



# Glueballs in QCD sum rules

Wei Chen

Sun Yat-Sen University

**Based on: PRD103 (2021) L091503; PRD104(2021) 094050; RPP86(2023) 026201**

**Collaborators: Hua-Xing Chen, Yan-Rui Liu, Xiang Liu, Shi-Lin Zhu**

Light-Cone 2024: Hadron Physics in the EIC era

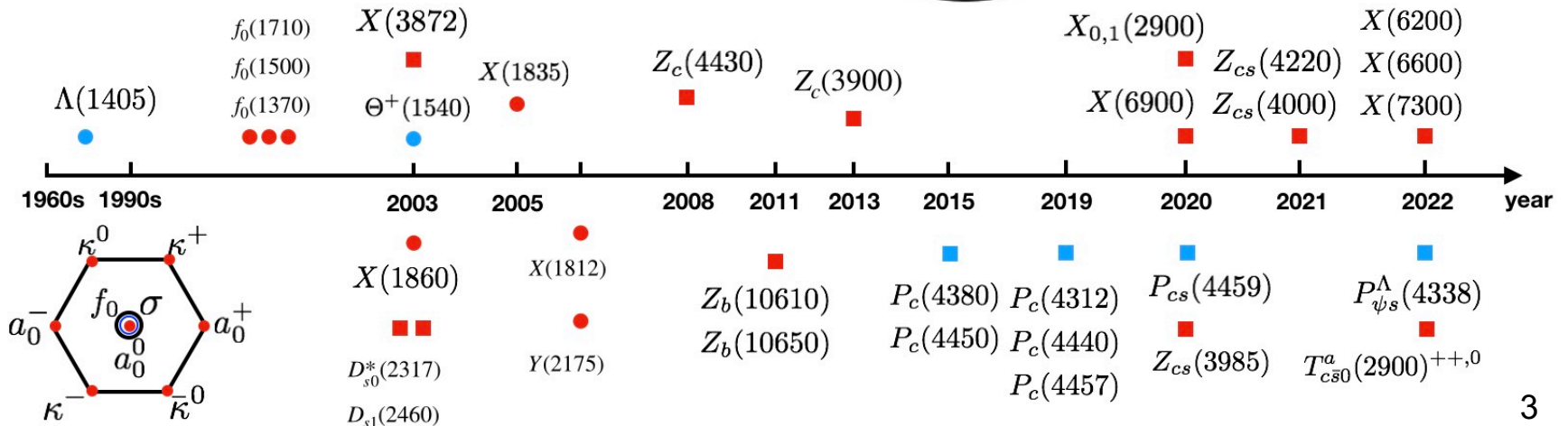
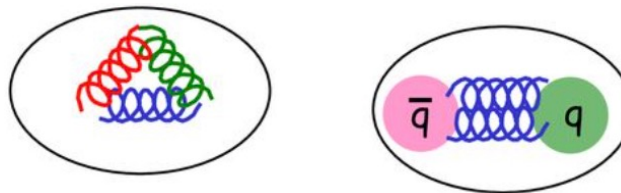
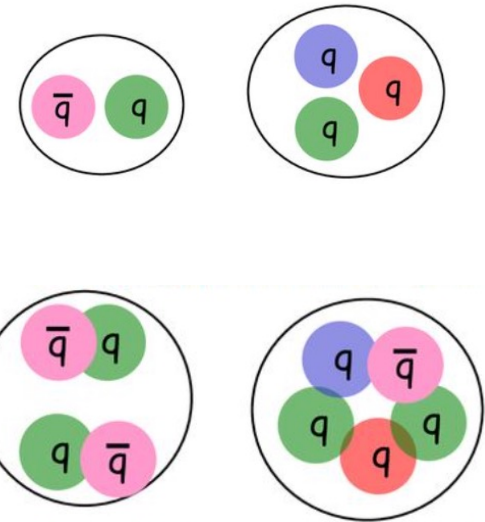
2024.11.25-29 IMP • Huizhou

# Outline

- **Research progresses on glueballs**
- **Two-gluon glueballs**
- **Three-gluon glueballs**
- **Summary**

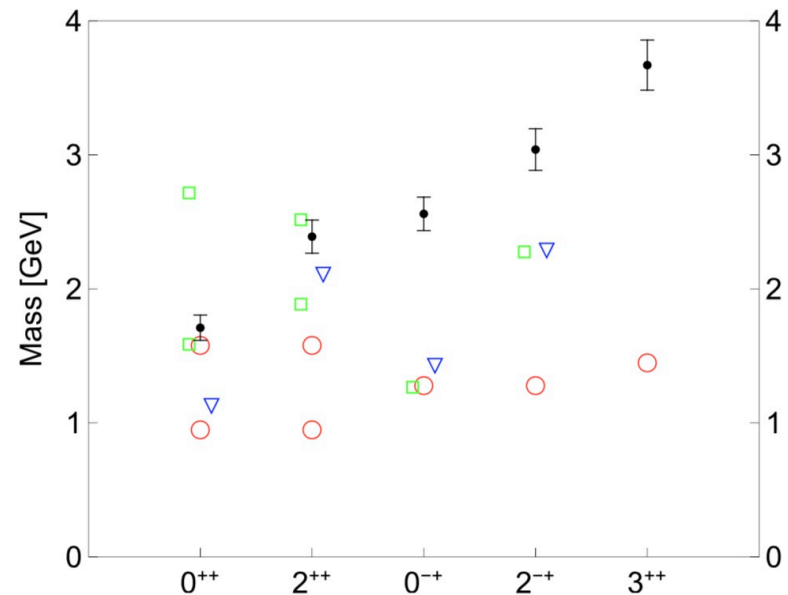
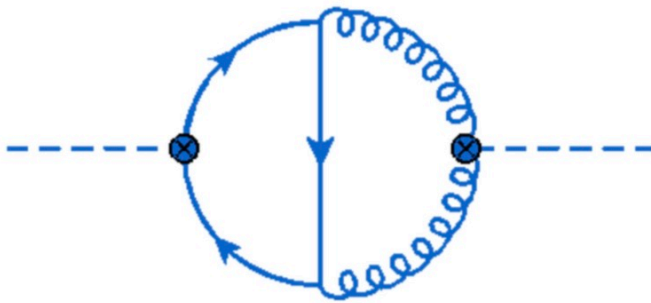
# Quark model and Exotic hadrons

- **Quark Model:**  $q\bar{q}$  mesons and  $qqq$  baryons
- **Exotic Hadrons:** hadrons beyond QM, such as multiquarks, hybrids, glueballs...
- **Hybrids and Glueballs:** very distinctive predictions of QCD!

