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Refs:

DC, P. Choudhary, B. Gurjar, R Kishore, T. Maji, C. Mondal, A. Mukherjee, PRD 108, 014009(2023); DC, P. Choudhary, B. Gurjar, T. Maji, C. Mondal, A. Mukhrejee, PRD 109, 1149040 (2024).

Gluon distributions in the proton

Light-Cone 2024 Huizhou November 25-29, 2024

Introduction

- One of the main goals of EIC/EicC is to understand the three dimensional structure of nucleons in terms of quarks and gluons as well as their spin and angular momentum distributions.
- Gluon distributions: crucial to understand low-x phenomena. Gluon PDFs are mainly small-x dominated.
- Form factors, PDFs, GPDs, TMDs, Wigner distributions.... Encode different informations.
- Gluon distributions are not yet well understood not enough theoretical studies.
- Large uncertainty in small-x, specially for polarized pdf \rightarrow not well constrained.
- Except lattice, they are mostly studied in different models.

In this talk, I'll mainly discuss gluon TMDs and GPDs.

TMDs: DY and SIDIS

• SIDIS and Drell-Yan processes are sensitive to TMDs.

Quark TMDs

 $=$ $H^{DY}(Q) \otimes f(Q^2, x_1, k_T) \otimes f(Q^2, x_2, k_T)$

• Azimuthal asymmetry of unpolarised quarks in transversely polarised proton: Sivers

- TMD factorization: DY: *dσ dQ*2*dydk*² *T*
- TMDs: 3D spatial structure of proton
- $\bullet \Rightarrow$ Transverse motion of partons, spin-transverse momentum correlations
- TMDS: Spin asymmetries
- effect.
- Final State Interaction (FSI) in SIDIS (Initial State Interaction for DY): gluon effect.

exchange between the struck quark and the remnant produces nonzero Sivers

* Talk by Marco Radici | Marco Radici | Brodsky, Hwang, Schmidt, PLB530, 99

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Gluon TMDs

- Gluon TMDs : experimentally/theoretically not yet well understood.
- Gluon distributions are small-x dominated. To study gluon TMDs, high energy and/or small x are required.
- Phenomenological models at small -x: Weiszacker-Williams(WW) [both gauge links are future pointing], dipole [one future and another past-pointing gauge link]
- EIC/EicC will probe the gluon distributions for both unpolarized and polarized proton.
- Gluon Sivers TMD: $e \, p^{\uparrow} \rightarrow eQ\bar{Q}X, e p \rightarrow eD\bar{D}X$ [back to back D meson pair production], $ep^{\uparrow} \rightarrow eJ/\Psi X$ C.Pisano 1912.13020
- Measurement of azimuthal asymmetries/transverse momentum distributions :gluon TMDs
- TMDs are not universal [due to FSI/ISI dependence] Sivers TMDs for quarks in SIDIS and DY differ by an overall negative sign.

D. Boer, 1601.01813

- Spectator model studies provide good insight into the different partonic distributions.
- Simplified, but insightful, help to understand the proton structure. GIMPINICU, DUC MOISICIUI, REIP LO UNUCISIANU CHE PIOLON SU UCLUI C. de proton structure.
	- consider the proton as a composite state of spin 1/2 spectator+ gluon(active parton). G distribution as a composite state of spin $1/2$ spectator + grudinactive parton). \mathbb{R} $\mathcal{A}(1)$ Z de de la posse \vert \vert \vert 1

¼ −1=2 have the

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|\mathbf{n}\mathbf{)}.
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$$
|P; \uparrow(\downarrow)\rangle = \int \frac{d^2 \mathbf{p}_{\perp} dx}{16\pi^3 \sqrt{x(1-x)}} \times \left[\psi_{+1+\frac{1}{2}}^{\uparrow(\downarrow)}(x, \mathbf{p}_{\perp}) \right] + 1, + \frac{1}{2}; x P^+, \mathbf{p}_{\perp} \rangle + \psi_{+1-\frac{1}{2}}^{\uparrow(\downarrow)}(x, \mathbf{p}_{\perp}) \right] + 1, -\frac{1}{2}; x P^+, \mathbf{p}_{\perp} \rangle
$$
\n
$$
+ \psi_{-1+\frac{1}{2}}^{\uparrow(\downarrow)}(x, \mathbf{p}_{\perp}) \Big| - 1, + \frac{1}{2}; x P^+, \mathbf{p}_{\perp} \rangle + \psi_{-1-\frac{1}{2}}^{\uparrow(\downarrow)}(x, \mathbf{p}_{\perp}) \Big| - 1, -\frac{1}{2}; x P^+, \mathbf{p}_{\perp} \rangle \Big],
$$

• $\Psi_{\lambda_g \lambda_X}(x, \mathbf{p}_{\perp})$ = LFWF corresponding to the two particle state $\psi_{\lambda,\lambda}^{\uparrow(\downarrow)}(x,\mathbf{p}_{\perp})$ P_{\perp}) $=$ LFWF corresponding to the ty $t = \mu$ is the corresponding to the th $\frac{\partial f(\mathcal{F})}{\partial g \partial x}(x,\mathbf{p}_{\perp})$ are \mathbf{p}_{\perp} are though that the theorem \mathbf{p}_{\perp} and the theorem \mathbf{p}_{\perp} two-particle state jλg; λX; xP^þ i put with proton proton en

acchetta, Celiberto, Radici, Taels, nents of the constituent gluon and spectrum and spectrum and spectrum and spectrum and spectrum and spectrum a ¼ ↑ð↓Þ. Here, λ^g and λ^X stand for the helicity compo-DC et al, PRD 108, 014009 Bacchetta, Celiberto, Radici, Taels, nents of the constituent gluon and spectrum and spectrum and spectrum and spectrum and spectrum and spectrum a
The constituent gluon and spectrum and spectrum and spectrum and spectrum and spectrum and spectrum and spectru

Light front model Model with active gluon

 $\begin{pmatrix} 1 & 2 & 3 & 3 \end{pmatrix}$ particle state $\mathcal{C}_g, \mathcal{C}_X, \mathcal{N}_I, \mathcal{P}_{\perp}$ $\begin{array}{ccc} \n\boxed{9} & 9 \\ \n\end{array}$ article state $\mathbb{P}_g, \mathbb{P}_X, \mathbb{P}_Y$, \mathbb{P}_Y particle state $\vert \lambda_g, \lambda_X; xP^+, \mathbf{p}_{\perp} \rangle$ particle state

