Vector meson spin alignments in different high energy reactions

Liang Zuo-tang
Shandong University

A recent short review:
J.H. Chen, ZTL, Y.G. Ma, X.L. Sheng, & Q. Wang, "*Vector meson's spin alignments in high energy reactions*", Sci. China-Phys. Mech. Astron. 68, 211001 (2025).

Outline

- Introduction
- The global vector meson spin alignment and quark spin correlations in relativistic heavy ion collisions (HIC)
- Vector meson alignment vs hyperon polarization in quark fragmentation
- Summary and outlook

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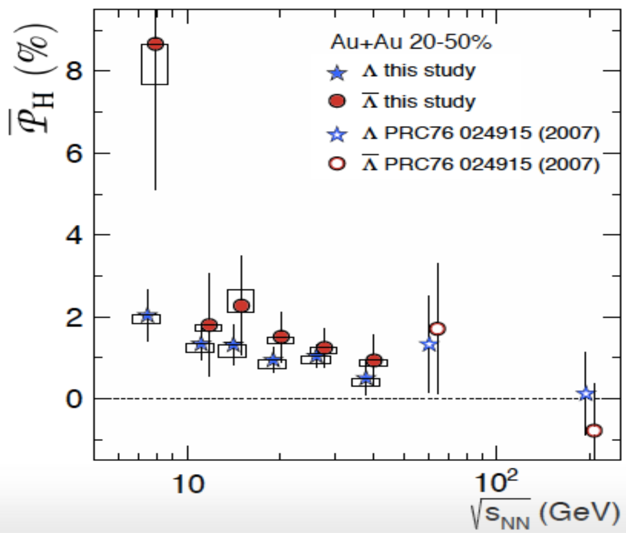
Introduction: Global Λ polarization in HIC has been observed

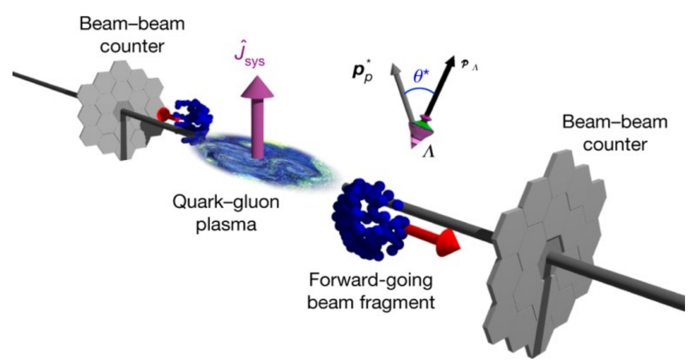
Nature 548, 62(2017)

LETTER

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration*





STAR

PRL 94, 102301 (2005) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2005

Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

Zuo-Tang Liang[1] and Xin-Nian Wang[2,1]

[1]*Department of Physics, Shandong University, Jinan, Shandong 250100, China*

[2]*Nuclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

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Introduction: Global ϕ meson spin alignment has been observed

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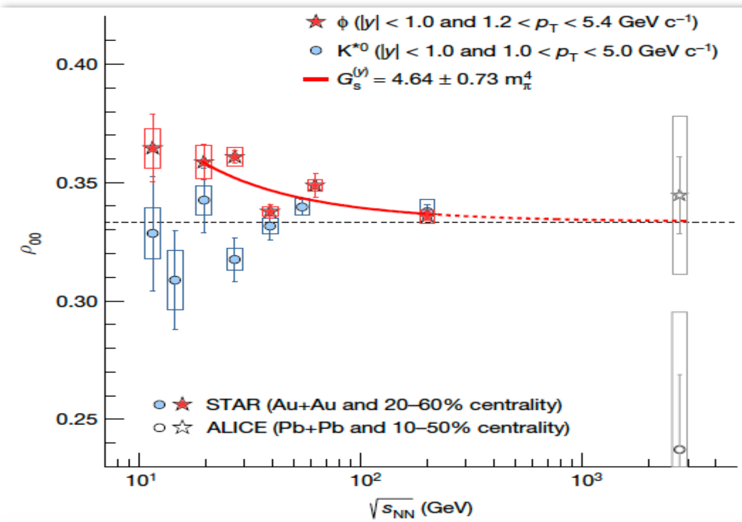
M.S. Abdallah *et al*., Nature 614, 244 (2023)

Article
Pattern of global spin alignment of ϕ and K^{*0} mesons in heavy-ion collisions

again in Nature !



- Global vector meson spin alignment confirmed
- However $\left|\rho_{00}^{V}-\frac{1}{3}\right| \gg P_\Lambda^2 \sim P_q^2$

Surprise ?

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Spin alignment of vector mesons in non-central $A+A$ collisions

Zuo-Tang Liang [a], Xin-Nian Wang [a,b]

PRL 94, 102301 (2005) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2005

Globally Polarized Quark-Gluon Plasma in Noncentral $A+A$ Collisions

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[1]*Department of Physics, Shandong University, Jinan, Shandong 250100, China*
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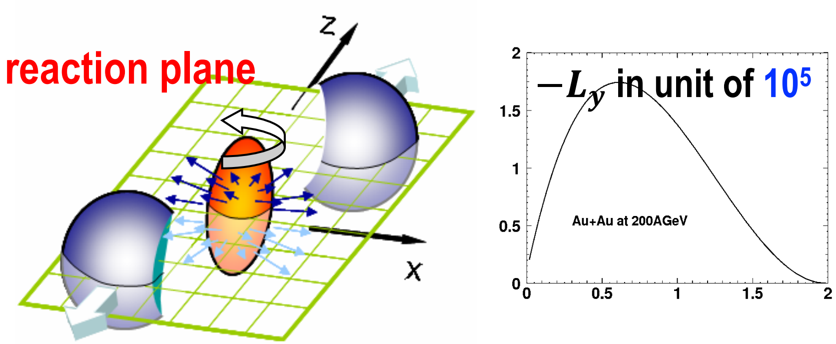
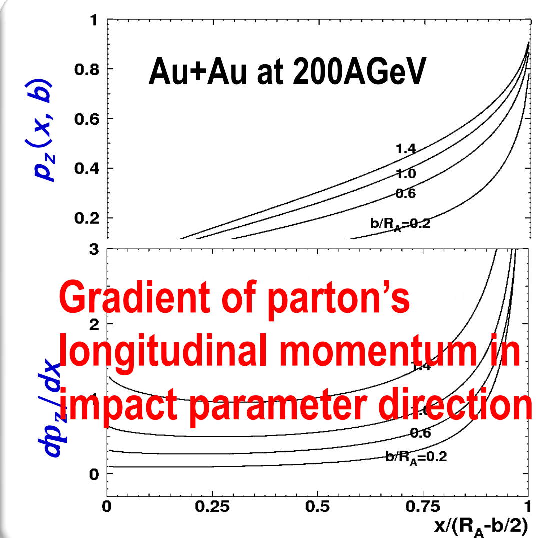
Introduction: The basic idea and result of the global polarization effect

Globally polarized quark gluon plasma (QGP) in relativistic heavy ion collisions



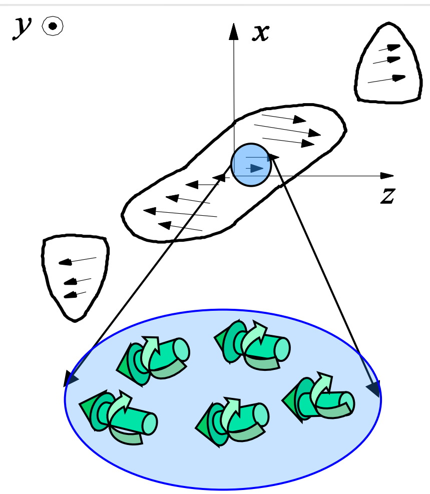
Huge orbital angular momentum

leads to



Gradient of parton's longitudinal momentum in impact parameter direction

QCD spin-orbit interactions



"global quark polarization"

hadronization (combination)

- Global hyperon polarization

$$P_H = P_{\bar{H}} = P_q = P_{\bar{q}}$$

PRL 94,102301 (2005)

- Global vector meson spin alignment

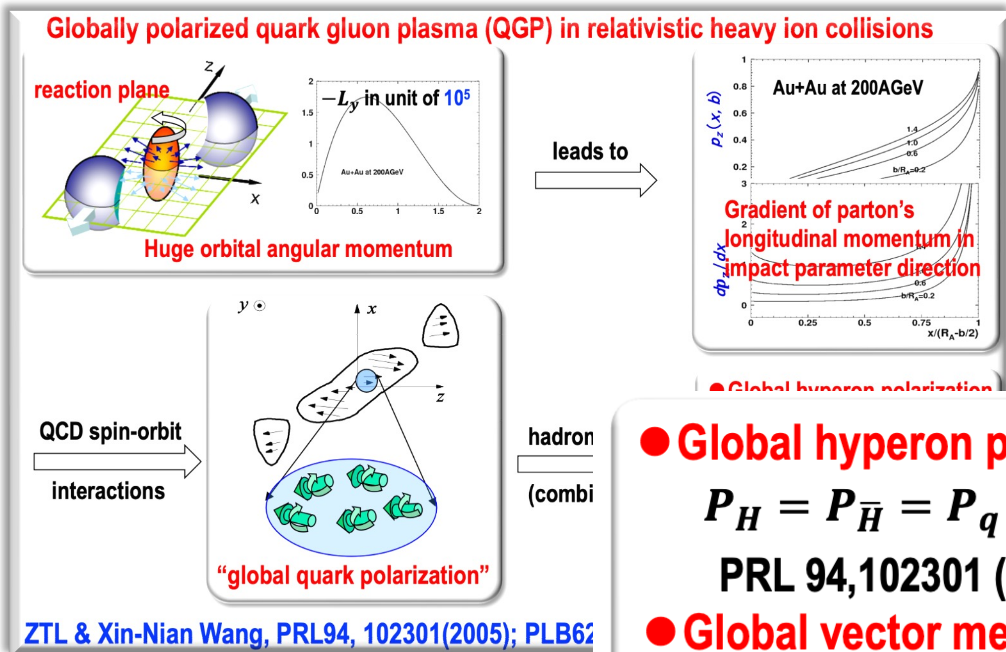
$$\rho_{00} = \frac{1 - P_q^2}{3 + P_q^2}$$

PLB 629, 20 (2005)

ZTL & Xin-Nian Wang, PRL94, 102301(2005); PLB629, 20 (2005).