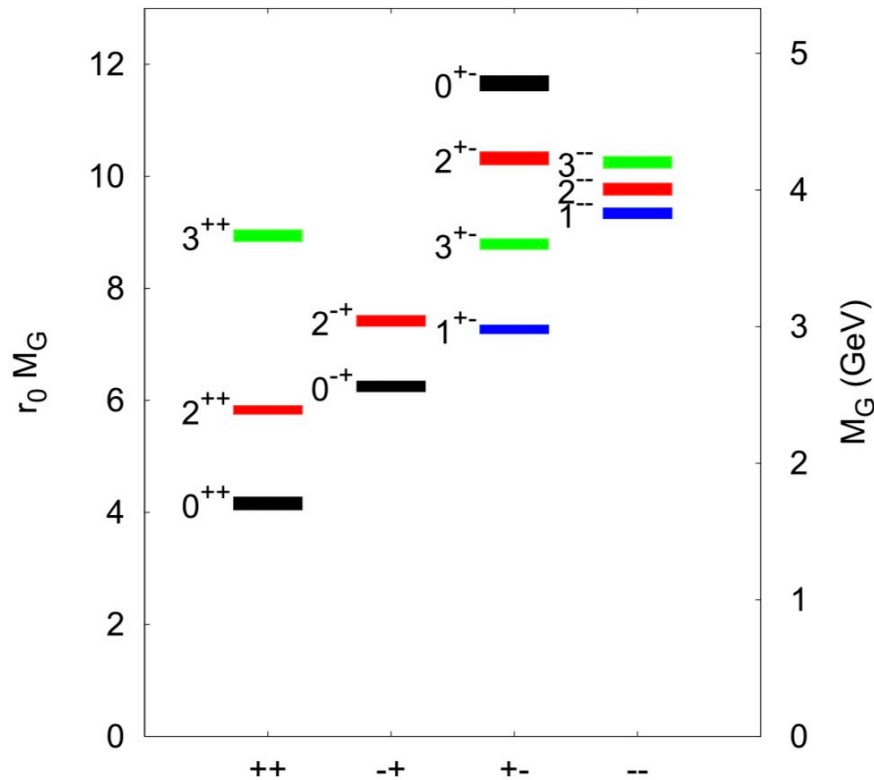


**Glueballs:** colorless bound states of gluons as gluons have a self-coupling

- Mixing with normal  $q\bar{q}$  states, it is hard to isolate the pure glueballs experimentally.
- No universal definition of constituent gluon: massless or massive?
- Plenty of theoretical studies in the past half century based on various methods.



# Mass spectrum of glueballs in LQCD



## Lightest glueballs:

$$J^{PC} = 0^{++}: 1710 \pm 50 \pm 80 \text{ MeV}$$

$$J^{PC} = 2^{++}: 2390 \pm 30 \pm 120 \text{ MeV}$$

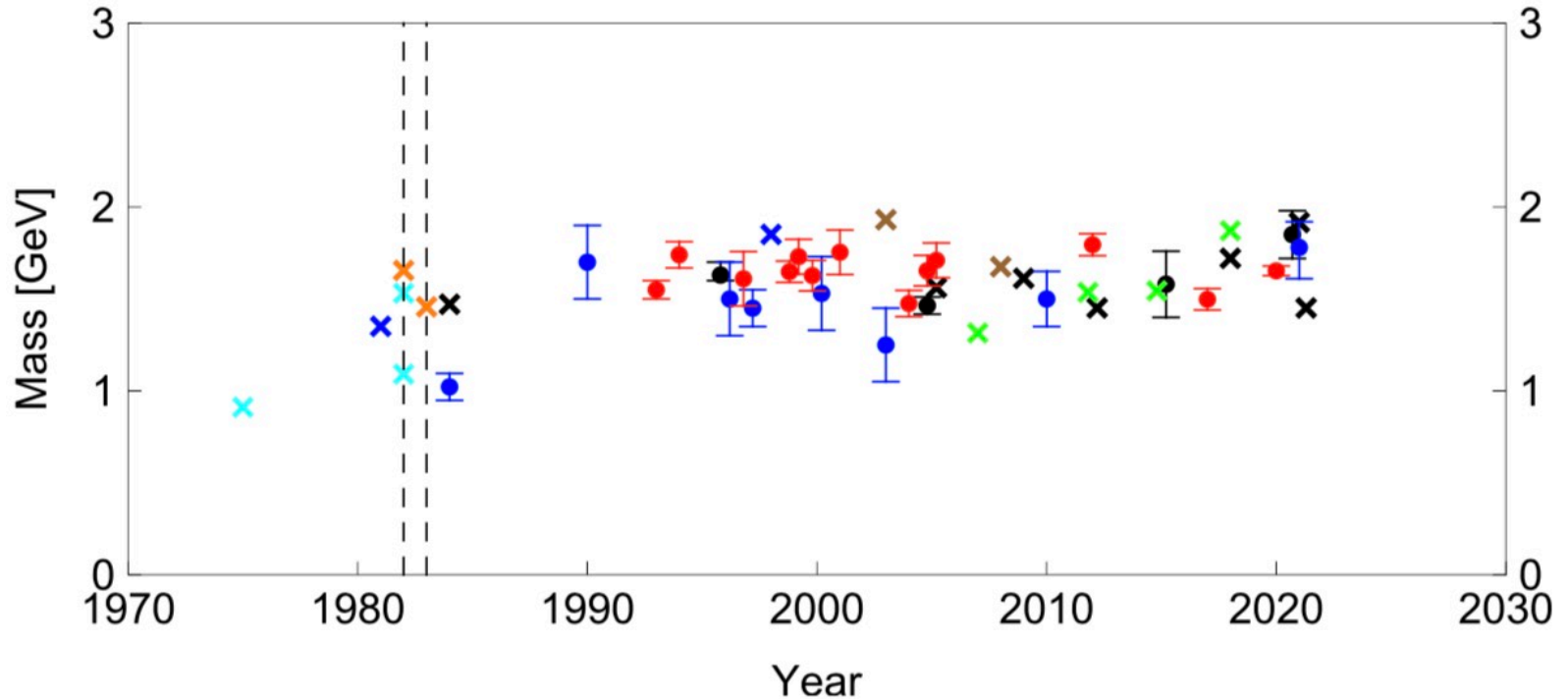
$$J^{PC} = 0^{-+}: 2560 \pm 35 \pm 120 \text{ MeV}$$

There are some candidates in these channels!

