From Pion to DiPion LCDAs

Shan Cheng (程山)

Hunan University

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Overview

I LCDAs in the "hard" processes

II Pion LCDAs and form factors

III Dipion LCDAs and B_{l4} decays

IV Conclusion and Prospect

LCDAs in the "hard" processes

Emergent phenomena of QCD

QCD is believed to confine, that is, its physical states are color singlets with internal quark and gluon degrees of freedom

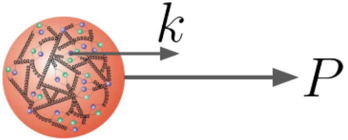
- QCD allow us to study hadron structures in terms of partons
- Factorization theorem to separate the hard partonic physics out of the hadronic physics (soft, nonperturbative objects)
- Define hadron structures by quantum field theories
- Identify theoretical observables in factorizable formulism

$$\frac{d\sigma}{d\Omega} = \int_x^1 \frac{d\zeta}{\zeta} \mathcal{H}(\zeta) f(\frac{x}{\zeta})$$

- The universal nonperturbative objects can be studied by QCD-based analytical (QCDSRs, χPT, instanton) and numerical approaches (LQCD)
- Also can be studied by performing global QCD analysis and fit, an inverse problem !
- CETQ, CT, MMHT, NNPDF, ABM, JAM, et.al.

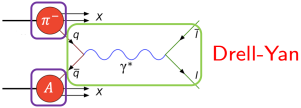
Pion PDF,TMD,GPD

Definitions of pion distribution



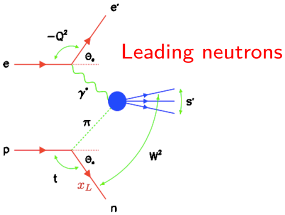
One dimension PDF

- △ $f_i(\zeta) = \int \frac{dz^-}{4\pi} e^{-i\zeta P^+ z^-} \langle \pi | \bar{\psi}_i(0, z^-, \mathbf{0}_T)\gamma^+ \psi_i(0) | \pi \rangle$
- △ $\zeta = \frac{k^+}{P^+}$, the parton momentum fraction
- △ $f_i(\zeta) \sim \sum_\alpha \int dk_T^2 \langle \pi | b^\dagger_{k,\alpha} b_{k,\alpha} (\zeta P^+, k_T, \alpha) | \pi \rangle$
  number operator
- △ transversal momentum distributions (TMD) $f(\zeta, k_T)$
- △ Generalized parton distributions (GPD) $f(\zeta, b_T)$



$$\sigma \propto \sum_{i,j} f_i^\pi(x_\pi, \mu) \otimes f_j^A(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Extracted from **fixed target πA data**



Deeply virtuality meson production

- △ TDIS at 12GeV JLab, leading proton observable, fixed target instead of collider (HERA);
- △ EIC, EIcC, great integrated luminosity to reduce the systematics uncertainties;
- △ COMPASS++/AMBER give π-induced DY data.

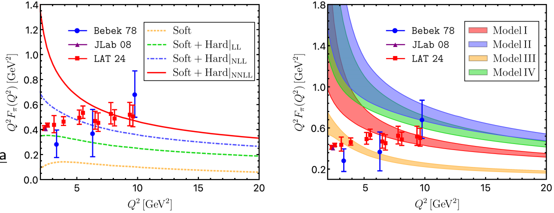
LCDAs in the "hard" processes

- In the "hard" processes, a certain hadron can be described by LCDAs at different (collinear) twist
- LCDAs is a use of conformal symmetry in massless QCD
- The structure of perturbative prediction for light-cone dominated processes reveals the underlying conformal symmetry of the QCD $\mathcal{L}$
  - pQCD is the calculation of the scale dependence (evolution equations) of physical observables
  - the evolution equations of DA, GPD can be understood as the RGE for the light-cone operators
- The conformal partial expansion of hadron distribution amplitudes
  - similar to partial-wave expansion of wave function in quantum mechanism
  - invariance of massless QCD under conformal trans. *VS* rotation symmetry
  - longitudinal $\otimes$ transversal dofs *VS* angular $\otimes$ radial dofs for spherically symmetry potential
  - transversal-momentum dependence (scale dependence) is governed by the RGE
  - longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the collinear subgroup of conformal group $SL(2,R) \cong SU(1,1) \cong SO(2,1)$
- This expansion was instrumental for the proof of the QCD factorization for the elastic and transition form factors

