Energy-momentum tensor distribution for transversely polarized nucleons

Ho-Yeon Won

Supervisor: Cédric Lorcé

CPHT, CNRS, École polytechnique, Institute Polytechnique de Paris, 91120 Palaiseau, France

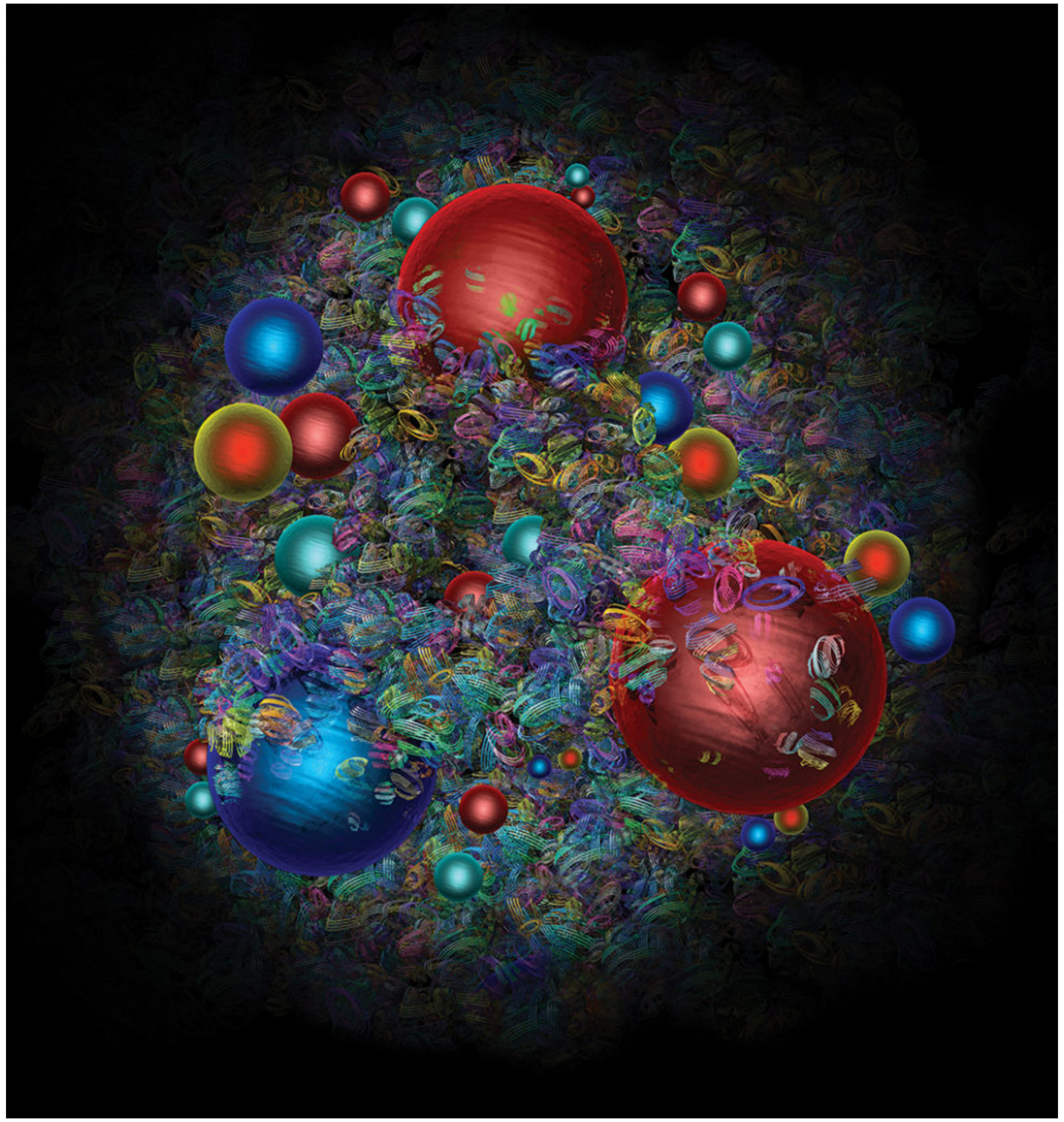
Contents

Introduction

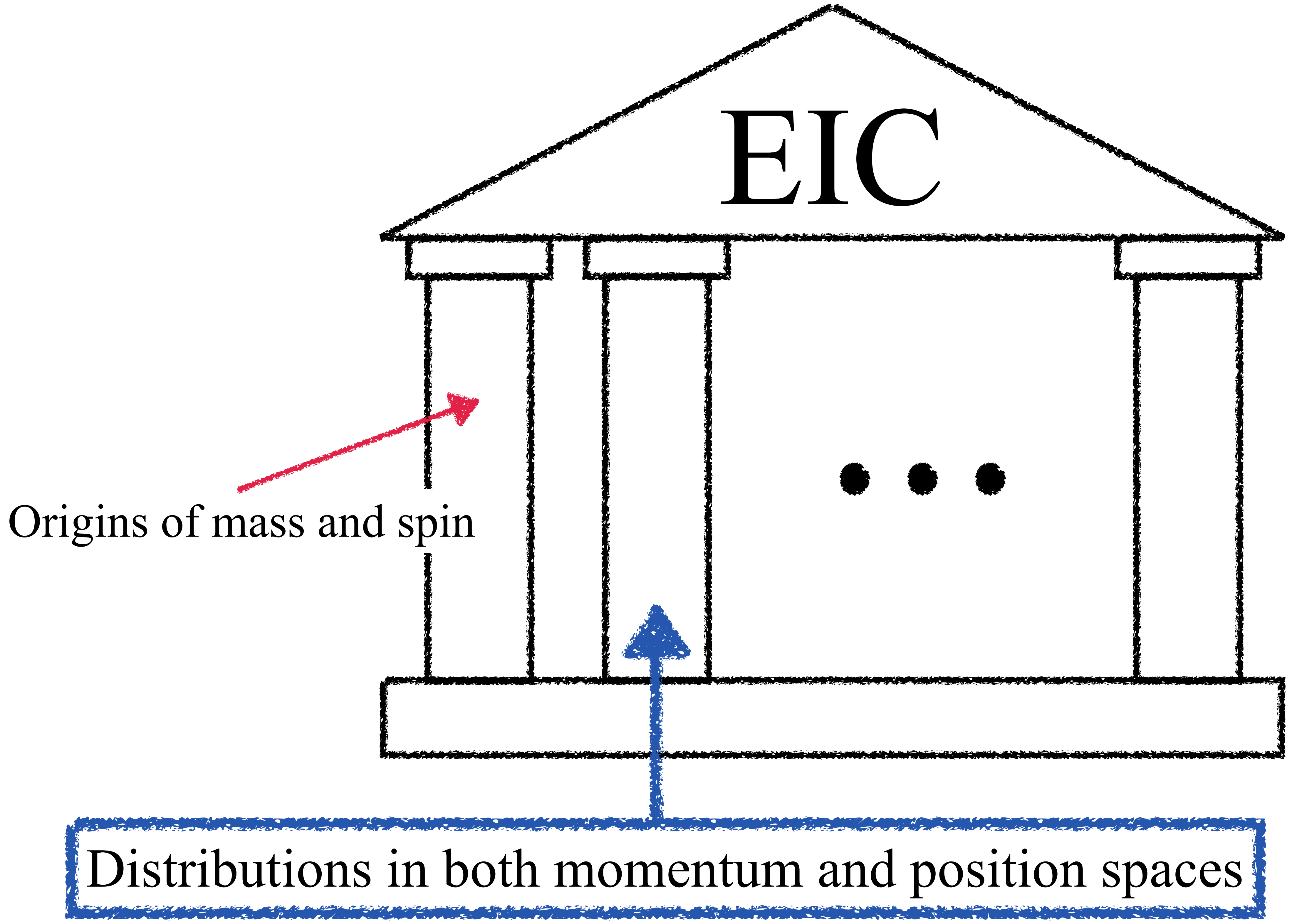
What is the nucleon?

Abdul Khalek et al., NPA 1026 (2022)

Modern understanding



Electron-Ion Collider (EIC) project

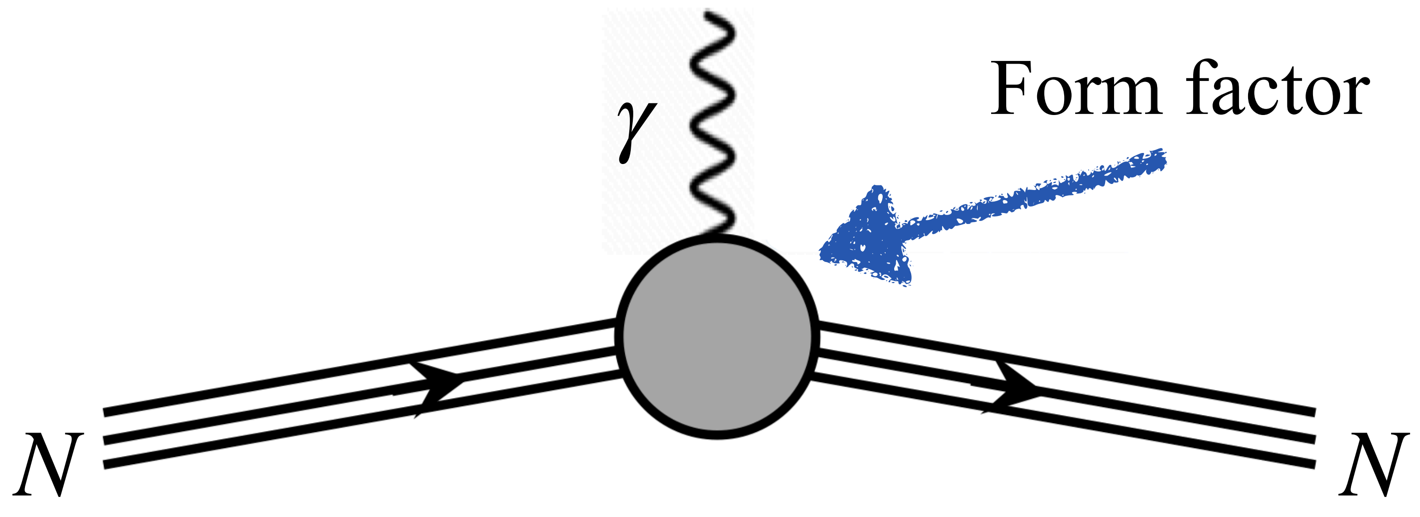


Images: Bensteele1995,CERN

Spatial distribution & energy-momentum tensor

Spatial distribution

Internal structure and form factor



$p = P - \frac{\Delta}{2}$ $\quad p' = P + \frac{\Delta}{2}$

$\mu_p \neq 1$, $\mu_n \neq 0$ → Implies the internal structure

Sachs, PR 126 (1962)
Jaffe, RRD 103 (2021)

Classical distribution

$$\rho(\boldsymbol{r}) := \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} G_E(t)$$

$t = -\Delta^2$

Particle size

Validity: $R \gg \bar{\lambda}$

Reduced Compton wavelength

However,

$$\sqrt{\langle r^2\rangle^Q_{\text{exp}}} = 0.84\ \text{fm} \quad \text{v.s.} \quad \bar{\lambda} = \frac{\hbar}{M_p c} \sim 0.21\ \text{fm}$$

PDG, PTEP 2022 (2022)

The internal dynamics of the nucleon is fully relativistic.

