Total gluon helicity without effective theory matching

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- Introduction to total gluon helicity
- Total gluon helicity from local matrix element in a fixed gauge
- Total gluon helicity from gauge invariant nonlocal matrix element

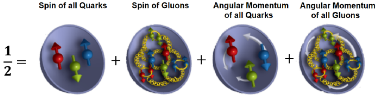
- Introduction to total gluon helicity

Total gluon helicity from local matrix element in a fixed gauge

Total gluon helicity from gauge invariant nonlocal matrix element

- Ji's decomposition(Ji, 1996):
  $$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2},$$
  (gauge invariant decomposition)

- Jaffe-Manohar decomposition(Jaffe,Manohar,1990):
  $$\frac{1}{2}\Delta\Sigma + S_g + \ell_q + \ell_g = \frac{\hbar}{2},$$
  (decomposition in IMF)



C.Alexandrou et al.,2020

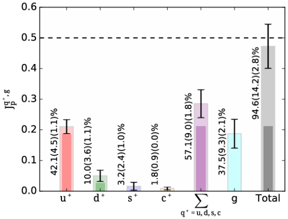


FIG. 24. The decomposition of the proton spin J. The color notation of the bars is as in Fig. 23. The second error quoted on the percentages is the systematic error from the Q^2 extrapolation needed in the determination of $B_{20}(0)$. The dashed horizontal line indicates the observed proton spin value, and the percentage is given relative to the total proton spin. Results are given in the $\overline{\mathrm{MS}}$ scheme at 2 GeV.

Introduction to total gluon helicity

From gluon helicity distribution to ΔG

First moment of the gluon polarized distribution gives total gluon helicity:

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle PS| F_a^{+\mu}(\xi^-) \mathcal{L}_{ab}(\xi^-, 0) \tilde{F}^+_{b,\mu}(0) |PS\rangle, \tilde{F}^{\alpha\beta} = 1/2\epsilon^{\alpha\beta\mu\nu} F_{\mu\nu}.$$

total gluon helicity ← $\left\langle PS \middle| (\vec{E}_a \times \vec{A}_{\perp,a})^z \middle| PS \right\rangle$ (Boost to IMF)

$A^+ = 0$ ↓ $\left\langle PS \middle| (\vec{E}_a \times \vec{A}_a)^z \middle| PS \right\rangle$

Gluon spin

$\nabla \cdot A = 0$ ↓ $\left\langle PS \middle| (\vec{E}_a \times \vec{A}_a)^z \middle| PS \right\rangle$

Implementable on lattice,

up to perturbative matching.

Ji,Zhang,Zhao,2013

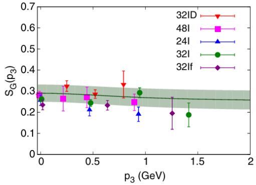


FIG. 4. The results extrapolated to the physical pion mass as a function of the absolute value of $\vec{p} = (0, 0, p_3)$, on all the five ensembles. All the results have been converted to $\overline{\text{MS}}$ at $\mu^2 = 10$ GeV^2. The data on several ensembles are shifted horizontally to enhance the legibility. The green band shows the frame dependence of the global fit [with the empirical form in Eq. (11)] of the results.

Yang el al., 2016

Introduction to total gluon helicity

Inconsistency between different matching proposals

- Matching between $\langle PS|\vec{E}\times\vec{A}|PS\rangle|_{\text{C.G.}}(=\Delta\tilde{G})$ and total gluon helicity:

  $\Delta\tilde{G} = C_{gg}\Delta G + C_{gq}\Delta\Sigma + h.t.,$

  $C_{gg} = 1 + a_s C_A \frac{7}{3}\ln\frac{P_z^2}{\mu^2} + \text{fin.},\quad C_{gq} = a_s C_F \frac{4}{3}\ln\frac{P_z^2}{\mu^2} + \text{fin.}.$
- Extract the gluon polarized distribution using LaMET, take its first moment,

  $\Delta\tilde{g}(x) = \int\frac{dy}{|y|}C_{gg}\left(\frac{x}{y},\frac{\mu}{yP_z}\right)\Delta g(y) + \int\frac{dy}{|y|}C_{gq}\left(\frac{x}{y},\frac{\mu}{yP_z}\right)\Delta q(y) + h.t.,$