Global vector meson spin alignment —— Why so interesting?

Theoretical predictions



- **Global hyperon polarization**

$$P_H = P_{\bar{H}} = P_q = P_{\bar{q}}$$

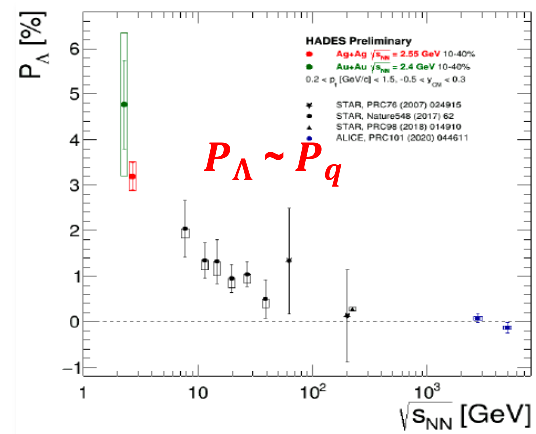
PRL 94,102301 (2005)

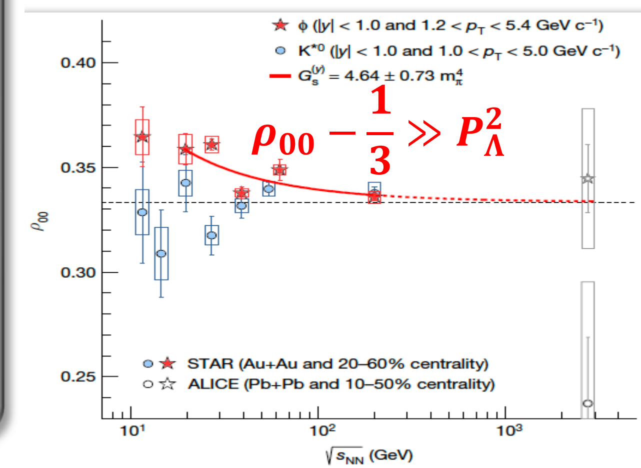
- **Global vector meson spin alignment**

$$\rho_{00} = \frac{1 - P_q^2}{3 + P_q^2}$$

PLB 629, 20 (2005)

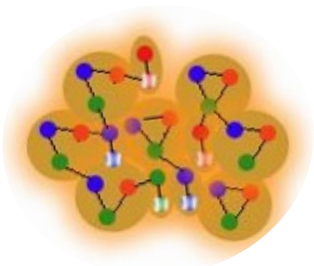
STAR experiments:





How can we understand it? What does it tell us?

Hadronization mechanism and spin transfer



QGP hadronization ⟹ combination / recombination / coalescence

$$\begin{pmatrix} q_1 + \bar{q}_2 \to M \\ q_1 + q_2 + q_3 \to B \\ \bar{q}_1 + \bar{q}_2 + \bar{q}_3 \to \bar{B} \end{pmatrix}$$

Spin density of h produced in quark combination is independent of transition matrix $\widehat{\mathcal{M}}$.

E.g., for $q_1 + \bar{q}_2 \to V$ $\quad \hat{\rho}^V = \widehat{\mathcal{M}}\hat{\rho}^{(q_1\bar{q}_2)}\widehat{\mathcal{M}}^\dagger \quad$ $\widehat{\mathcal{M}}$: the transition matrix

However $\quad \rho^V_{mm'} = \langle jm|\hat{\rho}^V|jm'\rangle = \langle jm|\widehat{\mathcal{M}}\hat{\rho}^{(q_1\bar{q}_2)}\widehat{\mathcal{M}}^\dagger|jm'\rangle$

$= N_j \langle jm|\hat{\rho}^{(q_1\bar{q}_2)}|jm'\rangle$

independent of $\widehat{\mathcal{M}}$

spin state of V

⟹ direct probe of $\hat{\rho}^{(q_1\bar{q}_2)}$ before hadronization

Proof: $\rho^V_{mm'} = \sum_{m_i m'_i} \langle jm|\widehat{\mathcal{M}}|m_i\rangle\langle m_i|\hat{\rho}^{(q_1\bar{q}_2)}|m'_i\rangle\langle m'_i|\widehat{\mathcal{M}}^\dagger|jm'\rangle \qquad |m_i\rangle \equiv |j_1 m_1, j_2 m_2\rangle$

$$\langle jm|\widehat{\mathcal{M}}|m_i\rangle = \sum_{j'm'}\langle jm|\widehat{\mathcal{M}}|j'm'\rangle\langle j'm'|m_i\rangle = \langle jm|\widehat{\mathcal{M}}|jm\rangle\langle jm|m_i\rangle = c_j\langle jm|m_i\rangle$$

Wigner-Eckhart theorem: $\quad \langle jm|\widehat{\mathcal{M}}|jm\rangle = \langle j\|\widehat{\mathcal{M}}\|j\rangle$

Global vector meson spin alignment —— calculations in 2005

ZTL & Xin-Nian Wang, PRL94, 102301 (2005); PLB629, 20 (2005).

Quark spin density matrix: $\hat{\rho}^{(q)} = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix}$ constant / average value

Hyperon: $q_1^{\uparrow} + q_2^{\uparrow} + q_3^{\uparrow} \to H$ $\hat{\rho}^{(q_1 q_2 q_3)} = \hat{\rho}^{(q_1)} \otimes \hat{\rho}^{(q_2)} \otimes \hat{\rho}^{(q_3)}$

$\rho^{H}_{mm'} = \langle j_H m' | \hat{\rho}^{(q_1 q_2 q_3)} | j_H m \rangle$ $P_H = \sum_{i=1-3} c_i P_{qi} = P_q$

no correlation

c_i: constant determined by C.G. coefficients

Vector meson: $q_1^{\uparrow} + \bar{q}_2^{\uparrow} \to V$ $\hat{\rho}^{(q_1 \bar{q}_2)} = \hat{\rho}^{(q_1)} \otimes \hat{\rho}^{(\bar{q}_2)}$

$\rho^{V}_{mm'} = \langle j_V m' | \hat{\rho}^{(q_1 \bar{q}_2)} | j_V m \rangle$ $\rho^{V}_{00} = \frac{1 - P_{q_1} P_{\bar{q}_2}}{3 + P_{q_1} P_{\bar{q}_2}} = \frac{1 - P_q^2}{3 + P_q^2}$

It was for the most simplified case: only spin degree of freedom

① P_q was taken as a constant, no fluctuation, no correlations
② no other degree of freedom

Global vector meson spin alignment —— correlations?

Consider fluctuation and/or other degree of freedom, at least,

for $q_1^{\uparrow} + q_2^{\uparrow} + q_3^{\uparrow} \to H$

$$P_H = \left\langle\left\langle \sum_i c_i P_{qi} \right\rangle_H\right\rangle_S = \sum_i c_i \langle P_{qi}\rangle = \langle P_q \rangle$$

for $q_1^{\uparrow} + \bar{q}_2^{\uparrow} \to V$

$$\rho_{00}^V = \frac{1-\langle P_q P_{\bar{q}}\rangle}{3+\langle P_q P_{\bar{q}}\rangle} \neq \frac{1-\langle P_q\rangle\langle P_{\bar{q}}\rangle}{3+\langle P_q\rangle\langle P_{\bar{q}}\rangle}$$

two folded average

$$\langle P_q P_{\bar{q}}\rangle = \left\langle \langle P_q P_{\bar{q}}\rangle_V \right\rangle_S$$

inside the meson V

over the system S

STAR Data indicates $\langle P_q P_{\bar{q}}\rangle \neq \langle P_q\rangle\langle P_{\bar{q}}\rangle$ simply means correlation!

By studying P_H, we study the average of quark polarization P_q;
by studying ρ_{00}^V, we study the correlation between P_q and $P_{\bar{q}}$.

⇨ **A window to study quark spin correlation in QGP**

Quark spin correlations in QGP in HIC?

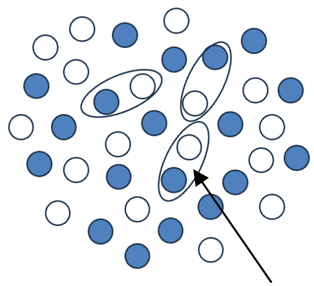
Correlations: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$

(1) local correlation:

$$\langle P_q P_{\bar{q}} \rangle_V \neq \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V$$

(2) long range correlation:

$$\langle \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V \rangle_S \neq \langle \langle P_q \rangle_V \rangle_S \langle \langle P_{\bar{q}} \rangle_V \rangle_S$$



two folded average

$$\langle P_q P_{\bar{q}} \rangle = \langle \langle P_q P_{\bar{q}} \rangle_V \rangle_S$$

inside the meson V

over the system S

Off-diagonal elements ?

$$\hat{\rho}^{(q)} = \frac{1}{2} \begin{pmatrix} 1 + P_{qz} & P_{qy} - iP_{qy} \\ P_{qx} + iP_{qx} & 1 - P_{qz} \end{pmatrix}$$

$$\langle P_{qx} \rangle = \langle P_{qy} \rangle = 0; \ \langle P_{qx}^2 \rangle \neq 0, \langle P_{qy}^2 \rangle \neq 0$$

⇒ a systematic study

- how to describe them?
- relationships to measurable quantities?
- where do they come from?

