Exploring the light-cone wave functions with Dyson-Schwinger Equations (in Euclidian space).

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2024.11.25@IMP CAS (Huizhou)《Light-Cone 2024: Hadron Physics in the EIC era》

Outline

- Introduction to DSEs & LFWFs
- Heavy flavor-asymmetric meson $q\bar{Q}$-LFWFs & Application
- Photon $q\bar{q}$-LFWFs & Application
- Summary

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Light-Cone Wave Functions

$$|M\rangle = \sum_{\lambda_1,\lambda_2} \int \frac{d^2\boldsymbol{k}_T}{(2\pi)^3} \frac{dx}{2\sqrt{x\bar{x}}} \frac{\delta_{ij}}{\sqrt{3}} \underline{\Phi_{\lambda_1,\lambda_2}(x,\boldsymbol{k}_T)}\, b^\dagger_{f,\lambda_1,i}(x,\boldsymbol{k}_T)\, d^\dagger_{h,\lambda_2,j}(\bar{x},\bar{\boldsymbol{k}}_T)|0\rangle + \phi_3 \,|\, q\bar{q}g\rangle + \dots$$

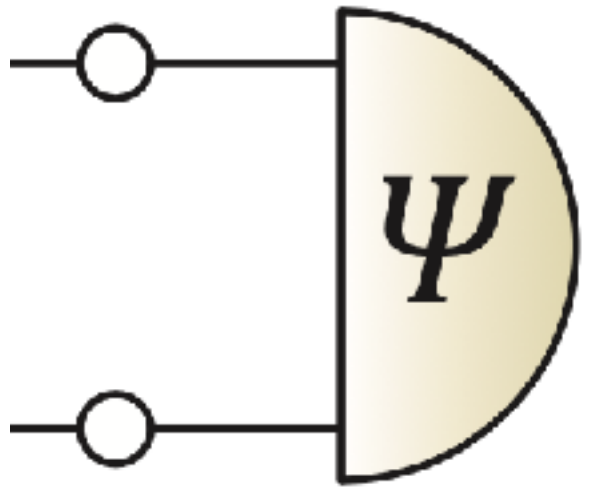
Light-Cone Wave Function

- Light-Cone wave functions in LF QCD are like Schrodinger wave functions in nonrelativistic quantum mechnics.
- Light-Cone wave functions = light-front wave functions (LFWFs).

From Bethe-Salpeter WFs to light-front WFs

- Bethe-Salpeter wave function is the transition amplitude of a hadron state into quark and antiquark legs in ordinary space-time coordinate.

$$\langle 0 | \psi_\alpha(x) \bar{\psi}_\beta(y) | h \rangle \xrightarrow{\text{Fourier Transform}} \langle b_\alpha^+ d_\beta^+ | h \rangle$$



- Light front wave function is the transition amplitude of a hadron state into certain Fock components in the light-front coordinate.

$$|h\rangle = \phi_1 | b_\alpha^+ d_\beta^+ \rangle + \phi_2 | b_\alpha^+ d_\beta^+ a_g^+ \rangle + \ldots$$

$$\phi_1 = \langle b_\alpha^+ d_\beta^+ | h \rangle$$

- The LF wave functions can be obtained by projecting the BS wave functions onto the light-front, i.e., changing from $(\xi^0, \xi^3) \rightarrow (\xi^+ = 1/\sqrt{2}(\xi^0 + \xi^3), \xi^- = 1/\sqrt{2}(\xi^0 - \xi^3))$, and set $\xi^+ = 0$(required by LFWFs)

From Bethe-Salpeter WFs to light-front WFs

- "...'t Hooft did not use the light–cone formalism and which nowadays might be called standard. Instead, he started from covariant equations... The light–cone Schrodinger equation was then obtained by projecting the Bethe–Salpeter equation onto hyper-surfaces of equal light–cone time."   (T. Heinzl arXiv:hep-th/0008096)

$$\int \frac{dk^-}{2\pi} \Psi_{BS}(k;p) = \frac{u^{(1)}(x_1, k_\perp)}{\sqrt{x_1}} \frac{u^{(2)}(x_2, -k_\perp)}{\sqrt{x_2}} \psi(x_i, k_\perp)$$

(G. Lepage and S. Brodsky, PRD 1980)

$$\psi(x, \mathbf{p}; s_1, s_2) = \frac{1}{2P^+} \int \frac{dp^-}{2\pi} \, \bar{u}(xP^+, \mathbf{p}; s_1)\gamma^+ \Phi(p) \gamma^+ v((1-x)P^+, -\mathbf{p}; s_2) \, .$$

(H. Liu and D. Soper, PRD1993)

$$\langle 0|\bar{d}_+(0)\gamma^+\gamma_5 u_+(\xi^-, \xi_\perp)|\pi^+(P)\rangle = i\sqrt{6}P^+\psi_0(\xi^-, \xi_\perp),$$

$$\langle 0|\bar{d}_+(0)\sigma^{+i}\gamma_5 u_+(\xi^-, \xi_\perp)|\pi^+(P)\rangle = -i\sqrt{6}P^+\partial^i\psi_1(\xi^-, \xi_\perp).$$

(M. Burkardt, X. Ji, F. Yuan, PLB 2002)

All equivalent!

Projection Formula

Covariant Bethe-Salpeter wave function

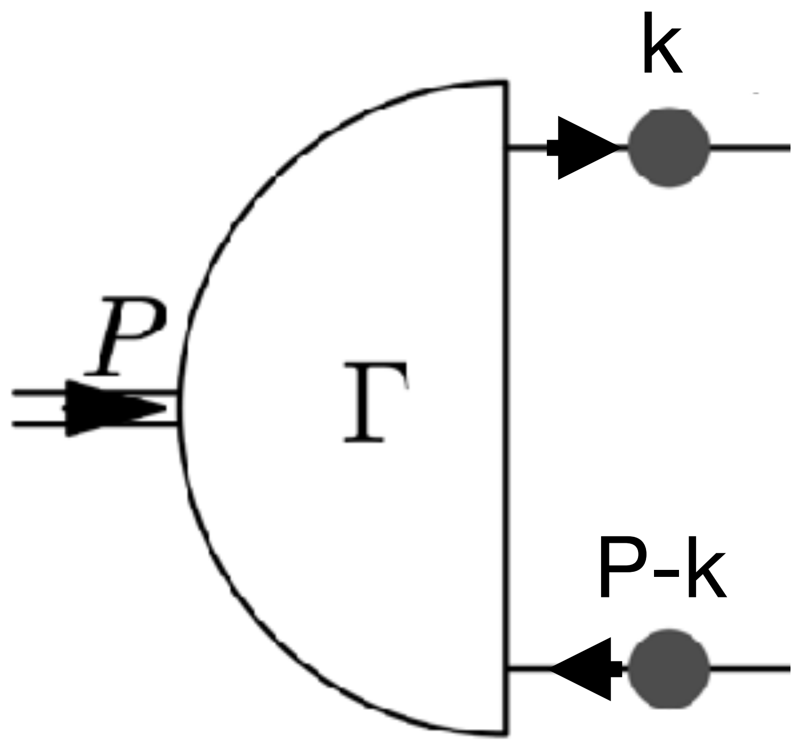
$$\phi_i(x, \vec{k}_T) \sim \int dk^- dk^+ \delta(xP^+ - k^+) \mathrm{Tr}[\Gamma_i \chi(k, P)]$$

LFWF

spin configurations

set $k^+ = xP^+$

Setting light front time $\xi^+ = 0$



(C.S., Y. Xie, M Li, X. Chen, et al, PRD(L) 2021)

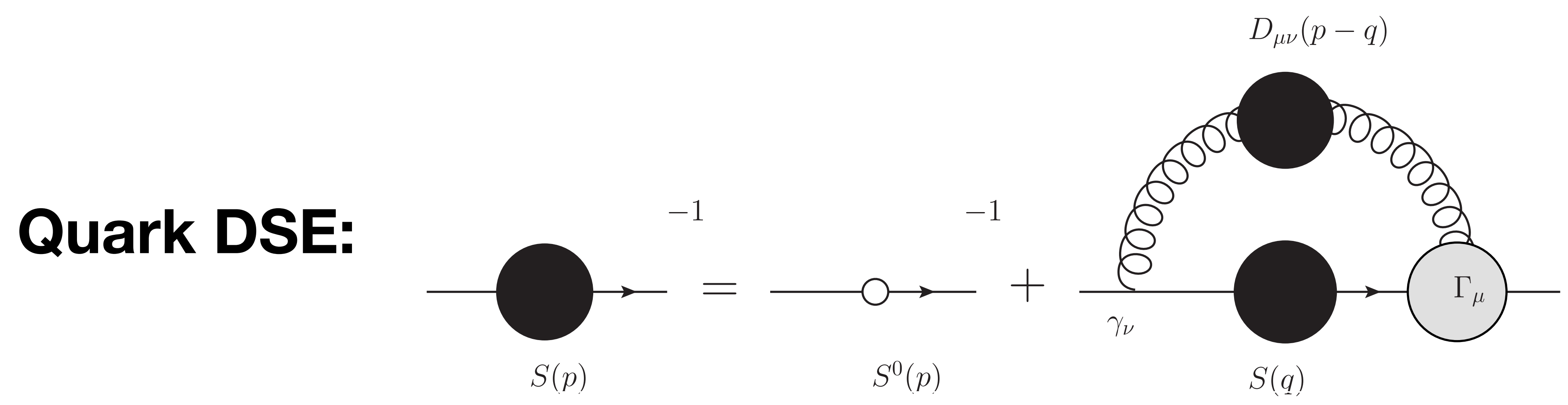
- $q\bar{q}$ LFWFs from various kinds of hadrons/particles can be extracted.
- $q\bar{q}$ LFWFs with all possible quark spin configurations can be extracted.
- $q\bar{q}$ LFWFs can be cleanly extracted from many Fock-states embedded.

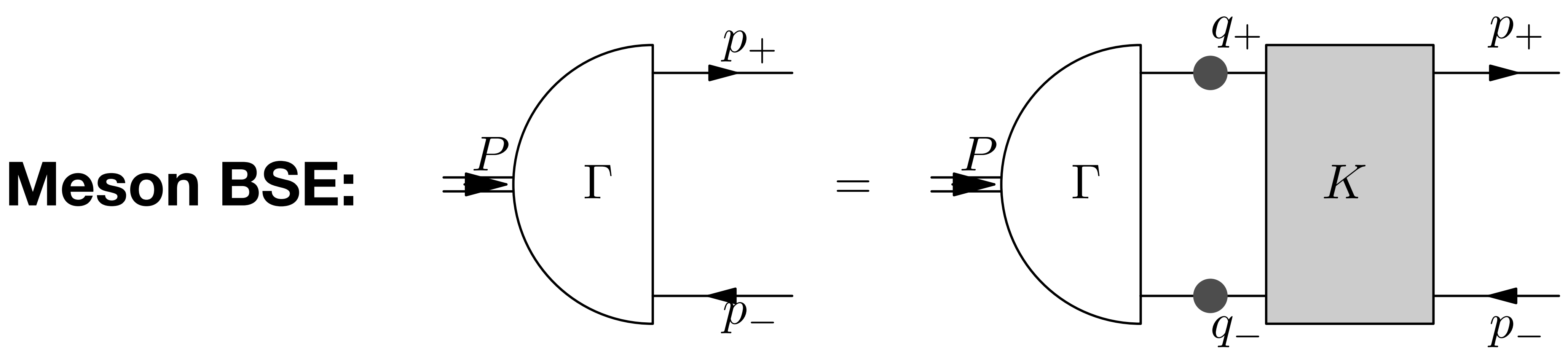
$$|h\rangle = \phi_2 |q\bar{q}\rangle + \phi_3 |q\bar{q}g\rangle + \phi_4 |q\bar{q}gg\rangle \ldots$$

- For Euclidean BS wave function, the formula holds at Mellin moments level.

