Lattice Parton Collaboration

北京航空航天大學
BEIHANG UNIVERSITY

Determining heavy meson LCDAs from lattice QCD

Based on 2403.17492, 2410.18654, 2411.07101 *et al.*

In collaboration with LPC members and J. Xu, S. Zhao, *et al.*

Qi-An Zhang

Beihang University (BUAA)

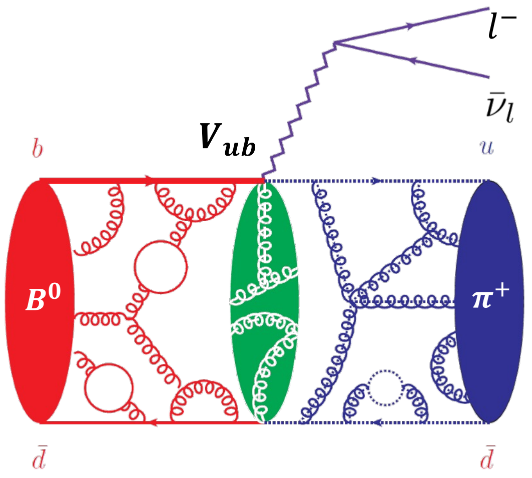
Nov. 25, 2024 @ Light Cone 2024, Huizhou

Outline

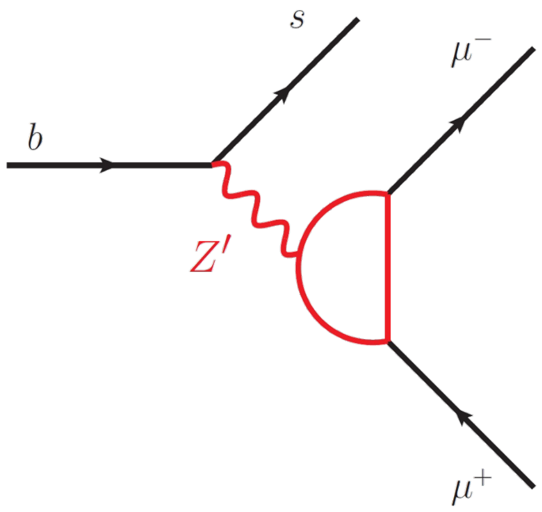
- Motivation
- Theoretical framework for the two-step factorization method
- Lattice QCD verification
- Phenomenological discussions
- Summary and prospect

Motivation

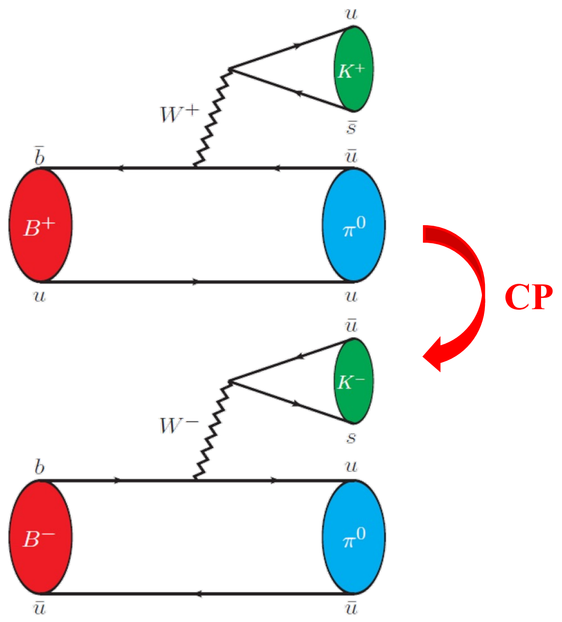
Heavy flavor physics is one of the frontier topics in particle physics:



Precisely testing standard model



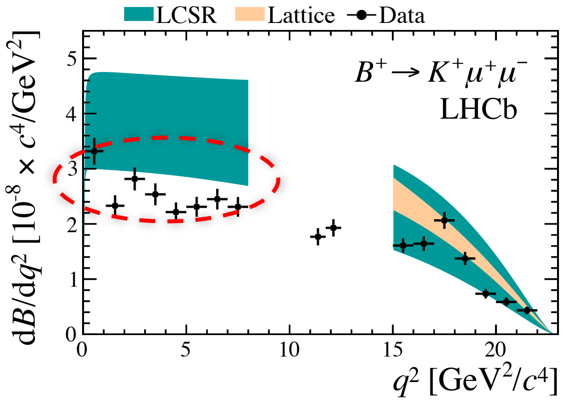
Indirect search for new physics

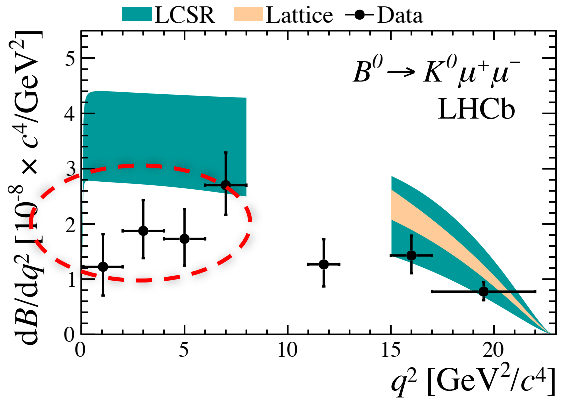


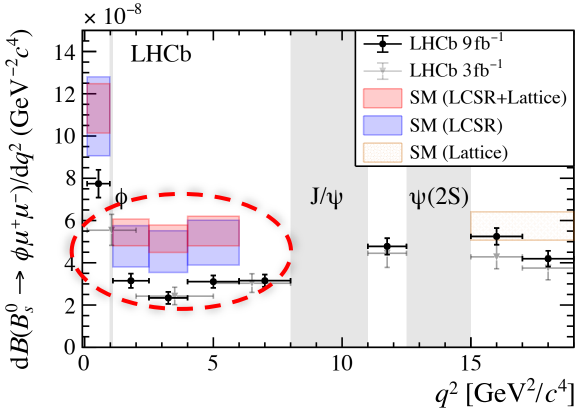
Study on CP violation

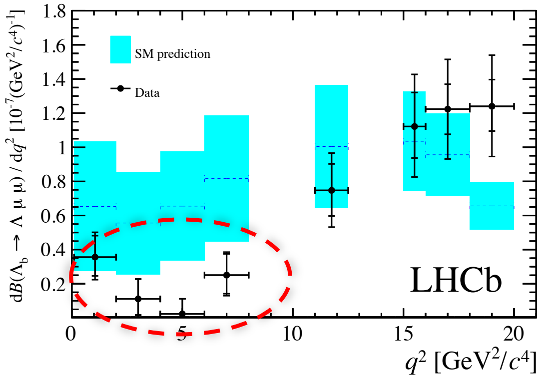
Motivation

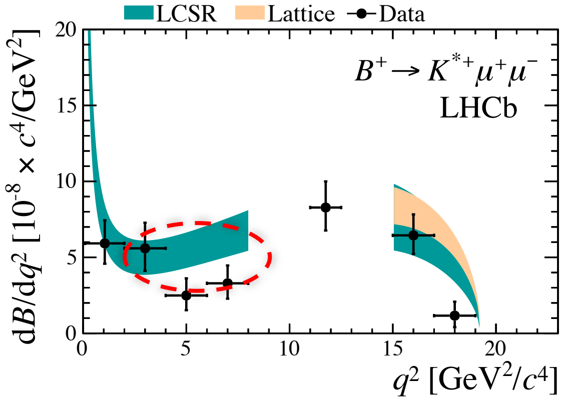
Current experimental results show deviations from theoretical prediction…

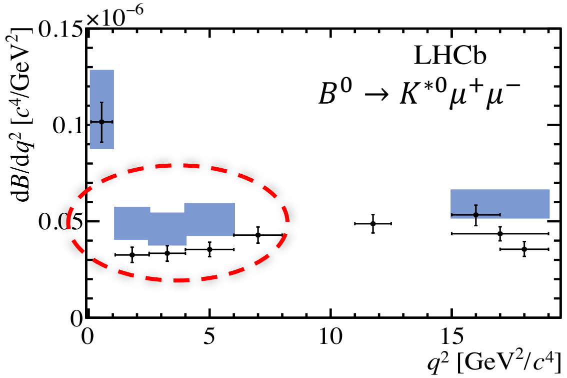


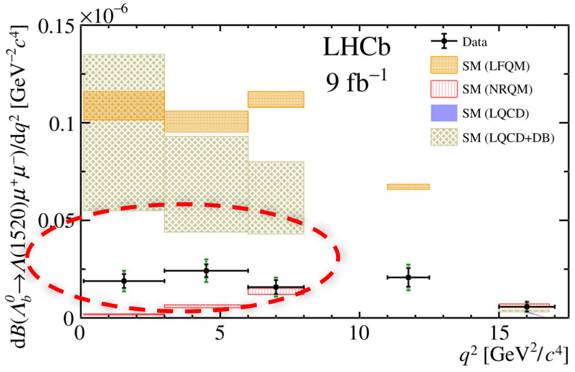












High precision calculations play a crucial role in the search for new physics!