Ji-peng Lv, Zi-han Yu, ZTL, Qun Wang, and Xin-Nian Wang, PRD 109, 114003 (2024)

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Definition of quark spin correlations —— decomposition

For single particle, we decompose the complete set $(\mathbb{I}, \hat{\boldsymbol{\sigma}}_i)$

$$\hat{\boldsymbol{\rho}}^{(1)} = \frac{1}{2}(\mathbb{I} + \boldsymbol{P}_{1i}\hat{\boldsymbol{\sigma}}_{1i}) \qquad \boldsymbol{P}_{1i} = \langle\hat{\boldsymbol{\sigma}}_{1i}\rangle = \mathrm{Tr}[\hat{\boldsymbol{\rho}}^{(1)}\hat{\boldsymbol{\sigma}}_{1i}]$$

For two particle system (12), the complete set $(\mathbb{I}_1, \hat{\boldsymbol{\sigma}}_{1i}) \otimes (\mathbb{I}_2, \hat{\boldsymbol{\sigma}}_{2i})$

we are used to

$$\hat{\boldsymbol{\rho}}^{(12)} = \frac{1}{2^2}\left(\mathbb{I}_1 \otimes \mathbb{I}_2 + \boldsymbol{P}_{1i}\hat{\boldsymbol{\sigma}}_{1i} \otimes \mathbb{I}_2 + \boldsymbol{P}_{2i}\mathbb{I}_1 \otimes \hat{\boldsymbol{\sigma}}_{2i} + \boldsymbol{t}_{ij}^{(12)}\hat{\boldsymbol{\sigma}}_{1i} \otimes \hat{\boldsymbol{\sigma}}_{2j}\right)$$

shortage: $\boldsymbol{t}_{ij}^{(12)} = \boldsymbol{P}_{1i}\boldsymbol{P}_{2j} \neq 0$ if $\hat{\boldsymbol{\rho}}^{(12)} = \hat{\boldsymbol{\rho}}^{(1)} \otimes \hat{\boldsymbol{\rho}}^{(2)}$

we propose

$$\hat{\boldsymbol{\rho}}^{(12)} = \hat{\boldsymbol{\rho}}^{(1)} \otimes \hat{\boldsymbol{\rho}}^{(2)} + \frac{1}{2^2}\boldsymbol{c}_{ij}^{(12)}\hat{\boldsymbol{\sigma}}_{1i} \otimes \hat{\boldsymbol{\sigma}}_{2j}$$

$$\boldsymbol{c}_{ij}^{(12)} = \langle\hat{\boldsymbol{\sigma}}_{1i}\hat{\boldsymbol{\sigma}}_{2j}\rangle - \langle\hat{\boldsymbol{\sigma}}_{1i}\rangle\langle\hat{\boldsymbol{\sigma}}_{2j}\rangle \qquad \boldsymbol{c}_{ij}^{(12)} = 0 \text{ if } \hat{\boldsymbol{\rho}}^{(12)} = \hat{\boldsymbol{\rho}}^{(1)} \otimes \hat{\boldsymbol{\rho}}^{(2)}$$

For three particle system (123)

$$\hat{\boldsymbol{\rho}}^{(123)} = \hat{\boldsymbol{\rho}}^{(1)} \otimes \hat{\boldsymbol{\rho}}^{(2)} \otimes \hat{\boldsymbol{\rho}}^{(3)} + \frac{1}{2^2}\left[\boldsymbol{c}_{ij}^{(12)}\hat{\boldsymbol{\sigma}}_{1i} \otimes \hat{\boldsymbol{\sigma}}_{2j} \otimes \hat{\boldsymbol{\rho}}^{(3)} + (1 \to 2 \to 3)\right] + \frac{1}{2^3}\boldsymbol{c}_{ijk}^{(123)}\hat{\boldsymbol{\sigma}}_{1i} \otimes \hat{\boldsymbol{\sigma}}_{2j} \otimes \hat{\boldsymbol{\sigma}}_{3k}$$

Definition of quark spin correlations —— α-dependence

Single particle: $\hat{\rho}^{(1)}(\alpha)=\frac{1}{2}[1+P_{1i}(\alpha)\hat{\sigma}_{1i}]$

Two particle system A=(12) at given (α_1,α_2):

$$\hat{\rho}^{(12)}(\alpha_1,\alpha_2)=\hat{\rho}^{(1)}(\alpha_1)\otimes\hat{\rho}^{(2)}(\alpha_2)+\frac{1}{2^2}c_{ij}^{(12)}(\alpha_1,\alpha_2)\hat{\sigma}_{1i}\otimes\hat{\sigma}_{2j}$$

Suppose A=(12) is at given α_{12} in the state $|\alpha_{12}\rangle$, the α_{12}-dependent spin density matrix of (12) is

$$\hat{\bar{\rho}}^{(12)}(\alpha_{12})=\langle\alpha_{12}|\,\hat{\rho}^{(12)}(\alpha_1,\alpha_2)|\alpha_{12}\rangle$$

average inside *A*

$$=\hat{\bar{\rho}}^{(1)}(\alpha_{12})\otimes\hat{\bar{\rho}}^{(2)}(\alpha_{12})+\frac{1}{2^2}\bar{c}_{ij}^{(12)}(\alpha_{12})\,\hat{\sigma}_{1i}\otimes\hat{\sigma}_{2j}$$

However, the correlation $\bar{c}_{ij}^{(12)}(\alpha_{12})\neq\left\langle c_{ij}^{(12)}(\alpha_1,\alpha_2)\right\rangle$ does not equal to $c_{ij}^{(12)}$ averaged inside *A*

instead $\bar{c}_{ij}^{(12)}(\alpha_{12}) = \langle c_{ij}^{(12)}(\alpha_1,\alpha_2)\rangle + \bar{c}_{ij}^{(12;0)}(\alpha_{12})$

"effective correlation" = "genuine correlation" + "induced correlation"

the observed — the original process — due to average over α_i

$$\bar{c}_{ij}^{(12;0)}(\alpha_{12})\equiv\langle P_{1i}(\alpha_1)P_{2j}(\alpha_2)\rangle-\langle P_{1i}(\alpha_1)\rangle\langle P_{1i}(\alpha_1)\rangle$$

Relationship between $\hat{\rho}^h$ and $\hat{\rho}^{(q_1-q_n)}$

For $q_1+\bar{q}_2 \to V$

in general, $\hat{\rho}^V = \hat{\mathcal{M}}\hat{\rho}^{(q_1\bar{q}_2)}\hat{\mathcal{M}}^\dagger$ $\hat{\mathcal{M}}$: the transition matrix

$$\rho^V_{mm'} = \langle jm|\hat{\mathcal{M}}\hat{\rho}^{(q_1\bar{q}_2)}\hat{\mathcal{M}}^\dagger|jm'\rangle = N_j\langle jm|\hat{\rho}^{(q_1\bar{q}_2)}|jm'\rangle$$

independent of $\hat{\mathcal{M}}$

For $q_1+q_2+q_3 \to H$

$$\rho^H_{mm'} = \langle jm|\hat{\mathcal{M}}\hat{\rho}^{(q_1q_2q_3)}\hat{\mathcal{M}}^\dagger|jm'\rangle = N_j\langle jm|\hat{\rho}^{(q_1q_2q_3)}|jm'\rangle$$

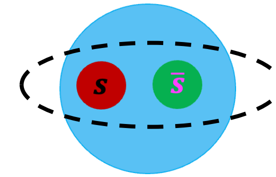
For $q_1+q_2+q_3+q_4+q_5+q_6 \to H_1+H_2$

$$\rho^{H_1H_2}_{m_1m_2m_1'm_2'} = \langle jm_1jm_2|\hat{\mathcal{M}}\hat{\rho}^{(q_1-q_6)}\hat{\mathcal{M}}^\dagger|jm_1'jm_2'\rangle = N_{jj}\langle jm_1jm_2|\hat{\rho}^{(q_1-q_6)}|jm_1'jm_2'\rangle$$

⇒ direct probe of $\hat{\rho}^{(q_1-q_n)}$ of quarks/antiquarks before hadronization

Spin density matrix for vector meson V

The spin alignment $\rho_{00}^{V}(\alpha_V) = \dfrac{1 + \bar{t}_{ii}^{(q_1\bar{q}_2)} - 2\bar{t}_{zz}^{(q_1\bar{q}_2)}}{3 + \bar{t}_{ii}^{(q_1\bar{q}_2)}}$



$$= \frac{1}{3 + \bar{t}_{ii}^{(q_1\bar{q}_2)}} \left(1 + \left\langle c_{xx}^{(q_1\bar{q}_2)} + c_{yy}^{(q_1\bar{q}_2)} - c_{zz}^{(q_1\bar{q}_2)} + P_{q_1x}P_{\bar{q}_2x} + P_{q_1y}P_{\bar{q}_2y} - P_{q_1z}P_{\bar{q}_2z} \right\rangle_V \right)$$

The off-diagonal element, e.g. $\mathrm{Re}\,\rho_{10}^{V} = \dfrac{\bar{P}_{q_1x} + \bar{P}_{\bar{q}_2x} + \bar{t}_{zx}^{(q_1\bar{q}_2)} + \bar{t}_{xz}^{(q_1\bar{q}_2)}}{\sqrt{2}\left(3 + \bar{t}_{ii}^{(q_1\bar{q}_2)}\right)}$

$$\bar{t}_{ij}^{(q_1\bar{q}_2)} \equiv \bar{c}_{ij}^{(q_1\bar{q}_2)} + \bar{P}_{q_1i}\bar{P}_{\bar{q}_2j} \qquad \bar{c}_{ij}^{(q_1\bar{q}_2)} = \left\langle c_{ij}^{(q_1\bar{q}_2)}(\alpha_1, \alpha_2) \right\rangle_V + \bar{c}_{ij}^{(q_1\bar{q}_2;0)}(\alpha_{12})$$

$$\bar{c}_{ij}^{(q_1\bar{q}_2;0)}(\alpha_{12}) = \langle P_{q_1i}(\alpha_1)P_{\bar{q}_2j}(\alpha_2)\rangle_V - \bar{P}_{q_1i}\bar{P}_{\bar{q}_2j}$$

depends on local spin correlations between q_1 and $\bar{q}_2$

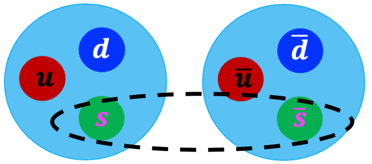
Sensitive to local spin correlations between q_1 and $\bar{q}_2$