Friar, Negele, ANP 8 (1975)
Lorcé, PRL 125 (2020)

Corrected 3-D distribution in BF

$$\rho_{\text{BF}}(\boldsymbol{r}) := \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \frac{\langle p'|\hat{j}^0(0)|p\rangle|_{\text{BF}}}{2P^0_{\text{BF}}},$$

$$= \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \frac{M}{P^0_{\text{BF}}} G_E(t)$$

$\tau = -\frac{\Delta^2}{4M^2}$

$\frac{M}{P^0_{\text{BF}}} = \frac{1}{\sqrt{1+\tau}}$

Recoil correction

Infinite mass or large-N_c limit

$$\frac{M}{P^0_{\text{BF}}} \approx 1 \longrightarrow \rho_{\text{BF}} \to \rho|_{\text{Sachs}}$$

2-D distribution in LF

$P_z \to \infty \Rightarrow \Delta^0 \approx \Delta_z \ll P^0$

$\boldsymbol{b}_\perp$: impact parameter space

$$\rho_{\text{LF}}(\boldsymbol{b}_\perp) := \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \frac{\langle p'|\hat{j}^0(0)|p\rangle|_{\text{IMF}}}{2P^+},$$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left[F_1(t) + \frac{(\boldsymbol{\sigma}\times i\boldsymbol{\Delta}_\perp)_z}{2M} F_2(t)\right]$$

Dirac form factor $\quad$ Pauli form factor

There is no recoil correction

Lorcé, Wang, PRD105 (2022)
Chen, Lorcé, PRD106 (2022)

Interpolation between 3D BF and 2D LF?

Image: C.Lorcé's talk

Phase-space formalism

Hadronic matrix element

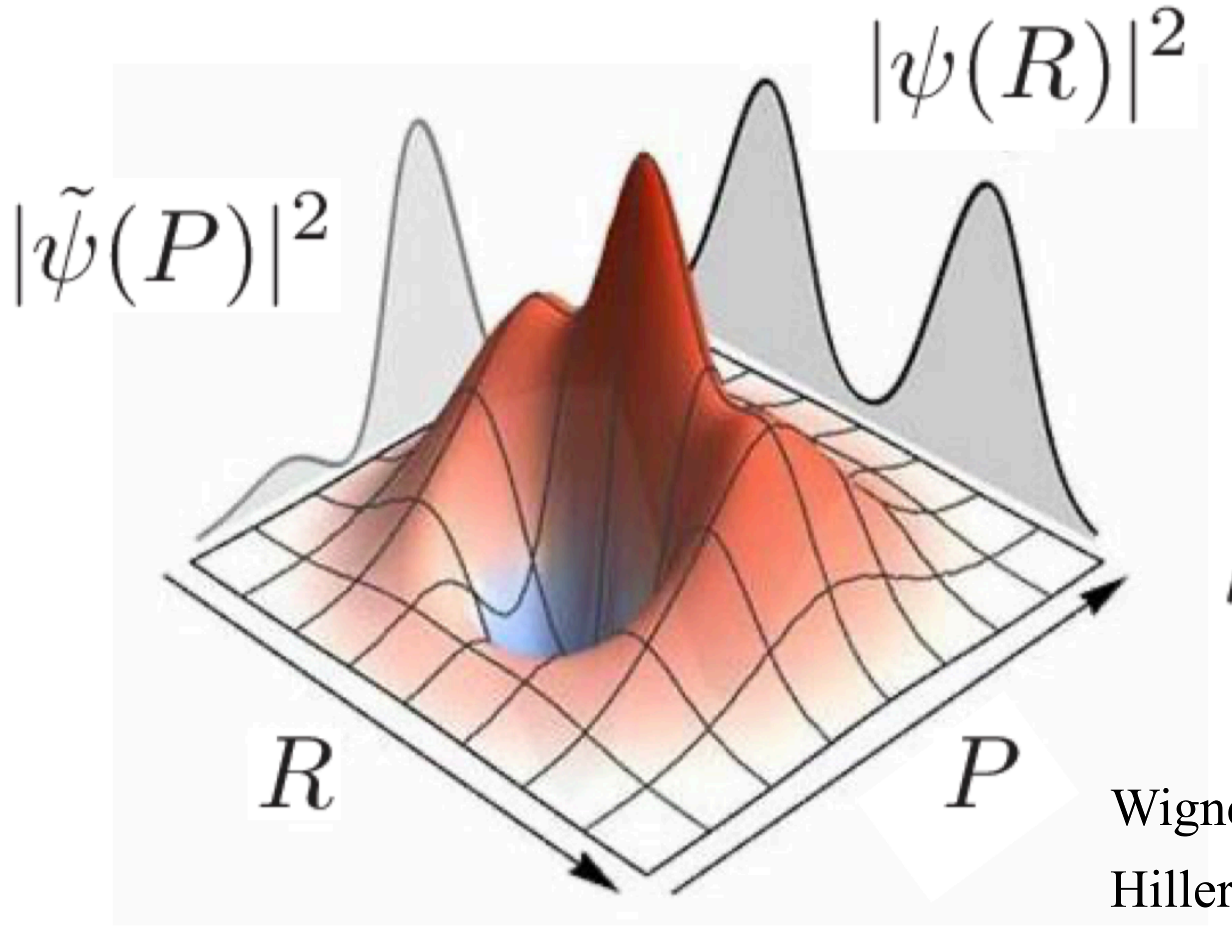
$$\langle\Psi|\widehat{\mathcal{O}}(\boldsymbol{r})|\Psi\rangle = \int \frac{d^3P}{(2\pi)^3}\int d^3R\, \rho_\Psi(\boldsymbol{R},\boldsymbol{P})\left\langle\widehat{\mathcal{O}}(\boldsymbol{r})\right\rangle_{\boldsymbol{R},\boldsymbol{P}}$$

Nucleon Wigner distribution

$$\rho_\Psi(\boldsymbol{R},\boldsymbol{P}) = \int \frac{d^3q}{(2\pi)^3}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{R}}\widetilde{\psi}^\dagger\left(\boldsymbol{P}+\frac{\boldsymbol{q}}{2}\right)\widetilde{\psi}\left(\boldsymbol{P}-\frac{\boldsymbol{q}}{2}\right)$$
$$= \int d^3Y\, e^{i\boldsymbol{P}\cdot\boldsymbol{Y}}\psi^\dagger\left(\boldsymbol{R}+\frac{\boldsymbol{Y}}{2}\right)\psi\left(\boldsymbol{R}-\frac{\boldsymbol{Y}}{2}\right)$$

Wave packet

$$\psi(\boldsymbol{r}) = \langle\boldsymbol{r}|\Psi\rangle$$
$$\widetilde{\psi}(\boldsymbol{p}) = \frac{1}{\sqrt{2p^0}}\langle\boldsymbol{p}|\Psi\rangle$$



Wigner, PR40 (1932)
Hillery, O'Connell, Scully, Wigner, PR106 (1984)
Bialynicki-Birula, Gornicki, Rafelski, PRD 44 (1991)

Quasi-probablistic interpretation

$$\int d^3R\, \rho_\Psi(\boldsymbol{R},\boldsymbol{P}) = \left|\widetilde{\psi}(\boldsymbol{P})\right|^2$$
$$\int \frac{d^3P}{(2\pi)^3}\, \rho_\Psi(\boldsymbol{R},\boldsymbol{P}) = |\psi(\boldsymbol{R})|^2$$

Matrix element of internal structure

$$\left\langle\widehat{\mathcal{O}}(\boldsymbol{r})\right\rangle_{\boldsymbol{R},\boldsymbol{P}} = \int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\boldsymbol{\Delta}\cdot\boldsymbol{R}}\frac{1}{\sqrt{2p_i^0}\sqrt{2p_f^0}}\left\langle P+\frac{\Delta}{2}\middle|\widehat{\mathcal{O}}(\boldsymbol{r})\middle|P-\frac{\Delta}{2}\right\rangle$$

Necessity of full information over the whole ***P*** variable

→ **Introducing the elastic frame**

Image: C.Lorcé's talk

Elastic frame

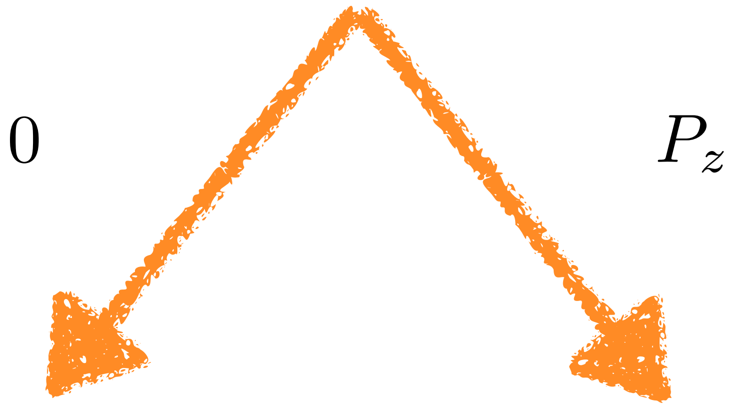
Durand, DeCelles, Marr, PR 126 (1962)

Polyzou, Glökle, Witala, FBS 54 (2013)

I. A generic frame, $\Delta_z = \Delta^0 = 0$

Connection between the rest (Breit) frame and the moving frame.

$$P = \frac{p' + p}{2} = (P^0, \mathbf{0}_\perp, P_z) \qquad \Delta = p' - p = (0, \mathbf{\Delta}_\perp, 0)$$



$P_z = 0$ $\qquad$ $P_z \to \infty$

Breit frame (BF)

$$P = (P^0, \mathbf{0}_\perp, 0), \quad \Delta = (0, \mathbf{\Delta}_\perp, 0)$$

Infinite momentum frame (IMF)

$$P \approx (P_z, \mathbf{0}_\perp, P_z), \quad \Delta = (0, \mathbf{\Delta}_\perp, 0)$$

cf. light-front frame
$P^+ \sim P_z, \quad P^- \approx 0$

Time-independent distribution

$$O(\boldsymbol{b}_\perp, P_z; s', s) = \int dr_z \left\langle \widehat{\mathcal{O}}(r) \right\rangle_{\boldsymbol{R},\boldsymbol{P}}$$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} \left. \frac{\langle p', s' | \widehat{\mathcal{O}}(0) | p, s \rangle}{2P^0} \right|_{\text{EF}},$$

