Gravitational form factors of charmonium on the light front

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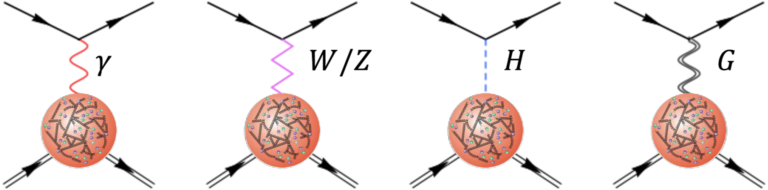
In collaboration with:
Xianghui Cao, Siqi Xu, Yang Li, Xingbo Zhao, James Vary

Based on:

- Xu, Cao, TH, Li, Zhao, Vary, Phys.Rev.D 109, 114024 (2024)
- TH, Cao, Xu, Li, Zhao, Vary, arXiv:2408.09689 [hep-ph]

Hadronic energy-momentum tensor



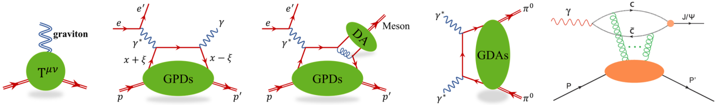
- Hadronic energy-momentum tensor encodes the energy-stress information of hadrons
- In principle, hadronic EMT can be probed in scattering off gravitons
- Factorizing hadronic matrix elements to get gravitational form factors (GFFs):

[Kobzarev:1962wt, Pagels:1966zza]

$$\langle p', s' | T^{\mu\nu}(0) | p, s\rangle = \frac{1}{M}\bar{u}_{s'}(p')\left[P^\mu P^\nu A(q^2) + \frac{1}{2} i P^{\{\mu}\sigma^{\nu\}\rho} q_\rho J(q^2) + \frac{1}{4}(q^\mu q^\nu - g^{\mu\nu} q^2 D(q^2))\right] u_s(p)$$

Experimental approach

review:[Polyakov:2018zvc, Burkert:2023wzr]



- Ji's sum rules: second Melin moments of GPDs, e.g., [Ji:1996nm, Polyakov:2002yz]

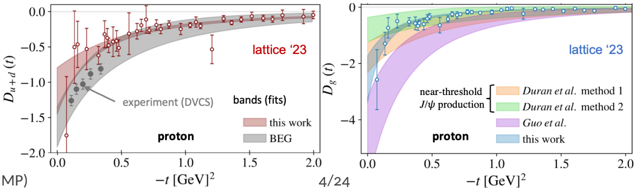
$$\int_{-1}^{1} \mathrm{d}x\, xH^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t), \quad \int_{-1}^{1} \mathrm{d}x\, xE^a(x,\xi,t) = B^a(t) - \xi^2 D^a(t)$$

  - Deeply virtual Compton scattering (DVCS) & hard exclusive meson production

    [Burkert:2018bqq, Burkert:2021ith]

- Di-photon pair production [Kumano:2017lhr]
- Near threshold vector meson production [Kharzeev:2021qkd, Duran:2022xag]

[Lattice '23: Hackett:2023nkr]



Last global unknown

- Conervation laws constrain GFFs except GFF D [Cotogno:2019xcl, Lorce:2019sbq]

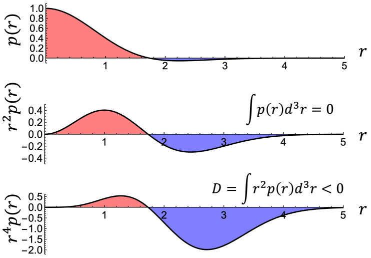
$$A(0)=1,\quad J(0)=\frac{1}{2},\quad \lim_{Q^2\to 0} Q^2 D(Q^2)=0 \Rightarrow \int \mathrm{d}^3 r \mathcal{P}(r)=0$$

  the von Laue condition implies hadrons are in mechanical equilibrium [Laue:1911lrk]

