

# Finite- $t$ and Mass Corrections in Deeply Virtual Compton Scattering

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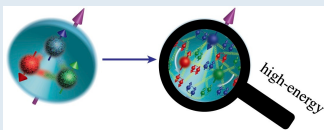
based on: [Vladimir Braun, YJ, and Alexander Manashov](#), to appear

**Light Cone 2024, Huizhou**

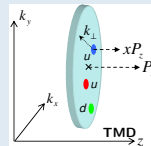
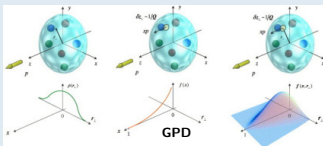


## Internal structures of hadrons

- Hadrons are composite particles with complicated internal structures



- PDFs only encode longitudinal information; GPDs and TMDs include transverse information



- GPDs are crucial for understanding the spin structure of hadrons

$$\sum_{s,j} \text{[Diagram of hadron spin structure]} = \frac{1}{2}$$

**Unsolved problem in physics:**

**?** How do the quarks and gluons carry the spin of protons?

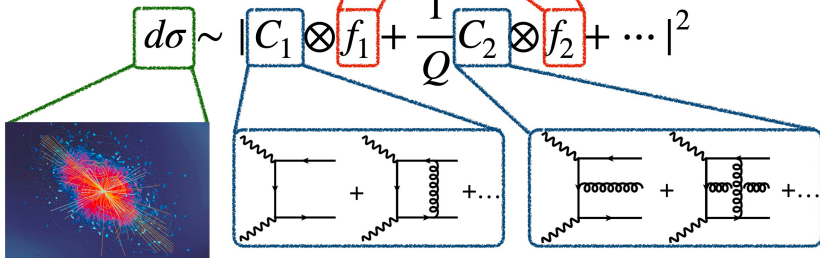


# Extracting structure information of hadrons from high-energy experiment

- Theoretical foundation for accessing structural information of hadrons: factorization

*DVCS*

$e + N \rightarrow e + N + \gamma$



## Deeply Virtual Compton Scattering (DVCS)

- Golden channel for extracting the Generalized Parton Distributions (GPDs)

$$\gamma^*(q) + N(p) \rightarrow \gamma(q') + N(p')$$

**Kinematics:**  $q'^2 = 0$ ,  $q^2 = -Q^2$ ,  $t = \Delta^2 = (p' - p)^2$ ,  $p^2 = p'^2 = m^2$ ,  $P_\mu = (p_\mu + p'_\mu)/2$

- DVCS amplitude defined as (hadronic)

$$\delta(p + q - p' - q') \mathcal{A}_{\mu\nu}(q, q', p) = i \int \frac{d^4x d^4y}{(2\pi)^4} e^{-iq \cdot x + iq' \cdot y} \langle p' | T \{ j_\mu^{\text{em}}(x) j_\nu^{\text{em}}(y) \} | p \rangle$$

decomposable into helicity amplitudes

[V. Braun, A. Manashov, B. Pirnay, (2012)]

$$\begin{aligned} \mathcal{A}_{\mu\nu}(q, q', p) = & \varepsilon_\mu^+ \varepsilon_\nu^{*+} \mathcal{A}^{++} + \varepsilon_\mu^- \varepsilon_\nu^{*-} \mathcal{A}^{--} + \varepsilon_\mu^0 \varepsilon_\nu^{*+} \mathcal{A}^{0+} + \varepsilon_\mu^0 \varepsilon_\nu^{*-} \mathcal{A}^{0-} \\ & + \varepsilon_\mu^+ \varepsilon_\nu^{*-} \mathcal{A}^{+-} + \varepsilon_\mu^- \varepsilon_\nu^{*+} \mathcal{A}^{-+} + q'_\nu \mathcal{A}_\mu^{(3)} \end{aligned}$$

Parity conservation dictates:

$$\mathcal{A}^{++} = \mathcal{A}^{--} \equiv \mathcal{A}^{(0)}, \quad \mathcal{A}^{0\pm} \equiv (\varepsilon_\mu^\pm P^\mu) \mathcal{A}^{(1)}, \quad \mathcal{A}^{\mp\pm} \equiv (\varepsilon_\mu^\pm P^\mu)^2 \mathcal{A}^{(2)},$$

