Basis Light-front Quantization Approach to Λ_b and Σ_b Baryons

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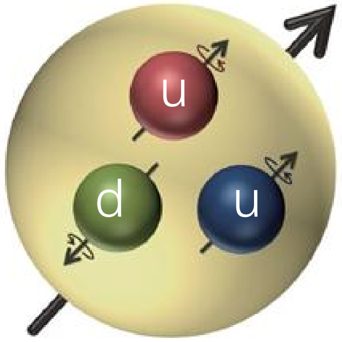
OUTLINE

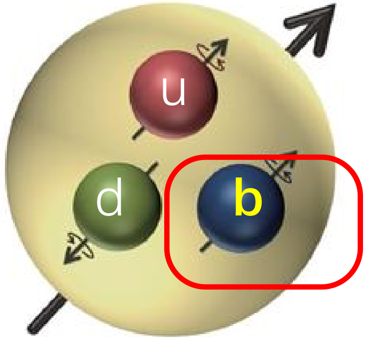
- Why do we care about Λ_b ?
- Basis Light-front Quantization
- Electromagnetic form factors
- Parton Distribution Functions
- Conclusion and prospect

Why do we care about Λ_b?

- Larger mass and shorter life compared to proton
- Difficult to measure the bound state in experiment
- Various decay processes are documented (hadronic decay, semileptonic decay...)



proton (0.938 GeV)



Λ_b(5.619 GeV)

Light cone distribution amplitudes
K. Huang et al. EPJC 83:272 (2023)

Light-front wave functions

Transition form factors

Basis Light-front Quantization

We adopt an effective light-front Hamiltonian and solve the matrix:

$$H_{\text{eff}}\,|\Psi\rangle = M^2\,|\Psi\rangle$$

The effective Hamiltonain is made of several constituents:

$$H_{\text{eff}} = H_{\text{k.e.}} + H_{\text{trans}} + H_{\text{longi}} + H_{\text{int}}$$

The baryon state can be expanded in the Fock space:

$$|B\rangle = \boxed{\psi_{(3q)}\,|qqq\rangle} + \psi_{(3q+g)}\,|qqqg\rangle + \psi_{(3q+q\bar{q})}\,|qqqq\bar{q}\rangle + \dots$$

valence quark

Basis Light-front Quantization

In longitudinal direction we choose plane wave basis:

$$k_i^+ = \frac{2\pi k_i}{L} \qquad x_i = k_i / \boxed{K_{\text{tot}}}$$

In transverse direction we choose 2-D harmonic oscillator basis:

$$\phi_{nm}(\vec{k}_\perp) = \frac{1}{b}\sqrt{\frac{4\pi n!}{(n+|m|)!}}\left(\frac{|\vec{k}_\perp|}{b}\right)^{|m|} \times e^{-\frac{1}{2}\vec{k}_\perp^2/b^2} L_n^{|m|}\left(\frac{\vec{k}_\perp^2}{b^2}\right) e^{im\theta},$$

$$\sum_{i=1}^{3} 2n_i + |m_i| + 1 \leq \boxed{N_{\text{max}}}.$$

truncation parameters

Total angular momentum projection: $\sum_{i=1}^{3}(\lambda_i + m_i) = \Lambda$

Basis Light-front Quantization

Table I: Basis truncation parameters and model parameters

N_{max}	K_{tot}	α_s	κ(GeV)	$m_{l/k}$(GeV)	$m_{l/g}$(GeV)	$m_{b/k}$(GeV)	$m_{b/g}$(GeV)
10	32	0.57±0.06	0.337	0.30	0.20	5.05	4.95

T. Peng et al. PRD 106, 114040 (2022)

Table II: Mass spectra

Baryons	M_{BLFQ}(MeV)	M_{exp}(GeV)	
Λ_b	5624±2	5619.60±0.17	ground state
Σ_b	5636±2	5810.56±0.25 (Σ_b^+) 5815.64±0.27 (Σ_b^-) ...(Σ_b^0)	first excited state

PDG: $\Lambda_b^0 = udb,\ \Sigma_b^0 = udb,\ \Sigma_b^+ = uub,\ \Sigma_b^- = ddb$

Numerical results to support the solved LFWFs

- Flavor electromagnetic FFs

$$\langle P+q,\uparrow|\frac{J^+(0)}{2P^+}|P,\uparrow\rangle = F_1(Q^2),$$
$$\langle P+q,\uparrow|\frac{J^+(0)}{2P^+}|P,\downarrow\rangle = -\frac{(q_1-\mathrm{i}q_2)}{2M}F_2(Q^2).$$

- Baryon electromagnetic FFs

$$F^{\mathrm{B}}_{1(2)}(Q^2) = \sum_q e_q F^q_{1(2)}(Q^2)$$
$$G^{\mathrm{B}}_E(Q^2) = F^{\mathrm{B}}_1(Q^2) - \frac{Q^2}{4M^2}F^{\mathrm{B}}_2(Q^2),$$
$$G^{\mathrm{B}}_M(Q^2) = F^{\mathrm{B}}_1(Q^2) + F^{\mathrm{B}}_2(Q^2).$$

- Parton Distribution Functions

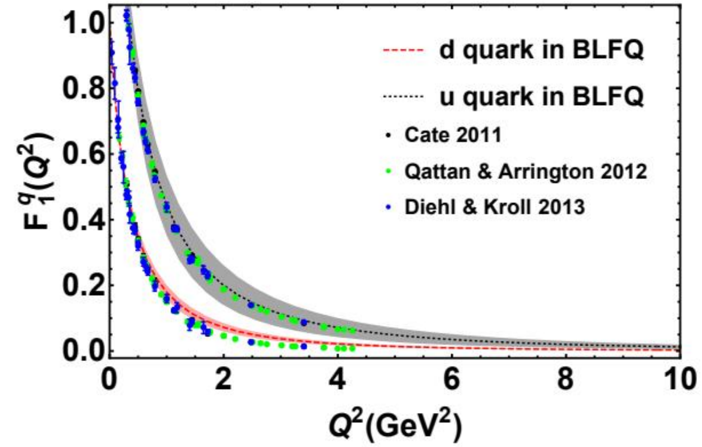
$$\Phi^{\Gamma(q)}(x) = \frac{1}{2}\int\frac{\mathrm{d}z^-}{4\pi}e^{ip^+z^-/2} \times \langle P,\Lambda|\bar{\psi}_q(0)\Gamma\psi_q(z^-)|P,\Lambda\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

With Γ being $\gamma^+, \gamma^+\gamma^5, i\sigma^{j+}\gamma^5$, one can have different PDFs.

