# Gluon distributions in the proton

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Refs:
DC, P. Choudhary, B. Gurjar, R Kishore, T. Maji, C. Mondal, A. Mukherjee, PRD 108, 014009(2023);
DC, P. Choudhary, B. Gurjar, T. Maji, C. Mondal, A. Mukhrejee, PRD 109, 1149040 (2024).

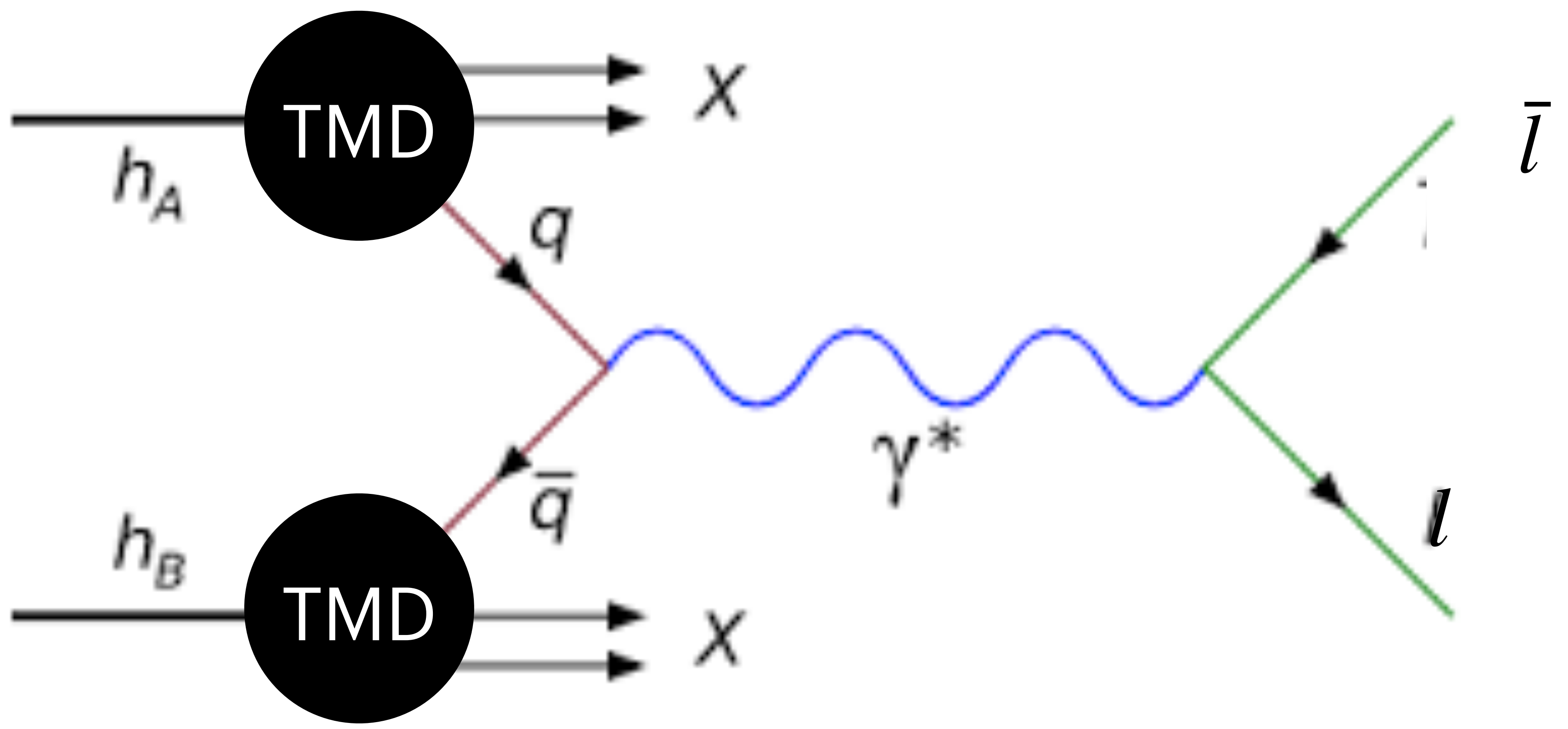
# Introduction

- One of the main goals of EIC/EicC is to understand the three dimensional structure of nucleons in terms of quarks and gluons as well as their spin and angular momentum distributions.
- Gluon distributions: crucial to understand low-x phenomena. Gluon PDFs are mainly small-x dominated.
- Form factors, PDFs, GPDs, TMDs, Wigner distributions…. Encode different informations.
- Gluon distributions are not yet well understood - not enough theoretical studies.
- Large uncertainty in small-x, specially for polarized pdf ⟶ not well constrained.
- Except lattice, they are mostly studied in different models.

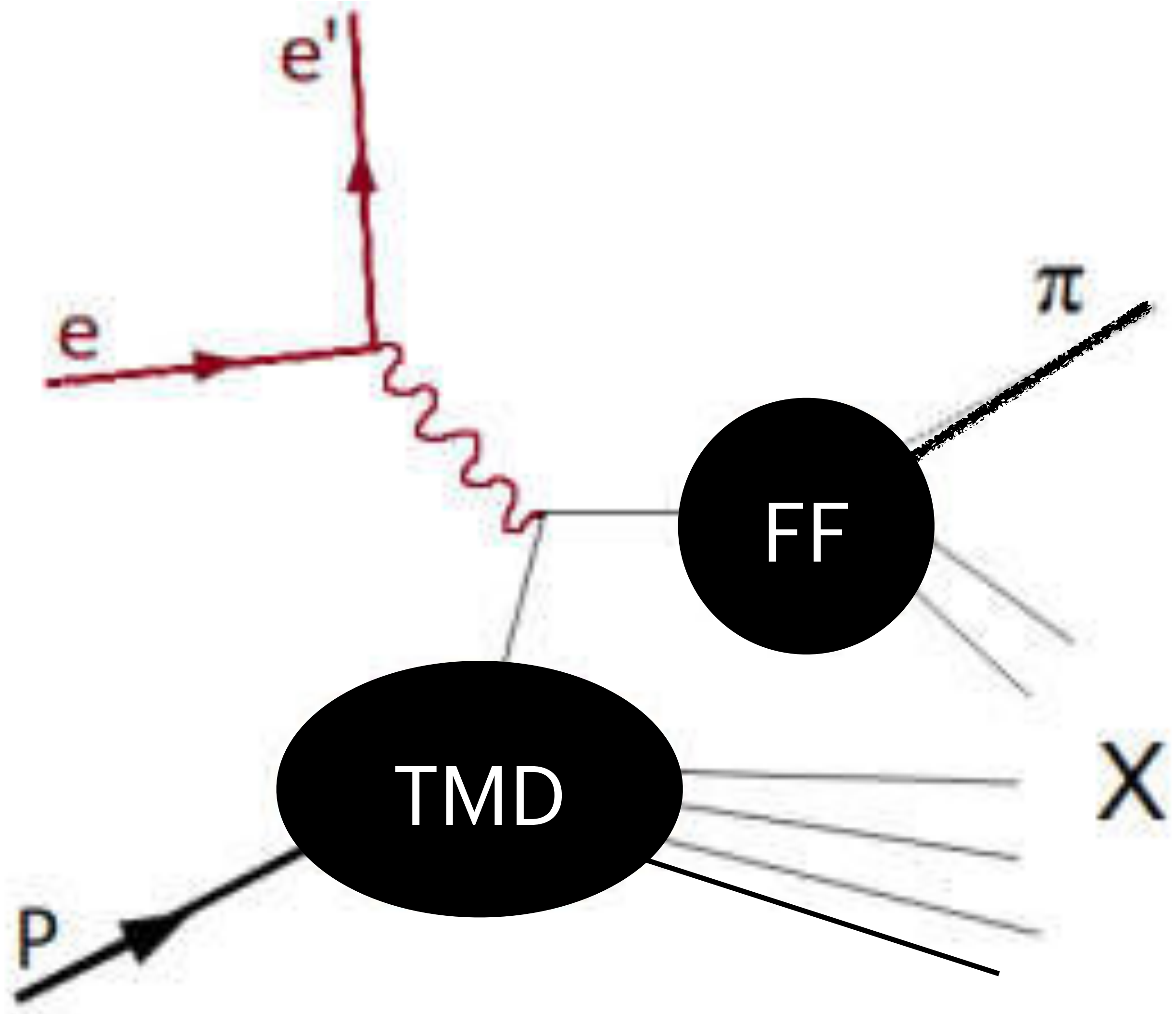
In this talk, I'll mainly discuss gluon TMDs and GPDs.

# TMDs: DY and SIDIS

- SIDIS and Drell-Yan processes are sensitive to TMDs.




DY




SIDIS

# Quark TMDs

- TMD factorization: DY: $\frac{d\sigma}{dQ^2 dy dk_T^2} = \int H^{DY}(Q) \otimes f(Q^2, x_1, k_T) \otimes f(Q^2, x_2, k_T)$
- TMDs: 3D spatial structure of proton
- —> Transverse motion of partons, spin-transverse momentum correlations
- TMDS: ➡ spin asymmetries
- Azimuthal asymmetry of unpolarised quarks in transversely polarised proton: Sivers effect.
- Final State Interaction (FSI) in SIDIS (Initial State Interaction for DY): gluon exchange between the struck quark and the remnant produces nonzero Sivers effect.

* Talk by Marco Radici

Brodsky, Hwang, Schmidt, PLB530, 99

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  Hard scattering — TMD

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# Gluon TMDs

- Gluon TMDs : experimentally/theoretically not yet well understood.
- Gluon distributions are small-x dominated. To study gluon TMDs, high energy and/or small x are required.
- Phenomenological models at small -x: Weiszacker-Williams(WW) [both gauge links are future pointing], dipole [one future and another past-pointing gauge link]
- EIC/EicC will probe the gluon distributions for both unpolarized and polarized proton.
- Gluon Sivers TMD: $e\ p^{\uparrow} \rightarrow eQ\bar{Q}X$, $ep \rightarrow eD\bar{D}X$ [back to back D meson pair production], $ep^{\uparrow} \rightarrow eJ/\Psi\ X$

C.Pisano 1912.13020
D. Boer, 1601.01813

- Measurement of azimuthal asymmetries/transverse momentum distributions :gluon TMDs
- TMDs are not universal [due to FSI/ISI dependence]- - Sivers TMDs for quarks in SIDIS and DY differ by an overall negative sign.

# Model with active gluon

- Spectator model studies provide good insight into the different partonic distributions.
- Simplified, but insightful, help to understand the proton structure.
- consider the proton as a composite state of spin 1/2 spectator+ gluon(active parton).

$$|P;\uparrow(\downarrow)\rangle = \int \frac{\mathrm{d}^2\mathbf{p}_\perp \mathrm{d}x}{16\pi^3\sqrt{x(1-x)}} \times \Big[\psi^{\uparrow(\downarrow)}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp)\Big|+1,+\frac{1}{2};xP^+,\mathbf{p}_\perp\Big\rangle + \psi^{\uparrow(\downarrow)}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp)\Big|+1,-\frac{1}{2};xP^+,\mathbf{p}_\perp\Big\rangle$$

$$+\psi^{\uparrow(\downarrow)}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp)\Big|-1,+\frac{1}{2};xP^+,\mathbf{p}_\perp\Big\rangle + \psi^{\uparrow(\downarrow)}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp)\Big|-1,-\frac{1}{2};xP^+,\mathbf{p}_\perp\Big\rangle\Big],$$

- $\psi^{\uparrow(\downarrow)}_{\lambda_g\lambda_X}(x,\mathbf{p}_\perp)$ = LFWF corresponding to the two particle state $|\lambda_g,\lambda_X;xP^+,\mathbf{p}_\perp\rangle$

Bacchetta, Celiberto, Radici, Taels, EPJC80, 733

DC et al, PRD 108, 014009

$$\psi^{\uparrow}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp) = -\sqrt{2}\frac{(-p^1_\perp + ip^2_\perp)}{x(1-x)}\varphi(x,\mathbf{p}^2_\perp),$$

$$\psi^{\uparrow}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp) = -\sqrt{2}\left(M - \frac{M_X}{(1-x)}\right)\varphi(x,\mathbf{p}^2_\perp),$$

$$\psi^{\uparrow}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp) = -\sqrt{2}\frac{(p^1_\perp + ip^2_\perp)}{x}\varphi(x,\mathbf{p}^2_\perp),$$

$$\psi^{\uparrow}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp) = 0,$$

With

$$\varphi(x,\mathbf{p}^2_\perp) = N_g\frac{4\pi}{\kappa}\sqrt{\frac{\log[1/(1-x)]}{x}}x^b(1-x)^a\exp\left[-\frac{\log[1/(1-x)]}{2\kappa^2x^2}\mathbf{p}^2_\perp\right].$$

The model parameters are fitted to the unpolarised gluon PDF(NNPDF3.0 data) at $Q_0 = 2$ GeV

DC et al. PRD 108, 014009

# Fixing the parameters

- 4 parameters in the model: $a, b, N_g, M_X$
- $N_g$ : fixed by normalization condition,
- Spectator mass $M_X > M$ (proton mass)
- Behaviour of the distribution is determined by $a$ and $b$
- The parameters in the model are fixed by fitting the unpolarised gluon pdf $f_1^g(x)$ with NNPDF3.0 NLO data at $Q_0 = 2$ GeV.

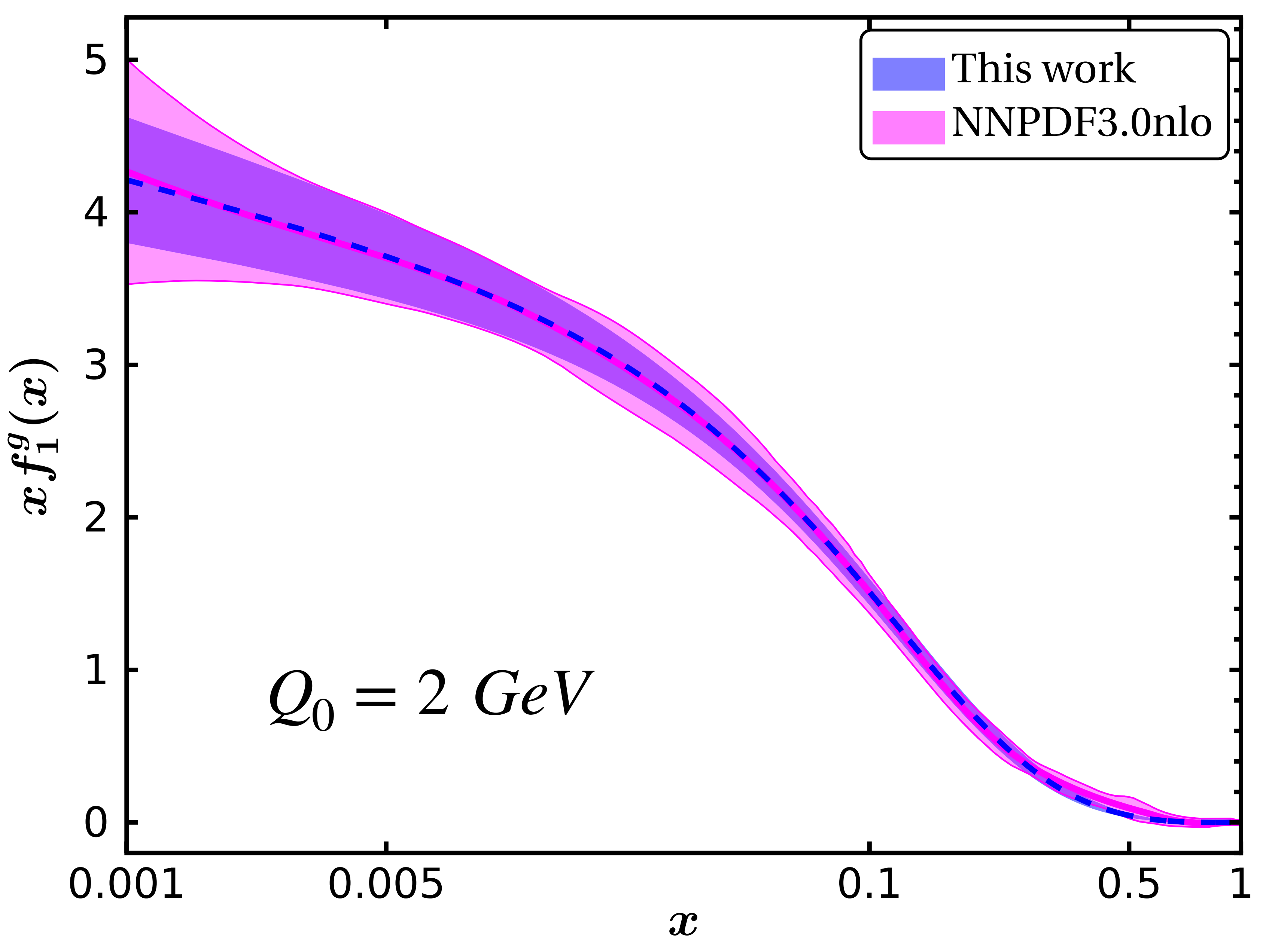
# Fitting the parameters

- Unpolarized gluon PDF:

$$f^g(x) \quad = 2N_g^2 x^{2b+1}(1-x)^{2a-2}\left[\kappa^2 \frac{(1+(1-x)^2)}{\log[1/(1-x)]} + (M(1-x)-M_X)^2\right]$$

- We take 300 NNPDF3.0NLO data points in the interval $0.001 < x < 1$

(Considered 100 replicas of the NNPDF data)




Large uncertainty in small-x region Excluded in our model.