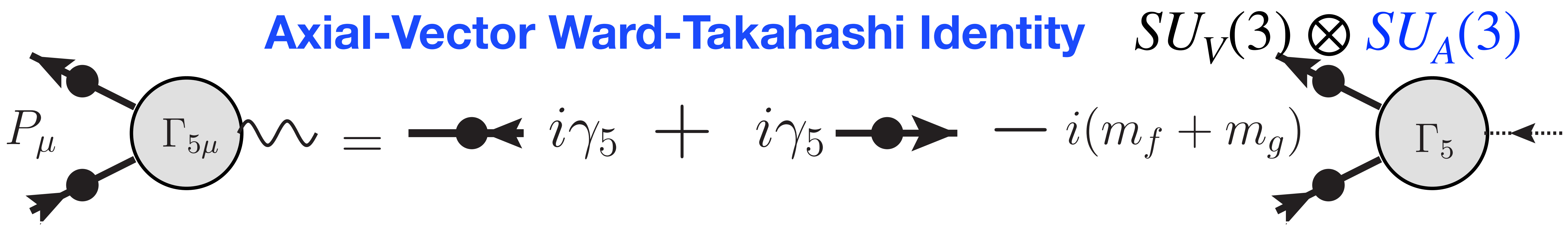
Bethe-Salpeter Wave Function From DSE

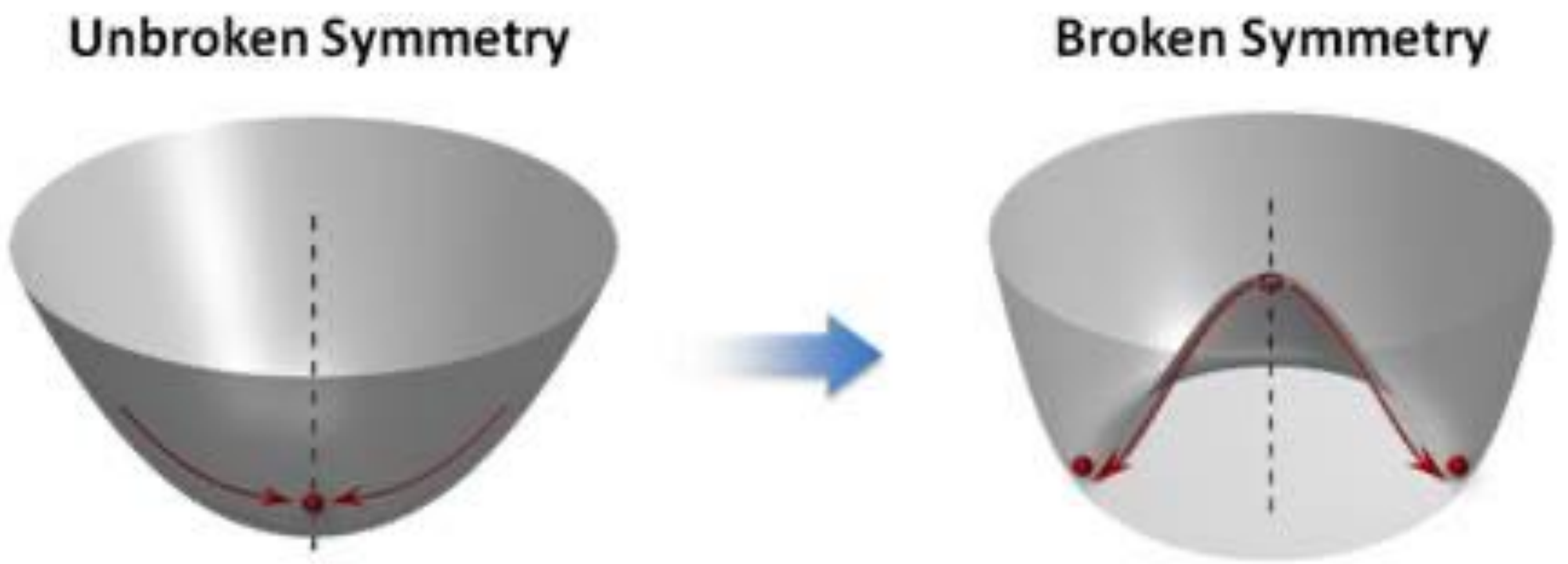
- The Bethe-Salpeter wave function is solved by aligning quark gap equation and meson BS equation, incorporating quark-gluon dynamics.





- The BS wave function has to respect QCD's chiral symmetry and manifests its dynamical breaking





$f_\pi E_\pi(k;0) = B(k^2)$ (P. Maris, C.D. Roberts and P. C. Tandy, PLB1998)

The Bethe-Salpeter wave function is tightly constrained by chiral symmetry.

Light and Heavy Pseudoscalar Mesons

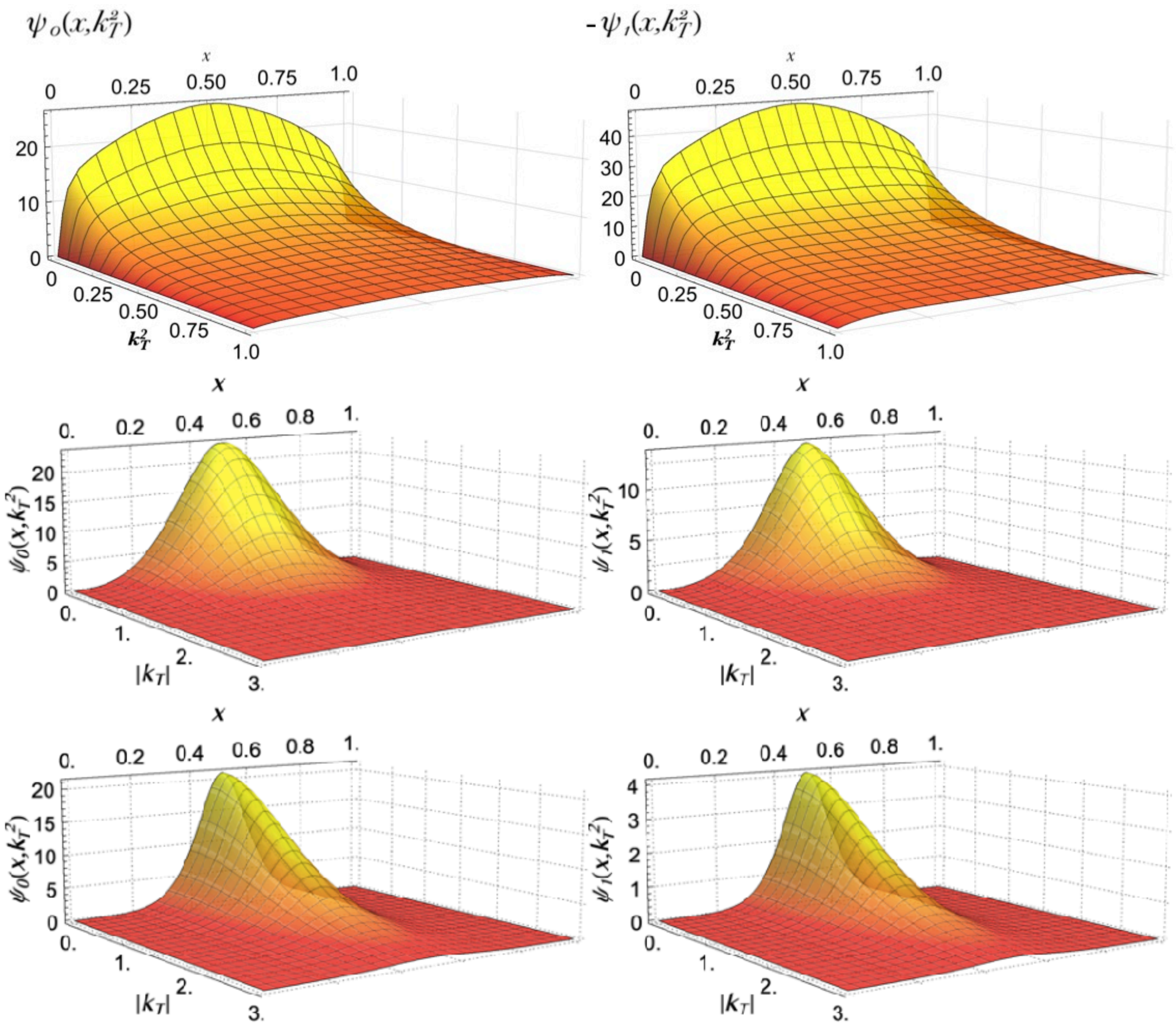
$\Lambda = \lambda + \lambda' + L_z$

s-wave $|L_z| = 0$ p-wave $|L_z| = 1$

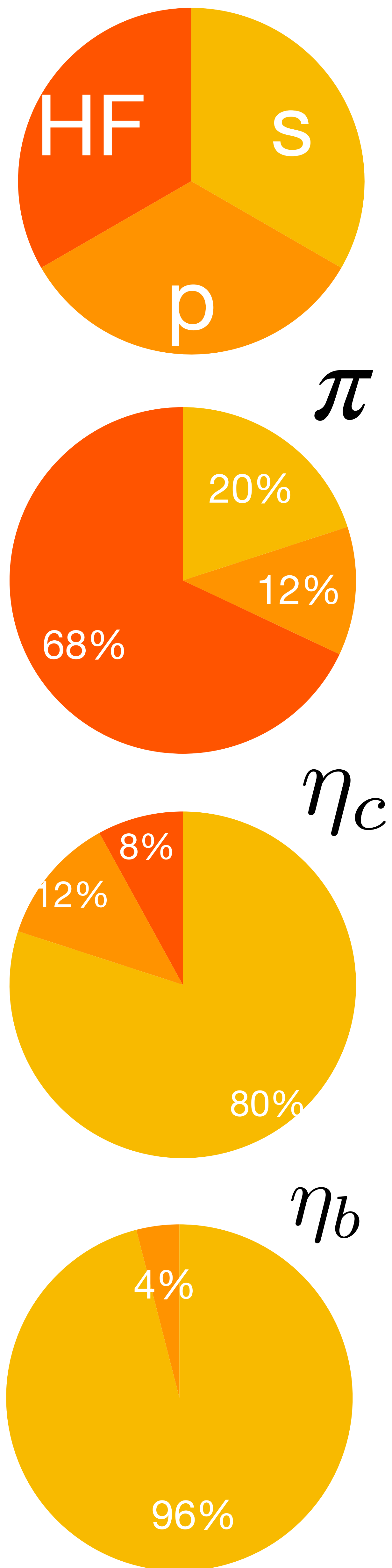
π

η_c

η_b



HF=Higher Fock states



- Like Schrodinger wave functions, LFWFs are probability amplitudes.
- p-wave components exists due to relativity.
- A strong indication of considerable higher Fock-states in light mesons.

Phys.Rev.Lett. 122 (2019) 8, 082301
Phys.Rev.D 101 (2020) 7, 074014
Phys.Rev.D 104 (2021) 9, 094016