  $C_{gg}\supset\delta(1-\frac{x}{y})+4a_sC_A\theta(x)\theta(y-x)\left\{\frac{(2x^2-3xy+2y^2)}{(x-y)y}\left(\ln\frac{\mu^2}{4y^2P_z^2}-\ln\frac{x(y-x)}{y^2}\right)+fin.\right\}.$
- The intrinsic momentum scale in factorization is the partonic momentum. However, in the factorization of $\Delta\tilde{G}$, the only available momentum scale is the hadronic momentum.
- $\int_{-\infty}^{\infty}dx\Delta\tilde{g}(x)\neq C_{gg}\Delta G + C_{gq}\Delta\Sigma + h.t.$, the r.h.s. is a convolution rather than a multiplication.

Introduction to total gluon helicity

Motivation

- (Previous proposal)
  $$\langle PS|\vec{E}\times\vec{A}|PS\rangle_{\text{C.G.}} \xrightarrow{\text{IMF}} \langle PS|\vec{E}\times\vec{A}|PS\rangle_{A^+=0} \equiv \Delta G,$$
  $\vec{E}\times\vec{A}$ is gauge-variant, for finite P_z, nontrivial matching exists.

- In Coulomb gauge, a nonperturbative renormalization is difficult to implement, the convergence of lattice perturbation theory is poor.

- To resolve above inconsistencies, we can choose appropriate operator O such that the matching between $\langle PS|O|PS\rangle$ and ΔG is trivial.

Introduction to total gluon helicity

- Total gluon helicity from local matrix element $\langle p|K^0|p\rangle_{\mathrm{C.G.}}$

Total gluon helicity from gauge invariant nonlocal matrix element

Gluon helicity from local matrix element in a fixed gauge

Our proposal

$\langle pS|K^0|pS\rangle|_{\nabla\cdot A=0,B}$ → boost → $\langle pS|K^0|pS\rangle|_{A^z=0,B} + h.t. = \Delta G_B + h.t.$

Renormalize the bare matrix element in RI/MOM scheme. ↓

$K^\mu \to K^\mu + \partial_\beta H^{\beta\mu}$ under a local gauge transformation

Cronström and Mickelsson,1983

$\langle pS|K^0|pS\rangle|_{\nabla\cdot A=0,R}$ = $\Delta G_R + h.t.$

solved using generalized covariant condition $A_\perp^\mu$ → boost → $A_{phys}^\mu = A^{\mu,a}(\xi) - \frac{1}{\partial^z}[(\partial^\mu A^{z,b})(\mathcal{L}^{ba}(\xi',\xi))]$

$\nabla\cdot A = 0$ ↓ ↓ $A^z = 0$

$A^\mu|_{\nabla\cdot A=0}$ → boost → $A^\mu|_{A^z=0}$

Ji,Zhang,Zhao,2013

Hatta,Ji,Zhao,2014

Gluon helicity from local matrix element in a fixed gauge

K^μ and its matrix element

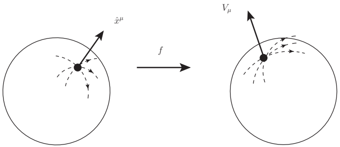
$K^\mu = \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[A_\nu F_{\alpha\beta} + \frac{2}{3} i g_s A_\nu A_\alpha A_\beta\right]$, after a gauge transformation:

$$K_\mu \to K_\mu - \frac{2i}{g}\varepsilon_{\mu\nu\alpha\beta}\partial^\alpha \mathrm{Tr}(U^{-1}\partial^\nu U A^\beta)$$
$$-\frac{2}{3g^2}\varepsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\{U^{-1}(\partial^\nu U)U^{-1}(\partial^\alpha U)U^{-1}(\partial^\beta U)\},$$

(Belavin et al.,1973)

"large" transform

Jacobian of $\mathbb{S}_3$ to $\mathbb{G}$, $\quad \nu \equiv \int d^4x Q(x) \sim \int_{\mathbb{S}^3} \varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left[U^\dagger(\partial_\nu U)U^\dagger(\partial_\alpha U)U^\dagger(\partial_\beta U)\right].$



l.h.s: topological charge.