LCDAs in the "hard" processes

- **Example I Pion electromagnetic form factor**
  - N^{2}LO factorization at leading power of Λ^2_{QCD}/Q^2 expansion, the light-cone projections on the leading-twist-collinear operators [Chen^2, Feng, Jia 2312.17228]
  - rigorous two-loop computation of leading-twist contribution in the hard-collinear factorization [Ji, Shi, Wang^3, Yu 2411.03658]

see talk from Prof. Yu Jia 14:00-14:30, 29th, Nov



Model I [Khodjamirian, et al 2011.11275] Model II [SC, et al 2007.05550]
Model III [Stefanis 2006.10576] Model IV [Cloet, et al 2407.00206]

  - the N^{2}LO QCD correction to the short-distance coefficient function is enormous
  - large uncertainty from a_2, a_4 in pion meson LCDAs

LCDAs in the "hard" processes

- **Example II Exclusive heavy-to-light decays $B \to \pi l^+ \nu$ and $B \to \rho l^+ \nu$**
  - proportional to $B \to \pi, \rho$ form factors obtained from LCSRs/LQCD
  - $|V_{ub}|$ extracted from $B^0 \to \pi^-, \rho^0 l^+ \nu$ has $\sim 3\sigma$ deviation [Belle II 2407.17403]

$$|V_{ub}|_{B\to\pi l\nu} = (3.93 \pm 0.19 \pm 0.13 \pm 0.19(\text{theo})) \times 10^{-3} \quad [\text{LQCD}]$$
$$|V_{ub}|_{B\to\pi l\nu} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16(\text{theo})) \times 10^{-3} \quad [\text{LQCD + LCSRs}]$$
$$|V_{ub}|_{B\to\rho l\nu} = (3.19 \pm 0.12 \pm 0.18 \pm 0.26(\text{theo})) \times 10^{-3} \quad [\text{LCSRs}]$$

  - LCSRs calculations do not consider the width effect of ρ in the $\pi\pi$ invariant mass spectral or optimistically estimates the uncertainty from B-meson LCDAs

- The introduction of Dipion LCDAs and the 2πDAs LCSRs
  [SC, Khodjamirian, Virto 1709.0173, SC 1901.06071]
  - high partial waves give few percent contributions to $B \to \pi\pi$ form factors
  - ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave
  - qualitatively explains the $|V_{ub}|$ tension obtained from $B \to \pi, \rho l\nu$

Pion LCDAs and form factors

Pion LCDAs

- Define the LCDAs with the Lorentz and gauge invariant ME

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi \int_0^1 du\, e^{i\zeta P\cdot x}\,[iP_\mu \phi(u) + \cdots]$$

- LCDAs are dimensionless functions of u and renormalization scale μ

◦ the probability amplitudes to find the π in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$

- expansion in power of large momentum transfer is governed by contributions from small transversal separations $x^2 \sim 1/\mu$ between constituents

$$\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu)\, C_n^{3/2}(u)$$

- $a_0^\pi = f_\pi \propto \langle 0|\bar{u}(0)\gamma_z\gamma_5 d(0)|\pi^-(P)\rangle$ and $a^\pi_{n\geqslant 2}(\mu_0)$ are universal parameters
- μ dependences in a_n^π **transversal dof** [Brodsky & Lepage '80, Balitsky & Braun '88]
- $C_n(u)$ are Gegenbauer polynomials in the local collinear conformal expansion **longitudinal dof** [Lepage & Brodsky '79, '80, Efremov & Radyushkin '80, Braun & Filyanov '90]