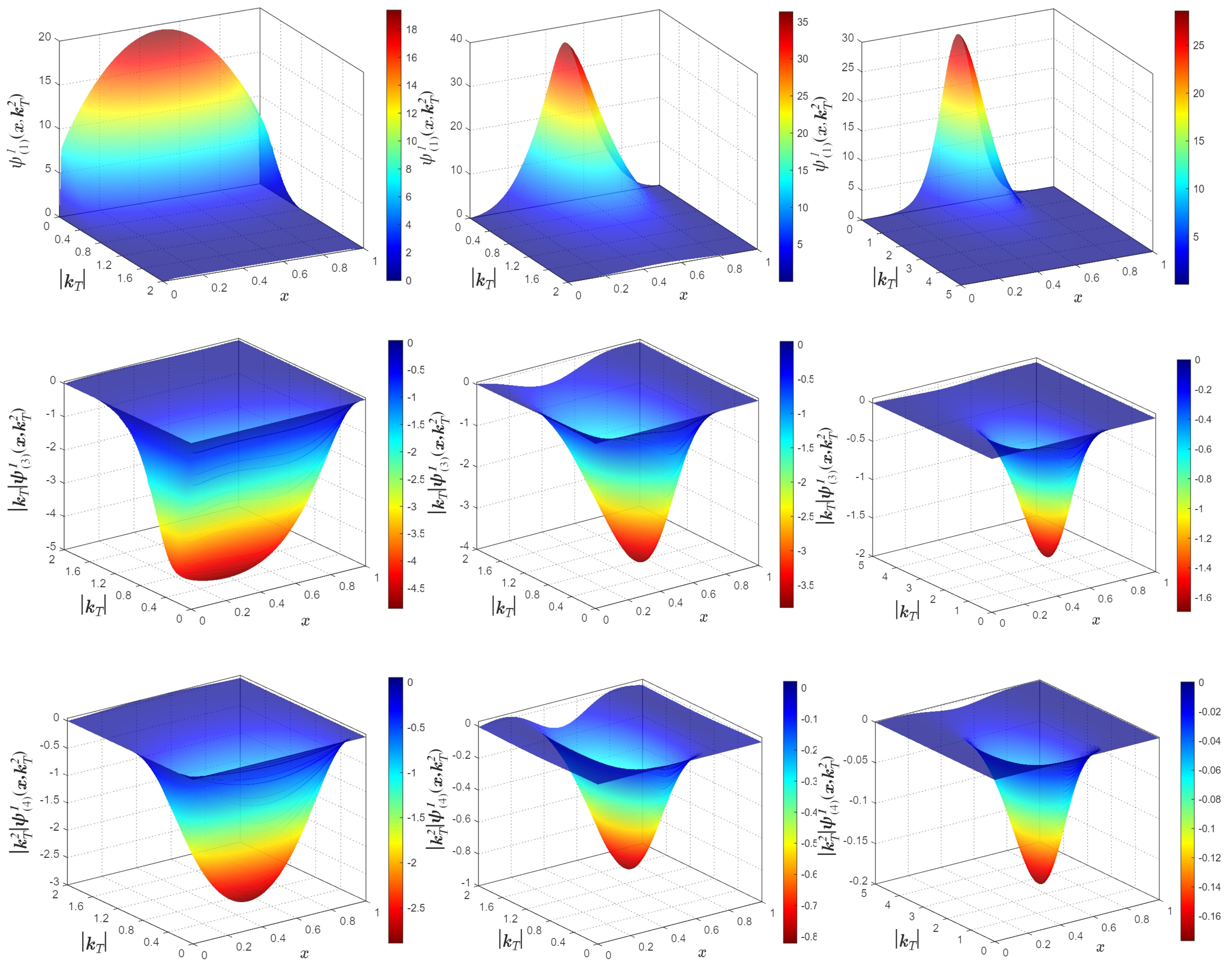
Light and Heavy Vector Mesons

Λ=1 ρ J/ψ Υ

s-wave $|L_z|$=0

p-wave $|L_z|$=1

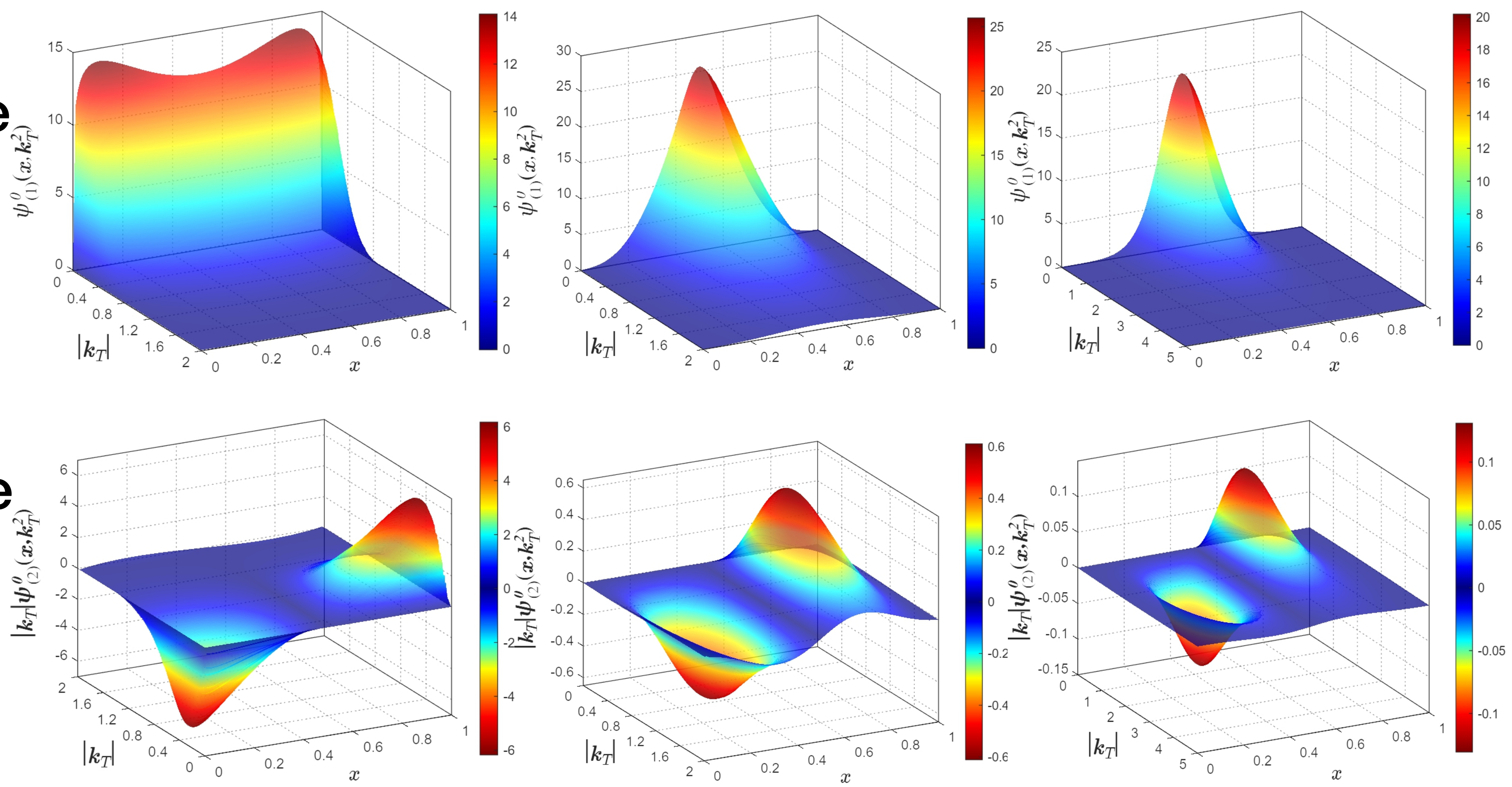
d-wave $|L_z|$=2



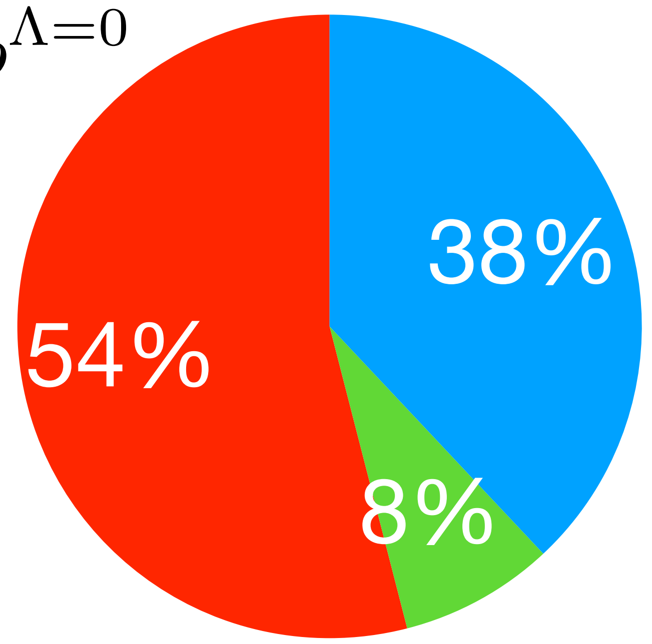
Λ=0 ρ J/ψ Υ

s-wave $|L_z|$=0

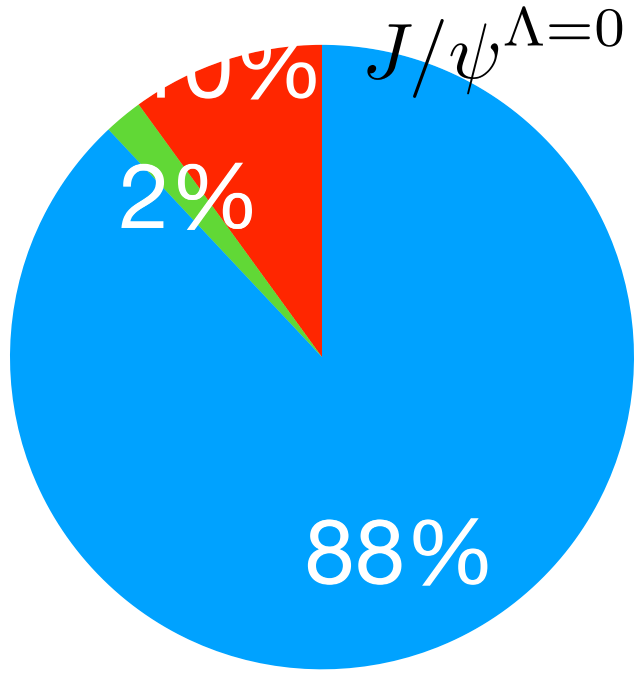
p-wave $|L_z|$=1



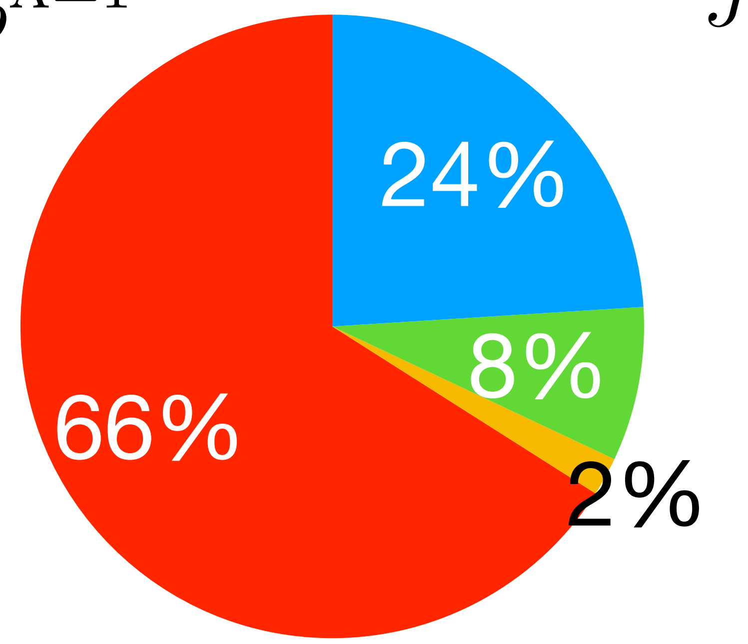
$\rho^{\Lambda=0}$



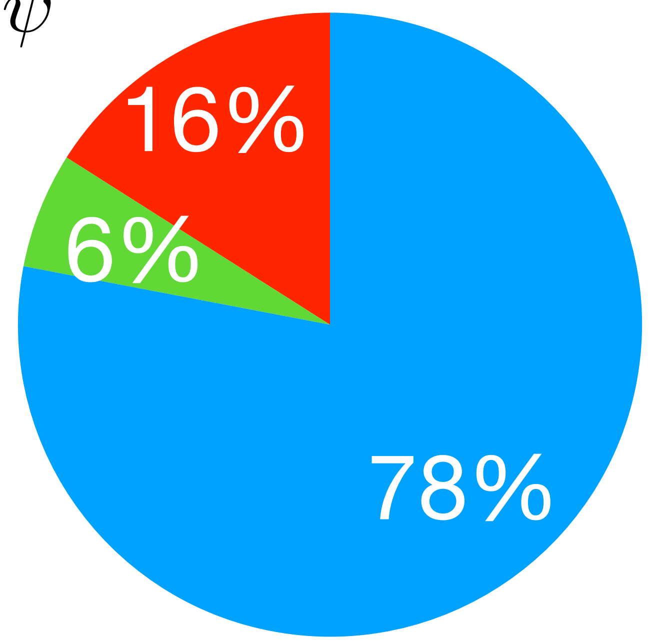
$J/\psi^{\Lambda=0}$



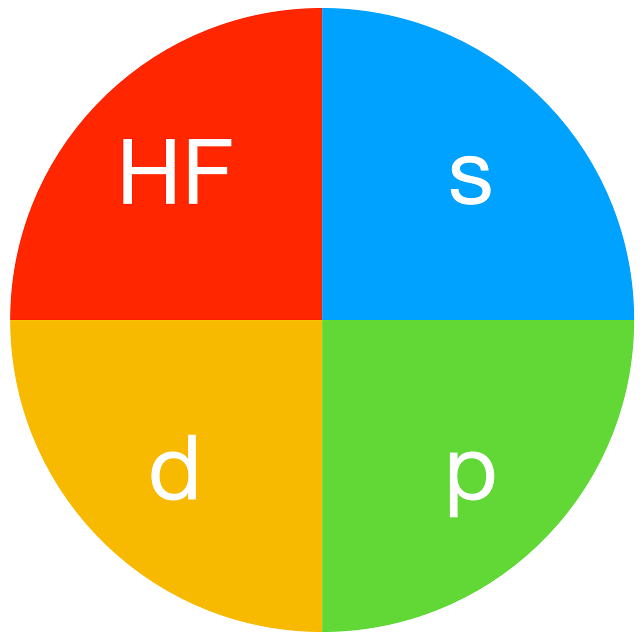
$\rho^{\Lambda=1}$



$J/\psi^{\Lambda=1}$



HF=Higher Fock



Phys.Rev.D 104 (2021) 9, L091902
Phys.Rev.D 106 (2022) 1, 014026
Phys.Rev.D 107 (2023) 7, 074009

