Generalized Bell-Clauser-Horne Inequalities and Quantum Nonlocality in Spin-Correlated Decays

Yang-Guang Yang (杨阳光)

Institute of Modern Physics, Chinese Academy of Sciences

Light-Cone 2024

Nov 25 – 29, 2024

Huizhou, Guangdong, China

Collaborators: Chen Qian, Cong-Feng Qiao, Qun Wang, Si-Hao Wu, Si-Qi Xu, Xing-Bo Zhao

Outline

- **Quantum Entanglement, Nonlocality and Bell-CH Inequalities**
- **Quantum Measurement Description of Hyperon Decays**
- **Finding New Methods of Constructing Bell-CH Inequalities**
- **Generalized New Class of Bell-CH Inequalities**
- **Numerical Results**
- **Summary**

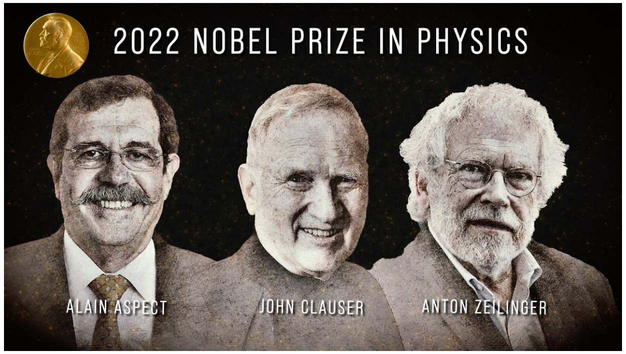
Quantum Entanglement and Nonlocality

- ✓ Quantum Entanglement: Phenomenon where two or more quantum systems exhibit correlations regardless of the distance between them. The measurement outcomes are interconnected, defying classical separability.
- ✓ Quantum Nonlocality: Phenomenon exceeding classical local realism, suggesting 'nonlocal' influence.
- ✓ Quantum Entanglement vs Quantum Nonlocality: Entanglement describes quantum correlations, while nonlocality means these correlations defy local realism.
- ✓ Bell Inequality: A Test for Quantum vs Classical Correlations.

<u>Historical Background and Experimental Verification:</u>

- ➢ EPR Paradox & Hidden Variable Theory (1935): Einstein questioned whether quantum mechanics provides a complete description of physical reality. A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935)
- ➢ Bell's Theorem (1964): Quantum physics is incompatible with local hidden variable theories (LHVT) using the famous Bell inequality. J. S. Bell, Physics 1, 195 (1964)
- ➢ Real-World Bell Experiments (1972-now): Finding violations of Bell inequalities which supported quantum mechanics, ruled out another potential explanation for entanglement, etc.

The Nobel Prize in Physics 2022 was awarded "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science" to be shared jointly between Alain Aspect, John Clauser, and Anton Zeilinger.



Quantum Entanglement and Nonlocality

- In quantum physics and quantum information theory, Bell inequalities probe entanglement between spatially-separated systems

✓ **Bell Inequality** J. S. Bell, Physics 1, 195 (1964)

✓ **Clauser-Horne-Shimony-Holt (Bell-CHSH) Inequality**

✓ **Clauser-Horne (Bell-CH) Inequality**

Spatial Correlation

J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969)

J. F. Clauser and M. A. Horne, Phys. Rev. D 10, 526 (1974)

- In contrast to Bell inequalities, Leggett-Garg inequalities test the correlations of a single system measured at different times

✓ **Leggett-Garg Inequality**

✓ **Temporal Bell-like Inequality**

Temporal Correlation

A. J. Leggett and A. Garg, Phys. Rev. Lett. 54, 857 (1985)

Bell Nonlocality ⊂ Quantum Entanglement

Quantum Measurement Description of Hyperon Decays

➢ **Qubit: Fundamental two-level quantum system for information encoding**

✓ **Spin-1/2 hyperons serve as effective qubits in particle physics, as their spin states can be observed through weak decay measurements**

a typical weak decay process $B \to B'M\ (\bar{B} \to \bar{B}'\bar{M})$

$$\frac{dN}{d\cos\theta} \propto 1 + \alpha \mathcal{P}_B \cos\theta$$

weak decay rules

➢ **Hyperon decay as quantum measurement**

✓ **Quantum measurement postulate: The post-measurement state can be taken as a quantum evolution generated by the measurement**