* Gluon mass $m_g = 0$

**Parameters $a, b$ depend on spectator mass($M_X$)

We choose $M_x = 0943\ MeV$ : close to proton mass

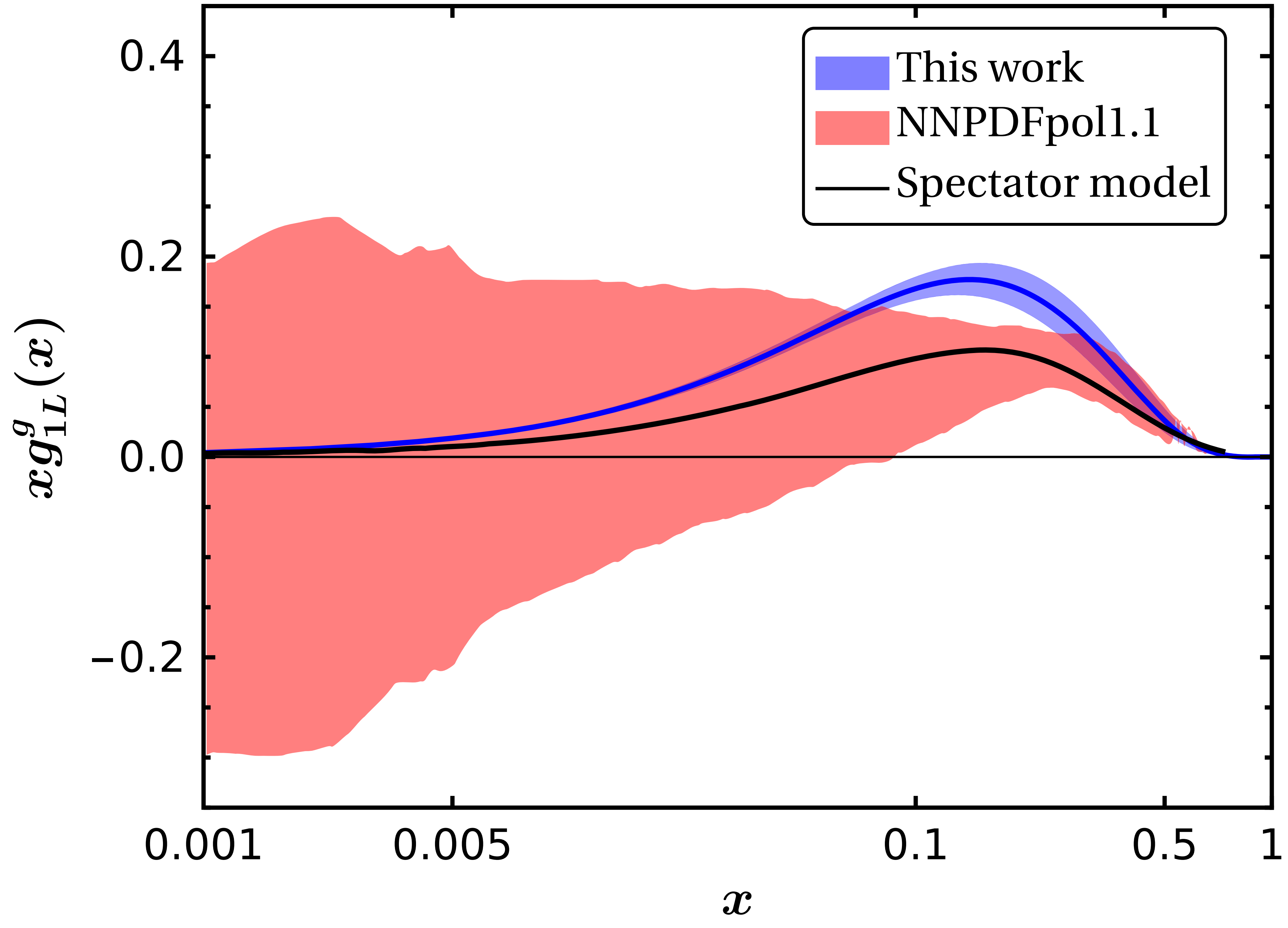
DC et al. PRD 108, 014009

- Model is very sensitive to small $x$, $x < 0.001$ region is excluded from the fit.
- Fitted parameters: $a = 3.88 \pm 0.22,\ b = -0.53 \pm 0.01$ ($2\sigma$ error)
- Except the unpolarized gluon PDF, everything else is our model prediction.
- Average longitudinal momentum = second Mellin moment of unpolarised pdf:
- $\langle x \rangle_g = \int_{0.001} dx\ x\ f_1^g(x) = 0.416^{+0.048}_{-0.041}$
- lattice result: $\langle x \rangle_g = 0.427(92)$ C. Alexandrou et al, PRD 101, 094512
- Spectator model [1]: $\langle x \rangle_g = 0.424$ A. Becchetta et al, EPJC 80, 733
- Spectator model[2]: $\langle x \rangle_g = 0.411$ Lu & Ma, PRD 94, 094022

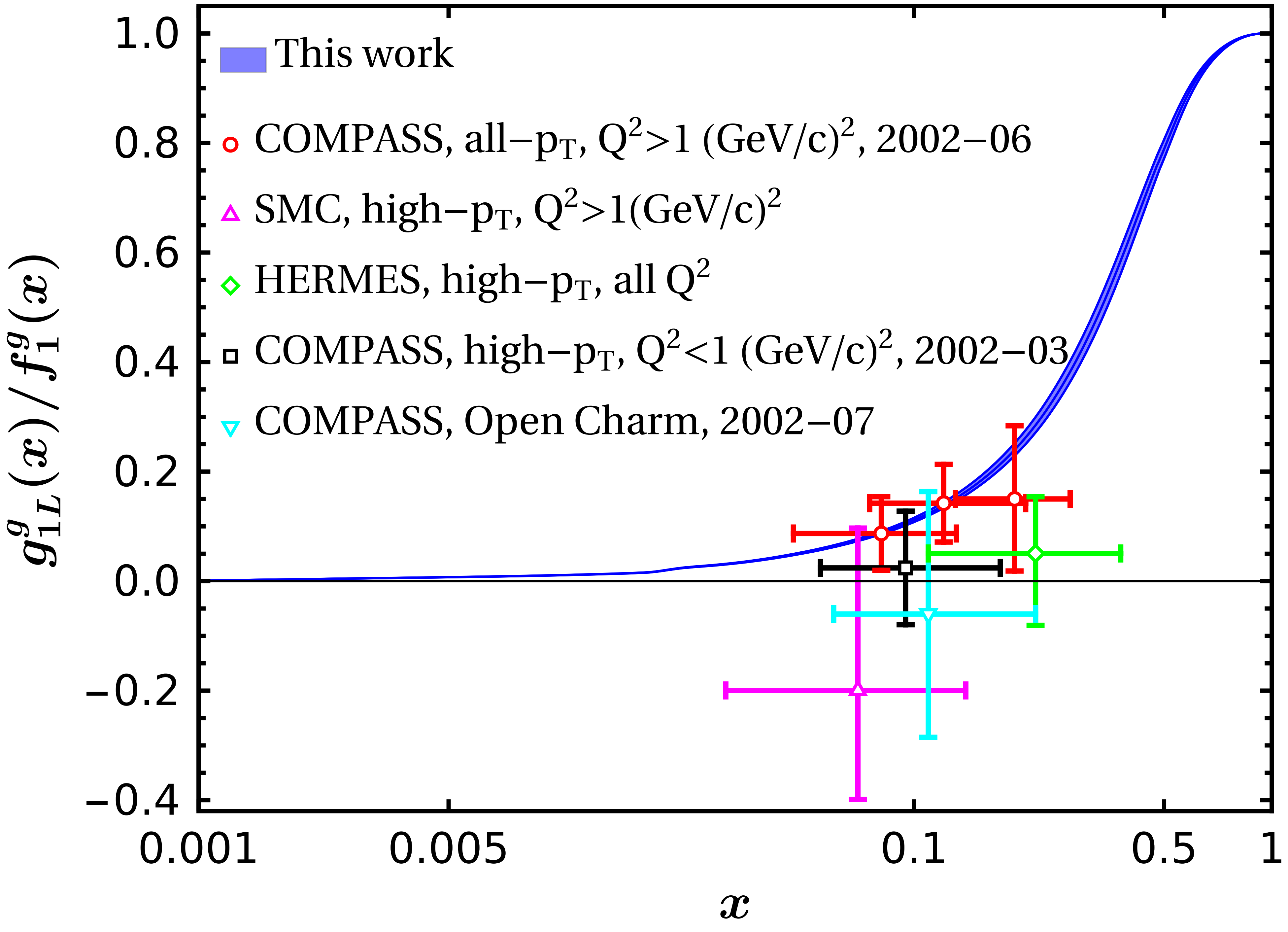
# Gluon helicity pdf

[Circularly polarised gluon in a longitudinally polarised proton]

**Gluon helicity pdf**




**Gluon helicity asymmetry**




- $$\lim_{x\to 0}\frac{g^g_{1L}(x)}{f^g_1(x)} = 0, \quad \text{and} \quad \lim_{x\to 1}\frac{g^g_{1L}(x)}{f^g_1(x)} = 1.$$

Model-independent constraints

# Gluon spin contribution:

| | Our prediction | Comparison |
|---|---|---|
| $\Delta G = \int_{0.05}^{0.3} dx \Delta g(x)$ | 0.28 | 0.20<br>[A.Adare et al[Phenix] PRD90] |
| $\Delta G = \int_{0.05}^{0.2} dx \Delta g(x)$ | 0.22 | 0.23<br>[Nocera et al. [NNPDF] NPB886] |
| $\Delta G = \int_{0.05}^{1} dx \Delta g(x)$ | 0.32 | 0.19<br>[Florian et al, PRL113] |

# Gluon TMDs

## Model results

- The correlator for gluon TMDs in SIDIS:

$$\Phi^{g[ij]}(x, \mathbf{p}_\perp; S) = \frac{1}{xP^+} \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ik\cdot\xi} \langle P; S | F_a^{+j}(0) \mathcal{W}_{+\infty,ab}(0;\xi) F_b^{+i}(\xi) | P; S \rangle \Big|_{\xi^+=0^+},$$

- 

- At leading twist 8 gluon TMDs: 4 are T-even and 4 are T-odd.

# Chiral even gluon TMDs:

$$\Phi^g(x, \mathbf{p}_\perp; S) = \delta_T^{ij} \Phi^{g[ij]}(x, \mathbf{p}_\perp; S)$$
$$= f_1^g(x, \mathbf{p}_\perp^2) - \frac{\epsilon_\perp^{ij} \mathbf{p}_\perp^i S_\perp^j}{M} f_{1T}^{\perp g}(x, \mathbf{p}_\perp^2)$$

$$\tilde{\Phi}^g(x, \mathbf{p}_\perp; S) = i\epsilon_T^{ij} \Phi^{g[ij]}(x, \mathbf{p}_\perp; S)$$
$$= \lambda g_{1L}^g(x, \mathbf{p}_\perp^2) + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M} g_{1T}^g(x, \mathbf{p}_\perp^2),$$

Goes to 0 as $p_\perp \to 0$

Chiral even TMDs $(f_1^g,\, g_{1L}^g,\, g_{1T}^g,$ and $h_1^{\perp g})$

# Chiral odd gluon TMDs:

$$\Phi_T^{g,ij}(x,\mathbf{p}_\perp;S)$$
$$= -\hat{\mathbf{S}}\Phi^{g[ij]}(x,\mathbf{p}_\perp;S)$$
$$= -\frac{\hat{\mathbf{S}}\mathbf{p}_\perp^i\mathbf{p}_\perp^j}{2M^2}h_1^{\perp g}(x,\mathbf{p}_\perp^2) + \frac{\lambda\hat{\mathbf{S}}\mathbf{p}_\perp^i\epsilon_\perp^{jk}\mathbf{p}_\perp^k}{2M^2}h_{1L}^{\perp g}(x,\mathbf{p}_\perp^2)$$
$$+ \frac{\hat{\mathbf{S}}\mathbf{p}_\perp^i\epsilon_\perp^{jk}S_\perp^k}{2M}\left(h_{1T}^g(x,\mathbf{p}_\perp^2) + \frac{\mathbf{p}_\perp^2}{2M^2}h_{1T}^{\perp g}(x,\mathbf{p}_\perp^2)\right)$$
$$+ \frac{\hat{\mathbf{S}}\mathbf{p}_\perp^i\epsilon_\perp^{jk}(2p_\perp^k\mathbf{p}_\perp.\mathbf{S}_\perp - S_\perp^k\mathbf{p}_\perp^2)}{4M^3}h_{1T}^{\perp g}(x,\mathbf{p}_\perp^2).$$

- 
- Chiral odd TMDs: $(f_{1T}^{\perp g}, h_{1L}^{\perp g}, h_{1T}^g, h_{1T}^{\perp g})$

# Unpolarised TMD $f_1^g(x, p_\perp^2)$

- overlap representation of light front wave functions:

$$f_1^g(x, \mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\Big[|\psi^{\uparrow}_{+1+1/2}(x, \mathbf{p}_\perp^2)|^2 + |\psi^{\uparrow}_{+1-1/2}(x, \mathbf{p}_\perp^2)|^2 + |\psi^{\uparrow}_{-1+1/2}(x, \mathbf{p}_\perp^2)|^2\Big].$$

- 

- With our wave functions we get:

$$f_1^g(x, p_\perp^2) = N^2 \frac{2}{\pi\kappa^2}\frac{ln[1/(1-x)]}{x} x^{2b}(1-x)^{2a}\Big[(M - \frac{M_X}{1-x})^2 + p_\perp^2 \frac{1+(1-x)^2}{x^2(1-x)^2}\Big] Exp[-\frac{ln[1/(1-x)]}{\kappa^2 x^2} p_\perp^2]$$

- When integrated over the transverse momentum, it reduces to the unpolarised pdf $f_1^g(x)$.