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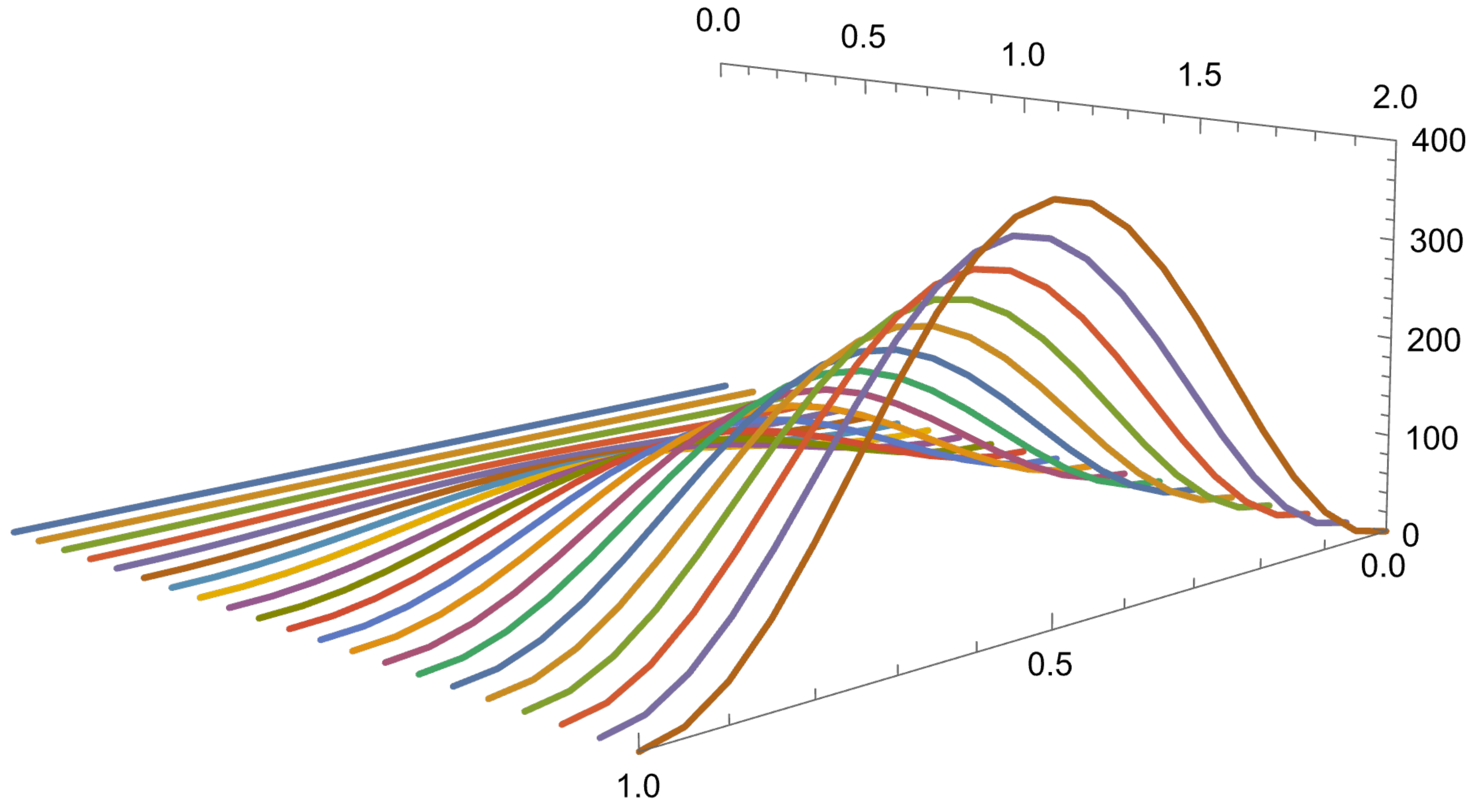
Calculation techniques (Numerical)

$$\phi_i(x,\vec{k}_T) \sim \int dk^- dk^+ \delta(xP^+ - k^+)\mathrm{Tr}[\Gamma_i \chi(k,P)]$$

D, B and B_c Mesons

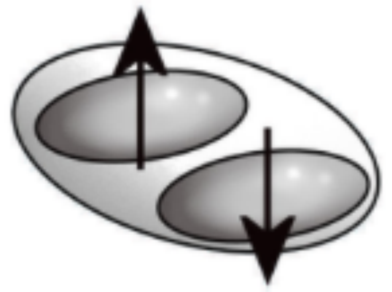
BS wave function: Yin-Zhen Xu, *JHEP* 07 (2024)

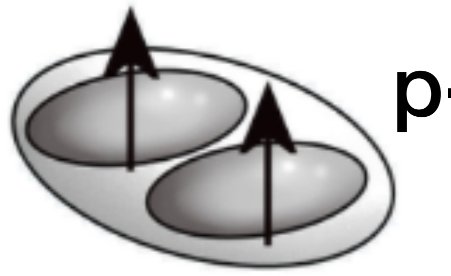


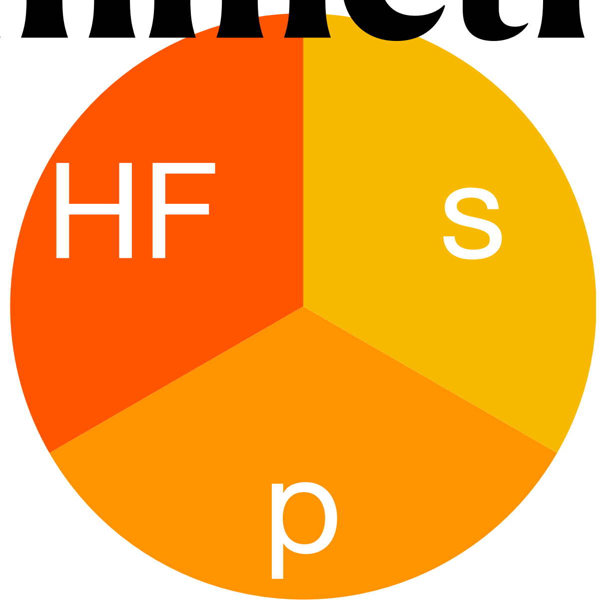
$$\langle x^m \rangle_{\vec{k}_T} = \int dx x^m \phi_i(x,\vec{k}_T) = \int dk_\parallel^2 \left(\frac{k^+}{P^+}\right)^m \cdots$$

- Compute the Mellin moments of LFWFs at every discretized $\vec{k}_T$
- For every $\vec{k}_T$, use $x^\alpha(1-x)^\beta(c_0 + c_1 + c_2 x^2)$ to fit the Mellin moments.
- Interpolate the LFWFs at every $\vec{k}_T$ to get the final WFs.