 $\frac{1}{\sqrt{2}}$, the proton with J $\frac{1}{\sqrt{2}}$, the proton with J $\frac{1}{\sqrt{2}}$, the proton with J $\frac{1}{\sqrt{2}}$ DC et al, PRD 108, 014009 $\overline{}$, we propose the light-front wave functions in the light-front wave funct

$$
\varphi(x, \mathbf{p}_{\perp}^2) = N_g \frac{4\pi}{\kappa} \sqrt{\frac{\log[1/(1-x)]}{x}} x^b (1-x)^a \exp\left[-\frac{\log[1/(1-x)]}{2\kappa^2 x^2} \mathbf{p}_{\perp}^2\right].
$$
With

The model parameters are fitted to the unpolarised gluon PDF(NNPDF3.0 data) at $\mathcal{Q}_0 = 2$ GeV 2 $F(N)$ ffi DF3.0 data) at ζ r
1952
1952 $\partial_0 = 2$ GeV The model parameters are fitted to the unpolarised sqluon PDF(NNPDF3.0 data) at $O_0 = 2$ GeV parameters and the normalization constant \mathcal{G}_0 and \mathcal{G}_0 and \mathcal{G}_0 \to \mathcal{G}_0 \to \mathcal{G}_0 \to \mathcal{G}_0 $Q_0 = 2$

and the Stability of the Stability of the Stability of the proton mass of the proton mass, $D = 0.0000$

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$$
\psi_{+1+\frac{1}{2}}^{\uparrow}(x, \mathbf{p}_{\perp}) = -\sqrt{2} \frac{(-p_{\perp}^{1} + ip_{\perp}^{2})}{x(1-x)} \varphi(x, \mathbf{p}_{\perp}^{2}),
$$

\n
$$
\psi_{+1-\frac{1}{2}}^{\uparrow}(x, \mathbf{p}_{\perp}) = -\sqrt{2} \left(M - \frac{M_{X}}{(1-x)} \right) \varphi(x, \mathbf{p}_{\perp}^{2}),
$$

\n
$$
\psi_{-1+\frac{1}{2}}^{\uparrow}(x, \mathbf{p}_{\perp}) = -\sqrt{2} \frac{(p_{\perp}^{1} + ip_{\perp}^{2})}{x} \varphi(x, \mathbf{p}_{\perp}^{2}),
$$

\n
$$
\psi_{-1-\frac{1}{2}}^{\uparrow}(x, \mathbf{p}_{\perp}) = 0,
$$

Fixing the parameters

- 4 parameters in the model: *a*, *b*,*Ng*, *MX*
- N_g : fixed by normalization condition,
- Spectator mass $M_X > M$ (proton mass) $M_X > M$
- Behaviour of the distribution is determined by *a* and *b*
- The parameters in the model are fixed by fitting the unpolarised gluon pdf *f* with NNPDF3.0 NLO data at $Q_0 = 2$ GeV.

g $\binom{3}{1}(x)$

• Unpolarized gluon PDF: $\overline{\mathbf{r}}$ n PDF: \blacksquare with the latest available gluon \mathcal{C} \mathbf{r} $\mathbf D$ $\overline{}$ 1ðxÞ 4.416
1604 - Alexander Alexander (d. 1914)

• We take 300 NNPDF3.0NLO data points in the interval • We take 300 NNPI g • We take 300 NNPDF3.0NLO data points in the interval $0.001 < x < 1$ 300 NNPDF3.0NLO data points

•

¹ðxÞ as,

^κ² ^ð¹ [−] ^ð¹ [−] ^xÞ²^Þ

mass properties of the proton, respectively. The parameters a

and b decide the behavior of the distributions in extreme

limits of x are crucial to fix. We determine these model

 \bullet

choose 300 data points with the interval 0.001 data points with the interval 0.001 data

Excluded in our model. Large uncertainty in small-x region Excluded in our model.

 * Gluon mass $m_{\stackrel{\scriptstyle g}{\scriptstyle g}}=0$ **Parameters a, b depend on spectator mass(M_{χ}) We choose $M_x = 0943$ *MeV*: close to proton mass

$$
f^{g}(x) = 2N_g^2 x^{2b+1} (1-x)^{2a-2} \left[\kappa^2 \frac{(1+(1-x)^2)}{\log[1/(1-x)]} + (M(1-x) - M_X)^2 \right]
$$

unpolarized PDF as,

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Z 1910

DC et al. PRD 108, 014009

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19Þ - það 19Þ - það

- Model is very sensitive to small x , $x < 0.001$ region is excluded from the fit.
- Fitted parameters: $a = 3.88 \pm 0.22$, $b = -0.53 \pm 0.01$ (2*σ* error)
- Except the unpolarized gluon PDF, everything else is our model prediction.
- Average longitudinal momentum = second Mellin moment of unpolarised pdf:

$$
\langle x \rangle_{g} = \int_{0.001} dx \ x f_1^{g}(x) = 0.416^{+0.048}_{-0.041}
$$

• lattice result: $\langle x \rangle_{g} = 0.427(92)$

- Spectator model [1]: $\langle x \rangle_g = 0.424$
- Spectator model^[2]: $\langle x \rangle_g = 0.411$

• C. Alexandrou et al, PRD 101, 094512

A. Becchetta et al, EPJC 80, 733

Lu & Ma, PRD 94, 094022

[Circularly polarised gluon in a longitudinally polarised proton] $\frac{1}{\sqrt{1-\frac{1$ collinear limit of the world the world the world the Boer-Mulders and the Boer-Mulders and the Boer-Mulders and
The Boer-Mulders and the B [Circularly polarised gluon in a longitudinally polarised proton]

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Cluon helicity pdf

Gluon helicity pdf TABLE II. Comparison of the numerical values of the average of the average of the spin contribution of the gluon has been reported in Refs. [96,97]. The spin contribution of the gluon has been reported in Refs. [96,97]. Th spin contribution with the available data at Q⁰ ¼ 2 GeV. Meanwhile, in Ref. [80], the gluon spin contribution is a set of the gluon spin contribution is a set of the g
The gluon spin contribution is a set of the gluon spin contribution is a set of the gluon spin contribution is mass is MX → M. ϵ icity ndf gear g it dans huge and the Boer-Mulders huge

The Gluon helicity pdf and the solution of the Gluon helicity asymmetry and Gluon helicity asymmetry and the system of the Superinten with the Gluon helicity asymmetry

¹ ðxÞ TMDs as a

Gluon spin contribution: G distributions in the proton in G is a light- \mathcal{O} 108, 014009 (2023) (2 Gluon spin contributions in the Phys. Rev. 2023. GRUON DISTRIBUTION IN THE PROTOCOME IN THE PROTOCOME IN A LIGHT- \mathcal{L}

 \overline{a} $J0.05$ $\overline{}$ $\overline{}$

Gluon TMDs

• The correlator for gluon TMDs in SIDIS: \bullet $\Phi^{g[ij]}$ $(x, \mathbf{p}_{\perp}; S) = \frac{1}{rP}$ xP^+ $\int d\xi^{-}$ 2π $d^2 \xi_{\perp}$

i.e., M \sim

• At leading twist 8 gluon TMDs: 4 are T-even and 4 are T-odd. Λ the ding twist Ω cluon TMD _{0: A and T .} • At leading twist 8 giuon i MDS: 4 are 1-6

Model results

$$
\frac{d^2 \xi_{\perp}}{(2\pi)^2} e^{ik\cdot\xi} \langle P; S| F_a^{+j}(0) \mathcal{W}_{+\infty,ab}(0;\xi) F_b^{+i}(\xi) |P;S\rangle \bigg|_{\xi^+=0^+},
$$

and 4 are T-odd.