$\boldsymbol{b}_\perp = \boldsymbol{r}_\perp - \boldsymbol{R}_\perp$

Lorcé, Mantovani, Pasquini, PLB 776 (2018)

Lorcé, Moutarde, Trawiński, EPJC 79 (2019)

Lorcé, EPJC 78 (2018)

Lorcé, PRL 125 (2020)

II. Relativistic spin dynamics

Spin-0 particle

$$\langle p' | \hat{T}^{\mu\nu}(0) | p \rangle = \Lambda^\mu_\alpha \Lambda^\nu_\beta \langle p'_{\text{BF}} | \hat{T}^{\alpha\beta}(0) | p_{\text{BF}} \rangle,$$

Trivial

Spin-$\frac{1}{2}$ particle

$$\langle p', s' | \hat{T}^{\mu\nu}(0) | p, s \rangle$$

$$= \sum_{s'_{\text{BF}}, s_{\text{BF}}} D_{s_{\text{BF}} s}(p_{\text{BF}}, \Lambda) D^*_{s'_{\text{BF}} s'}(p'_{\text{BF}}, \Lambda) \Lambda^\mu_\alpha \Lambda^\nu_\beta \langle p'_{\text{BF}}, s'_{\text{BF}} | \hat{T}^{\alpha\beta}(0) | p_{\text{BF}}, s_{\text{BF}} \rangle,$$

Complicated

Irreciprocal process

$p'^\mu = \Lambda^\mu_\nu p^\nu$

$|p, s\rangle \xrightarrow{U(\Lambda)} |p', s'\rangle$

$U(p)$: $|0,s\rangle \to |p, s\rangle$; $\quad U^{-1}(\Lambda p)$: $|p', s'\rangle \to \sum_{s'} D_{ss'}(p,\Lambda)|0,s'\rangle$

$$|0,s\rangle \neq \sum_{s'} D_{ss'}(p, \Lambda) |0,s'\rangle$$

Wigner rotation

$$[K^i, K^j] = -i\varepsilon^{ijk} J^k$$

Melosh rotation

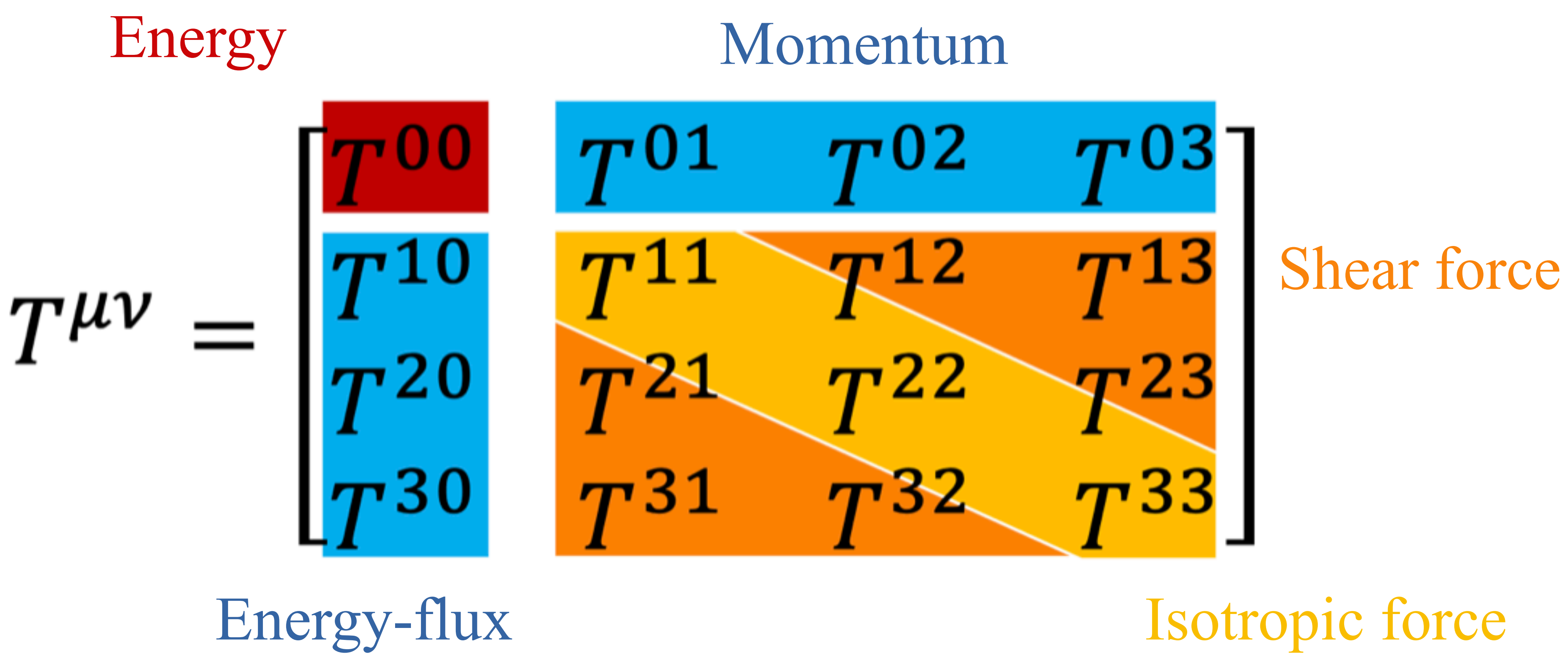
$$|p, \lambda\rangle = \sum_s \mathcal{M}_{s\lambda} |p, s\rangle$$

$$\lim_{p_z \to \infty} D(p, \Lambda) = \mathcal{M}$$

Energy-momentum tensor

Energy-momentum tensor (EMT)

Conserved current under space-time translations



V.D. Burkert et al., RMP 95 (2023)

Matrix elements of the EMT current

$$\langle p', s'|\hat{T}_a^{\mu\nu}(0)|p, s\rangle$$

$$= \bar{u}(p', s')\left[\frac{P^\mu P^\nu}{M} A_a(Q^2) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} D_a(Q^2) + M g^{\mu\nu}\bar{C}_a(Q^2) + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} J_a(Q^2) - \frac{iP^{[\mu}\sigma^{\nu]\rho}\Delta_\rho}{2M} S_a(Q^2)\right] u(p, s)$$

BF Matrix elements

$$\langle p', s'|\widehat{T}_a^{00}(0)|p, s\rangle\Big|_{\mathrm{BF}} = 2P^0_{\mathrm{BF}} M E_a(Q^2)\, \delta_{s'_{\mathrm{BF}} s_{\mathrm{BF}}},$$

Energy form factor

$$\langle p', s'|\widehat{T}_a^{03}(0)|p, s\rangle\Big|_{\mathrm{BF}} = 2P^0_{\mathrm{BF}} \frac{(\boldsymbol{\sigma}_{s'_{\mathrm{BF}} s_{\mathrm{BF}}} \times i\boldsymbol{\Delta})^k}{2M} \left[J_a(Q^2) - S_a(Q^2)\right],$$

Total angular momentum

$$\langle p', s'|\widehat{T}_a^{30}(0)|p, s\rangle\Big|_{\mathrm{BF}} = 2P^0_{\mathrm{BF}} \frac{(\boldsymbol{\sigma}_{s'_{\mathrm{BF}} s_{\mathrm{BF}}} \times i\boldsymbol{\Delta})^k}{2M} \left[J_a(Q^2) + S_a(Q^2)\right],$$

Intrinsic spin

$$\langle p', s'|\widehat{T}_a^{33}(0)|p, s\rangle\Big|_{\mathrm{BF}} = 2P^0_{\mathrm{BF}} M F_a(Q^2)\, \delta_{s'_{\mathrm{BF}} s_{\mathrm{BF}}},$$

Force

$$E = A + \overline{C} + \tau(A - 2J + D),$$
$$J = \frac{A + B}{2},$$
$$F = -\tau D - \overline{C},$$

Parametrization

$\mu = 2$ GeV

F_a	n_F	$F_q(0)$	Λ_{F_q} (GeV)	$F_G(0)$	Λ_{F_G} (GeV)
A_a	2	0.55	0.91	0.45	0.91
B_a	3	−0.07	0.80	0.07	0.80
D_a	3	−1.28	0.80	−2.24	0.80
$\bar{C}_a$	2	−0.11	0.91	0.11	0.91
S_a	2	0.33	1.00	−	−

C. Lorcé, Moutarde, Trawiński, EPJC 79 (2019)

Multi-pole ansatz

$$F_a(t) = \frac{F_a(0)}{(1 - t/\Lambda^2_{F_a})^{n_F}}$$

Spatial distributions of the EMT

Relativistic EMT quantities

Lorentz boost factors

$$\gamma_P = \frac{E_P}{M}, \quad \beta_P = \frac{P_z}{E_P}, \quad E_P = \sqrt{P_z^2 + M^2},$$

$$\int d^2 b_\perp \, T_a^{\mu\nu}(\boldsymbol{b}_\perp, P_z; s', s)$$

$$= \begin{pmatrix} M\gamma_P & 0 & 0 & M\gamma_P\beta_P \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ M\gamma_P\beta_P & 0 & 0 & M\gamma_P\beta_P^2 \end{pmatrix} \delta_{s's} = \frac{\mathfrak{P}^\mu \mathfrak{P}^\nu}{\mathfrak{P}^0}\delta_{s's},$$

$$\mathfrak{P} = M\gamma_P(1, \mathbf{0}_\perp, \beta_P)$$

$T^{00} \to \infty$ as $P_z \to \infty$ $\quad$ T^{00} **: relativistic inertia density**

The order of the Lorentz boost factor

In moving frame due to the Lorentz transformation and Wigner rotation,

$$\hat{j}^0 \sim P_z^1 \longrightarrow \frac{\langle p', s' | \hat{j}^0(0) | p, s\rangle}{2P^0} \sim P_z^0$$

$$\widehat{T}^{00} \sim P_z^2 \longrightarrow \frac{\langle p', s' | \widehat{T}^{00}(0) | p, s\rangle}{2P^0} \sim P_z^1$$

$P^0 \propto P_z^1$

→ It causes the *divergence* in the EMT distributions.