Pion LCDAs $\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$

- QCD definition $a_n^\pi(\mu) = \langle \pi | q(z)\bar{q}(z) + z_\rho \partial_\rho q(z)\bar{q}(z) + \cdots |0\rangle$

- **LQCD**: 0.334 ± 0.129[UKQCD '10], 0.135 ± 0.032[RQCD '19], $0.258^{+0.079}_{-0.052}$ [LPC '22]

  - default scale at 1 GeV   scale running

$$a_n(\mu) = a_n(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{\gamma_n^{(0)}-\gamma_0^{(0)}}{2\beta_0}}, \quad \gamma_n^{\perp(\parallel),(0)} = 8C_F\left(\sum_{k=1}^{n+1}\frac{1}{k} - \frac{3}{4} - \frac{1}{2(n+1)(n+2)}\right)$$

  - a_4^π is not available $\leftarrow$ the growing number of derivatives in $q\bar{q}$ operator

- **QCDSR**: 0.19 ± 0.06[Chernyak '84], $0.26^{+0.21}_{-0.09}$[Khodjamirian '04], $0.28^{+0.08}_{-0.08}$[Ball '06]

  - nonlocal vacuum condensate is introduced and modeled for $a_{n>2}^\pi$ [Bakulev '01]

- Dispersion relation as an **Inverse problem** [Li '20, Yu '22]

  quark-hadron duality → Laguerre Polynomials to construct spectral density

$$\{a_2, a_4, a_6, a_8\} = \{0.249, 0.134, 0.106, 0.096\}$$

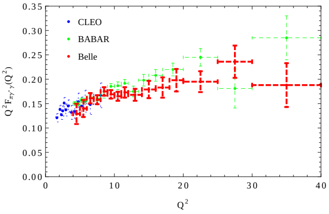
Pion LCDAs $\quad \phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$

- **Data-driven** with QCD calculations for the π involved exclusive processes

○ $F_{B\to\pi}$: 0.19 ± 0.19 [Ball '05], 0.16 [Khodjamirian '11], large error from B meson

○ $F_{\pi\gamma\gamma^*}$: 0.14 [Agaev '10] BABAR+CLEO, 0.10 [Agaev '12] Belle+CLEO

large uncertainty of $a^\pi_{n>2}$, discrepancy data at large Q^2



Method	a_2^π(2 GeV)	Refs.
LO QCDSR, CZ model	0.39	[30,31]
QCDSR	$0.18^{+0.15}_{-0.26}$	[32]
QCDSR	0.19 ± 0.06	[33]
QCDSR, NLC	0.13 ± 0.04	[34,35]
$F_{\pi\gamma\gamma^*}$, LCSRs	0.12 ± 0.04 (2.4 GeV)	[36]
$F_{\pi\gamma\gamma^*}$, LCSRs	0.21 (2.4 GeV)	[37]
$F_{\pi\gamma\gamma^*}$, LCSRs, R	0.19	[38]
$F_{\pi\gamma\gamma^*}$, LCSRs, R	0.31	[39]
$F_{\pi\gamma\gamma^*}$, LCSRs, NLO	0.096	[40]
$F_{\pi\gamma\gamma^*}$, LCSRs, NLO	0.068	[41]
F_π^{em}, LCSRs	$0.17 \pm 0.10 \pm 0.05$	[42]
F_π^{em}, LCSRs, R	0.14 ± 0.02	[43]
$F_{B\to\pi}$, LCSRs	0.13 ± 0.13	[44]
$F_{B\to\pi}$, LCSRs	0.11	[45,46]
LQCD, TWST, $N_f = 2$, CW	0.201 ± 0.114	[47]
LQCD, TWST, $N_f = 2+1$, DWF	0.233 ± 0.088	[48]
LQCD, MST, $N_f = 2$	0.136 ± 0.03	[27]
LQCD, MST, $N_f = 2+1$, CW	0.0762 ± 0.0127	[29]

[**SC** 1901.06071]
DiPion LCDAs

○ F_π: 0.24 ± 0.17 [Bebek '78] Wilson Lab+NA7, 0.20 ± 0.03 [Agaev '05] JLab

large uncertainty of $a^\pi_{n>2}$, data is available only in small spacelike q^2

