Gravitational form factors on the light-front

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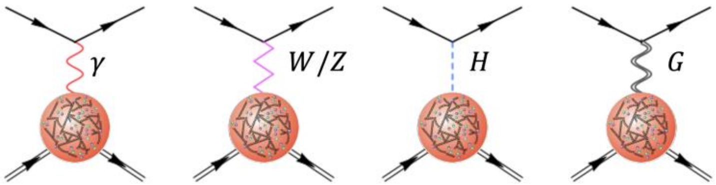
Institute of Modern Physics, Huizhou, November 26

Energy-momentum tensor



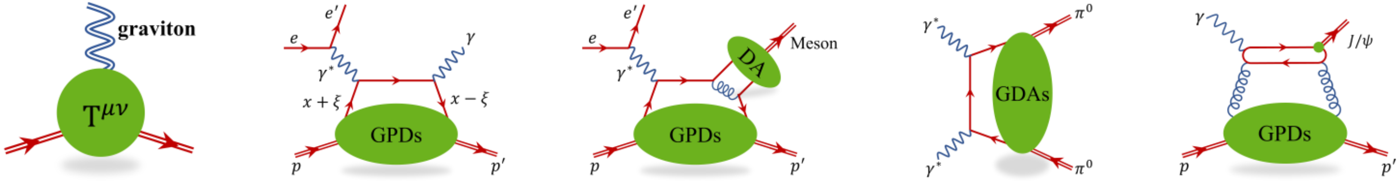
Gravitational form factors (GFFs) of the nucleon: [Kobzarev:1962wt, Pagels:1966zza]

$$\langle p', s'|\hat{T}_i^{\mu\nu}(0)|p, s\rangle =$$

$$\frac{1}{2M}\bar{u}_{s'}(p')\left[2P^\mu P^\nu A_i(q^2) + iP^{\{\mu}\sigma^{\nu\}\rho}q_\rho J_i(q^2) + \frac{1}{2}(q^\mu q^\nu - g^{\mu\nu}q^2)D_i(q^2) + 2g^{\mu\nu}\bar{c}_i(q^2)\right]u_s(p)$$

- Energy and momentum distribution
- Angular momentum distribution
- Stress distribution

How to access gravitational form factors



[Kumano:2017lhr, Duran:2022xag, Burkert:2023wzr]

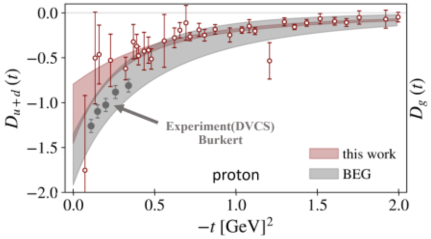
- Deeply virtual Compton scattering
- Deeply virtual meson production
- Two-photon pair production
- J/ψ threshold photoproduction

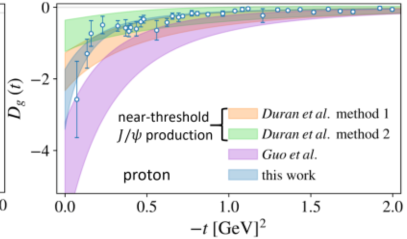
Ji's sum rule:

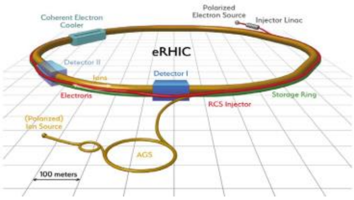
$$\int_{-1}^{1} dx x H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t), \quad \int_{-1}^{1} dx x E^{q,g}(x,\xi,t) = B^{q,g}(t) - \xi^2 D^{q,g}(t)$$

Here, $H^{q,g}$ and $E^{q,g}$ are generalized parton distributions.

