Holographic spin alignment for vector mesons

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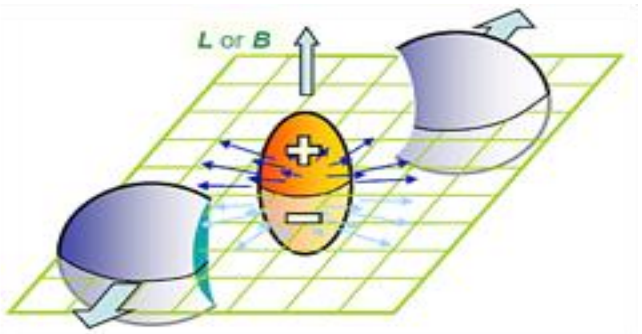
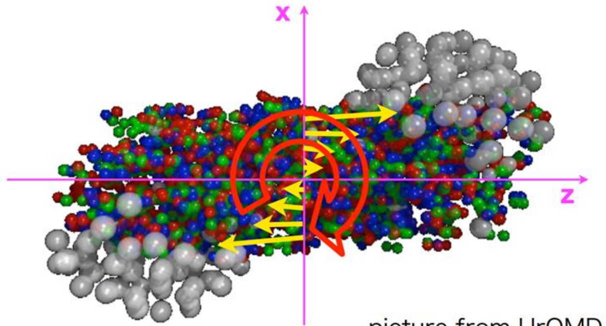
Based on: JHEP 08 (2024) 070 ; PRD 110 (2024) 5, 056047

LIGHT CONE 2024 Hadron Physics in the EIC era

Stongly interacting QCD under new extreme conditions



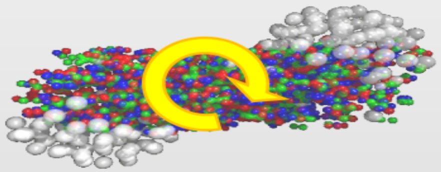
$B \sim 10^{18}$ Gauss



picture from UrQMD

$\omega \sim 10^{21} s^{-1}$

Initial orbital angular momentum → Vorticity field / Magnetic field → Polarized quark/gluon → Spin polarization for spin-1/2 or spin-3/2 baryons, Λ, Σ^0, Δ^{++}, Ω^-, ...



Spin alignment for vector mesons, ϕ, K^{*0}, ρ^0, ...

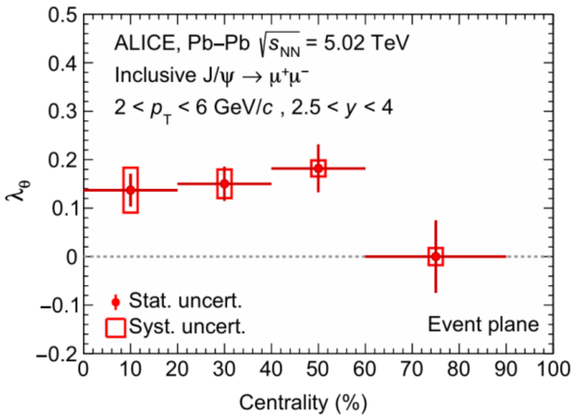
Spin aligments : See ZT Liang's talk on Nov.27

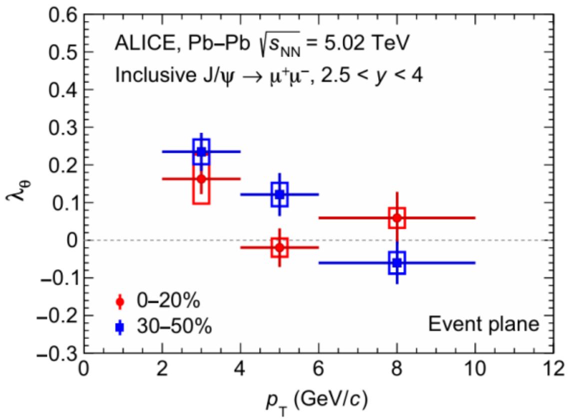
New QCD phase diagram: See YQ Zhao's talk on Nov.25

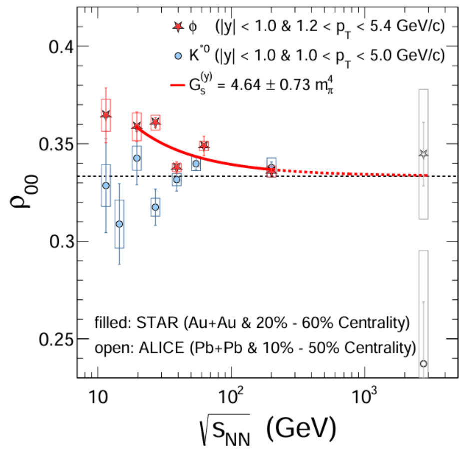
Experiment results

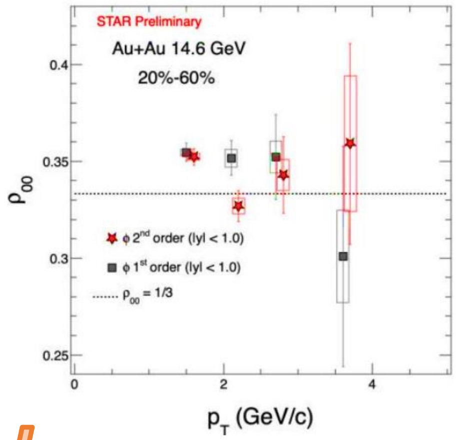
ALICE, Phys. Rev. Lett. 131 (4) (2023) 042303.

STAR, Nature 614 (7947) (2023) 244–248.









J/ψ

$$\boxed{\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}},}$$

$\rho_{00} < 1/3$

$\rho_{00} > 1/3$

ϕ

$$\boxed{\lambda_\theta = \frac{3\rho_{00}-1}{1-\rho_{00}}}$$

J/ψ are preferably to be transversely polarized.

ϕ are preferably to be longitudinally polarized.

$\boxed{\rho_{00} = 1/3 \text{ no spin alignment}}$

Theoretical Methods

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3. Y.-G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 97, 034917 (2018)
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5. F. Li, S. Liu, arXiv: 2206.11890
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7. S. Fang, S. Pu, D.-L. Yang, arXiv:2311.15197.
8. P. H. D. Moura, K. J. Goncalves, G. Torrieri, PRD 108, 034032 (2023)
9. X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817, 136325 (2021)
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14. X.-L. S, S.-Y. Yang, Y.-L. Zou, D. Hou, arXiv:2209.01872
15. Y.-Q. Zhao, X.-L. S, S.-W. Li, D. Hou, arXiv: 2403.07468
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18. A. Kumar, B. Muller, D.-L. Yang, PRD 108, 016020 (2023)
19. X.-L. S, S. Pu, Q. Wang, PRC 108, 054902 (2023).
20. X.-L. S, Y.-Q. Zhao, S.-W. Li, F. Becattini, D. Hou, arXiv: 2403.07522
21. X.-L. S, L. Oliva, Z.-T. Liang, Q. Wang, X.-N. Wang, PRL 131, 042304 (2023); PRD 109, 036004 (2024)

$$\boxed{\boldsymbol{\rho_{00}} \approx \frac{1}{3} + \overbrace{c_{hadro} + c_{EM} + c_F + c_A + c_h}^{\text{Smaller deviation from 1/3}} + \overbrace{\boldsymbol{c_{strong}}}^{\text{Larger deviation from 1/3 ?}}}$$

c_{strong} → Anisotropic strong force [4,15,18-21]

c_h → Helicity Polarization[17]

c_A → Anomalous color field [16]

c_F → Fragmentation [1]

c_{EM} → Electromagnetic fields [3,4,14,15]

c_{hadro} → Hydrodynamic gradient (vorticity; Acceleraction; shear tensor; second order;)[1-13]

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IV J/ψ meson in magnetized plasma

I — **Spin alignment**

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IV — J/ψ meson in magnetized plasma

Ⅰ、Spin alignment

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- S-matrix element($J/\psi \to l + \bar{l}$ and $\phi \to l + \bar{l}$.):

C. Gale and J. I. Kapusta, Nucl. Phys. B 357 (1991) 65–89

$$S_{fi} = \int d^4x d^4y \langle f, l\bar{l}|J_\mu(y) G_R^{\mu\nu}(x-y) J_\nu^l(x)|i\rangle$$

where

J_μ is the current that couples to V.M. ;

J_ν^l is the leptonic current

$$J_\nu^l(x) = g_{Ml\bar{l}} \bar{\psi}_l(x) \Gamma_\nu \psi_l(x)$$

- Propagators (vacuum):

$$G_{R/A}^{\mu\nu} = -\frac{\eta^{\mu\nu} + p^\mu p^\nu / p^2}{p^2 + m_V^2 \pm i m_V \Gamma}$$

- Retarded current-current correlation:

$$D^{\mu\nu}(x,p) \equiv \int d^4y \theta(y^0) \langle [J^\mu(y), J^\nu(0)] \rangle_{T(x)} e^{-ip\cdot y}$$

- Spectral function:

$$\varrho_{\alpha\beta}(x,p) \equiv -\mathrm{Im} D_{\alpha\beta}(x,p)$$

- Differential production rate:

$$n(x,p) = -\frac{2g_{Ml\bar{l}}^2}{3(2\pi)^5}\left(1 - \frac{2m_l^2}{p^2}\right)\sqrt{1 + \frac{4m_l^2}{p^2}} p^2 n_B(x,\omega) \times \left(\eta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) G_A^{\mu\alpha}(p) \varrho_{\alpha\beta}(x,p) G_R^{\beta\nu}(p),$$