$$P(\boldsymbol{p}) = \mathrm{Tr}\left(M_{\boldsymbol{p}} \rho_B M_{\boldsymbol{p}}^{\dagger}\right) = \frac{1}{4\pi}\left(1 + \alpha_B \boldsymbol{s}_B \cdot \hat{\boldsymbol{p}}\right)$$

measurement operator

$$M_{\boldsymbol{p}} = \frac{S + P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}}{\sqrt{4\pi\left(|S|^2 + |P|^2\right)}}$$

S-wave + P-wave

T.-D. Lee and C.-N. Yang, Phys. Rev. 108, 1645 (1957)

Quantum Measurement Description of Hyperon Decays

$$B\bar{B} \to B'M\bar{B}'\bar{M}$$

typical joint decay with spin correlation

described by spin density operator

spin state of two spin-1/2 particles

$$\rho_{B\bar{B}} = \frac{1}{4}\left(1 + \boldsymbol{s}_B \cdot \boldsymbol{\sigma} \otimes 1 + 1 \otimes \boldsymbol{s}_{\bar{B}} \cdot \boldsymbol{\sigma} + \sum_{ij} C_{ij}\sigma_i \otimes \sigma_j\right)$$

$$\boldsymbol{s}_B = \langle \boldsymbol{\sigma} \otimes 1 \rangle$$

$$\boldsymbol{s}_{\bar{B}} = \langle 1 \otimes \boldsymbol{\sigma} \rangle$$

$$C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$$

$$\rho_B = Tr_{\bar{B}}(\rho_{B\bar{B}}) = \frac{1}{2}(1 + \boldsymbol{s}_B \cdot \boldsymbol{\sigma})$$

$$\rho_{\bar{B}} = Tr_B(\rho_{B\bar{B}}) = \frac{1}{2}(1 + \boldsymbol{s}_{\bar{B}} \cdot \boldsymbol{\sigma})$$

The one particle density operator can be obtained by taking partial trace

$$P(\boldsymbol{p}, \bar{\boldsymbol{p}}) = Tr\left[(M_{\boldsymbol{p}} \otimes M_{\bar{\boldsymbol{p}}})\,\rho_{B\bar{B}}\left(M_{\boldsymbol{p}}^\dagger \otimes M_{\bar{\boldsymbol{p}}}^\dagger\right)\right]$$

quantum measurement postulate

A joint decay process can be regarded as parallel quantum measurement which gives the joint probability

S.-H. Wu, C. Qian, YGY and Q. Wang, arxiv: 2402.16574 (2024)

Bell & CHSH Inequality

- Bell Inequality: Violation of Bell inequality proves quantum entanglement cannot be explained by local hidden variables.

J. S. Bell, Physics 1, 195 (1964)

Bohm-EPR: Bell/EPR state

$$|E(\vec{a},\vec{b}) - E(\vec{a},\vec{c})| - E(\vec{b},\vec{c}) \leq 1$$ Bell version

$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B\right)$$

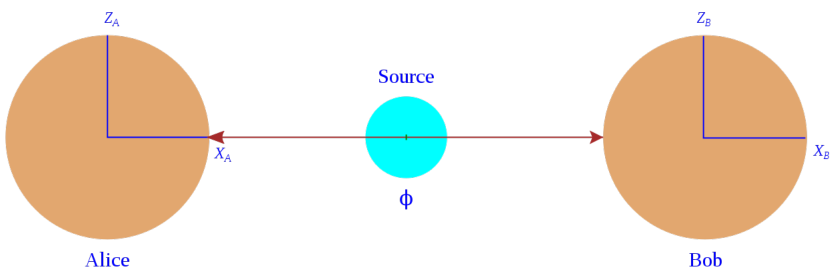
- Clauser-Horne-Shimony-Holt (CHSH) Inequality: Applies to a wide range of entangled states, including antisymmetrized and photon entangled states.