Gluon helicity from local matrix element in a fixed gauge

K^μ and its matrix element

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(Belavin et al.,1973)
"large" transform
→ Jacobian of $\mathbb{S}_3$ to $\mathbb{G}$,

(Cronström and Mickelsson,1983)
local gauge transform
→ $\partial^\alpha H_{\mu\alpha}$

Gluon helicity from local matrix element in a fixed gauge

K^μ and its matrix element

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(Belavin et al.,1973)
"large" transform

(Cronström and Mickelsson,1983)
local gauge transform

Jacobian of $\mathbb{S}_3$ to $\mathbb{G}$,

$\partial^\alpha H_{\mu\alpha}$

If three-gluon term in K^μ exists, the forward matrix element of K^μ will have a nonperturbative gauge dependence.

Gluon helicity from local matrix element in a fixed gauge

K^μ and its matrix element

$\langle p' | K^\mu | p \rangle$ contains a gauge-dependent massless pole due to the three-gluon term:

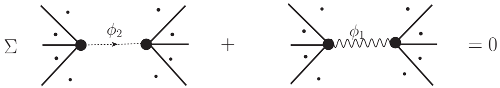
$$\langle p' | K^\mu | p \rangle_{n\cdot A=0} = 4S^\mu \Delta G(n,P) + \frac{n^\mu}{n\cdot q}\left(-4(q\cdot S)\Delta G(n,q,P) + \frac{i}{2}\langle P'S | F^{\mu\nu,a}\tilde{F}^a_{\mu\nu} | PS\rangle\right),$$

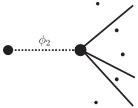
(Balitsky and Braun,1991)

$$\langle p' | K^\mu | p \rangle_{\partial\cdot A=0} = 4S^\mu a_1(q^2) + \frac{q^\mu}{q^2} b_1(q^2),$$

(Kogut and Susskind,1973;Veneziano,1979)

For massive Schwinger model, the pole is caused by the coupling between K^μ and a ghost(one of the Goldstone dipole), in covariant gauge.

$\langle f | O_{\text{inv}} | i \rangle =$ 

$\langle f | K^\mu | i \rangle = K^\mu$ 

Gluon helicity from local matrix element in a fixed gauge

K^μ and its matrix element

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For massive Schwinger model, the pole is caused by the coupling between K^μ and a ghost(one of the Goldstone dipole), in covariant gauge.

In axial gauge and $\mu = 0$ ($\langle p' | K^0 | p \rangle_{A^z=0}$), the pole term vanishes:

$$\langle p | K^0 | p \rangle_{A^z=0} = S^0 \Delta G + h.t. \leftarrow \langle p | K^0 | p \rangle_{\nabla\cdot A=0}$$

- The bare result $\lim_{q\to 0}\langle P'S|K^{0/z}|PS\rangle_{\text{C.G.}}$ (or $\langle PS|K^{0/z}|PS\rangle_{\text{C.G.}}$) needs to be renormalized, we adopt RI/MOM scheme:

$$\begin{pmatrix}\Delta G_{\mathrm{R}}^{\mathrm{RI}}\\ \Delta\Sigma_{\mathrm{R}}^{\mathrm{RI}}\end{pmatrix} = \begin{pmatrix} Z_{11}^{\mathrm{RI}} & Z_{12}^{\mathrm{RI}} \\ Z_{21}^{\mathrm{RI}} & Z_{22}^{\mathrm{RI}}\end{pmatrix}\begin{pmatrix}\Delta G_B \\ \Delta\Sigma_B\end{pmatrix},$$

The perturbative gauge-invariance of $\langle PS|K^{0/z}|PS\rangle$ has been used.