Redefinition of EMT distributions

Using the Lorentz transformation on a generic second-rank tensor,

Energy; Longitudinal momentum; Longitudinal energy-flux; Longitudinal normal force

$$t_a^{\mu\nu} = \begin{pmatrix} \gamma_P^2 \rho_a & \gamma_P \mathcal{P}_a^i & \gamma_P^2 \mathcal{P}_a^z \\ \gamma_P \mathcal{I}_a^i & t_a^{ij} & \gamma_P \Pi_a^{iz} \\ \gamma_P^2 \mathcal{I}_a^z & \gamma_P \Pi_a^{zi} & \gamma_P^2 \sigma_a^z \end{pmatrix} = \gamma_P T_a^{\mu\nu},$$

from the denominator $2P^0$

$i = 1, 2$

Our main targets

With the normalizations

$$\int d^2 b_\perp \sum_{a=q,G} \rho_a(\boldsymbol{b}_\perp, P_z; s', s) = M\delta_{s's}.$$

$$\int d^2 b_\perp \sum_{a=q,G} h_a(\boldsymbol{b}_\perp, P_z; s', s) = M\beta_P\delta_{s's},$$

$$\int d^2 b_\perp \sum_{a=q,G} \sigma_a^z(\boldsymbol{b}_\perp, P_z; s', s) = M\beta_P^2\delta_{s's}.$$

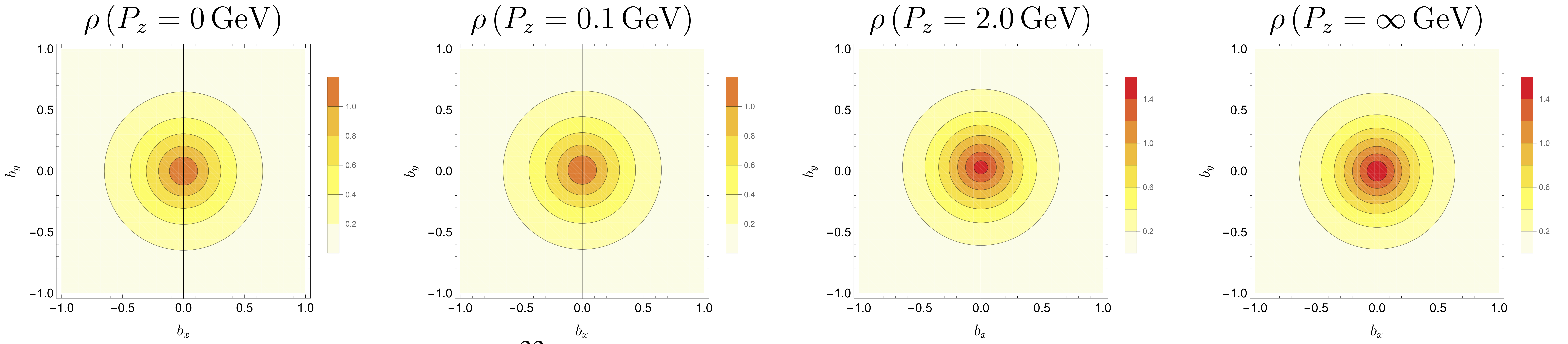
$h = \mathcal{P}^z, \mathcal{I}^z$

Results on the polarized EMT distributions

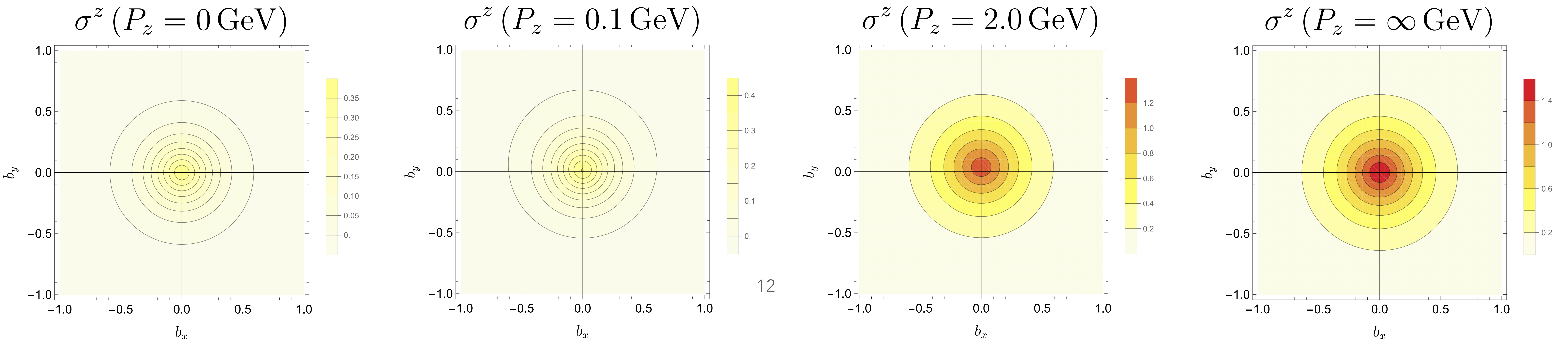
Energy and longitudinal normal force

$N \bullet\!\!\longrightarrow +x$ $\qquad P_z \to \infty$

Polarized energy distribution, T^{00}



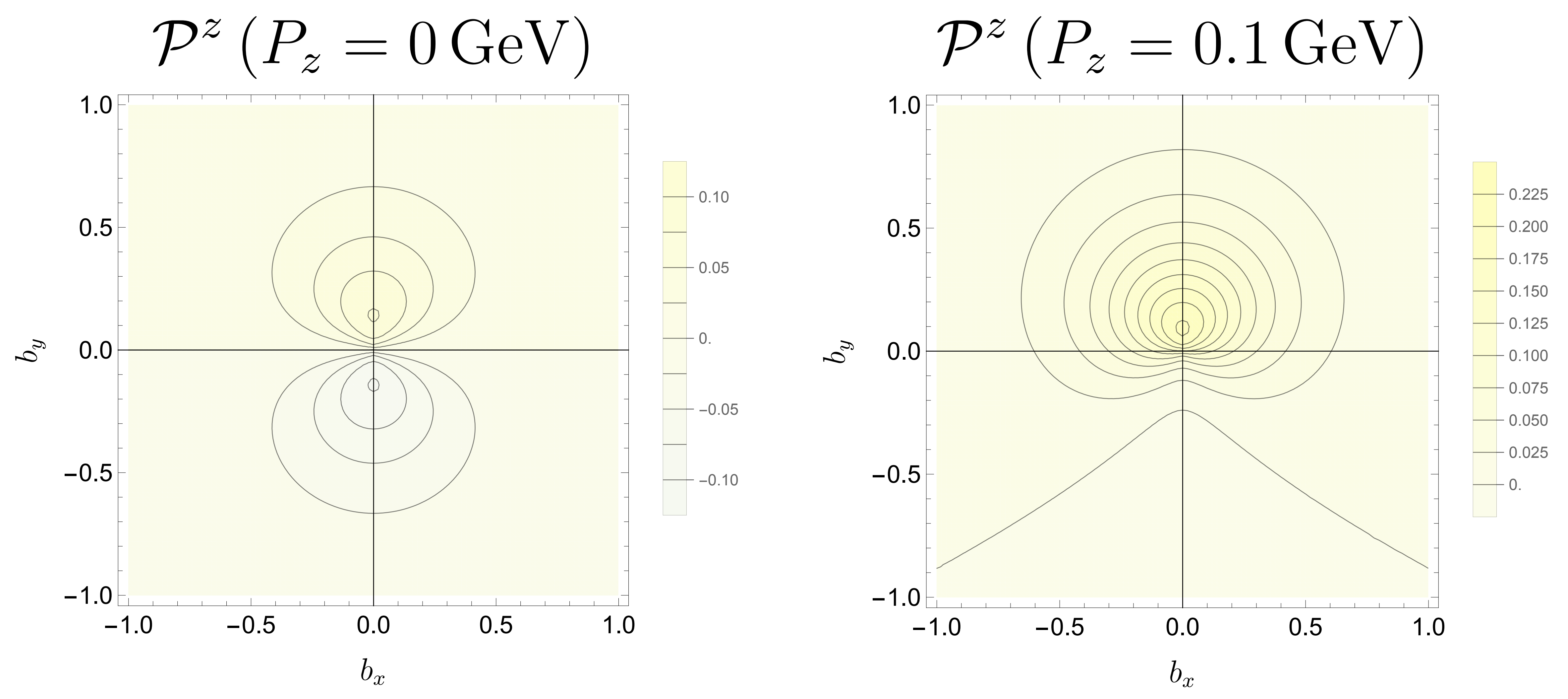
Polarized longitudinal normal force, T^{33}