CHSH version

$$|E(\vec{a}_1,\vec{b}_1) - E(\vec{a}_1,\vec{b}_2)| + E(\vec{a}_2,\vec{b}_1) + E(\vec{a}_2,\vec{b}_2) \leq 2$$

J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969)

error of detection is taken into account



hidden variable λ

E(x, y): expected value

$$\text{LHVT:} \quad E(\vec{a},\vec{b}) = \int d\lambda \rho(\lambda) A(\vec{a},\lambda) B(\vec{b},\lambda)$$

$$\text{QM:} \quad E(\vec{a},\vec{b}) = \langle\psi|\sigma_1\cdot\vec{a}\otimes\sigma_2\cdot\vec{b}|\psi\rangle$$

probability density $\int d\lambda\, \rho(\lambda) = 1$

$0 \leqslant \rho(\lambda) \leqslant 1$

QM will violate Bell & CHSH inequality in some parameter ranges !

Bell-CH Inequalities

- Clauser-Horne (CH) Inequality: Accounts for detector inefficiencies in experiments.

$$I_2 = Prob(\vec{a}_1, \vec{b}_1) + Prob(\vec{a}_1, \vec{b}_2) + Prob(\vec{a}_2, \vec{b}_1) - Prob(\vec{a}_2, \vec{b}_2) - Prob(\vec{a}_1) - Prob(\vec{b}_1),$$

original CH inequality

J. F. Clauser and M. A. Horne, Phys. Rev. D 10, 526 (1974)

A. Fine, Phys. Rev. Lett. 48, 291 (1982)

probability value

$Prob(\vec{a_i}, \vec{b_j})$

- Bell-CH Inequalities: Accounts for more measurement settings.

$$I_3 = Prob(\vec{a}_1, \vec{b}_1) + Prob(\vec{a}_1, \vec{b}_2) + Prob(\vec{a}_1, \vec{b}_3) + Prob(\vec{a}_2, \vec{b}_1) + Prob(\vec{a}_2, \vec{b}_2) - Prob(\vec{a}_2, \vec{b}_3) + Prob(\vec{a}_3, \vec{b}_1) - Prob(\vec{a}_3, \vec{b}_2) - Prob(\vec{a}_1) - 2Prob(\vec{b}_1) - Prob(\vec{b}_2),$$

$$I_4 = Prob(\vec{a}_1, \vec{b}_1) + Prob(\vec{a}_1, \vec{b}_2) + Prob(\vec{a}_1, \vec{b}_3) + Prob(\vec{a}_1, \vec{b}_4) + Prob(\vec{a}_2, \vec{b}_1) + Prob(\vec{a}_2, \vec{b}_2) + Prob(\vec{a}_2, \vec{b}_3) - Prob(\vec{a}_2, \vec{b}_4) + Prob(\vec{a}_3, \vec{b}_1) + Prob(\vec{a}_3, \vec{b}_2) - Prob(\vec{a}_3, \vec{b}_3) + Prob(\vec{a}_4, \vec{b}_1) - Prob(\vec{a}_4, \vec{b}_2) - Prob(\vec{a}_1) - 3Prob(\vec{b}_1) - 2Prob(\vec{b}_2) - Prob(\vec{b}_3),$$

Bell-CH inequalities, e.g. for 3-4 measurement settings

Two advantages of Bell-CH inequalities:

- Easily tested in physical experiment
- Mathematical structures as building blocks

M. Froissart, Nuovo Cimento Soc. Ital. Fis. B 64, 241 (1981)

A. Garg and N. D. Mermin, Phys. Rev. Lett. 49, 1220 (1982)

D. Collins and N. Gisin, J. Phys. A: Math. Gen. 37, 1775 (2004)

QM also violate Bell-CH inequalities

For any local realistic theories $I_i \leq 0$ has to be hold $(i = 2, 3, 4)$.

Real-world Bell-CH Inequalities' experiments involve event rates and allows for testing the validity of quantum nonlocality without relying on perfect detector efficiency assumptions.