Pion LCDAs from F_π

- Spacelike data is available in the narrow region $q^2 \in [-2.5, 0]$ GeV^2
- Perturbative QCD calculations are valid in the large $|q^2|$
- The mismatch destroys the direct extracting programme from $F_\pi(q^2 < 0)$
- Timelike form factor $F_\pi(q^2 > 0)$ provides another opportunity
  - △ $e^+e^- \to \pi^+\pi^-(\gamma)$, $4m_\pi^2 \leqslant q^2 \lesssim 9$ GeV^2 [BABAR '12]
  - △ $\tau \to \pi\pi\nu_\tau$, $4m_\pi^2 \leqslant q^2 \leqslant 3.125$ GeV^2 [Belle '08]
  - △ $e^+e^-(\gamma) \to \pi^+\pi^-$, $0.6 \leqslant Q^2 \leqslant 0.9\,\mathrm{GeV}^2$ with ISR [BESIII '16]
- TL measurement and SL predictions are related by dispersion relation
- The standard dispersion relation and The modulus representation

$$F_\pi(q^2 < s_0) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}F_\pi(s)}{s - q^2 - i\epsilon}$$

⇓ [**SC**, Khodjamirian, Rosov 2007.05550]

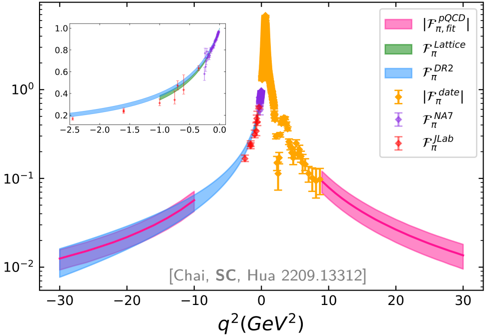
$$F_\pi(q^2 < s_0) = \exp\left[\frac{q^2\sqrt{s_0 - q^2}}{2\pi}\int_{s_0}^{\infty}\frac{ds\,\ln|F_\pi(s)|^2}{s\sqrt{s - s_0}\,(s - q^2)}\right]$$

$$|\mathcal{F}_\pi(s)|^2 = \Theta(s_{\mathrm{max}} - s)\,|\mathcal{F}^{\mathrm{data}}_{\pi,\mathrm{Inter.}}(s)|^2 + \Theta(s - s_{\mathrm{max}})\,|\mathcal{F}^{\mathrm{pQCD}}_\pi(s)|^2$$

Pion LCDAs from F_π

- $a_2 = 0.270 \pm 0.047,\ a_4 = 0.179 \pm 0.060,\ a_6 = 0.123 \pm 0.086$
  - Pion deviates from the purely asymptotic one, a_2^π is not enough
  - consists well with $0.258^{+0.079}_{-0.052}$ [LPC '22], $0.249^{+0.005}_{-0.006}$ [Li '22]

- The state-of-the-art pQCD calculation and modular dispersion relation

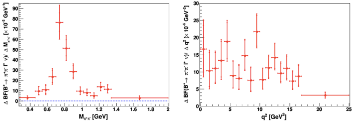


Dipion LCDAs at leading twist

$B \to \pi^- \pi^0 l^+ \nu$ decays

$B \to \pi\pi$ form factors and $B \to [\rho^+ \to]\, \pi^+\pi^0 l\bar{\nu}$ decay

- H_{l4} decays have rich observables, nontrivial tests of SM [Faller '14]
- Different exclusive $b \to u$ processes help in the $|V_{ub}|$ **determination**



[Belle 2005.07766]