Y. Burnier, M. Laine, M. Vepsalainen,, JHEP 02 (2009) 008.

L. D. McLerran, T. Toimela, Phys. Rev. D 31 (1985) 545.

H. A. Weldon, Phys. Rev. D 42 (1990) 2384–2387.

Spin space

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- **Spectral function:**

$$\varrho^{\mu\nu}(x,p)=\sum_{\lambda,\lambda'=0,\pm1} v^{\mu}(\lambda,p)v^{*\nu}(\lambda',p)\tilde{\varrho}_{\lambda\lambda'}(x,p)$$

- **Polarization vectors:**

$$v^{\mu}(\lambda,p)=\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_{\lambda}}{M},\boldsymbol{\epsilon}_{\lambda}+\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_{\lambda}}{M(\omega+M)}\mathbf{p}\right)$$

Orthonormality conditions:

$$\eta_{\mu\nu}v^{\mu}(\lambda,p)v^{*\nu}(\lambda',p)=\delta_{\lambda\lambda'}$$

Completeness condition :

$$\sum_{\lambda} v^{\mu}(\lambda,p)v^{*\nu}(\lambda,p)=(\eta^{\mu\nu}+p^{\mu}p^{\nu}/p^{2})$$

- **Dilepton production rate:**

$$n_{\lambda}(x,p)=-\frac{2g_{Ml\bar{l}}^{2}}{3(2\pi)^{5}}\left(1-\frac{2m_{l}^{2}}{p^{2}}\right)\times\sqrt{1+\frac{4m_{l}^{2}}{p^{2}}}\frac{p^{2}n_{B}(x,\omega)\tilde{\varrho}_{\lambda\lambda}(x,p)}{(p^{2}+m_{V}^{2})^{2}+m_{V}^{2}\Gamma^{2}}.$$

- **Spin alignment:**

$$\rho_{00}(x,\mathbf{p})\equiv\frac{\int d\omega\, n_{0}(x,p)}{\sum_{\lambda=0,\pm1}\int d\omega\, n_{\lambda}(x,p)}$$

I Spin alignment

II **Correlation function**

III J/ψ and ϕ mesons in hot plasma

IV J/ψ meson in magnetized plasma

II、 Correlation function from AdS/CFT

- **AdS/CFT dictionary:**

$$Z_{\text{QFT}}[A_\mu^{(0)}] = Z_{\text{gravity}}[A_\mu]$$

where

$$Z_{\text{QFT}}[A_\mu^{(0)}] = \left\langle \exp\left\{ \int_{\partial\mathcal{M}} J^\mu A_\mu^{(0)} d^4x \right\} \right\rangle,$$

$$Z_{\text{gravity}}[A_\mu] = \exp\{-S_{\text{bulk}}[A_\mu]\}$$

- **Bulk geometry:**

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{\zeta\zeta} d\zeta^2$$

$$0 \leq \zeta \leq \zeta_h$$

- **Two-point correlators**

$$\langle [J^\mu, J^\nu] \rangle = D^{\mu\nu} \propto \frac{\delta^2 Z_{\text{QFT}}}{\delta A_\mu^{(0)} \delta A_\nu^{(0)}} = \frac{\delta^2 Z_{\text{gravity}}}{\delta A_\mu^{(0)} \delta A_\nu^{(0)}}$$

- **Bulk mesonic action:** T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843882.

$$S_{\text{bulk}} = -\int d^4x d\zeta Q(\zeta) F_{MN} F^{MN}$$

- **Equation of motion:** T. Sakai, S. Sugimoto, Prog. Theor. Phys. 114 (2005) 1083–1118. A. Karch, E. Katz, D. T. Son, M. A. Stephanov, Phys. Rev. D 74(2006) 015005.

$$\partial_M [Q(\zeta) F^{MN}] = 0$$

Radial gauge condition: $A_\zeta = 0$

Fourier transformation: $A_\mu(x,\zeta) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} A_\mu(p,\zeta)$

II、 Correlation function in spin space

➢ **Electric fields:**

$$E_i(p,\zeta) \equiv -p_0 A_i(p,\zeta) + p_i A_0(p,\zeta)$$

➢ **Boundary condition:**

$$\lim_{\zeta\to 0} \tilde{E}_i(j,p,\zeta) = \delta_{ij}$$

➢ **Equation of motion :**

$$\partial_\zeta^2 E_i(p,\zeta) + \frac{[\partial_\zeta Q(\zeta) g^{\zeta\zeta}]}{Q(\zeta) g^{\zeta\zeta}}[\partial_\zeta E_i(p,\zeta)] - \frac{p^2}{g^{\zeta\zeta}} E_i(p,\zeta) + (-p_0 g_{i\mu} + p_i g_{0\mu})(\partial_\zeta g^{\mu\nu})[\partial_\zeta A_\nu(p,\zeta)] = 0,$$

➢ **Correlation function:**

D. T. Son and A. O. Starinets, JHEP 09 (2002) 042

$$D^{\mu\nu}(p) = \lim_{\zeta\to 0} g^{\zeta\zeta} g^{\mu\alpha} Q(\zeta) \frac{\delta[\partial_\zeta A_\alpha(p,\zeta)]}{\delta A_\nu(p,\zeta)}\bigg|_{A_\mu(p,0)=0}$$

$$D^{\mu 0}(p) = -\lim_{\zeta\to 0} \frac{p_j}{p_0}\left(g^{\mu k} - \frac{p^\mu p^k}{p^2}\right)\frac{1}{\zeta}\partial_\zeta \tilde{E}_k(j,p,\zeta),$$

$$D^{\mu i}(p) = \lim_{\zeta\to 0}\left(g^{\mu k} - \frac{p^\mu p^k}{p^2}\right)\frac{1}{\zeta}\partial_\zeta \tilde{E}_k(i,p,\zeta).$$

➢ **Ward identity:**

$$p_\mu D^{\mu\nu} = p_\mu D^{\nu\mu} = 0$$

➢ **Spectral function:**

$$\tilde{\varrho}_{\lambda\lambda}(p) = v_\mu^*(\lambda,p) v_\nu(\lambda,p) \mathrm{Im} D^{\mu\nu}(p)$$