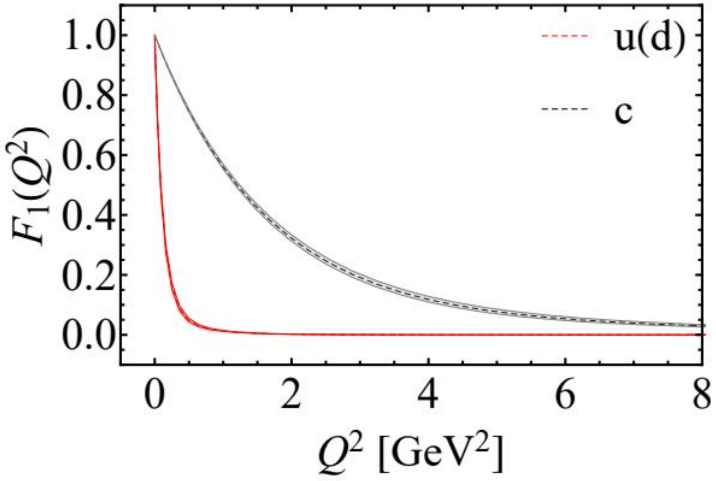
- Proton results: S. Xu et al. PRD 104, 094036 (2021)
- Λ and Λ_c results: T. Peng et al. PRD 106, 114040 (2022)

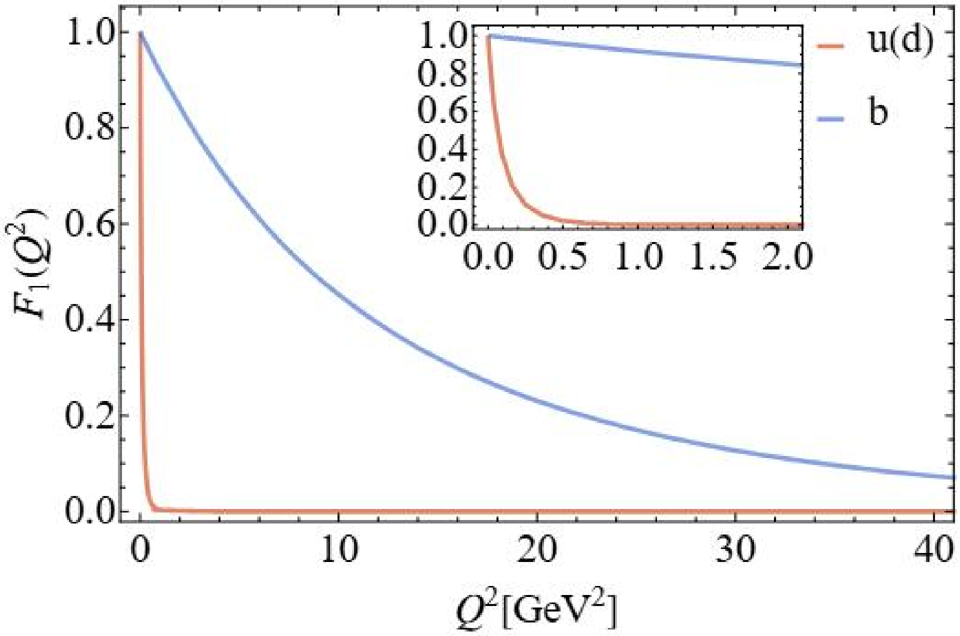
Flavor Dirac form factors

proton
(**uud**)



Λ_c
(**udc**)



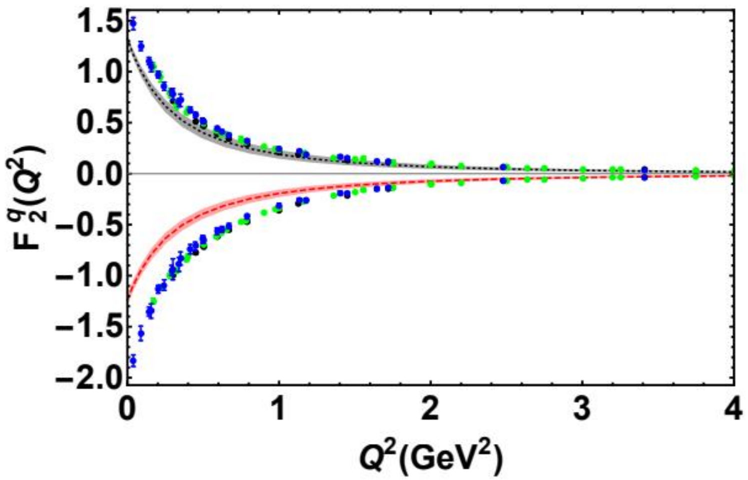


Λ_b
(**udb**)

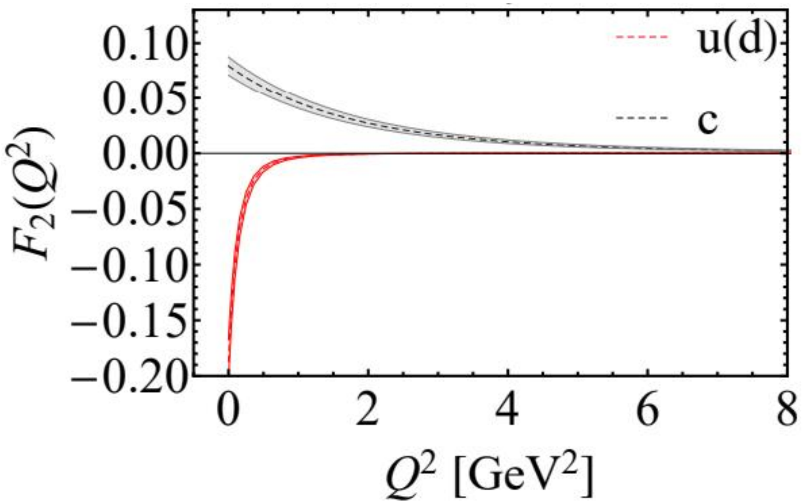
- Heavier quark contributes more to Dirac FFs.
- Heavy quark distribution increases when the mass increases.

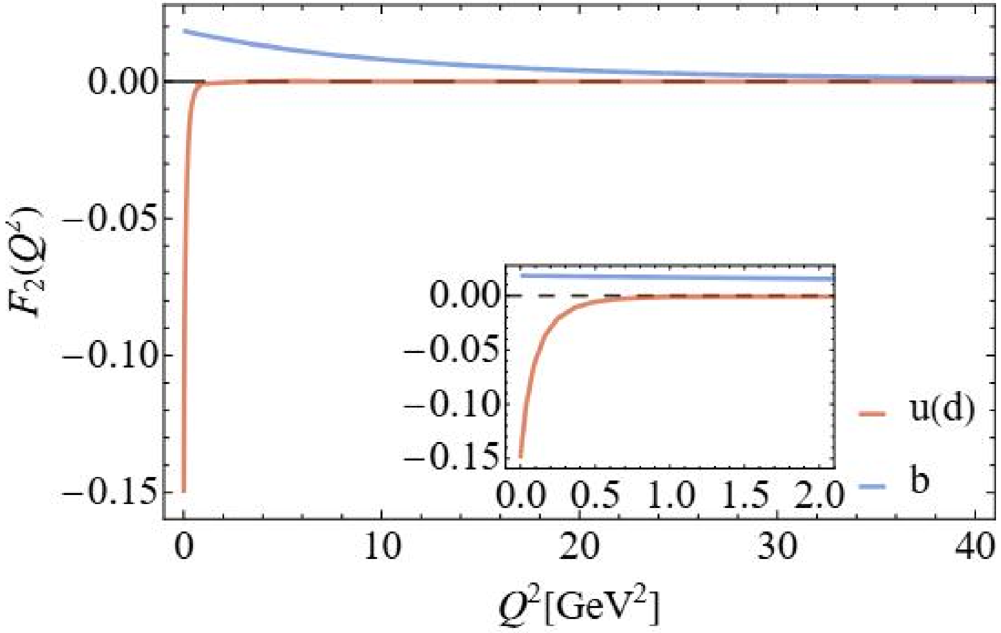
Flavor Pauli form factors

proton
(**uud**)



Λ_c
(**udc**)





Λ_b
(**udb**)

- Light quarks almost dominate in samll Q^2 region.
- Along with the increase of Q^2, contributions from light quarks drop rapidly and being lower than that from heavy quark.