#

Chiral even gluon TMDs: Chiral even gluon TMDs: parameters and the normalization constant \mathbf{C} hiral even obtain WITH AT EVEN BILION. FORD STABILITY OF THE PROTON MASS, MASS, MITHOLD MASS, MA

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$$
= f_1^g(x, \mathbf{p}_\perp^2) - \frac{\epsilon_\perp^{ij} \mathbf{p}_\perp^i S_\perp^j}{M} f_{1T}^{\perp g}(x, \mathbf{p}_\perp^2)
$$

$$
\tilde{\Phi}^g(x, \mathbf{p}_\perp; S) = i \epsilon_T^{ij} \Phi^{g[ij]}(x, \mathbf{p}_\perp; S)
$$

$$
= \lambda g_{1L}^g(x, \mathbf{p}_\perp^2) + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M} g_{1T}^g(x, \mathbf{p}_\perp^2),
$$

Chiral even TMDs

$$
\frac{(f_1^g, g_{1L}^g, g_{1T}^g, \text{ and } h_1^{\perp g})}{M}
$$

 $\mathcal{L} = \{ \mathcal{L}^{\mathcal{L}} \}$

$$
\Phi^{g}(x, \mathbf{p}_{\perp}; S) = \delta_{T}^{ij} \Phi^{g[ij]}(x, \mathbf{p}_{\perp}; S)
$$
\n
$$
= f_{1}^{g}(x, \mathbf{p}_{\perp}^{2}) - \frac{\epsilon_{\perp}^{ij} \mathbf{p}_{\perp}^{i} S_{\perp}^{j}}{M} f_{1T}^{\perp g}(x, \mathbf{p}_{\perp}^{2})
$$
\n
$$
\tilde{\Phi}^{g}(x, \mathbf{p}_{\perp}; S) = i \epsilon_{T}^{ij} \Phi^{g[ij]}(x, \mathbf{p}_{\perp}; S)
$$
\n
$$
= \lambda g_{1L}^{g}(x, \mathbf{p}_{\perp}^{2}) + \frac{\mathbf{p}_{\perp} \cdot \mathbf{S}_{\perp}}{M} g_{1T}^{g}(x, \mathbf{p}_{\perp}^{2}),
$$
\n
$$
\mathbf{R}_{\perp} \mathbf{R
$$

$$
f(x,\mathbf{p}_{\perp}^{2}).
$$

Chiral odd gluon TMDs: the Chiral III. GLUON TMDs

 \bullet

$$
= h_{1L}^{\perp g}(x, \mathbf{p}_{\perp}^2)
$$

$$
h_{1T}^{\perp g}(x, \mathbf{p}_{\perp}^2)
$$

$$
,h_{1T}^{\perp g})
$$

Unpolarised TMD *f* T van die Broek van die Soos-provinsie van die Soos-provinsie van die Soos-provinsie van die Soos-provinsie van
Die Soos-provinsie van die Soos-provinsie van die Soos-provinsie van die Soos-provinsie van die Soos-provinsi gluon transverse momentum dependent distributions are distributions are distributions are distributions are dis obtained by using the symmetry operator $\mathcal{L}_\mathcal{D}$, we use $\mathcal{L}_\mathcal{D}$ is the symmetry operator $\mathcal{L}_\mathcal{D}$ ˆ , which is a set of \mathbb{R}^n , which is a set of \mathbb{R}^n ¹⁶π³ ^g μη ϵ \overline{a} $\overline{}$

• overlap representation of light front wave functions: ¼ ient $\mathfrak{u}\mathfrak{u}\mathfrak{v}$ n of light front wave functions.

1
1
1

• With our wave functions we get: \overline{a} fi <u>2</u> In ctions we? $\frac{1}{\sqrt{2\pi}}$ $\begin{array}{c}\n\text{ns we get:} \\
-x) \Big|_{\partial h} = \sqrt{2 \pi} \Big|_{\partial h} =$ A $\frac{1}{\pi k^2}$ $\frac{1}{x}$ $\frac{x}{y}$ *f* $g_1^g(x, p_\perp^2) = N^2 \frac{2}{\pi k}$ *πκ*² $ln[1/(1-x)]$ *x* $x^{2b}(1-x)^{2a}$

•

• When integrated over the transverse momentum, it reduces to the unpolarised pdf

^T ^¼ [−]gij and ^ϵ

ij

 $f_1^g(x)$. $\frac{1}{2}\int_{1}^{8}(x)$. *g* $\binom{2}{1}$

 $\binom{3}{1}$ (*x*, *p*² $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ \sim as the overlap representation of the proton light-front wave $f_1^{\circ}(x, p_1^{\circ})$ functions as $\mathbf{1}_{\mathbb{R}^3}$ λ0 ^gλ^X ^ðx; ^p² ⊥Þψ[↑] ^λgλ^X ^ðx; ^p² ⊥Þ

$$
f_1^g(x, \mathbf{p}_\perp^2) = \frac{1}{16\pi^3} \Big[|\psi_{+1+1/2}^{\uparrow}(x, \mathbf{p}_\perp^2)|^2 + |\psi_{+1-1/2}^{\uparrow}(x, \mathbf{p}_\perp^2)|^2 + |\psi_{-1+1/2}^{\uparrow}(x, \mathbf{p}_\perp^2)|^2 \Big].
$$

$$
(x-1)^{2a} \left[(M - \frac{M_X}{1-x})^2 + p_\perp^2 \frac{1 + (1-x)^2}{x^2(1-x)^2} \right] Exp[-\frac{\ln[1/(1-x)]}{\kappa^2 x^2} p_\perp^2]
$$