Hyperon polarization & spin correlations

Λ polarization

$$P_\Lambda(\alpha_\Lambda) = \bar{P}_{sz} - \frac{1}{\bar{C}_\Lambda}\left[\bar{c}_{iiz}^{(uds)} + \bar{c}_{iz}^{(us)}\bar{P}_{di} + \bar{c}_{iz}^{(ds)}\bar{P}_{ui}\right] \qquad \bar{C}_\Lambda = 1 - \bar{t}_{ii}^{(ud)}$$

influences from quark spin correlations



$\Lambda\bar{\Lambda}$ spin correlation

$$C_{zz}^{\Lambda\bar{\Lambda}}(\alpha_\Lambda, \alpha_{\bar{\Lambda}}) \approx P_{\Lambda z}(\alpha_\Lambda)P_{\bar{\Lambda}z}(\alpha_{\bar{\Lambda}}) + \bar{c}_{zz}^{(s\bar{s})} - \frac{\bar{P}_{sz}}{\bar{C}_\Lambda}\left[\bar{c}_{iz}^{(d\bar{s})}\bar{P}_{ui} + \bar{c}_{iz}^{(u\bar{s})}\bar{P}_{di}\right] - \frac{\bar{P}_{\bar{s}z}}{\bar{C}_{\bar{\Lambda}}}\left[\bar{c}_{zi}^{(s\bar{d})}\bar{P}_{\bar{u}i} + \bar{c}_{zi}^{(s\bar{u})}\bar{P}_{\bar{d}i}\right]$$

if only two particle spin correlations are considered

$$\bar{c}_{zz}^{(s\bar{s})} = \left\langle c_{zz}^{(s\bar{s})}\right\rangle_{\Lambda\bar{\Lambda}}$$

only long range, no induced contributions

Sensitive to the long range spin correlation between s and $\bar{s}$.

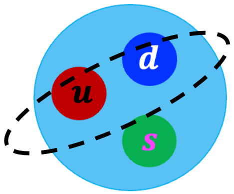
Ji-peng Lv, Zi-han Yu, ZTL, Qun Wang, and Xin-Nian Wang, PRD 109, 114003 (2024)

Polarizations of spin-3/2 baryons, e.g., S_L, S_{LL}, S_{LLL}

$$S_L = \frac{1}{2\overline{C}_3}\left(5\sum_{j=1}^{3}\overline{P}_{q_j z} + \overline{t}_{zii}^{\{q_1q_2q_3\}}\right) \longrightarrow \frac{1}{2\overline{C}_3}\left(5P_{qz} + \overline{t}_{zii}^{(qqq)}\right)$$

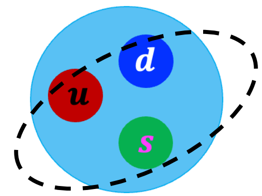
⟶ quark polarization

$$S_{LL} = \frac{1}{\overline{C}_3}\left[\left(3\overline{t}_{zz}^{(q_1q_2)} - \overline{t}_{ii}^{(q_1q_2)}\right) + (1 \leftrightarrow 2 \leftrightarrow 3)\right] \longrightarrow \frac{3}{\overline{C}_3}\left(3\overline{t}_{zz}^{(qq)} - \overline{t}_{ii}^{(qq)}\right)$$



⟶ local spin correlations of two quarks

$$S_{LLL} = \frac{9}{10\overline{C}_3}\left(5\overline{t}_{zzz}^{(q_1q_2q_3)} - 3\overline{t}_{zii}^{\{q_1q_2q_3\}}\right) \longrightarrow \frac{9}{10\overline{C}_3}\left(5\overline{t}_{zzz}^{(qqq)} - 3\overline{t}_{zii}^{(qqq)}\right)$$



⟶ local spin correlations of three quarks

Sensitive to the local two or three quark spin correlations

$$\overline{C}_3 = \mathrm{Tr}\hat{\rho} = 3 + \overline{t}_{ii}^{(q_1q_2)} + (1 \leftrightarrow 2 \leftrightarrow 3) \longrightarrow 3\left(1 + \overline{t}_{ii}^{(qq)}\right)$$

$$\overline{t}_{ijk}^{(q_1q_2q_3)} \equiv \overline{c}_{ijk}^{(q_1q_2q_3)} + \overline{c}_{ij}^{(q_1q_2)}\overline{P}_{q_3k} + \overline{c}_{jk}^{(q_2q_3)}\overline{P}_{q_1i} + +\overline{c}_{ki}^{(q_3q_1)}\overline{P}_{q_2j} + \overline{P}_{q_1i}\overline{P}_{q_2j}\overline{P}_{q_3k}$$

$$\overline{t}_{ijk}^{\{q_1q_2q_3\}} \equiv \overline{t}_{ijk}^{(q_1q_2q_3)} + \overline{t}_{ijk}^{(q_2q_3q_1)} + \overline{t}_{ijk}^{(q_3q_1q_2)} \qquad \overline{t}_{ij}^{(q_1\bar{q}_2)} \equiv \overline{c}_{ij}^{(q_1\bar{q}_2)} + \overline{P}_{q_1i}\overline{P}_{\bar{q}_2j}$$

Zhe Zhang, Ji-Peng Lv, Zi-han Yu, and ZTL, PRD 110, 074019 (2024).

Measurables and sensitive quark spin quantities

Hadron	Measurables	Sensitive quantities
Spin 1/2 (hyperon H)	Hyperon polarization P_H	average quark polarization $\langle P_q \rangle$
	Hyperon spin correlation $c_{H_1 H_2}, c_{H_1 \bar{H}_2}$	long range quark spin correlations $c_{qq}, c_{q\bar{q}}$
Spin 1 (Vector mesons)	Spin alignment ρ_{00}	local quark spin correlations $c_{q\bar{q}}$
	Off diagonal elements $\rho_{m'm}$	local quark spin correlations $c_{q\bar{q}}$
Spin 3/2 $J^P = \frac{3}{2}^+$ baryons	Hyperon polarization P_{H^*} or S_L	average quark polarization $\langle P_q \rangle$
	Rank 2 tensor polarization S_{LL}	local quark spin correlations c_{qq}
	Rank 3 tensor polarization S_{LLL}	local quark spin correlations c_{qqq}

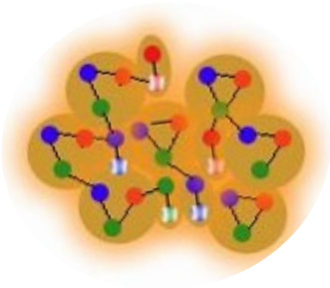
⇨ **Systematic studies of quark spin correlations in QGP!**

Ji-peng Lv, Zi-han Yu, ZTL, Qun Wang, and Xin-Nian Wang, PRD 109, 114003 (2024);
Zhe Zhang, Ji-Peng Lv, Zi-han Yu, and ZTL, PRD 110, 074019 (2024).

Outline

- Introduction
- The global vector meson spin alignment and quark spin correlations in relativistic heavy ion collisions (HIC)
- Vector meson alignment vs hyperon polarization in quark fragmentation
- Summary and outlook

Polarization and hadronization mechanism



QGP hadronization ⟹ combination / recombination / coalescence

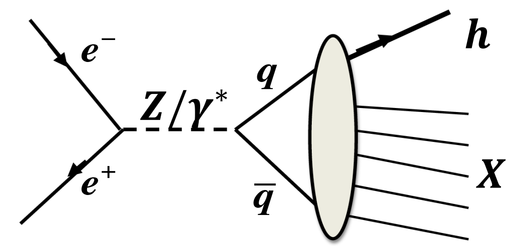
$$\begin{pmatrix} q_1 + \bar{q}_2 \to M \\ q_1 + q_2 + q_3 \to B \\ \bar{q}_1 + \bar{q}_2 + \bar{q}_3 \to \bar{B} \end{pmatrix}$$

direct probe to spin density of quarks and/or anti-quarks

Fragmentation

$$q \to h + X$$

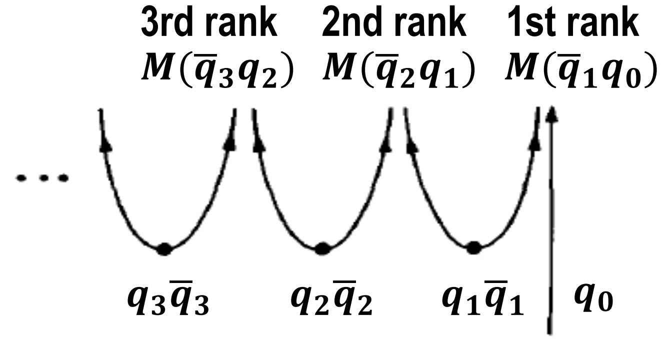
e.g.: $e^+e^- \to h + X$



Field-Feynman recursive cascade picture for $q_0 \to hX$

$$q_0 \to q_0 + (\bar{q}_1 q_1) \to M(q_0\bar{q}_1) + q_1$$
$$q_1 \to q_1 + (\bar{q}_2 q_2) \to M(q_1\bar{q}_2) + q_2$$

… … … … …

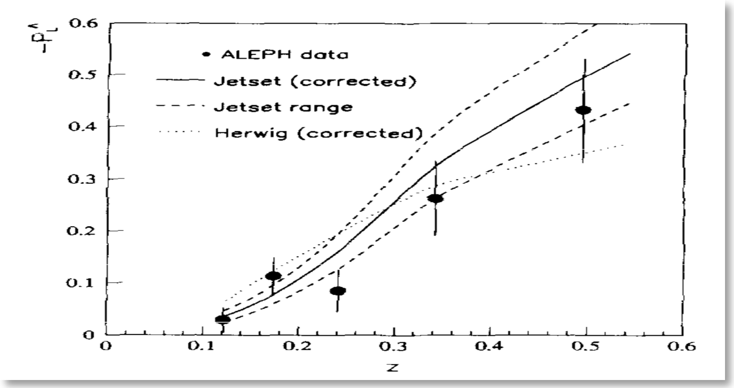


R.D. Field, R.P. Feynman, NPB136, 1-76 (1978)