Y. Chen et. al., PRD73(2006)014516

# Scalar glueball:

H.X.Chen et. al., RPP86(2023) 026201



Average mass predictions:

$$M_{|gg;0^{++}\rangle} \approx 1650 \text{ MeV}$$

# Candidates: scalar light mesons

$$M_{|gg;0^{++}\rangle} \approx 1650 \text{ MeV}$$

Name	$f_0(500)$	$f_0(1370)$	$f_0(1710)$	$f_0(2020)$	$f_0(2200)$
$M$ (MeV)	$410 \pm 20$ $400 \rightarrow 550$	$1370 \pm 40$ $1200 \rightarrow 1500$	$1700 \pm 18$ $1704 \pm 12$	$1925 \pm 25$ $1992 \pm 16$	$2200 \pm 25$ $2187 \pm 14$
$\Gamma$ (MeV)	$480 \pm 30$ $400 \rightarrow 700$	$390 \pm 40$ $100 \rightarrow 500$	$255 \pm 25$ $123 \pm 18$	$320 \pm 35$ $442 \pm 60$	$150 \pm 30$ $\sim 200$
Name	$f_0(980)$	$f_0(1500)$	$f_0(1770)$	$f_0(2100)$	$f_0(2330)$
$M$ (MeV)	$1014 \pm 8$ $990 \pm 20$	$1483 \pm 15$ $1506 \pm 6$	$1765 \pm 15$	$2075 \pm 20$ $2086^{+20}_{-24}$	$2340 \pm 20$ $\sim 2330$
$\Gamma$ (MeV)	$71 \pm 10$ $10 \rightarrow 100$	$116 \pm 12$ $112 \pm 9$	$180 \pm 20$	$260 \pm 25$ $284^{+60}_{-32}$	$165 \pm 25$ $250 \pm 20$

Production rates in the gluon-rich  $J/\psi$  radiative decay processes:

$$\mathcal{B}(J/\psi \rightarrow \gamma f_0(1500)) \sim 0.29 \times 10^{-3},$$

$$\mathcal{B}(J/\psi \rightarrow \gamma f_0(1710)) \sim 2.2 \times 10^{-3},$$

BESIII: *Natl. Sci. Rev.* 8 (2021)  
nwab198

$$\mathcal{B}(J/\psi \rightarrow \gamma |gg; 0^{++}\rangle) = (3.8 \pm 0.9) \times 10^{-3}. \quad \text{PRL110(2010)021601}$$

# Mixing scheme:

PLB826(2022)36906

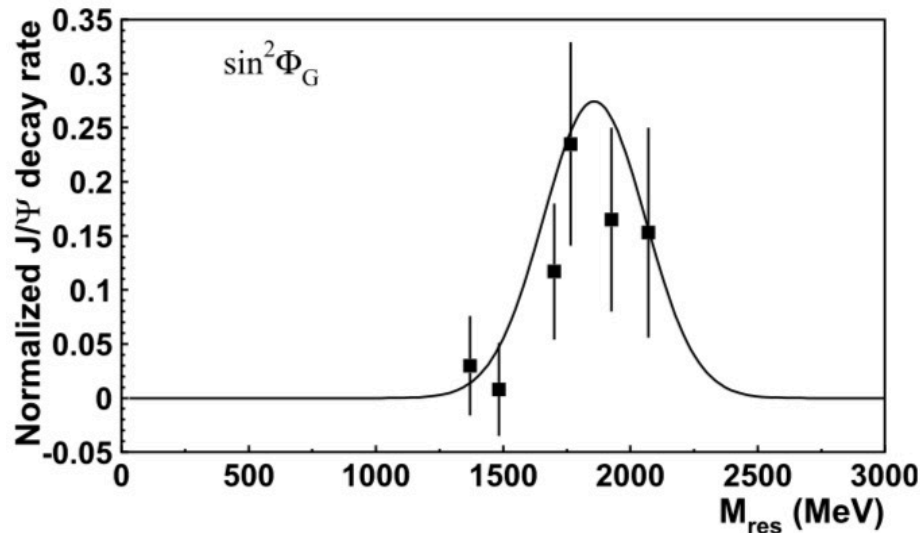
$$\mathcal{M} = \begin{pmatrix} M_{gg} & f & \sqrt{2}f \\ f & M_{s\bar{s}} & 0 \\ \sqrt{2}f & 0 & M_{n\bar{n}} \end{pmatrix}$$

$$H' = (|n\bar{n}\rangle \cos \varphi' - |s\bar{s}\rangle \sin \varphi') \cos \phi^H + |gg\rangle \sin \phi^H,$$

$$L' = (|n\bar{n}\rangle \sin \varphi' + |s\bar{s}\rangle \cos \varphi') \cos \phi^L + |gg\rangle \sin \phi^L,$$

$$|n\bar{n}\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle}{\sqrt{2}}, \quad f = \langle s\bar{s} | V | gg \rangle = \langle n\bar{n} | V | gg \rangle / \sqrt{2}$$

$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(1770)$	$f_0(2020)$	$f_0(2100)$
$(5 \pm 4)\%$	$< 5\%$	$(12 \pm 6)\%$	$(25 \pm 10)\%$	$(16 \pm 9)\%$	$(17 \pm 8)\%$