- Helicity TMD: Circularly polarized gluon in longitudinally polarized proton

- $$g^g_{1L}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left[|\psi^{\uparrow}_{+1+1/2}(x,\mathbf{p}_\perp^2)|^2 + |\psi^{\uparrow}_{+1-1/2}(x,\mathbf{p}_\perp^2)|^2 - |\psi^{\uparrow}_{-1+1/2}(x,\mathbf{p}_\perp^2)|^2\right]$$

$$= N_g^2\frac{2ln[1/(1-x)]}{\pi\kappa^2 x}x^{2b}(1-x)^{2a}\left[\left(M-\frac{M_X}{1-x}\right)^2 + p_\perp^2\frac{1-(1-x)^2}{x^2(1-x)^2}\right]exp[-C(x)p_\perp^2]$$

- Gluon helicity pdf. $g^g_{1L}(x) = \int d^2p_\perp g^g_{1L}(x,p_\perp^2)$

$$C(x) = \frac{\log[1/(1-x)]}{\kappa^2 x^2}.$$

- Worm-gear TMD: circularly polarized gluon in transversely polarized proton

$$\frac{\mathbf{p}_\perp.\mathbf{S}_\perp}{M}g^g_{1T}(x,\mathbf{p}_\perp^2) = -\frac{1}{16\pi^3}i\epsilon_T^{\mu\nu}\sum_{\lambda_g\lambda_g'\lambda_X}\epsilon_\mu^{\lambda_g'*}\epsilon_\nu^{\lambda_g}\psi^{\uparrow *}_{\lambda_g'\lambda_X}(x,\mathbf{p}_\perp^2)\psi^{\downarrow}_{\lambda_g\lambda_X}(x,\mathbf{p}_\perp^2)$$

$$= \frac{1}{16\pi^3}\frac{i}{2}\sum_{\lambda_g\lambda_g'\lambda_X}(\epsilon_1^{\lambda_g'*}\epsilon_2^{\lambda_g} - \epsilon_2^{\lambda_g'*}\epsilon_1^{\lambda_g})[\psi^{\uparrow *}_{\lambda_g'\lambda_X}(x,\mathbf{p}_\perp^2)\psi^{\downarrow}_{\lambda_g\lambda_X}(x,\mathbf{p}_\perp^2) + \psi^{\downarrow *}_{\lambda_g'\lambda_X}(x,\mathbf{p}_\perp^2)\psi^{\uparrow}_{\lambda_g\lambda_X}(x,\mathbf{p}_\perp^2)]$$

- In our model:

$$g^g_{1T}(x,\mathbf{p}_\perp^2) = -\frac{4M}{\pi\kappa^2}N_g^2(M(1-x)-M_X)\log[1/(1-x)]x^{2b-2}(1-x)^{2a-1}\exp[-C(x)\mathbf{p}_\perp^2].$$

-

- Boer-Mulders TMD: linearly polarized gluon inside unpolarised proton [interference between $\pm 1$ gluon helicities]

- $$\frac{\mathbf{p}_\perp^2}{2M^2} h_1^{\perp g}(x, \mathbf{p}_\perp^2) = \frac{1}{2} \eta_T^{\mu\nu} \sum_{\lambda_N \lambda_g \neq \lambda'_g \lambda_X} \frac{1}{16\pi^3} \left[ \epsilon_\mu^{\lambda'_g *} \epsilon_\nu^{\lambda_g} \psi_{\lambda'_g \lambda_X}^{*\lambda_N}(x, \mathbf{p}_\perp) \psi_{\lambda_g \lambda_X}^{\lambda_N}(x, \mathbf{p}_\perp) \right],$$

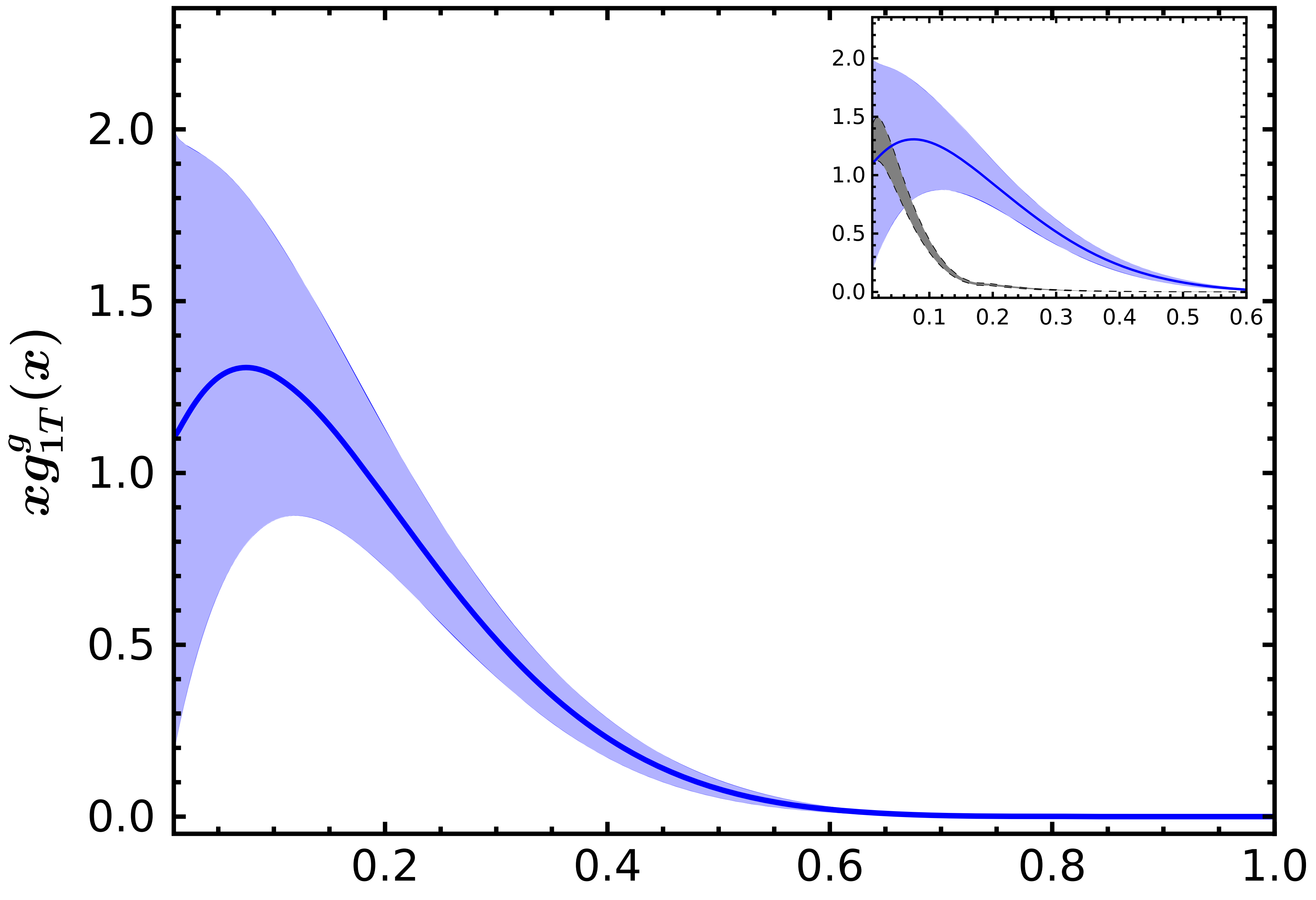
- Analytic form in our model:

$$h_1^{\perp g}(x, \mathbf{p}_\perp^2) = \frac{8M^2}{\pi\kappa^2} N_g^2 \log[1/(1-x)] x^{2b-3} (1-x)^{2a-1} \exp[-C(x)\mathbf{p}_\perp^2].$$

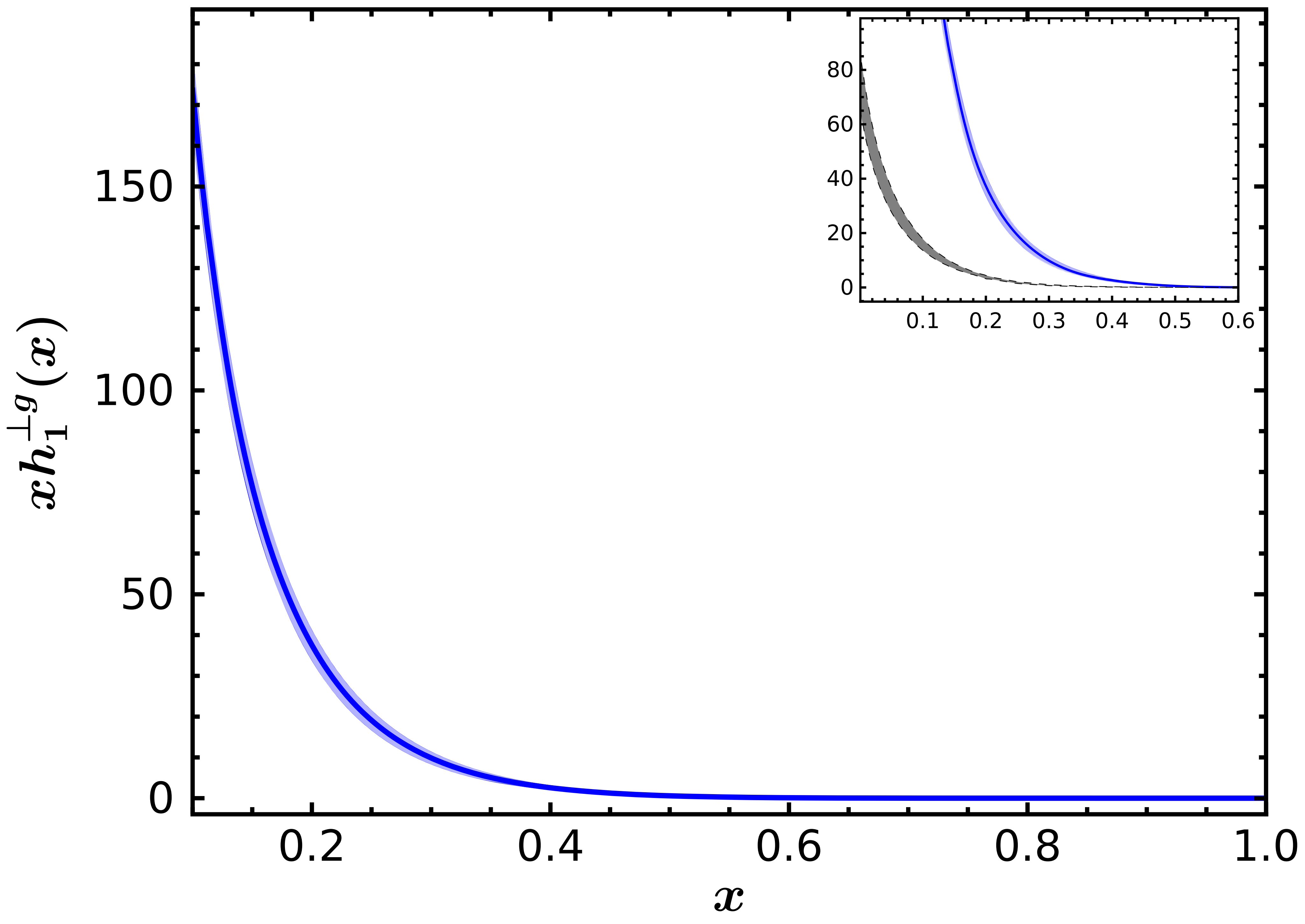
- Corresponding PDFs :

- $$g_{1T}^g(x) = \int d^2\mathbf{p}_\perp g_{1T}^g(x, \mathbf{p}_\perp^2) \qquad h_1^{\perp g}(x) = \int d^2\mathbf{p}_\perp h_1^{\perp g}(x, \mathbf{p}_\perp^2)$$

Gluon worm gear pdf
[circularly polarised gluon in a transversely polarised proton]




Gluon Boer-Mulders pdf
[transversley polarised gluon in an unpolarised proton]