The conversion factor from RI/MOM scheme to $\overline{\text{MS}}$ scheme is derived at one-loop:

$$R_{11}^{\overline{MS},\mathrm{RI}}(\mu_R^2,\mu^2) = 1 + \frac{\alpha_s}{4\pi}\left[\beta_0 \ln\left(\frac{\mu_R^2}{\mu^2}\right) - \frac{367}{36}C_A + \frac{10}{9}n_f\right],$$

$$R_{12}^{\overline{\mathrm{MS}},\mathrm{RI}}(\mu_R^2,\mu^2) = \frac{\alpha_s}{4\pi}C_F\left[3\ln\left(\frac{\mu_R^2}{\mu^2}\right) - 6\right],$$

$$R_{21}^{\overline{\mathrm{MS}},\mathrm{RI}}(\mu_R^2,\mu^2) = -2\frac{\alpha_s}{4\pi}C_F,$$

$$R_{22}^{\overline{MS},\mathrm{RI}}(\mu_R^2,\mu^2) = 1.$$

Martinelli et al., 1995.

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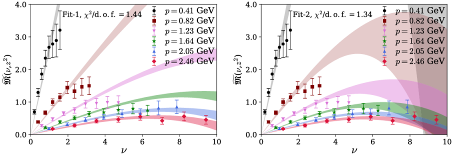
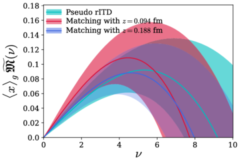
Total gluon helicity from nonlocal matrix element

- Besides local matrix element in a fixed gauge, one can choose specific gauge-invariant nonlocal matrix element, the matching between its first moment and ΔG is trivial in $\overline{\mathrm{MS}}$ scheme.





- Previous lattice calculations of the gluon helicity distribution have large systematic uncertainty, the effect of matching is also large (Egerer et al.,2022).

Total gluon helicity from nonlocal matrix element

Basic workflow

$$\tilde{h}(z, P_z, \tfrac{1}{a}) \xrightarrow{\text{hybrid scheme}} \tilde{h}_R^{\text{hyb.}}(z, P_z) \xrightarrow{Z_T} \tilde{h}_R^{\overline{\text{MS}}}(z, P_z) \xrightarrow{\int dz} \Delta G$$

$$(\langle PS | F^{3\mu}(z)\mathcal{L}(z,0)\tilde{F}^{0}_{\mu}(0) | PS \rangle)$$

Total gluon helicity from nonlocal matrix element

Basic workflow

$$\tilde{h}(z, P_z, \tfrac{1}{a}) \xrightarrow{\text{hybrid scheme}} \tilde{h}_R^{\text{hyb.}}(z, P_z) \xrightarrow{Z_T} \tilde{h}_R^{\overline{\text{MS}}}(z, P_z) \xrightarrow{\int dz} \Delta G$$

$$(\langle PS | F^{3\mu}(z)\mathcal{L}(z,0)\tilde{F}^0_\mu(0) | PS \rangle)$$

$$\tilde{h}_R^{\mathrm{hyb.}}(z, P_z) = \frac{\tilde{h}(z,P_z,1/a)}{\tilde{h}(z,P_z=0,1/a)}\theta(z_s - |z|) + T_s \mathrm{e}^{-\delta m|z|}\tilde{h}(z, P_z, 1/a)\theta(|z| - z_s),$$

Total gluon helicity from nonlocal matrix element

Basic workflow

$$\tilde{h}(z, P_z, \tfrac{1}{a}) \xrightarrow{\text{hybrid scheme}} \tilde{h}_R^{\text{hyb.}}(z, P_z) \xrightarrow{Z_T} \tilde{h}_R^{\overline{\text{MS}}}(z, P_z) \xrightarrow{\int dz} \Delta G$$

$$(\langle PS|F^{3\mu}(z)\mathcal{L}(z,0)\tilde{F}^0_\mu(0)|PS\rangle)$$

$$Z_T(\mu, z, z_s) = \theta(z_s - |z|)\left(1 + 2a_s C_A\left(\frac{7}{3}\ln\frac{\mu^2 z^2}{4e^{-2\gamma_E}} + \frac{7}{3}\right)\right),$$