I Spin alignment

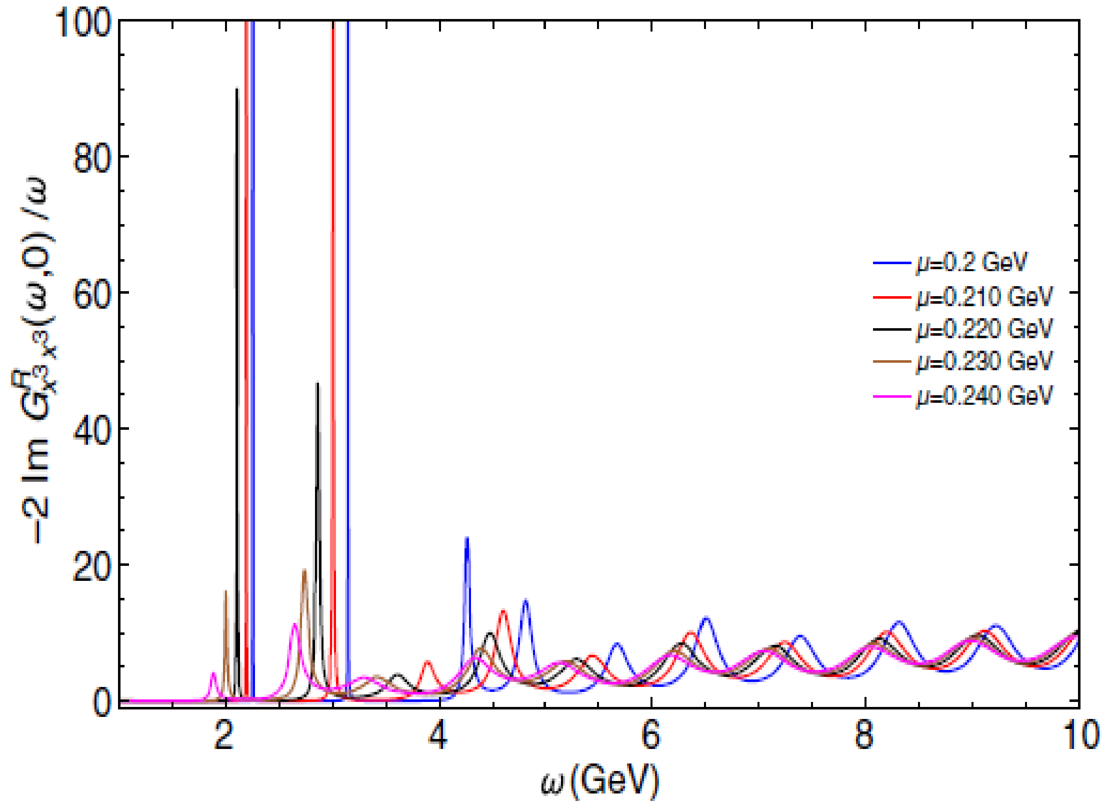
II Correlation function

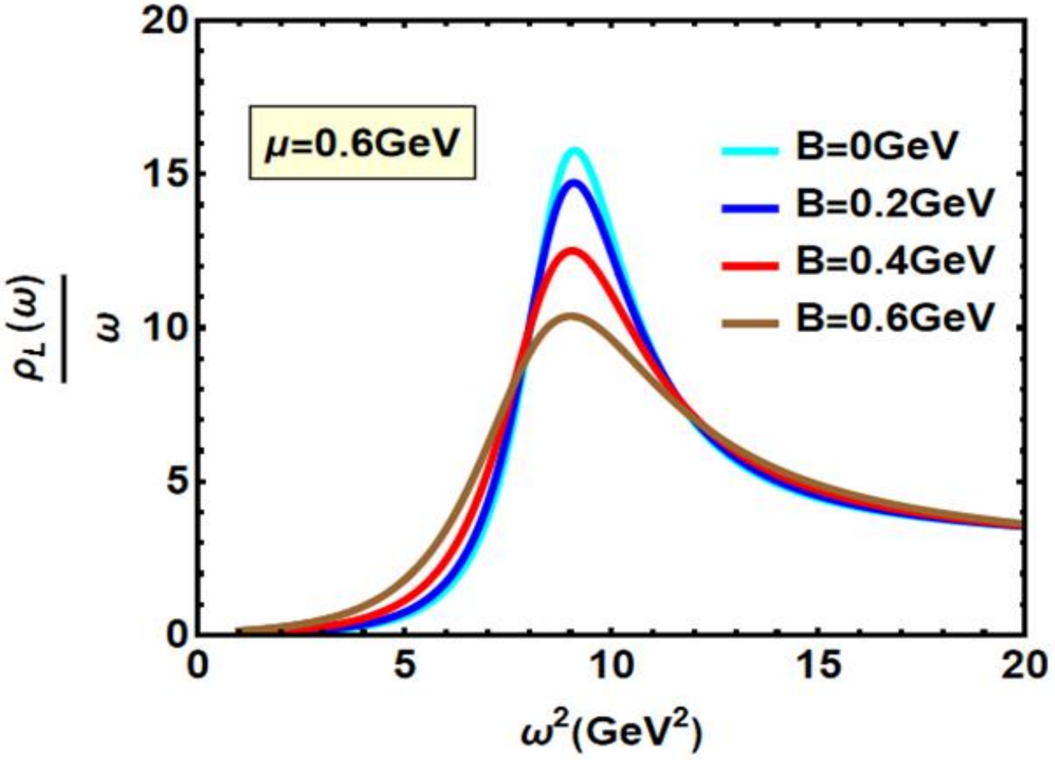
III J/ψ and ϕ mesons in hot plasma

IV J/ψ meson in magnetized plasma

The spectral function of heavy vector mesons

Mamani , Hou, Braga, PRD 105, 126020 (2022)



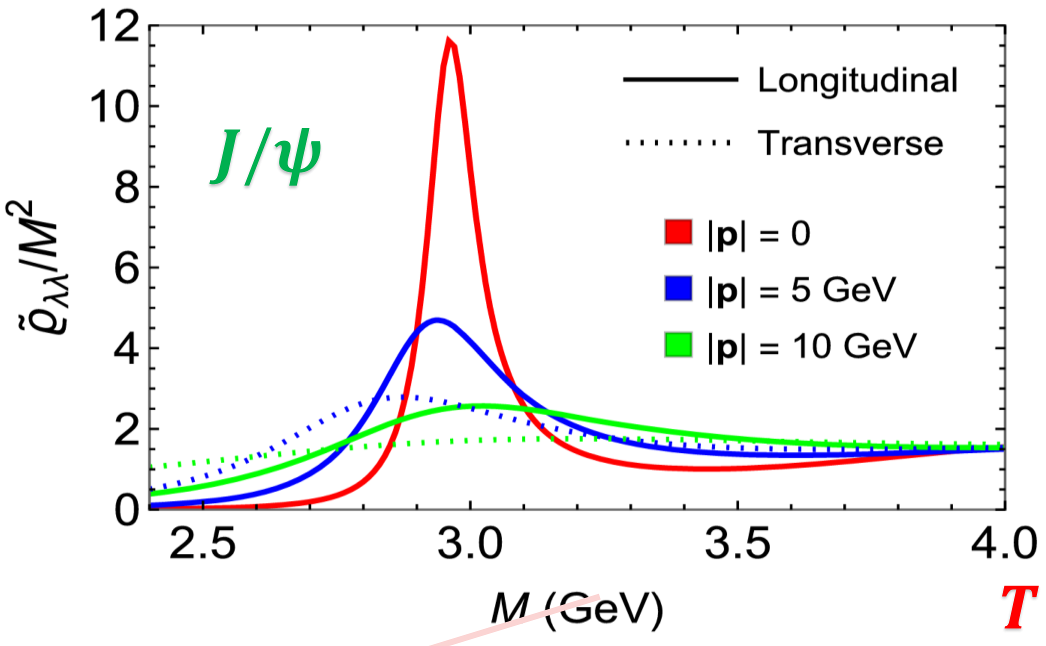


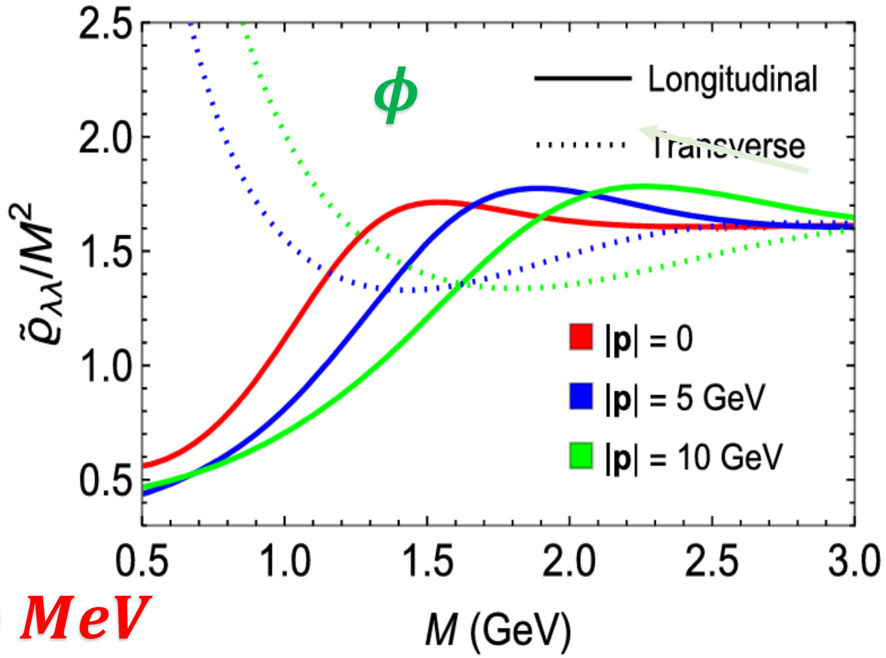
Yan-Qing Zhao, Defu Hou, Eur.Phys.J.C 82 (2022) 12, 1102 • e-Print: 2108.08479

Ⅲ、 J/ψ and ϕ mesons in hot plasma

Motion of J/ψ relative to a thermal background breaks symmetry between longitudinally polarized state and transversely polarized state

Spectral function





$T = 150\ MeV$

At zero momentum, spectral functions for all spin states are degenerate because of the rotation symmetry.

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Spin alignment in the helicity frame