- Gluon helicity pdf. $\int_{1}^{8} f(x) dx = \int_{1}^{8}$ μ μ \perp δ ₁ h elicity pdf. $g_{1L}^g(x)$ $\overline{\lambda}$ $=$ $L^{(n)}$ g_{1}^g $d^2 p_{\perp} g_{1L}^g$ • Gluon helicity pdf. $g_{1L}^g(x) = \int d^2 p_\perp g_{1L}^g(x, p_\perp^2)$ $g(x) = d^2 p_{\perp} g_{1L}^g(x, p_{\perp}^2)$ $C(x) = \frac{\log(1/x)}{2}$ polarized proton in the set of \mathcal{S} p⊥:S[⊥] \bullet Glu uon helicity pdf. ^g $g_{1L}^g(x)$ \mathbf{F} $\int_{\mathcal{X}}$ \int \overline{D} $\overline{\mathcal{Q}}^g$ $(x, p²)$ $\mathcal{D}_\perp^>$ $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$
	- Worm-gear TMD: circularly polarized gluon in transversely polarized proton de de la de la deceni<mark>a de la c</mark>ontine de la distinccion de la contempo de la contempo de la contempo de la conte
La propie<mark>nta</mark>

• In our model: g d. el: \blacksquare In our model.

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• Helicity TMD: Circularly polarized gluon in longitudinally polarized proton 160 ized g \bullet Helicity TMD: Circularly polarized gluon in Jongitudinally polarized proton in longitudinally polari After gradin in tongitudinary polarized proton

• Helicity TMD: Circularly polarized gluon in longitudinally polarized proton
\n
$$
g_{1L}^{g}(x, \mathbf{p}_{\perp}^{2}) = \frac{1}{16\pi^{3}} \Big[|\psi_{+1+1/2}^{\uparrow}(x, \mathbf{p}_{\perp}^{2})|^{2} + |\psi_{+1-1/2}^{\uparrow}(x, \mathbf{p}_{\perp}^{2})|^{2} - |\psi_{-1+1/2}^{\uparrow}(x, \mathbf{p}_{\perp}^{2})|^{2} \Big]
$$

! ^M [−] MX "

$$
= N_g^2 \frac{2ln[1/(1-x)]}{\pi \kappa^2 x} x^{2b} (1-x)^{2a} \Big[\left(M - \frac{M_X}{1-x}\right)^2 + p_\perp^2 \frac{1 - (1-x)^2}{x^2 (1-x)^2} \Big] exp[-C(x)p_\perp^2]
$$

• Gluon helicity pdf. $g_{1L}^g(x) = \int d^2 p_\perp g_{1L}^g(x, p_\perp^2)$

$$
C(x) = \frac{\log[1/(1-x)]}{\kappa^2 x^2}.
$$

 Ω

 $(x, \mathbf{p}_{\perp}^2) \psi_{\lambda_g \lambda_X}^{\dagger}(x, \mathbf{p}_{\perp}^2)$ $\binom{2}{1}$ 2 by \sqrt{x} $\sqrt{x^2}$ $\mathbf{L}=\mathbf{L}+\mathbf{$

 $\frac{\lambda_g}{\lambda_g} \big)[\psi^{\uparrow *}_{\lambda_g \lambda_X}(x,{\bf p_\perp^2}) + \psi^{\downarrow *}_{\lambda_g' \lambda_X}(x,{\bf p_\perp^2}) \psi^{\uparrow}_{\lambda_g \lambda_X}(x,{\bf p_\perp^2})]$ $\mathcal{L}[\psi_{\lambda'_{\alpha}\lambda_{X}}^{\uparrow*}(x,\mathbf{p}_{\perp}^{2})\psi_{\lambda_{\alpha}\lambda_{X}}^{\downarrow}(x,\mathbf{p}_{\perp}^{2}) + \psi_{\lambda'_{\alpha}\lambda_{X}}^{\downarrow*}(x,\mathbf{p}_{\perp}^{2})\psi_{\lambda_{\alpha}\lambda_{X}}^{\uparrow}(x,\mathbf{p}_{\perp}^{2})]$ \mathbf{p}_{\perp}^{2}) $\psi_{\lambda_{g}\lambda_{\lambda}}^{\downarrow}$ \overline{z} $\sqrt{2}$ λgλ⁰ ^gλ^X $\hat{\mathbf{z}}$ $\binom{2}{1}$ – $+\psi_{\lambda'_a\lambda_X}^{\dagger*}(x,\mathbf{p}_\perp^2)\psi_{\lambda_a\lambda_X}^{\dagger}(x,\mathbf{p}_\perp^2)$

$$
\frac{\mathbf{p}_{\perp}.\mathbf{S}_{\perp}}{M} g_{1T}^g(x, \mathbf{p}_{\perp}^2) = -\frac{1}{16\pi^3} i \epsilon_T^{\mu\nu} \sum_{\lambda_g \lambda_g' \lambda_X} \epsilon_{\mu}^{\lambda_g} \epsilon_{\nu}^{\lambda_g} \psi_{\lambda_g' \lambda_X}^{\dagger *}(x, \mathbf{p}_{\perp}^2) \psi_{\lambda_g \lambda_X}^{\dagger}(x, \mathbf{p}_{\perp}^2)
$$
\n
$$
= \frac{1}{16\pi^3} \frac{i}{2} \sum_{\lambda_g \lambda_g' \lambda_X} (\epsilon_1^{\lambda_g} \epsilon_2^{\lambda_g} - \epsilon_2^{\lambda_g' \epsilon} \epsilon_1^{\lambda_g}) [\psi_{\lambda_g' \lambda_X}^{\dagger *}(x, \mathbf{p}_{\perp}^2) \psi_{\lambda_g \lambda_X}^{\dagger}(x, \mathbf{p}_{\perp}^2) + \psi_{\lambda_g' \lambda_X}^{\dagger *}(x, \mathbf{p}_{\perp}^2) \psi_{\lambda_g \lambda_X}^{\dagger}(x, \mathbf{p}_{\
$$

$$
g_{1T}^g(x, \mathbf{p}_\perp^2) = -\frac{4M}{\pi\kappa^2} N_g^2(M(1-x) - M_X) \log[1/(1-x)]x^{2b-2}(1-x)^{2a-1} \exp[-C(x)\mathbf{p}_\perp^2].
$$

•

• Boer-Mulders TMD: linearly polarized gluon inside unpolarised proton [interference between ±1 gluon helicities] ⊥ • Boer-Mulders TN N η $\overline{\mathbf{L}}$ $\frac{1}{2}$ pola $\overline{\mathbf{z}}$

$$
\frac{\mathbf{p}_\perp^2}{2M^2}h_1^{\perp g}(x,\mathbf{p}_\perp^2)=\frac{1}{2}\eta_T^{\mu\nu}\sum_{\lambda_N\lambda_g\neq \lambda'_g\lambda_X}\frac{1}{16\pi^3}\Big[\epsilon^{\lambda'_g*}_\mu\epsilon^{\lambda_g}_\nu\psi^{*\lambda_N}_{\lambda'_g\lambda_X}(x,\mathbf{p}_\perp)\psi^{\lambda_N}_{\lambda_g\lambda_X}(x,\mathbf{p}_\perp)\Big],
$$