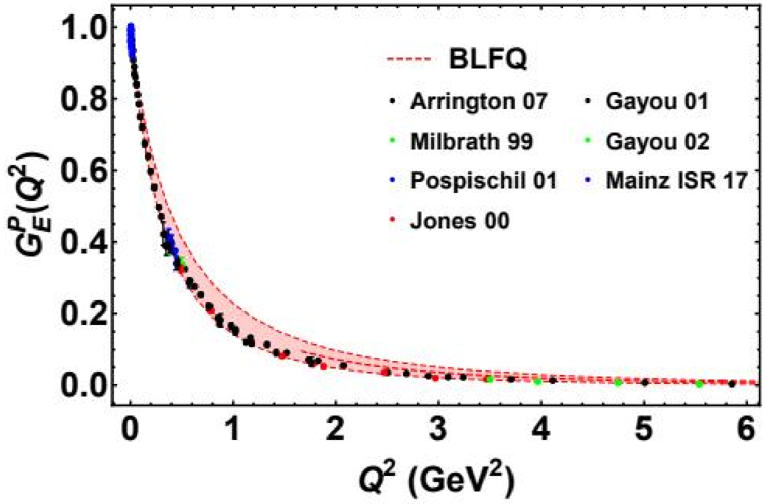
Baryon electromagnetic form factors

$$e_u = \frac{2}{3}, \quad e_d = -\frac{1}{3}$$
$$e_s = \frac{2}{3}, \quad e_c = -\frac{1}{3}$$
$$e_b = -\frac{1}{3}$$

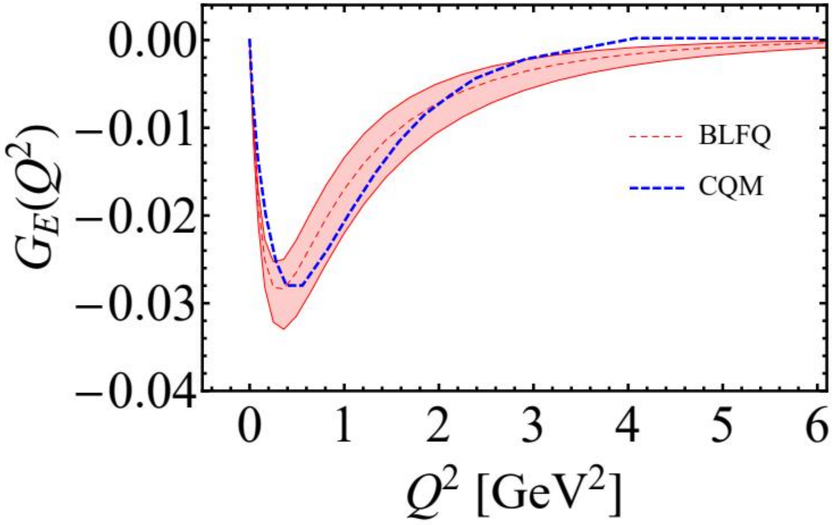
- ➢ Sach's electric FFs G_E

G_E and G_M are influenced by the constituent charges, so we choose Λ (uds) to compare with.

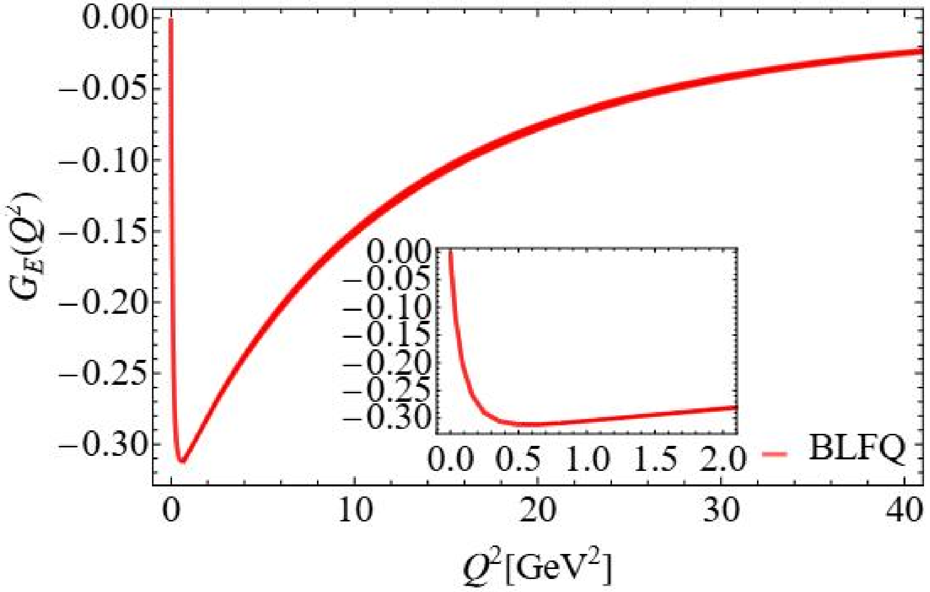
proton (**uud**)



Λ (**uds**)



Λ_b (**udb**)



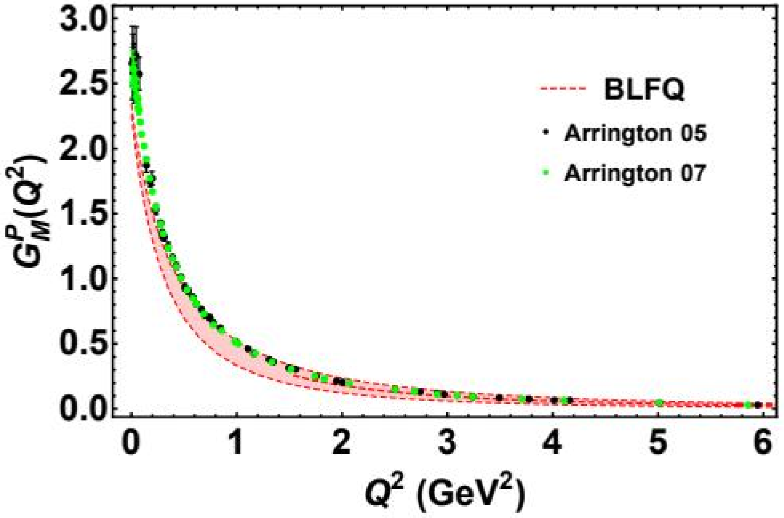
- Heavier quark mass brings a rapider peak at small Q^2 region and slower going back to near zero with Q^2 increases.