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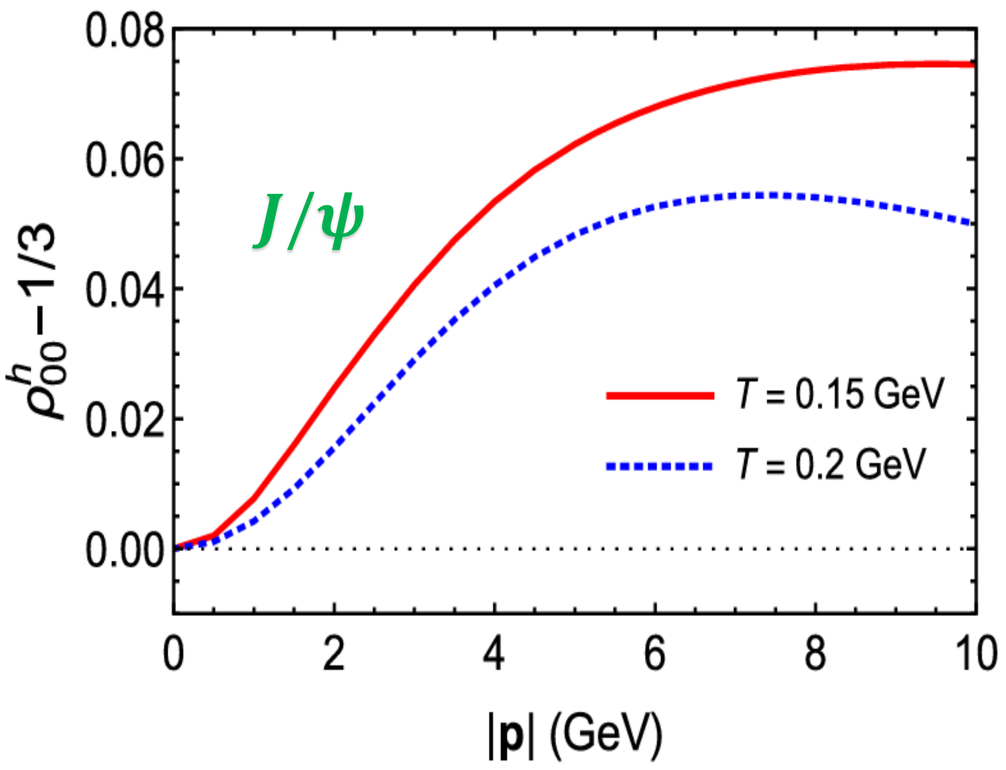
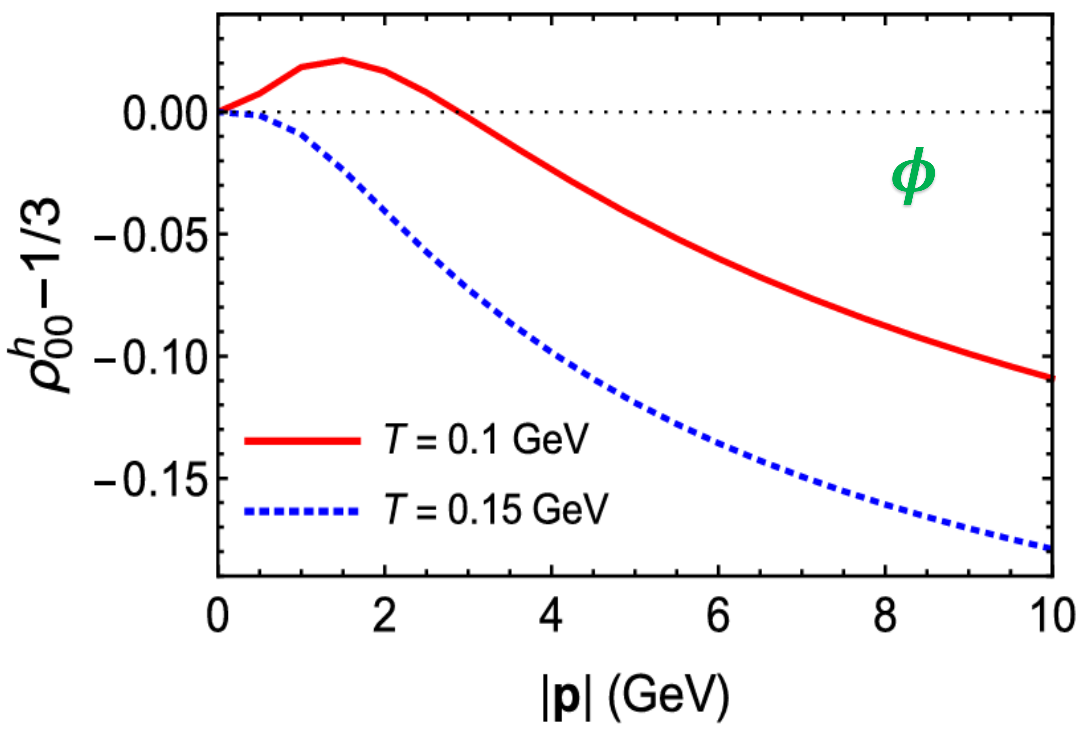




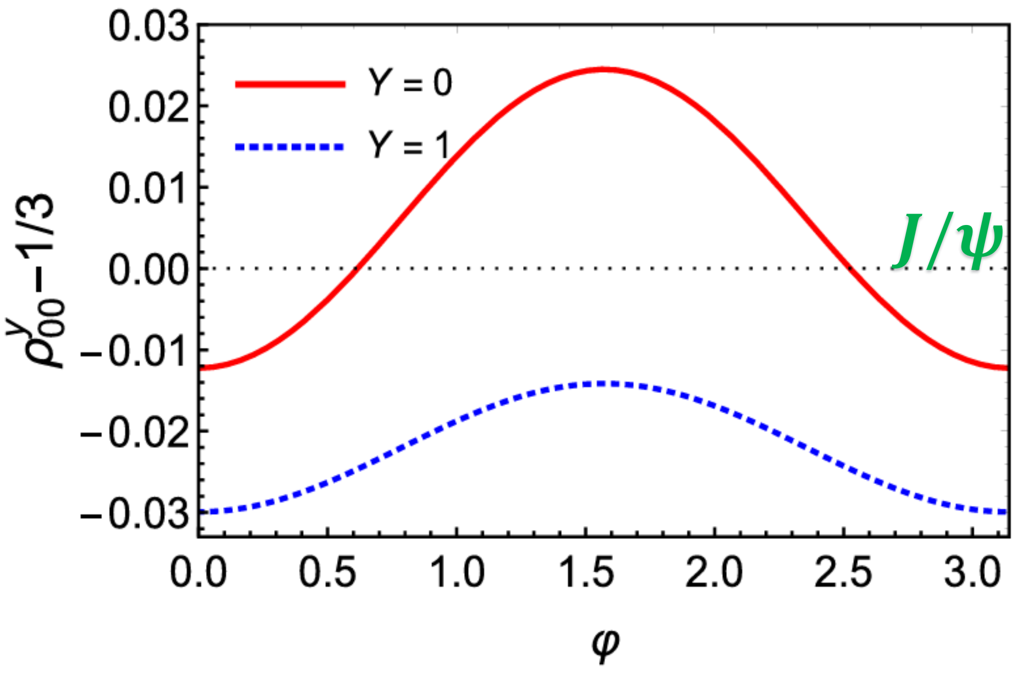
A positive deviation from $1/3$, corresponding to a negative λ_θ parameter.

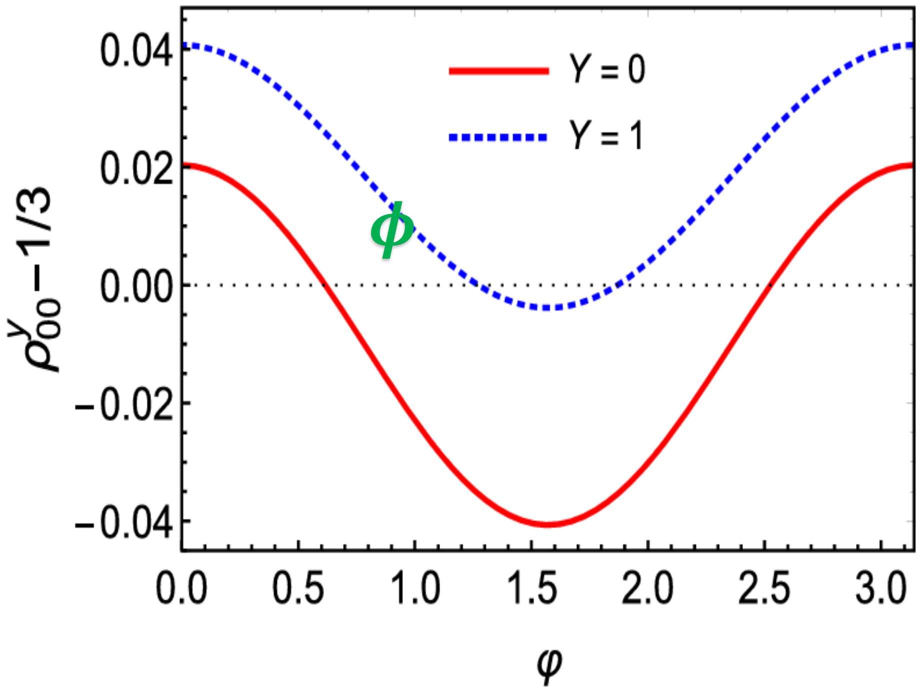
A negative deviation from $1/3$, absolute value becomes larger at larger $|p|$.

ρ^h_{00} at $T = 0.1$ GeV has a nonmonotonic dependence to $|p|$.

Global Spin alignment of J/ψ and ϕ

$p_T = 2\ GeV, T = 150\ MeV$





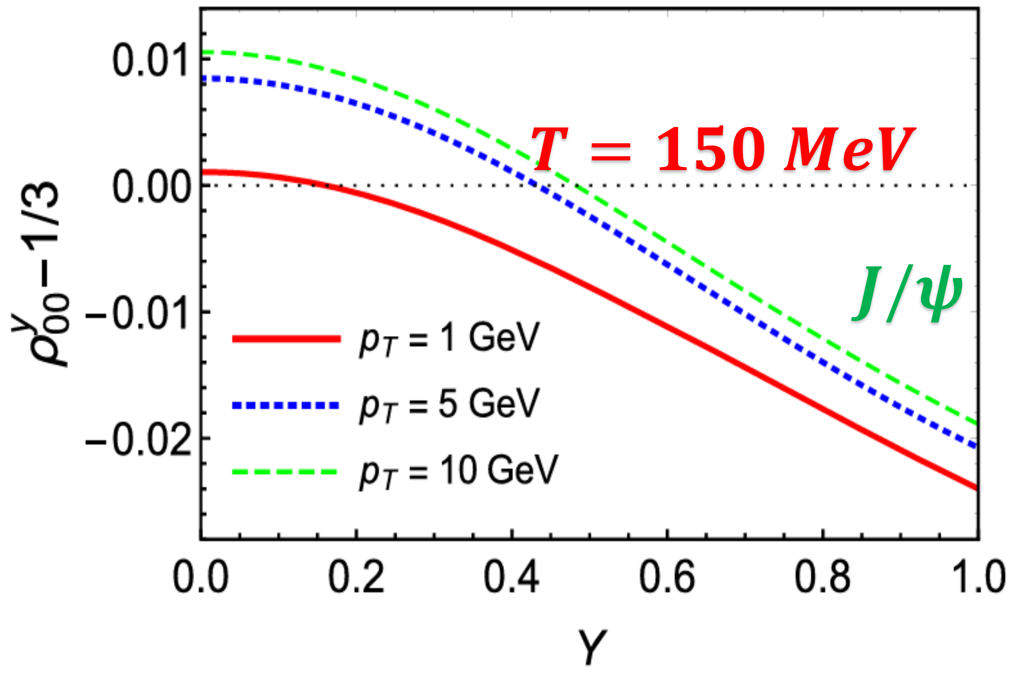
at center rapidity $Y = 0$, the spin alignment shows a negative deviation from $1/3$ when $\varphi = 0$, then increases with φ and reaches the maximum value at $\varphi = \pi/2$,which is larger than $1/3$.