**Fractional glueball contents:**

**$(78 \pm 18)\%$**

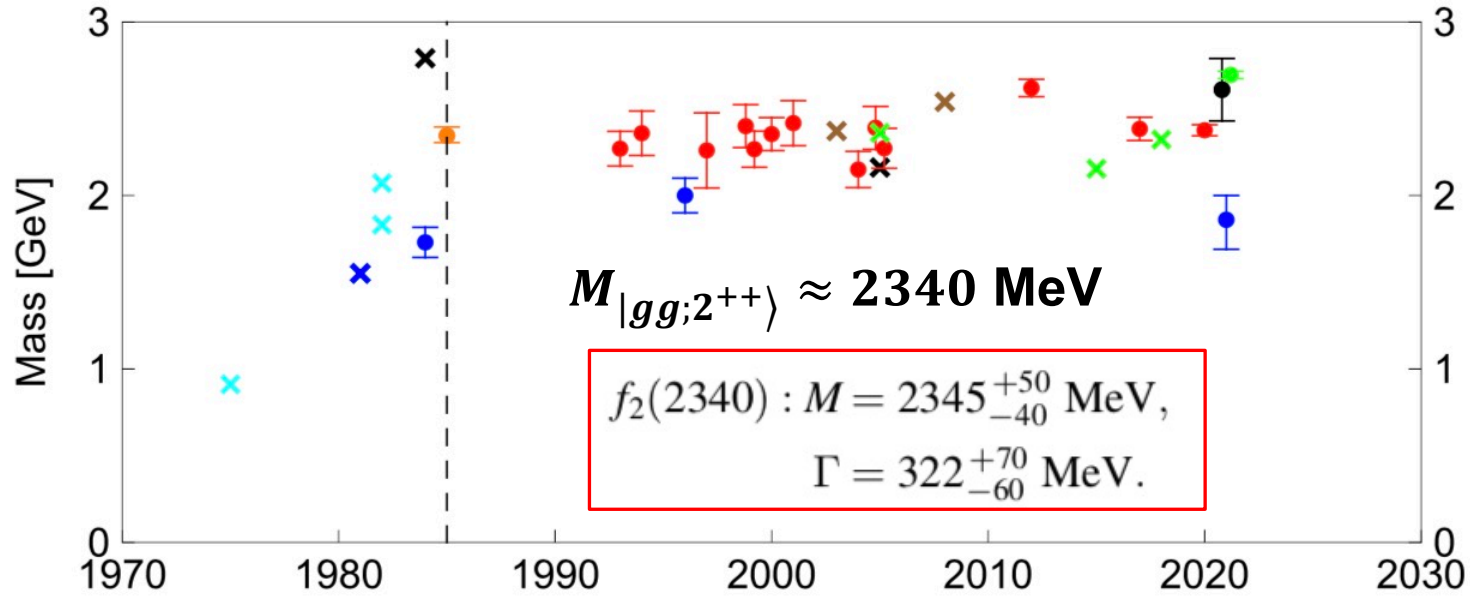
**$M \approx 1865 \text{ MeV}$**

**$\Gamma \approx 370 \text{ MeV}$**



# Tensor glueball:

H.X.Chen et. al., RPP86(2023) 026201



$$\mathcal{B}(J/\psi \rightarrow \gamma f_2(2340) \rightarrow \gamma K\bar{K}) = (5.54^{+0.34+3.82}_{-0.40-1.49}) \times 10^{-5},$$

BESIII's measurements

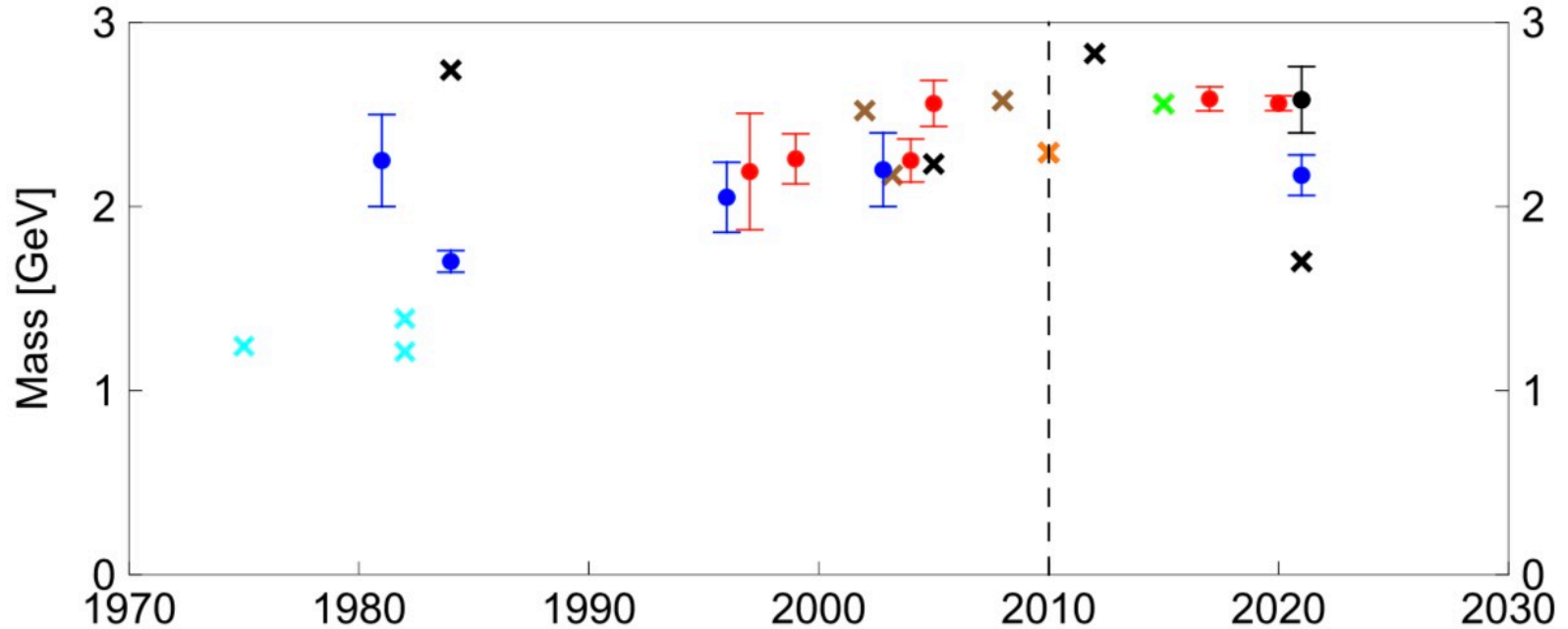
$$\mathcal{B}(J/\psi \rightarrow \gamma f_2(2340) \rightarrow \gamma \eta\eta) = (5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5},$$

$$\mathcal{B}(J/\psi \rightarrow \gamma f_2(2340) \rightarrow \gamma \phi\phi) = (1.91 \pm 0.14^{+0.72}_{-0.73}) \times 10^{-5},$$

$$\mathcal{B}(J/\psi \rightarrow \gamma |gg;2^{++}\rangle) = (1.1 \pm 0.2 \pm 0.1) \times 10^{-2}$$