- Polyakov et al. conjectured that $D<0$ for mechanically stable systems [Polyakov:2018zvc]

$$D=\int \mathrm{d}^3 r\, r^2 \mathcal{P}(r) \overset{???}{<} 0$$

electromagnetic	$G_E(0) = Q = 1.602176487(40)\times 10^{-19}\mathrm{C}$ $G_M(0) = \mu = 2.792847356(23)\mu_N$
weak	$G_A(0) = g_A = 1.2694(28)$ $G_P(0) = g_p = 8.06(55)$
gravitational	$mA(0) = m = 938.272013(23)\ \mathrm{MeV}/c^2$ $J(0) = J = \frac{1}{2}$ $D(0) = D = ?$



Light-front densities

- Light-front density is a true 2D distribution

$$\mathcal{O}_{\mathsf{LF}}(\vec{r}_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \langle P + \tfrac{1}{2}\vec{q}|\hat{O}(0)_\perp|P - \tfrac{1}{2}\vec{q}\rangle, \quad \hat{O}(\vec{x}_\perp) = \frac{1}{2}\int \mathrm{d}x^- O(x)$$

- Light-front densities show what the probes “see” in high-energy collision experiments [Burkardt:2000za]

$$\mathcal{T}^{\alpha\beta}(\vec{r}_\perp; P) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} t^{\alpha\beta}(\vec{q}_\perp; P)$$

where, the hadronic matrix elements are,

$$t^{\alpha\beta}(\vec{q}_\perp; P) = \frac{1}{2P^+}\langle P{+}\tfrac{1}{2}q|\, T^{\alpha\beta}(0)|P{-}\tfrac{1}{2}q\rangle$$

light-front coordinates:

$$v^\pm = v^0 \pm v^3$$
$$\vec{v}_\perp = (v^1, v^2)$$

Energy and momentum densities

Momentum $(\mu = +, 1, 2)$ and energy densities $(\nu = -)$:

$$\int \mathrm{d}^3 x T^{+\mu}(x) = P^\mu$$

$$\mathcal{P}^\mu(r_\perp) \equiv \mathcal{T}^{+\mu}(r_\perp; P) = P^\mu \mathcal{A}(r_\perp),$$

$$\mathcal{P}^-(r_\perp) \equiv \mathcal{T}^{+-}(r_\perp; P) = \frac{P_\perp^2 \mathcal{A}(r_\perp) + \mathcal{M}^2(r_\perp)}{P^+}$$

where (for spin-0),

$$\mathcal{A}(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} A(q_\perp^2),$$

$$\mathcal{M}^2(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} \Big[(M^2 + \frac{1}{4} q_\perp^2) A(q_\perp^2) + \frac{1}{2} q_\perp^2 D(q_\perp^2) \Big]$$

- $\mathcal{A}(r_\perp)$ can be interpreted as the matter/momentum density
- Notice $P^- = (P_\perp^2 + M^2)/P^+$, $\mathcal{M}^2(r_\perp)$ can be interpreted as the distribution of the invariant mass squared

Physical densities

[Li:2022ldb,Li:2024vgv]

Inspired by the EMT of relativistic spin medium, we identify the hadronic EMT as,

$$\mathcal{T}^{\alpha\beta} \equiv \mathcal{E}\mathcal{U}^\alpha\mathcal{U}^\beta - \mathcal{P}\Delta^{\alpha\beta} + \tfrac{1}{2}\partial_\sigma(\mathcal{U}^{\{\alpha}\mathcal{S}^{\beta\}\sigma}) + \mathit{\Pi}^{\alpha\beta}$$