[Ji:1996nm]







[Lattice'23:Hackett:2023nkr]

Gravitational form factor D: the last global unknown

$D = D(0)$ is not constrained by global properties of hadrons [Polyakov:2018zvc]

em: $\partial_\mu J^\mu_{\mathrm{em}} = 0$	$\langle N'\vert J^\mu_{\mathrm{em}}\vert N\rangle \longrightarrow$	Q	$= 1.602176487(40) \times 10^{-19}\mathrm{C}$
		μ	$= 2.792847356(23)\mu_N$
weak: PCAC	$\langle N'\vert J^\mu_{\mathrm{weak}}\vert N\rangle \longrightarrow$	g_A	$= 1.2694(28)$
		g_p	$= 8.06(55)$
gravity: $\partial_\mu T^{\mu\nu}_{\mathrm{grav}} = 0$	$\langle N'\vert T^{\mu\nu}_{\mathrm{grav}}\vert N\rangle \longrightarrow$	m	$= 938.272013(23)\,\mathrm{MeV}/c^2$
		J	$= \frac{1}{2}$
		D	$= ?$

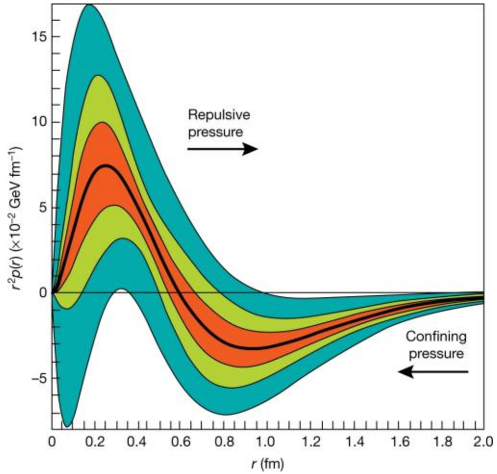
$D(q^2)$ is related to the pressure and shear forces inside hadrons

$$p(r) = \frac{1}{6M}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}r^2\frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r) \qquad s(r) = -\frac{1}{4M}r\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r)$$

Hadron stability conditions:

- Force equilibrium (von Laue condition): $\partial_\mu T^{\mu\nu} \Rightarrow \int d^3r p(r) = 0$
- Local stability conjecture: $D \propto -\int d^3r r^2 \mathrm{d}F_r/\mathrm{d}S_r < 0?$

where $\mathrm{d}F_r/\mathrm{d}S_r = 2s(r)/3 + p(r)$ [Perevalova:2016dln]



[Burkert:2018bqq]

Light-front wave function representation

Diagonal representation for charge form factor and GFF $A(q^2)$

[Drell:1969km, West:1970av, Brodsky:1980zm]

$$F_1(q^2) = \sum_j \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\})\psi_n(\{x_i, \boldsymbol{r}_{i\perp}\}) e_j e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp}$$

[Brodsky:2000ii]

$$A(q^2) = \sum_j \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\})\psi_n(\{x_i, \boldsymbol{r}_{i\perp}\}) x_j e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp}$$

One-body densities:

$$\rho_{\mathrm{ch}}(r_\perp) = \Big\langle \sum_j e_j \delta^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}_{j\perp}) \Big\rangle$$

$$\mathcal{A}(r_\perp) = \Big\langle \sum_j x_j \delta^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}_{j\perp}) \Big\rangle$$

where the quantum average is defined as,

$$\langle \hat{O} \rangle = \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) \hat{O} \psi_n(\{x_i, \boldsymbol{r}_{i\perp}\})$$

Light-front wave function representation for D

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| Reviews

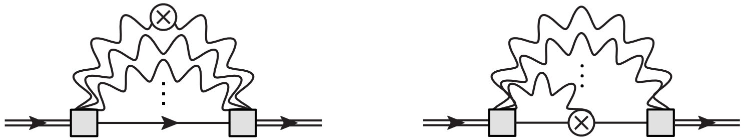
Forces inside hadrons: Pressure, surface tension, mechanical radius, and all that

Maxim V. Polyakov and Peter Schweitzer

https://doi.org/10.1142/S0217751X18300259 | **Cited by:** 241 **(Source: Crossref)**

$\hat{T}_{++}$ of the EMT. Being related to the stress tensor $\hat{T}_{ij}$ the form factor $D(t)$ naturally "mixes" good and bad light-front components and is described in terms of transitions between different Fock state components in overlap representation. As a quantity intrinsically nondiagonal in a Fock space, it is difficult to study the D-term in approaches based on light-front wave functions. This is due to the rela-