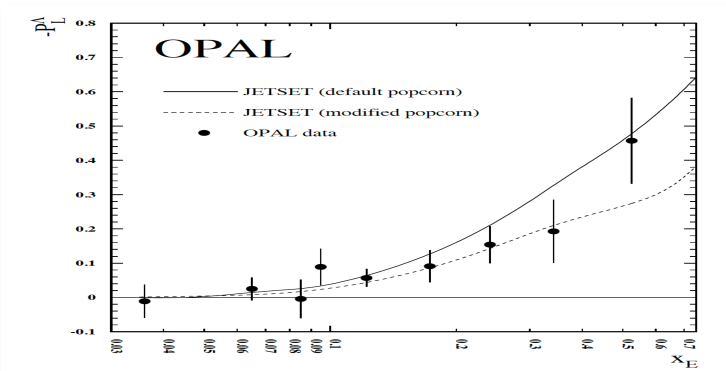
Hadron polarization in $e^+e^- \to Z^0 \to hX$ at LEP

Λ polarization in $e^+e^- \to Z^0 \to \Lambda X$

ALEPH, PLB 374, 319 (1996)



OPAL, EPJC 2, 49 (1998)

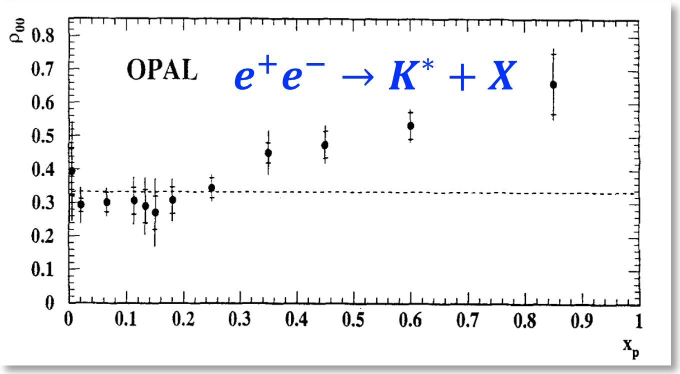


Spin alignment in $e^+e^- \to Z^0 \to VX$

DELPHI, PLB 406, 271 (1997)

Particle	x_p range	ρ_{00}	Re ρ_{1-1}	Im ρ_{1-1}
$K^{*0}(892)$	$0.1 \leqslant x_p \leqslant 0.3$	0.33 ± 0.05	0.00 ± 0.02	-0.01 ± 0.02
ϕ	$0.05 \leqslant x_p \leqslant 0.3$	0.30 ± 0.04	0.00 ± 0.02	0.00 ± 0.02
ρ^0		0.42 ± 0.04	0.00 ± 0.02	0.00 ± 0.02
$K^{*0}(892)$	$x_p \geqslant 0.3$	0.41 ± 0.07	0.01 ± 0.03	-0.01 ± 0.03
ϕ		0.27 ± 0.04	0.00 ± 0.02	0.00 ± 0.02
ρ^0		0.43 ± 0.05	0.01 ± 0.02	-0.01 ± 0.02
$K^{*0}(892)$	$x_p \geqslant 0.4$	0.46 ± 0.08	0.00 ± 0.03	-0.03 ± 0.03
ϕ		0.30 ± 0.04	0.01 ± 0.02	-0.01 ± 0.02
ρ^0		0.48 ± 0.06	0.02 ± 0.03	0.00 ± 0.03
$K^{*0}(892)$	$x_p \geqslant 0.5$	0.47 ± 0.10	-0.02 ± 0.04	-0.06 ± 0.04
ϕ		0.36 ± 0.06	0.02 ± 0.03	0.00 ± 0.03
ϕ	$x_p \geqslant 0.7$	0.55 ± 0.10	0.02 ± 0.04	0.00 ± 0.04

OPAL, PLB 412, 210 (1997)



due to polarization of the initial quark produced at the e^+e^- annihilation vertex?

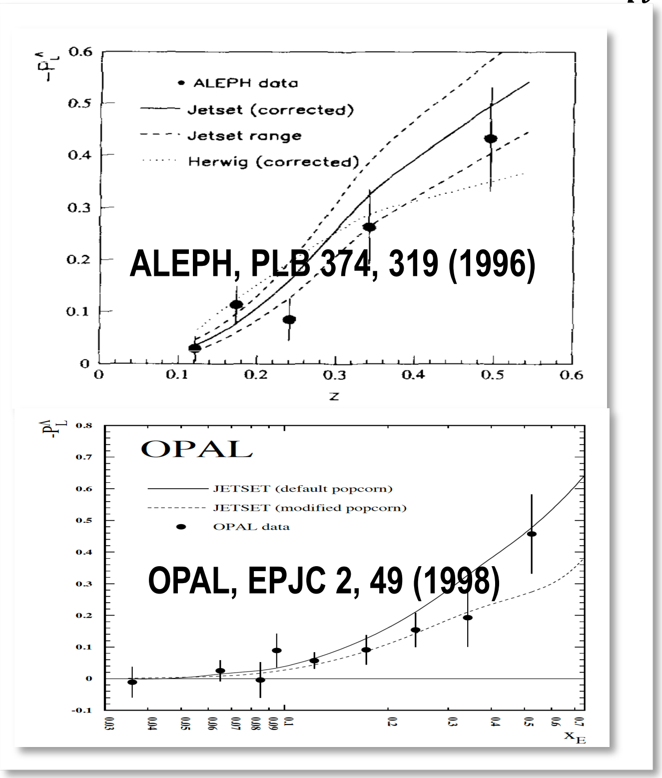
Hadron polarization in fragmentation processes

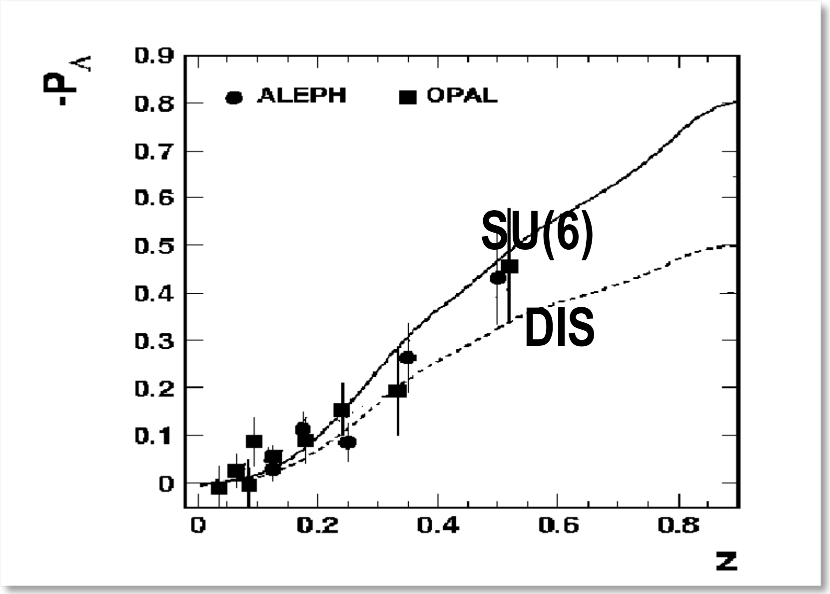
Earlier phenomenological studies :

① attributes to the polarization of the initial quark

② only first rank hadrons, i.e. only those containing initial quarks, are polarized

$$P_H^{1st\_rank} = P_q \frac{\Delta Q}{N_{q_v}} \qquad P_H^{higher\_rank} = 0$$





C. Boros, ZTL, PRD 57, 4491 (1998)
with decay contributions etc.

G. Gustafson and J. Hakkinen, PLB 303 (1993) 350.

Hadron polarization in fragmentation processes

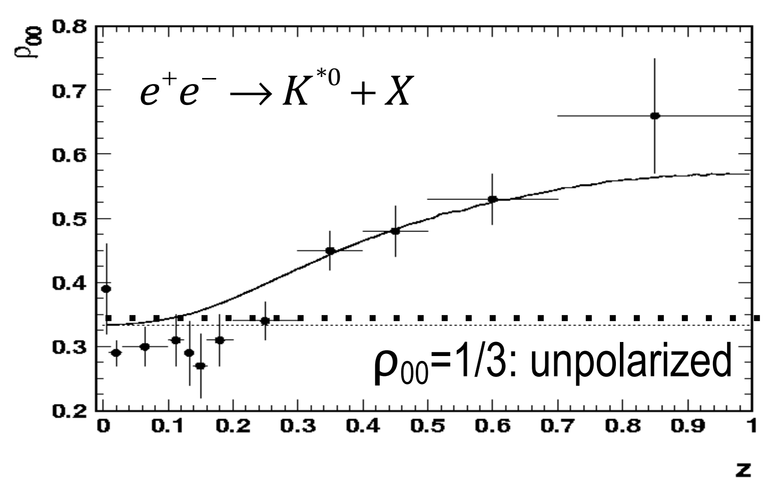
Earlier phenomenological studies :

① attributes to the polarization of the initial quark

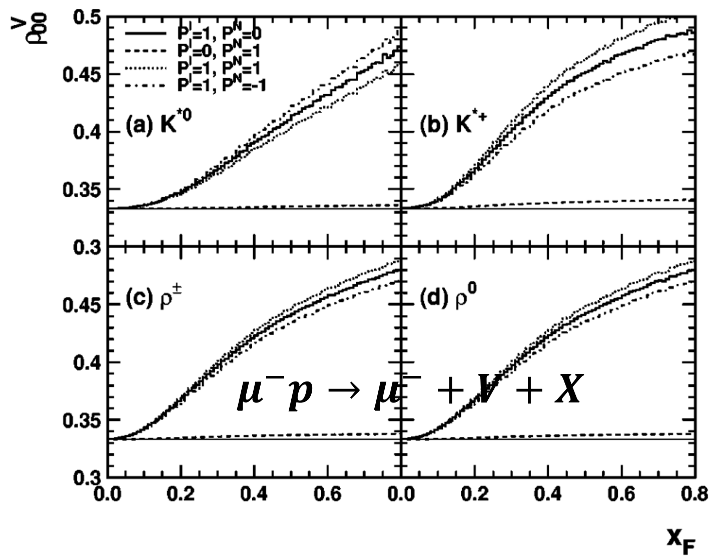
② only first rank hadrons, i.e. only those containing initial quarks, are polarized

if extend to vector meson spin alignment, we need

$$\rho_{00}^{1st\_rank} = \frac{1+\beta P_q^2}{3-\beta P_q^2} \qquad \rho_{00}^{higher\_rank} = \frac{1}{3}$$