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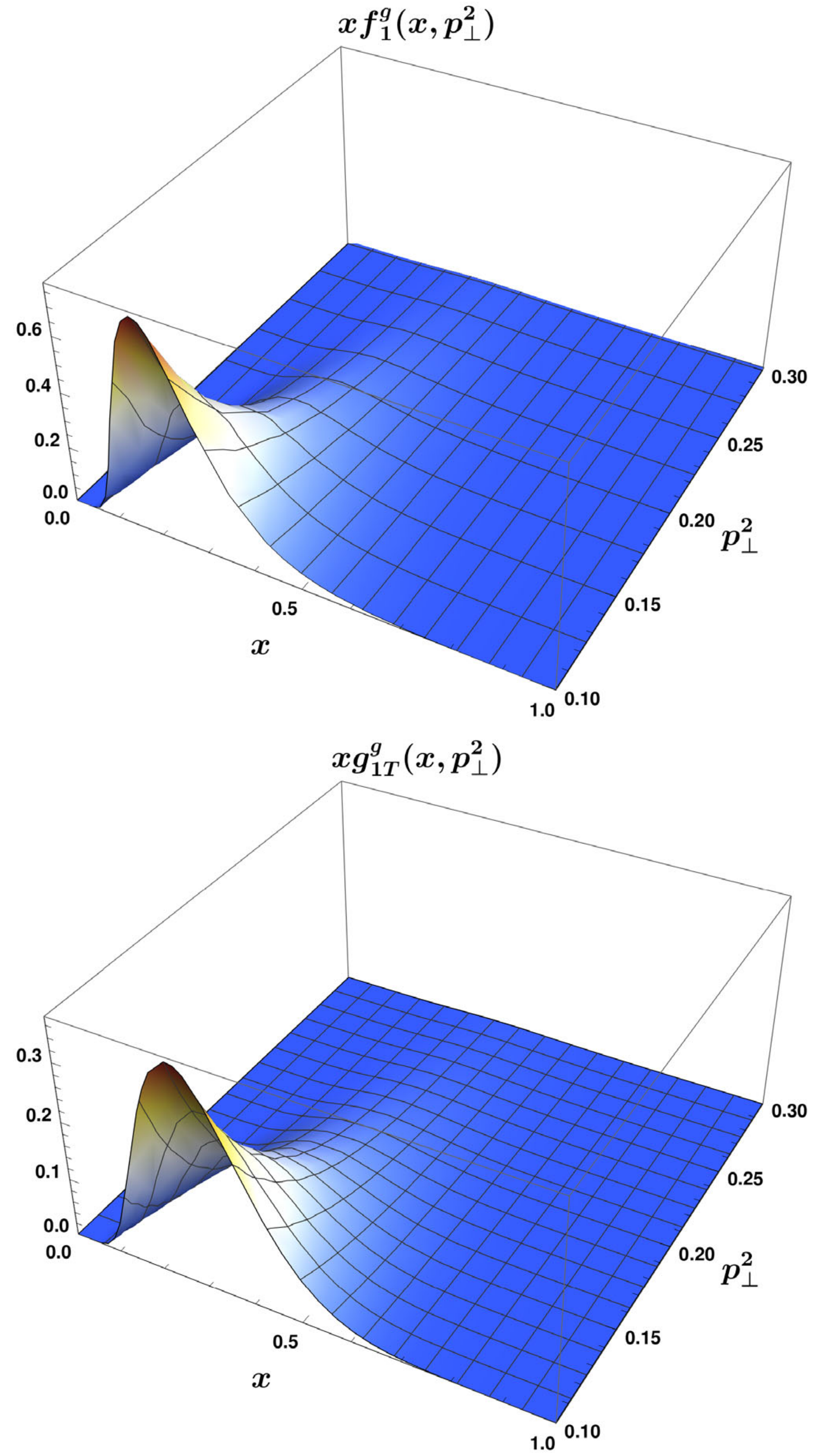
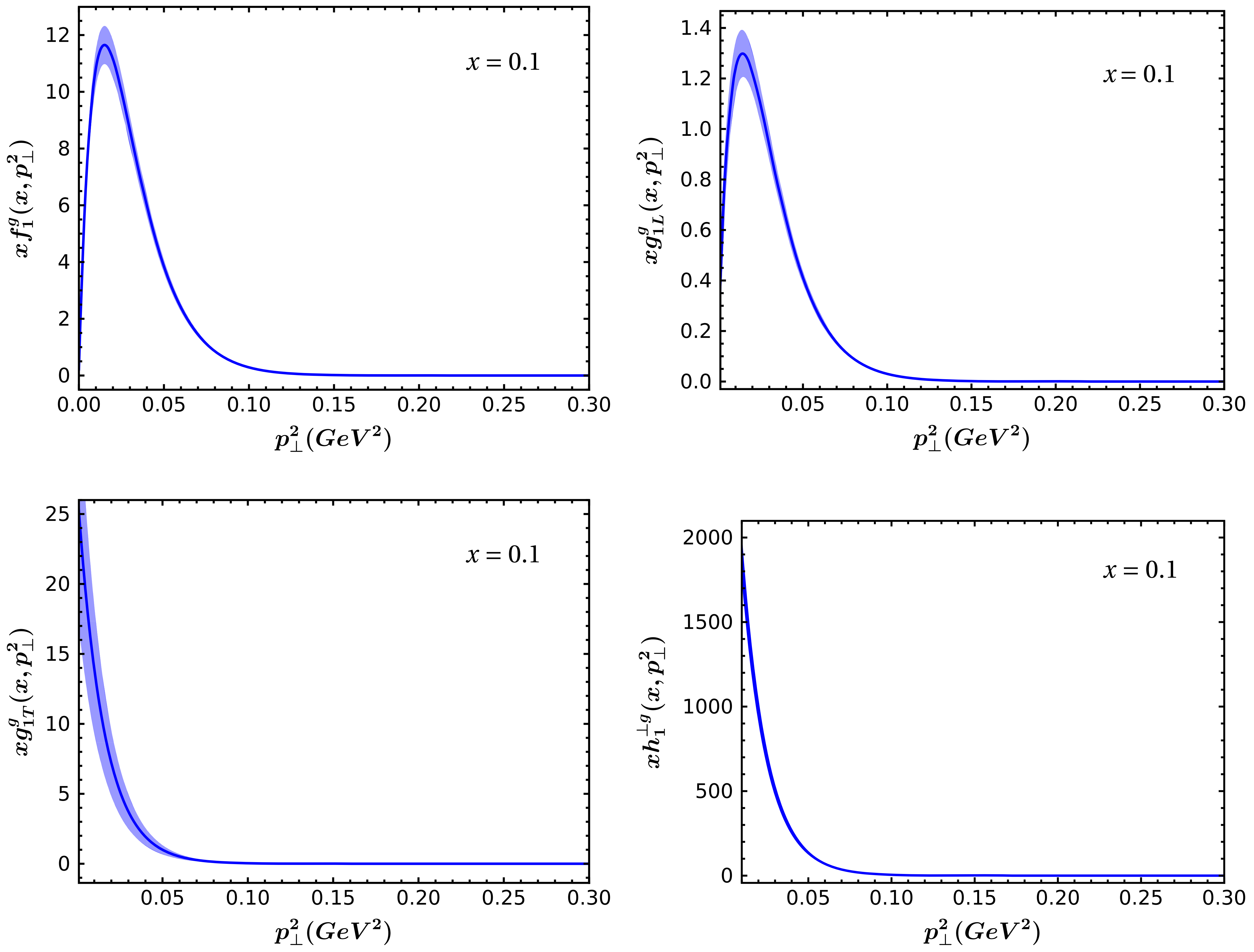
Inset: compared with the predictions of V.E. Lyubovitskij and I. Schmidt, PRD 103, 094017

# Gluon TMDs:

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For x=0.1

# TMD relations:

- Positivity bound:

$$f_1^g(x,\mathbf{p}_\perp^2) > 0, \quad f_1^g(x,\mathbf{p}_\perp^2) \geq |g_{1L}^g(x,\mathbf{p}_\perp^2)|.$$

$$f_1^g(x,\mathbf{p}_\perp^2) \geq \frac{|\mathbf{p}_\perp|}{M}|g_{1T}^g(x,\mathbf{p}_\perp^2)|,$$

$$f_1^g(x,\mathbf{p}_\perp^2) \geq \frac{|\mathbf{p}_\perp|^2}{2M^2}|h_1^{\perp g}(x,\mathbf{p}_\perp^2)|.$$

Model-independent relations

- 

- Mulders-Rodrigues relations put more stringent conditions on TMDS:

$$\sqrt{[g_{1L}^g(x,\mathbf{p}_\perp^2)]^2 + \left[\frac{|\mathbf{p}_\perp|}{M}g_{1T}^g(x,\mathbf{p}_\perp^2)\right]^2} \leq f_1^g(x,\mathbf{p}_\perp^2),$$

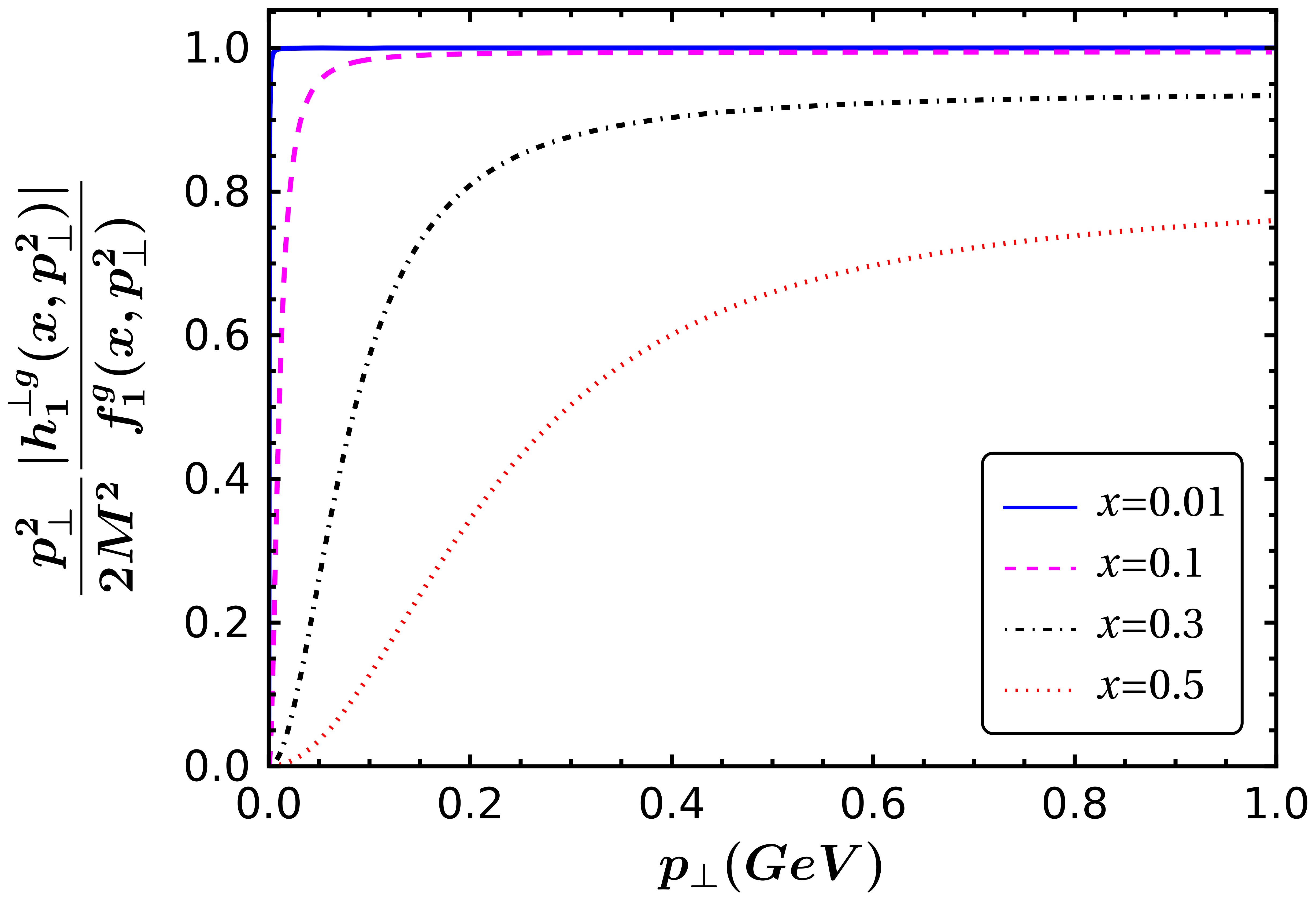
$$\sqrt{[g_{1L}^g(x,\mathbf{p}_\perp^2)]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2}h_1^{\perp g}(x,\mathbf{p}_\perp^2)\right]^2} \leq f_1^g(x,\mathbf{p}_\perp^2),$$

- $$\sqrt{\left[\frac{|\mathbf{p}_\perp|}{M}g_{1T}^g(x,\mathbf{p}_\perp^2)\right]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2}h_1^{\perp g}(x,\mathbf{p}_\perp^2)\right]^2} \leq f_1^g(x,\mathbf{p}_\perp^2).$$

Equality relation:

$$[f_1^g(x,\mathbf{p}_\perp^2)]^2 = [g_{1L}^g(x,\mathbf{p}_\perp^2)]^2 + \left[\frac{|\mathbf{p}_\perp|}{M}g_{1T}^g(x,\mathbf{p}_\perp^2)\right]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2}h_1^{\perp g}(x,\mathbf{p}_\perp^2)\right]^2,$$

# Positivity bound



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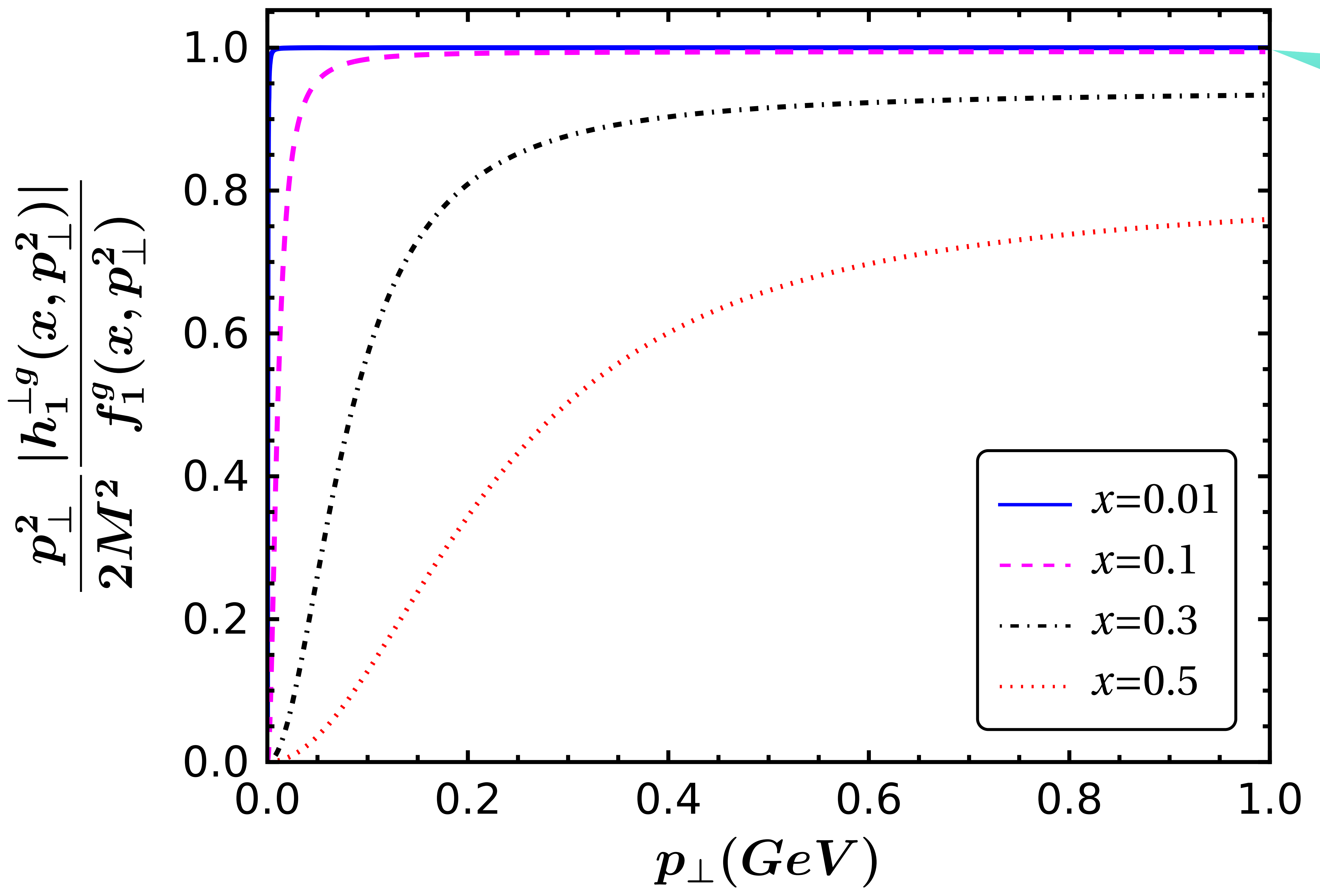