[Polyakov:2018zvc]



diagonal non-diagonal

Renormalization plays a vital role in the cancelation of non-diagonal terms

Scalar Yukawa model

$$\mathcal{L} = \partial_\mu N^\dagger \partial^\mu N - m^2 N^\dagger N + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi - \frac{1}{2}\mu^2\pi^2 + g_0 N^\dagger N \pi + \delta m^2 N^\dagger N$$

$$\Downarrow$$

$$\hat{T}^{\mu\nu} = \partial^{\{\mu} N^\dagger \partial^{\nu\}} N - g^{\mu\nu}\left[\partial_\sigma N^\dagger \partial^\sigma N - (m^2 - \delta m^2) N^\dagger N\right] - g^{\mu\nu} g_0 N^\dagger N \pi + \partial^\mu \pi \partial^\nu \pi - \frac{1}{2} g^{\mu\nu}\left(\partial^\rho \pi \partial_\rho \pi - \mu_0^2 \pi^2\right)$$

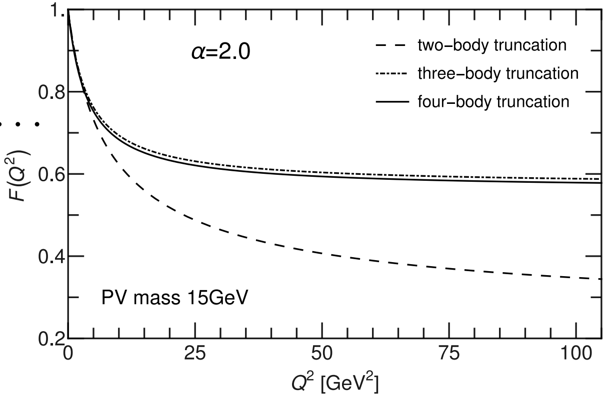
where $m = 0.94$GeV, $\mu = 0.14$GeV. g_0 and δm^2 are bare parameters.

- N: mock nucleon, π: mock pion

[Gross:2001ha]

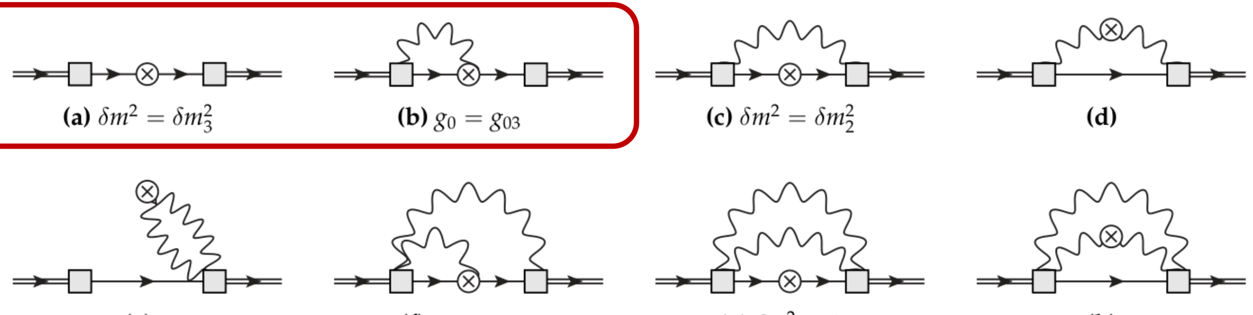
- Quenched approximation: to avoid vacuum instability
- Fock sector expansion: $|p\rangle = |N\rangle + |N\pi\rangle + |N\pi\pi\rangle + |N\pi\pi\pi\rangle \cdots$
- Solved up to $|N\pi\pi\pi\rangle$ sector at non-perturbative couplings
- Fock sector dependent renormalization [Karmanov:2008br]
- Fock sector expansion converged up to $|N\pi\pi\rangle$ sector

$\alpha = g^2/(16\pi m^2), \quad Q^2 = -q^2$



[Li, Karmanov, Maris & Vary:2015iaw]

Energy-momentum tensor renormalization



- Light-front wave functions (LFWFs) & sector dependent counterterms from [Li, Karmanov & Vary:2015iaw,2016yzu]
- Light-front graphical rules extended to non-perturbative regime using LFWFs [Carbonell:1998rj]
- All divergences cancel out with sector dependent counterterms, e.g. $(a) + (b)$:

$$t_a^{\alpha\beta} = Z[2P^\alpha P^\beta + (\frac{1}{2}q^2 - \delta m_3^2)g^{\alpha\beta} - \frac{1}{2}q^\alpha q^\beta]$$

$$t_b^{\alpha\beta} = -\sqrt{Z} g^{\alpha\beta} \int \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} g_{03} \psi_2(x, k_\perp) = g^{\alpha\beta} Z \delta m_3^2$$