Baryon electromagnetic form factors

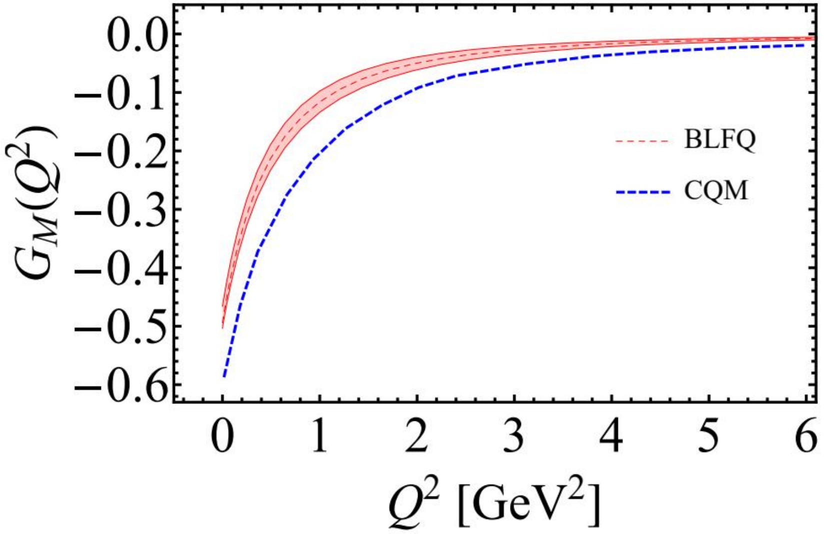
$$e_u = \frac{2}{3}, \quad e_d = -\frac{1}{3}$$
$$e_s = \frac{2}{3}, \quad e_c = -\frac{1}{3}$$
$$e_b = -\frac{1}{3}$$

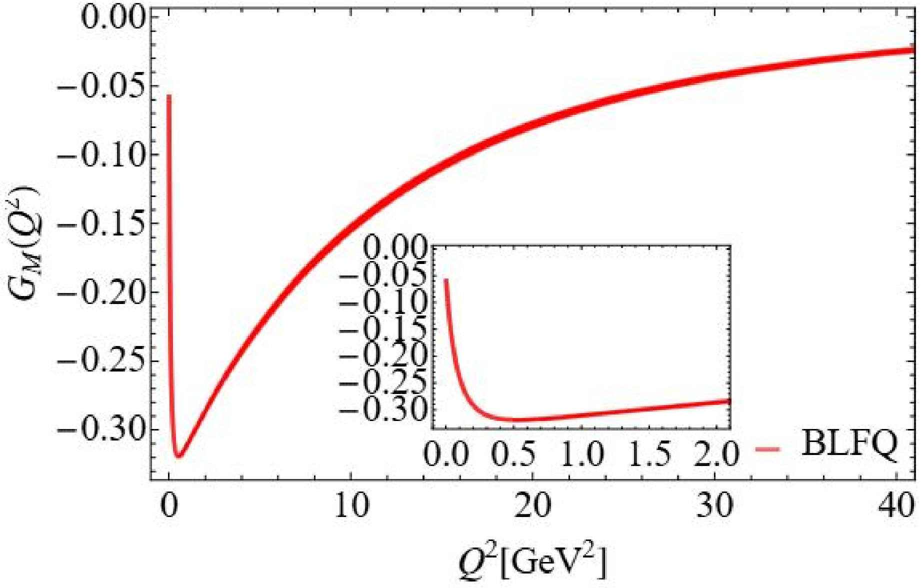
➢ Sach's magnetic FFs G_M

proton
(**uud**)



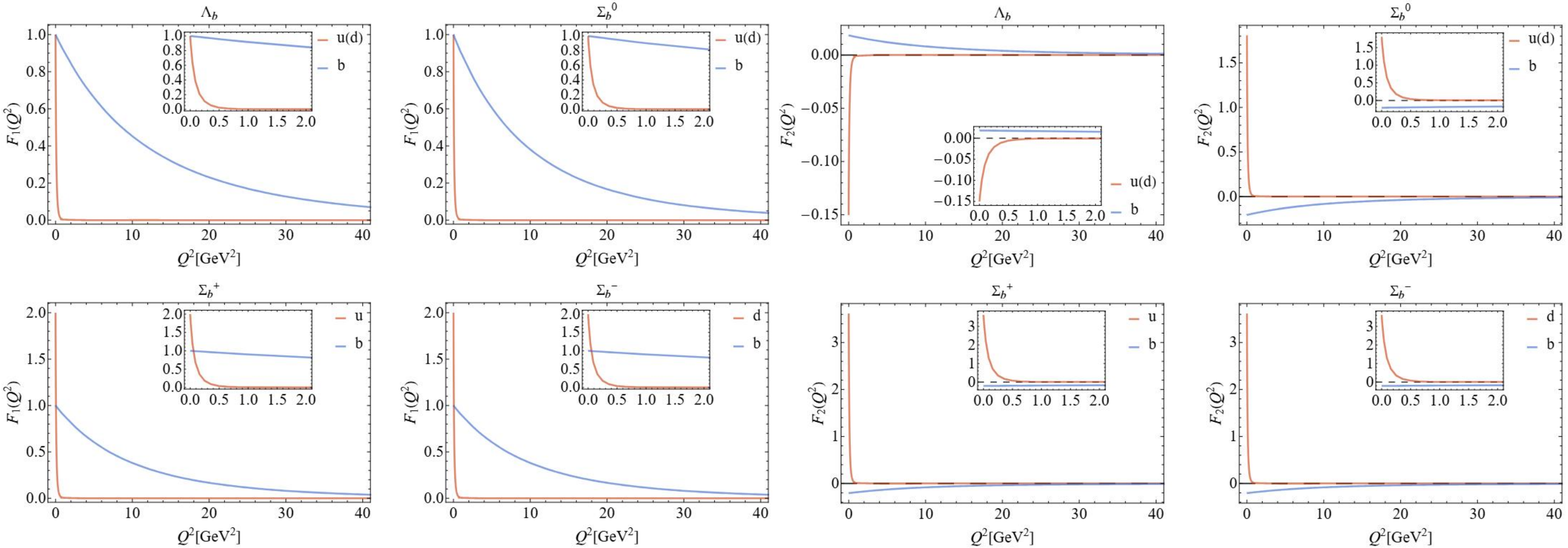
Λ
(**uds**)