- Analytic form in our model: 2 $\sqrt{11/1}$ κ^2 ights $\ddot{}$ $h_1^{\perp g}(x, \mathbf{p}_\perp^2) = \frac{8M^2}{\pi \kappa^2}$ $\frac{\partial W}{\partial x} N_g^2 \log[1/(1-x)]x^{2b-3}(1-x)^{2a-1} \exp[-C(x)]p_{\perp}^2$ $\mathcal{L}\mathcal{L}$ $\frac{101111100001}{8042}$ (x) | x^{2D-3} (1)
- Corresponding PDFs : 16π
3 Γ • Corresponding $\mathbf{\alpha}$ g PDF: \mathbf{S} : $t \cdot \theta$

 \bullet

$$
g_{1T}^{g}(x) = \int d^{2} \mathbf{p}_{\perp} g_{1T}^{g}(x, \mathbf{p}_{\perp}^{2}) \qquad h_{1}^{\perp g}(x) = \int d^{2} \mathbf{p}_{\perp} h_{1}^{\perp g}(x, \mathbf{p}_{\perp}^{2})
$$

on inside unpolarised pro

$$
(1-x)^{2a-1}\exp[-C(x)\mathbf{p}_\perp^2].
$$

 $2\sqrt{ }$ $\lfloor \frac{2}{\cdot} \rfloor.$ L) transversely polarised proton] The Changes of the C Gluon Boer-Mulders pdf [transversley polarised gluon in an unpolarised proton]

Gluon worm gear pdf [circularly polarised gluon in a

Gluon TMDs:

•

 $For x=0.1$ Γ O Γ X=O.I

 \blacksquare

• Mulders-Rodrigues relations put more stringent conditions on TMDS: \bullet ¹^Tðx; ^p² [⊥]^Þ (left) and the Boer-Mulders TMD, ^h[⊥]^g ¹ ^ðx; ^p² [⊥]^Þ (right), Rodrigues rel relations put more strin

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g

TMD relations: ¹ðx; ^p²

[⊥]^Þ [≥] ^j^g

1
1920 - Paul Barnett, paul Barnett
1920 - Paul Barnett, paul
1920 - Paul Barnett, paul Barnett, paul Barnett, paul Barnett, paul Barnett,

[⊥]Þj: ^ð28^Þ

relations and l anendent $\begin{array}{c|c}\n\hline\n\end{array}$ $\overline{}$ Model-independent

"

j
Literature
Literature

ions put more stringent conditions on TMDS: \bullet An interesting sum rule has been derived in Ref. [86] int conditions on TMDs;

$$
\sqrt{[g_{1L}^g(x, \mathbf{p}_\perp^2)]^2 + \left[\frac{|\mathbf{p}_\perp|}{M} g_{1T}^g(x, \mathbf{p}_\perp^2)\right]^2} \leq f_1^g(x, \mathbf{p}_\perp^2),
$$
\n
$$
\sqrt{[g_{1L}^g(x, \mathbf{p}_\perp^2)]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2} h_1^{\perp g}(x, \mathbf{p}_\perp^2)\right]^2} \leq f_1^g(x, \mathbf{p}_\perp^2),
$$
\n
$$
\sqrt{\left[\frac{|\mathbf{p}_\perp|}{M} g_{1T}^g(x, \mathbf{p}_\perp^2)\right]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2} h_1^{\perp g}(x, \mathbf{p}_\perp^2)\right]^2} \leq f_1^g(x, \mathbf{p}_\perp^2).
$$

$$
f_1^g(x, \mathbf{p}_\perp^2) \ge |g_{1L}^g(x, \mathbf{p}_\perp^2)|.
$$

$$
p_{\perp}^{2}) \ge \frac{|\mathbf{p}_{\perp}|}{M} |g_{1T}^{g}(x, \mathbf{p}_{\perp}^{2})|,
$$

\n
$$
p_{\perp}^{2}) \ge \frac{|\mathbf{p}_{\perp}|^{2}}{2M^{2}} |h_{1}^{\perp g}(x, \mathbf{p}_{\perp}^{2})|.
$$

\nModel-independent relations

$$
\frac{g_{1T}^q(x, \mathbf{p}_\perp^2)^2}{h_1^{\perp g}(x, \mathbf{p}_\perp^2)} \leq f_1^g(x, \mathbf{p}_\perp^2),
$$
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$$
\frac{h_1^{\perp g}(x, \mathbf{p}_\perp^2)^2}{h_1^{\perp g}(x, \mathbf{p}_\perp^2)} \leq f_1^g(x, \mathbf{p}_\perp^2),
$$
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$$
\frac{h_1^{\perp g}(x, \mathbf{p}_\perp^2)^2}{h_1^{\perp g}(x, \mathbf{p}_\perp^2)} \leq f_1^g(x, \mathbf{p}_\perp^2).
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$$
\frac{h_1^{\perp g}(x, \mathbf{p}_\perp^2)^2}{h_1^{\perp g}(x, \mathbf{p}_\perp^2)} \leq f_1^g(x, \mathbf{p}_\perp^2).
$$

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Positivity bound

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 Ξ unpolarized or longitudinally (circularly) polarized alongitudinally polarized alongitudinally polarized alongitudinally Ξ (p_1)

Gluon densities polarization and nucleon spin state. The unpolarized gluon density in an unpolarizated nucleon in an unpolarizated nucleon is calculated nucleon in an uncleon is cal s distributions are cylindrically symmetrically sym

• Unpolarised gluon density in an unpolarised proton = probability of finding the gluon with momentumm (x, p_\perp) as follows:

of gluons at various x for various combinations of the
The internal interna

$$
x\rho_g(x, p_x, p_y) = x f_1^g(x, \mathbf{p}_\perp^2),
$$

Gluon densities polarization and nucleon spin state. The unpolarized gluon density in an unpolarizated nucleon in an unpolarizated nucleon is calculated nucleon in an uncleon is cal s distributions are cylindrically symmetrically sym

• Unpolarised gluon density in an unpolarised proton = probability of finding the gluon with momentumm (x, p_\perp)

of gluons at various x for various combinations of the
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$$
x\rho_g(x, p_x, p_y) = x f_1^g(x, \mathbf{p}_\perp^2),
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Symmetric