New Method of Constructing Bell-CH Inequalities

New method I: Rearrangement inequalities

$$I_2 = x_1(y_1+y_2) + x_2(y_1-y_2) - x_1 Y - y_1 X$$

$$0 \le x_- \le x_1, \dots, x_m \le x_+ \le X$$
$$0 \le y_- \le y_1, \dots, y_n \le y_+ \le Y$$

$$\Rightarrow \quad I_2 \leqslant I_2^{(0)} \leqslant 0 \qquad I_2^{(0)} = -(x_1 y_1 + x_2 y_2) + x_+ y_- + x_- y_+$$

Alice: 2 measurements & Bob: 2 measurements

$$x_+ \equiv \max\{x_1, \dots, x_m\} \qquad y_+ \equiv \max\{y_1, \dots, y_n\}$$
$$x_- \equiv \min\{x_1, \dots, x_m\} \qquad y_- \equiv \min\{y_1, \dots, y_n\}$$

$$I_3 = x_1(y_2+y_3) + x_2(y_1+y_3) + x_3(y_1+y_2) - x_2 y_2 - x_3 y_3 - (x_1+x_2)Y - (y_1+y_2)X$$

$$\Rightarrow \quad I_3 \leqslant I_3^{(0)} \leqslant 0 \qquad I_3^{(0)} = -(x_1 y_1 + x_2 y_2 + x_3 y_3) + x_+ y_- + x_- y_+ + (x_1 + x_2 + x_3 - x_+ - x_-)(y_1 + y_2 + y_3 - y_+ - y_-)$$

Alice: 3 measurements & Bob: 3 measurements

C. Qian, YGY, Q. Wang and C.-F. Qiao, Phys. Rev. A 103, 062203 (2021)

New Method of Constructing Bell-CH Inequalities

New method II: Linear inequalities

a *special CH type inequalities family*

$$I_{mm}(x_1,\cdots,x_m|y_1,\cdots,y_m)=\sum_{j=1}^{m}\sum_{i=1}^{m+1-j}x_iy_j-\sum_{i=2}^{m}x_iy_{m+2-i}-\sum_{i=1}^{m-1}(m-i)x_iY-y_1X$$

$$I_{mm}(x_1,\cdots,x_m|y_1,\cdots,y_m)\leqslant 0$$

43-Type

$$\begin{array}{c|ccc} & -1 & 0 & 0 \\ \hline -2 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ 0 & 1 & -1 & -1 \end{array} \leqslant \begin{array}{c|} \theta \\ \hline 1 \\ 1 \\ -1 \\ -1 \end{array} \begin{array}{c|ccc} & -1 & 0 & 0 \\ \hline -1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ -2 & 1 & 1 & 0 \\ -1 & 1 & -1 & 0 \end{array} + \begin{array}{c|} \theta \\ \hline -1 \\ -1 \\ 1 \\ 1 \end{array} \begin{array}{c|ccc} & -1 & 0 & 0 \\ \hline -2 & 1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}$$

$$\begin{array}{c|ccc} & -2 & -1 & 0 \\ \hline -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{array} \leqslant \begin{array}{c|ccc} \theta & 1 & -1 & 0 \\ \hline \end{array} \begin{array}{c|ccc} & -1 & -2 & 0 \\ \hline -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{array} + \begin{array}{c|ccc} \theta & -1 & 1 & 0 \\ \hline \end{array} \begin{array}{c|ccc} & -2 & -1 & 0 \\ \hline -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{array}$$

$$\begin{array}{c|ccc} & -1 & -1 & 0 \\ \hline -2 & 2 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & -1 \end{array} \leqslant \begin{array}{c|ccc} \theta & 0 & 1 & -1 \\ \hline \end{array} \begin{array}{c|ccc} & -1 & 0 & -1 \\ \hline -2 & 2 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \end{array} + \begin{array}{c|ccc} \theta & 0 & 1 & -1 \\ \hline \end{array} \begin{array}{c|ccc} & -1 & -1 & 0 \\ \hline -2 & 2 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \end{array}$$