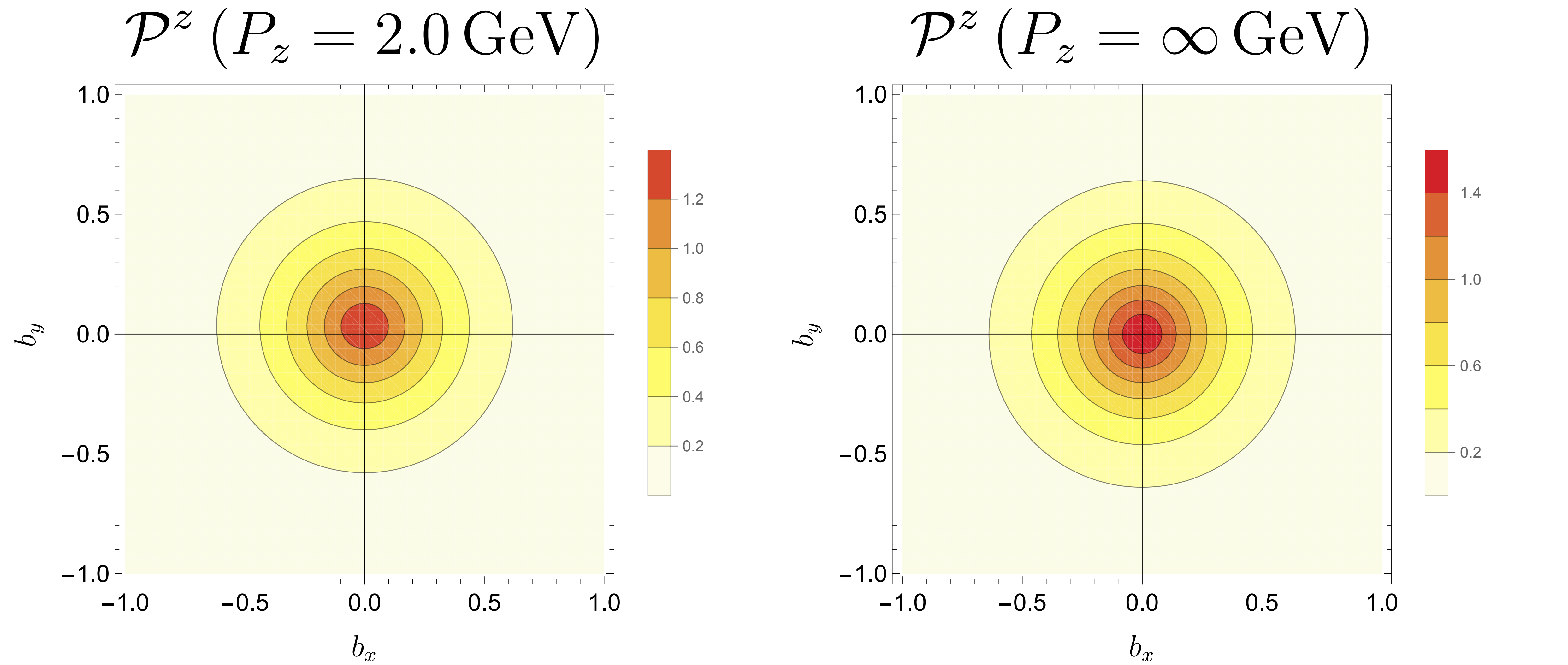


Longitudinal momentum and longitudinal energy-flux

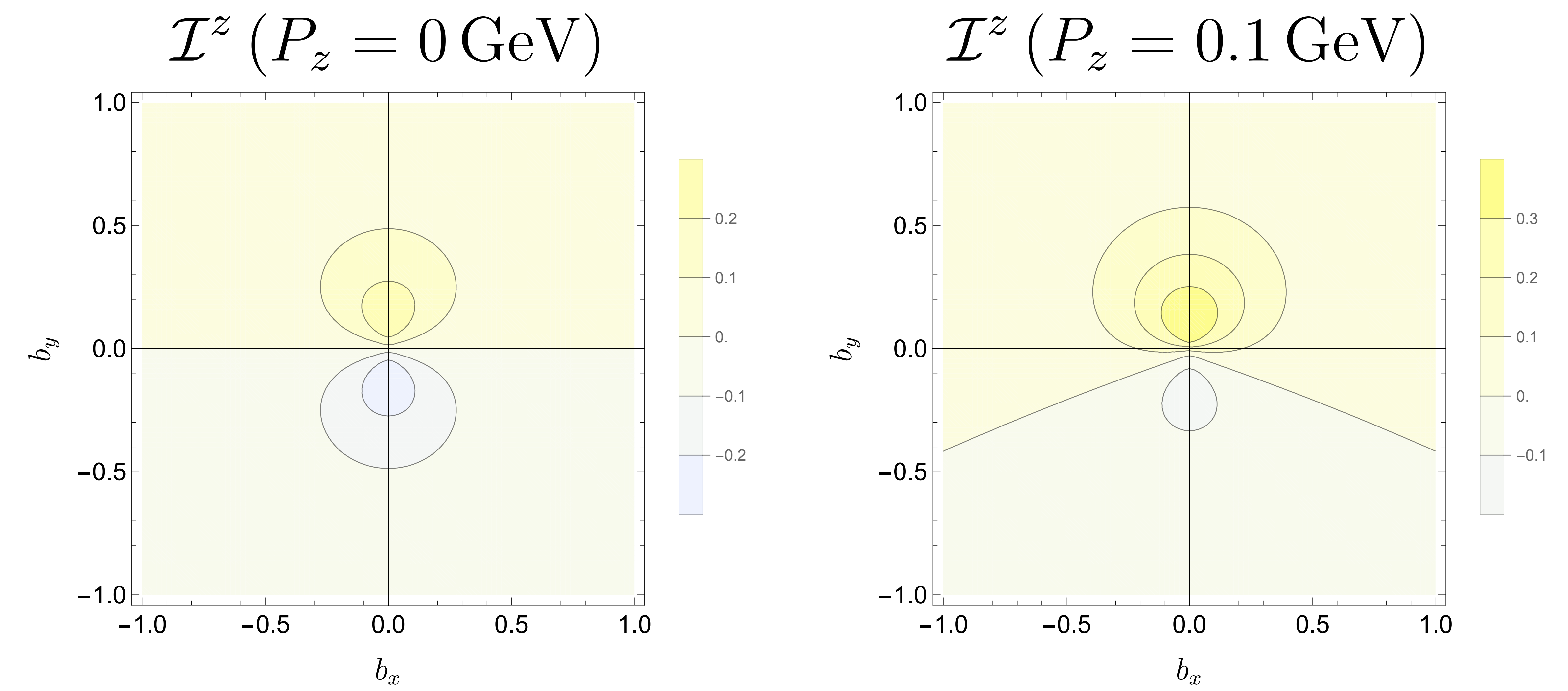
$N \bullet\!\!\longrightarrow +x$ $\qquad$ $P_z \to \infty$