LFWFs of 0^- heavy flavor asymmetric Meson



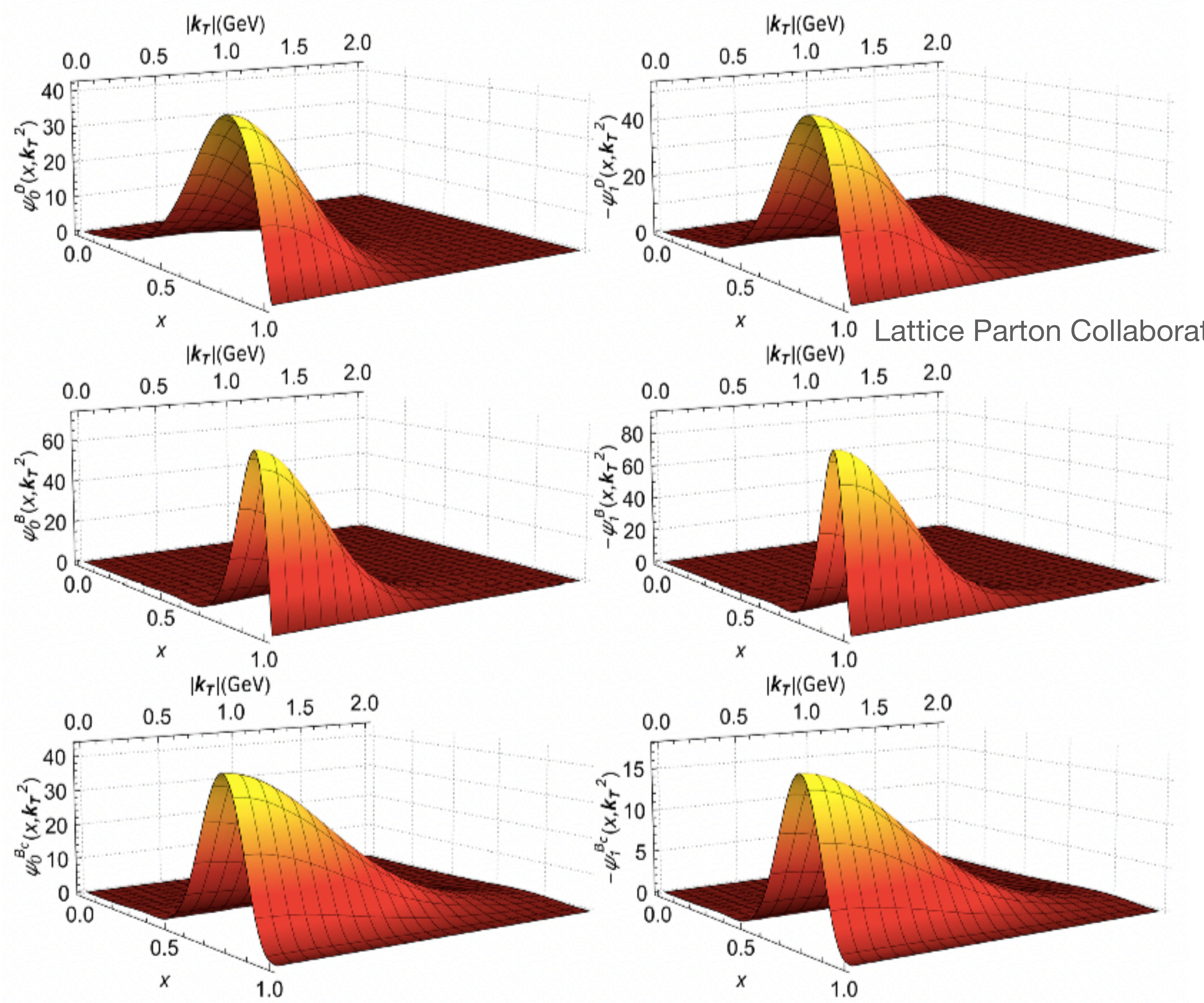




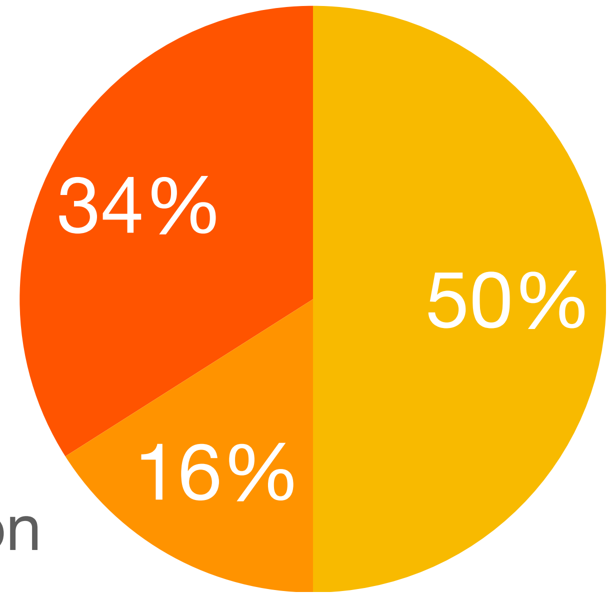
D

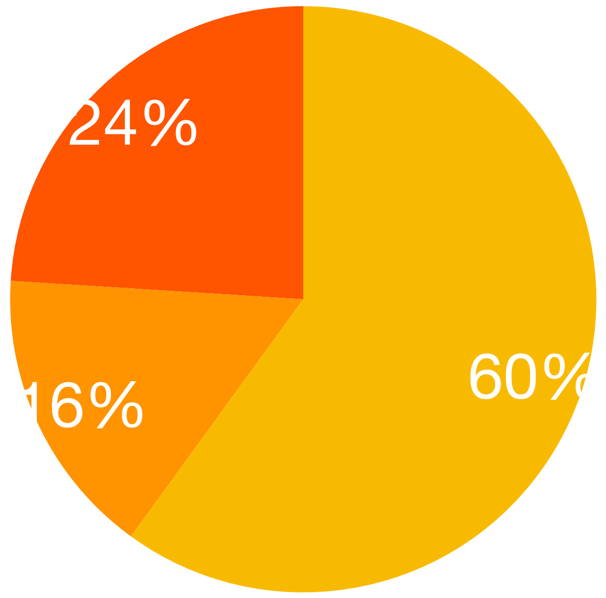
B

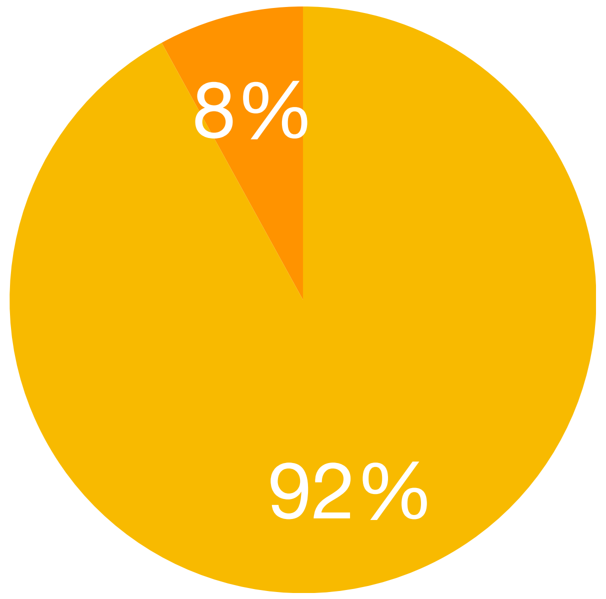
B_c



Lattice Parton Collaboration





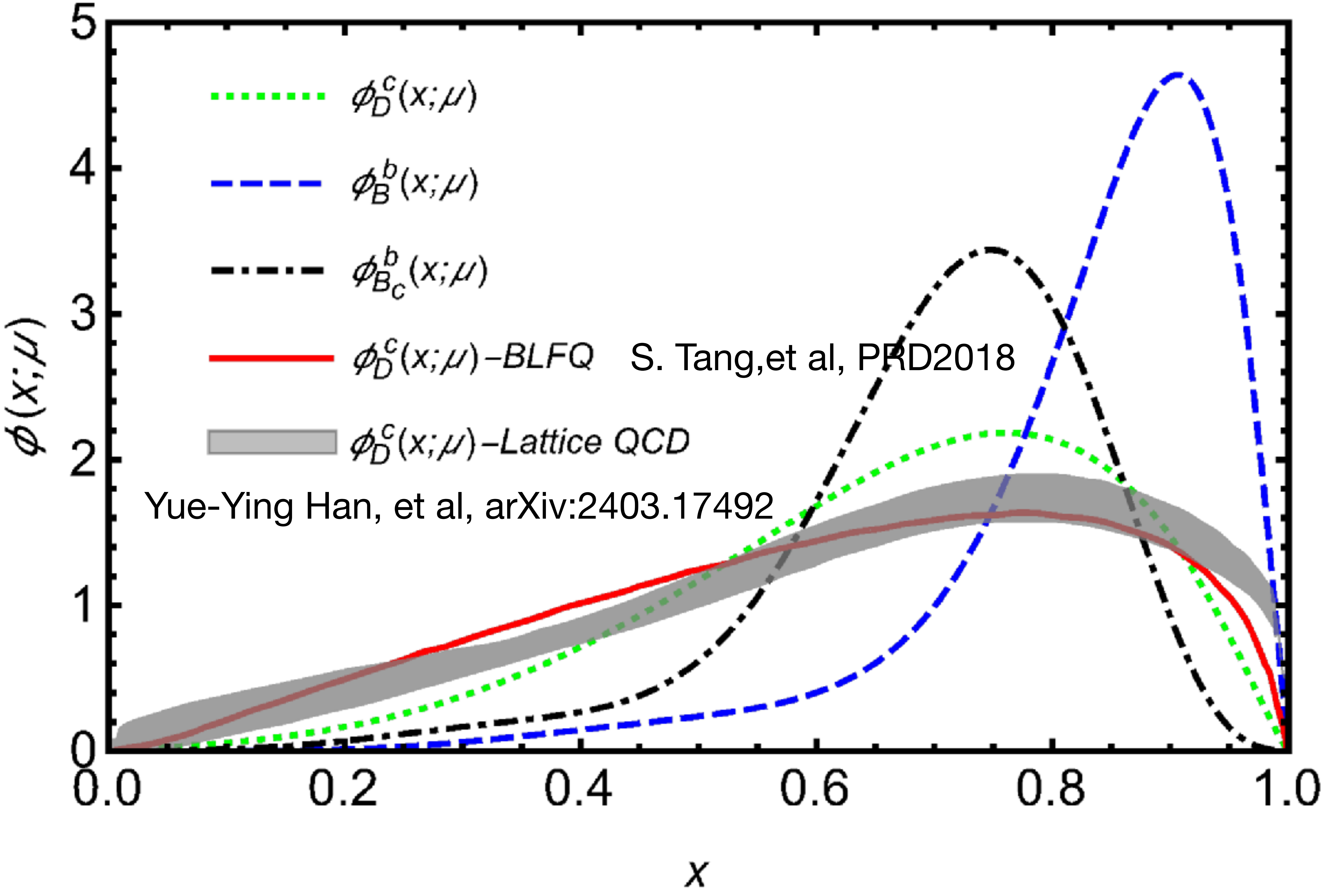


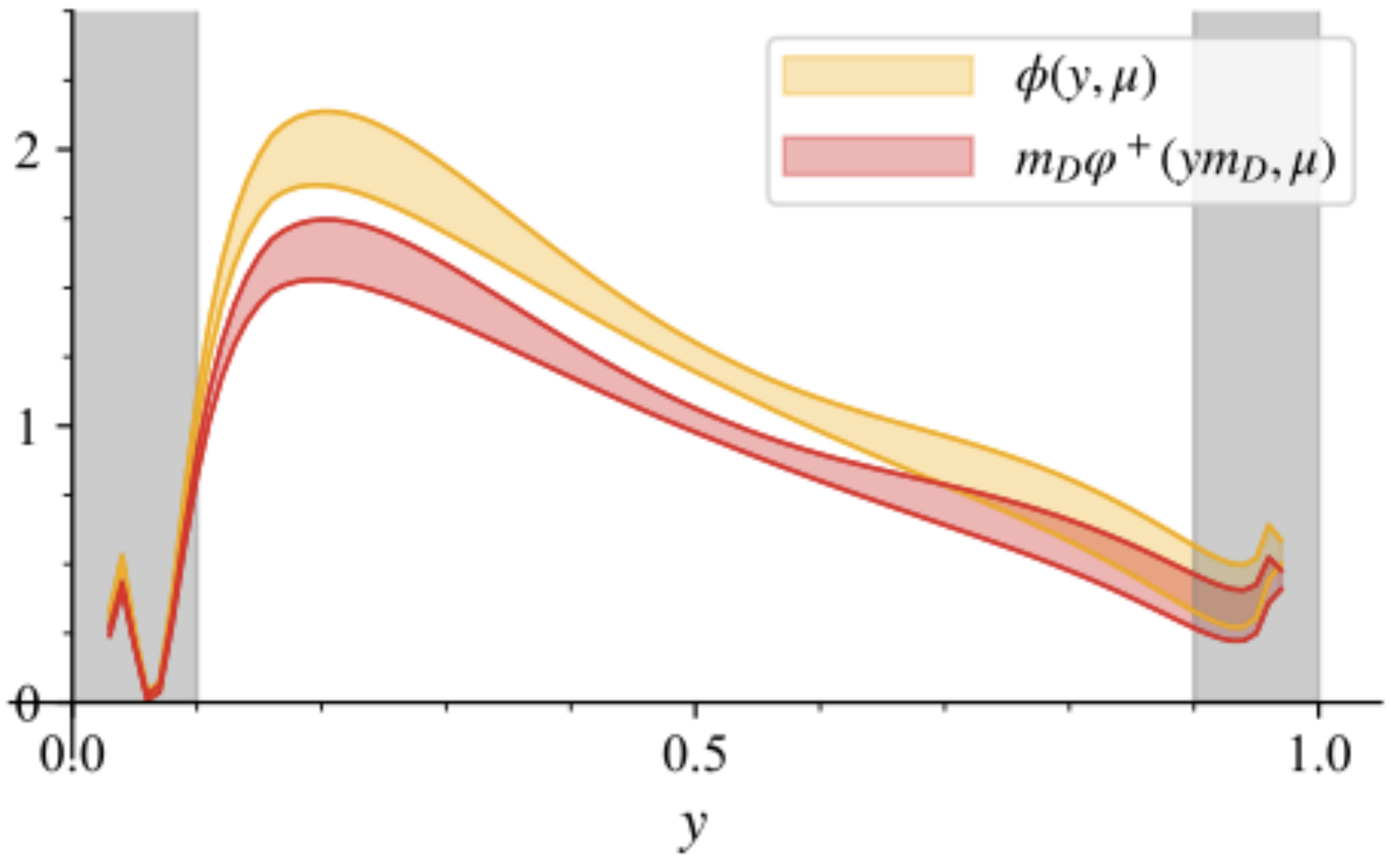
- Narrow x–distribution
- Narrow k_T–distribution
- Exhibiting a duality embodying characteristics from both light mesons and heavy quarkonium.

CS, P Liu, Y Du and W Jia, *Phys.Rev.D* 110 (2024) 9, 094010

Parton Distribution Amplitude

$$\phi(x, Q^2) = \int^{Q^2} dk_T^2 \psi_0(x, k_T^2)$$





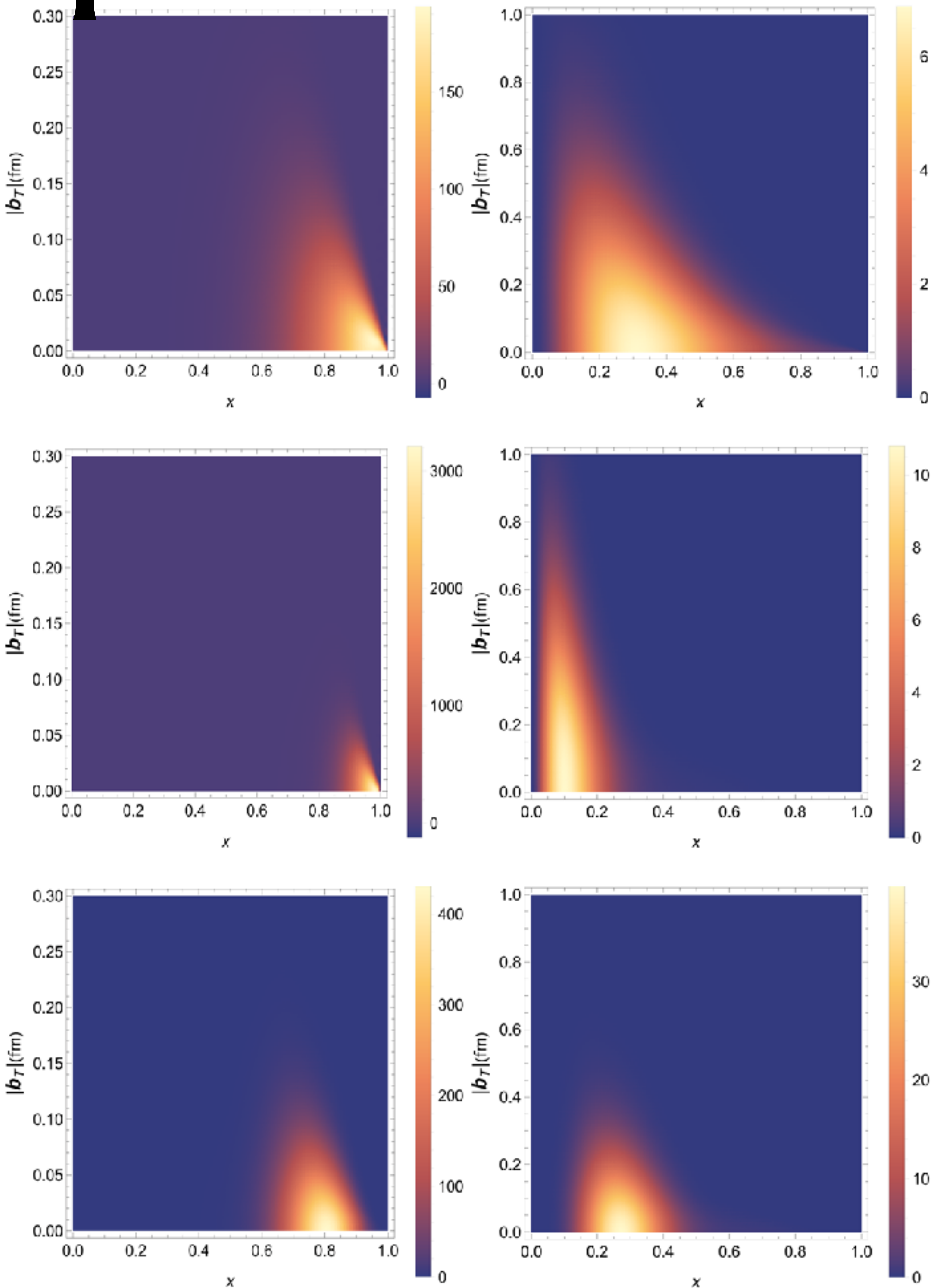
Lattice Parton Collaboration :arXiv:2410.18654

