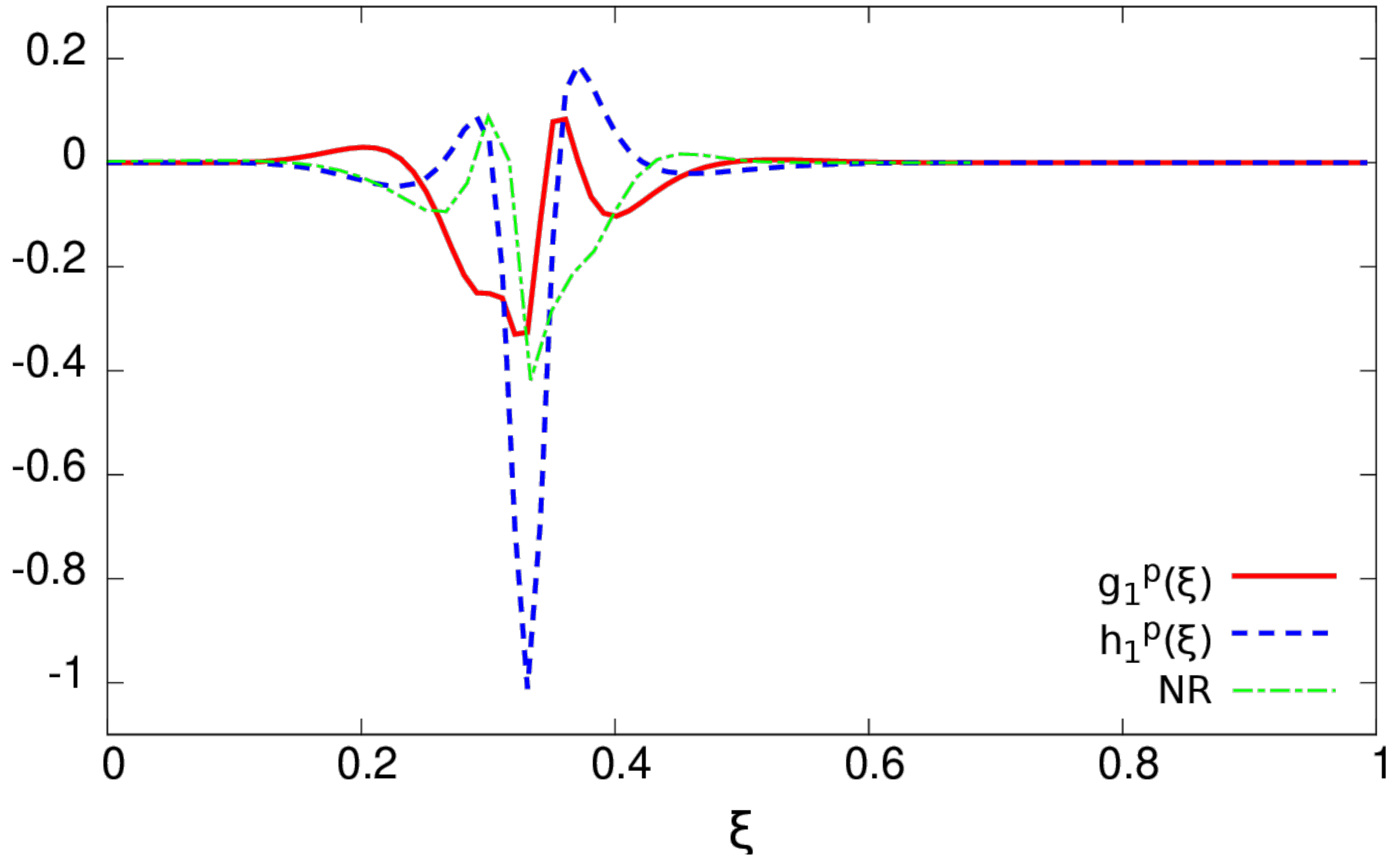



unpolarized distribution

E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485

# LC momentum distributions

PROTON

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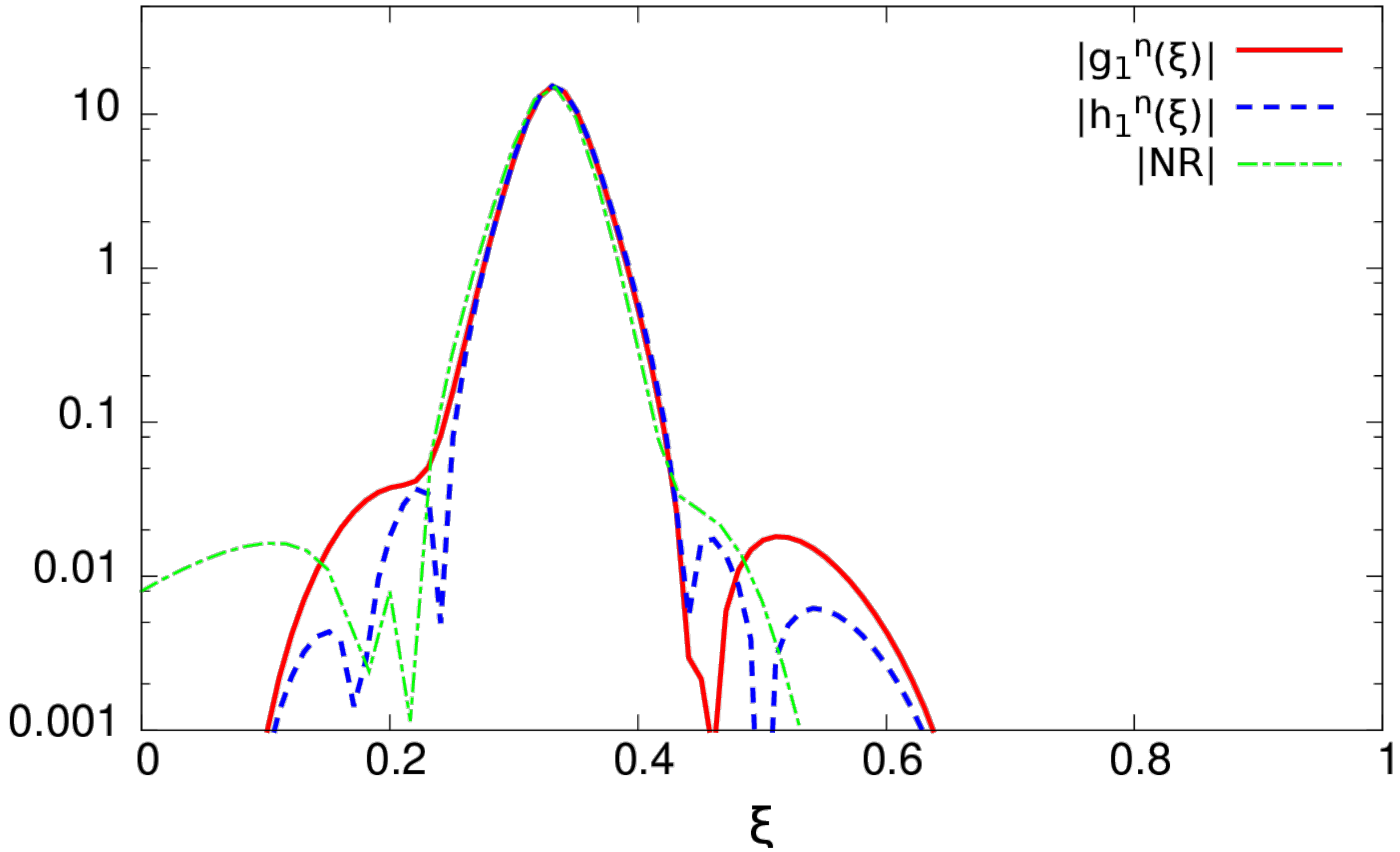



E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485

$g_1^n(\xi)$ longitudinal-polarization distribution

$h_1^n(\xi)$ transverse-polarization distribution

- They would be the same in a NR framework;
- Crucial for the extraction of the neutron information from DIS and SIDIS off $^3$He.
  Work in progress to LF update our NR results $\longrightarrow$ important for JLab12, EIC

## Effective polarizations

Key role in the extraction of neutron polarized structure functions and neutron Collins and Sivers single spin asymmetries, from the corresponding quantities measured for $^3$He

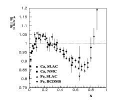
Effective longitudinal polarization (axial charge for the nucleon)

$$p_{||}^{\tau} = \int_0^1 dx \int d\mathbf{p}_\perp \; \Delta f^{\tau}(x, |\mathbf{p}_\perp|^2)$$

Effective transverse polarization (tensor charge for the nucleon)

$$p_{\perp}^{\tau} = \int_0^1 dx \int d\mathbf{p}_\perp \; \Delta_T' f^{\tau}(x, |\mathbf{p}_\perp|^2)$$

| Effective polarizations | proton | neutron |
|---|---|---|
| LF longitudinal polarization | -0.02299 | 0.87261 |
| LF transverse polarization | -0.02446 | 0.87314 |
| non relativistic polarization | -0.02118 | 0.89337 |



- The difference between the LF polarizations and the non relativistic results are up to 2% in the neutron case (larger for the proton ones, but it has an overall small contribution), and should be ascribed to the intrinsic coordinates, implementing the Macro-locality, and not to the Melosh rotations involving the spins.
- N.B. Within a NR framework: $p_{||}^{\tau}(NR) = p_{\perp}^{\tau}(NR)$

# Backup Slides: effective polarizations

The BT Mass operator for A=3 nuclei - II

The NR mass operator is written as

$$M^{NR} = 3m + \sum_{i=1,3} \frac{k_i^2}{2m} + V_{12}^{NR} + V_{23}^{NR} + V_{31}^{NR} + V_{123}^{NR}$$

and must obey to the commutation rules proper of the Galilean group, leading to translational invariance and independence of total 3-momentum.
Those properties are analogous to the ones in the BT construction. This allows us to consider the standard non-relativistic mass operator as a sensible BT mass operator, and embed it in a Poincaré covariant approach.