Ξ unpolarized or longitudinally (circularly) polarized alongitudinally polarized alongitudinally polarized alongitudinally Ξ (p_1) \mathcal{L} and \mathcal{L}

- Boer-Mulders density: linearly polarized gluon density \bullet $x\rho_{q}^{\leftrightarrow}$ $g^{\Leftrightarrow}(x,p_x,p_y)=\frac{1}{2}$ 2 $\sqrt{2}$ $xf_1^g(x, \mathbf{p}_\perp^2) + \frac{p_x^2 - p_y^2}{2M^2}xh_1^{\perp g}$
- Spherical symmetry gets distorted due to the second term(Boer-Mulders TMD)...shows dipolar structure in momentum space.

$$
xh_1^{\perp g}(x,\mathbf{p}_\perp^2)
$$

- Boer-Mulders density: linearly polarized gluon density • $x\rho_{q}^{\leftrightarrow}$ $g^{\Leftrightarrow}(x,p_x,p_y)=\frac{1}{2}$ 2 $\sqrt{2}$ $xf_1^g(x, \mathbf{p}_\perp^2) + \frac{p_x^2 - p_y^2}{2M^2}xh_1^{\perp g}$
- Spherical symmetry gets distorted due to the second term(Boer-Mulders TMD)...shows dipolar structure in momentum space. Γ

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is given as a set of $\mathcal{O}(\log n)$

$$
xh_1^{\perp g}(x,\mathbf{p}_\perp^2)
$$

- Boer-Mulders density: linearly polarized gluon density • $x\rho_{q}^{\leftrightarrow}$ $g^{\Leftrightarrow}(x,p_x,p_y)=\frac{1}{2}$ 2 $\sqrt{2}$ $xf_1^g(x, \mathbf{p}_\perp^2) + \frac{p_x^2 - p_y^2}{2M^2}xh_1^{\perp g}$
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is given as a set of $\mathcal{O}(\log n)$

$$
xh_1^{\perp g}(x, \mathbf{p}_\perp^2)
$$

• Worm-gear density: circularly polarized gluon density in a transversely polarized proton • Worm-gear density: circularly polarized gluon density in a transversely polarized the "which density" which description of "which density" polarized the "which density" in a transversely polarized the "which density" in i

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• Helicity density: circularly polarized gluon density in a longitudinally polarized proton. o Holigity dopoity givenlarly no

$$
x\rho_g^{\circ}\sigma/+(x, p_x, p_y) = \frac{1}{2} [xf_1^g(x, \mathbf{p}_\perp^2) + xg_{1L}^g(x, \mathbf{p}_\perp^2)]
$$

$$
x \rho_g^{\sigma/\leftrightarrow}(x, p_x, p_y) = \frac{1}{2} [xf_1^g(x, \mathbf{p}_\perp^2) - \frac{p_x}{M} x g_{1T}^g(x, \mathbf{p}_\perp^2)]
$$

⊥Þþxgg [⊥]Þ& ð26^Þ

• Worm-gear density: circularly polarized gluon density in a transversely polarized proton • Worm-gear density: circularly polarized gluon density in a transversely polarized the "which density" which description of "which density" polarized the "which density" in a transversely polarized the "which density" in i

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$$
x\rho_{g}^{(3/\leftrightarrow)}(x, p_x, p_y) = \frac{1}{2} [xf_1^g(x, p_\perp^2) - \frac{p_x}{M} x g_{1T}^g(x, p_\perp^2)]
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• Helicity density: circularly polarized gluon density in a longitudinally polarized proton. o Holigity dopoity givenlarly no $\frac{1}{\sqrt{2}}$

$$
x\rho_g^{\circled{5}/+}(x, p_x, p_y) = \frac{1}{2} [xf_1^g(x, \mathbf{p}_\perp^2) + xg_{1L}^g(x, \mathbf{p}_\perp^2)]
$$

[⊥]Þ& ð26^Þ

- GPDs appear in exclusive processes e.g., DVCS/ vector meson production
- are off-forward matrix elements of the bilocal operator and functions of (x, ξ, t) . • encode spatial as well as spin structure of the nucleon.
-
- don't have probabilistic interpretation.
- for skewness $\xi = 0$, in impact parameter space can have probabilistic interpretation. μ all Have prof
- In the forward limit GPDs \rightarrow PDFs.

GPDs

- GPDs appear in exclusive processes e.g., DVCS/ vector meson production
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-
- don't have probabilistic interpretation.
- for skewness $\xi = 0$, in impact parameter space can have p interpretation. (handbag diagrams dominate for large virtualities of the virtual photon), involves a quark propagator l,
- In the forward limit GPDs \rightarrow PDFs.

GPDs

(a) (b)

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-
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- In the forward limit GPDs \rightarrow PDFs.

GPDs

- Since nonperturbative QCD evaluations are not yet feasible, it is important to constraint the GPDs by using different model predictions. ?) reduces Since nonperturba
- II. GLUON GERMANIA G In the light-cone gauge *A*⁺ = 0, the o↵-forward matrix elements of the bilocal currents of light-front correlation In the light cone gauge $(A^+ = 0)$, 4 helicity conserving gluon (
- We analyze the gluon GPDs in our model for both $\xi = 0$ and $\xi \neq 0$ (in experiments $\xi \neq 0$). • In the light cone gauge $(A^+ = 0)$, 4 helicity conserving gluon GPDs:

• 4 gluon helicity flip GPDs: α *F F* α **F** is the dual field strength tensor and a summation of 1*, 2 is in CPDs.* 4 SIMOII Helicity hip GPDs: **g** the *iii* of the symmetrization operator in *i* and *j* an

•

constituents masses is considered than the proton mass, i.e., α *X also necessary for the stability for the stab* PDF data set at initial scale *µ*⁰ = 2 GeV and can be found in Ref. [55]. For the stability of the proton, the sum of of proton. **Explain why**. With *a* = 0 and *b* = 0, the modified form of light-front wave function, '(*x,* p²

$$
\frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | F^{+i}(-\frac{z}{2}) F^{+i}(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+ = 0, z_T = 0} = \frac{1}{2P^+} \bar{u}(p', \lambda') \left[H^g \gamma^+ + E^g \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} \right] u(p, \lambda),
$$

$$
-\frac{i}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | F^{+i}(-\frac{z}{2}) \bar{F}^{+i}(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+ = 0, z_T = 0} = \frac{1}{2P^+} \bar{u}(p', \lambda') \left[\tilde{H}^g \gamma^+ \gamma_5 + \tilde{E}^g \frac{\gamma_5 \Delta^+}{2M} \right] u(p, \lambda),
$$

$$
-\frac{1}{P^+}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle p',\lambda'|\mathbf{S}F^{+i}(-\frac{z}{2})F^{+j}(\frac{z}{2})\ket{p,\lambda}\Big|_{z^+=0,\,\mathbf{z}_T=0}=\mathbf{S}\frac{1}{2P^+}\frac{P^+\Delta^j-\Delta^+P^j}{2MP^+}\times\bar{u}(p',\lambda')\Bigg[H_T^g\,i\sigma^{+i}+\tilde{H}_T^g\frac{P^+\Delta^i-\Delta^+P^i}{M^2}+\frac{F_T^g}{2M}\frac{\gamma^+\Delta^i-\Delta^+\gamma^i}{2M}+\tilde{E}_T^g\frac{\gamma^+P^i-P^+\gamma^i}{M}\Bigg]u(p,\lambda)
$$