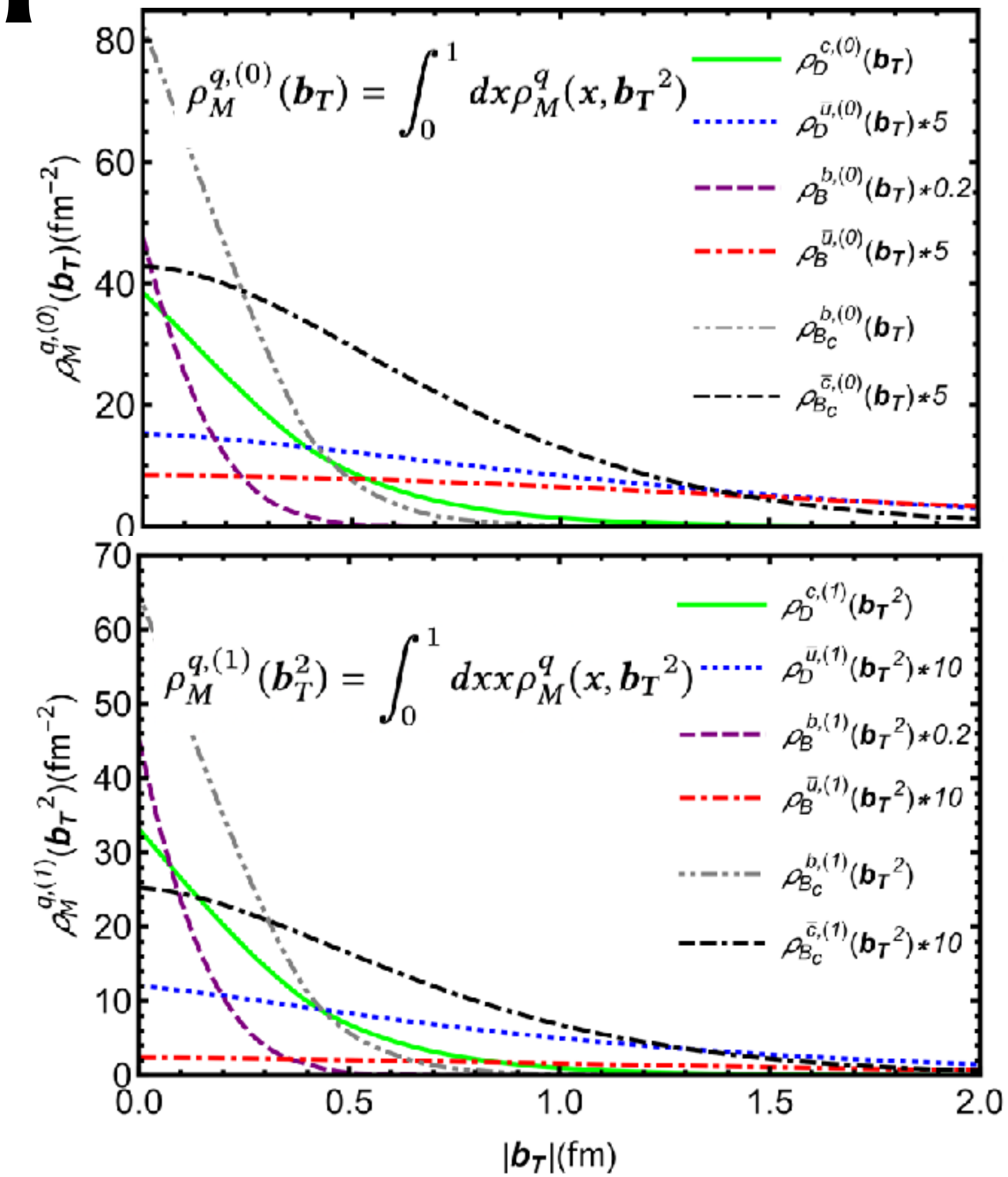
Impact Parameter Dependent GPD

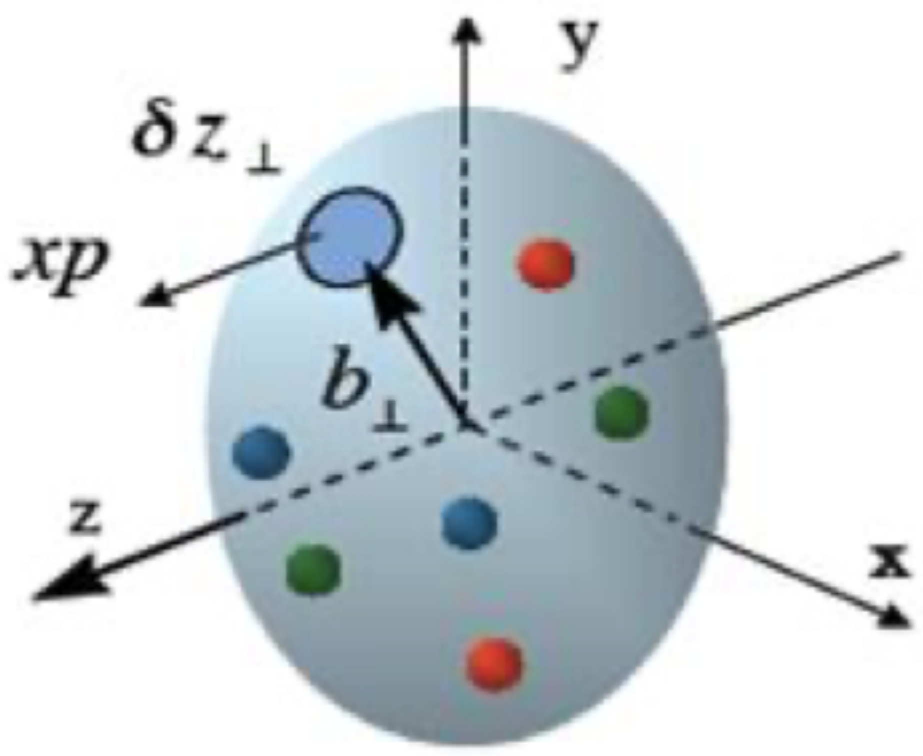
D

B

B_c



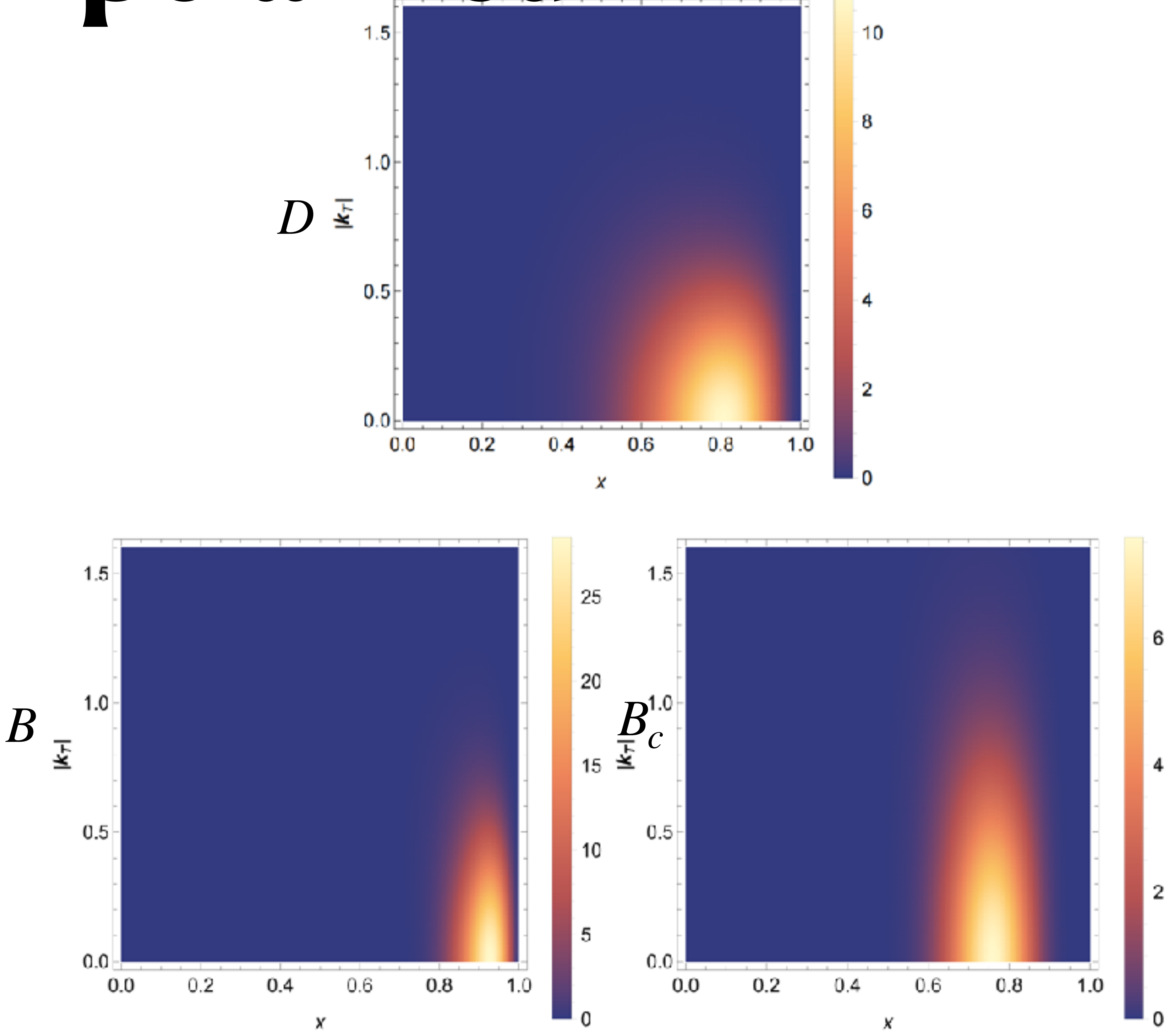


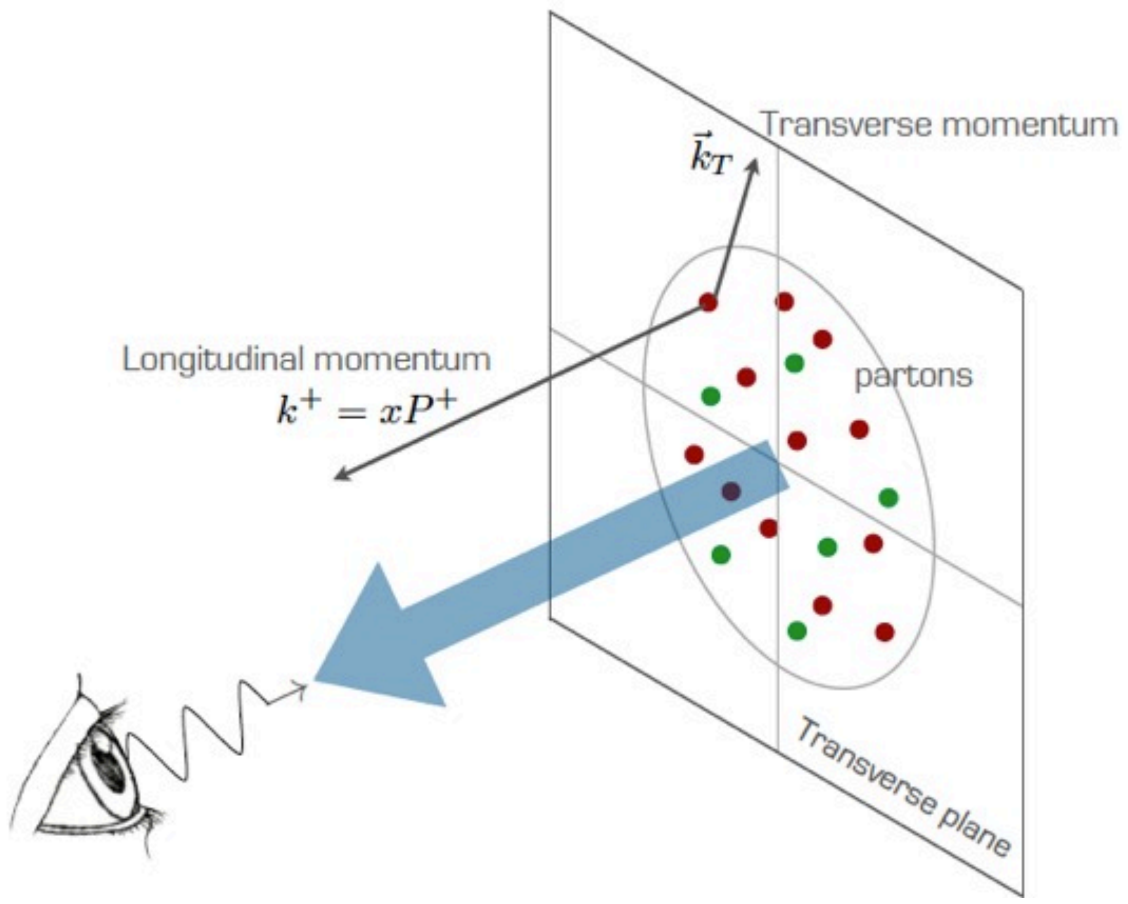


$\rho(x, |\vec{b}_T|)$

- Heavier quarks are spatially more centered, and carries most light-cone momentum.
- Lighter quarks are spread out, yet carries electric charge.
- In $B(b\bar{u})$, we find $\langle r^2_{E,\mathrm{LC}}\rangle_B = (0.14\ \mathrm{fm})^2 \ll \langle r^2_{c,LC}\rangle_{B^-} = (0.38\ \mathrm{fm})^2$!!!

Unpolarized TMD PDF





$$f_1(x, \overrightarrow{k}_T^2)$$

- $\langle|\overrightarrow{k}_T|\rangle = \int dx d^2\overrightarrow{k}_T f_1^q(x, \overrightarrow{k}_T^2)|\overrightarrow{k}_T|$ for D, B and B_c mesons are 0.43, 0.42 and 0.65 GeV, as compared to 0.39, 0.65, 1.0 GeV for π, η_c and η_b.
- The mean transverse momentum inside $Q\bar{q}$ is close to that in $q\bar{q}$!