Graphical construction of Bell-CHSH inequalities

$$\begin{array}{c|ccc} & C_{y_1} & \cdots & C_{y_n} \\ \hline C_{x_1} & C_{x_1y_1} & \cdots & C_{x_1y_n} \\ \cdots & \cdots & \cdots & \cdots \\ C_{x_m} & C_{x_my_1} & \cdots & C_{x_my_n} \end{array} \qquad \begin{array}{c|} \theta \\ \hline C_{x_1} \\ \cdots \\ C_{x_m} \end{array} \Rightarrow \theta(\textstyle\sum_{i=1}^{m} C_{x_i}x_i)$$

$$\theta(\textstyle\sum_{j=1}^{n} C_{y_j}y_j) \Leftarrow \begin{array}{c|ccc} \theta & C_{y_1} & \cdots & C_{y_n} \\ \hline \end{array}$$

$$Y\sum_{i=1}^{m}C_{x_i}x_i+X\sum_{j=1}^{n}C_{y_j}y_j+\sum_{i=1}^{m}\sum_{j=1}^{n}C_{x_iy_j}x_iy_j$$

22-Type, 33-Type

$$\begin{array}{c|cc} & -1 & 0 \\ \hline -1 & 1 & 1 \\ 0 & 1 & -1 \end{array} \leqslant \begin{array}{c|} \theta \\ \hline 1 \\ -1 \end{array} \begin{array}{c|cc} & -1 & 0 \\ \hline 0 & 1 & 0 \\ -1 & 1 & 0 \end{array} + \begin{array}{c|} \theta \\ \hline -1 \\ 1 \end{array} \begin{array}{c|cc} & -1 & 0 \\ \hline -1 & 1 & 0 \\ 0 & 1 & 0 \end{array}$$

$$\begin{array}{c|ccc} & -1 & 0 & 0 \\ \hline -2 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \end{array} \leqslant \begin{array}{c|} \theta \\ \hline 1 \\ -1 \\ 0 \end{array} \begin{array}{c|ccc} & -1 & 0 & 0 \\ \hline -1 & 1 & 1 & 0 \\ -2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array} + \begin{array}{c|} \theta \\ \hline -1 \\ 1 \\ 0 \end{array} \begin{array}{c|ccc} & -1 & 0 & 0 \\ \hline -2 & 1 & 1 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}$$

More than 100 pages' proofs of 257 Bell-CH inequalities!

C. Qian, YGY, Q. Wang and C.-F. Qiao, Phys. Rev. A 103, 062203 (2021)

New Class of Bell-CH Inequalities

$$I_{2,CH} = P(x_1,y_1)+P(x_1,y_2)+P(x_2,y_1)-P(x_2,y_2)-P(x_1)-P(y_1) \leqslant 0$$

CH inequality

$$I_{2,CH} \leqslant I_{2,CH}^{(0)} \leqslant 0$$

original inequality

$$I_{2,CH}^{(0)} = -\frac{1}{2}\int \{[P(x_1,\lambda)-P(x_2,\lambda)][P(y_1,\lambda)-P(y_2,\lambda)] + |[P(x_1,\lambda)-P(x_2,\lambda)][P(y_1,\lambda)-P(y_2,\lambda)]|\}\,\rho(\lambda)d\lambda$$

based on rearrangement inequalities method

Generalized Bell-CH inequality based on rearrangement inequality

$$I_{2,CH} \leqslant I_{2,CH}^{(0)} \leqslant -\frac{1}{2}\{[P(x_1,y_1)-P(x_1,y_2)-P(x_2,y_1)+P(x_2,y_2)] + |P(x_1,y_1)-P(x_1,y_2)-P(x_2,y_1)+P(x_2,y_2)|\}$$

tighter than original CH inequality with LHVT

new CH type inequality

The new inequality may play a powerful role in specific scenarios!

C. Qian, YGY, Q. Wang and C.-F. Qiao, Phys. Rev. A 103, 062203 (2021)

New Class of Bell-CH Inequalities

a *special CH type inequalities family* $I_{mm}(x_1,\cdots,x_m|y_1,\cdots,y_m)=\sum_{j=1}^{m}\sum_{i=1}^{m+1-j}x_iy_j-\sum_{i=2}^{m}x_iy_{m+2-i}-\sum_{i=1}^{m-1}(m-i)x_iY-y_1X$