- We consider $x \geq \xi$ only: particle number conserving process.
- In the forward limit GPDs \longrightarrow PDFs
- Unpolarised gluon GPD $H^g(x, \xi = 0, t = 0) = f^g(x)$ = unpolarised gluon pdf
- Helicity dependent GPD $\tilde{H}^{g}(x,\xi=0,t=0) = g_{1L}^{g}(x)$ = helicity pdf

$$
{}^{g}(x, \xi = 0, t = 0) = f^{g}(x) = \text{unpolarised gluon pdf}
$$

$$
\tilde{H}^{g}(x, \xi = 0, t = 0) = g^{g}_{1L}(x) = \text{helicity pdf}
$$

Check for your calculations!

FIG. 1: Our predictions for the 3D representation of gluon GPDs as a function of gluon longitudinal momentum

 xE^g , $x\tilde{E}^g$, xH_T^8 are of similar behaviour(with different magnitudes)

Gluon GPDs at $-t = 3$ GeV^2 as functions of (x, ξ)

Magnitudes of the peaks depend on skewness ξ

2D plots

GPDs in impact parameter space The two dimensional Fourier transverse with respect to the GPDs at $\frac{1}{2}$

the GPD in impact parameter space.

•

$$
\mathcal{F}(x, b_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{-i\Delta_{\perp} \cdot b_{\perp}} F^g(x, \xi = 0, t = -\Delta_{\perp}^2)
$$

Transverse distance of the struck quark from the CoM

- conditions but also o里r a competitive and substantial probabilities and substantial probabilities interpretatio
The competition interpretation [36, 91]. The competition [36, 91]. The competition [36, 91]. The competition [
	- quarks. At larger x, the radius decreases, becomes point-like at $x = 1$. $\langle b_\perp^2$ $\langle \frac{2}{\pi} \rangle$: transverse size of the nucleon. At small x

• 2D Fourier transform with respect to the transverse momentum transferred at $\xi = 0$ gives \mathbf{r}

from the CoM

• GPDs in impact parameter have probabilistic interpretation. M. Burkardt, IJMPA18, 187(2003) M. Burkardt, IJMPA18, 187(2003)

• $\langle b_\perp^2 \rangle$: transverse size of the nucleon. At small x, gluons show larger transverse radius than $\frac{1}{\sqrt{2}}$, there the station interfact the strain $\frac{1}{\sqrt{2}}$, then $\frac{1}{\sqrt{2}}$ at $x = 1$ quarks. At larger x, the radius decreases, becomes point-like at $x = 1$. x , the radius decreases, becomes point-like at $x = 1$

 $\left(\frac{2}{\sqrt{2}}\right)$

GPDs in impact parameter space (contd)

• $\langle b_\perp^2 \rangle$: transverse size of the nucleon. At small *x*, gluons show slightly larger $at x = 1.$ \hat{L}): transverse size of the nucleon. At small *x*, gluons show slightly

Transverse radius Γ exhibit comparable shapes Γ and display a negative distribution across the entire range of

transverse radius than quarks. At larger x , the radius decreases, becomes point-like challenge arises when a probability of \mathbf{r} arks. At larger x , the radius decreases, beco ger transverse average radii in contrast to the quark of the quark α \ddotsc as \ddotsc increases, the transverse size of the trans es point-like

According to Ji's sum rule [63], the total angular momentum *J^g*

• According to Ji's sum rule:

 \bullet

$$
J_z^g = \frac{1}{2} \int dx x \left[H^g(x,0,0) + E^g(x,0,0) \right]
$$

- Our estimate $J_z^g = 0.058$ consistent with BLFQ result of $J_z^g = 0.066$ σ_{at} collaboration, which all σ_{at} is σ_{at} and σ_{at} σ_{at} *B.* Lin et al. 2308. 08275 $\frac{1}{2}$ Explain the similarity and di $\frac{1}{2}$ that in an analogous gluon spectator with our model $\frac{1}{2}$ J_z^g *z* $= 0.058$ consistent with BLFQ result of J_z^g *z* $= 0.066$
- Helicity GPD gives the spin contribution of gluon: \bullet Helicity GPD gives the spin contribution of gluon: α $\Delta G = \int dx g_{1L}(x) = \int dx H^{g}(x,0,0)$ \mathcal{S} is interesting to recognize that the reco $\tilde{\c{H}}^g$ (*x*,0,0)
- Separation of gluon spin and OAM is not unique! separa
- Spin asymmetries in polarised scattering experiments are directly proportional to the gluon intrinsic spin! n etries in polarised scattering experiments are directly proportional to the
	- Two definitions of OAM: Kinetic and canonical OAM. ^{*l*} *L W* _d de *x* \mathbf{H} $\$ *g*(*x*)*g(<i>x*)*g)<i><i>g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g***(***x***)***g***)***g*

X. Ji, PRL 78, 610

• Our result: $\begin{array}{ccc} \text{c} & \text{f} & \text{f} & \text{g} & \text{h} \end{array}$ Our result: $L_z^g = -0.42$

$\{x \left[H^g(x,0,0) + E^g(x,0,0) \right] - \widetilde{H}^g(x,0,0) \}$

• Kinetic OAM: \sim $\frac{2}{J}$ defined in terms of GPDs.

•

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$$
L_z^g = \frac{1}{2} \int \{x \left[H^g(x, 0, 0) + E^g \right]
$$
 Kinetic OAM[.]

Kinetic OAM:

Unintegrated kinetic OAM as a function of x

• Canonical OAM in the light cone gauge is defined by GTMDs as ω Canonical Ω ^M, in the light canonical of gould, the calonical orbital angular momentum of Ω **z** can be obtained from \overline{G} Janomical UANI in the light cone gauge is uchned by

$$
\ell_z^g(x) = -\int d^2 \mathbf{p}_{\perp} \frac{\mathbf{p}_{\perp}^2}{M^2} F_{1,4}^g(x, 0, \mathbf{p}_{\perp})
$$

 $\mathbf{p}_{\perp}, 0, 0)$.