- Other parameterizations available, exact relations between them known

[A. Belitsky, D. Müller, YJ, (2014)], [V. Braun, A. Manashov, D. Müller, B. Pirnay, (2014)]



## Helicity amplitudes in DVCS

- Helicity amplitudes  $\mathcal{A}^{ij}$  can be expanded in  $1/Q$ ,

$$\begin{aligned}\mathcal{A}^{++} &= \mathcal{A}_0^{(0)} + \frac{t}{Q^2} \mathcal{A}_{1,1}^{(0)} + \frac{m^2}{Q^2} \mathcal{A}_{1,2}^{(0)} + \dots, \\ \mathcal{A}^{0+} &= \frac{Q}{q \cdot q'} \left( \mathcal{A}_0^{(1)} + \frac{t}{Q^2} \mathcal{A}_{1,1}^{(1)} + \frac{m^2}{Q^2} \mathcal{A}_{1,2}^{(1)} + \dots \right), \\ \mathcal{A}^{-+} &= \frac{\mathcal{O}(1)}{q \cdot q'} \left( \mathcal{A}_0^{(2)} + \frac{t}{Q^2} \mathcal{A}_{1,0}^{(2)} + \frac{m^2}{Q^2} \mathcal{A}_{1,1}^{(2)} + \dots \right),\end{aligned}$$

$$\mathcal{A}_{j,z}^{(i)} \sim \ln(\mu^2/Q^2), \quad q \cdot q' \sim Q^2$$

- subleading corrections are generated from two sources:

$$\mathcal{A}_{j>0,z}^{(i)} = \mathcal{A}_{j,z}^{(i),\text{kin}} + \mathcal{A}_{j,z}^{(i),\text{dyn}}$$

- dynamical corrections carries **genuine higher-twist** information of the hadron, while kinematic ones are induced by **twist-two GPDs**
- Each term is a convolution of coefficient function  $C_{j,z}^{(i)}$  and GPD  $H_k$ , schematically\*:

$$\mathbb{A}_{j,z}^{(i),\text{kin}} = C_{j,z}^{(i)} \otimes H_2, \quad \mathbb{A}_{j,z}^{(i),\text{dyn}} = C_{j,z}^{(i,k)} \otimes H_k, k > 2, \quad \text{*if factorization holds}$$

- DIS case:  $t = 0 \mapsto$  Nachtmann correction, (1973)



## Kinematic power corrections

- We study the kinematic corrections induced by **twist-two GPDs to NNLP**  $\mathcal{A}^{(i)}$  at tree-level
- GPDs are defined in terms of light-ray operators, twist-two vector case:

$$\langle p' | O_2(z_1 n, z_2 n) | p \rangle = 2P_+ \int_{-1}^1 dx e^{-iP_+[z_1(\xi-x)+z_2(\xi+x)]} F_q(x, \xi, t),$$

$$O_2(z_1 n, z_2 n) = \frac{1}{2} \left( \bar{q}(z_1 n) \gamma_+ q(z_2 n) - \bar{q}(z_2 n) \gamma_+ q(z_1 n) \right),$$

$$n_\mu = q'_\mu, \quad \xi = \frac{p_+ - p'_+}{p_+ + p'_+}, \quad \tilde{n}_\mu = -q_\mu + \frac{Q^2}{(Q^2 + t)} q'_\mu, \quad a_+ \equiv a \cdot n$$

[V. Braun, A. Manashov, B. Pirnay, (2012) ]

actual expansion parameter  $1/(n \cdot \tilde{n}) = 2/(t + Q^2)$

- Two-point nonlocal operator admits local OPE** ( $x^2$  not necessary on the lightcone):

$$O_2(z_1 x, z_2 x) = \sum_{N=1} \frac{2(2N+1)}{N!} z_{12}^{N-1} \int_0^1 du (u\bar{u})^N [\mathcal{O}_N(z_{21}^u x)]_{lt},$$

$$[\mathcal{O}_N(y)]_{lt} = x^{\mu_1} \cdots x^{\mu_N} [\mathcal{O}_{\mu_1 \cdots \mu_N}(y)]_{lt}, \quad \text{conformal operator}$$