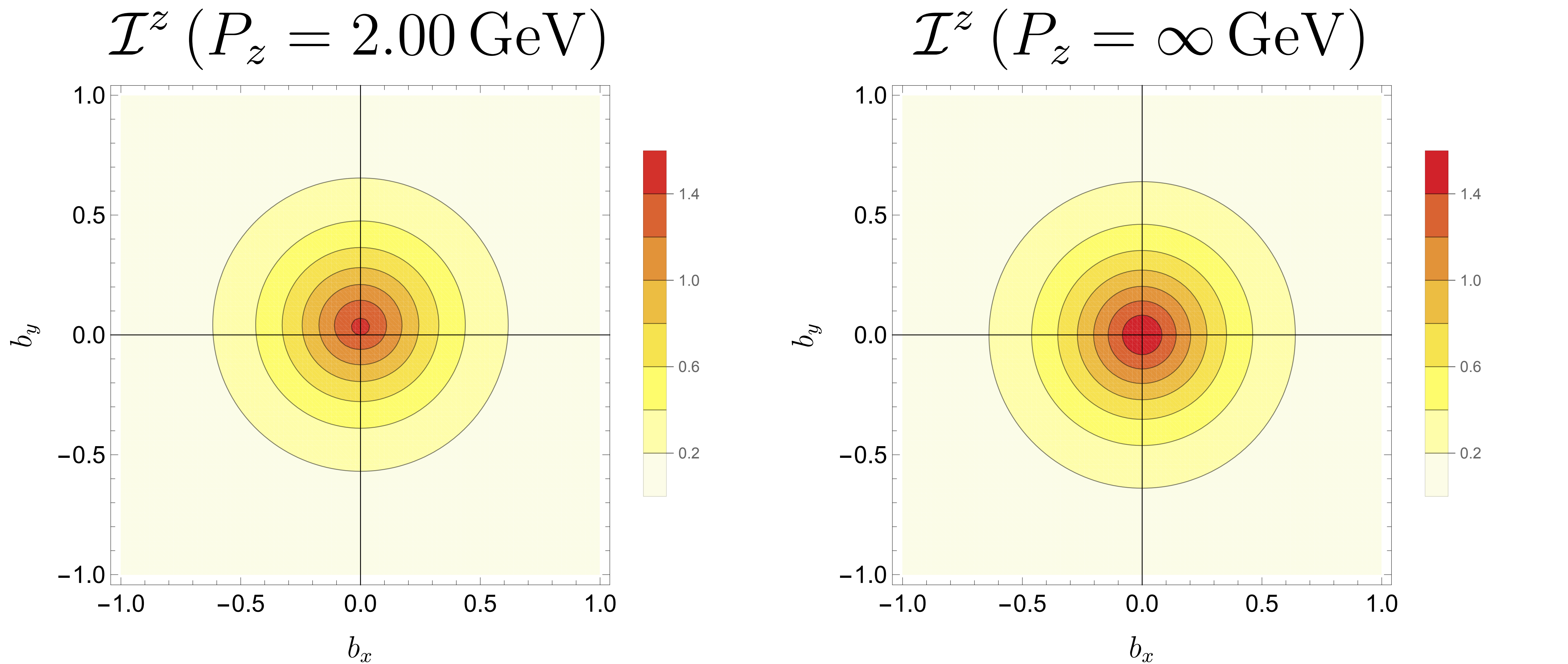
Polarized longitudinal momentum, T^{03}



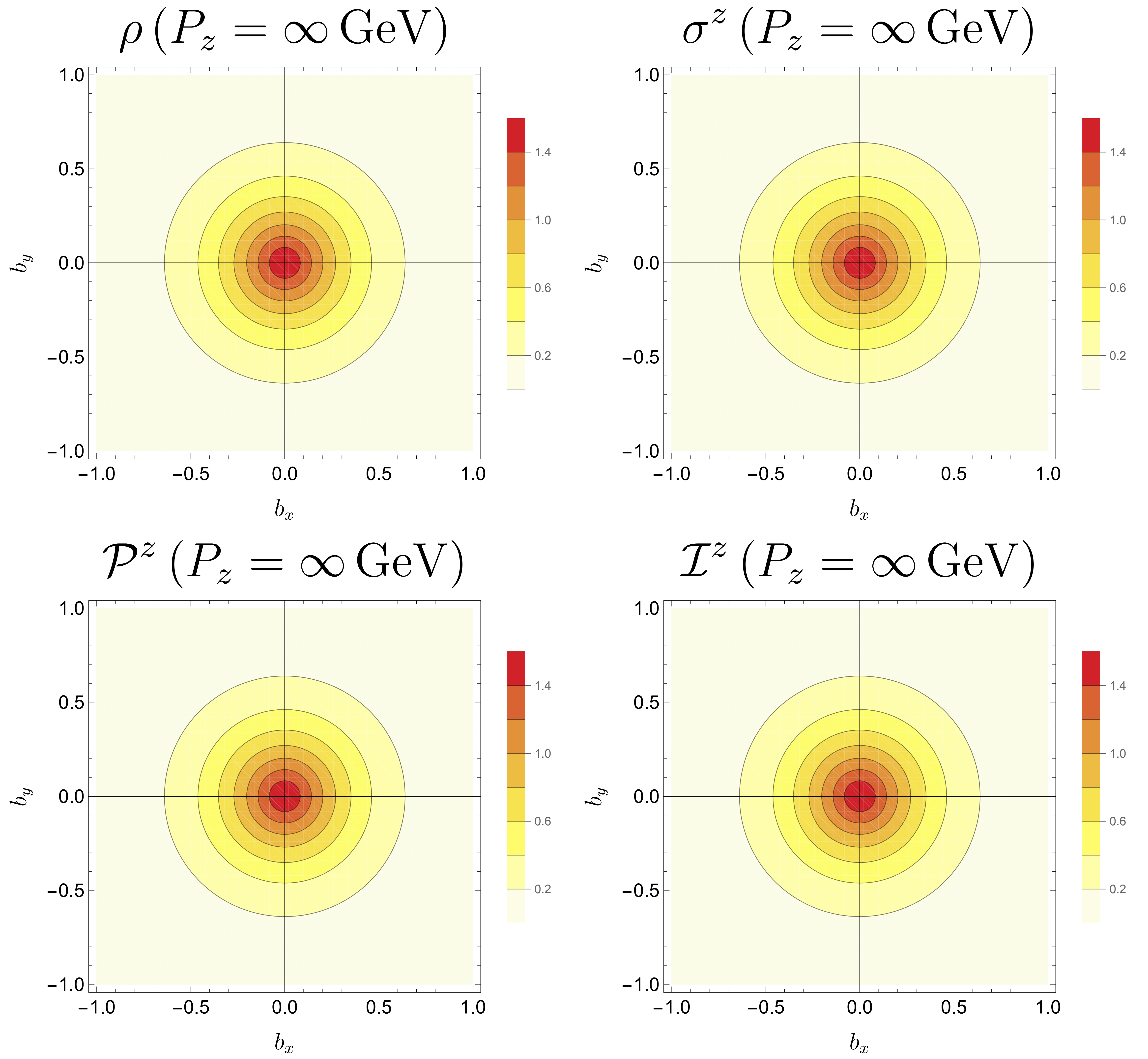


Polarized longitudinal energy-flux, T^{30}





Infinite momentum frame



In the EMT distributions,

∵ Relativistic pivot

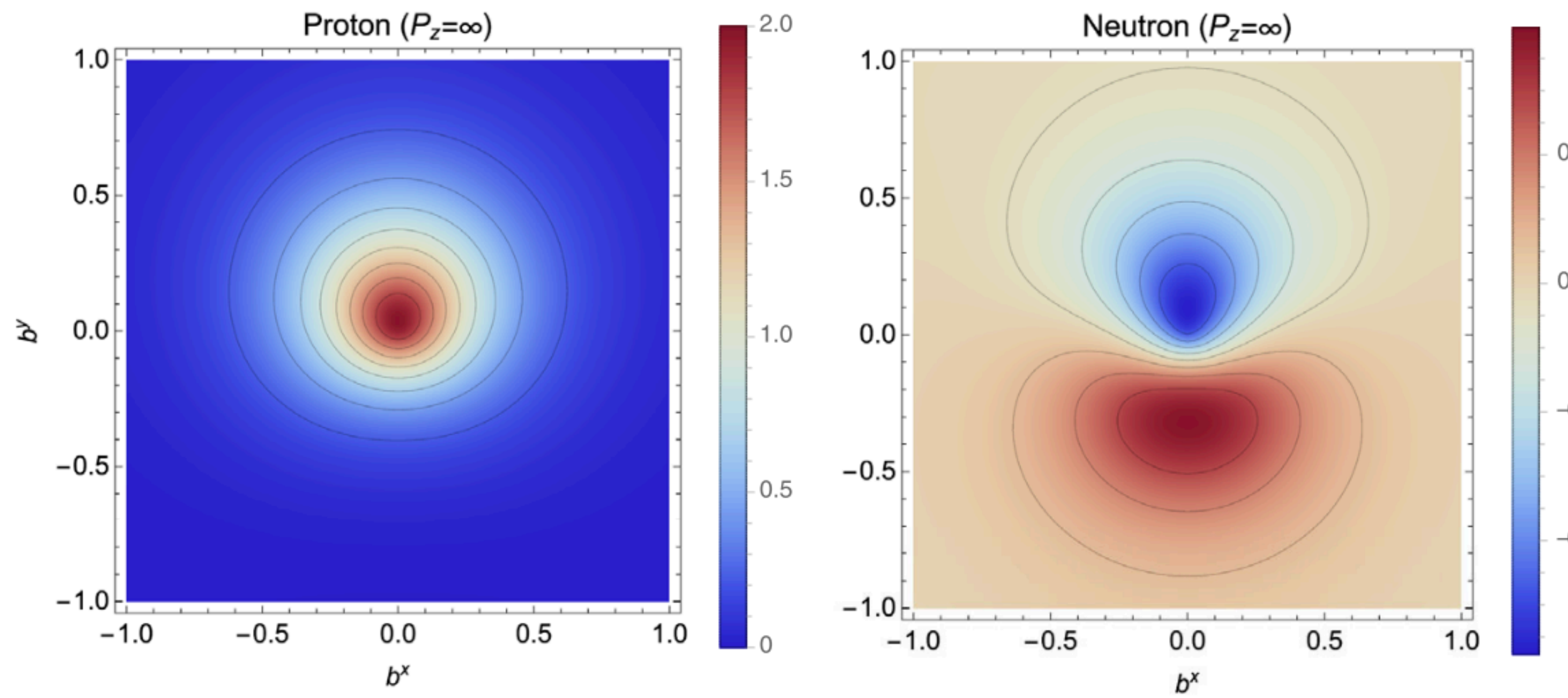
$$\rho\,(\boldsymbol{b}_\perp, \infty) = \mathcal{P}^z\,(\boldsymbol{b}_\perp, \infty) = \mathcal{I}^z\,(\boldsymbol{b}_\perp, \infty) = \sigma^z\,(\boldsymbol{b}_\perp, \infty)$$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left[A(t) + \frac{(\boldsymbol{\sigma}\times i\boldsymbol{\Delta}_\perp)_z}{2M} B(t)\right]$$

∵ Multi-pole mass

In the electric distribution,

$$\rho^{\mathrm{ch}}_{\mathrm{IMF}}\,(\boldsymbol{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left[F_1(t) + \frac{(\boldsymbol{\sigma}\times i\boldsymbol{\Delta}_\perp)_z}{2M} F_2(t)\right]$$



J.-Y. Kim et al., PRD 104 (2021)

Conclusion

Conclusion

- We study the **energy**, **longitudinal momentum**, **longitudinal energy-flux**, and **longitudinal normal force** distributions in the elastic frame.
- The phase-space formalism is realized by the **elastic frame**, which is a generic frame for defining the time-independent distributions in both rest and infinite momentum frames.
- It also considers the relativistic spin effect emerged as **Wigner rotation**.
- **In the IMF, these EMT distributions coincide because the relativistic centers of energy and canonical center merge in the transverse plane.**
- This work can be expanded to study the **P-odd EMT distributions** including spin-orbit correlation.

Thank you for listening

Back up

Physical spin states in the relativistic sense

$$|p,s\rangle = \sum_{s_{\rm BF}} U(\Lambda)\,|p_{\rm BF}, s_{\rm BF}\rangle D_{s_{\rm BF}s}\left(p_{\rm BF}, \Lambda\right)$$

$$\to D_{s_{\rm BF}s}\left(p_{\rm BF}, \Lambda\right) = \langle 0,s\,|\,U^{-1}\left(\Lambda p_{\rm BF}\right) U(\Lambda)\, U\left(p_{\rm BF}\right)|\,0,s_{\rm BF}\rangle$$

Matrix elements of quark P-odd EMT operator

$$< p', s'|\,\widehat{T}^{\mu\nu}_{q5}(0)\,|p,s> = \bar{u}\left(p',s'\right)\left[\frac{P^{\{\mu}\gamma^{\nu\}}\gamma_5}{2}\tilde{A}_a\left(Q^2\right) + \frac{P^{\{\mu}\Delta^{\nu\}}\gamma_5}{2}\tilde{B}_a\left(Q^2\right)\right.$$

$$\left. + \frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{2}\tilde{C}_a\left(Q^2\right) + \frac{P^{[\mu}\Delta^{\nu]}\gamma_5}{2}\tilde{D}_a\left(Q^2\right) + Mi\sigma^{\mu\nu}\gamma_5\tilde{F}_a\left(Q^2\right)\right] u\left(p,s\right)$$

Back up

EMT distributions in the EF

$$g_a\left(\boldsymbol{b}_\perp, P_z; s', s\right) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left[\delta_{s's}\widetilde{g}_a^U\left(Q^2, P_z\right) + \frac{\left(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta}_\perp\right)_z}{2M}\widetilde{g}_a^T\left(Q^2, P_z\right)\right],$$

Energy

$$\widetilde{\rho}_a^U\left(Q^2, P_z\right) = \frac{M}{\sqrt{P_z^2+M^2}}\frac{P^0\left[P^0+M\left(1+\tau\right)\right]}{\left(P^0+M\right)\left(1+\tau\right)}\left\{E_a\left(Q^2\right) + \frac{2\tau P_z^2}{P^0\left[P^0+M\left(1+\tau\right)\right]}J_a\left(Q^2\right) + \left(\frac{P_z}{P^0}\right)^2 F_a\left(Q^2\right)\right\}$$

$$\widetilde{\rho}_a^T\left(Q^2, P_z\right) = \frac{M}{\sqrt{P_z^2+M^2}}\frac{P^0 P_z}{\left(P^0+M\right)\left(1+\tau\right)}\left\{-E_a\left(Q^2\right) + \frac{2\left[P^0+M\left(1+\tau\right)\right]}{P^0}J_a\left(Q^2\right) - \left(\frac{P_z}{P^0}\right)^2 F_a\left(Q^2\right)\right\},$$

$$E_a\left(Q^2\right) = A_a\left(Q^2\right) - \tau B_a\left(Q^2\right) + \bar{C}_a\left(Q^2\right) + \tau D_a\left(Q^2\right),$$

$$J_a\left(Q^2\right) = \frac{1}{2}\left[A_a\left(Q^2\right) + B_a\left(Q^2\right)\right],$$

$$F_a\left(Q^2\right) = -\tau D_a\left(Q^2\right) - \bar{C}_a\left(Q^2\right),$$

Back up

Longitudinal momentum

$$\widetilde{\mathcal{P}}_a^{z,U}\left(Q^2,P_z\right)=\frac{M}{\sqrt{P_z^2+M^2}}\frac{P_z\left[P^0+M\left(1+\tau\right)\right]}{\left(P^0+M\right)\left(1+\tau\right)}$$
$$\times\left\{E_a\left(Q^2\right)+\frac{\tau P^2}{P^0\left[P^0+M\left(1+\tau\right)\right]}\left[\left(\frac{2P_z^2}{P^2}+1\right)J_a\left(Q^2\right)-S_a\left(Q^2\right)\right]+F_a\left(Q^2\right)\right\},$$
$$\widetilde{\mathcal{P}}_a^{z,T}\left(Q^2,P_z\right)=\frac{M}{\sqrt{P_z^2+M^2}}\frac{P_z^2}{\left(P^0+M\right)\left(1+\tau\right)}$$
$$\times\left\{-E_a\left(Q^2\right)+\frac{P^2\left[P^0+M\left(1+\tau\right)\right]}{P^0P_z^2}\left[\left(\frac{2P_z^2}{P^2}+1\right)J_a\left(Q^2\right)-S_a\left(Q^2\right)\right]-F_a\left(Q^2\right)\right\},$$