We can extract both the Breit-frame densities and the 2D light-front densities from above decomposition, and the light-front densities are,

energy density: $\mathcal{E}(r_\perp) = M\int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp}\Big\{\Big(1+\frac{q_\perp^2}{4M^2}A(q_\perp^2)\Big) + \frac{q_\perp^2}{4M^2}\Big[D(q_\perp^2) - 2J(q_\perp^2)\Big]\Big\}$

pressure: $\mathcal{P}(r_\perp) = -\frac{1}{6M}\int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} q_\perp^2 D(q_\perp^2)$

spin density: $\mathcal{S}^{\alpha\beta}(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp}\Big\{i\sigma^{\alpha\beta}\sqrt{1+\frac{q_\perp^2}{4M^2}} - \frac{U^{[\alpha}q^{\beta]}}{2M}\Big\}J(q_\perp^2)$

shear density: $\mathit{\Pi}^{\alpha\beta}(r_\perp) = \frac{1}{4M}\int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp}(q^\alpha q^\beta + \frac{q_\perp^2}{3}\Delta^{\alpha\beta})D(q_\perp^2)$

where, $U^\alpha = i\overleftrightarrow{\partial}^\alpha/\sqrt{4M^2 - q^2}, \quad \Delta^{\alpha\beta} = g^{\alpha\beta} - U^\alpha U^\beta$

Covariant analysis of EMT on the light cone

[Cao:2024rul]

The hadronic matrix elements of spin-0 particles read,

$$\langle p'|T_i^{\alpha\beta}(0)|p\rangle = 2P^\alpha P^\beta A_i(-q^2) + \frac{1}{2}(q^\alpha q^\beta - q^2 g^{\alpha\beta})D_i(-q^2) + 2M^2 g^{\alpha\beta}\bar{c}_i(-q^2) + \frac{M^4\omega^\alpha\omega^\beta}{(\omega\cdot P)^2}S_{1i}(-q^2) + (V^\alpha V^\beta + q^\alpha q^\beta)S_{2i}(-q^2)$$

where, $P=(p'+p)/2$, $q=p'-p$. $\omega^\mu=(\omega^+,\omega^-,\vec{\omega}_\perp)=(0,2,\vec{0})$ indicates the orientation of the quantization surface. Vector V^α is defined as $V^\alpha=\varepsilon^{\alpha\beta\rho\sigma}P_\beta q_\rho\omega_\sigma/(\omega\cdot P)$.

- In light-front dynamics, some of the Poincaré symmetries are not manifest, which are likely broken in practical calculations
- Emergence of spurious form factors $S_{1,2}$ due to the violation of dynamical Lorentz symmetries in practical calculations, which usually contain uncanceled divergences

Components to extract GFFs

- Hadronic matrix elements within Drell-Yan-Breit frame $(q^+ = 0, \vec{P}_\perp = 0)$:

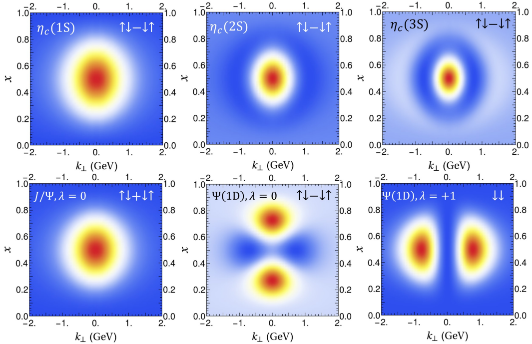
$$
\begin{aligned}
(2P^+)t_i^{++} &= 2(P^+)^2 A_i(q_\perp^2), \\
(2P^+)t_i^{+-} &= 2(M^2 + \frac{1}{4}q_\perp^2)A_i(q_\perp^2) + q_\perp^2 D_i(q_\perp^2) + 4M^2 \bar{c}_i(q_\perp^2), \\
(2P^+)t_i^{12} &= \frac{1}{2} q^1 q^2 D_i(q_\perp^2), \\
(2P^+)(t_i^{11} + t_i^{22}) &= -\frac{1}{2} q_\perp^2 D_i(q_\perp^2) - 4M^2 \bar{c}_i(q_\perp^2) + 2 q_\perp^2 S_{2i}(q_\perp^2) \\
(2P^+)t_i^{--} &= 2(\frac{M^2 + \frac{1}{4}q_\perp^2}{P^+})^2 A_i(q_\perp^2) + \frac{4M^4}{(P^+)^2} S_{1i}(q_\perp^2)
\end{aligned}
$$