Q.H. Xu, C.X. Liu and ZTL, PRD 63, 111301 (2001)



Q.H. Xu, and ZTL, PRD 66, 017301 (2002)

Hadron polarization in fragmentation processes

However, in QCD quantum field theory, fragmentation is described by fragmentation functions (FFs) defined via the quark-quark correlator

Un-integrated:

$$\hat{\Xi}(k;p,S) = \frac{1}{4\pi}\sum_X \int d^4\xi\, e^{-ik\xi}\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,0)|0\rangle\langle 0|\psi\,(0)|hX\rangle$$

One dimensional:

$$\hat{\Xi}(z;p,S) = \frac{1}{4\pi}\sum_X \int d\xi^-\, e^{-ip\xi^-/z}\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,0)|0\rangle\langle 0|\psi\,(0)|hX\rangle$$

Three dimensional (transverse momentum dependent):

$$\hat{\Xi}(z,k_\perp;p,S) = \frac{1}{4\pi}\sum_X \int d^2\xi_\perp d\xi^-\, e^{-ip\xi^-/z}e^{ik_\perp\cdot\xi_\perp}\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,0)|0\rangle\langle 0|\psi\,(0)|hX\rangle$$

Hadron polarization in fragmentation processes

FFs defined via the quark-quark correlator

e.g., one dimensional FFs:

We expand the quark-quark correlator $\hat{\Xi}(z;p,S)$ in terms of the Γ-matrices

$$\hat{\Xi}(z;p,S) = \Xi(z;p,S) + i\gamma_5\tilde{\Xi}(z;p,S) + \gamma^\alpha\Xi_\alpha(z;p,S) + i\gamma_5\gamma^\alpha\tilde{\Xi}_\alpha(z;p,S) + i\gamma_5\sigma^{\alpha\beta}\Xi_{\alpha\beta}(z;p,S)$$

We make the Lorentz decomposition, e.g.,

$$z\Xi_\alpha(z;p,S) = p^+\bar{n}_\alpha[D_1(z) + S_{LL}D_{1LL}(z)] - M\tilde{S}_{T\alpha}D_T(z) + MS_{LT\alpha}D_{LT}(z) + \frac{M^2}{p^+}n_\alpha[D_3(z) + S_{LL}D_{3LL}(z)]$$

We obtain, e.g., $D_1(z) + S_{LL}D_{1LL}(z) = \frac{1}{p^+}zn^\alpha\Xi_\alpha(z;p,S) = \frac{1}{4p^+}z\mathrm{Tr}\gamma^+\hat{\Xi}(z;p,S)$

See e.g., K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD94, 034003 (2016).

Description of polarization of particles with different spins

Spin 1/2 hadrons:

The spin density matrix is 2x2: $\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2}(1 + \vec{S}\cdot\vec{\sigma})$

Vector polarization: $S^{\mu} = (0, \vec{S}_T, \lambda)$

Spin 1 hadrons:

The spin density matrix is 3x3: $\rho = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}(1 + \frac{3}{2}\vec{S}\cdot\vec{\Sigma} + 3T^{ij}\Sigma^{ij})$

Vector polarization: $S^{\mu} = (0, \vec{S}_T, \lambda)$

Tensor polarization: scalar S_{LL} vector $S_{LT} = (0, S_{LT}^x, S_{LT}^y, 0)$ tensor $S_{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{TT}^{xx} & S_{TT}^{xy} & 0 \\ 0 & S_{TT}^{xy} & S_{TT}^{yy} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$$\hat{\rho} = \begin{pmatrix} \frac{1+S_{LL}}{3} + \frac{S_L}{2} & \frac{(S_{LT}^x - iS_{LT}^y) + (S_T^x - iS_T^y)}{2\sqrt{2}} & \frac{S_{TT}^{xx} - iS_{TT}^{xy}}{2} \\ \frac{(S_{LT}^x + iS_{LT}^y) + (S_T^x + iS_T^y)}{2\sqrt{2}} & \frac{1-2S_{LL}}{3} & \frac{(-S_{LT}^x + iS_{LT}^y) + (S_T^x - iS_T^y)}{2\sqrt{2}} \\ \frac{S_{TT}^{xx} + iS_{TT}^{xy}}{2} & \frac{(-S_{LT}^x - iS_{LT}^y) + (S_T^x + iS_T^y)}{2\sqrt{2}} & \frac{1+S_{LL}}{3} - \frac{S_L}{2} \end{pmatrix}$$

$$\rho_{00} = \frac{1 - S_{LL}}{3}$$

See e.g. A. Bacchetta and P.J. Mulders, PRD62, 114004 (2000)

The vector meson spin alignment v.s. the longitudinal spin transfer

$$\psi_{L/R} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi$$

The vector meson spin alignment $D_{1LL}(z)$

$$D_1(z) + S_{LL} D_{1LL}(z) = \frac{1}{8\pi} \sum_X \int z d\xi^- \, e^{-ip^+\xi^-/z} \sum_{\lambda_q = L,R} \left\langle hX \middle| \bar{\psi}_{\lambda_q}(\xi)\gamma^+ \middle| 0 \right\rangle \left\langle 0 \middle| \psi_{\lambda_q}(0) \middle| hX \right\rangle$$

independent of the spin λ_q of the fragmenting quark!

The longitudinal spin transfer $G_{1L}(z)$

$$S_L G_{1L}(z) = \frac{1}{8\pi} \sum_X \int z d\xi^- \, e^{-ip^+\xi^-/z} [\langle hX|\bar{\psi}_L(\xi)\gamma^+|0\rangle\langle 0|\psi_L(0)|hX\rangle - \langle hX|\bar{\psi}_R(\xi)\gamma^+|0\rangle\langle 0|\psi_R(0)|hX\rangle]$$

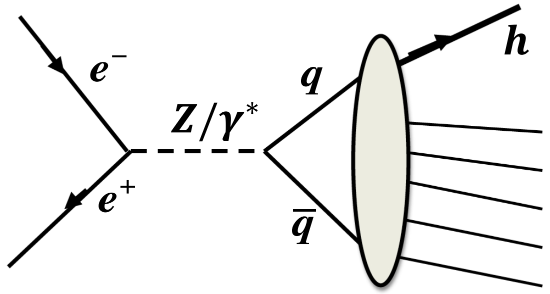
dependent of the spin λ_q of the fragmenting quark!

The longitudinal polarization of Λ in $e^+e^- \to Z^0 \to \Lambda X$ indeed originates from the polarization of the initial quark, but the vector spin alignment of V in $e^+e^- \to Z^0 \to VX$ does not!

Hadron polarization in $e^+e^- \to hX$

Vector meson spin alignment:

$$\langle S_{LL}\rangle(z,Q) = \frac{1}{2}\frac{\sum_q W_q(Q)\, D_{1LLq}(z,Q)}{\sum_q W_q(Q) D_{1q}(z,Q)}$$



Hyperon polarization:

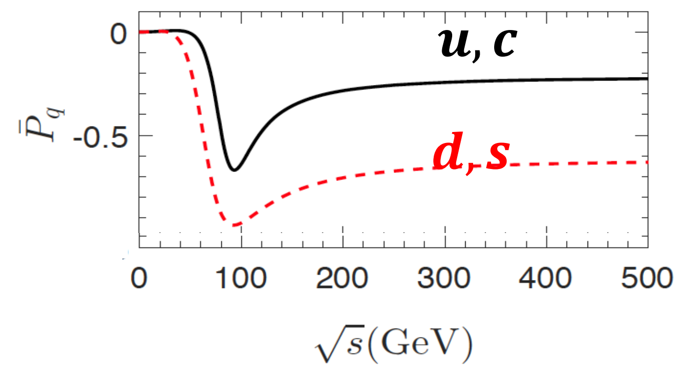
$$P_{L\Lambda}(z,Q) = \frac{\sum_q P_q(Q) W_q(Q) G_{1Lq}(z,Q)}{\sum_q W_q(Q) D_{1q}(z,Q)}$$

$P_q(Q)$: quark polarization

$$W_q(Q) = \frac{2}{3}\left(e_q^2 + \chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q\right)$$

$$\chi = s^2 / \left[\left(s - M_Z^2\right)^2 + \Gamma_Z^2 M_Z^2\right] \sin^4 2\theta_W$$

$$\chi_{int}^q = -2 e_q \chi (1 - M_Z^2/s)$$



K.B. Chen, S.Y. Wei, W.H. Yang and ZTL, PRD94, 034003 (2016);
K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017).

Hyperon polarization in $e^+e^- \to H + X$

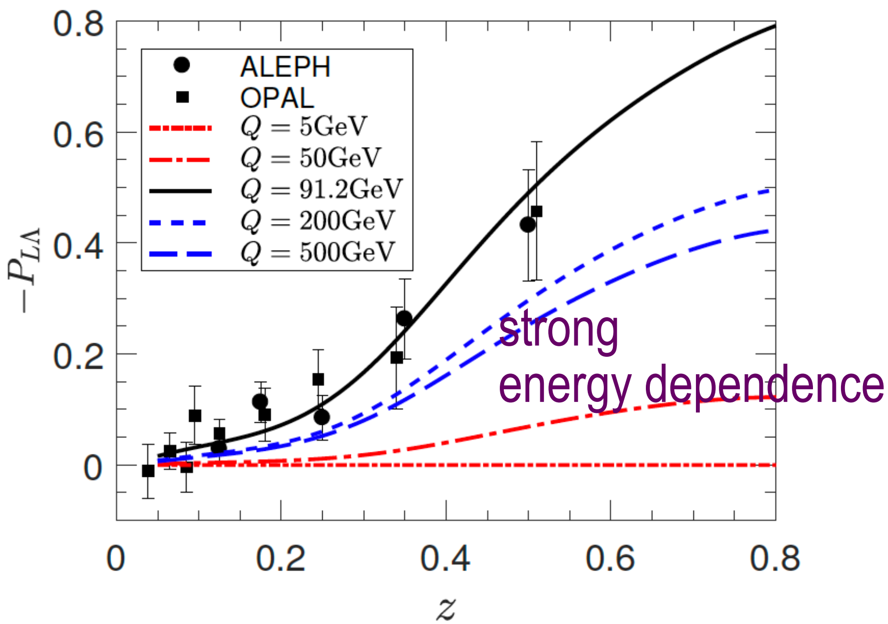
Parameterization at a initial scale:

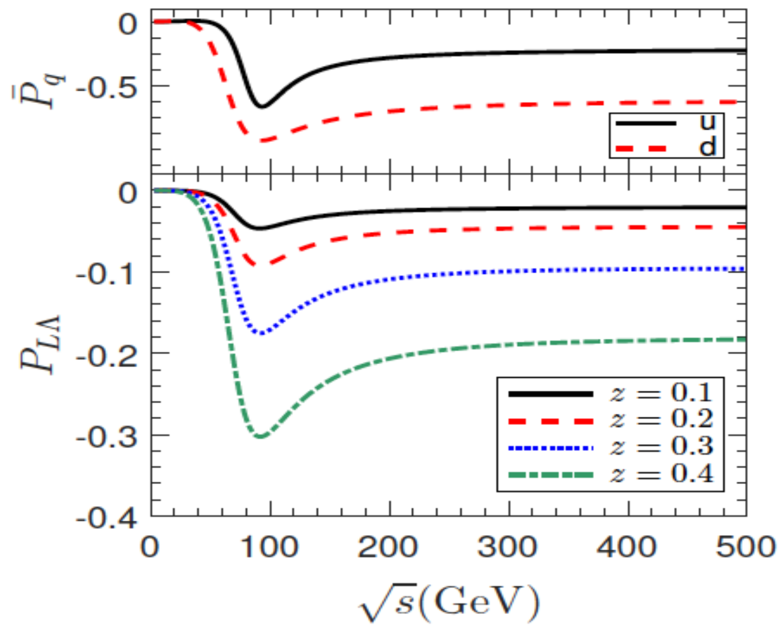
$$G_{1L}^{s\to\Lambda}(z,\mu_0) = z^a D_1^{s\to\Lambda}(z,\mu_0)$$

$$G_{1L}^{u/d\to\Lambda}(z,\mu_0) = N z^a D_1^{u/d\to\Lambda}(z,\mu_0)$$

QCD Evolution:
(DGLAP equation)

$$\frac{\partial}{\partial \ln Q^2} G_{1L}^{i\to h}(z,Q^2) = \frac{\alpha_s}{2\pi}\sum_j \int_z^1 \frac{dy}{y} G_{1L}^{j\to h}\left(\frac{z}{y},Q^2\right)\Delta P_{ij}(y,\alpha_s)$$





K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017).

Vector meson spin alignment in $e^+e^- \to V + X$

Two scenarios of parameterization at an initial scale.

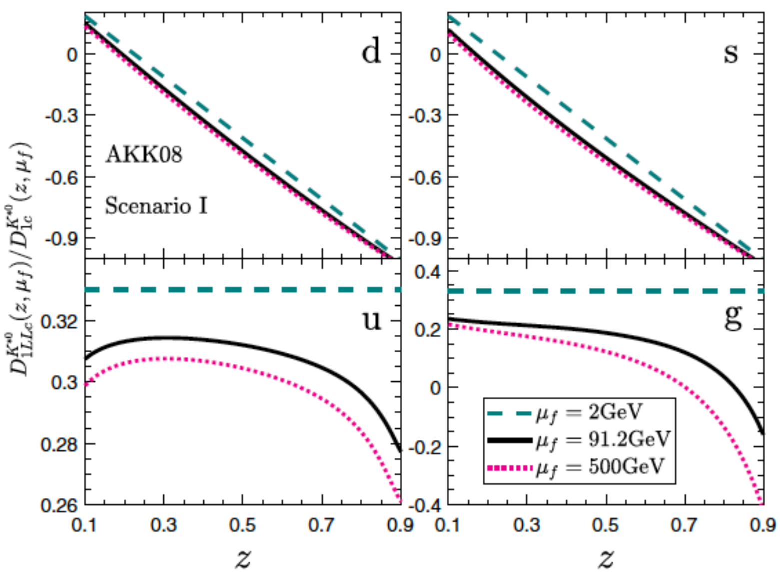
Scenario I: $$D_{1LL}^{\text{favored}}(z,\mu_0) = c_1(a_1 z + 1)\, D_1^{\text{favored}}(z,\mu_0)$$

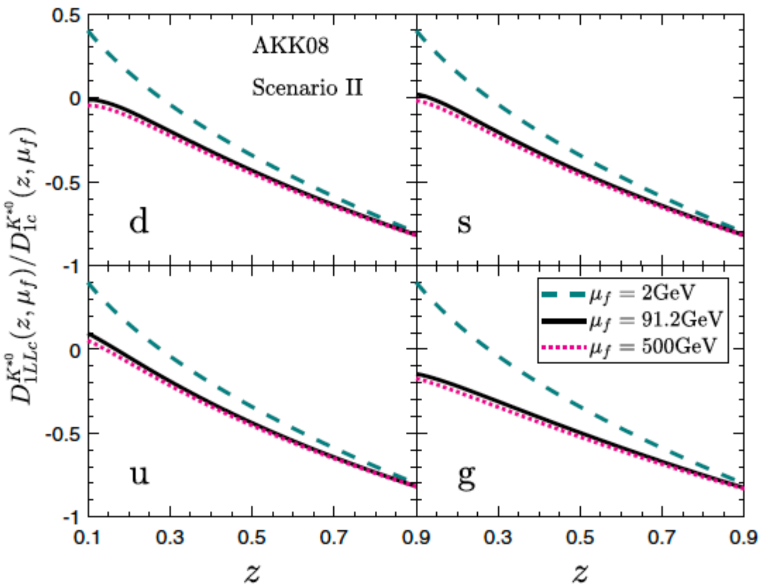
$$D_{1LL}^{\text{unfavored}}(z,\mu_0) = c_1 D_1^{\text{unfavored}}(z,\mu_0)$$

Scenario II: $$D_{1LL}(z,\mu_0) = c_2(a_2 z^{1/2} + 1)\, D_1(z,\mu_0)$$

favored, e.g.:
$d \to K^{*0}(d\bar{s}) + X$

unfavored, e.g.:
$u \to K^{*0}(d\bar{s}) + X$

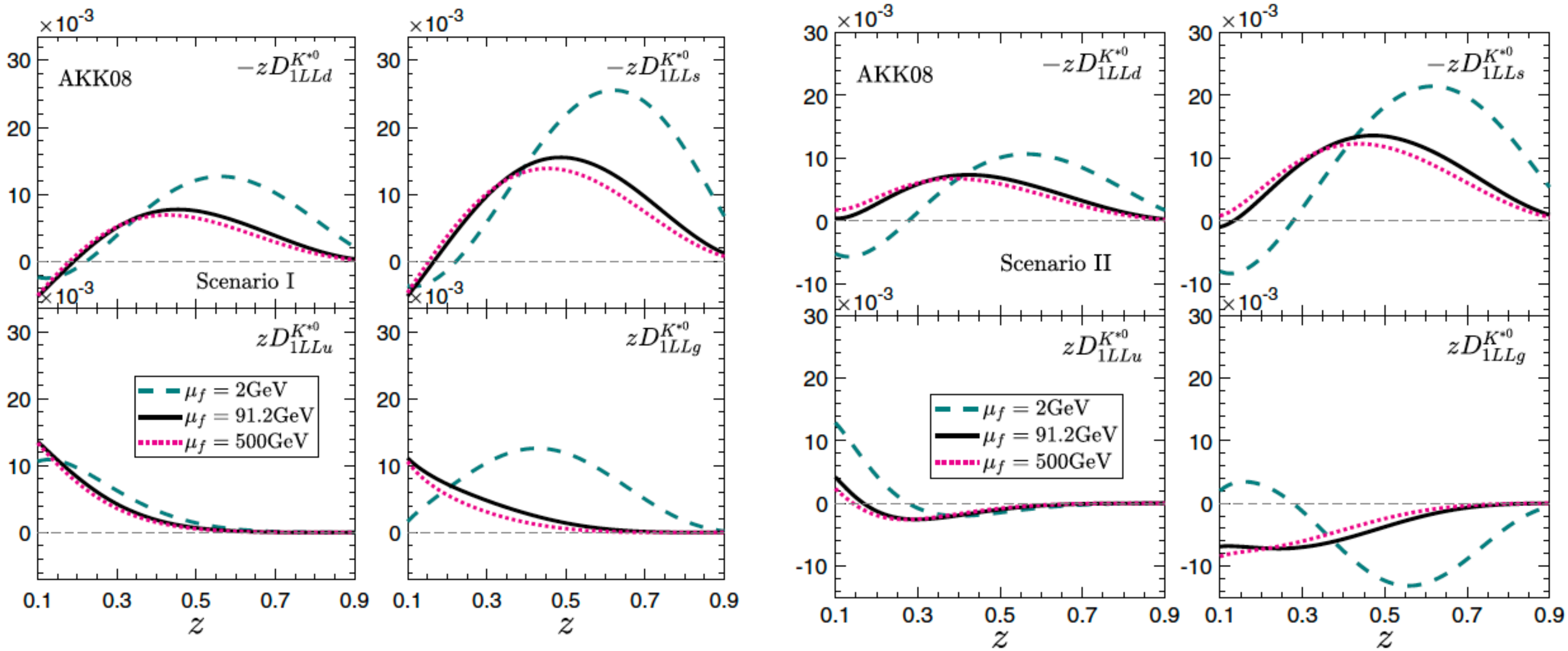




K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Vector meson spin alignment in $e^+e^- \to V + X$