Motivation

Uncertainties originating from *B* meson LCDAs dominate the primary errors in theoretical calculation.

- For example: $B \to \pi,\ K^*$ form factors from LCSRs:

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035
Cui, Huang, Shen, Wang, JHEP 03 (2023) 140

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

$$f^+_{B\to\pi}(0) = 0.122 \times \Big[1 \pm 0.07\Big|_{S_0^\pi} \pm 0.11\Big|_{\Lambda_q} \pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2} \pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}$$
$$^{+0.06}_{-0.10}\Big|_{\mu_h} \pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big].$$

λ_B and σ_n: the first inverse and inverse-log moments,

$\varphi_B^\pm$: uncertainties from different parameterizations of the *B* meson LCDA.

Without reliable *B* LCDA, it is impossible to discuss precision calculation!

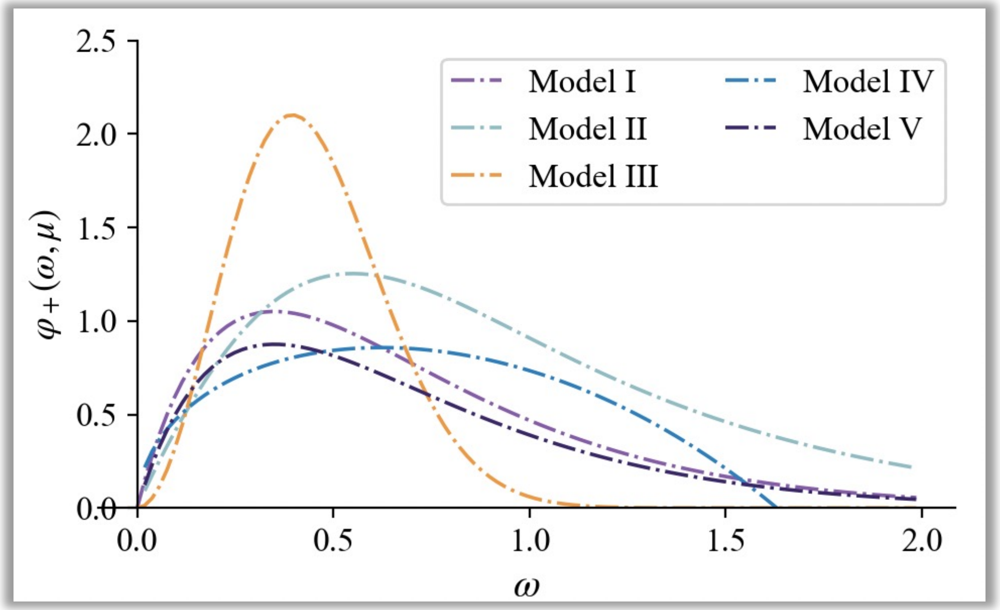
Model dependence of B meson LCDA

Grozin, Neubert, PRD 55, 272 (1997)

Braun, Ivanov, Korchemsky, PRD 69, 034014 (2004)

Beneke, Braun, Ji, Wei, JHEP 07, 154 (2018)

- B meson LCDA is only available through model parametrizations, lacking first-principle prediction.
- Predictions from different models vary significantly.
- This model dependence contribute to the largest theoretical uncertainties in $B \to K^*$ form factor:



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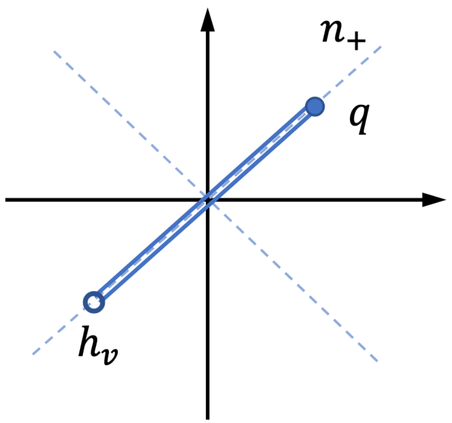
Challenges in first principle calculation

The definition of leading twist heavy meson LCDA:

$$i\tilde{f}_H(\mu)m_H\varphi^+(\omega,\mu) = \int_{-\infty}^{+\infty}\frac{dt}{2\pi}e^{i\omega n_+\cdot vt} \times \langle 0|\bar{q}\,(tn_+)\,\not{n}_+\gamma_5 W_c\,(tn_+,0)\,h_v(0)|H(v)\rangle$$



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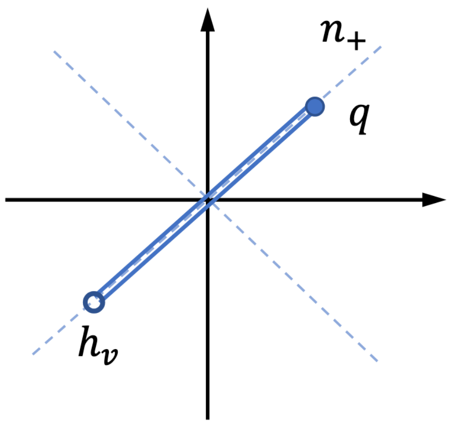

Challenge 1: the HQET field h_v

- Simulating the boosted h_v on the lattice will encounter significant signal-to-noise problem.

Mandula, Ogilvie, PRD 45, 2183-2187 (1992), NPB 34, 480-482 (1994);
S. Meinel, doi:10.17863/CAM.16088

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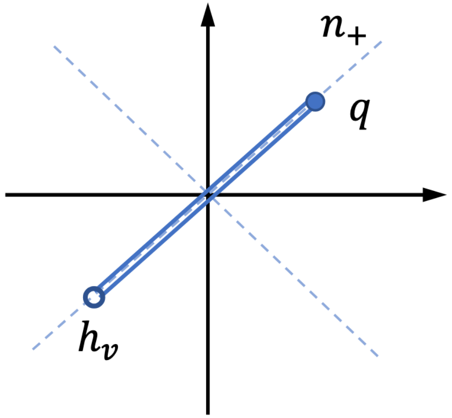

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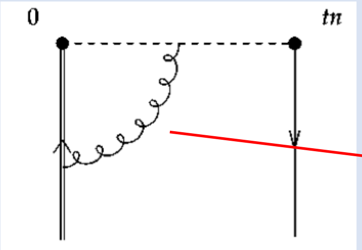
Challenge 2: h_v + light-like Wilson line



$$O_v^{\rm ren}(t,\mu) = \frac{4}{\hat{\epsilon}}\ln(it\mu)O_v^{\rm bare}(t) + \cdots$$

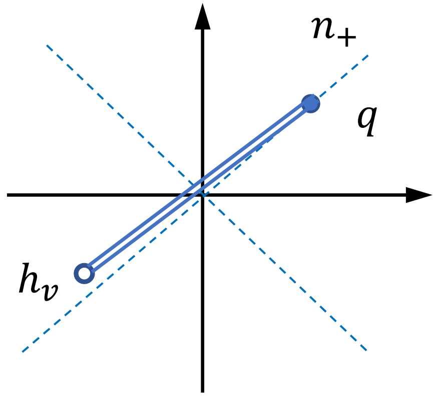
Braun, Ivanov, Korchemsky, PRD69, 034014 (2004)

- Cusp divergence: No local limit!
- Cannot obtain φ_B through its moments.

How to solve these problems?