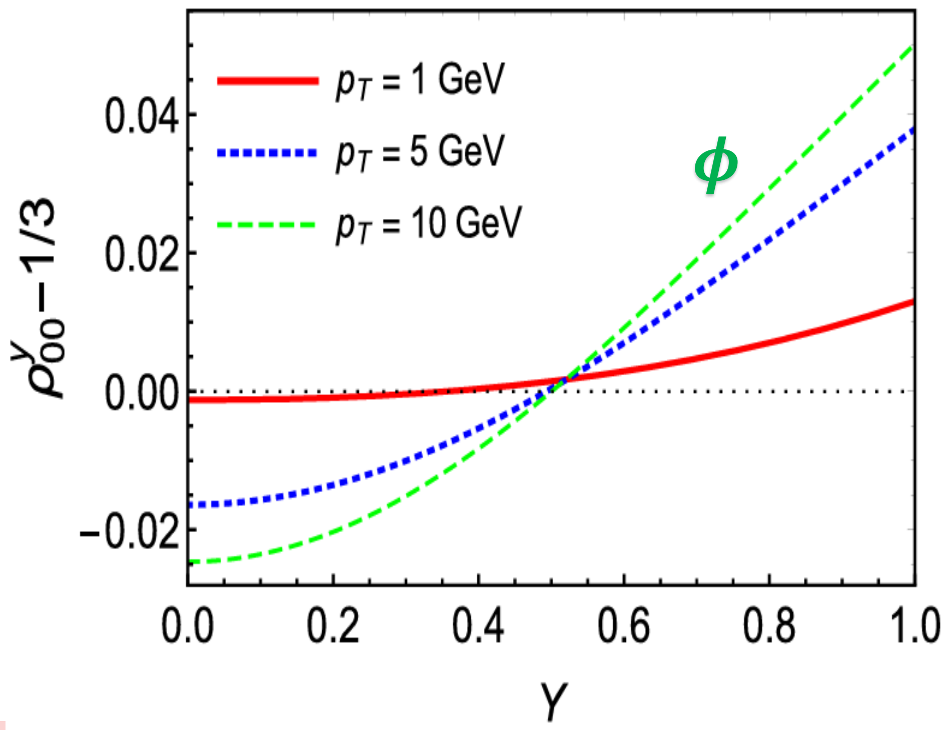
At a more forward rapidity $\mathrm{Y} = 1$, the spin alignment is always smaller than $1/3$.

at $Y = 0$, ρ_{00}^{y} is larger than $1/3$ at $\varphi = 0$ and decreases to a minimum value at $\varphi = \pi/2$, which is smaller than $1/3$.

The results at $\mathrm{Y} = 1$ have a positive shift compared to results at $Y = 0$.

Global Spin alignment





ρ_{00}^{y} increases with increasing p_T.

For all cases, $\rho_{00}^{y} > 1/3$ at center rapidity and decreases to a negative value at a larger rapidity. We can naturally expect that $\rho_{00}^{y} < 1/3$ in a more forward rapidity region $2.5 < Y < 4$ qualitatively agrees with the ALICE experiment.

$\rho_{00}^{y} < 1/3$ when $Y < 0.5$.

Absolute values for deviations from $1/3$ become larger at larger p_T, whose magnitude can be as large as $\mathcal{O}(10^{-2})$, which is the same magnitude as those observed in STAR experiments.

I Spin alignment

II Correlation function

III J/ψ and ϕ mesons in hot plasma

IV J/ψ meson in magnetized plasma

IV、 J/ψ meson in magnetized plasma

Yan-Qing Zhao ,Xin-Li Sheng ,Si-Wen Li DF Hou , JHEP08 (2024)070

➢ **Action:**

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}\right)+\frac{1}{8\pi G_5}\int d^4x\sqrt{-\gamma}\left(K-\frac{3}{L}\right)+S_{\mathrm{f}},$$

$$S_{\mathrm{f}} = -\frac{N_c}{16\pi^2}\int d^4x\int_0^{\zeta_h} d\zeta Q(\zeta)\mathrm{Tr}\left(F_L^2+F_R^2\right),$$

➢ **Holographic background metric:**

$$ds^2=\frac{L^2}{\zeta^2}\left(-f(\zeta)dt^2+h_T(\zeta)(dx^2+dy^2)+h_P(\zeta)dz^2+\frac{d\zeta^2}{f(\zeta)}\right).$$

$$f(\zeta)=1-\frac{\zeta^4}{\zeta_h^4}+\frac{2}{3}\frac{e^2B^2}{1.6^2}\zeta^4\ln\frac{\zeta}{\zeta_h}+\mathcal{O}(e^4B^4),$$

$$h_T(\zeta)=1-\frac{4}{3}\frac{e^2B^2}{1.6^2}\zeta_h^4\int_0^{\zeta/\zeta_h}\frac{y^3\ln y}{1-y^4}dy+\mathcal{O}(e^4B^4),$$

$$h_P(\zeta)=1+\frac{8}{3}\frac{e^2B^2}{1.6^2}\zeta_h^4\int_0^{\zeta/\zeta_h}\frac{y^3\ln y}{1-y^4}dy+\mathcal{O}(e^4B^4),$$

➢ **Hawking temperature:**

$$T=\frac{1}{4\pi}\left|\frac{4}{\zeta_h}-\frac{2}{3}\frac{e^2B^2}{1.6^2}\zeta_h^3\right|.$$

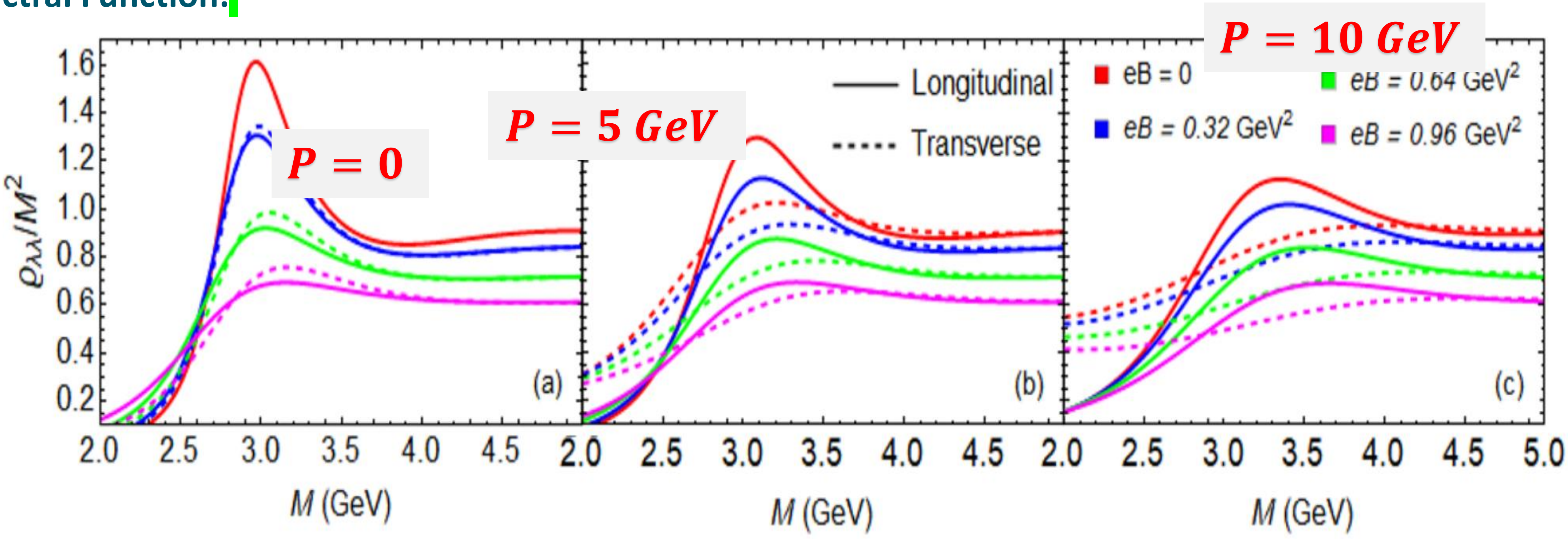
➢ **Magnetic field constraint conditions**

$$eB<\sqrt{\frac{3}{2}}\frac{1.6}{\zeta_h^2}\approx\frac{1.96}{\zeta_h^2}$$

- Magnetic field parallel to momentum

$T = 0.2\ \mathrm{GeV}$

- Spectral Function:



A nonzero magnetic field or a nonzero momentum will induce a separation between longitud. transversely polarized modes.