Covariant decomposition of EMT on the light-front

[Carbonell:1998rj, Karmanov:2002qu]

$$\langle p'|\,\hat{T}_i^{\alpha\beta}(0)\,|p\rangle = 2P^\alpha P^\beta A_i(q^2) + \frac{1}{2}(q^\alpha q^\beta - q^2 g^{\alpha\beta})D_i(q^2) + 2M^2 g^{\alpha\beta}\bar{c}_i(q^2) + \frac{M^4\omega^\alpha\omega^\beta}{(\omega\cdot P)^2}S_{1i}(q^2) + (V^\alpha V^\beta + q^\alpha q^\beta)S_{2i}(q^2)$$

where $P = (p+p')/2$, $q = p' - p$, $V^\alpha = \epsilon^{\alpha\beta\rho\sigma}P_\beta q_\rho \omega_\sigma/(\omega\cdot P)$. $\omega^\mu = (\omega^+, \omega^-, \boldsymbol{\omega}_\perp) = (0,2,0)$ is a null vector indicating the light-front direction.

- $S_{1,2}(q^2)$ are two spurious GFFs due to the violation of the Lorentz symmetry
- $\hat{T}_i^{++}$, $\hat{T}_i^{12}$ and $\hat{T}_i^{+-}$ are three "good currents" which are free of spurious form factors

$$t_i^{++} = 2(P^+)^2 A_i(q_\perp^2), \quad t_i^{--} = 2\Big(\frac{M^2 + \frac{1}{4}q_\perp^2}{P^+}\Big)^2 A_i(q_\perp^2) + \frac{4M^4}{(P^+)^2}S_{1i}(q_\perp^2)$$

$$t_i^{12} = \frac{1}{2}q_\perp^1 q_\perp^2 D_i(q_\perp^2), \quad t_i^{11} + t_i^{22} = -\frac{1}{2}q_\perp^2 D_i(q_\perp^2) - 4M^2\bar{c}_i(q_\perp^2) + 2q_\perp^2 S_{2i}(q_\perp^2)$$

$$t_i^{+-} = 2(M^2 + \frac{1}{4}q_\perp^2)A_i(q_\perp^2) + q_\perp^2 D_i(q_\perp^2) + 4M^2\bar{c}_i(q_\perp^2)$$

[Cao:2024rul]

Light-front wave function representation

[Cao:2023ohj, Cao:2024fto]

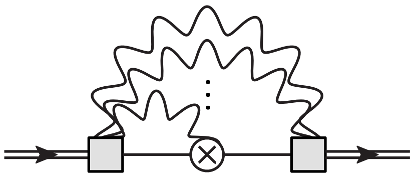
$$t^{12} = \frac{1}{2}\Big\langle \sum_j e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{j\perp}} \frac{i\overleftrightarrow{\nabla}_{j1}\overleftrightarrow{\nabla}_{j2} - q_1 q_2}{x_j} \Big\rangle$$

$$t^{+-} = 2\Big\langle \underbrace{\sum_j e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{j\perp}} \frac{-\frac{1}{4}\overleftrightarrow{\nabla}^2_{j\perp} + m_j^2 - \frac{1}{4}q_\perp^2}{x_j}}_{\text{kinetic part}} + \underbrace{V e^{i\boldsymbol{r}_{N\perp}\cdot\boldsymbol{q}_\perp}}_{\text{potential part}} \Big\rangle$$

where $V = M^2 - \sum_j(-\nabla^2_{j\perp} + m_j^2)/x_j$ in the scalar Yukawa model. The quantum average is defined as

$$\langle\hat{O}\rangle = \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\})\hat{O}\psi_n(\{x_i, \boldsymbol{r}_{i\perp}\})$$

- Modify V in phenomenological models
- $e^{i\boldsymbol{r}_{N\perp}\cdot\boldsymbol{q}_\perp} \xrightarrow{\text{F.T.}} \delta^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}_{N\perp})$ indicates the location of interaction



Macroscopic interpretation

[Li:2024vgv]

$$\langle\psi|\hat{T}_i^{\alpha\beta}(x)|\psi\rangle = e_i u^\alpha u^\beta - p_i \Delta^{\alpha\beta} + \pi_i^{\alpha\beta} + g^{\alpha\beta}\Lambda_i$$

where u^α is the media four velocity with $u^\alpha u_\alpha = 1$, $\Delta^{\alpha\beta} = g^{\alpha\beta} - u^\alpha u^\beta$.