- Early explorations:
  - Use off light-cone Wilson line, while retaining the HQET field h_v.

    Wang, Wang, Xu, Zhao, PRD 102, 011502 (2020)
    Xu, Zhang, PRD 106, 114019 (2022)
    Hu, Xu, Zhao, EPJC 84, 502 (2024)
    Zhao, Radyushkin, PRD 103, 054022 (2021)



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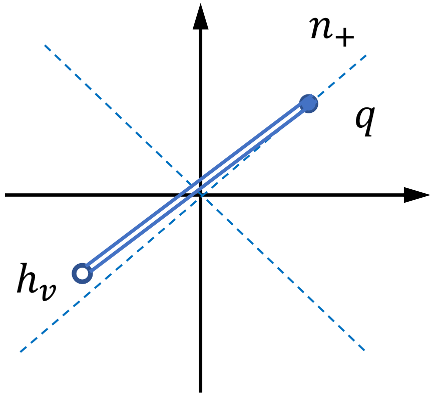


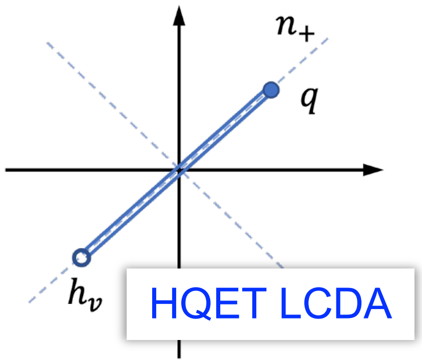
  - Solves the issue of cusp divergence, but not consider the feasibility of lattice implementation.

- We propose a new lattice-feasible approach: **A two-step factorization method**

A two-step factorization method

STEP 1:

- h_v is obtained with the $m_Q \to \infty$ limit;
- To avoid difficulties, transition from the HQET field to QCD, i.e. shift m_Q from ∞ to finite;
- Treat m_Q as a parameter $(m_Q \gg \Lambda_{\text{QCD}})$, its evolution in different regions follows different theories.



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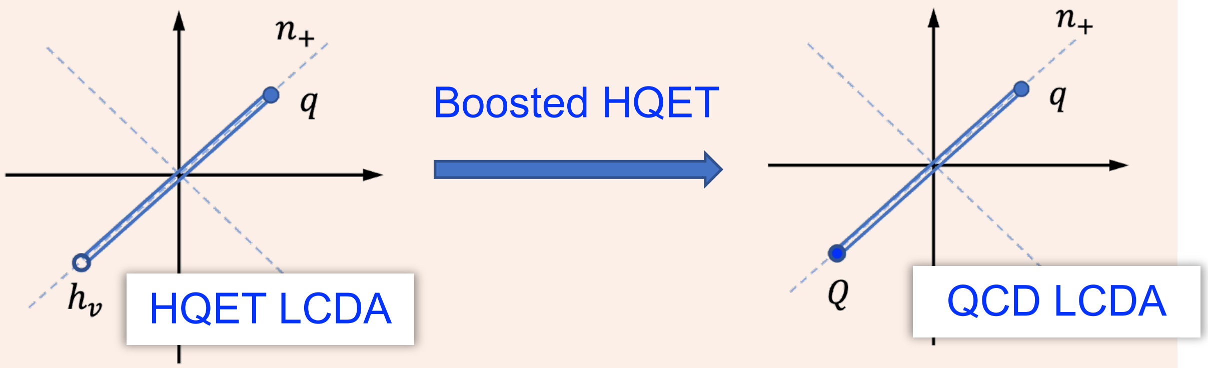
The matching from $m_Q \to \infty$ to finite can be achieved through an effective theory:



Ishaq, Jia, Xiong, Yang, PRL125(2020)132001

Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

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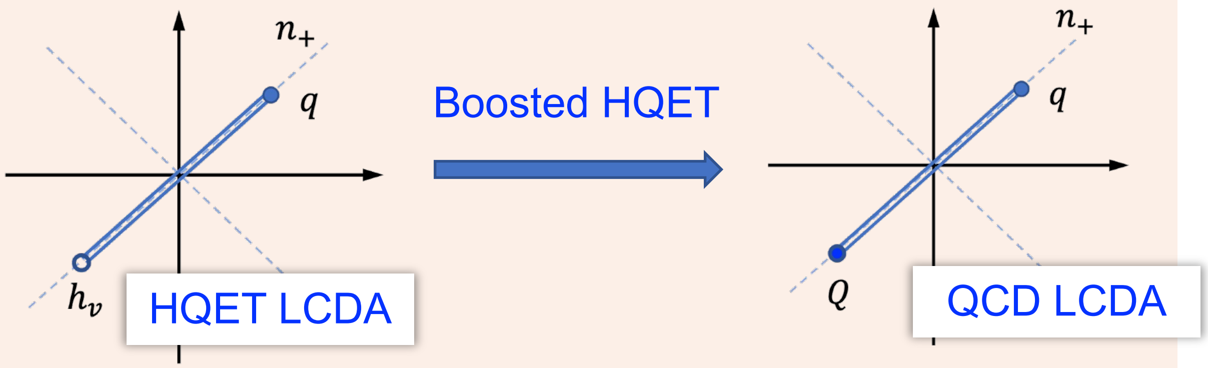

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Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

The evolution of m_Q between different finite values is governed by a heavy quark mass renormalization group: Wang, Xu, QAZ, Zhao, 2411.07101

$$m_Q \frac{\partial}{\partial m_Q} \phi\left(u, m_Q ; \mu\right) - u \frac{\partial}{\partial u} \phi\left(u, m_Q ; \mu\right) - \left(1+\gamma\left(m_Q, \mu\right)\right) \phi\left(u, m_Q ; \mu\right) = 0$$

its solution

$$\phi\left(u, m_Q ; \mu\right) \approx \exp \left[\frac{2 C_F}{\beta_0} \ln \frac{\alpha_s\left(m_Q\right)}{\alpha_s\left(m_{Q_0}\right)} - \frac{4 \pi C_F}{\beta_0^2}\left(\frac{1}{\alpha_s\left(m_{Q_0}\right)} \ln \frac{\alpha_s(\mu)}{\alpha_s\left(m_{Q_0}\right) e} - \frac{1}{\alpha_s\left(m_Q\right)} \ln \frac{\alpha_s(\mu)}{\alpha_s\left(m_Q\right) e}\right)\right] \frac{m_Q}{m_{Q_0}} \phi_0\left(u \frac{m_Q}{m_{Q_0}}\right) .$$

A two-step factorization method

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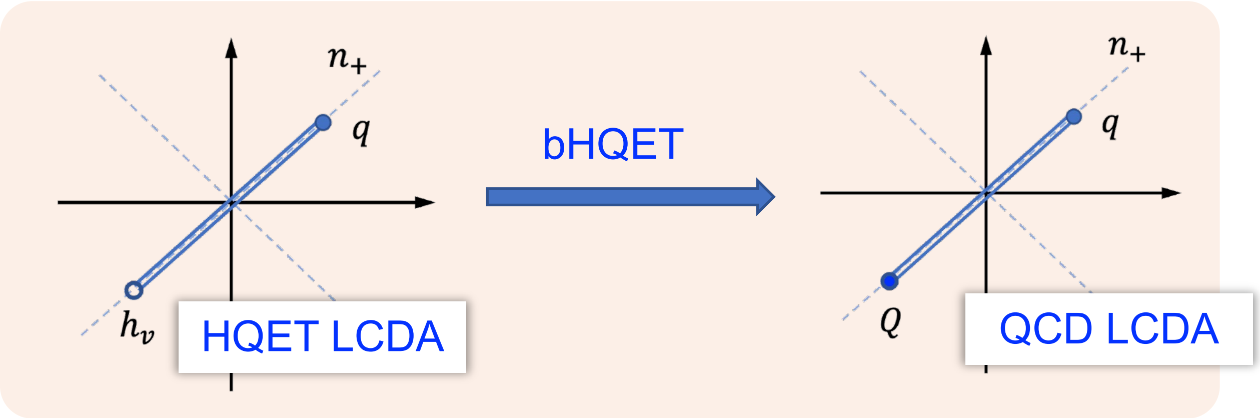

- Without h_v, the issues of lattice implementation of HQET field and cusp divergence are both resolved;
- The heavy quark field in QCD defines the QCD LCDA, which is also an important input for describing the final state heavy mesons in exclusive processes;
- Both bHQET matching and mass RGE are perturbative ($m_Q \gg \Lambda_{\text{QCD}}$), ensuring that IR behavior remains unchanged.