Saturation occurs for small x

# Gluon densities

- Unpolarised gluon density in an unpolarised proton = probability of finding the gluon with momentumm $(x, p_\perp)$

$$x\rho_g(x, p_x, p_y) = xf_1^g(x, \mathbf{p}_\perp^2),$$

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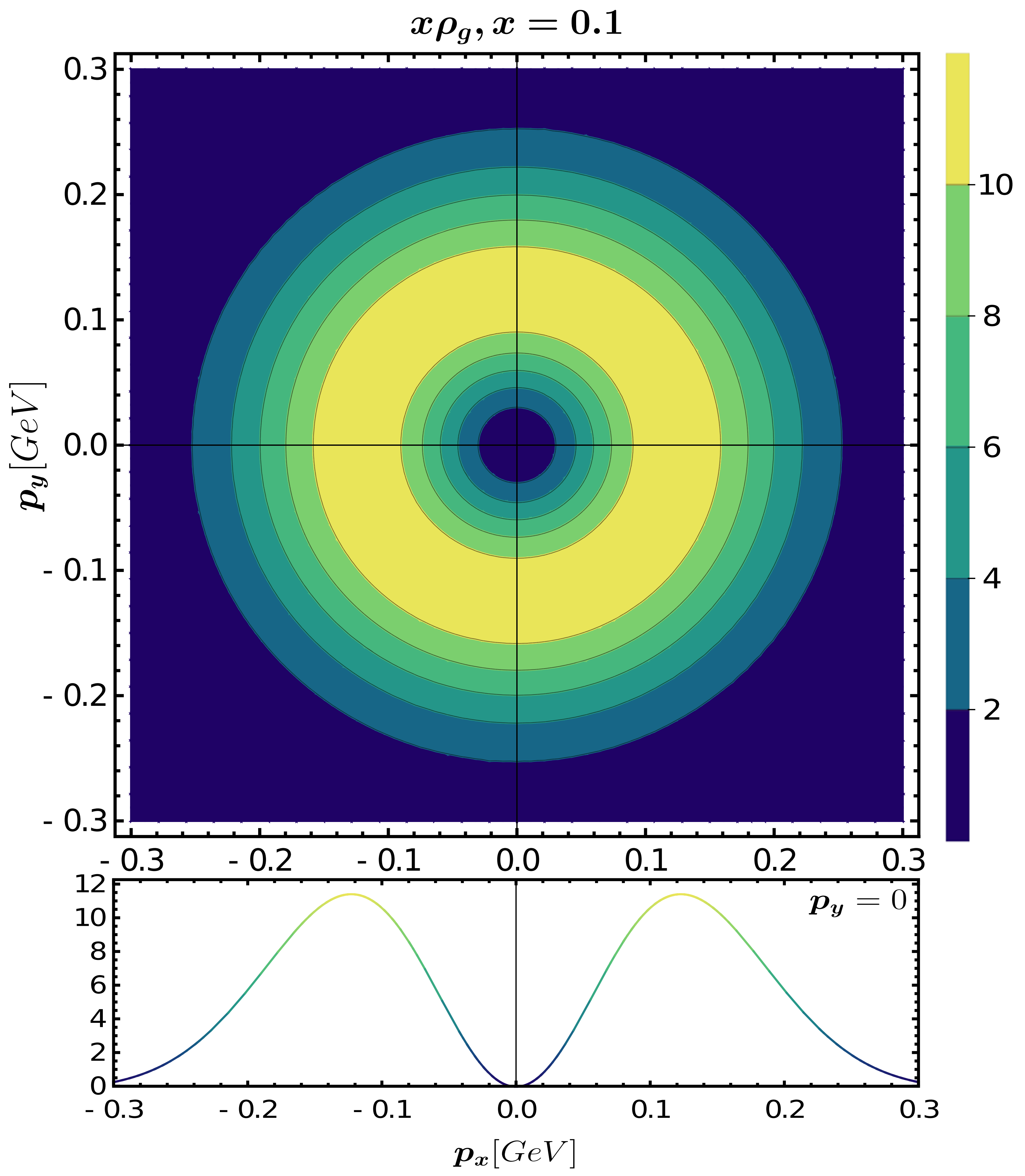
Symmetric



- Boer-Mulders density: linearly polarized gluon density
- $$x\rho_g^{\leftrightarrow}(x, p_x, p_y) = \frac{1}{2}\left[xf_1^g(x, \mathbf{p}_\perp^2) + \frac{p_x^2 - p_y^2}{2M^2}xh_1^{\perp g}(x, \mathbf{p}_\perp^2)\right]$$
- Spherical symmetry gets distorted due to the second term(Boer-Mulders TMD)...shows dipolar structure in momentum space.

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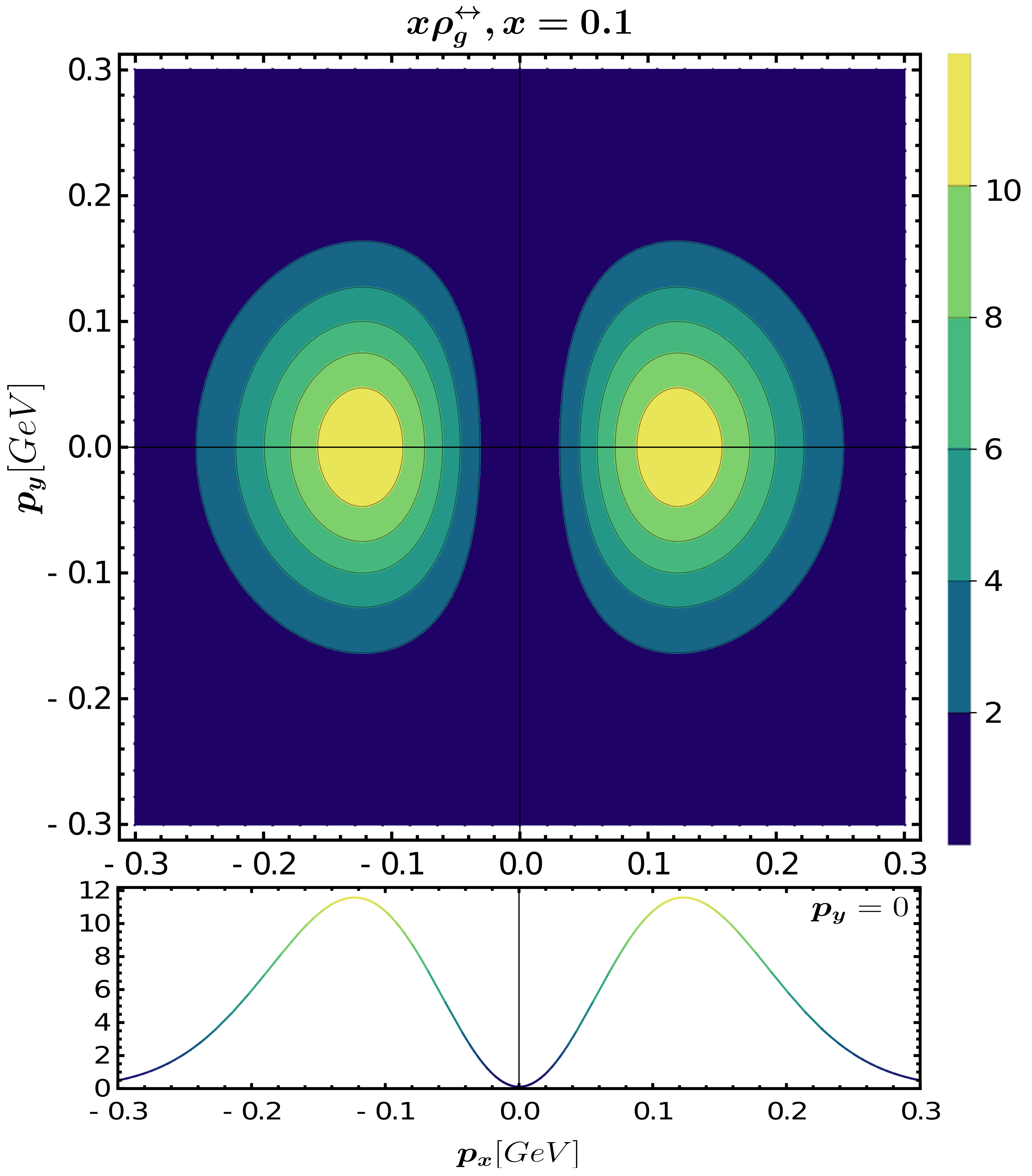

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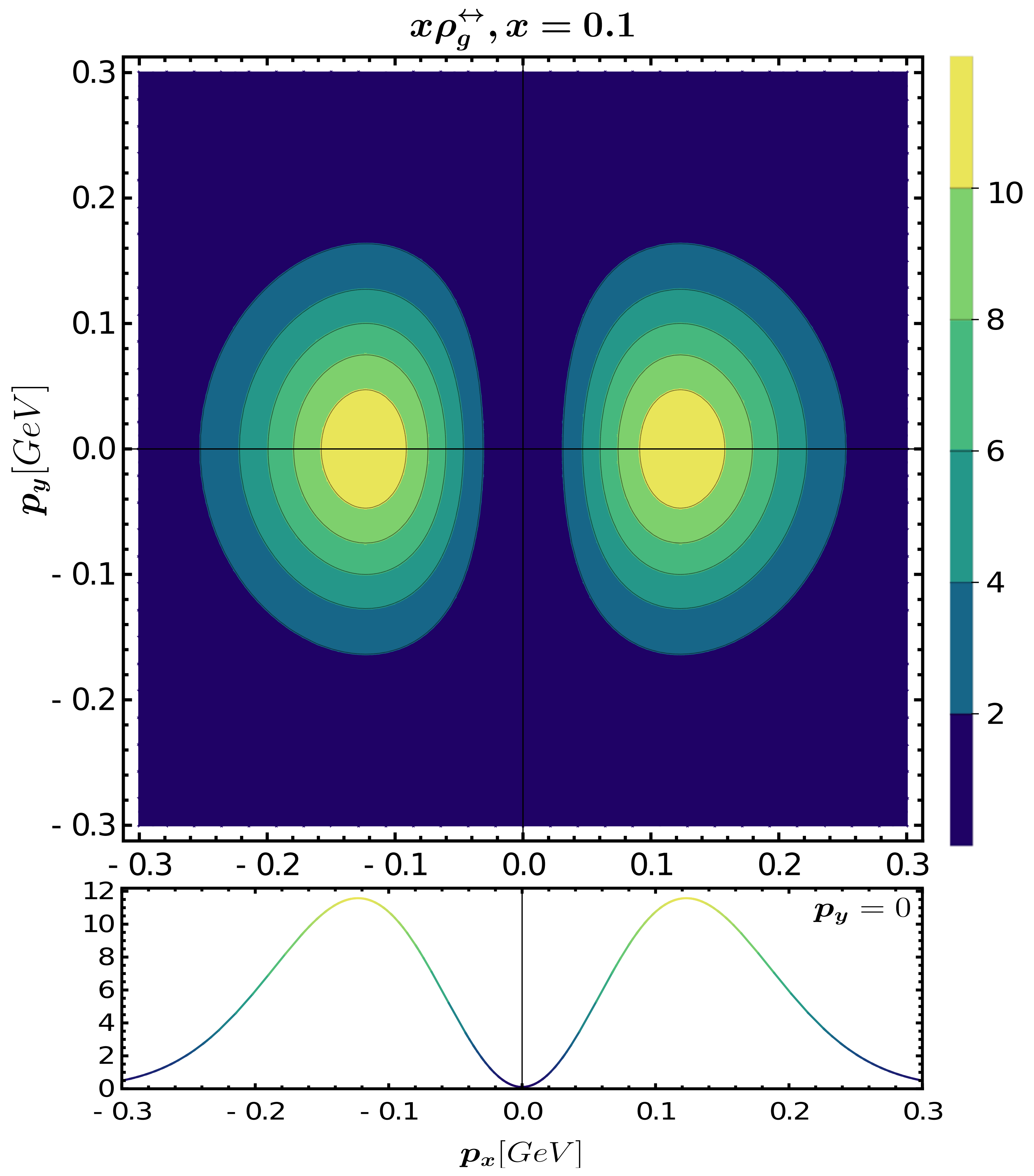


Results are similar to Bacchetta et al EPJC80

- Helicity density: circularly polarized gluon density in a longitudinally polarized proton.

- $$x\rho_g^{\circlearrowleft/+}(x,p_x,p_y) = \frac{1}{2}[xf_1^g(x,\mathbf{p}_\perp^2) + xg_{1L}^g(x,\mathbf{p}_\perp^2)]$$

- Worm-gear density: circularly polarized gluon density in a transversely polarized proton

- $$x\rho_g^{\circlearrowleft/\leftrightarrow}(x,p_x,p_y) = \frac{1}{2}[xf_1^g(x,\mathbf{p}_\perp^2) - \frac{p_x}{M}xg_{1T}^g(x,\mathbf{p}_\perp^2)]$$

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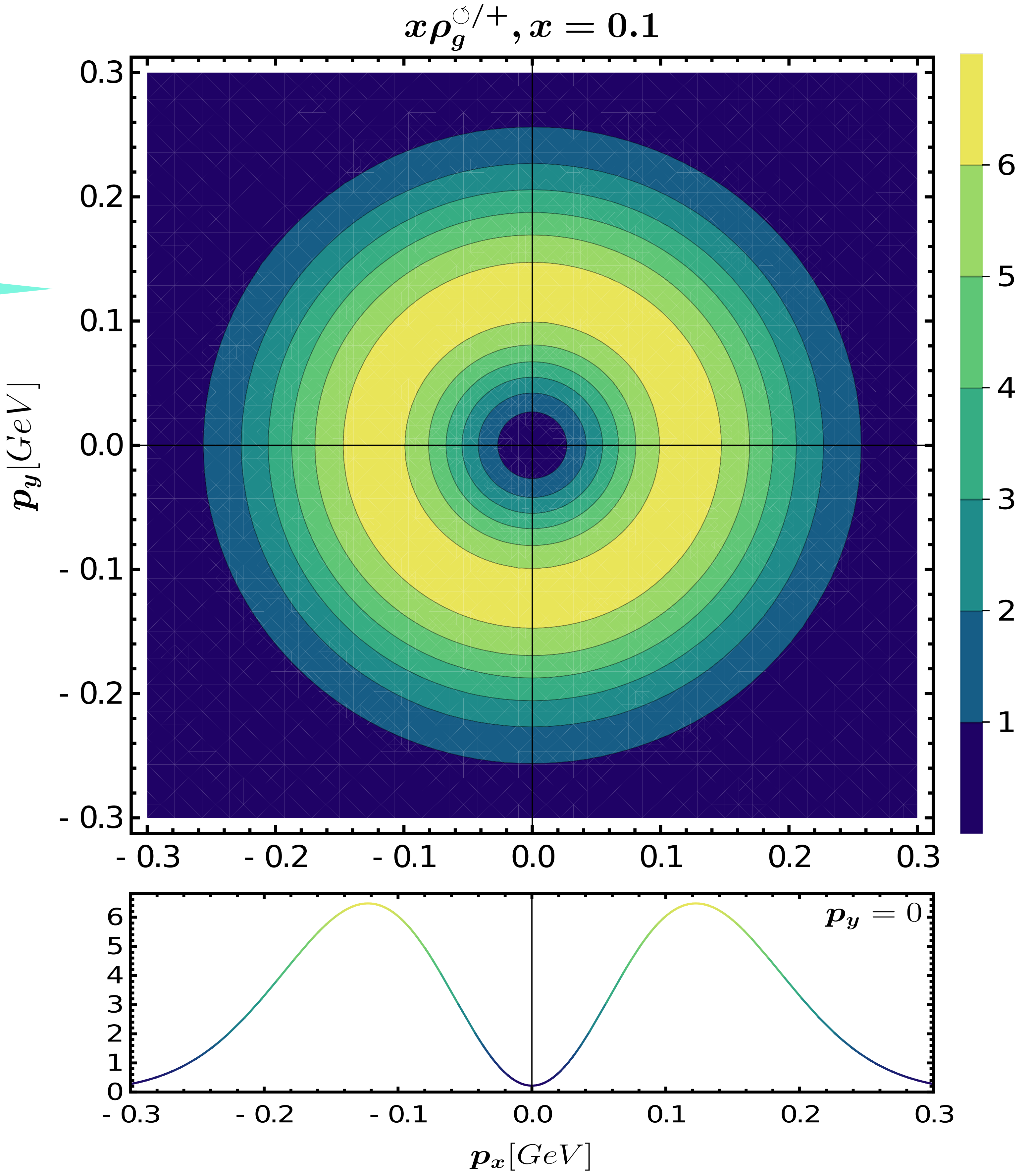
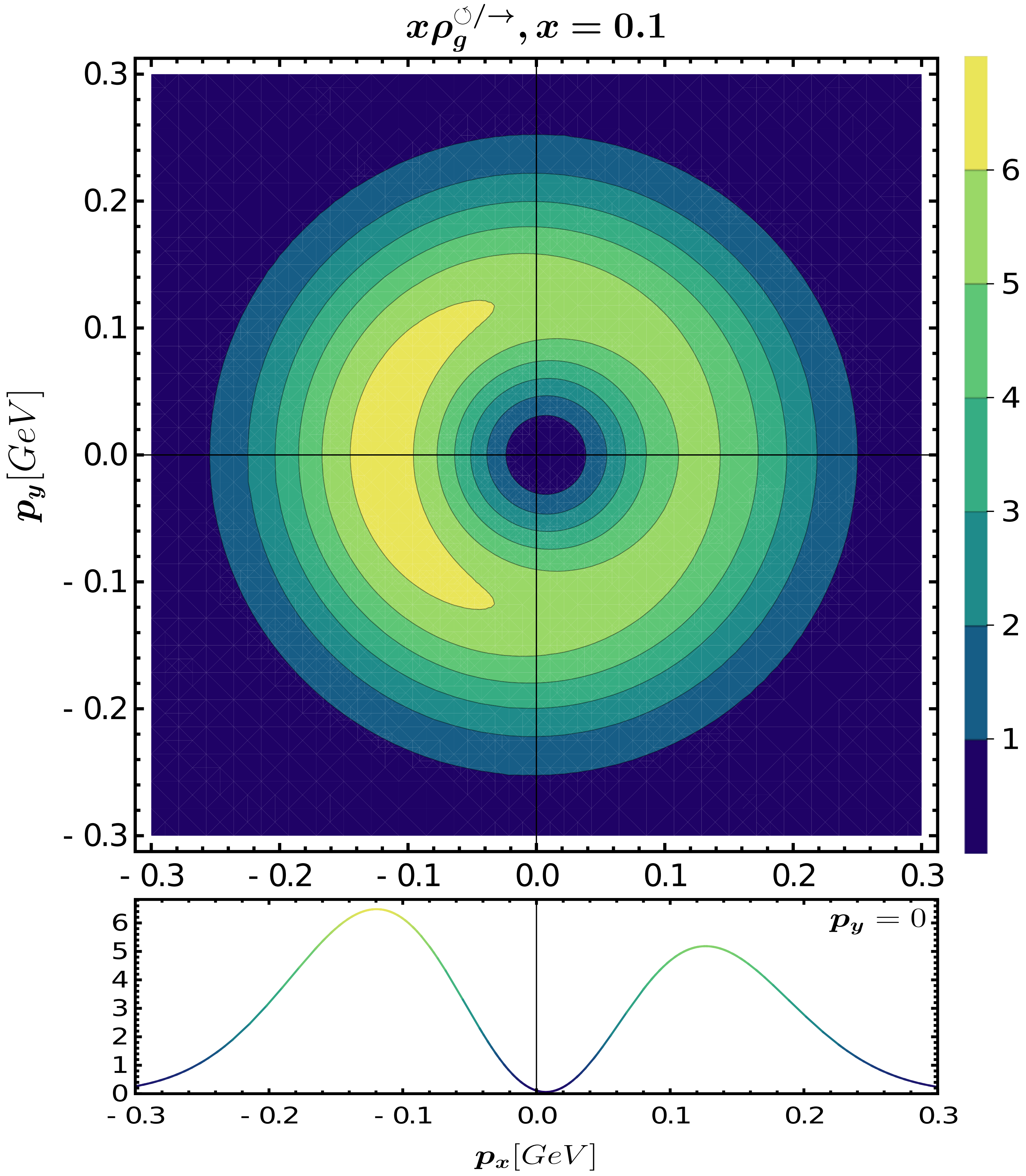
$$x\rho_g^{\circlearrowleft/+}(x,p_x,p_y)=\frac{1}{2}[xf_1^g(x,\mathbf{p}_\perp^2)+xg_{1L}^g(x,\mathbf{p}_\perp^2)]$$