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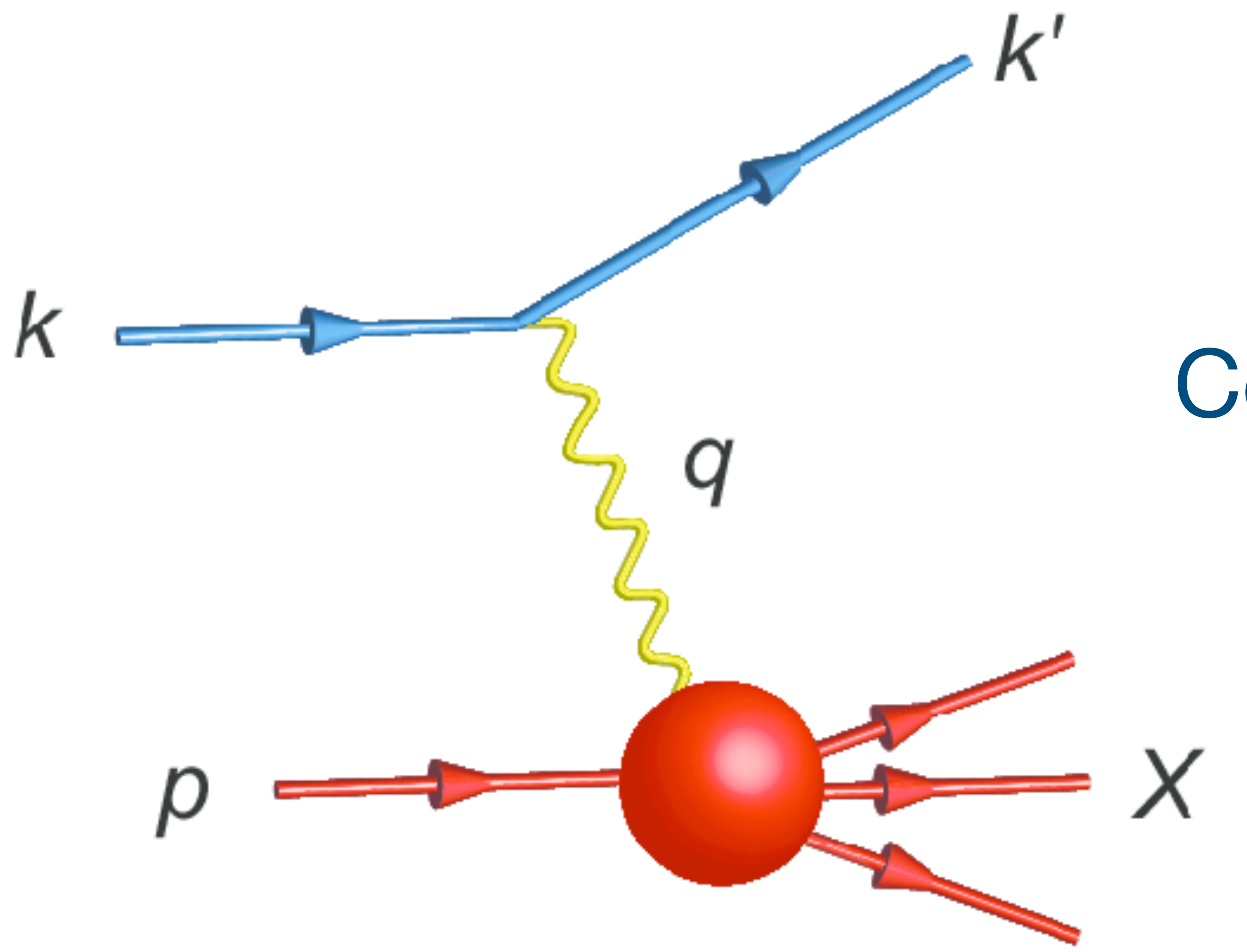
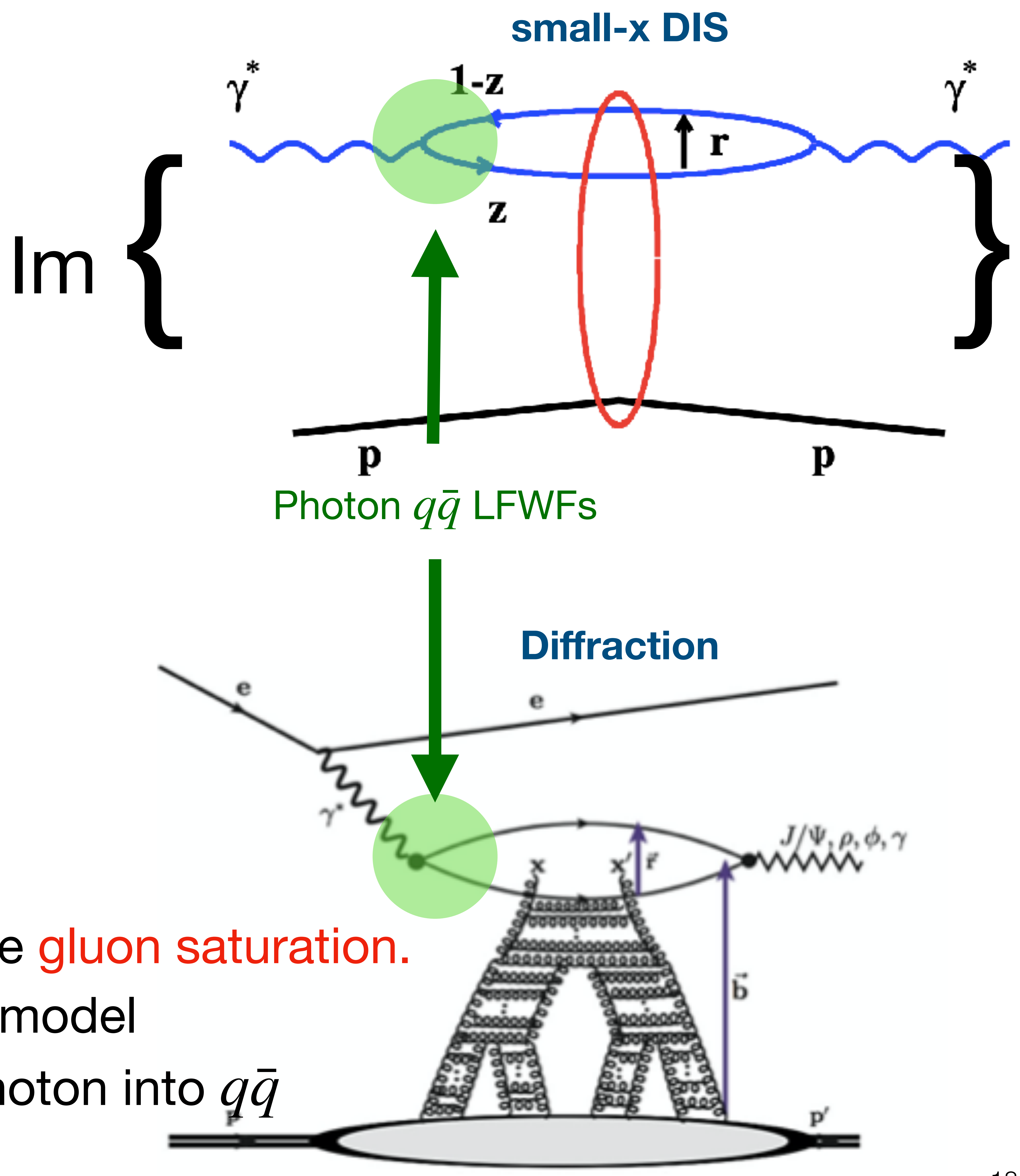
Color Dipole Model and small-x DIS



Color Dipole Model (small-x)



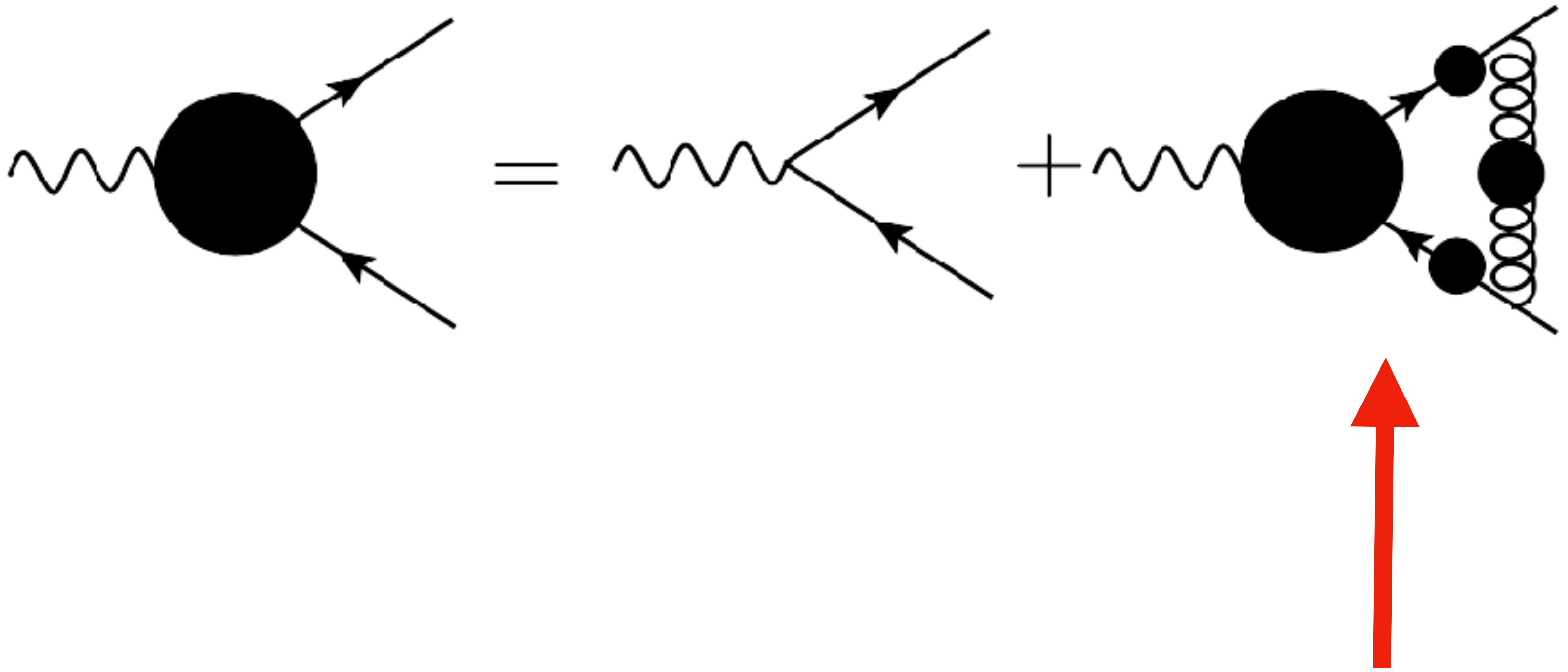
Kowalski_PRD2006

- Color dipole model is an important approach to investigate gluon saturation.
- Photon $q\bar{q}$ LFWFs are indispensable input of color dipole model
- Photon $q\bar{q}$ LFWFs describe the transition amplitude of photon into $q\bar{q}$

QCD in quasi-real photon

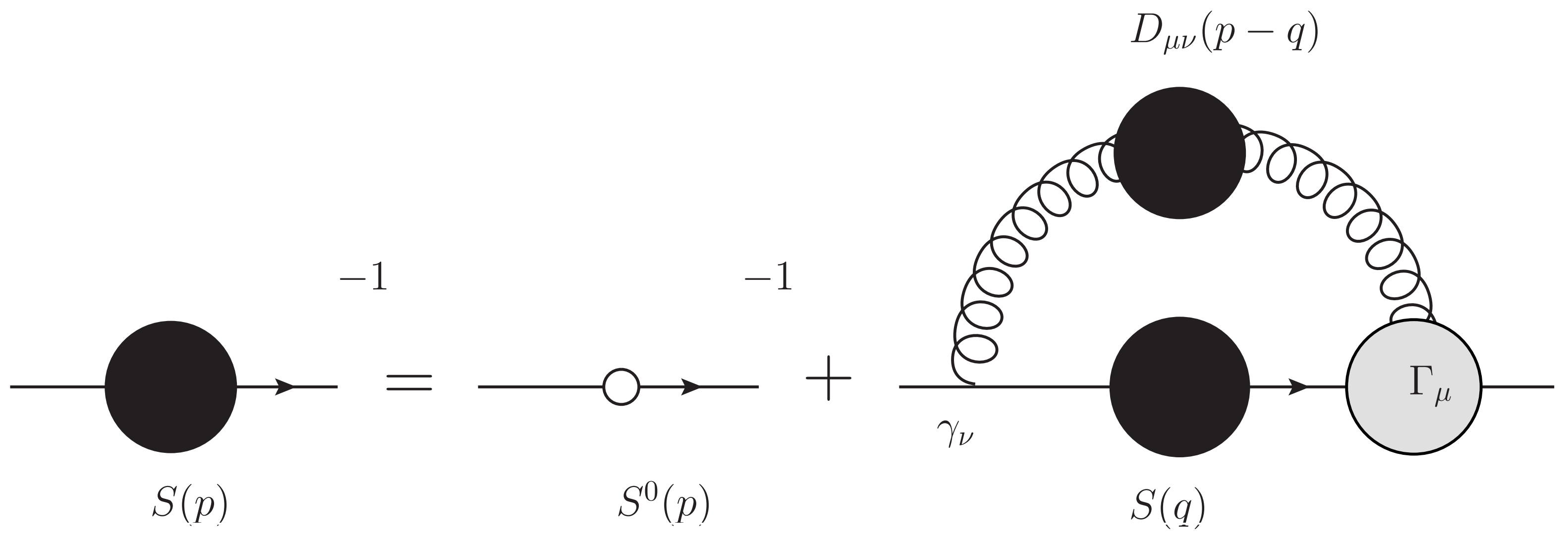
$$|\gamma^*_{\text{phys}}\rangle = |\gamma^*_{\text{bare}}\rangle + |e^+e^-\rangle_{\gamma^*} + \sum_{f=u,d,s\ldots} |q_f\bar{q}_f\rangle_{\gamma^*} + \ldots$$