The fragmentation functions $D_{1LLq}^{K^*}(z, \mu_f)$ in $q \to K^* + X$



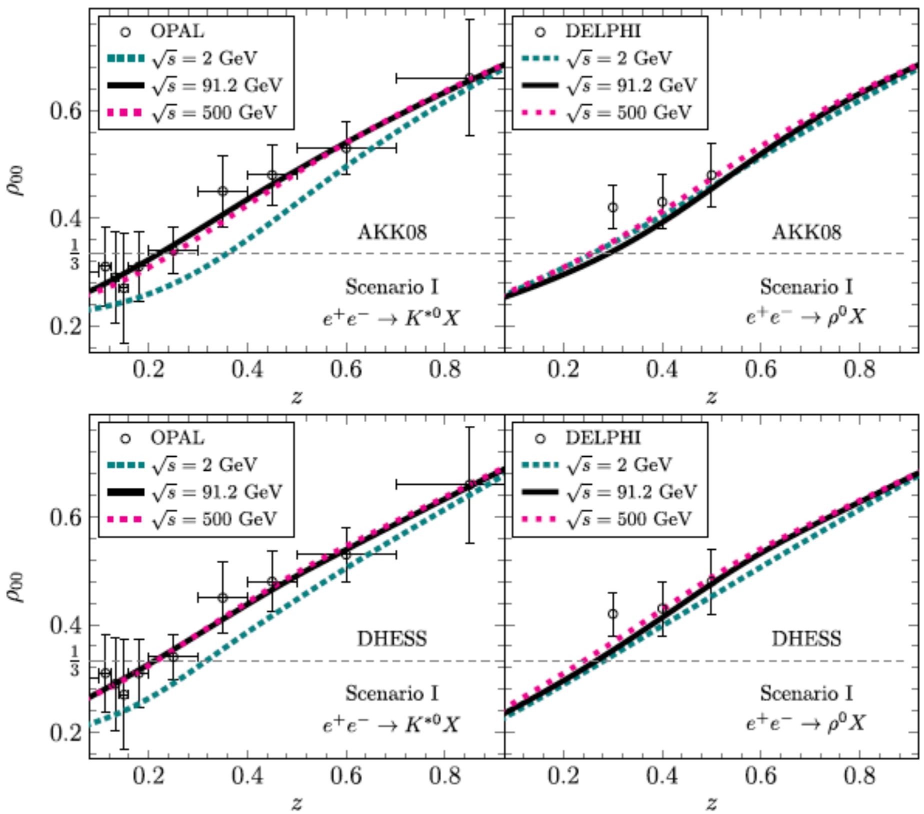
different for different q or g in $q/g \to K^{*+} + X$

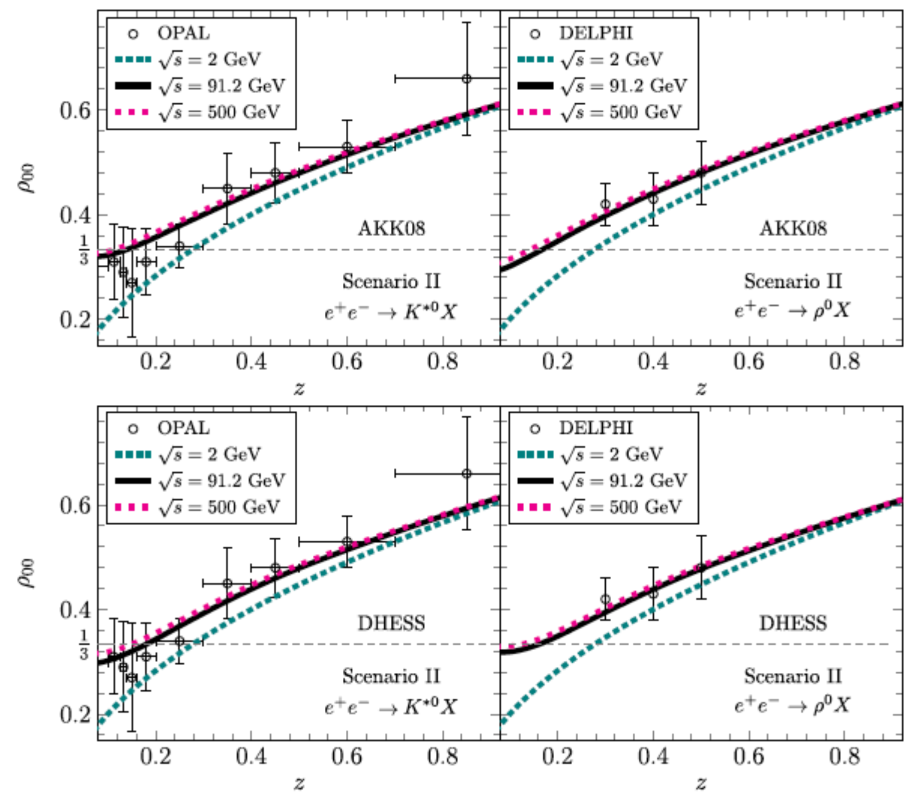
K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Vector meson spin alignment in $e^+e^- \to V + X$

Spin alignment in $e^+e^- \to \rho$ or $K^* + X$

weak energy dependence



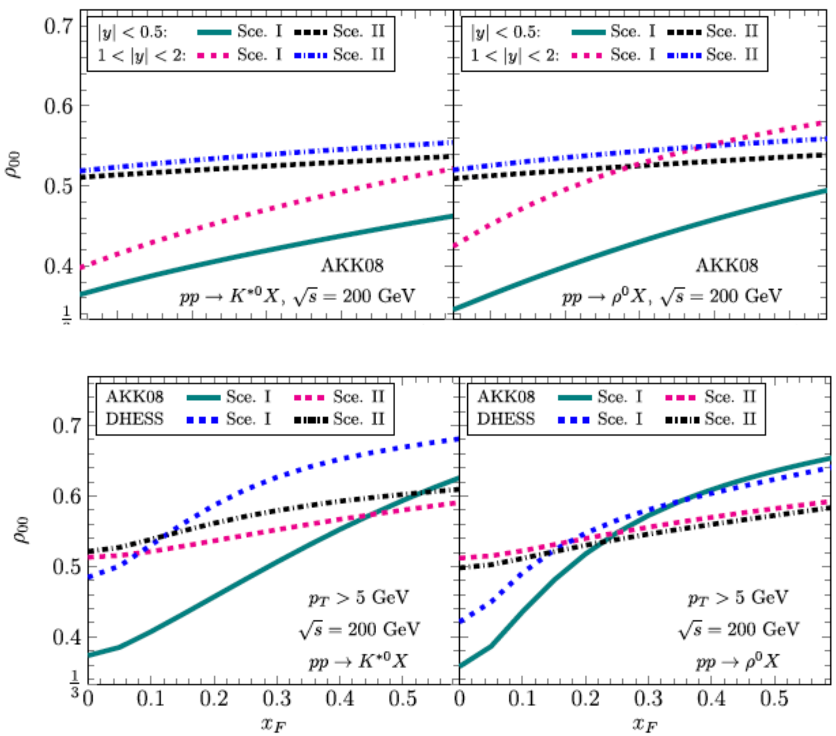


K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017).

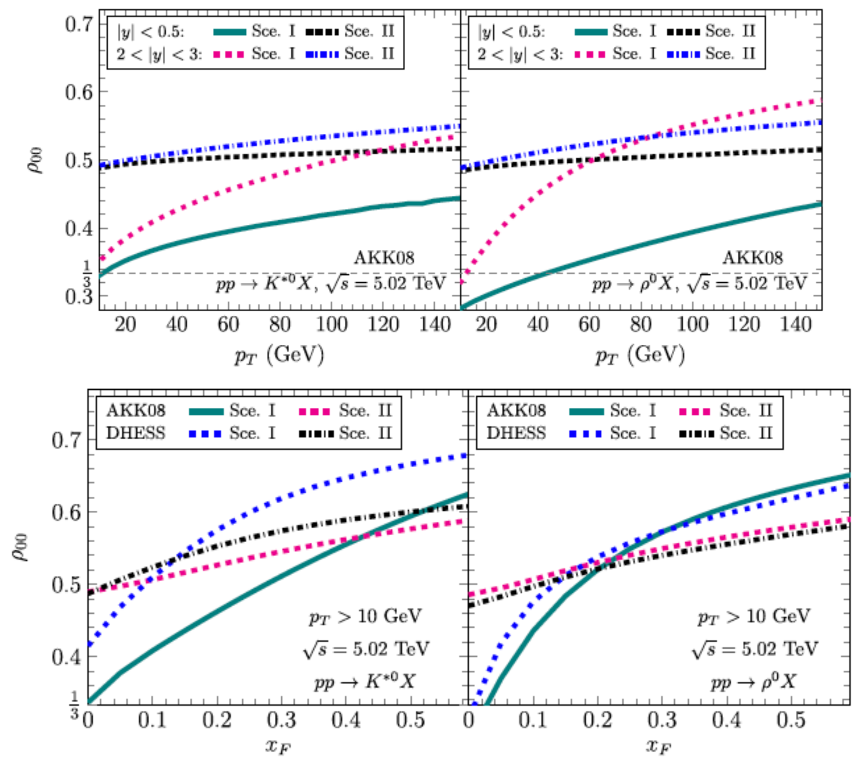
K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Spin alignment in $pp \to VX$

$\sqrt{s} = 200\text{GeV}$



$\sqrt{s} = 5.02\text{TeV}$



$\rho_{00} > 1/3$ and increase with increasing p_T or x_F

tested in future experiments e.g. at RHIC and LHC

K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Outline

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- The global vector meson spin alignment and quark spin correlations in relativistic heavy ion collisions (HIC)
- Vector meson alignment vs hyperon polarization in quark fragmentation
- Summary and outlook

Summary and outlook

- Global hyperon polarization and global vector meson spin alignment have been observed in relativistic heavy ion collisions (HIC).
- The global hyperon polarization is a measure of the average value of the global quark polarization in the system, while the global vector meson spin alignment measures the correlation between quark and anti-quark polarization.
- Vector meson spin alignments, tensor polarizations of spin 3/2 baryons are sensitive to local quark spin correlations; spin correlations of hyperon-hyperon or hyperon-antihyperon are sensitive to long range correlations.
- Vector meson spin alignment in fragmentation mechanism is independent on the spin of the initial quark. Predictions have been made for different high energy reactions that can be tested by future experiments.

A recent short review:
J.H. Chen, ZTL, Y.G. Ma, X.L. Sheng, and Q. Wang, "*Vector meson's spin alignments in high energy reactions*", Sci. China-Phys. Mech. Astron. 68, 211001 (2025).

Thank you for your attention!