- ○ $B \to \rho l \bar{\nu}_l$ $(1.63 \pm 0.20) \times 10^{-4}$
  [BABAR '11, Belle '13, Belle II '24]
- ○ first measurement of $B^+ \to \pi^+\pi^- l^+ \bar{\nu}_l$
  $(2.3 \pm 0.4) \times 10^{-4}$ [Belle '20]
- ○ First measurement of $D^0 \to \pi^+\pi^- e^+ e^-$ [LHCb-PAPER-2024-047, prelim.]
- ○ $(4.53 \pm 1.38) \times 10^{-7}$ in ρ/ω and $(3.84 \pm 0.96) \times 10^{-7}$ in ϕ
- ○ **$c \to u$-typed FCNC** upper limit 0.7×10^{-5} [BES III '18]
- ○ $D^0 \to K^-\pi^0\mu^+\nu$ S-wave accounts $\sim 2.06\%$, $(0.729 \pm 0.014 \pm 0.011)\,\%$ [BESIII 2403.10877]

- Dynamics of B_{l4} is governed by the $B \to \pi\pi$ form factors
- A big task for the practitioners of QCD-based methods
- ○ First Lattice QCD study of the $B \to \pi\pi l \bar{\nu}$ transition amplitude in the region of large q^2 and $\pi\pi$ invariant mass near the ρ resonance [Leskovec et.al. 2212.08833[hep-lat]]

$B \to \pi\pi$ form factors and $B \to [\rho^+ \to] \, \pi^+\pi^0 l\bar{\nu}$ decay

- DiPion LCDAs will shine a light on **the width effect encountered in FP** (multibody B decays, $B \to [\pi\pi] \, l\nu$, $b \to sll$, $c \to ull$, $D\pi$ system $\cdots$)

- How large of ρ contribution in P-wave $B \to \pi\pi$ transition ? How about the contributions from high partial waves ?

- $B \to \pi\pi$ **form factors** [Hambrock, Khodjamirian '15]

$$
\begin{aligned}
i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle &= F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma \\
&+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right) \\
&+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)
\end{aligned}
$$

- ◦ $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function
- ◦ $q\cdot k = (m_B^2 - q^2 - k^2)/2$ and $q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta - 1)$
- ◦ $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$, θ_π is the angle between the 3-momenta of the neutral pion and the B-meson in the dipion rest frame

Dipion LCDAs at leading twist

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov '99, Diehl '98]

$$\langle\pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu\tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

- ○ $n^2 = 0$, index f, f' respects the (anti-)quark flavor, a, b indicates the electric charge
- ○ coefficient $\kappa_{+-/00} = 1$ and $\kappa_{+0} = \sqrt{2}$, $k = k_1 + k_2$ is the invariant mass of dipion state
- ○ $\tau = 1/2, \tau^3/2$ corresponds to the isoscalar and isovector 2πDAs
- ○ higher twist $\propto 1, \gamma_\mu\gamma_5$ have not been discussed yet, γ_5 vanishes due to P-parity conservation

- Three independent kinematic variables

- ○ momentum fraction u carried by anti-quark respecting to the total momentum of DiPion state
- ○ longitudinal momentum fraction carried by one pion $\zeta = k_1^+/k^+$, $2q\cdot\bar{k}\,(\propto 2\zeta - 1)$ and k^2

- Normalization conditions $\int_0^1 du\, \Phi_{\parallel}^{I=1}(u,\zeta,k^2) = (2\zeta-1)F_\pi(k^2)$

$$\int_0^1 du\,(2u-1)\Phi_{\parallel}^{I=0}(u,\zeta,k^2) = -2M_2^{(\pi)}\zeta(1-\zeta)F_\pi^{\rm EMT}(k^2)$$

- ○ $F_\pi^{em}(0) = 1$, $F_\pi^{\rm EMT}(0) = 1$, $M_2^{(\pi)}$ is the moments of SPDs

Dipion LCDAs at leading twist

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=0,\text{even}}^{\infty}\sum_{l=1,\text{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=1,\text{odd}}^{\infty}\sum_{l=0,\text{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2,\mu)$ have similar scale dependence as the a_n of π, ρ, f_0 mesons

- Soft pion theorem relates the chirarlly even coefficients with a_n^π

$$\sum_{\ell=1}^{n+1} B_{n\ell}^{\parallel, I=1}(0) = a_n^\pi\,, \quad \sum_{\ell=0}^{n+1} B_{n\ell}^{\parallel, I=0}(0) = 0$$