$$I_{mm}(x_1,\cdots,x_m|y_1,\cdots,y_m)\leqslant 0$$

original corresponding one

new class of CH type inequalities

based on linear inequalities method

$$\max\left\{I^{(1)}_{k-1,k-1;Q}+\sum_{i=1}^{k-1}[P(x_2,y_i)-P(x_2)],\, I^{(2)}_{k-1,k-1;Q}+\sum_{i=1}^{k-1}[P(x_1,y_i)-P(x_1)]\right\}\leqslant 0,$$

$$I^{(1)}_{k-1,k-1;Q}\equiv\sum_{j=1}^{k-1}\sum_{i=1,i\neq 2}^{k-j}P(x_i,y_j)-\sum_{i=3}^{k-1}P(x_i,y_{k+1-i})-\sum_{i=1,i\neq 2}^{k-2}(k-1-i)P(x_i)-P(y_1),$$

$$I^{(2)}_{k-1,k-1;Q}\equiv\sum_{j=1}^{k-1}\sum_{i=2}^{k-j}P(x_i,y_j)-\sum_{i=3}^{k-1}P(x_i,y_{k+1-i})-\sum_{i=2}^{k-2}(k-1-i)P(x_i)-P(y_1).$$

the new Bell-CH inequalities have less measurement settings than original ones

Generalized Bell-CH inequalities based on linear inequalities

C. Qian, YGY, Q. Wang and C.-F. Qiao, Phys. Rev. A 103, 062203 (2021)

Robust with Resistance to Noise

resistance to noise $\lambda_{\max}$

$$\rho = \lambda|\psi(\theta_{\max})\rangle\langle\psi(\theta_{\max})| + (1-\lambda)\frac{I}{4}$$

parameterized quantum states

$$|\psi(\theta)\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$$

No.	Name	μ_{max}	μ^a_{max}	μ^b_{max}
1	I_{3322}	0.8	0.7836	-
2	I^4_{4422}	0.9728	0.864	0.7071
3	I^{14}_{4422}	0.8298	0.8034	0.7981
4	I^{16}_{4422}	0.8791	0.8691	0.8579
5	I^{18}_{4422}	0.9508	0.9096	0.7863
6	J^3_{4422}	0.838	0.8267	-

No.	Name	μ_{max}	μ^a_{max}	μ^b_{max}
24	N^{10}_{4422}	0.8333	0.822	-
25	A_{10}	0.8082	0.7918	-
26	A_{11}	0.7933	0.7918	-
27	A_{13}	0.8128	0.7836	0.7836
28	A_{16}	0.8278	0.7751	0.7601
29	A_{34}	0.7956	0.7659	-

Numerical Test of the Generalized Bell-CH inequalities

Shown violated & robust with small resistance to noise
in parameterized quantum states

C. Qian, YGY, Q. Wang and C.-F. Qiao, Phys. Rev. A 103, 062203 (2021)

Tested 2-body Bell-CHSH inequality in joint decays

Test of the CHSH inequality through the spin correlation in the hyperon-antihyperon system

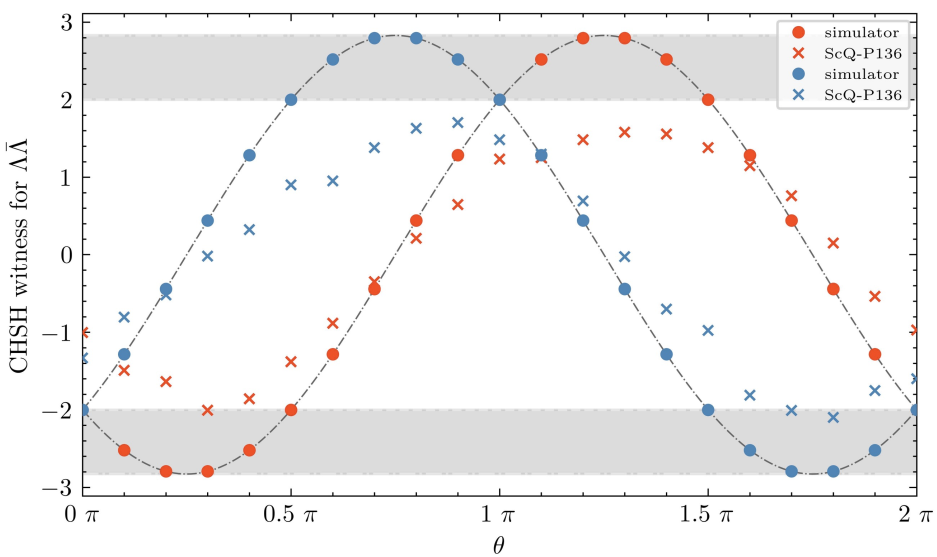
red $\langle \mathcal{B}_{\rm CHSH} \rangle_{\Psi^-} = \langle \hat{a}_1, \hat{b}_1 \rangle + \langle \hat{a}_1, \hat{b}_2 \rangle - \langle \hat{a}_2, \hat{b}_1 \rangle + \langle \hat{a}_2, \hat{b}_2 \rangle$