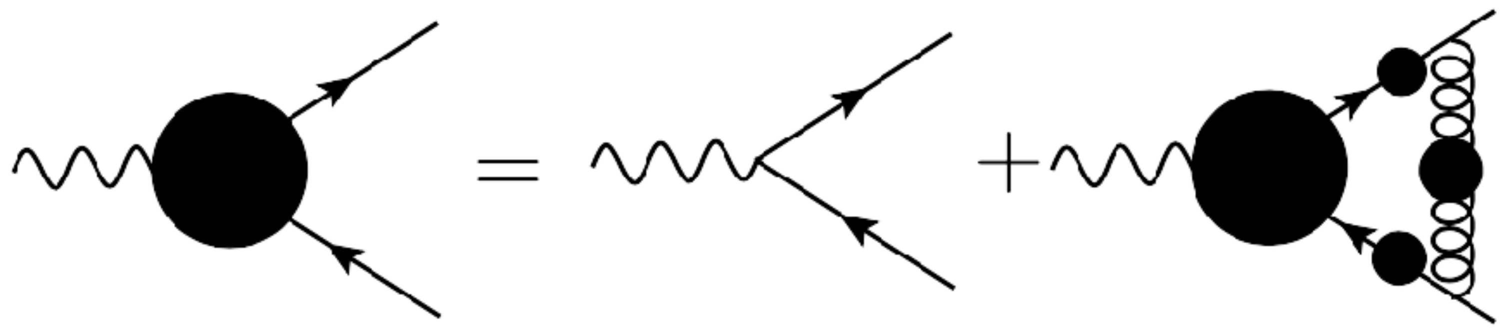
- At high virtuality, photon LFWFs can be calculated perturbatively.
- For low virtuality photon, such as quasi-real photon, there are significant nonperturbative QCD effects. (For instance, VMD)



QCD effect matters for quasi-real photons!

Photon Bethe-Salpeter Wave Function





Contact Interaction Model:

$$g^2 D_{\mu\nu}(k-q) = \delta_{\mu\nu}\frac{4\pi\alpha_{\mathrm{IR}}}{m_G^2}$$

$$S_f^{-1}(k) = i\gamma\cdot k + m_f + \frac{4}{3}\frac{4\pi\alpha_{\mathrm{IR}}}{m_G^2}\int\frac{d^4q}{(2\pi)^4}\gamma_\mu S_f(q)\gamma_\mu,$$

$$\Gamma_\mu^{\gamma^*,(f)}(k;Q) = \gamma_\mu - \frac{4}{3}\frac{4\pi\alpha_{\mathrm{IR}}}{m_G^2}\int\frac{d^4q}{(2\pi)^4} \times\gamma_\alpha S_f(q)\Gamma_\mu^{\gamma^*,(f)}(q;Q)S_f(q-Q)\gamma_\alpha,$$

$$S_f(p)^{-1} = i\gamma\cdot p + M_f,$$

$$\Gamma_\mu^{\gamma^*,(f)}(Q) = \gamma_\mu^T P_T^{(f)}(Q^2) + \gamma_\mu^L P_L^{(f)}(Q^2)$$

Calculation techniques (Analytical)

$$\Phi^{\Lambda,(f)}_{\lambda,\lambda'}(x,\boldsymbol{k}_T) = -\frac{1}{2\sqrt{3}}\int \frac{dk^- dk^+}{2\pi}\delta(xQ^+ - k^+)\ \mathrm{Tr}\Big\{\Gamma_{\lambda,\lambda'}\gamma^+ S_f(k)[e_f e\Gamma^{\gamma^*,(f)}(k;Q)\cdot\epsilon_\Lambda(Q)]S_f(k-Q)\Big\}$$

$$\langle x\rangle^m \equiv \int_0^1 dx x^m \Phi^0_{+,-}(x,\boldsymbol{k}_T)$$

$$= -\frac{e_f e P_T(Q^2)}{2\sqrt{3}}\int \frac{d^2\boldsymbol{k}_\parallel}{2\pi}\left(\frac{k^+}{Q^+}\right)^m \frac{1}{|Q^+|}\mathrm{Tr}\Big[(I+\gamma^5)\gamma^+ S(k)[\Gamma^{\gamma^*}(k;Q)\cdot\epsilon_0(Q)]S(k-Q)\Big]$$

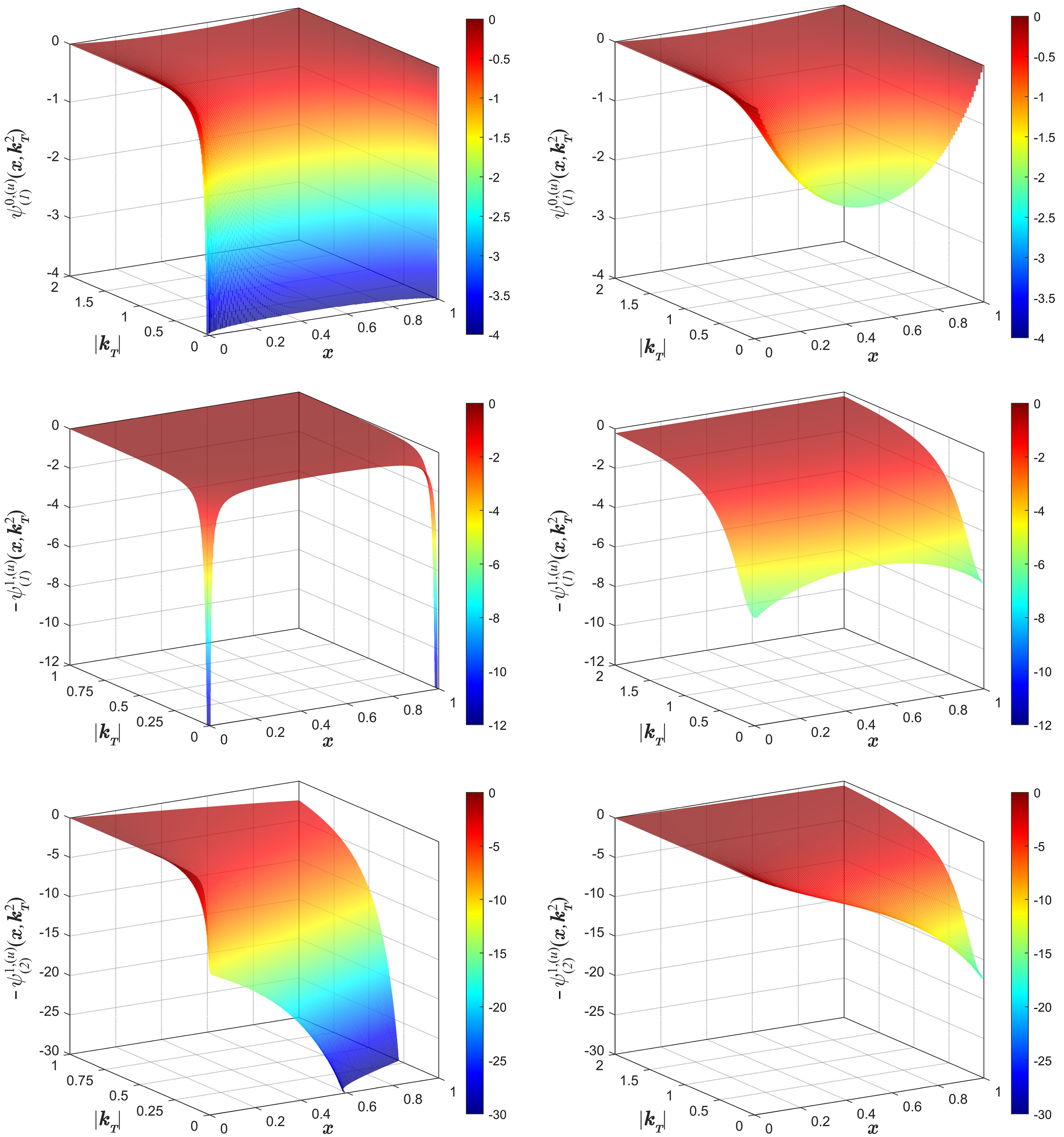
$$= -\frac{e_f e P_T(Q^2)}{2\sqrt{3}|Q\cdot n|}\int \frac{d^2\boldsymbol{k}_\parallel}{2\pi}\left(\frac{k_\parallel\cdot n}{Q\cdot n}\right)^m \frac{\mathrm{Tr}[\not{n}(-i\not{k}+M)\not{\epsilon}_0(-i\not{k}+i\not{Q}+M)]}{(k^2+M^2)(k^2-2k\cdot Q+Q^2+M^2)}$$

$$= \frac{2\sqrt{N_c}e_f e P_T(Q^2)}{Q}\int_0^1 du' u'^m \int \frac{d^2\boldsymbol{k}_\parallel}{2\pi}\frac{k_\perp^2+M^2-u'(1-u')Q^2}{[k_\parallel^2+Q^2u'(1-u')+M^2+k_\perp^2]^2}$$

$$= \int_0^1 du' u'^m \frac{\sqrt{N_c}e_f e P_T(Q^2)}{Q}\left(1-\frac{2u'(1-u')Q^2}{Q^2u'(1-u')+M^2+k_\perp^2}\right).$$

- For analytical BS WFs, the LF WFs can be determined unambiguously.

Photon LFWFs



- $Q^2 \approx -(0.5\text{GeV})^2$
- Big difference between the perturbative (left) and nonperturbative (right) result.
- Limited to low virtuality due to the simplified contact interaction model.
- Experiment support?

Photon LFWF& small-x DIS

- Color Dipole Model study of small-x DIS $\sigma \sim |\phi_{\gamma^*}^{q\bar{q}}|^2 \otimes \sigma_{q\bar{q},N}$

- We propose modified photon LFWFs incorporating nonperturbative effects.

$$|\Psi'^{(f)}_{T,L}(r,z;Q^2)|^2 = F_{\rm part}(Q^2)|\Psi^{(f),{\rm np}}_{T,L}(r,z;Q^2)|^2 + [1-F_{\rm part}(Q^2)]|\Psi^{(f),{\rm p}}_{T,L}(r,z;Q^2)|^2$$

$$F_{\rm part}(Q^2) = \frac{Q_0^{2n}}{(Q^2+Q_0^2)^n}.$$

- We re-fit HERA DIS reduced cross section $\sigma_r(x,y,Q^2) = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2}F_L(x,Q^2)$

LFWFs [Eqs. (30-35,49)]	Q^2/GeV^2	γ_s	N_0	x_0	λ	Q_0^2	n	$\chi^2/\text{d.o.f}$
Pert.	[0.85, 50]	0.6290	0.4199	2.395×10^{-4}	0.1962	-	-	265.8/223 = 1.192
Pert.	[0.25, 50]	0.3869	0.7556	7.047×10^{-7}	0.1052	-	-	678.4/282 = 2.406
Pert.+Nonpert.	[0.25, 50]	0.6177	0.4596	1.326×10^{-4}	0.1875	1.052	3.970	337.9/280 = 1.207

$\sigma_{q\bar{q},N}$ model

$F_{\rm part}$ model

- Conclusion: including nonperturbative QCD effect in photon LFWFs can acommodate small-x DIS data at lower Q^2

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Summary

- The $q\bar{q}$ light-cone wave functions are explored with Euclidean DSEs studies.
- Heavy flavor asymmetric mesons exhibit novel parton picture.
- Nonperturbative QCD affects photon LFWFs and small-x DIS.

Outlook

- More mesons $q\bar{q}$LFWFs to be explored and tested in exclusive productions.
- Refine the photon LFWFs with realistic DSE, bridging the gap between low and high Q^2.
- Refine the color dipole model study and its search for gluon saturation phenomenon.

Thank you!