- Worm-gear density: circularly polarized gluon density in a transversely polarized proton

$$x\rho_g^{\circlearrowleft/\leftrightarrow}(x,p_x,p_y)=\frac{1}{2}[xf_1^g(x,\mathbf{p}_\perp^2)-\frac{p_x}{M}xg_{1T}^g(x,\mathbf{p}_\perp^2)]$$

symmetric




Circular symmetry is broken

# GPDs

- GPDs appear in exclusive processes e.g., DVCS/ vector meson production
- are off-forward matrix elements of the bilocal operator and functions of $(x, \xi, t)$.
- encode spatial as well as spin structure of the nucleon.
- don't have probabilistic interpretation.
- for skewness $\xi = 0$, in impact parameter space can have probabilistic interpretation.
- In the forward limit GPDs —> PDFs.

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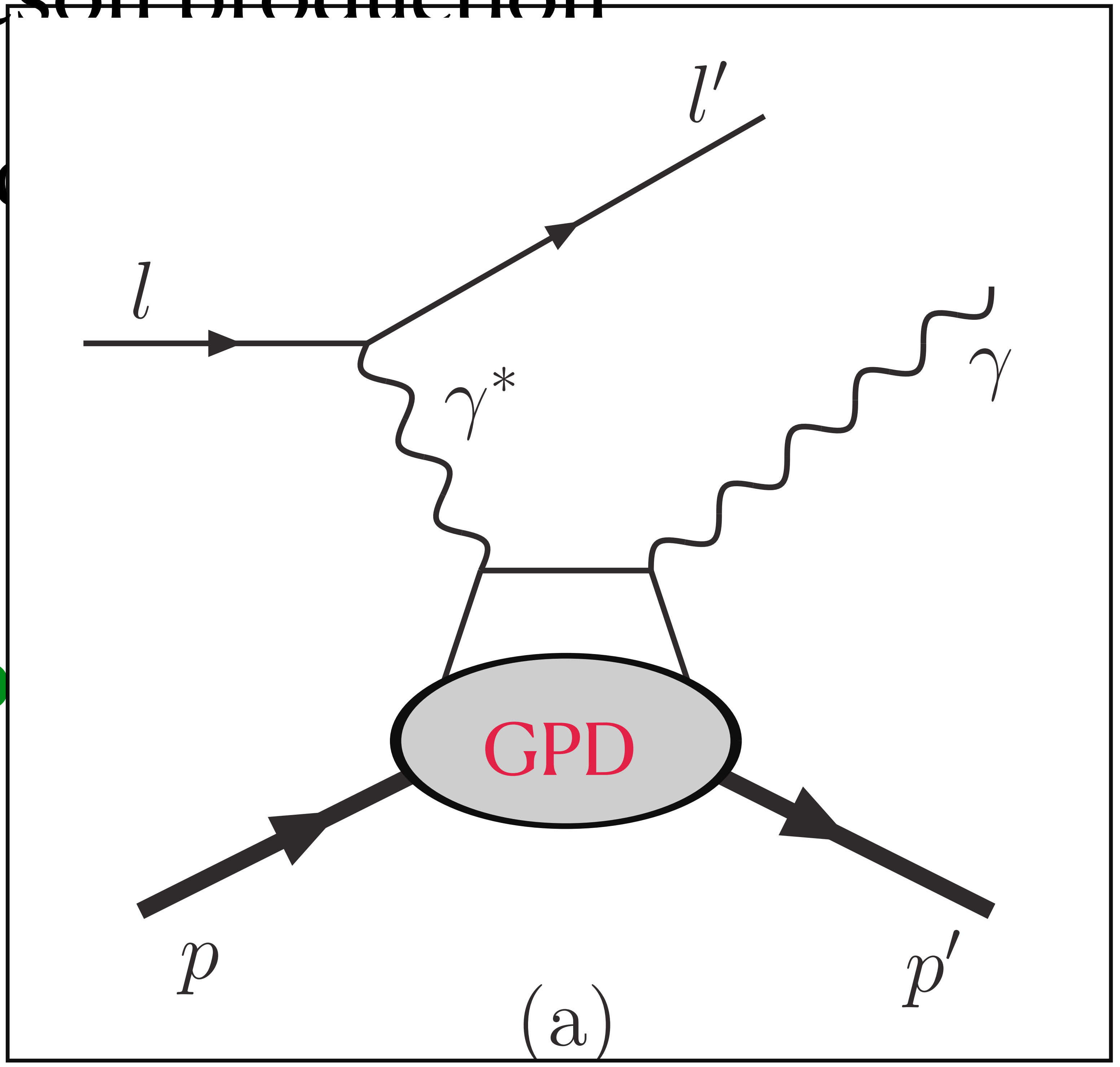

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# Gluon GPDs

- Since nonperturbative QCD evaluations are not yet feasible, it is important to constraint the GPDs by using different model predictions.
- We analyze the gluon GPDs in our model for both $\xi = 0$ and $\xi \neq 0$ ( in experiments $\xi \neq 0$).
- In the light cone gauge $(A^+ = 0)$ , 4 helicity conserving gluon GPDs:

$$\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p', \lambda' |\, F^{+i}(-\tfrac{z}{2})\, F^{+i}(\tfrac{z}{2})\, | p, \lambda\rangle\Big|_{z^+=0,\, \mathbf{z}_T=0} = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[H^g\gamma^+ + E^g\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\right]u(p,\lambda),$$

$$-\frac{i}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p', \lambda' |\, F^{+i}(-\tfrac{z}{2})\, \tilde{F}^{+i}(\tfrac{z}{2})\, | p, \lambda\rangle\Big|_{z^+=0,\, \mathbf{z}_T=0} = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\tilde{H}^g\gamma^+\gamma_5 + \tilde{E}^g\frac{\gamma_5\Delta^+}{2M}\right]u(p,\lambda),$$

- 4 gluon helicity flip GPDs:

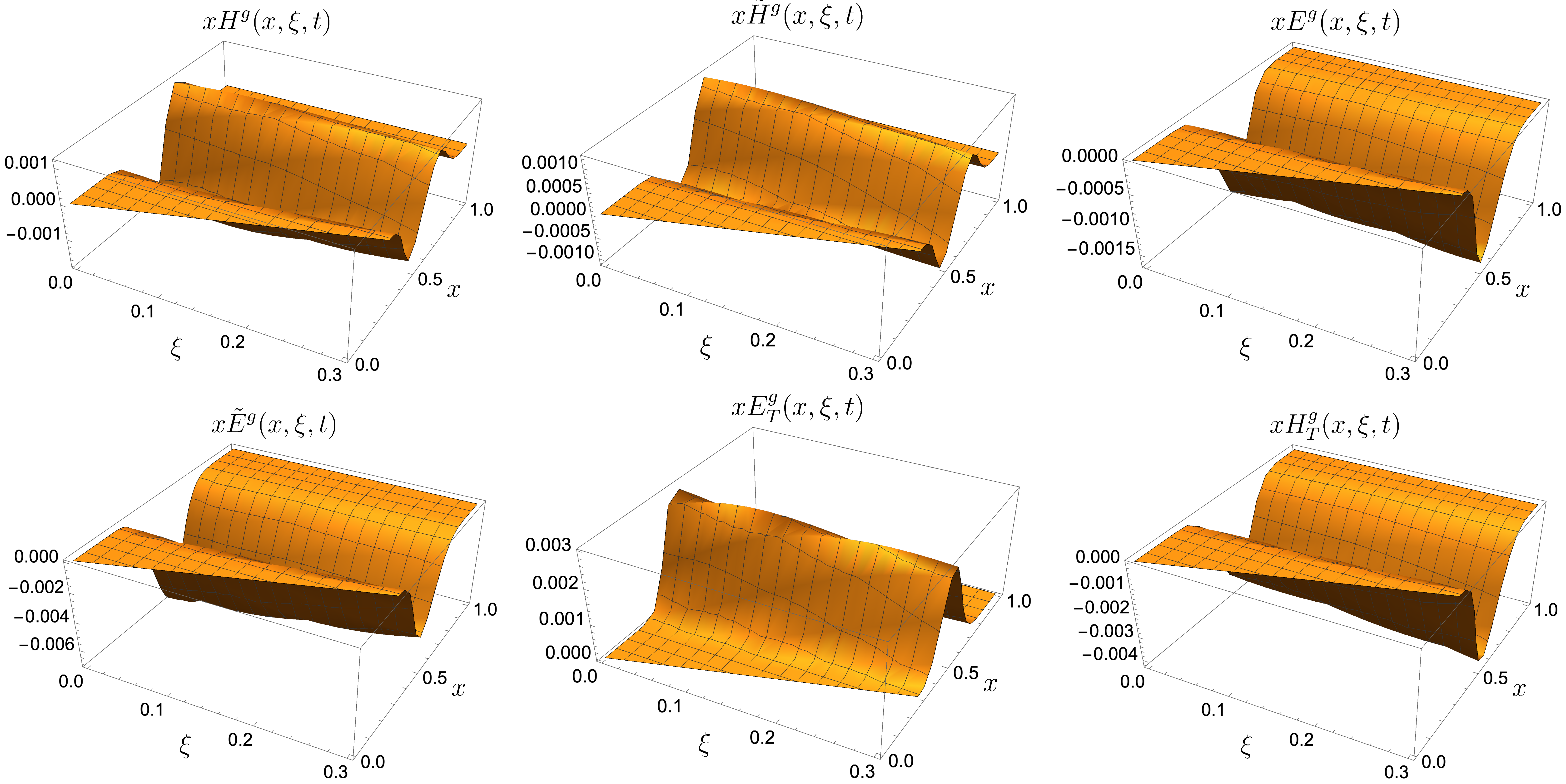
$$-\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p', \lambda' |\, \mathbf{S} F^{+i}(-\tfrac{z}{2})\, F^{+j}(\tfrac{z}{2})\, | p, \lambda\rangle\Big|_{z^+=0,\, \mathbf{z}_T=0} = \mathbf{S}\,\frac{1}{2P^+}\,\frac{P^+\Delta^j - \Delta^+P^j}{2MP^+}$$

$$\times\, \bar{u}(p',\lambda')\left[H_T^g\, i\sigma^{+i} + \tilde{H}_T^g\frac{P^+\Delta^i - \Delta^+P^i}{M^2} + E_T^g\frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2M} + \tilde{E}_T^g\frac{\gamma^+P^i - P^+\gamma^i}{M}\right]u(p,\lambda)$$

- We consider $x \geq \xi$ only: particle number conserving process.
- In the forward limit GPDs $\longrightarrow$ PDFs
- Unpolarised gluon GPD $H^g(x, \xi = 0, t = 0) = f^g(x)$ = unpolarised gluon pdf
- Helicity dependent GPD $\tilde{H}^g(x, \xi = 0, t = 0) = g^g_{1L}(x)$ = helicity pdf

Check for your calculations!