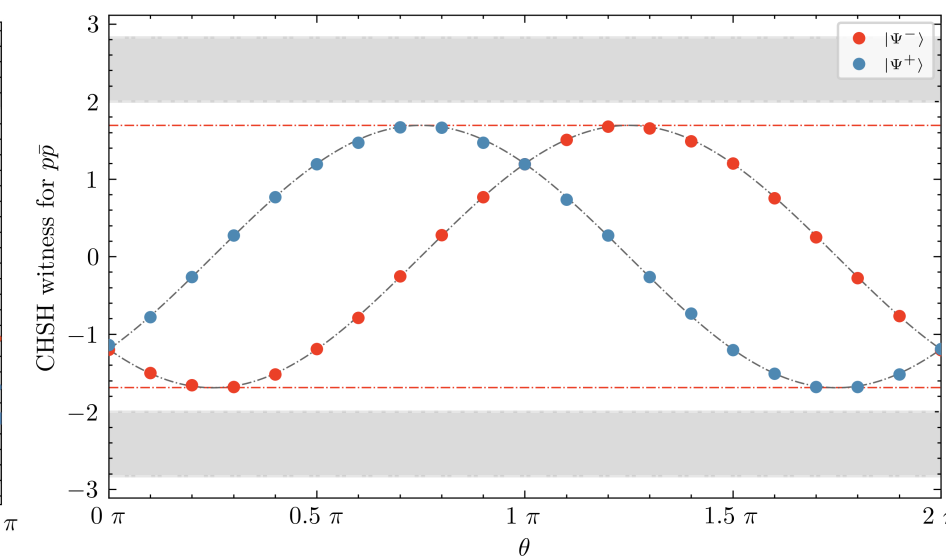
blue $\langle \mathcal{B}_{\rm CHSH} \rangle_{\Psi^+} = \langle \hat{a}_1, \hat{b}_1 \rangle + \langle \hat{a}_1, \hat{b}_2 \rangle + \langle \hat{a}_2, \hat{b}_1 \rangle - \langle \hat{a}_2, \hat{b}_2 \rangle$

$$\langle \mathcal{B}_{\rm CHSH} \rangle = \langle \boldsymbol{\sigma} \cdot \hat{a}_1 \otimes \boldsymbol{\sigma} \cdot \hat{b}_1 \rangle + \langle \boldsymbol{\sigma} \cdot \hat{a}_2 \otimes \boldsymbol{\sigma} \cdot \hat{b}_1 \rangle + \langle \boldsymbol{\sigma} \cdot \hat{a}_1 \otimes \boldsymbol{\sigma} \cdot \hat{b}_2 \rangle - \langle \boldsymbol{\sigma} \cdot \hat{a}_2 \otimes \boldsymbol{\sigma} \cdot \hat{b}_2 \rangle$$

spin-0 charmonia joint decay $\eta_c/\chi_{c0} \to \Lambda\bar{\Lambda} \to p\pi^- \bar{p}\pi^+$

quantum region $[-2\sqrt{2}, -2) \cup (2, 2\sqrt{2}]$





Simulation performed on simulators and the superconducting quantum computer **Quafu** developed in Beijing Academy of Quantum Information Sciences (BAQIS), the simulation results are in agreement with theoretical expectation that the spin correlation in decay daughters decreases in decay processes.