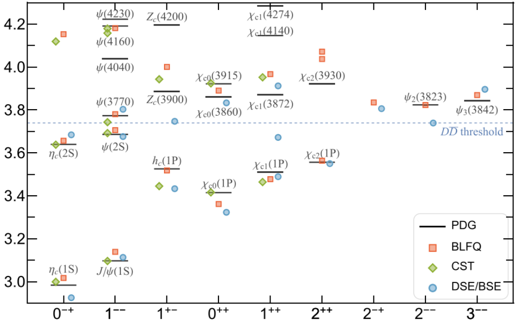
- Identify T^{++}, T^{+i}, T^{12} and T^{+-} as the “good” currents that are free of spurious form factors

Charmonium: “hydrogen atom” of QCD

- Charmonium is an ideal system to probe the properties of the strong force, which consists of a pair of charm and anti-charm quark $c\bar{c}$.
- We consider the spin-0 charmonium and adopt charmonium wave functions from basis light-front quantization (BLFQ) calculations, which was successfully applied to compute a number of hadronic observables including mass spectra and radiative transitions

[Li:2015zda,Li:2017mlw]



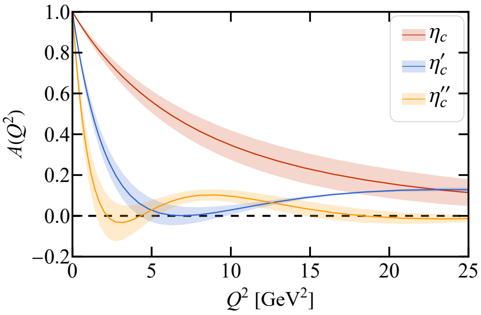


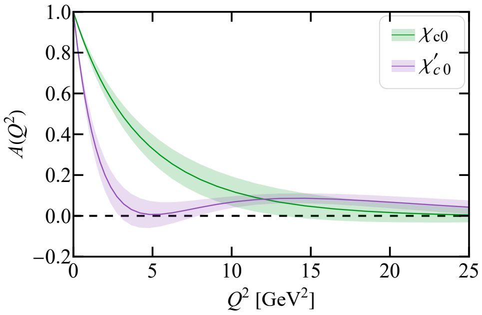
Light-front wavefunction representation of GFF A

- LFWF representation of A reads, [Brodsky:2000ii]

$$A(q_\perp^2) = \sum_n \sum_{s_i} \int [\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}]_n |\widetilde{\psi}_n(x_i, \vec{r}_{i\perp})|^2 \sum_j x_j e^{i\vec{r}_{j\perp}\cdot\vec{q}_\perp}$$

  The valence parton $x_j \sim O(1)$ mainly contributes to GFF $A(q_\perp^2)$
- All particles' calculation results satisfy the constraint $A(0) = 1$





Light-front wavefunction representation of GFF D

- Total GFF D can be extracted both from T^{+-} and T^{12}
- T^{+-} is the light-front energy density and should be well renormalized

$$\int \mathrm{d}^3 x T^{+-}(x) = P^-$$

- Decomposition of t^{+-}:

$$t^{+-}(Q^2) = \underbrace{t_0^{+-}(Q^2)}_{\text{kinetic part}} + \underbrace{t_{\text{int}}^{+-}(Q^2)}_{\text{potential part}}$$

- The LFWF representation of the kinetic part is,

$$(2P^+)t_0^{+-}(Q^2) = \sum_n \sum_{\{s_i\}} \int [\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}]_n \widetilde{\psi}_n^*(\{x_i, \vec{r}_{i\perp}, s_i\}) \sum_j e^{i\vec{r}_{j\perp}\cdot\vec{q}_\perp} \frac{-\frac{1}{4}\overleftrightarrow{\nabla}_{j\perp}^2 + m_j^2 - \frac{1}{4}q_\perp^2}{x_j} \widetilde{\psi}_n(\{x_i, \vec{r}_{i\perp}, s_i\})$$