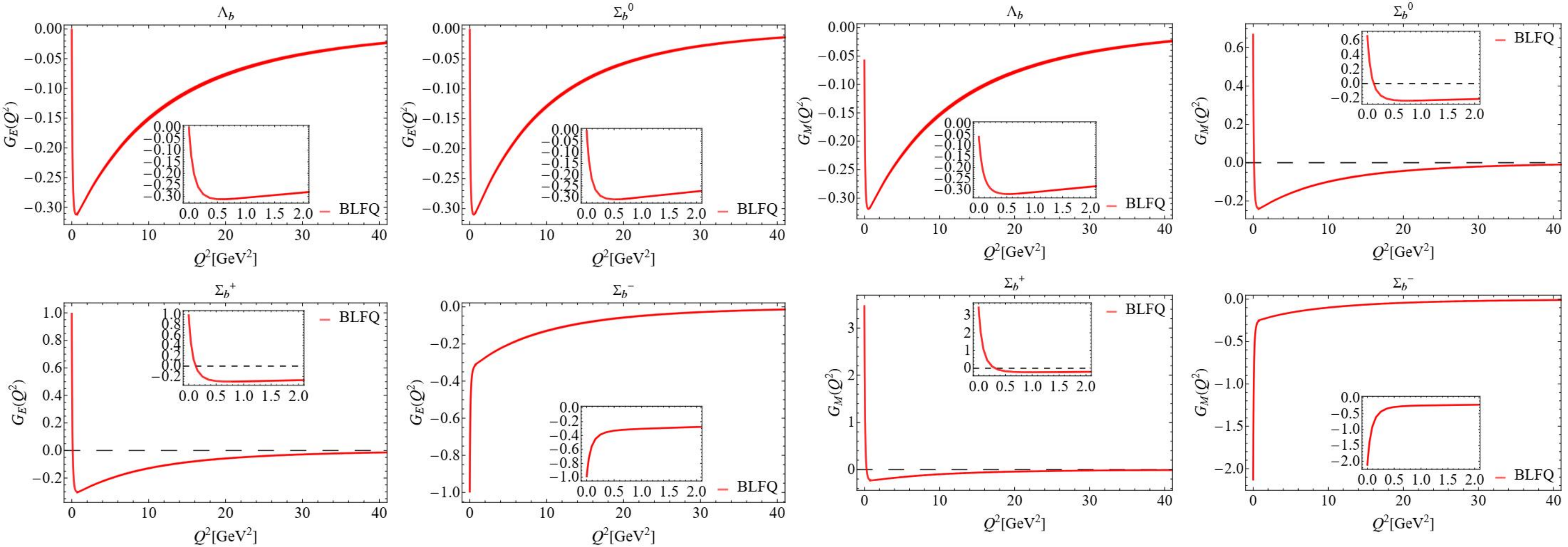




Λ_b
(**udb**)

- For Λ_b: The peak at the small Q^2 region is influenced by the lower Pauli FFs and higher Dirac FFs of the heavy quark.


Λ_b
$\Sigma_b{}^0$
Λ_b
$\Sigma_b{}^0$
$\Sigma_b{}^+$
$\Sigma_b{}^-$
$\Sigma_b{}^+$
$\Sigma_b{}^-$
$F_1(Q^2)$
$F_2(Q^2)$
$Q^2[\mathrm{GeV}^2]$
u(d)
u
d
b



Λ_b
$\Sigma_b{}^0$
Λ_b
$\Sigma_b{}^0$
$\Sigma_b{}^+$
$\Sigma_b{}^-$
$\Sigma_b{}^+$
$\Sigma_b{}^-$
$G_E(Q^2)$
$G_M(Q^2)$
Q^2[GeV^2]
BLFQ

Magnetic moments

Baryons	μ_{BLFQ}	[1]	[2, 3]	[4]	[3]	[5]	[6]
Λ_b	−0.0562±0.0002	−0.0620	−0.060	...	−0.066	−0.060	...
Σ_b^0	0.6719±0.0023	0.5653	0.640	0.659	0.422	0.603	0.390
Σ_b^+	3.4809±0.0085	2.1989	2.500	2.575	1.622	2.250	1.590
Σ_b^-	−2.1372±0.0040	−1.0684	−1.220	−1.256	−0.778	−1.150	−0.810

$$\mu = G_M^B(0)$$

Charge radius and magnetic radius

Baryons	r_E^2	Λ	r_M^2	Λ
Λ_b	0.8944±0.0145	0.07	−13.7106±0.1743	0.52
Σ_b^0	1.0066±0.0165	0.07	4.1193±0.0589	0.82
Σ_b^+	4.0509±0.0664	0.79	3.1865±0.0426	0.79
Σ_b^-	2.0375±0.0334	0.65	2.5996±0.0332	0.70

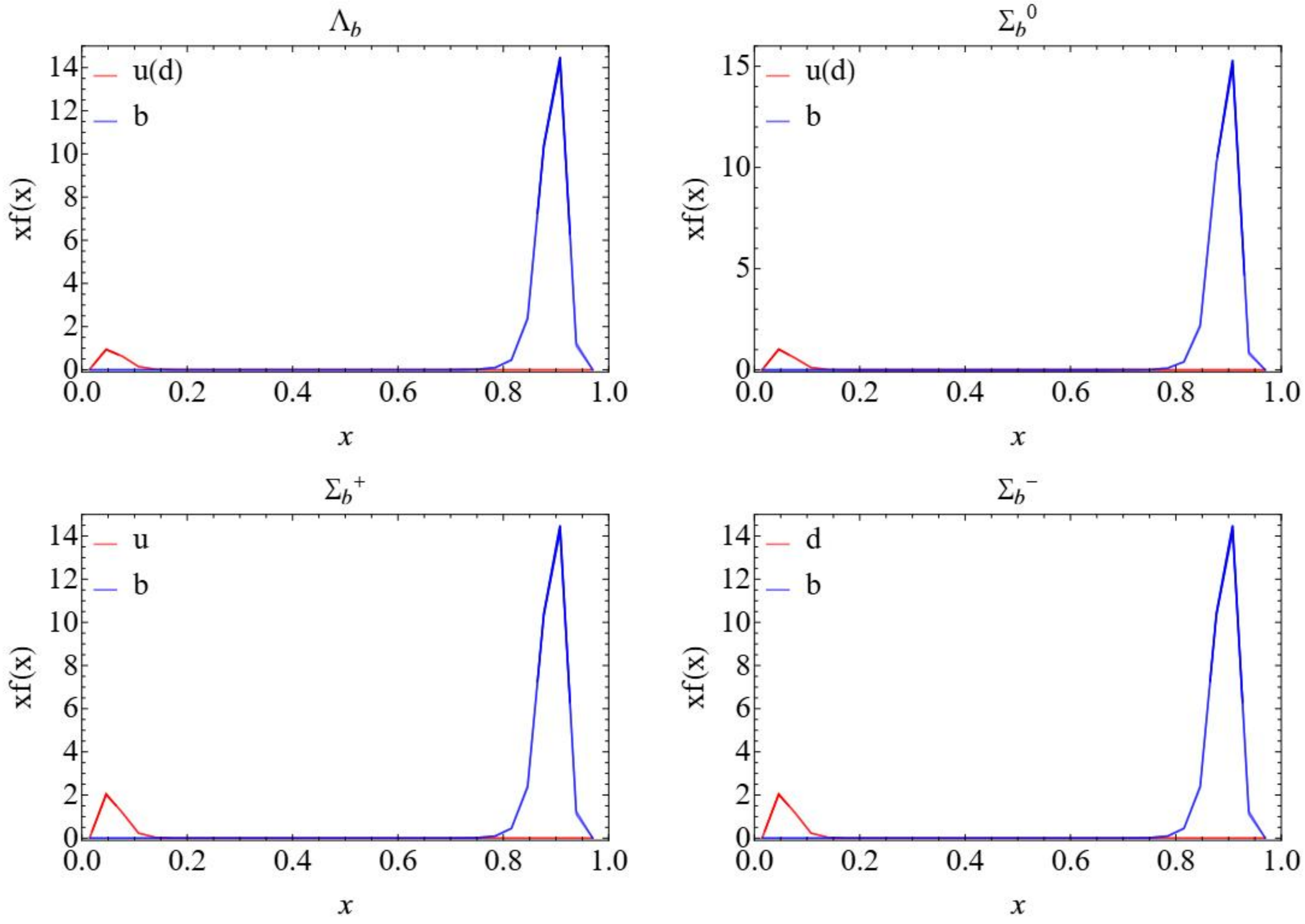
$$\left\langle r_E^2 \right\rangle^B = -\frac{6}{G_E^B(0)} \frac{dG_E^B(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