A two-step factorization method

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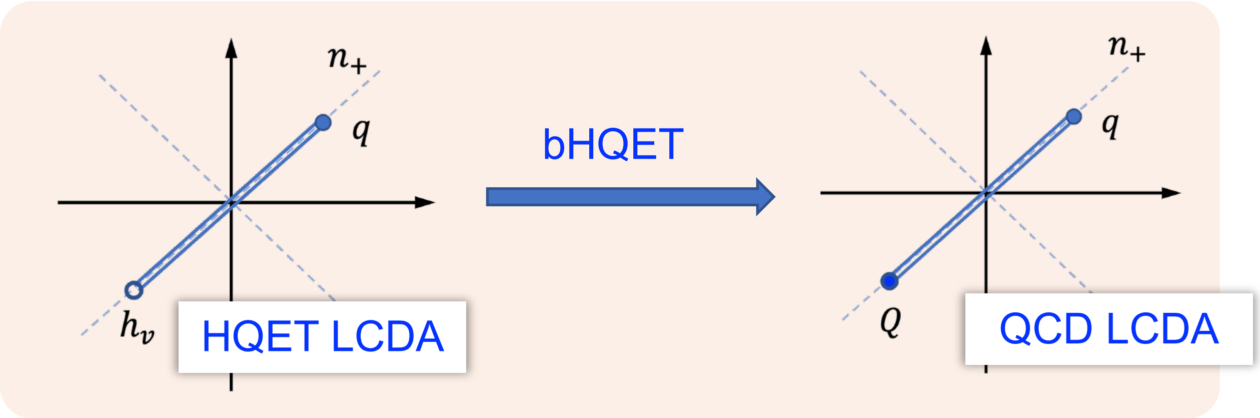

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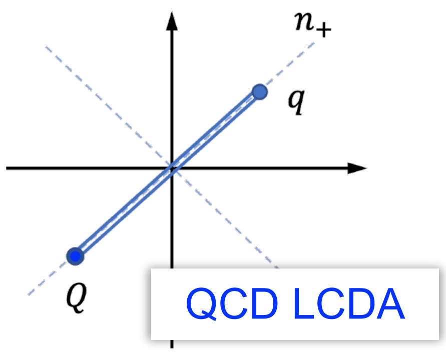
🤔 The following question is, how the QCD LCDA can be implemented on the lattice?

A two-step factorization method

STEP 2:

- QCD LCDA involves a matrix element of light-like nonlocal operator;
- Lattice QCD is a theory defined in Euclidean space, cannot directly simulate real-time correlations;
- Large-momentum effective theory (LaMET) provides a connection between Euclidean lattice and light-cone observables. Ji, PRL 110, 262002 (2013); Sci. China Phys. Mech. Astron. 57, 1407-1412 (2014)



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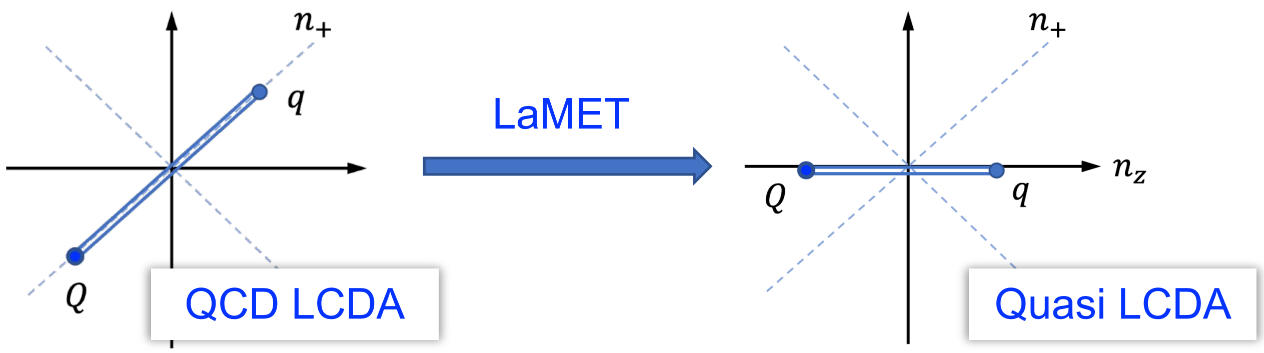


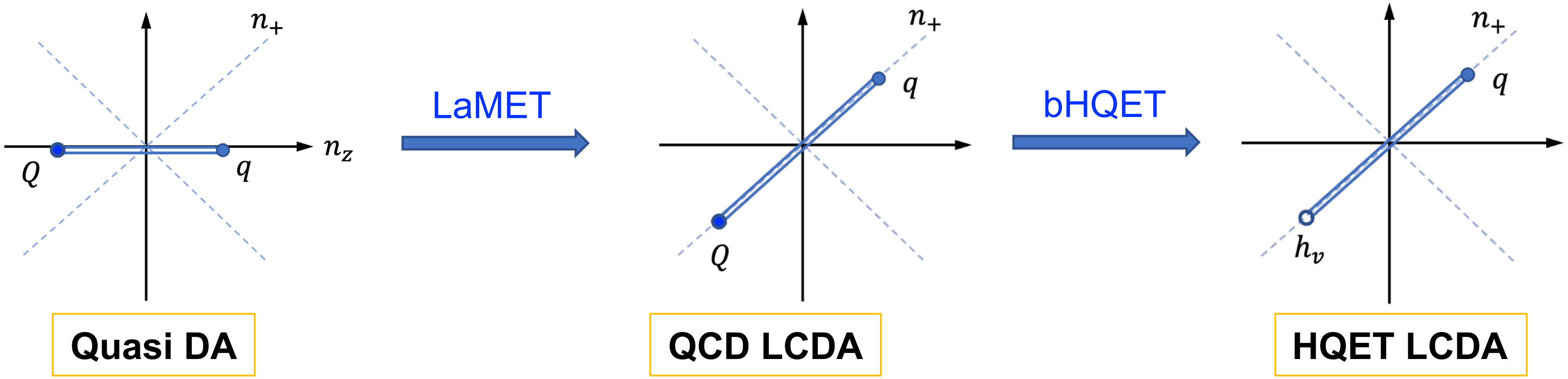
Quasi DA:

$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z z} \times \langle 0 \,| \bar{q}(z) \Gamma W_c(z,0) Q(0) |\, H(P^z) \rangle_R \,.$$

Equal-time correlation matrix element, lattice QCD calculable.

A two-step factorization method

A **chained factorization formula** to determine both heavy meson QCD LCDA and HQET LCDA from the first-principle:



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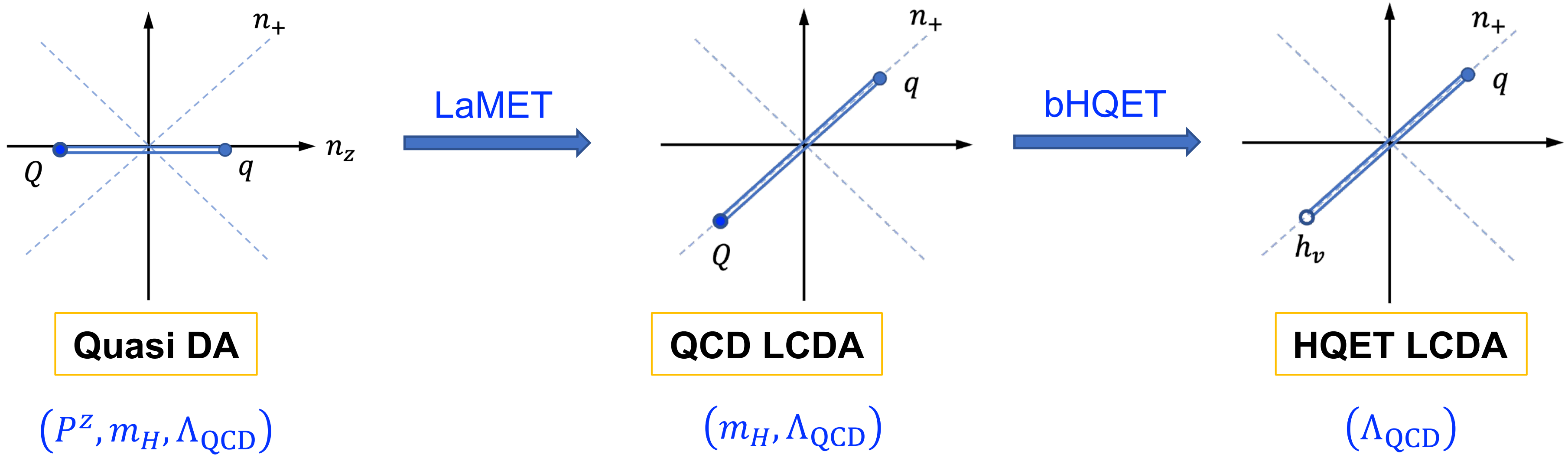