Wee partons $(x_j \ll 1)$ contribute to the kinetic energy

Difficulties in calculating the potential part

- Origin from the hamiltonian formalism, we lack the operator representation of T^{+-}_{int}. Thus we can't calculate its LFWF representation directly
- Notice that T^{+-}_{int} is exact the light-front potential energy density, we can construct it by localizing the potential energy operator

$$\underbrace{O \quad \to \quad \sum_i O_i \delta^3(r - r_i)}_{\text{QMBT}}$$

- In relativistic quantum theory, particles can only be localized on the transverse plane tangential to the light cone, which suffices to specify the hadronic one-body densities (OBDs)

$$\underbrace{O \quad \to \quad \sum_i O_i \delta^2(r_\perp - r_{i\perp})}_{\text{QFT}}$$

Potential energy density

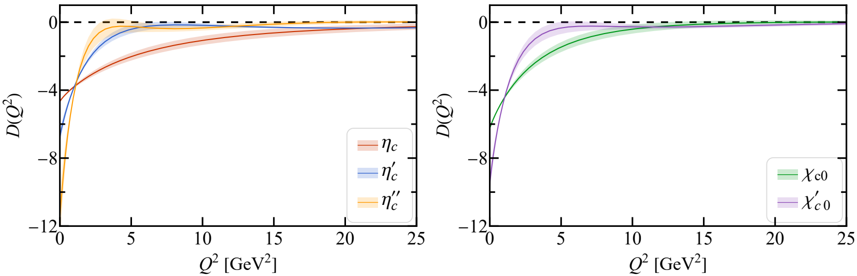
- We adopt the impulse ansatz that all interactions happen at the same instant in light-front time. This ansatz is expected to be a good approximation for small-size systems such as charmonium
- Thus we can construct the potential energy distribution by localizing the effective potential operator:

$$(2P^+) t^{+-}_{\text{int}}(Q^2) = \sum_n \frac{1}{n} \sum_{\{s_i\}} \int [\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}]_n \times \sum_j \widetilde{\psi}^*_n(\{x_i, \vec{r}_{i\perp}, s_i\}) e^{i\vec{r}_{j\perp}\cdot\vec{q}_\perp} v_n(\{x_i, -i\frac{\overleftrightarrow{\nabla}_{i\perp}}{2}\}) \widetilde{\psi}_n(\{x_i, \vec{r}_{i\perp}, s_i\})$$

where, the potential energy is expressed in terms of the mass eigenvalue and the kinetic energy $v_n = M^2 - \sum_{j=1}^n (-\frac{1}{4}\overleftrightarrow{\nabla}^2_{j\perp} + m_j^2)/x_j$

Gravitational form factor D from T^{+-}

Recall the relation $(2P^+)t^{+-} = M^2(q_\perp^2) = M^2 + \frac{1}{4}A(q_\perp^2) + \frac{1}{2}q_\perp^2 D(q_\perp^2)$, we can extract GFF D from mass squared factor



- For all different particles, D-term satisfies $D < 0$ and is finite

Gravitational form factor D from T^{12}

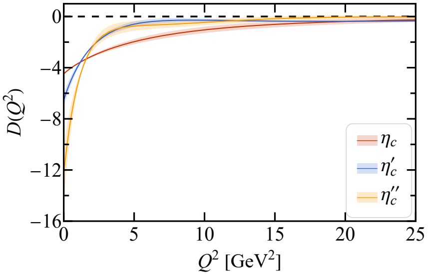
- T^{12} is a “good” component to extract GFF D,

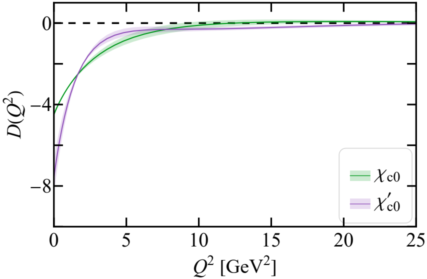
$$(2P^+)t^{12} = \frac{1}{2} q^1 q^2 D(q_\perp^2)$$