Total gluon helicity from nonlocal matrix element

Basic workflow

$$\tilde{h}(z, P_z, \tfrac{1}{a}) \xrightarrow{\text{hybrid scheme}} \tilde{h}_R^{\text{hyb.}}(z, P_z) \xrightarrow{Z_T} \tilde{h}_R^{\overline{\text{MS}}}(z, P_z) \xrightarrow{\int dz} \Delta G$$

$$(\langle PS | F^{3\mu}(z)\mathcal{L}(z,0)\tilde{F}^0_\mu(0) | PS \rangle)$$

Matching coefficients between $\tilde{h}_R^{\overline{\text{MS}}}(z, P_z)$ and $h_R^{\overline{\text{MS}}}(z, P_z)$:

$$C_{gg}(\alpha, z, \mu) = \delta(\alpha) + 2a_s C_A \Big\{ \Big(4\alpha\bar{\alpha} + 2\Big[\frac{\bar{\alpha}^2}{\alpha}\Big]_+\Big)(\mathrm{L_z} - 1) + 6\alpha\bar{\alpha} - 4\Big[\frac{\ln(\alpha)}{\alpha}\Big]_+ + (-3\mathrm{L_z} + 2)\delta(\alpha) \Big\},$$

$$C_{gq}(\alpha, z, \mu) = \frac{-2ia_s C_F}{z} \Big\{ -2\alpha(\mathrm{L_z} + 1) - 4\bar{\alpha} + \mathrm{L_z}\delta(\alpha) \Big\}.$$

Matching between $\Delta\tilde{G}$ and ΔG is trivial:

$$\Delta\tilde{G}(P_z, \mu) = \frac{1}{2P_z}\int_0^\infty dz \tilde{h}(z, P_z, \mu)$$

$$= \int d\lambda \int_0^1 \frac{d\alpha}{\bar{\alpha}} \Big[C_{gg}\big(\alpha, \tfrac{\lambda}{\bar{\alpha}P_z}, \mu\big) h_g(\lambda, \mu) + C_{gq}\big(\alpha, \tfrac{\lambda}{\bar{\alpha}P_z}, \mu\big) h_q(\lambda, \mu) \Big] + h.t. = \Delta G + h.t.,$$

Total gluon helicity from nonlocal matrix element

Relation to the topological current

- $\langle PS|F^{3\mu}(z)\mathcal{L}(z,0)\tilde{F}^{0}_{\mu}(0)|PS\rangle$ has a simple relation with $\langle K^0\rangle$, in the axial gauge:

  $\int_0^\infty dz\langle PS|m^{3\mu;0\mu}|PS\rangle\Big|_{A^z=0} = \langle PS|A^i B^i|PS\rangle|_{A^z=0} = \langle PS|K^0|PS\rangle|_{A^z=0}$.
  The matching is trivial to all orders in α_s, due to the perturbatively gauge invariance of $\langle PS|K^\mu|PS\rangle$.

- We can construct integral of other correlations, which reduce to $\langle PS|K^\mu|PS\rangle$ in specific choice of gauge and μ:

$$\int_0^\infty dz\langle PS|\tfrac{1}{2}\left(m^{0\mu;0\mu}+m^{3\mu;3\mu}\right)|PS\rangle = \langle PS|\tfrac{1}{2}\vec{A}\cdot\vec{B}|PS\rangle|_{A^0=0} + \langle PS|\tfrac{1}{2}\epsilon_{ij}\left(F^{i0}A^j - \tfrac{1}{2}A^0F^{ij}\right)|PS\rangle\Big|_{A^z=0},$$

$$\int_0^\infty dz\langle PS|m^{0\mu;3\mu}|PS\rangle = \langle PS|\epsilon_{ij}F^{i0}A^j|PS\rangle\Big|_{A^0=0}.$$

Hatta et al. 2014

Summary

- We have proposed two approaches to extract the total gluon helicity from lattice QCD. The matching coefficients are trivial in $\overline{\text{MS}}$ scheme, thus resolving the inconsistency between calculations based on local and nonlocal matrix elements.
- The conversion factor is calculated in the state-of-art scheme (RI/MOM scheme for local matrix element and hybrid scheme for nonlocal matrix element).
- For lattice calculations of gluon orbital angular momentum, whether there exists a proposal with trivial matching relation is under exploration.