- LaMET: $\Lambda_{\text{QCD}}, m_H \ll P^z$ and integrate out P^z
- bHQET: $\Lambda_{\text{QCD}} \ll m_H$ and integrate out m_H

$\Rightarrow$ Introduce a hierarchy $\boldsymbol{\Lambda_{\text{QCD}} \ll m_H \ll P^z}$

A two-step factorization method

The hierarchy $\Lambda_{\mathrm{QCD}} \ll m_H \ll P^z$ imposes limitations on lattice calculation:

- Currently, direct simulation of *B* meson is not practical.

A two-step factorization method

The hierarchy $\Lambda_{\rm QCD} \ll m_H \ll P^z$ imposes limitations on lattice calculation:

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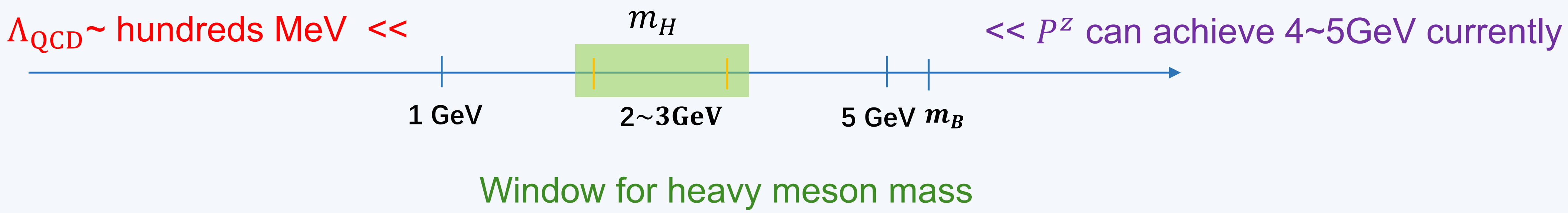


- *D* meson can be realized on the lattice;
- Heavy quark flavor symmetry ensures that the HQET LCDA is independent of heavy quark mass;
- m_H (m_D or m_B) only contributes to the power corrections.

Lattice QCD verification

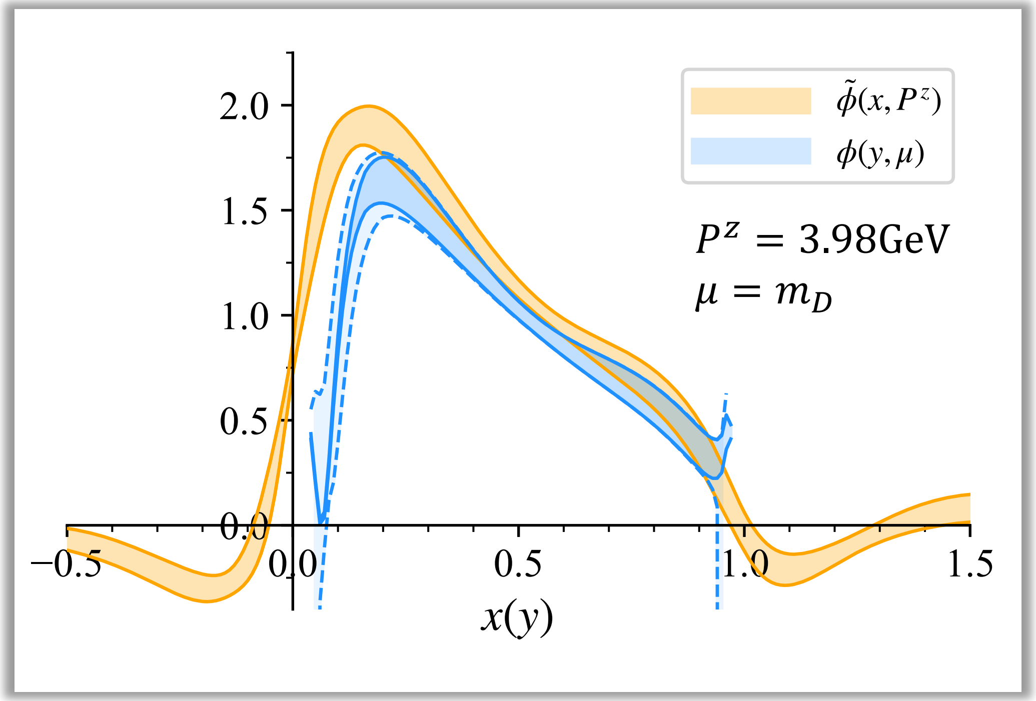
- A numerical simulation on the finest CLQCD ensemble ($a = 0.05187$ fm);

  CLQCD Collaboration, PRD 109, 054507 (2024)

- Simulate the *D* meson quasi DA with $m_D \simeq 1.92\text{GeV}$, up to $P^z \simeq 3.98\text{GeV}$;

$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}(z, P^z)$$

- The state-of-the-art techniques in self renormalization scheme and physics-inspired long-range extrapolation are adopted.



Lattice QCD verification

Matching formula in LaMET:

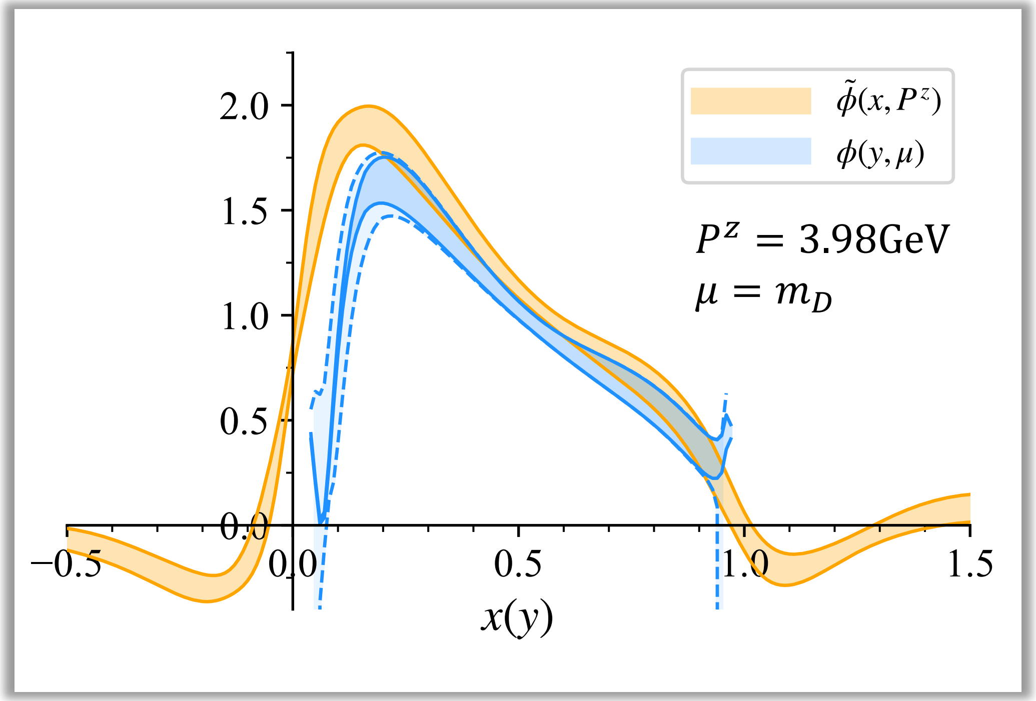
$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right)\phi(y,\mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