# Pseudoscalar glueball:

RPP86(2023) 026201



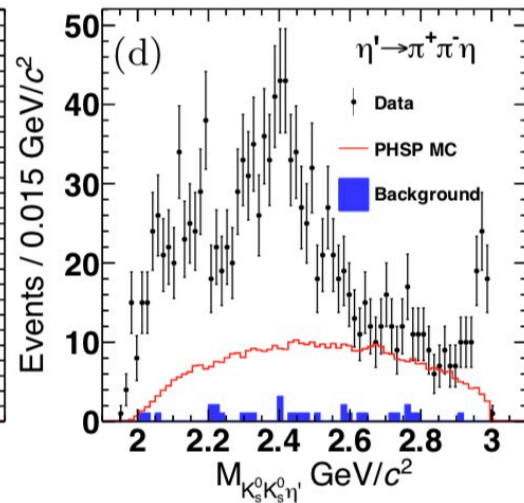
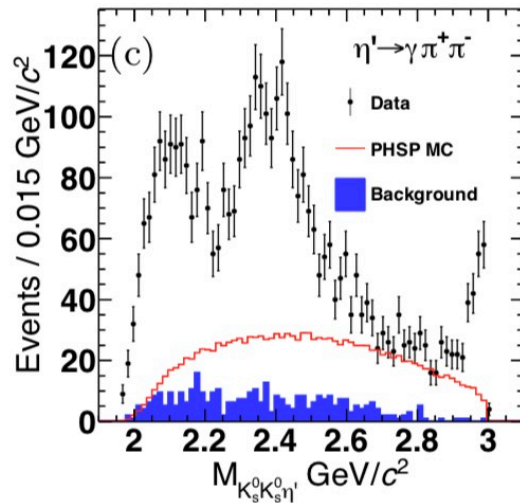
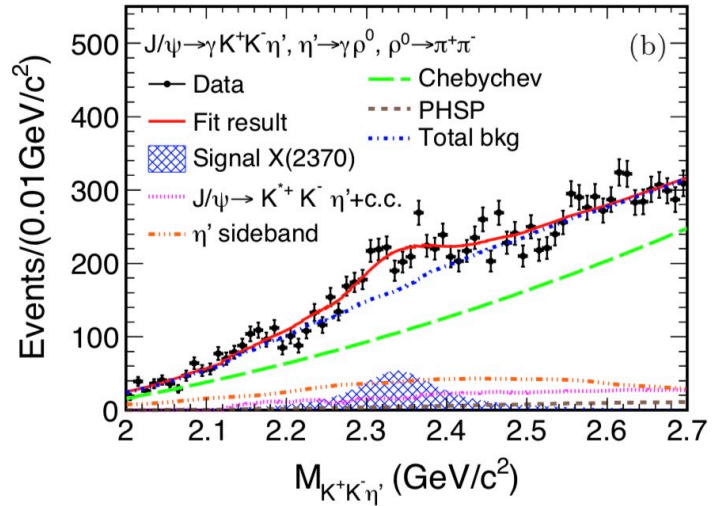
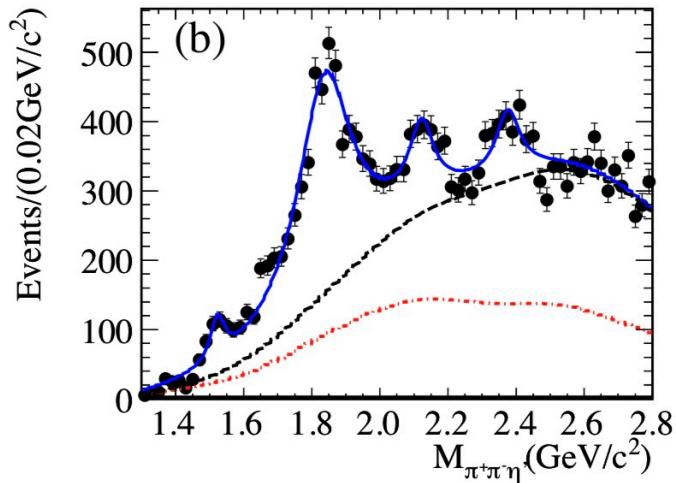
Average mass predictions:

$$M_{|gg;0^{-+}\rangle} \approx 2360 \text{ MeV}$$

# Pseudoscalar candidate X(2370): BESIII

PRL106(2011)072002:  $J/\psi \rightarrow \gamma\pi\pi\eta'$

EPJC80(2020)746:  $J/\psi \rightarrow \gamma K K \eta'$



PRL132(2024)181901:

$J/\psi \rightarrow \gamma K_s K_s \eta'$

$M = 2395 \pm 11^{+26}_{-94} \text{ MeV}$

$\Gamma = 188^{+18+124}_{-17-33} \text{ MeV}$

$J^{PC} = 0^{-+}$

## Production rates in the radiative $J/\psi$ decays:

$$\mathcal{B}(J/\psi \rightarrow \gamma X(2370) \rightarrow \gamma K^+ K^- \eta')$$

$$= (1.79 \pm 0.23 \pm 0.65) \times 10^{-5},$$

EPJC80(2020)746;  
PRD93(2016)112011

$$\mathcal{B}(J/\psi \rightarrow \gamma X(2370) \rightarrow \gamma K_S^0 K_S^0 \eta')$$

$$= (1.18 \pm 0.32 \pm 0.39) \times 10^{-5}.$$

$$\mathcal{B}(J/\psi \rightarrow \gamma X(2500) \rightarrow \gamma \phi \phi) = (1.7 \pm 0.2_{-0.8}^{+0.2}) \times 10^{-5}.$$

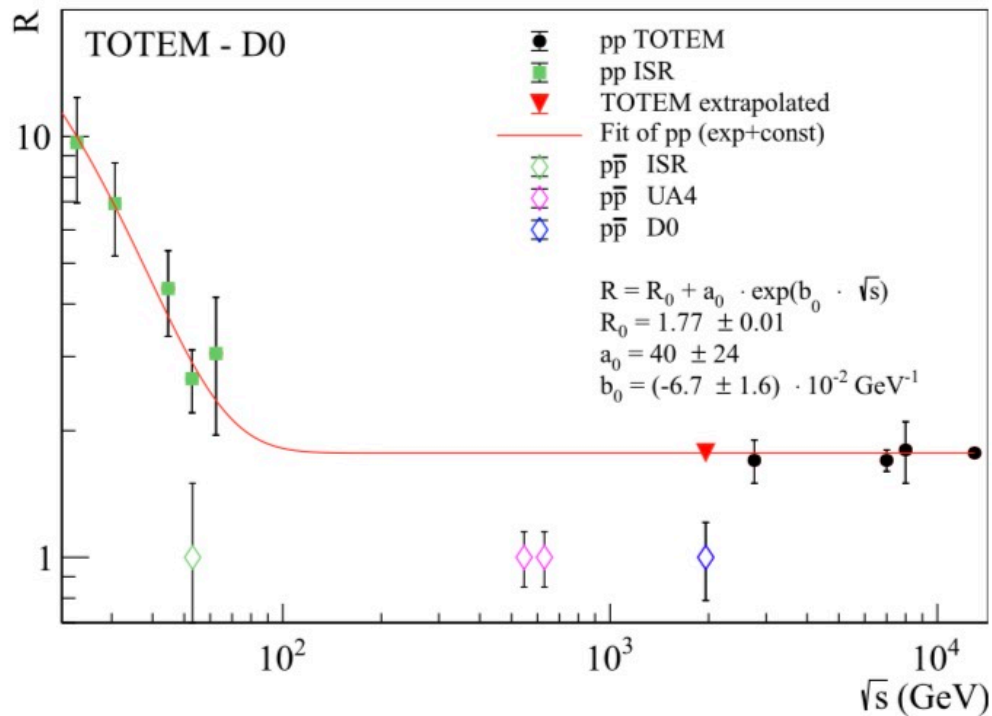
Close to the LQCD prediction of the pseudoscalar glueball

$$\mathcal{B}(J/\psi \rightarrow \gamma |gg; 0^{-+}\rangle) = (2.31 \pm 0.80) \times 10^{-4}.$$