**convention:**  $z_{12} \equiv z_1 - z_2 = 1$ ,  $z_{21}^u \equiv \bar{u}z_2 + uz_1$

- kinematic corrections are induced by:** hard to separate from genuine higher-twist in old technique

$$[\partial_y^{\mu_1} \mathcal{O}_{\mu_1 \cdots \mu_N}(y)]_{lt}, \quad [\partial_y^2 \mathcal{O}_{\mu_1 \cdots \mu_N}(y)]_{lt}, \quad \cdots$$



## kinematic correction

- consider local OPE of  $\langle p_1 | \{j_\mu(z_1)j_\nu(z_2) | p_2 \rangle$ , kinematic corrections are induced by:

$$[\partial_y^{\mu_1} \mathcal{O}_{\mu_1 \dots \mu_N}(y)]_{lt}, \quad [\partial_y^2 \mathcal{O}_{\mu_1 \dots \mu_N}(y)]_{lt}, \quad \dots \implies \langle p_1 | [ [\dots] ] | p_2 \rangle \neq 0$$

- they must be included to restore **EM gauge invariance** for nonforward scattering!
- “kinematic approximation”  $\Leftrightarrow$  setting all genuine higher-twist operators to zero **at all scales**
  - consistent under quantum corrections? evolution?
  - yes! genuine and kinematic higher-twist (all twist-2 descendant) evolve autonomously
    - but not true for each genuine higher-twists operators/GPDs: mixing
- vanish for onshell quark, subtlety in separating kinematic and genuine higher-twists

[V. Braun, A. Manashov, J. Rohrwild (2009), YJ, A Belitsky, (2014)]

[Ferrara, Grillo, Parisi, Gatto, (1971) - (1973)]

$$\partial^\mu O_{\mu\nu} = \boxed{2i\bar{q}gF_{\nu\mu}\gamma^\mu q}$$

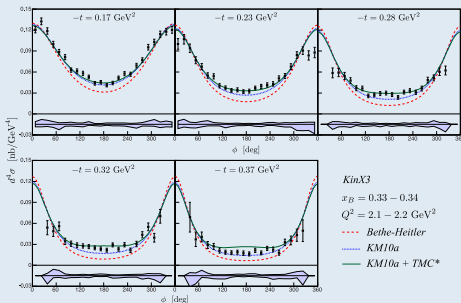
$$O_{\mu\nu} = \frac{1}{2}[\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q + (\mu \leftrightarrow \nu)]$$

must use RHS, but hard to distinguish RHS from genuine twist-4 operator



## Motivation

- Theory side:
    - necessary to remove frame dependence, restore **EM gauge and translation symmetry**
    - a systematic framework for higher-power kinematic contribution
    - verify factorization at **NNLP** order
    - application of conformal symmetry to higher-power corrections
  - Phenomenological side:
    - provide theory support for nuclei DVCS measurements (significant  $m^2$  corrections)
- [ M. Hattawy et al. [CLAS], Phys. Rev. Lett. 119, no.20, 202004 (2017) ]
- mismatch between theory and experiment, resolvable by higher-power corrections?



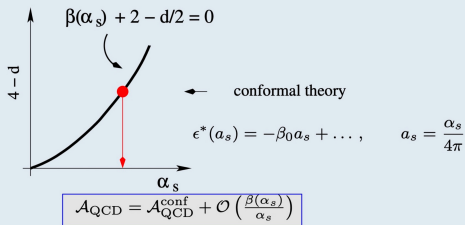
[M. Defurne et al. [Hall A Collaboration], (2015)]





# Application of conformal technique in QCD: general idea

QCD is not a conformal theory, but



- “conformal QCD”: Wilson-Fisher critical point  $\beta(\alpha_s, \epsilon) = 0$  with  $d = 4 - 2\epsilon$
- used to compute kernels (-1 loop), RG-resummation, and CFs: two-loop by “hand”

e.g., [V. Braun, YJ, A. Manashov, (2019), V. Braun, YJ, A. Manashov (2017), V. Braun, A. Manashov, S. Moch, J. Schoenleber]