- Matching kernel at NLO in α_s

  Liu, Wang, Xu, QAZ, Zhao, PRD 99, 094036 (2019)

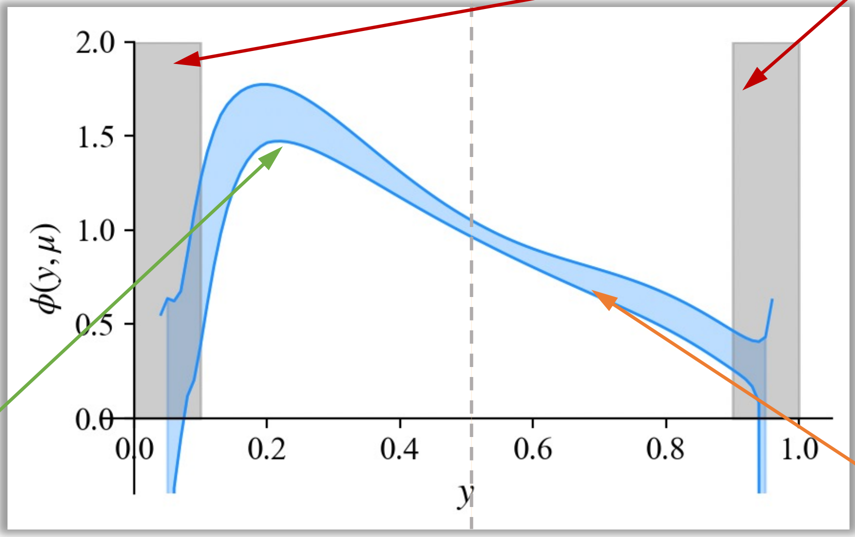
  Han, Hua, Ji, Lu, Wang, Xu, QAZ, Zhao, 2410.18654

- RG resummation is adopted to associate the lattice scale $2xP^z$ / $2(1-x)P^z$ and $\overline{\rm MS}$ scale $\mu = m_D$.



Lattice QCD verification

QCD LCDA of D meson



Peak region: $y \sim \frac{\Lambda_{\rm QCD}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA.

Ishaq, Jia, Xiong, Yang, PRL125(2020)132001
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

End-point region:

- LaMET matching kernel suffer large power corrections.
- Lattice QCD predictions fail

Tail region: $y \sim 1$

- Contain only hard-collinear physics, perturbative calculable;
- Suppressed in LCDA.

Lattice QCD verification

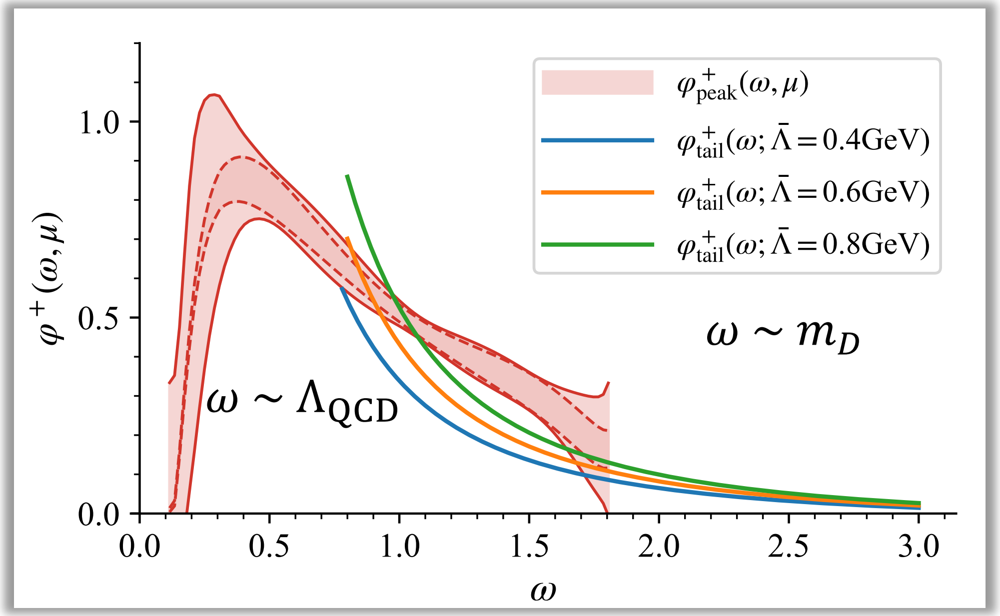
➢ Peak region:

- A multiplicative factorization from QCD LCDA to HQET LCDA:

$$\varphi^+_{\rm peak}(\omega,\mu) = \frac{f_H}{\widetilde{f}_H}\frac{1}{\mathcal{J}_{\rm peak}}\phi(y,\mu;m_H) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$$

Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

- Nonperturbative, determined from lattice QCD.



Lattice QCD verification

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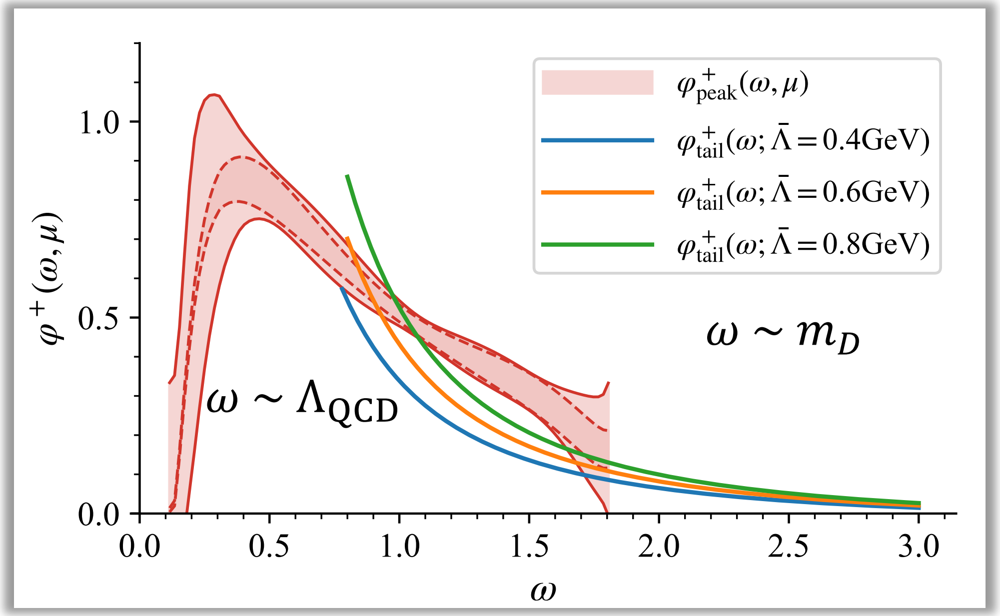
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

  - Nonperturbative, determined from lattice QCD.

- Tail region:
  - Perturbative result at 1-loop order:

$$\varphi^+_{\rm tail}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

Lee, Neubert, PRD 72, 094028 (2005)



Lattice QCD verification

Finally, we obtain the final result of HQET LCDA.