PRD9100(2019)054511

## Evidence for a t-channel exchanged odderon

PRL127(2021)062003



The discrepancy suggests the existence of a t-channel exchanged odderon with  $C=-$ . **If exist, it should be a three-gluon glueball!**

# QCD sum rule calculations of glueballs

The relativistic two-gluon and three-gluon operators:

$$J_0 = g_s^2 G_a^{\mu\nu} G_{\mu\nu}^a,$$

$$\tilde{J}_0 = g_s^2 G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a,$$

$$J_1^{\alpha\beta} = g_s^2 G_a^{\alpha\mu} \tilde{G}_\mu^{a,\beta} - \{\alpha \leftrightarrow \beta\},$$

$$J_2^{\alpha_1\alpha_2,\beta_1\beta_2} = \mathcal{S}[g_s^2 G_a^{\alpha_1\beta_1} G^{a,\alpha_2\beta_2}],$$

$$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = \mathcal{S}[g_s^2 G_a^{\alpha_1\beta_1} \tilde{G}^{a,\alpha_2\beta_2}].$$

**C=+**

$$\eta_0 = f^{abc} g_s^3 G_a^{\mu\nu} G_{b,\nu\rho} G_{c,\mu}^\rho,$$

$$\tilde{\eta}_0 = f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} \tilde{G}_{b,\nu\rho} \tilde{G}_{c,\mu}^\rho,$$

$$\eta_1^{\alpha\beta} = f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta},$$

$$\tilde{\eta}_1^{\alpha\beta} = f^{abc} g_s^3 \tilde{G}_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta},$$

$$\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2} = f^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} G_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = f^{abc} \mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} \tilde{G}_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}].$$

**All spin-1 two- and three-gluon operators with C=+ vanish!**

Three-gluon operators:

C=-

$$\xi_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta},$$

$$\tilde{\xi}_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta},$$

$$\xi_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc} \mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} \tilde{G}_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$\tilde{\xi}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} G_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$\xi_3^{\dots} = d^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\beta_2} G_c^{\alpha_3\beta_3}],$$

$$\tilde{\xi}_3^{\dots} = d^{abc} \mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\beta_2} \tilde{G}_c^{\alpha_3\beta_3}],$$

PRD103 (2021) L091503;

PRD104(2021) 094050

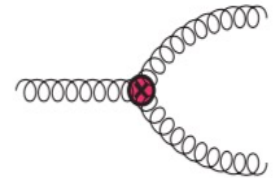
## Glueball sum rules:

Two-point correlation function:

$$\begin{aligned}\Pi^{\alpha\beta,\alpha'\beta'}(q^2) &\equiv i \int d^4x e^{iqx} \langle 0 | \mathbf{T} [J_1^{\alpha\beta}(x) J_1^{\alpha'\beta'\dagger}(0)] | 0 \rangle \\ &= (g^{\alpha\alpha'} g^{\beta\beta'} - g^{\alpha\beta'} g^{\beta\alpha'}) \Pi(q^2),\end{aligned}$$

Glueball field strength tensor

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_{b,\mu} A_{c,\nu},$$



Full gluon field propagator in the fixed point gauge

$$\begin{aligned}\langle 0 | \mathbf{T} [A_\mu^a(x) A_\nu^b(y)] | 0 \rangle &= \frac{\delta^{ab} g_{\mu\nu}}{4\pi^2 (x-y)^2} \\ &+ \frac{g_s \ln(-(x-y)^2)}{8\pi^2} f^{abc} G_{c,\mu\nu} \\ &- \frac{g_s g_{\mu\nu} x^\alpha y^\beta}{8\pi^2 (x-y)^2} f^{abc} G_{c,\alpha\beta}(0).\end{aligned}$$



OPEs are calculated up to  $d=8$  condensates



(a)



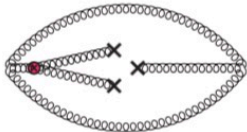
(b-1)



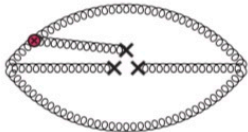
(b-2)



(c-1)



(c-2)



(c-3)



(c-4)



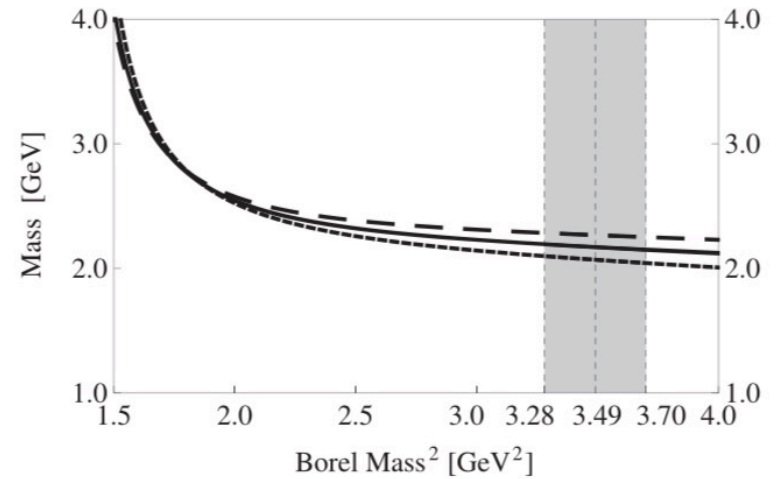
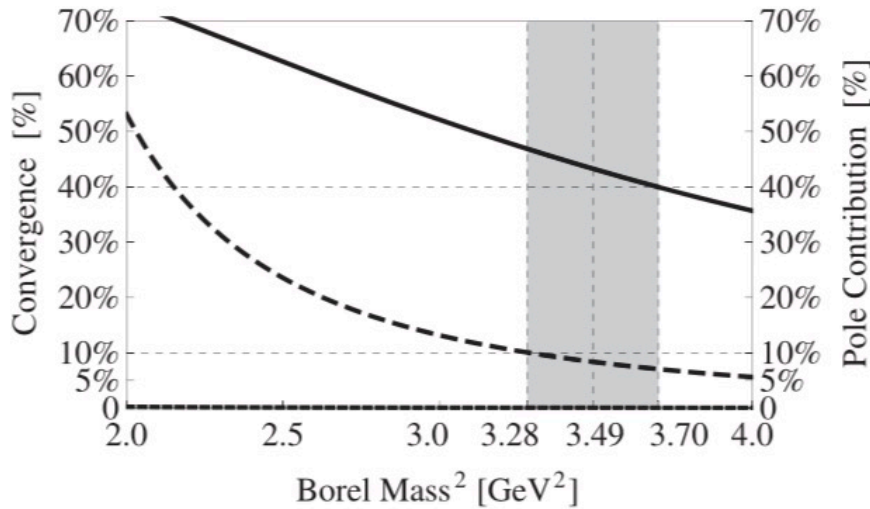
(c-5)