$$\left\langle r_M^2 \right\rangle^B = -\frac{6}{G_M^B(0)} \frac{dG_M^B(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

[1]A. Hazra et al. PRD 104, 053002 (2021) [2]J. Franklin et al. PRD 24, 2910 (1981)
[3]A. Bernotas et al. arXiv:1209.2900 (2012) [4]N. Barik et al. PRD 28, 2823 (1983)
[5]V. Simonis et al. arXiv: 1803.01809 (2018) [6]L. Meng et al. PRD 98, 094013 (2018)

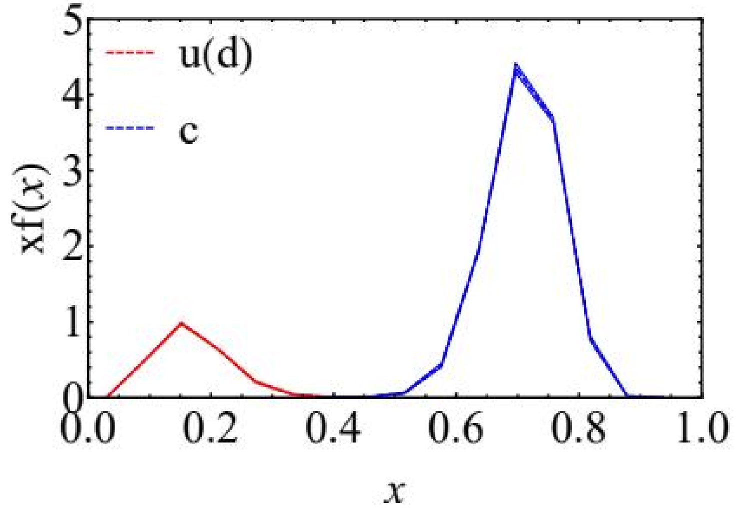
Unpolarized PDFs f(x)

$$f^q(x) = \sum_{\lambda_i} \int [\mathrm{d}\mathcal{X}\, \mathrm{d}\mathcal{P}_\perp] \times \Psi^{\uparrow *}_{\{x_i, \vec{p}_{i\perp}, \lambda_i\}} \Psi^{\uparrow}_{\{x_i, \vec{p}_{i\perp}, \lambda_i\}} \delta(x - x_q)$$



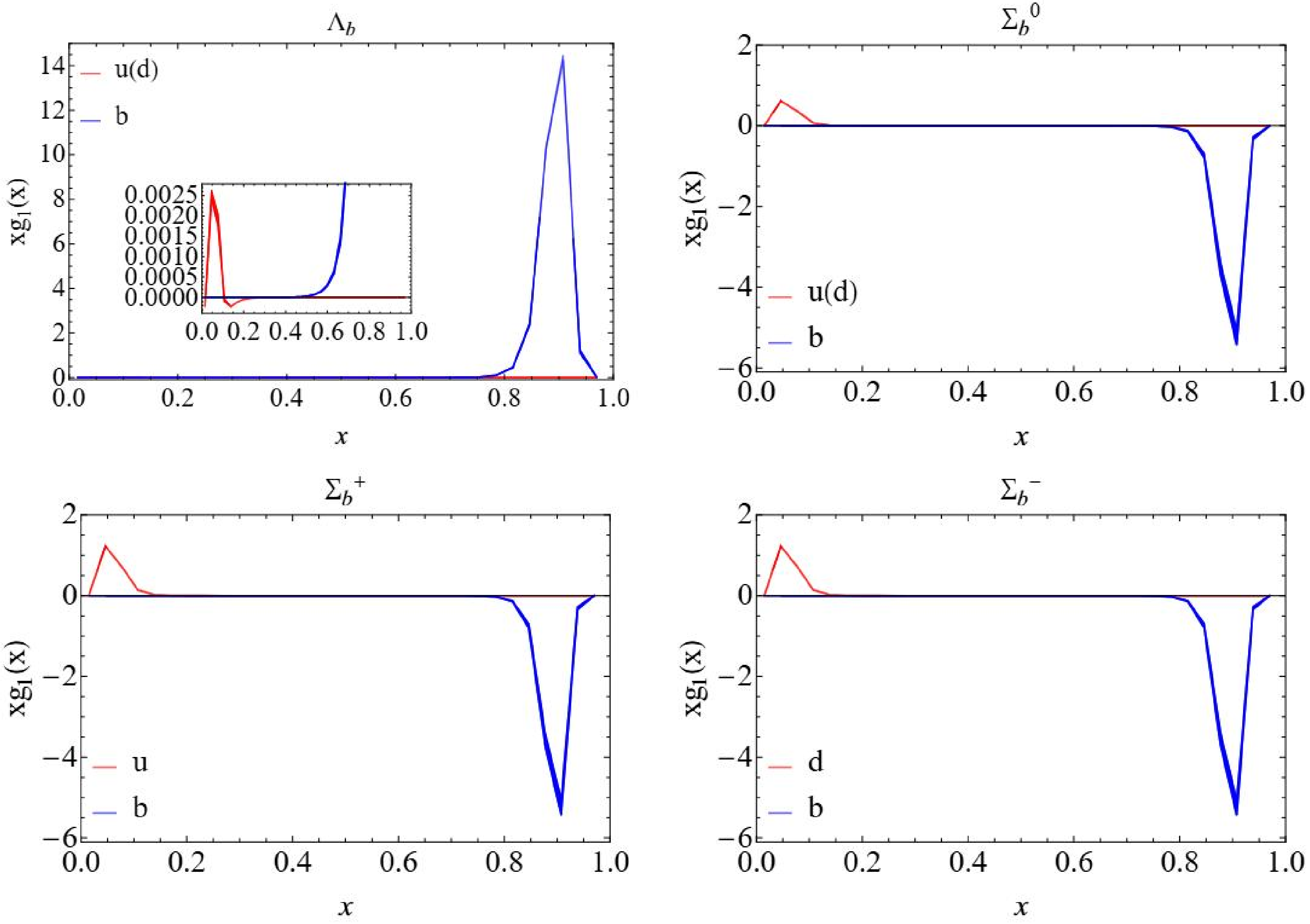
- Heavier quark gives more contribution.
- Heavier quark concentrates on larger x.