## Kinematic corrections: local form

- OPE expansion of current correlator (scalar current for demonstration):

vector-vector current: [V. Braun, YJ, A. Manashov (2021)]

$$\begin{aligned}
 T\{j(x_1)j(x_2)\} &= \sum_N \left\{ A_N^{\mu_1 \dots \mu_N} \underbrace{\mathcal{O}_{\mu_1 \dots \mu_N}^N}_{\text{twist-2 operators}} + B_N^{\mu_1 \dots \mu_N} \underbrace{\partial^\mu \mathcal{O}_{\mu, \mu_1 \dots \mu_N}^N}_{\text{descendants of twist-2}} \right. \\
 &\quad \left. + C_N^{\mu_1 \dots \mu_N} \partial^2 \underbrace{\mathcal{O}_{\mu_1 \dots \mu_N}^N}_{\text{twist-2 operators}} + D_N^{\mu_1 \dots \mu_N} \underbrace{\partial^\mu \partial^\nu \mathcal{O}_{\mu, \nu, \mu_1 \dots \mu_N}^N}_{\text{descendants}} + \dots \right\} \\
 &\quad + \text{genuine higher-twist contributions} \\
 &\equiv \sum_N C_N^{\mu_1 \dots \mu_N}(x, \partial) \mathcal{O}_{\mu_1 \dots \mu_N}^N + \text{genuine higher-twist contributions}
 \end{aligned}$$

- employ conformally covariant OPE to extract  $C_N^{\mu_1 \dots \mu_N}(x, \partial)$ : [S. Ferrara, A. F. Grillo, and Gatto, 1971-1973]  
 CFs of descendants are related to the CFs of the twist-2 operators by conformal symmetry

$$A_N^{\mu_1 \dots \mu_N} \xrightarrow{O(4,2)} C_N^{\mu_1 \dots \mu_N}(x, \partial)$$



## conformal triangles

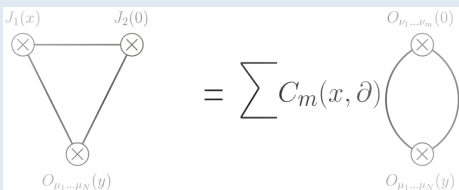
## A. M. Polyakov, 1970

$$\langle O_1(x_1)O_2(x_2) \rangle = \frac{\text{const}}{|x_1 - x_2|^{2\Delta_1}} \delta_{\Delta_1 \Delta_2},$$

$$\langle O_1(x_1)O_2(x_2)O_2(x_3) \rangle = \frac{\text{const}}{|x_1 - x_2|^{\Delta_1 + \Delta_2 - \Delta_3} |x_1 - x_3|^{\Delta_1 + \Delta_3 - \Delta_2} |x_2 - x_3|^{\Delta_2 + \Delta_3 - \Delta_1}},$$

$\Delta_i$  are scaling dimensions: canonical + anomalous

- $A_N, B_N, \dots$  are extracted by exploiting conformal properties of 3-point function



- most efficient to apply shadow operator formalism to extract  $C_m(x, \partial)$  to all orders!
  - four(one) independent Lorentz structures for vector(axial-vector) case
  - in general, calculation carried out at  $d^*$ -dimensions (critical point), here  $d^* = 4$
- normalization (const) fixed by DIS coefficient functions
  - agree with previous lower-order result (different approach)

[V. Braun and A. Manashov, (2012)]



## CFs for kinematic corrections: local OPE of leading twist and descendants

- A glimpse of local coefficient functions

- vector case:

[V. Braun, YJ, A. Manashov (2021)]

$$\begin{aligned}
 [j^\mu(x_1)j^\nu(x_2)]_V &= \sum_N \frac{r_{N,V}}{(-x_{12}^2 + i0)^2} \int_0^1 du (u\bar{u})^N \left[ (N+1)g^{\mu\nu} \left( 1 - \frac{1}{4} \frac{u\bar{u}}{N+1} x_{12}^2 \partial^2 \right) \right. \\
 &\quad + \frac{1}{2N} x_{12}^2 (\partial_1^\mu \partial_2^\nu - \partial_1^\nu \partial_2^\mu) + \left( 1 - \frac{1}{4} \frac{u\bar{u}}{N} x_{12}^2 \partial^2 \right) \left( \frac{\bar{u}}{u} x_{21}^\mu \partial_1^\nu + \frac{u}{\bar{u}} x_{12}^\nu \partial_2^\mu \right) \\
 &\quad - \frac{1}{4} \frac{u\bar{u}}{N(N+1)} x_{12}^2 \partial^2 (x_{21}^\nu \partial_1^\mu + x_{12}^\mu \partial_2^\nu) \\
 &\quad \left. - \frac{x_{12}^\mu x_{12}^\nu}{N+1} u\bar{u} \partial^2 \left( 1 - \frac{1}{4} \frac{u\bar{u}}{N+2} x_{12}^2 \partial^2 \right) \right] \mathcal{O}_{N,V}^{(0)}(x_{21}^u) + \dots, \quad x_{21}^u = \bar{u}x_2 + ux_1,
 \end{aligned}$$