Longitudinal normal force

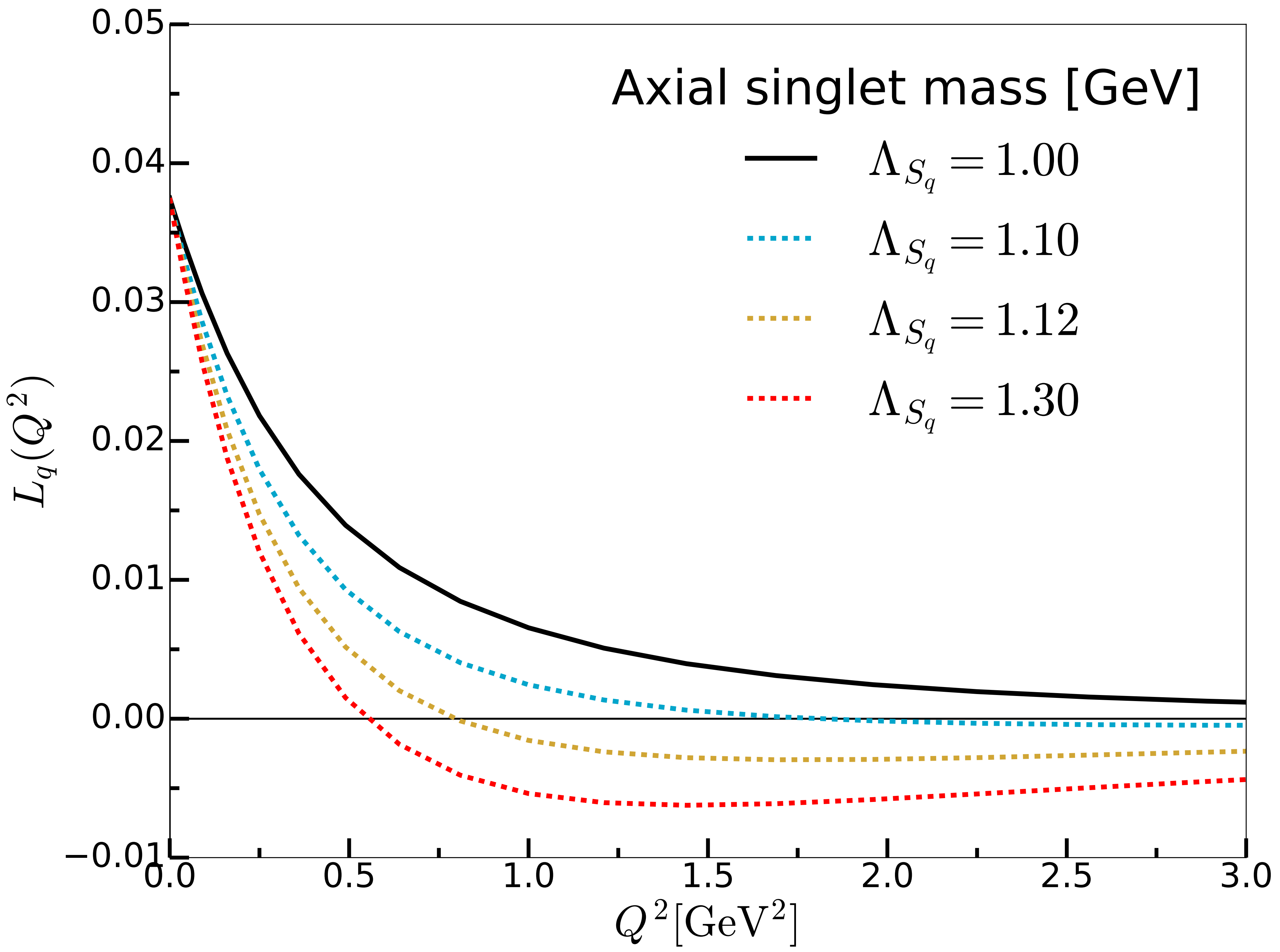
$$\widetilde{\sigma}_a^{z,U}\left(Q^2,P_z\right)=\frac{M}{\sqrt{P_z^2+M^2}}\frac{P^0\left[P^0+M\left(1+\tau\right)\right]}{\left(P^0+M\right)\left(1+\tau\right)}\left[\left(\frac{P_z}{P^0}\right)^2E_a\left(Q^2\right)+\frac{2\tau P_z^2}{P^0\left[P^0+M\left(1+\tau\right)\right]}J_a\left(Q^2\right)+F_a\left(Q^2\right)\right]$$
$$\widetilde{\sigma}_a^{z,T}\left(Q^2,P_z\right)=\frac{M}{\sqrt{P_z^2+M^2}}\frac{P^0P_z}{\left(P^0+M\right)\left(1+\tau\right)}\left[-\left(\frac{P_z}{P^0}\right)^2E_a\left(Q^2\right)+\frac{2\left[P^0+M\left(1+\tau\right)\right]}{P^0}J_a\left(Q^2\right)-F_a\left(Q^2\right)\right].$$

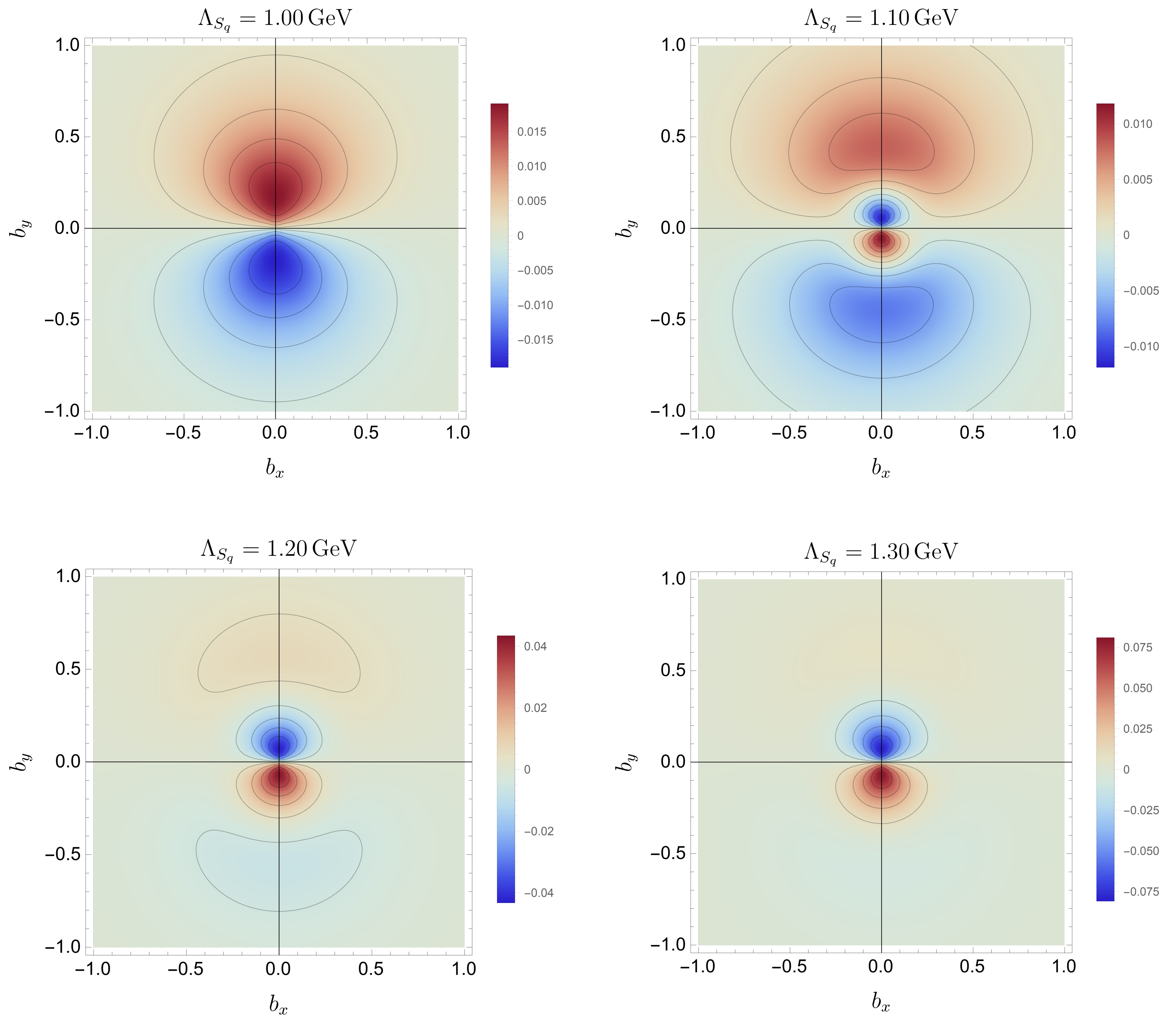
Back up

Longitudinal OAM distribution

For transversely polarized nucleon along the *x*-axis

$$\mathcal{P}_a^{P,z}(\boldsymbol{b}_\perp, 0) = M \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} \frac{i\Delta_y}{2M} L_a(Q^2),$$