Λ_c (**udc**)



Helicity PDFs $g_1(x)$

$$g_1^q(x) = \sum_{\{\lambda_i\}} \int [\mathrm{d}\mathcal{X}\,\mathrm{d}\mathcal{P}_\perp] \times \lambda_1 \Psi^{\uparrow *}_{\{x_i,\vec{p}_{i\perp},\lambda_i\}} \Psi^{\uparrow}_{\{x_i,\vec{p}_{i\perp},\lambda_i\}} \delta(x - x_1)$$



- Different spin structure
- For Λ_b (ground state):

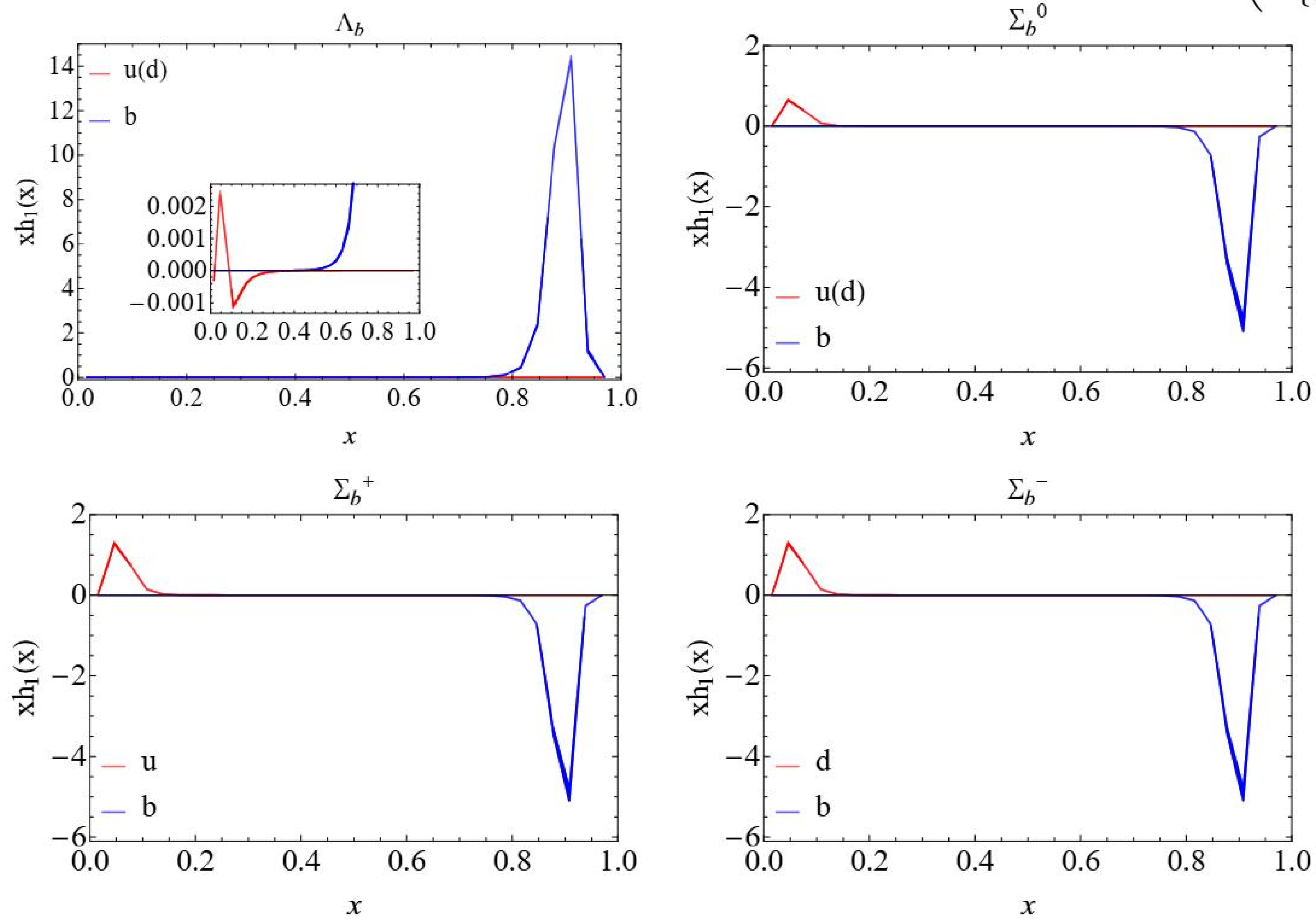
b quark spin dominates the total spin of the bound state.

- For Σ_b (first excited state):

light quarks also have contributions.

Transversity PDFs $h_1(x)$

$$h_1^q(x) = \sum_{\{\lambda_i\}} \int [\mathrm{d}\mathcal{X}\, \mathrm{d}\mathcal{P}_\perp] \times \left(\Psi^{\uparrow *}_{\{x_i, \vec{p}_{i\perp}, \lambda'_i\}} \Psi^{\downarrow}_{\{x_i, \vec{p}_{i\perp}, \lambda_i\}} + (\uparrow \leftrightarrow \downarrow) \right) \delta(x - x_1)$$



- Similar to $g_1(x)$.