- axial-vector case:

$$\begin{aligned}
 [j^\mu(x_1)j^\nu(x_2)]_A &= \frac{r_{N,A}}{(-x_{12}^2 + i0)^2} \int_0^1 du (u\bar{u})^N \left\{ \epsilon^{\mu\nu}{}_{\beta\gamma} x_{12}^\beta \left[ N \left( \frac{u}{\bar{u}} \partial_2^\gamma - \frac{\bar{u}}{u} \partial_1^\gamma \right) \right. \right. \\
 &\quad \left. \left. - \frac{1}{4} \frac{u\bar{u} x_{12}^2 \partial^2}{(N+1)} (\partial_2^\gamma - \partial_1^\gamma + (N+1) \left( \frac{u}{\bar{u}} \partial_2^\gamma - \frac{\bar{u}}{u} \partial_1^\gamma \right)) \right] \right. \\
 &\quad \left. - (x_{12}^\nu \epsilon^\mu{}_{\alpha\beta\gamma} + x_{12}^\mu \epsilon^\nu{}_{\alpha\beta\gamma}) x_{12}^\alpha \left( 1 - \frac{1}{4} \frac{u\bar{u} x_{12}^2 \partial^2}{N+1} \right) \partial_1^\beta \partial_2^\gamma \right\} \mathcal{O}_{A,N}^{(0)}(x_{21}^u) + \dots
 \end{aligned}$$



## Kinematic corrections: nonlocal form

- resumming back into nonlocal operator  $\mapsto$  GPD/DD

[V. Braun, Y.J. A. Manashov (2022)]

$$\begin{aligned}
 \mathbb{A}_V^{\mu\nu} &= \text{T}\{j^\mu(x)j^\nu(0)\} \\
 &= \frac{1}{i\pi^2} \left\{ \frac{1}{(-x^2 + i0)^2} \int_0^1 dv \left[ [g^{\mu\nu}(x\partial) - x^\mu\partial^\nu]O(v,0) - x^\nu(\partial^\mu - i\Delta^\mu)O(1,v) \right] \right. \\
 &\quad - \frac{1}{(-x^2 + i0)} \int_0^1 du \int_0^u dv \left[ \frac{i}{2}(\Delta^\nu\partial^\mu - \Delta^\mu\partial^\nu)O(u,v) - \frac{\Delta^2\bar{u}}{4}x^\mu\partial^\nu O(u,v) \right] \\
 &\quad \left. + \frac{\Delta^2}{2} \frac{x^\mu x^\nu}{(-x^2 + i0)^2} \int_0^1 du u \int_0^u dv O(u,v) + \dots \right\},
 \end{aligned}$$

$O(z_1, z_2)$  non-local twist-2 light-ray operators  $\mapsto$  GPD/DD, also  $\Delta \cdot \partial_x O(z_1, z_2), \dots$

- many highly nontrivial cancellations must happen to resum local  $\mapsto$  nonlocal
  - need “intertwining operator” to relate operators of different conformal spin
- similar for axial-vector case



## Phenomenology: nucleon DVCS

- **Fourier transform to momentum space: final prediction to NNLP\***

$$\mathcal{A}_{\mu\nu}(q, q', p) = \varepsilon_{\mu}^{+} \varepsilon_{\nu}^{*+} \mathcal{A}^{++} + \varepsilon_{\mu}^{-} \varepsilon_{\nu}^{*-} \mathcal{A}^{++} + \varepsilon_{\mu}^{0} \varepsilon_{\nu}^{*+} \mathcal{A}^{0+} + \varepsilon_{\mu}^{0} \varepsilon_{\nu}^{*-} \mathcal{A}^{0+} \\ + \varepsilon_{\mu}^{+} \varepsilon_{\nu}^{*-} \mathcal{A}^{-+} + \varepsilon_{\mu}^{-} \varepsilon_{\nu}^{*+} \mathcal{A}^{-+}$$