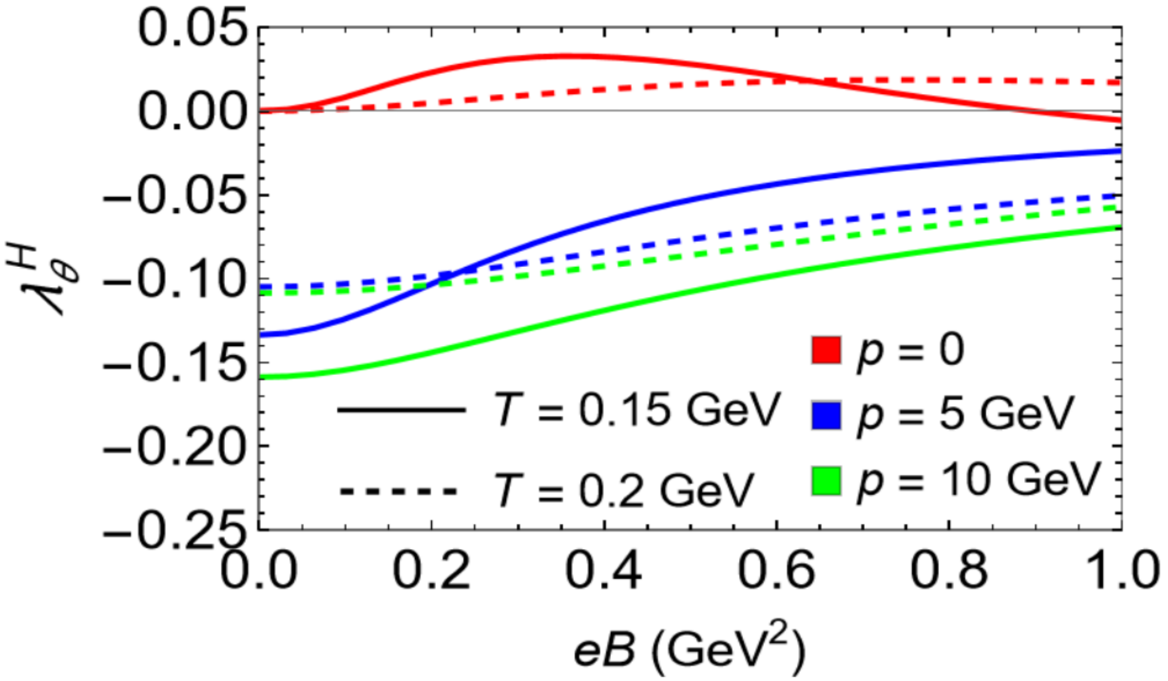
- **Magnetic field parallel to momentum**

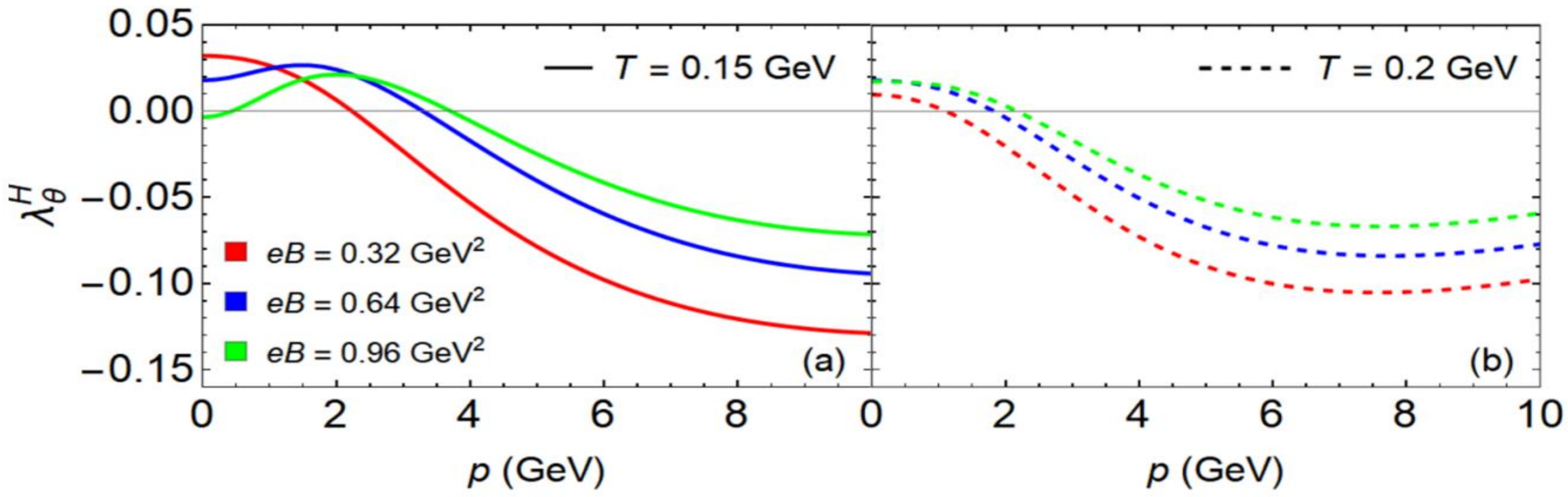
- **Spin alignment:**

$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}},$$

High T→monotonic ;

Low T → non-monotonic.

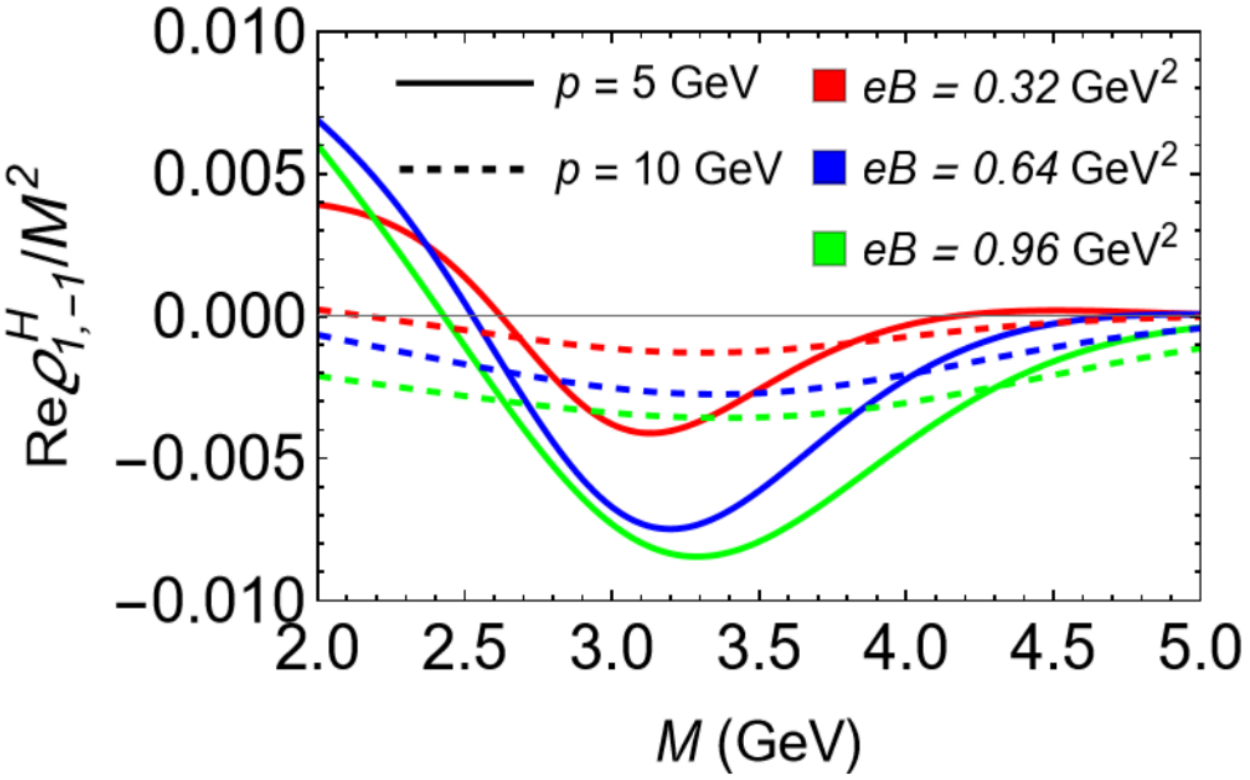




➢ **Magnetic field perpendicular to momentum**

➢ **Spectral Function:**

$T = 0.2$ GeV



$\mathrm{Re}\,\varrho^{\mathrm{H}}_{1,-1}$ reaches a minimum value at $M \sim 3$ GeV - 3.5 GeV.

$\mathrm{Re}\,\varrho^{\mathrm{H}}_{1,-1}$ is two orders of magnitude smaller than the diagonal elements $\varrho_{\lambda\lambda}$.