- 2πDAs relate to the skewed parton distributions (SPDs) by crossing
  - moments of SPDs M_N^π is expressed in terms of $B_{nl}(k^2=0)$ in the forward limit

- In the vicinity of the resonance, 2πDAs reduce to the DAs of ρ/f_0
  - relation between the a_n^ρ and the coefficients $B_{n\ell}$
  - f_ρ relates to the imaginary part of $B_{nl}(m_\rho^2)$ divided by the strong coupling $g_{\rho\pi\pi}$

Dipion LCDAs at leading twist

- **What's the evolution from $4m_\pi^2$ to large k^2** $\mathcal{O}(m_c^2)$, furtherly to $\mathcal{O}(m_b\lambda_{\rm QCD})$?
  - Watson theorem of π-π scattering amplitudes implies an intuitive way to express the imaginary part of 2πDAs, leads to the Omnés solution of N-subtracted DR for the coefficients

$$B^I_{n\ell}(k^2) = B^I_{n\ell}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B^I_{n\ell}(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{\delta^I_\ell(s)}{s^N(s-k^2-i0)}\right]$$

- 2πDAs in a wide k^2 range is given by δ^I_ℓ and a few subtraction constants

- The subtraction constants of $B_{n\ell}(s)$ at low s (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

- firstly studied in the low-energy EFT based on instanton vacuum [Polyakov '99]
- updated with the kinematical constraints and the new a_2^π, a_2^ρ [SC '19,'23]

- We are now at leading twist, subleading twist LCDAs are not known yet

$B \to \pi\pi$ form factors

- Starting with the correlation function

$$F_\mu(k_1,k_2,q) = i\int d^4x e^{iq\cdot x}\langle\pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5(0)\}|0\rangle$$
$$\equiv \varepsilon_{\mu\nu\rho\sigma}q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})}$$

- Take $F_\perp^l(q^2,k^2,\zeta)$ as an example

$$\frac{F_\perp^l(q^2,k^2,\zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2}f_B m_B^2 f_{2\pi}^\perp(2\zeta-1)}\int_{u_0}^1 \frac{du}{u}\Phi_\perp^l(u,\zeta,k^2)\, e^{-\frac{s(u)+m_B^2}{M^2}}$$

  - the Chiral-odd DiPion LCDAs $\Phi_\perp^{ab,ff'}(u,\zeta,k^2)$
  - partial wave expansion $F_{\perp,\parallel}(k^2,q^2,\zeta) = \sum_\ell \sqrt{2\ell+1}\, F_{\perp,\parallel}^{(\ell)}(k^2,q^2)\, P_\ell^{(1)}(\cos\theta_\pi)/\sin\theta_\pi$

- The leading result is obtained by using the orthogonality relation of the Legender polynomials

$$F_\perp^{(\ell)}(k^2,q^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^\perp}\frac{\sqrt{\lambda_B}m_b}{m_B^2 f_B}e^{\frac{m_B^2}{M^2}}\sum_{n=0,2,\cdots}\sum_{\ell'=1,3}^{n+1} I_{\ell\ell'}B_{n\ell'}^\perp(k^2,\mu)J_n^\perp(q^2,k^2,M^2,s_0^B)$$

$$I_{\ell\ell'} \equiv -\frac{\sqrt{2\ell+1}(\ell-1!)}{2(\ell+1)!}\int_{-1}^1\frac{dz}{z}\sqrt{1-z^2}P_\ell^{(1)}(z)P_{\ell'}^{(0)}(z)$$

$$J_n^\perp(q^2,k^2,M^2,s_0^B) = \int_{u_0}^1 du e^{\frac{-s}{M^2}}6(1-u)C_n^{3/2}(2u-1)$$

  - $I_{\ell\ell'} = 0$ when $\ell > \ell'$, $I_{11} = 1/\sqrt{3}$, $I_{13} = -1/\sqrt{3}$, $I_{15} = 4/(5\sqrt{3})$
  - $\ell' = 1$, asymptotic DAs, P-wave term retains in the DiPion LCDAs

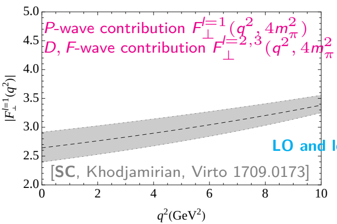
$B \to \pi\pi$ form factors

- How large of $P-$wave contribution to $B \to \pi\pi$ FFs $(\ell = 1)$?