- **Fourier transform to momentum space: final prediction to NNLP\***

$$\mathcal{A}^{++} \sim 1 + \frac{1}{Q^2} + \frac{1}{Q^4} + \dots, \\ \mathcal{A}^{0+} \sim \frac{1}{Q} + \frac{1}{Q^3} + \dots, \\ \mathcal{A}^{-+} \sim \frac{1}{Q^2} + \frac{1}{Q^4} + \dots,$$

- **Further contributions can be calculated if necessary**



## A glimpse to the final analytic results

- **structure of  $\mathcal{A}^{-+}$ , an example:**

[V. Braun, YJ, A. Manashov (2024), to appear]

$$\mathcal{A}_{1/Q^4}^{-+} = -\frac{3\kappa|P_\perp|^2}{2(q \cdot q')^2} \left( t \left( 1 + \frac{1}{3\xi} D_\xi \right) - \frac{1}{12} |P_\perp|^2 D_\xi^2 \right) \\ \times D_\xi^2 \int_{-1}^1 \frac{dx}{\xi} \mathcal{H}(x, \xi, t) \left( \frac{1}{\bar{x}_\xi} (\text{Li}_2(x_\xi) - \zeta_2) - \ln \bar{x}_\xi \right) + \dots$$

$$D_\xi = \xi^2 \partial_\xi, \quad x_\xi = \frac{x + \xi}{2\xi} + i0, \quad \bar{x}_\xi = 1 - x_\xi, \quad |P_\perp|^2 = -m^2 - t(1 - \xi^2)/(4\xi^2).$$

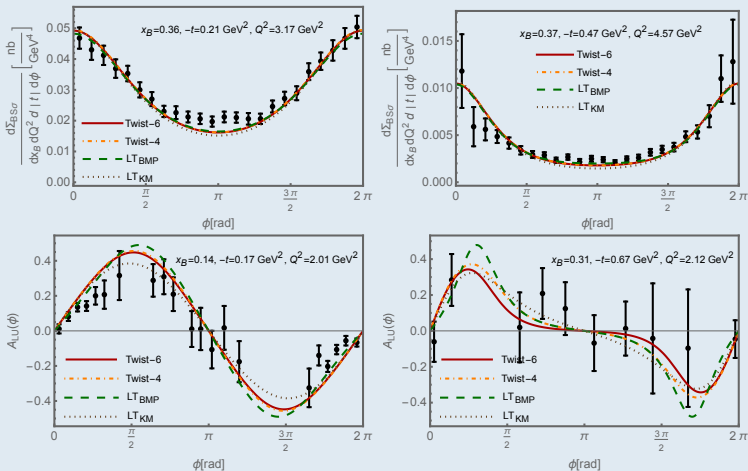
- $\mathcal{A}^{-+}$  and  $\mathcal{A}^{0+}$  **relatively simple**,  $\mathcal{A}_{++}$  **complicated; factorization holds!**
  - **factorization violating  $1/q'^2, \ln q'^2$  cancel in final result, highly nontrivial for  $\mathcal{A}^{++}$**
  - **only  $\text{Li}_2, \ln^2$  appears in final result**
  - **agree with previous lower-order result**

[V. Braun, A. Manashov, B. Pirnay, Phys. Rev. D 86 (2012) 014003]



## Numerics

- Expansion parameter  $1/Q^2 \rightarrow 1/(qq') = 2/(Q^2 + t)$ !



GPD model: GK12; lower panel: electron helicity dependent cross section





## Conclusions and outlooks

- **Conclusions**

- **NNLP corrections are small with expansion  $1/(qq') = -\frac{2}{Q^2+t}$ : resummation**
- **kinematic corrections to CFFs in BMJ frame are generally larger**
- **result respects all symmetries to  $1/Q^5$  order**
- **tree-level factorization holds to NNLP**

- **Outlooks**

- **more extensive numerical analysis for JLAB/EIC data**
- **embedding into public codes**
- **$\alpha_s$  corrections, kinematic corrections involving gluon GPD: long term**
- **apply to other exclusive processes: target mass correction**

[P. Ball, V. Braun, NPB543 (1999) 201]