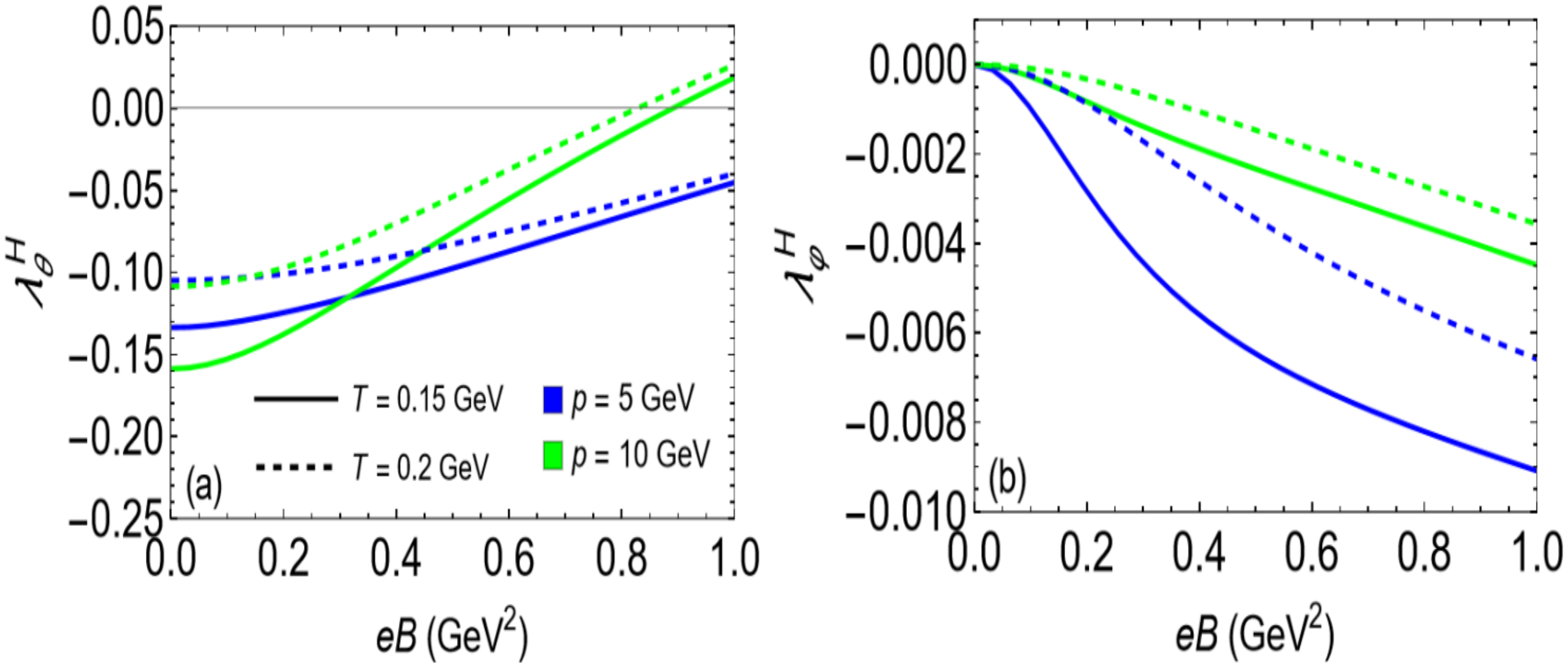
- **Magnetic field perpendicular to momentum**

Yan-Qing Zhao , Xin-Li Sheng , Si-Wen Li DF Hou , JHEP08 (2024) 070

- **Spin alignment:**

$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}},$$

$$\lambda_\varphi = \frac{2\mathrm{Re}\rho_{1,-1}}{1 + \rho_{00}},$$



the direction of magnetic field does not affect the qualitative behavior for the B-dependence of $\lambda_\theta^{\mathrm{H}}$.

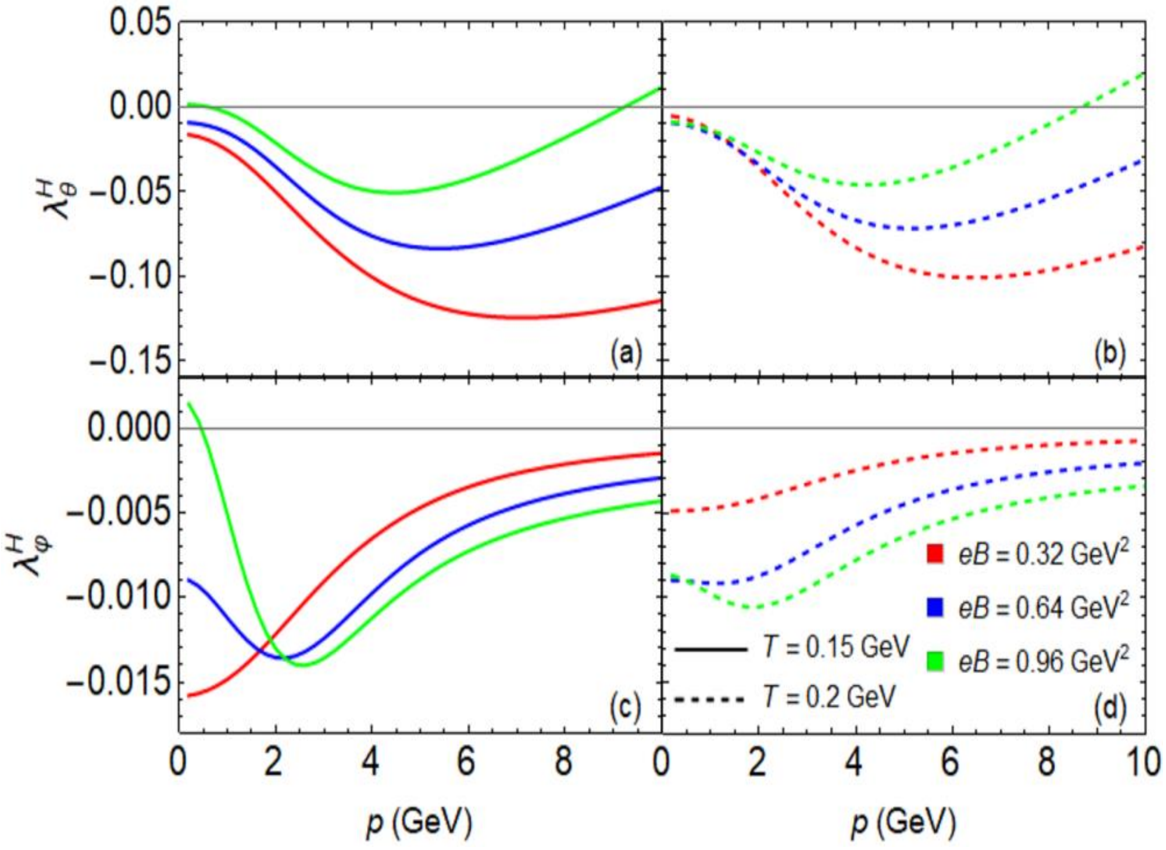
- Magnetic field perpendicular to momentum

$$\mathbf{p} = (p, 0, 0)$$

- Spin alignment:

$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}},$$

$$\lambda_\varphi = \frac{2\mathrm{Re}\rho_{1,-1}}{1 + \rho_{00}},$$



in most of the considered regions, both $\lambda_\theta^{\mathrm{H}}$ and $\lambda_\varphi^{\mathrm{H}}$ have negative values.