- Physical densities:

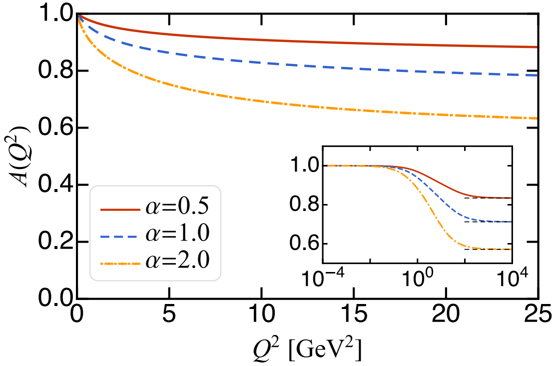
Energy density: $\mathcal{E}_i(x) = M \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot x} \left\{ A_i(q^2) - \frac{q^2}{4M^2}[A_i(q^2) + D_i(q^2)] \right\}$

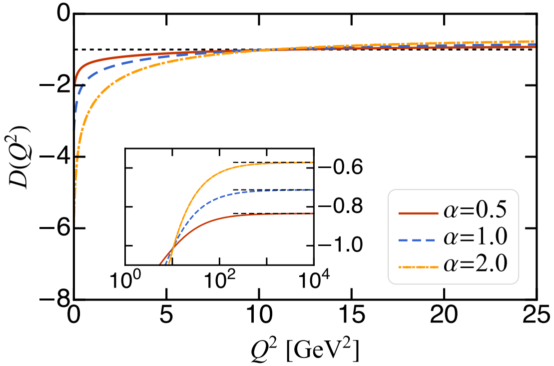
Cosmological constant: $\Lambda_i = M \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot x} \bar{c}(q^2)$

Pressure: $\mathcal{P}_i(x) = \frac{1}{6M} \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot x} q^2 D(q^2)$

Shear tensor: $\Pi_i^{\alpha\beta}(x) = \frac{1}{4M} \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot x} (q^\alpha q^\beta - \frac{q^2}{3}\Delta^{\alpha\beta}) D(q^2)$

Strongly-coupled scalar nucleon





[Cao:2023ohj]

$\alpha = g^2/(16\pi m^2)$

$Q^2 = -q^2$

- For small coupling, $D(Q^2)$ is close to -1, the free scalar particle's result
- In the forward limit ($Q^2 = 0$),

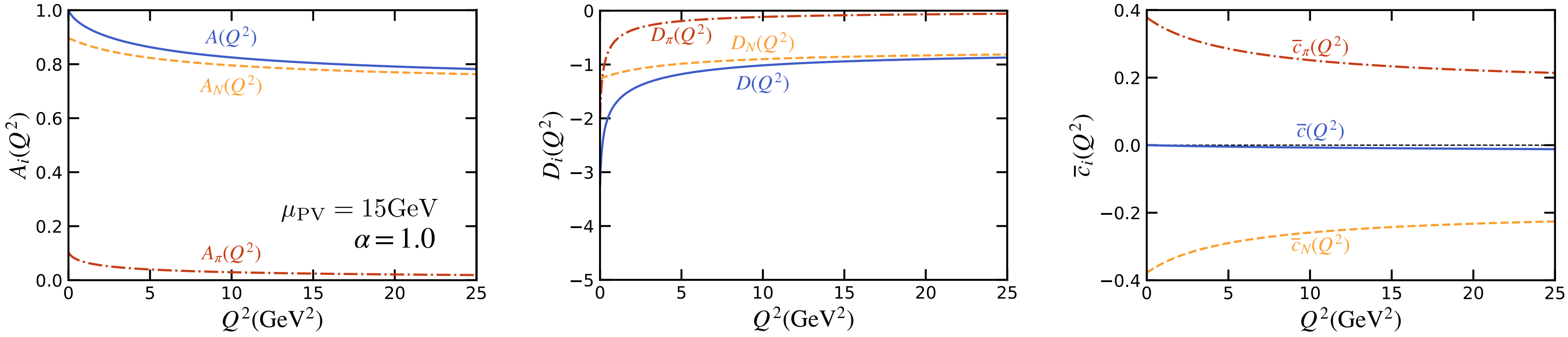
$$\lim_{Q^2=0} A(Q^2) = 1, \qquad \underbrace{\lim_{Q^2\to 0} Q^2 D(Q^2) = 0}_{\text{von Laue condition}}$$