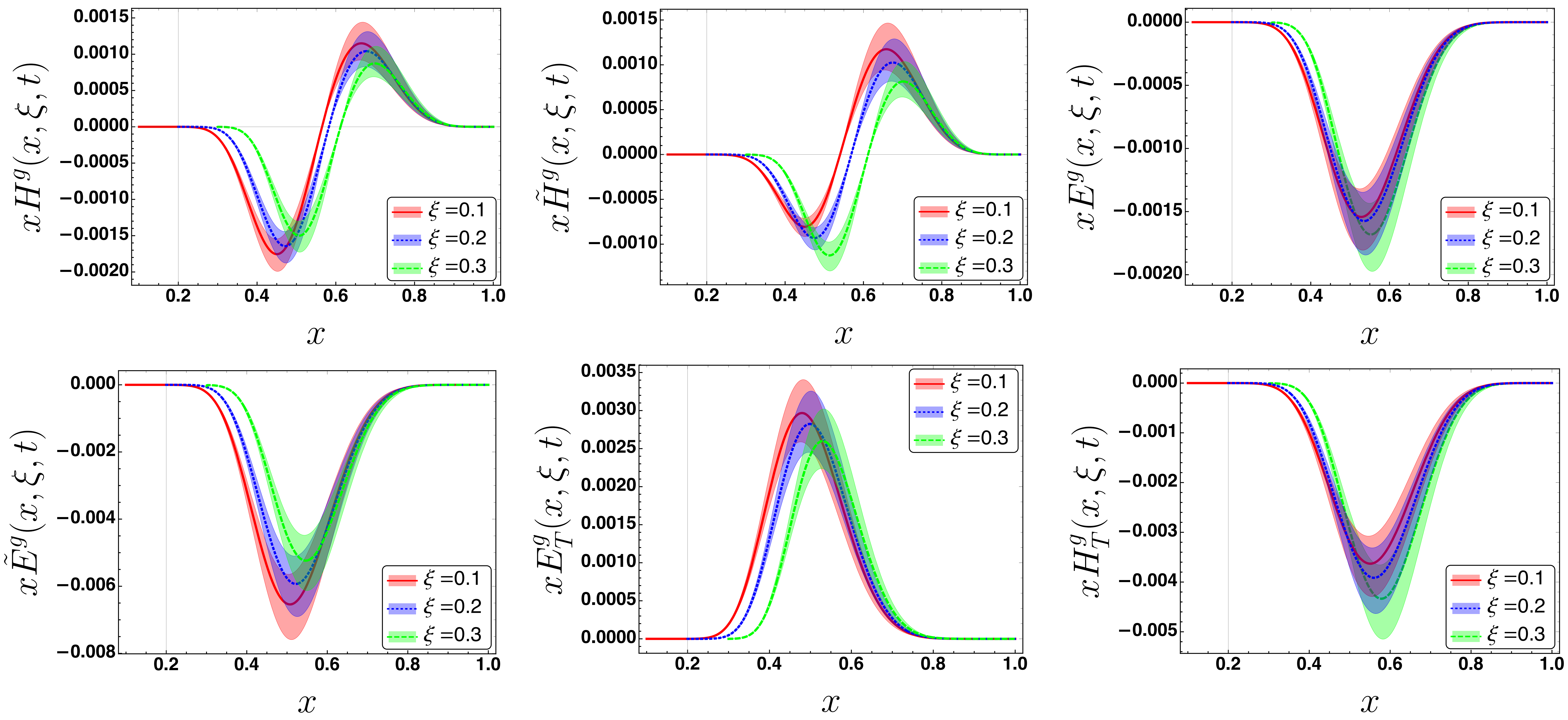
# Gluon GPDs at $-t = 3\ GeV^2$ as functions of $(x, \xi)$




In our model $\tilde{H}^g_T = 0$

$xE^g$, $x\tilde{E}^g$, $xH^g_T$ are of similar behaviour(with different magnitudes)

# 2D plots




Magnitudes of the peaks depend on skewness $\xi$

# GPDs in impact parameter space

- 2D Fourier transform with respect to the transverse momentum transferred at $\xi = 0$ gives the GPD in impact parameter space.
- $$\mathcal{F}(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp . b_\perp} F^g(x, \xi = 0, t = -\Delta_\perp^2)$$

  Transverse distance of the struck quark from the CoM

- GPDs in impact parameter have probabilistic interpretation. M. Burkardt, IJMPA18, 187(2003)
- $\langle b_\perp^2 \rangle$: transverse size of the nucleon. At small $x$, gluons show larger transverse radius than quarks. At larger $x$, the radius decreases, becomes point-like at $x = 1$.

# GPDs in impact parameter space (contd)

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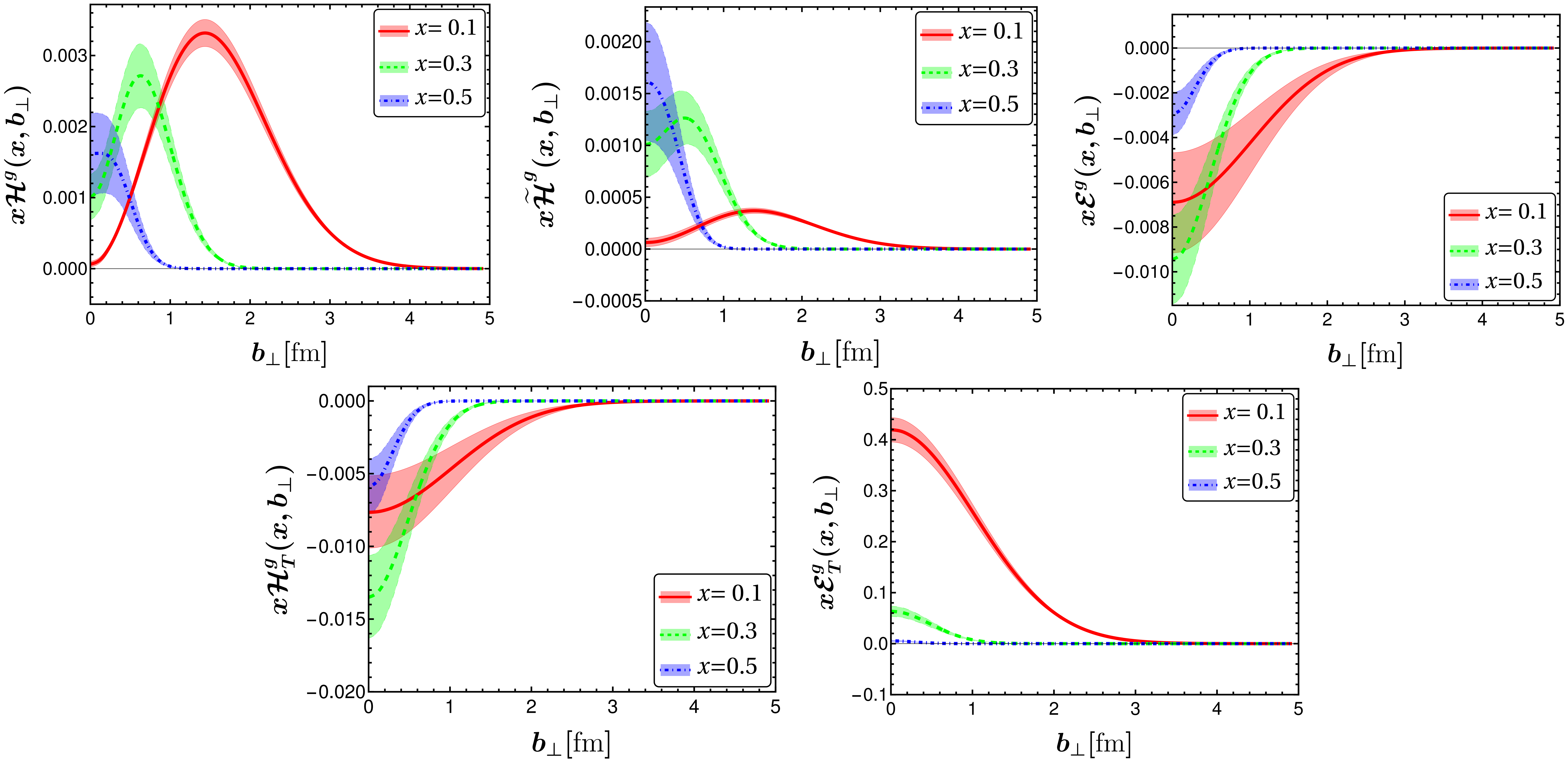
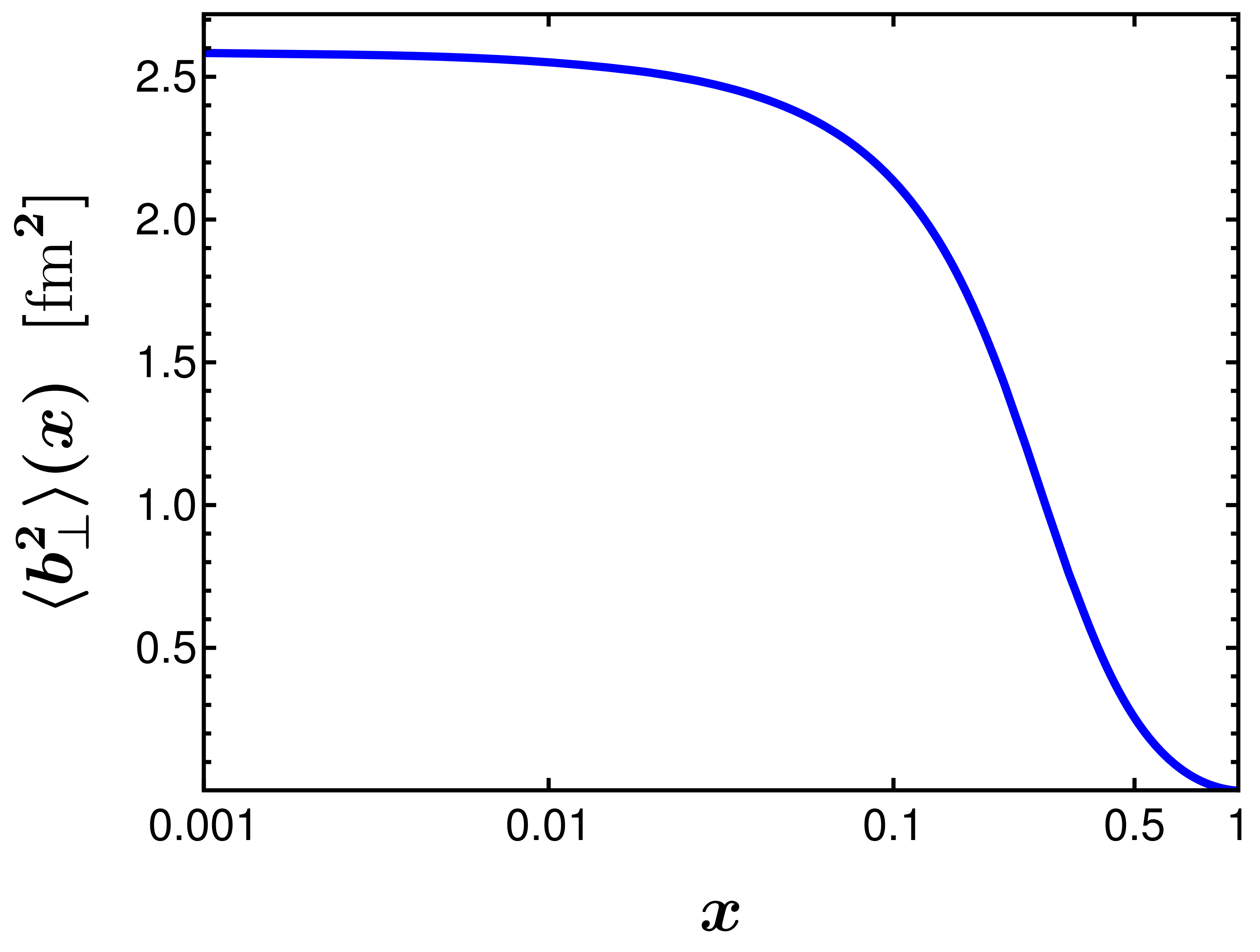


# Transverse radius

- $\langle b_\perp^2 \rangle$: transverse size of the nucleon. At small $x$, gluons show slightly larger transverse radius than quarks. At larger $x$, the radius decreases, becomes point-like at $x = 1$.



# Orbital angular momentum

- According to Ji's sum rule:

X. Ji, PRL 78, 610

- $$J_z^g = \frac{1}{2}\int dx\,x\,[H^g(x,0,0) + E^g(x,0,0)]$$

- Our estimate $J_z^g = 0.058$ consistent with BLFQ result of $J_z^g = 0.066$

B. Lin et al. 2308. 08275

- Helicity GPD gives the spin contribution of gluon:

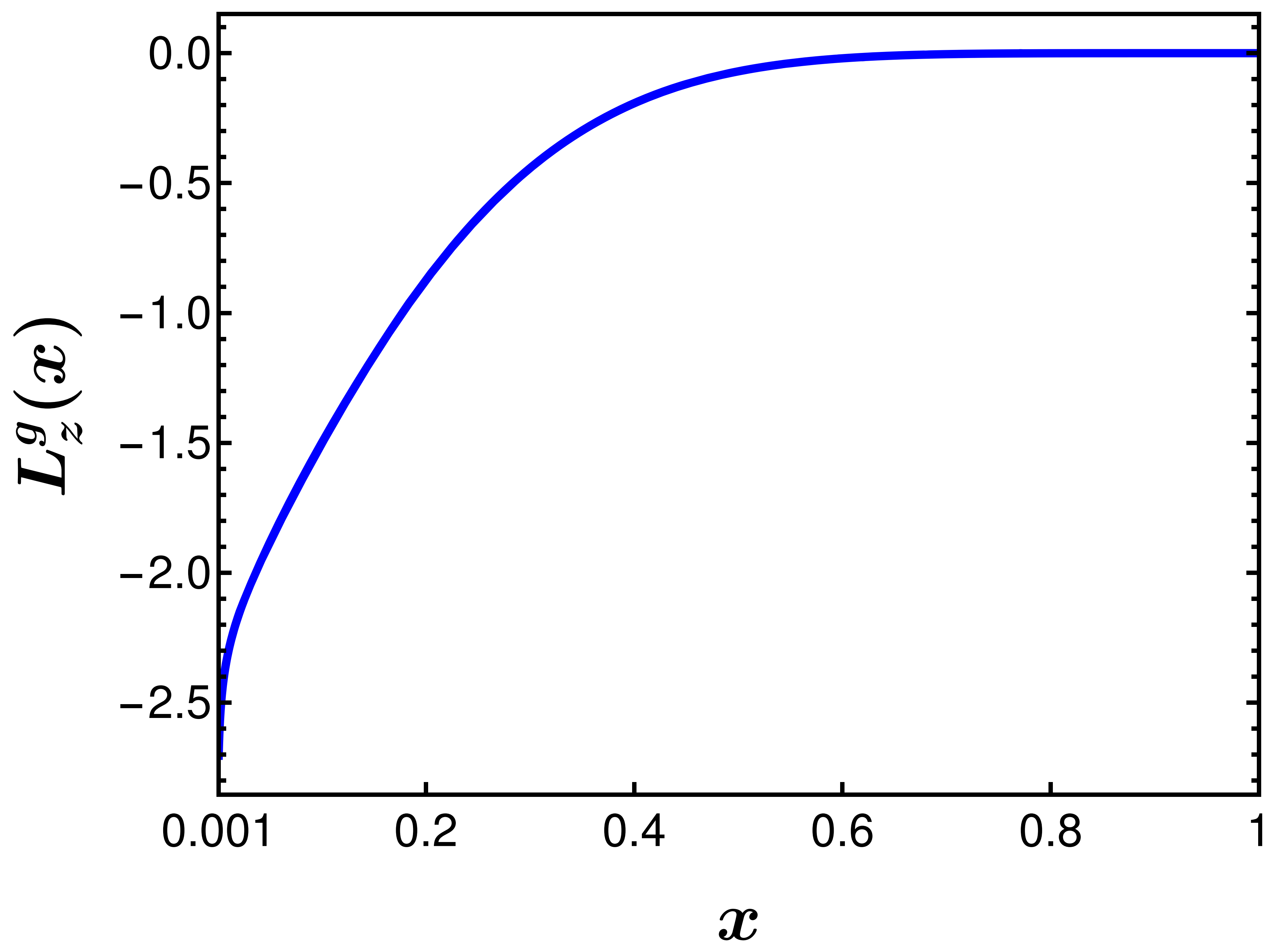
$$\Delta G = \int dx\; g_{1L}(x) = \int dx\; \tilde{H}^g(x,0,0)$$

- Separation of gluon spin and OAM is not unique!

- Spin asymmetries in polarised scattering experiments are directly proportional to the gluon intrinsic spin!