S.-H. Wu, C. Qian, YGY and Q. Wang, arxiv: 2402.16574 (2024)

Tested 3-body Bell-CH inequality among proton spin states

state of baryon $|\Psi\rangle = \varphi_{(qqq)}|qqq\rangle + \varphi_{(qqqg)}|qqqg\rangle + \varphi_{(qqqq\bar{q})}|qqqq\bar{q}\rangle \cdots$

light-front Hamiltonian

$$P^-_{QCD}(P^{\pm} = P^0 \pm P^3)$$

eigenvalue equation

$$P^-_{QCD}P^+|\Psi\rangle = M^2_\Psi|\Psi\rangle$$

Fock sectors with three valence quarks

$|qqq\rangle$ spin state

Basis light-front quantization (BLFQ) provides a relativistic and nonperturbative approach for solving many-body quantum problems in QFT

3-body Bell-CH inequalities

$$\begin{aligned}&-3P(\boldsymbol{a}_1)-2P(\boldsymbol{b}_1)-P(\boldsymbol{c}_1)+2P(\boldsymbol{a}_1,\boldsymbol{b}_1)+3P(\boldsymbol{a}_1,\boldsymbol{b}_2)+2P(\boldsymbol{a}_2,\boldsymbol{b}_1)\\&-2P(\boldsymbol{a}_2,\boldsymbol{b}_2)+P(\boldsymbol{a}_1,\boldsymbol{c}_1)+P(\boldsymbol{a}_2,\boldsymbol{c}_1)+P(\boldsymbol{b}_1,\boldsymbol{c}_1)+P(\boldsymbol{a}_1,\boldsymbol{b}_1,\boldsymbol{c}_1)-2P(\boldsymbol{a}_2,\boldsymbol{b}_1,\boldsymbol{c}_1)\\&+P(\boldsymbol{b}_2,\boldsymbol{c}_1)-3P(\boldsymbol{a}_1,\boldsymbol{b}_2,\boldsymbol{c}_1)+P(\boldsymbol{a}_2,\boldsymbol{b}_2,\boldsymbol{c}_1)+3P(\boldsymbol{a}_1,\boldsymbol{c}_1)-P(\boldsymbol{a}_2,\boldsymbol{c}_2)+2P(\boldsymbol{b}_1,\boldsymbol{c}_2)\\&-4P(\boldsymbol{a}_1,\boldsymbol{b}_1,\boldsymbol{c}_2)-P(\boldsymbol{a}_2,\boldsymbol{b}_1,\boldsymbol{c}_2)-2P(\boldsymbol{b}_2,\boldsymbol{c}_2)-P(\boldsymbol{a}_1,\boldsymbol{b}_2,\boldsymbol{c}_2)+3P(\boldsymbol{a}_2,\boldsymbol{b}_2,\boldsymbol{c}_2)\\&\leqslant 0\end{aligned}$$

spin entanglement among 3 quarks! ✓ Violation

C. Qian, S.-Q. Xu, X.-B. Zhao and YGY, to be appeared

Summary

- **Quantum Entanglement, Nonlocality & Bell-CH Inequalities:**

 - Quantum entanglement & nonlocality are fundamental to quantum mechanics.

 - Quantum nonlocality can experimentally validated through Bell-CH inequalities' violations, revealing non-local correlations beyond classical physics.

- **Generalized Quantum Measurement Framework for Hyperon Decays:**

 - A proposed quantum measurement framework analyzes spin-1/2 hyperon decays.

 - Applied to joint decay of correlated $\Lambda\bar{\Lambda}$ pairs.

- **Generalized Bell-CH Inequalities with New Inequality Derivation Methods:**

 - Using rearrangement and linear inequalities expand the class of Bell-CH inequalities, uncovering violations by specific quantum-entangled states.

 - Shown violated & robust with small resistance to noise in parameterized quantum states.

- **Tested 2-body Bell-CHSH inequality in joint decay of correlated $\Lambda\bar{\Lambda}$ pairs with quantum simulations.**
- **Tested 3-body Bell-CH inequality among proton spin states using wave function of proton based on BLFQ.**

Thank you very much!

Backup: Bounds of CH inequality

$$P(A_1,B_1)+P(A_1,B_2)+P(A_2,B_1)-P(A_2,B_2)-P(A_1)-P(B_1)\le 0$$