- For large Q^2,

$$\lim_{Q^2\to\infty} A(Q^2) = Z, \qquad \lim_{Q^2\to\infty} D(Q^2) = -Z$$

revealing a pointlike core, consistent with the physical picture of the model

Dissecting the strongly-coupled scalar nucleon



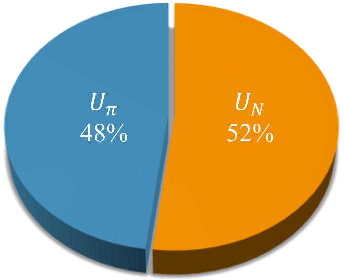
- A nonvanishing but small $\bar{c}(q^2)$ because of Fock space truncation [Cao:2024fto]

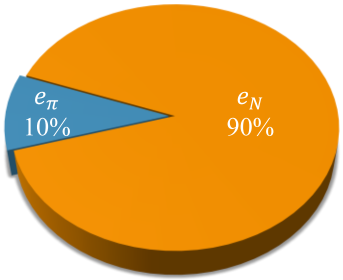
- Mass decomposition: [Lorce:2017xzd]

$$e_i = \int d^2 r_\perp \mathcal{E}(r_\perp) = A_i(0)$$

$$\lambda_i = \int d^2 r_\perp \Lambda_i(r_\perp) = \bar{c}_i(0)$$

$$U_i = e_i + \lambda_i$$





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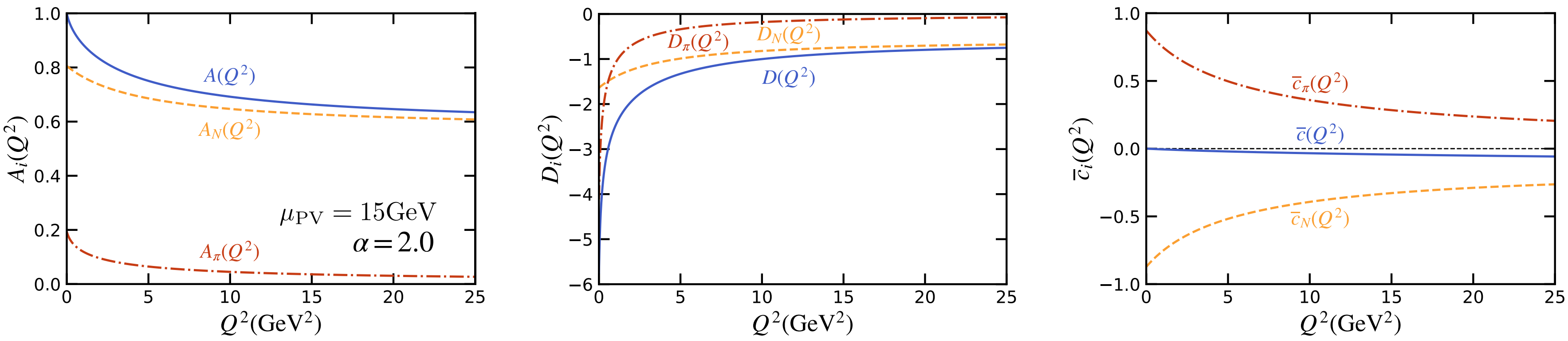