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT} \sim M^{NR}$$

The 2-body phase-shifts contain the relativistic dynamics, and the Lippmann-Schwinger equation, like the Schrödinger one, has a suitable structure for the BT construction. Therefore what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.

The eigenfuntions of $M^{NR}$ do not fulfill the cluster separability, but we take care of Macro-locality in the spectral function.

# LF spectral function decomposition

The LF spin-dependent spectral function (SF), for a nucleus with polarization S, can be macroscopically decomposed in terms of the available vectors:

- the unit vector $\hat{n}$ , $\perp$ to the hyperplane $n^\mu x_\mu = 0$. Our choice is $n^\mu \equiv \{1,0,0,1\} \Rightarrow \hat{n} \equiv \hat{z}$
- the polarization vector **S**
- the transverse (wrt the $\hat{z}$ axis) momentum component of the constituent, i.e. $\mathbf{k}_\perp(123) = \mathbf{p}_\perp(Lab) = \boldsymbol{\kappa}_\perp(1;23)$

$$\mathcal{P}^{\mathcal{T}}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S) = \frac{1}{2}\left[\mathcal{B}^{\mathcal{T}}_{0,\mathcal{M}} + \boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{F}}^{\mathcal{T}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,\mathbf{S})\right]_{\sigma'\sigma}$$

unpolarized SF $\mathcal{B}^{\mathcal{T}}_{0,\mathcal{M}} = Tr\left[\mathcal{P}^{\mathcal{T}}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right]$

$\boldsymbol{\mathcal{F}}^{\mathcal{T}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,\mathbf{S}) = Tr\left[\hat{\mathcal{P}}^{\mathcal{T}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\,\boldsymbol{\sigma}\right]$ pseudovector

$$\boldsymbol{\mathcal{F}}^{\mathcal{T}}_{\mathcal{M}}(x,\mathbf{k}_\perp;\epsilon,\mathbf{S}) = \mathbf{S}\mathcal{B}^{\mathcal{T}}_{1,\mathcal{M}}(\ldots) + \hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^{\mathcal{T}}_{2,\mathcal{M}}(\ldots) + \hat{\mathbf{k}}_\perp(\mathbf{S}\cdot\hat{z})\mathcal{B}^{\mathcal{T}}_{3,\mathcal{M}}(\ldots) + \hat{z}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^{\mathcal{T}}_{4,\mathcal{M}}(\ldots) + \hat{z}(\mathbf{S}\cdot\hat{z})\mathcal{B}^{\mathcal{T}}_{5,\mathcal{M}}(\ldots)$$

$$x = \kappa^+(1;23)/\mathcal{M}_0(1;23)$$

The scalar functions $\mathcal{B}^{\mathcal{T}}_{i,\mathcal{M}}(\ldots)$ depend, for $\mathcal{J} = 1/2$, on $|\mathbf{k}_\perp|$, $x$, $\epsilon$

# LF spectral function and momentum distribution

By integrating the LF SF on $\kappa^-$, equivalent to the integration on the $\epsilon$ ≡ internal energy of the spectator system, one straightforwardly gets the LF spin-dependent momentum distribution

$$\mathcal{N}^{\tau}_{\sigma'\sigma}(x, \mathbf{k}_\perp; \mathcal{M}, \mathbf{S}) = \frac{1}{2}\{b_{0,\mathcal{M}}(\dots) + \boldsymbol{\sigma} \cdot \mathbf{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})\}_{\sigma'\sigma}$$

$$b_{0,\mathcal{M}}(\dots) \rightarrow \int d\epsilon \; \mathcal{B}^{\tau}_{i,\mathcal{M}}(\dots)$$

$$\mathbf{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S}) \rightarrow \int d\epsilon \; \boldsymbol{\mathcal{F}}^{\tau}_{\mathcal{M}}(x, \mathbf{k}_\perp; \epsilon, \mathbf{S})$$

The decomposition is useful to get:

an explicit interplay between transverse momentum component and spin dofs

relations between Transverse-momentum distributions (TMDs) in the *valence sector*