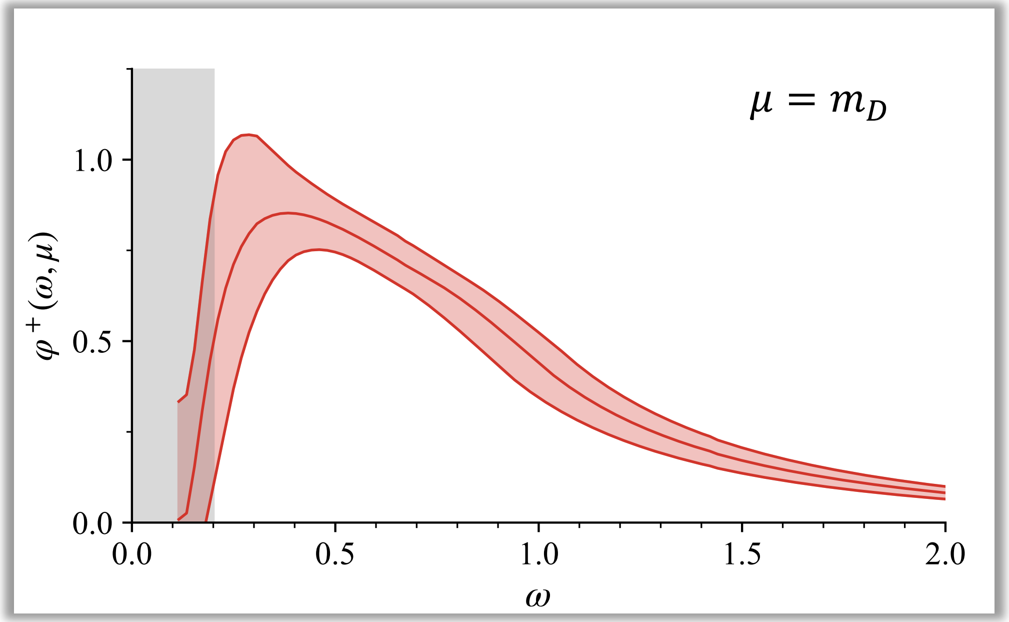
- A verification of the two-step factorization method, the numerical result is still preliminary.
- Considered the systematic errors in lattice analysis:

  From extrapolation, scale uncertainty, ……

- Some key systematic errors are still absent:

  Only one lattice spacing,

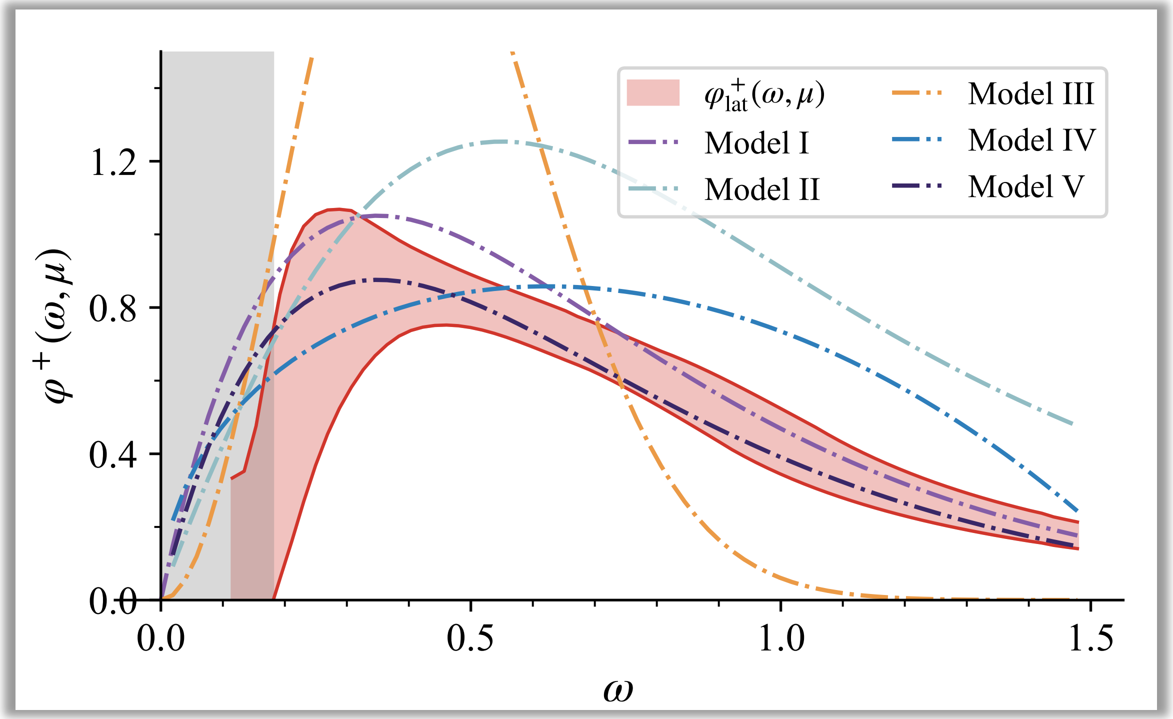
  Power corrections within two matchings are still significant, ……



Although current result is preliminary, it still warrants some phenomenological discussions

Discussions I: comparison with models

- The model dependence contributes the largest systematic error in the form factors.
- Our result is basically consistent with most of the model estimates, and will also provide a first-principle constrains on the existing models.



- For theoretical calculations, result from first-principles will help to REMOVE the primary uncertainties arising from the model parametrizations.

Discussions II: Inverse and inverse-logarithmic moments

Significant uncertainties from λ_B and σ_1:

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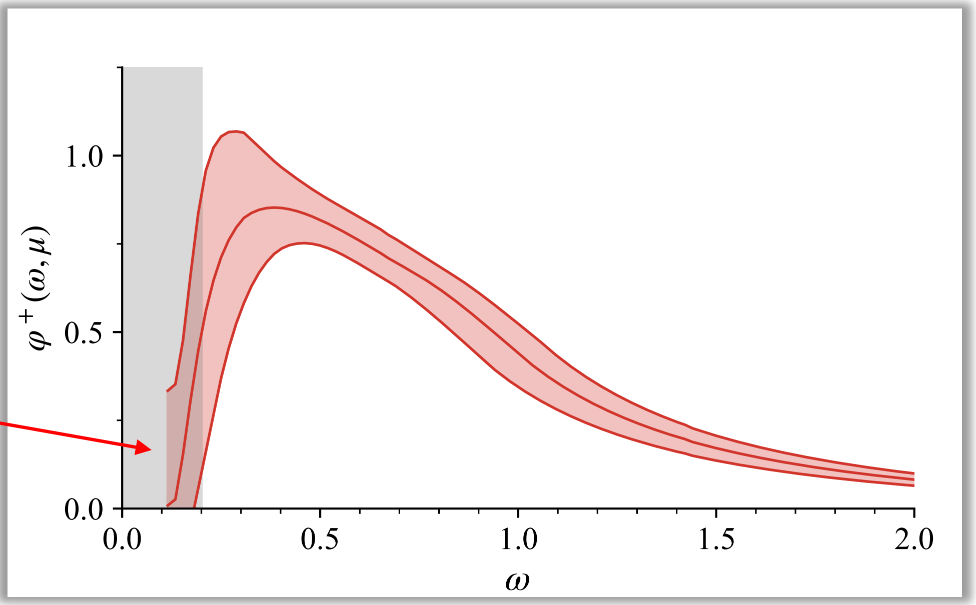
$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Definition of Inverse and inverse-logarithmic moments:

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\varphi^+(\omega,\mu),$$

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\ln\left(\frac{\mu}{\omega}\right)^{(n)}\varphi^+(\omega,\mu).$$

The power corrections at small ω makes the integral non-computable.



Discussions II: Inverse and inverse-logarithmic moments

A model-independent parametrization form:

$$\varphi^+(\omega,\mu) = \sum_{n=1}^{N} c_n \frac{\omega^n}{\omega_0^{n+1}} e^{-\omega/\omega_0}$$