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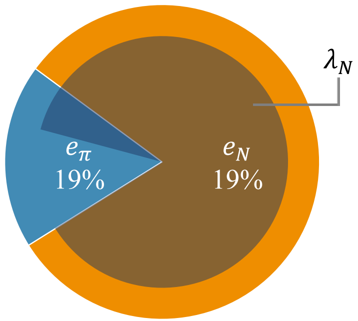
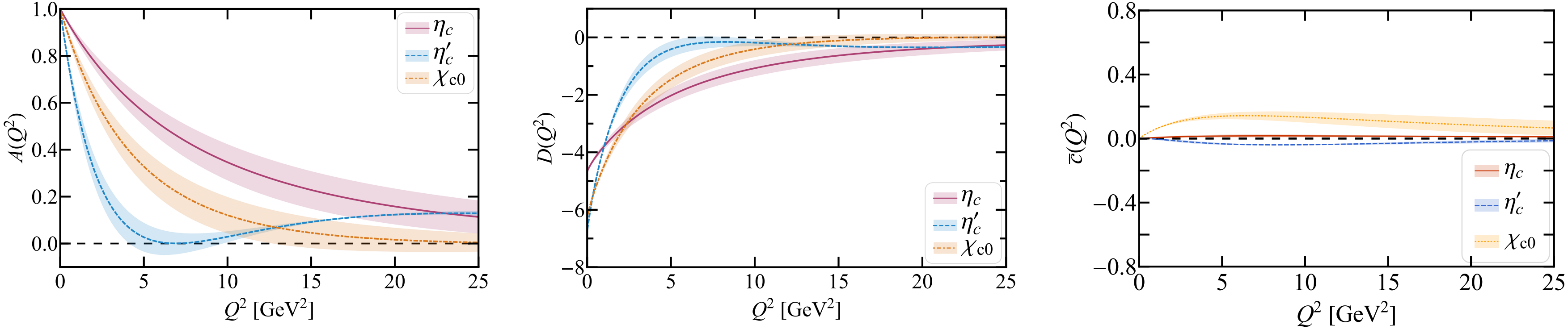


Charmonium gravitational form factors

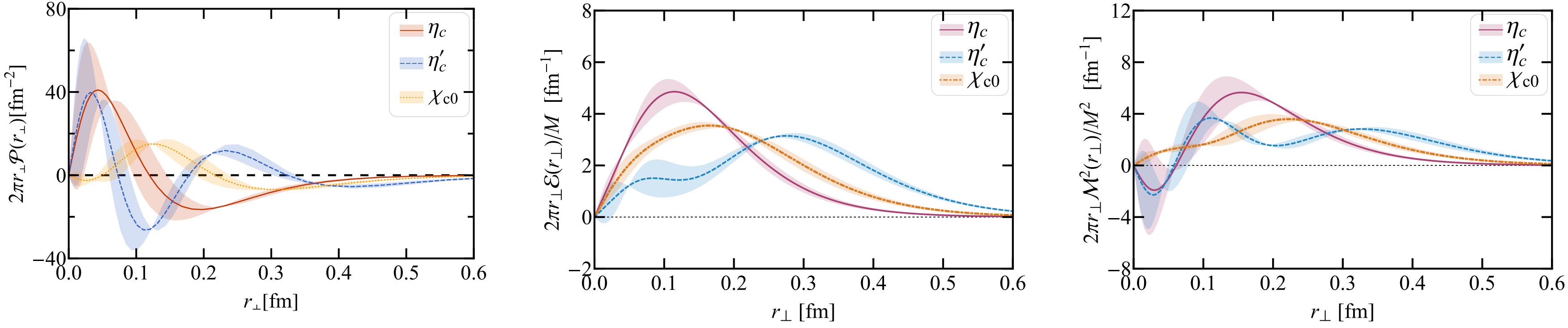
See Tianyang Hu's talk for detail

- Adopt charmonium wave functions from basis light-front quantization

[Li:2017mlw]
[Xu:2024hfx, Hu:2024edc]



- Extracted pressure, energy density and invariant mass squared density



Summary

- We obtain a non-perturbative light-front wave function representation of the gravitational form factor $D(q^2)$
- We calculate the gravitational form factors of a strongly-coupled scalar nucleon using these light-front wave function representations and obtain contributions of individual components
- We also apply these representations to a phenomenological model of charmonium and extract related hadronic densities

Based on:

Cao, Li, Vary, PRD 108, 056026 (2023)
Xu, Cao, Hu, Li, Zhao, Vary, PRD 109, 114024 (2024)
Cao, Xu, Li, Chen, Zhao, Karmanov, Vary, JHEP (2024) 95
Cao, Li, Vary, PRD 110, 076025 (2024)
Hu, Cao, Xu, Li, Zhao, Vary, arXiv:2408.09689

Thank you