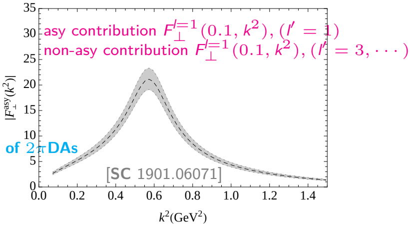
$$R_\ell \equiv F_\perp^{(\ell>1)}(k^2, q^2)/F_\perp^{(\ell=1)}(k^2, q^2)$$

- How much ρ contained in $P-$wave $B \to \pi\pi$ FFs $(\ell = 1, \ell' = 1)$?

◦ Short-distance part of the correlation: $\mu = 3$ GeV without NLO correction
$f_B = 207^{+9}_{-17}$ MeV, $M^2 = 16.0 \pm 4.0$ GeV^2 $\leftrightarrow$ $s_0^B = 37.5 \pm 2.5$ GeV^2 [P. Gelhausen '13]



LO and leading twist of 2πDAs

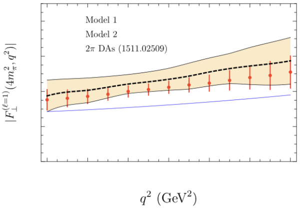


⋆ High partial waves give few percent contributions to $B \to \pi\pi$ form factors

⋆ ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave

$B \to \pi\pi$ form factors

- 30% smaller than the result obtained from B-meson LCSRs
  [SC, Khodjamirian and Virto 1701.01663]
  - high twist contributions ?
  - large uncertainty from B-meson LCDAs ?



- Comparison with the B meson LCSRs

$$F_\mu(k_1, k_2, q) = i \int d^4x e^{iq\cdot x} \langle \pi^+(k_1)\pi^0(k_2)| \mathrm{T}\{j_\mu^{V-A}(x), j_5^B(0)\}|0\rangle$$

$$F_{\mu\nu}(k, q) = i \int d^4x\, e^{ik\cdot x} \langle 0| \mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \mathrm{T}\{j_\mu^{V-A}(x)\}|\bar{B}^0(q+k)\rangle$$

- At the current accuracy, they give same order plots of $B \to \pi^+\pi^0$ FFs
- For the $P-$wave FFs, they both predict sizable non-ρ contribution ($\sim 20\%$) $\rho', \rho'', \cdots$ and NR background
- B-meson LCSRs can not predict the contributions from higher partial waves, but it indeed exist while is small in the DiPion LCSRs
- B-meson LCSRs rely on the resonance model (an inverse problem), DiPion LCSRs is currently limited by the poor knowledge of DiPion LCDAs

Conclusion and Prospect

- The introduction of DiPion LCDAs provides an opportunity to study the width effects and the structures of nonstable mesons in H_{l4} processes
  - a new booster on the accurate calculation in flavor physics
  - improvement study in the CKM determinations and the flavor anomalies

- DiPion LCDAs study is at leading twist so far   QCD definitions and double expansion
  - determine the parameters by low energy effective theory and data constraints
  - evolution of k^2 from the threshold to large scale $\mathcal{O}(m_c^2, m_b\lambda_{QCD})$
  - universal phase shift in $\pi\pi$ scattering and heavy decay ?

- Go further to high twist LCDAs, not only to match the precise measurement
  - $B \to \pi\pi l\nu, B \to [\rho\rho \to] \to 4\pi,\ D_s \to \pi\pi l\nu, D \to K\pi\mu\nu,\ D \to \pi\pi e^+e^-$ et al.

Thanks a lot for your patience.