Application to heavy-ion collisions

- magnetic field along the y-direction,

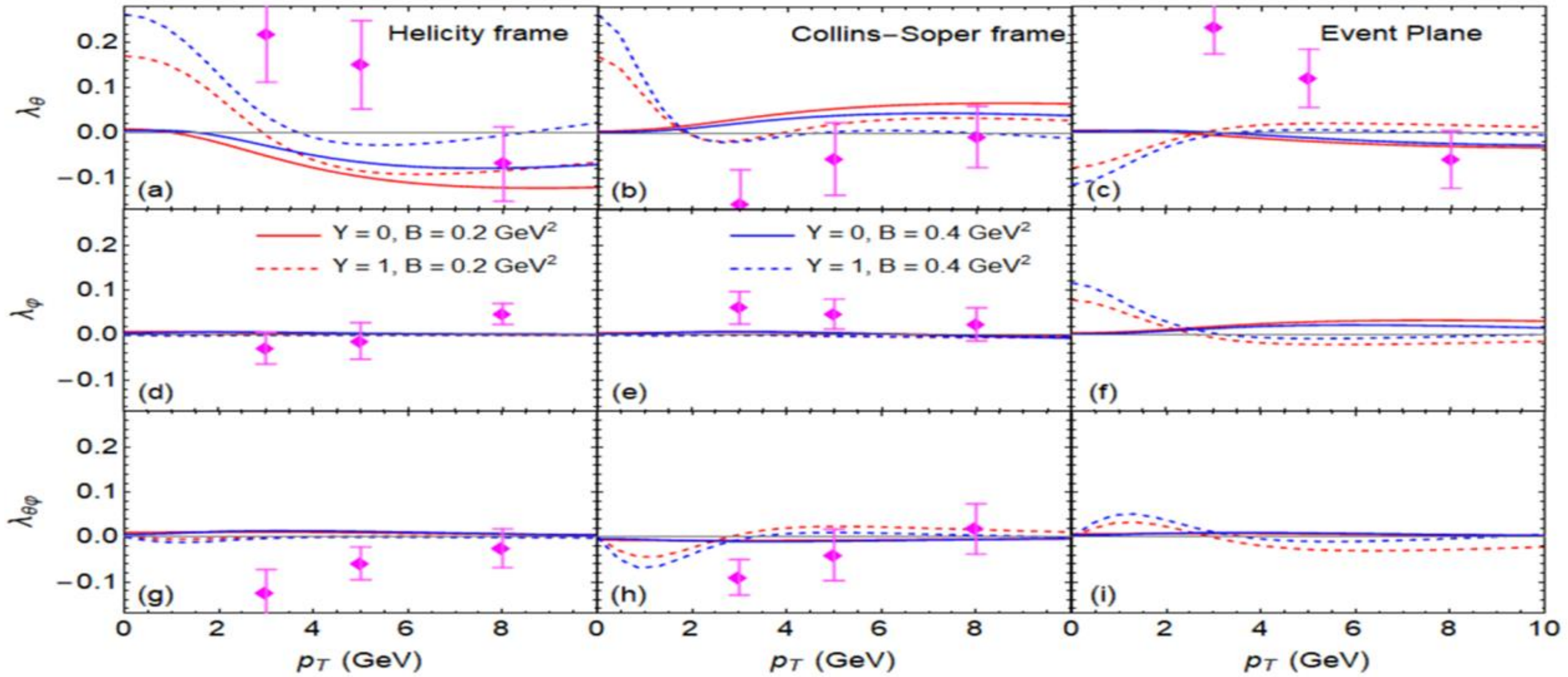
$$\boxed{T = 0.15\ \mathrm{GeV}}$$

$$\boxed{\mathbf{p} = \left(p_T\cos\varphi, p_T\sin\varphi, \sqrt{M^2 + p_T^2}\sinh(Y)\right)}$$

- Spin alignment: angular distribution of the dacay products

$$W(\theta^*,\varphi^*) \propto \frac{1}{3+\lambda_\theta}\Big(1 + \lambda_\theta \cos^2\theta^* + \lambda_\varphi \sin^2\theta^* \cos 2\varphi^* + \lambda_{\theta\varphi} \sin 2\theta^* \cos\varphi^*$$
$$+\lambda_\varphi^\perp \sin^2\theta^* \sin 2\varphi^* + \lambda_{\theta\varphi}^\perp \sin 2\theta^* \sin\varphi^*\Big)$$

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}}, \qquad \lambda_\varphi = \frac{2\mathrm{Re}\rho_{1,-1}}{1+\rho_{00}}, \qquad \lambda_{\theta\varphi} = \frac{\sqrt{2}\mathrm{Re}(\rho_{01} - \rho_{0,-1})}{1+\rho_{00}},$$



For the helicity frame and the Collins-Soper frame, we find that the λ_θ parameter is dominant when measuring along the event plane direction, all three parameters $\lambda_\theta^{\mathrm{EP}}$, $\lambda_\varphi^{\mathrm{EP}}$, and $\lambda_{\theta\varphi}^{\mathrm{EP}}$ are of the same order.

ALICE Collaboration, S. Acharya et al., Phys. Rev. Lett. 131 no. 4,(2023) 042303

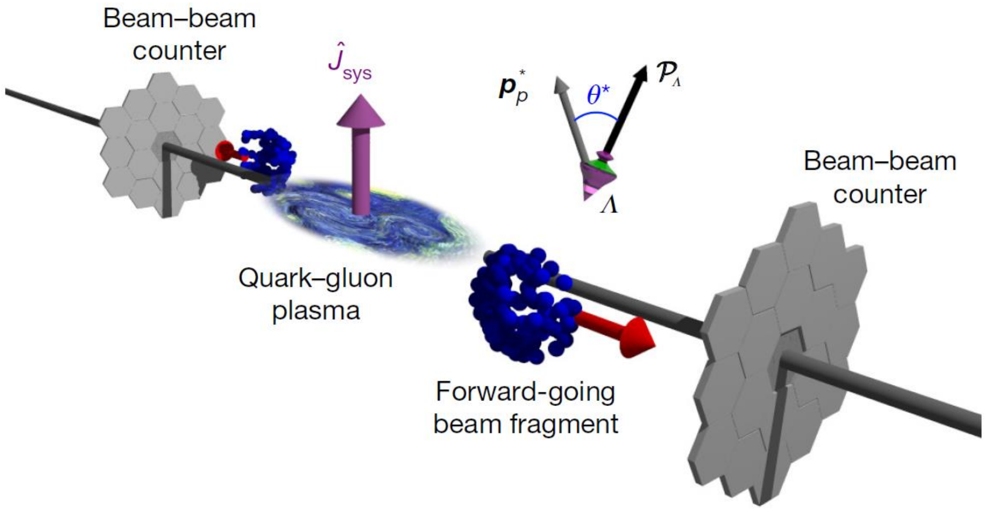
V Summary

- We develop a general framework for spin alignment ρ00 for vector mesons by gauge/gravity duality.
- The spin alignment can be purely induced by the motion of vector meson relative to the background.
- The holographic prediction shows that J/ψ and ϕ have opposite behaviours. $J/\psi(\phi)$ are preferably to be transversely(longitudinally) polarized.
- The meson's spin alignment is a non-perturbative property in the strongly interacting matter.
- Magnetic field induces $\lambda_\theta^H > 0$ when the meson's p is very small, while $\lambda_\theta^H < 0$ when p is large enough.
- show qualitative agreement with experimental data for λ_θ and λ_φ in the helicity and Collins-Soper frames.
- significant differences between our results for $\lambda_{\theta\varphi}^H$ and λ_θ^{EP} with experiments .

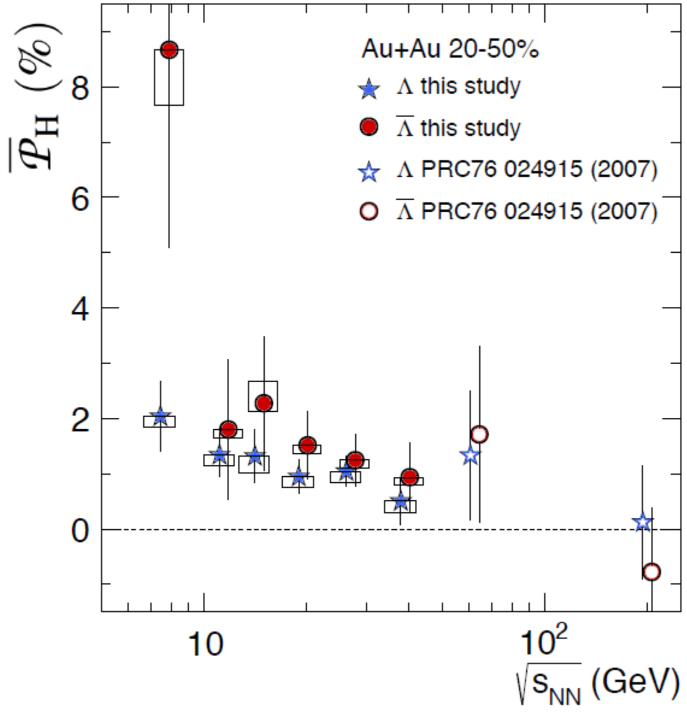
Thank you all very much ！

Global spin polarization: Experiments

- First measurement of Λ polarization by STAR@ RHIC



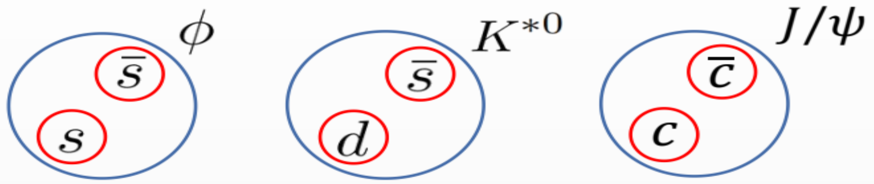
STAR, Nature 548,62 (2017)



Liang, Wang, PRL(2005)
Betz, Gyaulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini et al, Annals Phys (2013)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)

Vorticity interpretation of global Λ polarization works well!

- Spin alignment for a vector meson ($J^P = 1^-$) is 00-element ρ_{00} of its normalized spin density matrix, probability of spin-0 state



$$\rho^{S=1}_{rs} = \begin{pmatrix} \rho_{+1,+1} & \rho_{+1,0} & \rho_{+1,-1} \\ \rho_{0,+1} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,+1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix} = \frac{1}{3} + \frac{1}{2} P_i \Sigma_i + T_{ij} \Sigma_{ij}$$

Vector polarization (3 components, not measurable)

Tensor polarization (5 components, measurable)

- Measured through polar angle distribution of decay products

Processes	Examples	Polar angle distribution $W(\theta)$	Spin is converted to
Strong p-wave decay	$K^{*0} \to K^+ + \pi^-$ $\phi \to K^+ + K^-$	$\frac{3}{4}\left[1 - \rho_{00} + (3\rho_{00} - 1)\cos^2\theta\right]$	OAM
Dilepton decay	$J/\psi \to \mu^+ + \mu^-$	$\frac{3}{8}\left[1 + \rho_{00} + (1 - 3\rho_{00})\cos^2\theta\right]$	Spin

K. Schilling, P. Seyboth, G. E. Wolf, NPB 15, 397 (1970) [Erratum-ibid. B 18, 332 (1970)].
P. Faccioli, C. Lourenco, J. Seixas, H. K. Wohri, EPJC 69, 657-673 (2010)

Theoretical Methods