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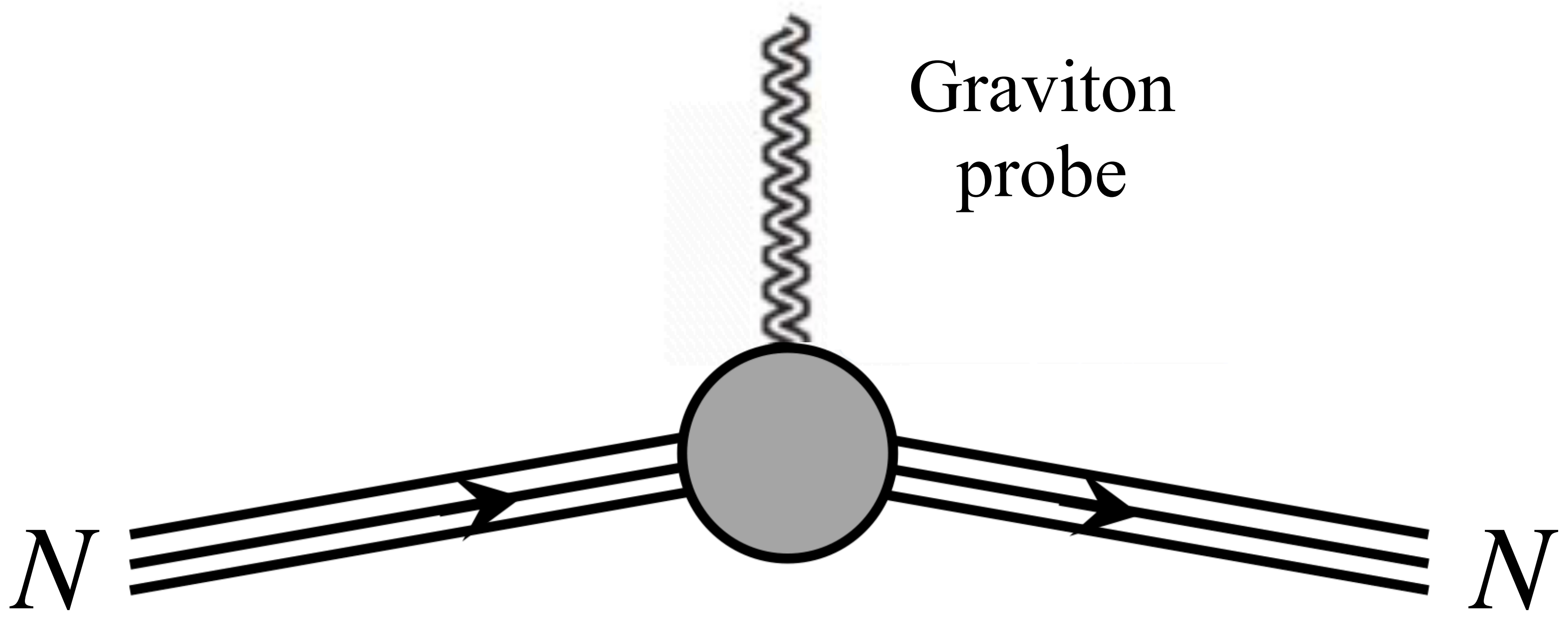
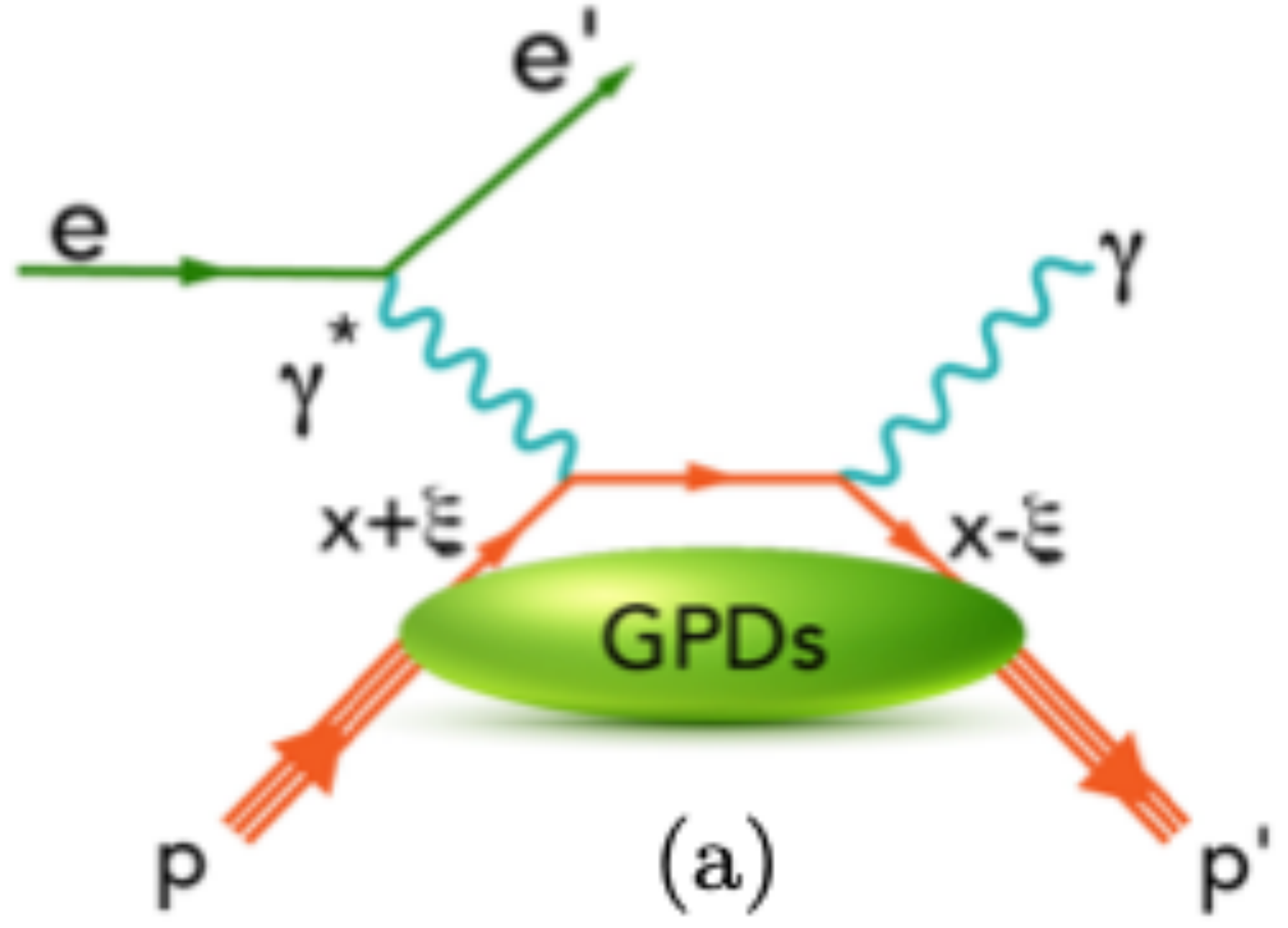
Direct measure



Too weak gravitation

Indirect measure



Deeply virtual Compton scattering

X.-D. Ji, PRL 78 (1997) 610

V.D. Burkert et al., Rev.Mod.Phys. 95 (2023) 4

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Matrix elements of non-local quark operator; Generalized Parton Distributions (GPDs)

$$\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix(P\cdot z)} < p', s' | \overline{\psi}_q \left(-\frac{\lambda n}{2}\right) \gamma^\mu n_\mu \psi_q \left(\frac{\lambda n}{2}\right) | p, s > \Bigg|_{z=\lambda n}$$

$$= \frac{1}{2(P\cdot n)} \overline{u}\left(p', s'\right) \left[H^q\left(x, \xi, t\right) \gamma^\mu n_\mu + E^q\left(x, \xi, t\right) \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} \right] u\left(p, s\right)$$

In the DVCS, the actual observables are Compton form factors (CFFs) at leading order α_s

$$\mathrm{Re}\mathscr{H}\left(\xi, t\right) + i\mathrm{Im}\mathscr{H}\left(\xi, t\right) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H_q\left(x, \xi, t\right)$$

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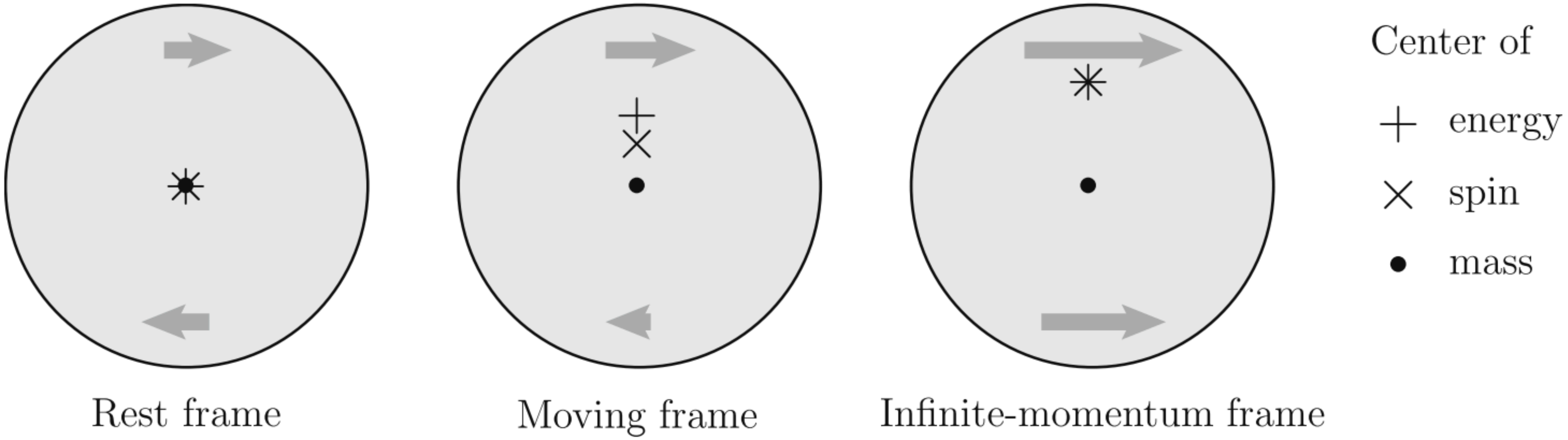
C. Lorcé, EPJC 81 (2023)

Relativistic pivots



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C. Lorcé's DIS 2024 talk

Center	Position operator	Canonical relation $[R^i, P^j] = i\delta^{ij}$	Vector under rotations $[J^i, R^j] = i\epsilon^{ijk} R^k$	Compatibility of components $[R^i, R^j] = 0$	Four-vector transformation $R'^\mu = \Lambda^\mu{}_\nu R^\nu$	
P^+	$R^i_\perp = \frac{1}{P^+}\int \mathrm{d}x^-\,\mathrm{d}^2x_\perp\, x^i_\perp T^{++} \overset{x^+=0}{=} -\frac{B^i_\perp}{P^+}$	✓	✓	✓	✗	2D
Energy	$R^i_E = \frac{1}{P^0}\int \mathrm{d}^3x\, x^i T^{00} \overset{x^0=0}{=} -\frac{K^i}{P^0}$	✓	✓	✗	✗	
Mass	$R^\mu_M = \Lambda^\mu{}_\nu R^\nu_E\vert_{\text{rest}}$	✓	✓	✗	✓	3D
Canonical	$R^i_c = \frac{P^0 R^i_E + M R^i_M}{P^0 + M}$	✓	✓	✓	✗	

Relativistic center of	$\langle \boldsymbol{R}_X \rangle$	$\langle \boldsymbol{S}_X \rangle = \langle \boldsymbol{J} \rangle - \langle \boldsymbol{R}_X \times \boldsymbol{P} \rangle$
Energy ($X = E$)	$\boldsymbol{\mathcal{R}} + \frac{\boldsymbol{p}\times\boldsymbol{s}}{2p^0(p^0+m)}$	$\frac{m}{2p^0}\left(\boldsymbol{s} + \frac{\boldsymbol{p}(\boldsymbol{p}\cdot\boldsymbol{s})}{m(p^0+m)}\right)$
Mass ($X = M$)	$\boldsymbol{\mathcal{R}} - \frac{\boldsymbol{p}\times\boldsymbol{s}}{2m(p^0+m)}$	$\frac{p^0}{2m}\left(\boldsymbol{s} - \frac{\boldsymbol{p}(\boldsymbol{p}\cdot\boldsymbol{s})}{p^0(p^0+m)}\right)$
Spin ($X = c$)	$\boldsymbol{\mathcal{R}}$	$\frac{1}{2}\boldsymbol{s}$