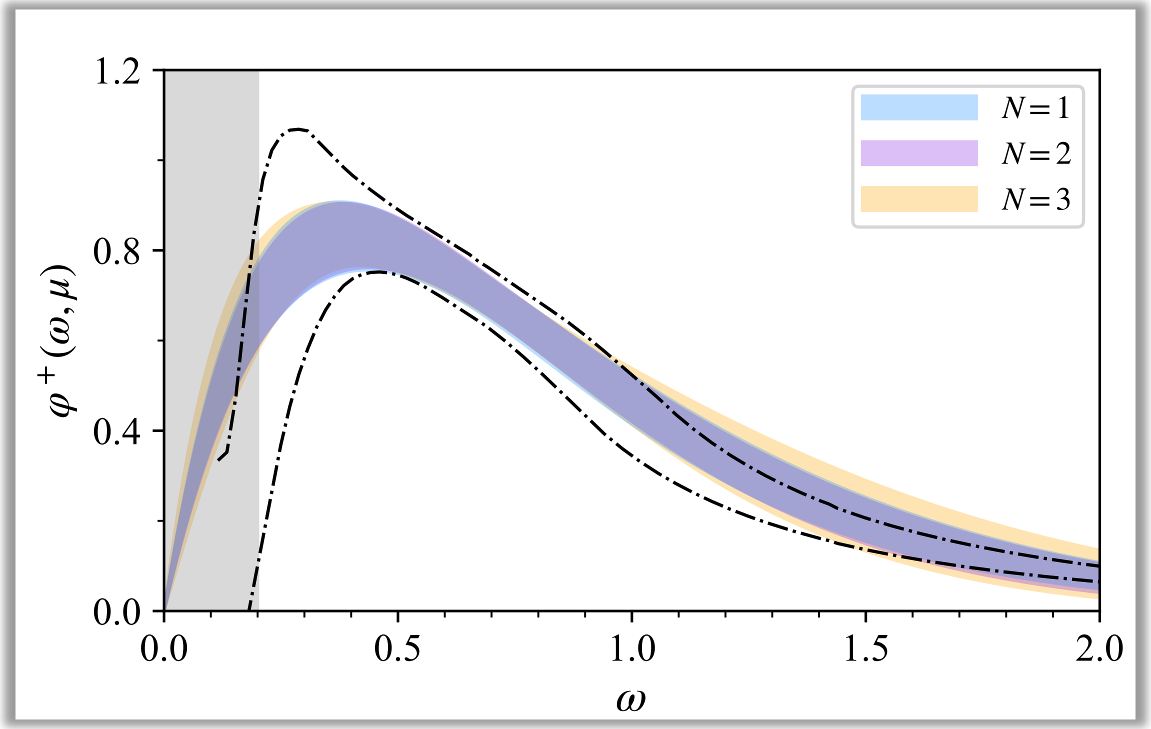
$$= \frac{c_1\omega}{\omega_0^2}\left[1 + c_2'\frac{\omega}{\omega_0} + c_3'\left(\frac{\omega}{\omega_0}\right)^2 + \cdots\right] e^{-\omega/\omega_0},$$

Fit results up to the N-th order:

$$N=1:\ \omega_0 = 0.403(44),\ c_1 = 0.932(73);$$

$$N=2:\ \omega_0 = 0.352(82),\ c_1 = 0.69(37),\ c_2' = 0.17(32);$$

$$N=3:\ \omega_0 = 0.32(15),\ c_1 = 0.63(44),\ c_2' = 0.12(37), c_3' = 0.04(19).$$



Discussions II: Inverse and inverse-logarithmic moments

Numerical results of λ_B and $\sigma_B^{(1)}$ at $\mu = 1$GeV:

		λ_B (GeV)	$\sigma_B^{(1)}$
Our results	*N=1*	0.389(35)	1.63(8)
	N=2	0.393(37)	1.62(7)
	N=3	0.381(59)	1.63(12)
Experiment	*Belle 2018*	> 0.24	
Other theoretical approach	*Khodjamirian, Mandal, Mannel, 2020*	0.383(153)	
	Gao, Lu, Shen, Wang, Wei, 2020	$0.343^{+0.064}_{-0.079}$	
	Lee, Neubert, 2005	0.48(11)	1.6(2)
	Braun, Ivanov, Korchemsky, 2004	0.46(11)	1.4(4)
	Grozin, Neubert, 1997	0.35(15)	
	Mandal, Nandi, Ray, 2024	0.338(68)	

Discussions III: Impact on $B \to V$ form factors

An accurate λ_B will significantly improve the prediction for the $B \to K^*$ form factors:

	GLSWW		Our result
λ_B:	0.343^{+64}_{-79}	$\to$	$0.389(35)$
Error of $\mathcal{V}(0)$:	0.23	$\to$	0.11

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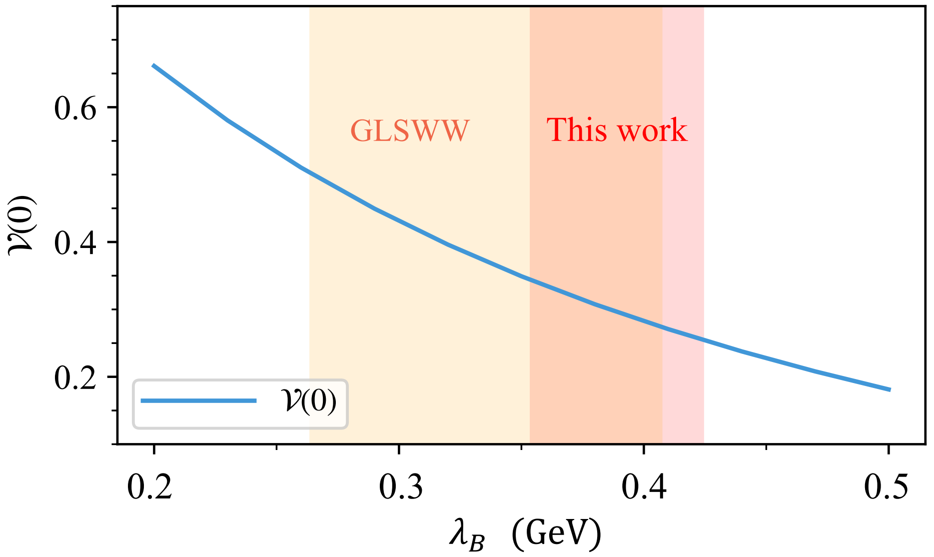

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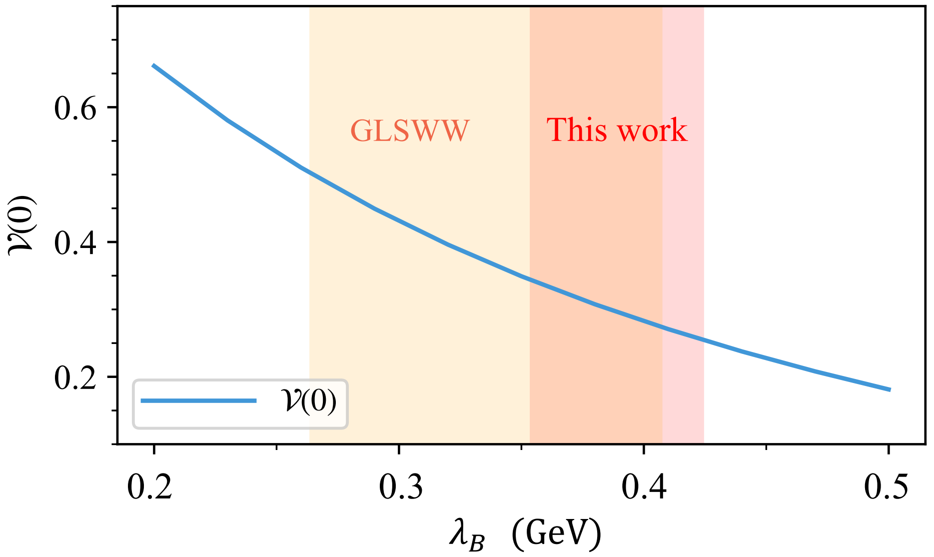

We are looking forward to a more precise analysis of the form factors and accordingly physical observables.

$$\mathcal{V}_{B\to K^*}(0) = 0.359 \left.{}^{+0.141}_{-0.085}\right|_{\lambda_B} \left.{}^{+0.019}_{-0.019}\right|_{\sigma_1} \left.{}^{+0.001}_{-0.062}\right|_{\mu} \left.{}^{+0.010}_{-0.004}\right|_{M^2} \left.{}^{+0.016}_{-0.017}\right|_{s_0} \left.{}^{+0.153}_{-0.079}\right|_{\varphi_\pm(\omega)},$$

Our results can:

- REDUCE the errors from λ_B and $\sigma_B^{(n)}$;
- REMOVE the errors from model dependence.

Summary and Prospect

- ✓ We present a first lattice-implementable method to extract the heavy meson LCDA, and implement it on a CLQCD ensemble.
- ✓ Although the results are preliminary, they can be continually improved.
- ✓ The phenomenological implications demonstrate that our results will significantly advance the theoretical studies towards the frontier of high precision.

More importantly, improving the reliability of our results for the next stage:

- How to properly control the power corrections within two step factorization?
- More systematic lattice QCD calculations: more a, larger P^z, …

Thanks for your attention!