- Two definitions of OAM: Kinetic and canonical OAM.

- Kinetic OAM: $$L_z^g = \frac{1}{2}\int \{x\,[H^g(x,0,0) + E^g(x,0,0)] - \widetilde{H}^g(x,0,0)\}$$ defined in terms of GPDs.




Unintegrated kinetic OAM as a function of x

- 
- Our result: $L_z^g = -0.42$
- 

# Canonical OAM

- Canonical OAM in the light cone gauge is defined by GTMDs as

$$\ell_z^g(x) = -\int d^2\mathbf{p}_\perp \frac{\mathbf{p}_\perp^2}{M^2} F_{1,4}^g(x, 0, \mathbf{p}_\perp, 0, 0).$$

# GTMDs

- GTMDs : higher dimensional distributions ⟷ Wigner distributions.

* TMDs can be obtained from GTMDs at $\Delta_\perp = 0$ limit

** GPDs in impact parameter space are obtained by integrating GTMDs over $p_\perp$

- GTMD correlator:
- $$W(x, \xi = 0, p_\perp, \Delta_\perp) = \frac{1}{xP^+} \int \frac{dz^- d^2 z^\perp}{(2\pi)^3} e^{ip.z} \langle p + \frac{\Delta_\perp}{2} | F^{+i}(-z/2) \mathscr{W} F^{+j} | p - \frac{\Delta_\perp}{2} \rangle |_{z^+=0}$$

Chirally even gluon GTMDs: $F_{1,1},\ F_{1,4},\ G_{1,1},\ G_{1,4}$

- $$\frac{i(\mathbf{p}_\perp \times \mathbf{\Delta}_\perp)_z}{M^2} F^g_{1,4} = \frac{1}{2(2\pi)^3}\,\frac{1}{2} \sum_{\Lambda,\lambda,\mu} \mathrm{sign}(\Lambda)\left[\psi^{\Lambda *}_{\lambda,\mu}(\hat{x}, \hat{\boldsymbol{p}}'_\perp)\psi^{\Lambda}_{\lambda,\mu}(\hat{x}, \hat{\boldsymbol{p}}_\perp)\right]$$

GTMD $F_{1,4}$ describes the distortion of unpolarised parton in a longitudinally polarised nucleon

- $$-\frac{i(\mathbf{p}_\perp \times \mathbf{\Delta}_\perp)_z}{M^2} G^g_{1,1} = \frac{1}{2(2\pi)^3}\,\frac{1}{2} \sum_{\Lambda,\lambda,\mu} \mathrm{sign}(\mu)\left[\psi^{\Lambda *}_{\lambda,\mu}(\hat{x}, \hat{\boldsymbol{p}}'_\perp)\psi^{\Lambda}_{\lambda,\mu}(\hat{x}, \hat{\boldsymbol{p}}_\perp)\right]$$

$G_{1,1}$ : distortion of longitudinally polarised gluon inside a unpolarised nucleon

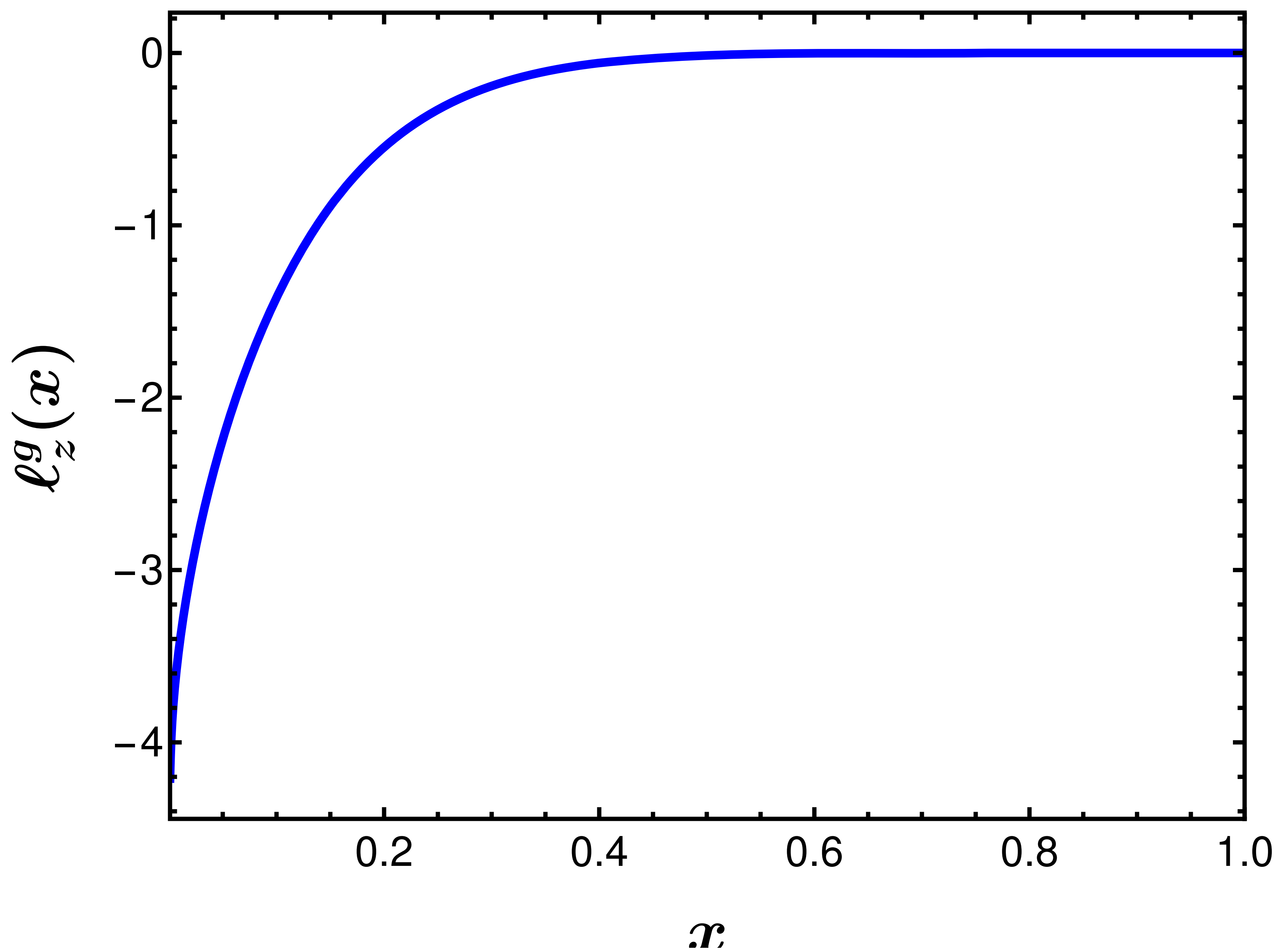
$\Lambda$ =proton helicity
$\lambda$= quark helicity
$\mu$=gluon helicity

**Canonical OAM:**

$$\ell_z^g(x) = -\int d^2\mathbf{p}_\perp \frac{\mathbf{p}_\perp^2}{M^2} F_{1,4}^g(x, 0, \mathbf{p}_\perp, 0, 0).$$

- Our model result:

$$\ell_z^g(x) = -N_g^2\kappa^2 \frac{1-(1-x)^2}{x^2(1-x)^2} x^{2b+3}(1-x)^{2a+1} \frac{1}{\log[\frac{1}{1-x}]}$$

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- Integrated value: canonical OAM $l_z^g \approx -0.38$ [consistent with another spectator model result $l_z^g = -0.33$ ]

# Spin-orbit correlation

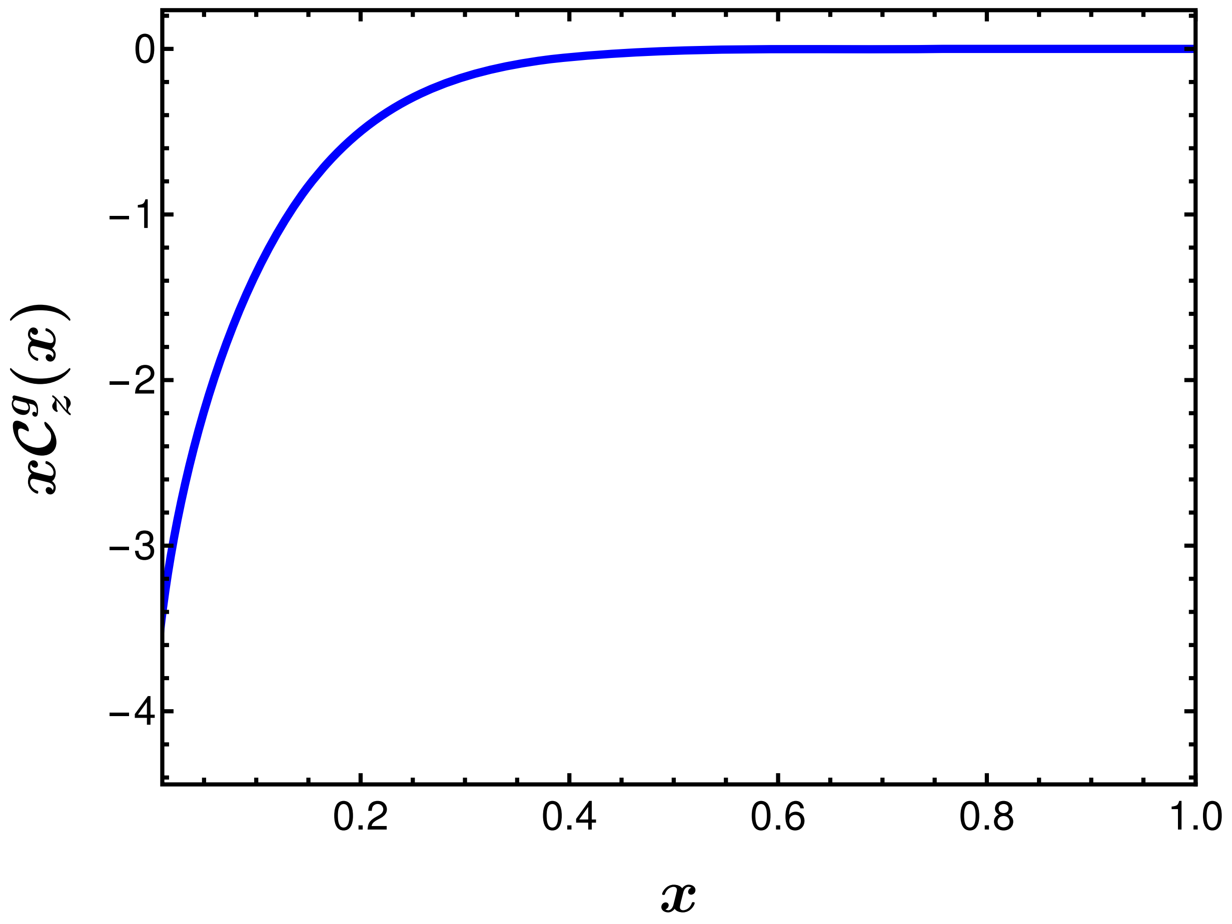
$$C_z^g(x) = \int d^2\mathbf{p}_\perp \frac{\mathbf{p}_\perp^2}{M^2} G_{1,1}^g(x, 0, \mathbf{p}_\perp, 0, 0)$$

- $G_{1,1}$ gives the spin-OAM correlation

- $C_z^g < 0$ : spin and OAM are anti-aligned
- $C_z^g > 0$ : spin and OAM are aligned.

Lorce, Pasquini, PRD84, 014015

Model predicts $C_z^g < 0$

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# Summary and conclusions

- To understand the three dimensional structure and partonic level description of spin/OAM , we need to investigate both quark and gluons (and sea quarks too!).
- Gluon distributions are not yet well understood/studied.
- We presented the study of different gluon distributions in a simple model of proton.
- gluon contributions to spin/OAM .
- We require more experiments , lattice results, better models with gluons…

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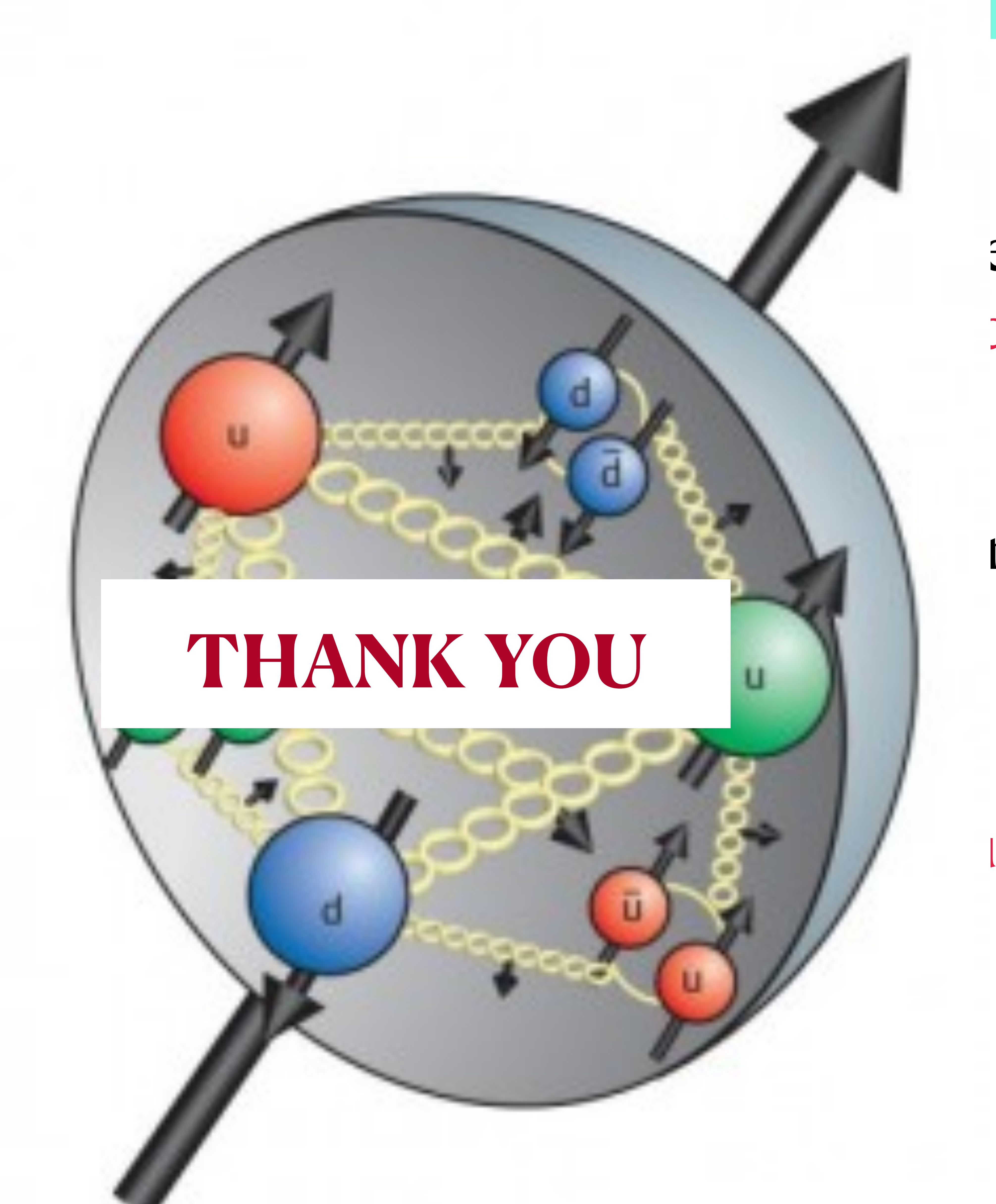


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