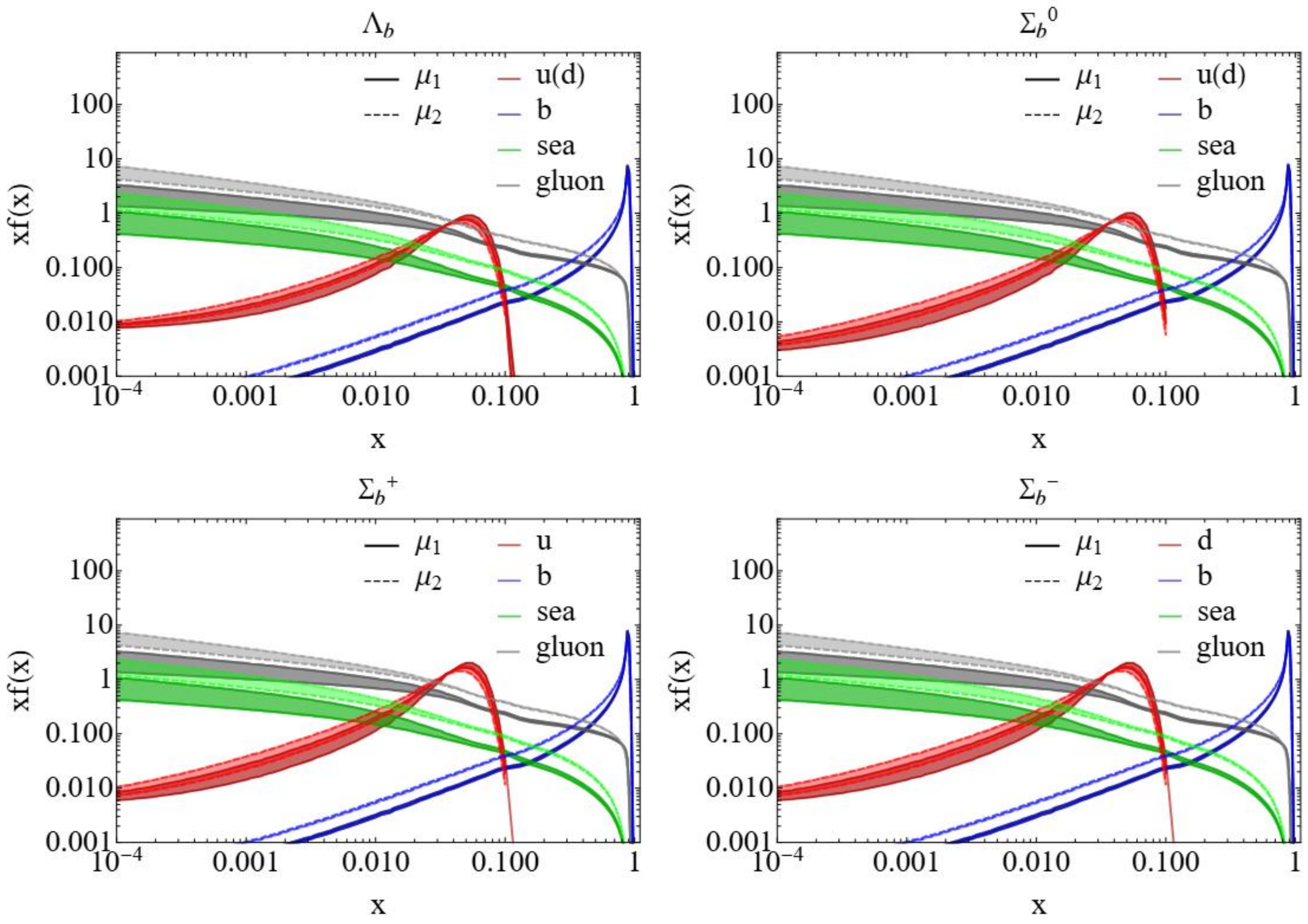
Evolved PDFs

- DGLAP evolution

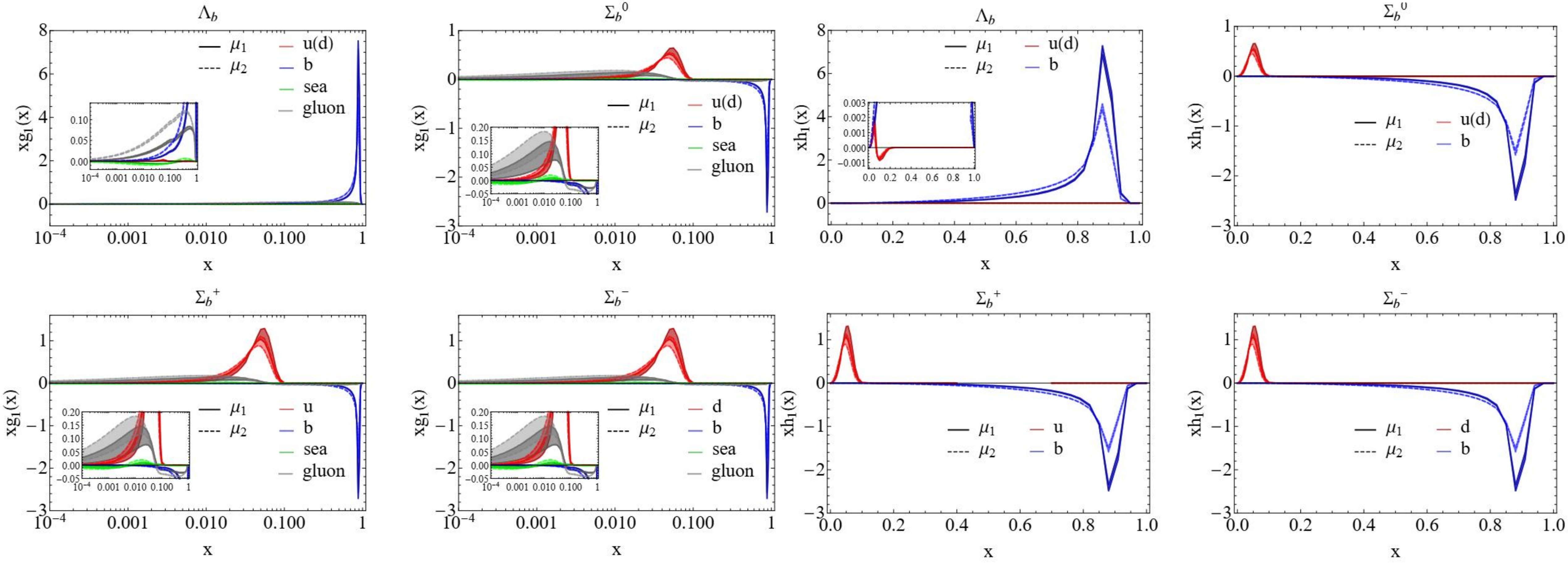
Initial scale (GeV)	
μ_h	1.90 (UV cutoff)
m_q	5.05

J. Lan et al. PRD 102, 014020 (2020)

Final scale (GeV)	
μ_1	20 (EicC)
μ_2	80 (eRHIC)



Evolved PDFs



Conclusion and prospect

- We obtain the masses comparable to experiment and the LFWFs of Λ_b and its isospin triplet baryons.
- Our prediction of their EM properties is in agreement with other theoretical calculations.
- We investigated their longitudinal structure with PDFs.

With the LFWFs of Λ_b and proton, we will calculate TFFs.

Thank you

$$H_{\text{LF}}\,|\Psi\rangle = M^2\,|\Psi\rangle$$

$$H_{\text{LF}} = H_{\text{eff}} + H'.$$

$$H_{\text{eff}} = \sum_{i=1}^{3} \frac{\vec{k}_{i\perp}^2 + m_i^2}{x_i} + \frac{1}{2}\sum_{i\neq j}^{3} V_{i,j}^{\text{conf}} + \frac{1}{2}\sum_{i\neq j}^{3} V_{i,j}^{\text{OGE}},$$

$$V_{i,j}^{\text{conf}} = \kappa^4\left[\vec{r}_{ij\perp} - \frac{\partial_{x_i}(x_i x_j \partial_{x_j})}{(m_i+m_j)^2}\right]$$

$$V_{i,j}^{\text{OGE}} = \frac{4\pi C_F \alpha_s}{Q_{ij}^2}\bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma^\mu u_{s_j}(k_j)$$

$$H' = \lambda_L(H_{\text{c.m.}} - 2b^2 I)$$

$$H_{\text{c.m.}} = \left(\sum_{i=1}^{3}\vec{k}_{i\perp}\right)^2 + b^4\left(\sum_{i=1}^{3} x_i \vec{r}_{i\perp}\right)^2,$$