(d)

# Pseudoscalar glueball

PRD104(2021) 094050



Mass prediction:

$$M_{|GG;0^{-+}\rangle} = 2.17 \pm 0.11 \text{ GeV.}$$

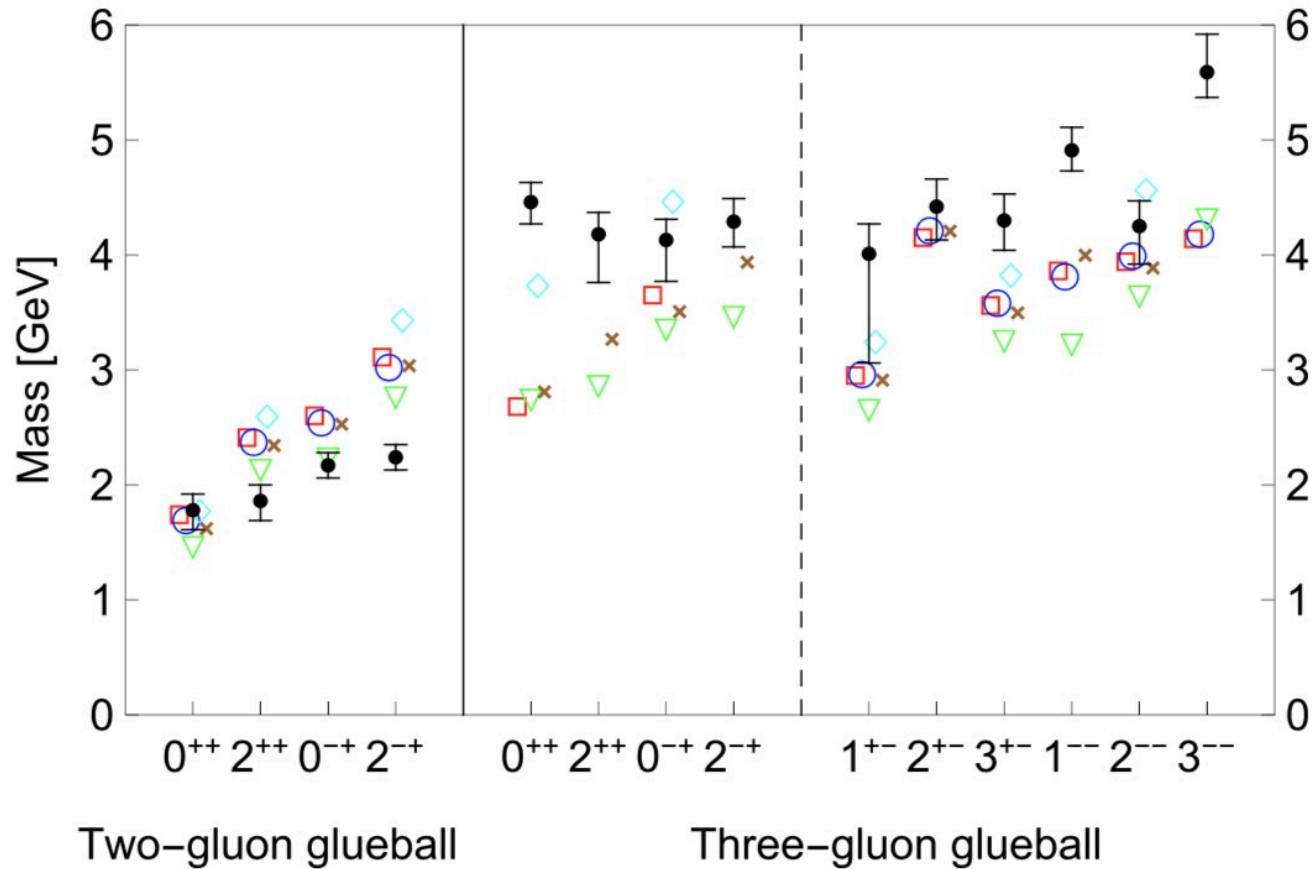
# Glueball mass spectrum

Glueball	Current	$s_0^{\min}$ [GeV <sup>2</sup> ]	Working Regions			Mass [GeV]
			$s_0$ [GeV <sup>2</sup> ]	$M_B^2$ [GeV <sup>2</sup> ]	Pole [%]	
$ \text{GG}; 0^{++}\rangle$	$J_0$	7.8	$9.0 \pm 1.0$	3.70–4.19	40–48	$1.78^{+0.14}_{-0.17}$
$ \text{GG}; 2^{++}\rangle$	$J_2^{\alpha_1\alpha_2\beta_1\beta_2}$	8.5	$10.0 \pm 1.0$	3.99–4.60	40–50	$1.86^{+0.14}_{-0.17}$
$ \text{GG}; 0^{-+}\rangle$	$\tilde{J}_0$	8.2	$9.0 \pm 1.0$	3.28–3.70	40–47	$2.17^{+0.11}_{-0.11}$
$ \text{GG}; 2^{-+}\rangle$	$\tilde{J}_2^{\alpha_1\alpha_2\beta_1\beta_2}$	8.1	$10.0 \pm 1.0$	3.27–4.20	40–55	$2.24^{+0.11}_{-0.11}$
$ \text{GGG}; 0^{++}\rangle$	$\eta_0$	31.6	$33.0 \pm 3.0$	7.25–7.61	40–44	$4.46^{+0.17}_{-0.19}$
$ \text{GGG}; 2^{++}\rangle$	$\eta_2^{\alpha_1\alpha_2\beta_1\beta_2}$	16.0	$35.0 \pm 3.0$	4.77–9.04	40–90	$4.18^{+0.19}_{-0.42}$
$ \text{GGG}; 0^{-+}\rangle$	$\tilde{\eta}_0$	17.0	$33.0 \pm 3.0$	4.48–8.13	40–88	$4.13^{+0.18}_{-0.36}$
$ \text{GGG}; 2^{-+}\rangle$	$\tilde{\eta}_2^{\alpha_1\alpha_2\beta_1\beta_2}$	33.1	$35.0 \pm 3.0$	8.10–8.53	40–44	$4.29^{+0.20}_{-0.22}$
$ \text{GGG}; 1^{+-}\rangle$	$\xi_1^{\alpha\beta}$	9.0	$34.0 \pm 4.0$	3.16–9.09	40–99	$4.01^{+0.26}_{-0.95}$
$ \text{GGG}; 2^{+-}\rangle$	$\xi_2^{\alpha_1\alpha_2\beta_1\beta_2}$	32.7	$35.0 \pm 4.0$	7.53–8.09	40–46	$4.42^{+0.24}_{-0.29}$
$ \text{GGG}; 3^{+-}\rangle$	$\xi_3^{\alpha_1\alpha_2\alpha_3\beta_1\beta_2\beta_3}$	30.2	$33.0 \pm 4.0$	7.69–8.40	40–47	$4.30^{+0.23}_{-0.26}$
$ \text{GGG}; 1^{--}\rangle$	$\tilde{\xi}_1^{\alpha\beta}$	31.2	$34.0 \pm 4.0$	5.81–6.77	40–51	$4.91^{+0.20}_{-0.18}$