• GTMDs : higher dimensional distributions Wigner distributions.

•
$$
W(x, \xi = 0, p_{\perp}, \Delta_{\perp}) = \frac{1}{xP^+} \int \frac{dz^- d^2 z^{\perp}}{(2\pi)^3} e^{ip \cdot z} \langle p + \frac{\Delta_{\perp}}{2} | F^{+i}(-z/2) \mathcal{W} F^{+j} | p - \frac{\Delta_{\perp}}{2} \rangle |_{z^+}
$$

Chirally even gluon GTMDs: $F_{1,1}$, $F_{1,4}$, $G_{1,1}$, $G_{1,4}$

* TMDs can be obtained from GTMDs at $\Delta_{\perp} = 0$ limit ** GPDs in impact parameter space are obtained by integrating GTMDs over *p*⊥

• GTMD correlator:

2(2⇡)³

 $\mathcal{L}(\mathcal{L})$

p2

⌘

 $\mathrm{sign}(\Lambda)$ $\lceil \psi^{\Lambda*}_{\lambda,\iota} \rceil$ $\overset{\Lambda *}{\lambda},\mu\,(\hat{x},\hat{\boldsymbol{p}}')$ $\langle \bot \rangle \psi_{\lambda}^{\Lambda}$ $\Lambda^{A}_{\lambda,\mu}(\hat{x},\hat{\bm{p}}_{\perp})$ $\begin{array}{c} \hline \end{array}$ ⇤⇤

where **we**, and *µ denoted the proton, quark and gluon helicities, respectively. By employing the proton, quark and gluon helicities, respectively. By employing the proton LFWFs from LFWFs from LFWFs from LFWFs from LFWFs ,* (29)

$$
\frac{i(\mathbf{p}_{\perp}\times\mathbf{\Delta}_{\perp})_z}{M^2}F_{1,4}^g=\frac{1}{2(2\pi)^3}\frac{1}{2}\sum_{\Lambda,\lambda,\mu}\text{sign}(1)
$$

 \mathbb{R}^2

 \bullet

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di *i* <u>2010 11011</u> $\overline{\mathbf{u}}$ $\frac{d}{dx}$ h
h sed parton in a longitudinally p i arised nucleon | *i* Un dispondense parton in a fongitudinality GTMD *F*1,4 describes the distortion of unpolarised parton in a longitudinally polarised nucleon

 $\Delta =$ proton helicity μ =gluon helicity 4 $\Lambda =$ proton helicity. λ = quark helicity <u>μ=gluon helicity</u>

<u></u> Λ =proton helicity = quark helicity *λ*

$$
-\frac{i(\mathbf{p}_{\perp}\times\mathbf{\Delta}_{\perp})_z}{M^2}G_{1,1}^g=\frac{1}{2(2\pi)^3}\frac{1}{2}\sum_{\Lambda,\lambda,\mu}\mathrm{sign}(\mu)\left[\psi^{\Lambda*}_{\lambda,\mu}(\hat{x},\hat{\boldsymbol{p}}_{\perp}^{\prime})\psi^{\Lambda}_{\lambda,\mu}(\hat{x},\hat{\boldsymbol{p}}_{\perp})\right]
$$

µ = **longitud** *µ G*1,1 : distortion of longitudinally polarised gluon inside a unpolarised nucleon

⇤*,,µ*

"⇤

"*,µ*(ˆ*x, ^p*

ˆ0

?) "

"*,µ*(ˆ*x, ^p*

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?)

h

sign(*µ*) = "*,µ*(ˆ*x, ^p* ?) " "*,µ*(ˆ*x, ^p* ?) "⇤ ˆ0 ˆ

i

?

¹*^x*] (37) $\frac{1}{1-x}$

$$
\ell_z^g(x) = -\int d^2 \mathbf{p}_{\perp} \frac{\mathbf{p}_{\perp}^2}{M^2} F_{1,4}^g(x, 0, \mathbf{p}_{\perp}, 0, 0).
$$

egrated value: canonical OAM $l_z^g \approx -0.38$ [consistent with another spectator \sim canonical OAM $18 \approx -0.38$ Econsistent with another spectator

• Our model result:

•

panel whereas the spin-orbit correlation function \sim del result $l_z^{\circ} = -0.5$. $\overline{}$ • Integrated value: canonical OAM $l_z^g \approx -0.38$ [consistent with another spectator model result $l_z^g = -0.33$] • Integrated value: canonical OAM $l_z^g \approx -0.38$ [consistent with another spectator panel where the spin-orbit l^g = 0.22 km \sim $\mu_{\overline{z}} = -0.55$ and $\mu_{\overline{z}} = -0.35$ and ζ canonical UANI $\iota_{\zeta} \approx -0.58$ [consistent with *l g* $z^{2} \approx -0.38$ *l g z* $=-0.33$

- $G_{1,1}$ gives the spin-OAM correlation and the fourth chiral even GTMD *G^g*
- $C_z^g < 0$: spin and OAM are anti-aligned $\frac{2}{z}$ < 0 :
- $C_z^g > 0$: spin and OAM are aligned. $\frac{2}{z} > 0$: $d\mathbf{I}\mathbf{U}\mathbf{V}\mathbf{I}$ alternative alignment.

•

Spin-orbit correlation Similarly, the spin-orbit correlation factor for the gluons can be obtained by using the gluon GTMD *G^g*

$$
C_z^g(x) = \int d^2 \mathbf{p}_{\perp} \frac{\mathbf{p}_{\perp}^2}{M^2} G_{1,1}^g(x,0,\mathbf{p}_{\perp},0,0)
$$

 $\frac{1}{2}$ Model predicts $C_z^g < 0$

gluon canonical OAM, `*^g*

^z(*x*) over *x* one can obtained the numerical value of gluon

Summary and conclusions

- To understand the three dimensional structure and partonic level description of spin/OAM , we need to investigate both quark and gluons (and sea quarks too!).
- Gluon distributions are not yet well understood/studied.
- We presented the study of different gluon distributions in a simple model of proton.
- gluon contributions to spin/OAM.
- We require more experiments, lattice results, better models with gluons...

Summary and conclusions

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EIC/EicC

Summary and conclusions

- To understand the three dime spin/OAM, we need to investigate both quarks too!).
- Gluon distributions are not ye
- We presented the study of dif $\left\{ \begin{array}{ccc} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\$ proton.
- gluon contributions to spin/ C
- We require more experiments \setminus \setminus

EIC/EicC

THANK YOU