$ \text{GGG}; 2^{--}\rangle$	$\tilde{\xi}_2^{\alpha_1\alpha_2\beta_1\beta_2}$	19.7	$36.0 \pm 4.0$	5.80–9.47	40–81	$4.25^{+0.22}_{-0.33}$
$ \text{GGG}; 3^{--}\rangle$	$\tilde{\xi}_3^{\alpha_1\alpha_2\alpha_3\beta_1\beta_2\beta_3}$	35.8	$38.0 \pm 4.0$	6.15–7.22	40–49	$5.59^{+0.33}_{-0.22}$

PRD103 (2021) L091503;

PRD104(2021) 094050

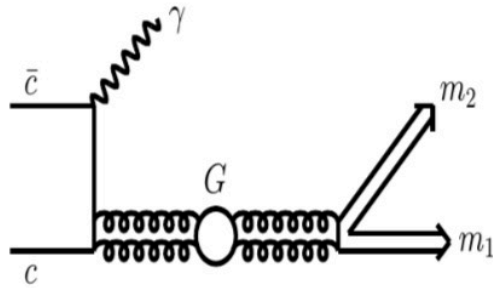
# Comparing to LQCD's results



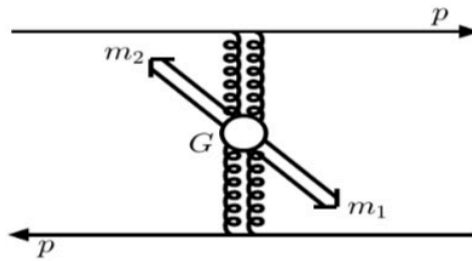
RPP86(2023) 026201

# Productions and decays

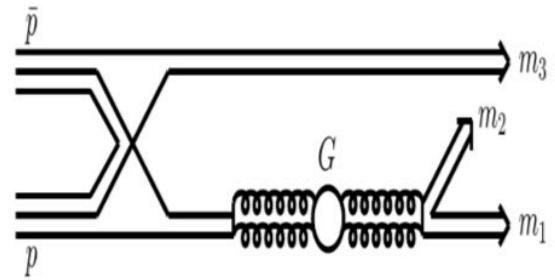
*Phys. Rept.*, 454:1–202, 2007



*J/ψ* radiative decay

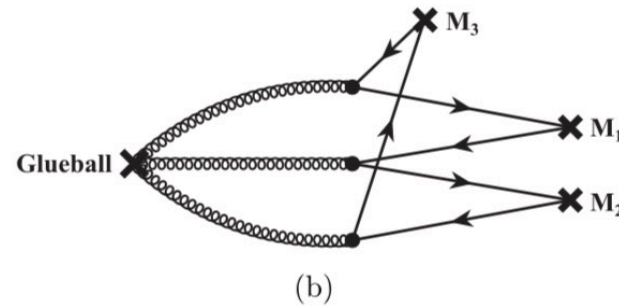
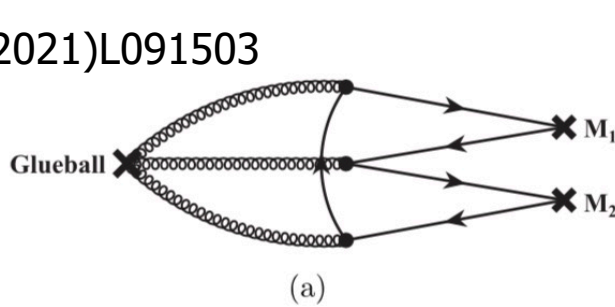


double pomeron exchange



*p* $\bar{p}$  annihilation

PRD103(2021)L091503



$0^- \rightarrow$	$PPP, VVP, VVV$	(S-wave)	&	$VP, VV$	(P-wave),
$0^+ \rightarrow$	$VPP, VVP, VVV$	(P-wave)	&	$PP, VV$	(S-wave),
$1^- \rightarrow$	$VPP, VVP, VVV$	(S-wave)	&	$PP, VP, VV$	(P-wave),
$1^+ \rightarrow$	$PPP, VPP, VVP, VVV$	(P-wave)	&	$VP, VV$	(S-wave),
$2^- \rightarrow$	$VVP, VVV$	(S-wave)	&	$VP, VV$	(P-wave),
$2^+ \rightarrow$	$VPP, VVP, VVV$	(P-wave)	&	$VV$	(S-wave),
$3^- \rightarrow$	$VVV$	(S-wave)	&	$VV$	(P-wave),
$3^+ \rightarrow$	$VVP, VVV$	(P-wave)	&	$VP, VV$	(D-wave).



# Summary

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- The existence of glueball is one of the most distinctive prediction of QCD, and essential to the confirmation of the theory!
- **We systematically calculated the mass spectra of the two-gluon and three-gluon glueballs.**
- **The ground state of spin-1 glueballs with  $C=+$  do not exist in the relativistic framework.**
- Some gluon-rich processes are expected to detect glueballs.

Thank you