- The light-front wave function representation reads,

$$(2P^+)t^{12} = \frac{1}{2}\sum_n \sum_{\{s_i\}} \int [\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}]_n \widetilde{\psi}_n^*(\{x_i, \vec{r}_{i\perp}, s_i\}) \sum_j e^{i\vec{r}_{j\perp}\cdot\vec{q}_\perp} \frac{i\overleftrightarrow{\nabla}^1_{j\perp} i\overleftrightarrow{\nabla}^2_{j\perp} - q^1_\perp q^2_\perp}{x_j} \widetilde{\psi}_n(\{x_i, \vec{r}_{i\perp}, s_i\})$$

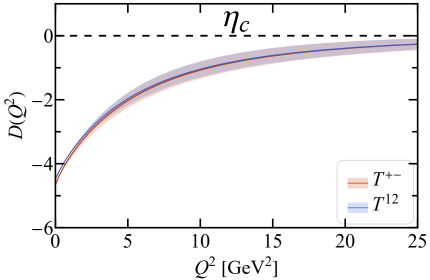
- GFF D extracted from T^{12} also satisfies the von Laue condition

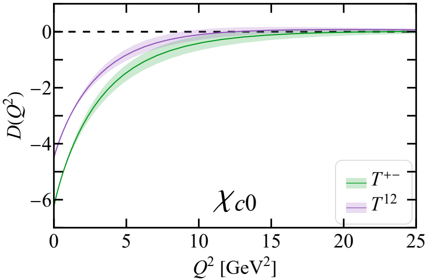




GFF D extracted from T^{+-} vs T^{12}

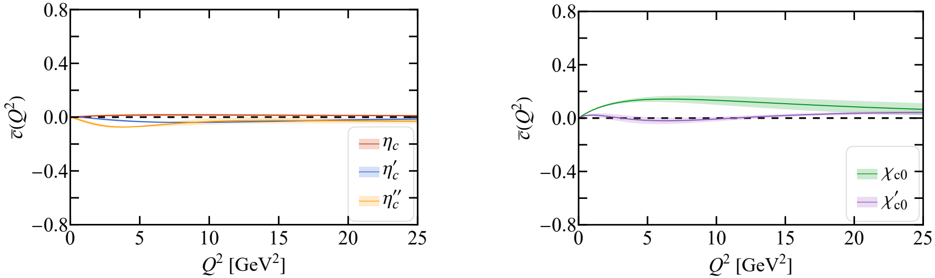
- The truncations in calculations and the ansatz introduced break the current conservation, leading to $\bar{c} = \sum_i \bar{c}_i \neq 0$
- The difference between $D(q_\perp^2)$ extracted from T^{+-} and T^{12} reflects the degree of current conservation violation
- The S-wave charmonium η_c retains more Poincaré symmetries than the P-wave one χ_{c0}





Gravitational form factor $\bar{c}$

- $\bar{c}_i$ represents the force between the i-th constituent and the rest of the system. A vanishing total $\bar{c}$ means all inter-particle forces balance out for an isolated system
- A non-vanishing $\bar{c}$ implies that the net force acting on the system is non-vanishing, closely resembling the effect of the cosmological constant $g^{\mu\nu}\Lambda$ [Teryaev:2016edw]
- Our results show that this form factor vanishes in the forward limit $Q^2 = 0$, but remains a small value in the off-forward situation

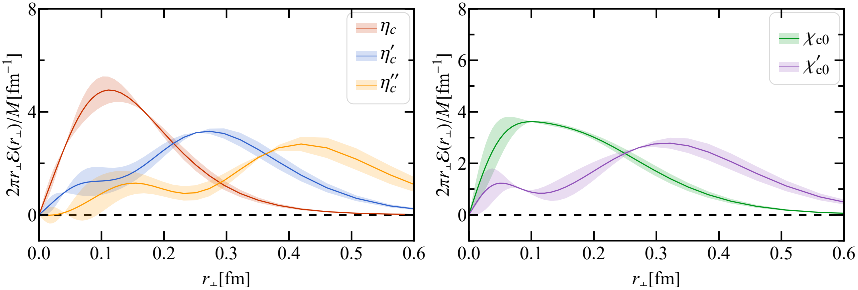


Energy density $\mathcal{E}(r_\perp)$

Energy density $\mathcal{E}(r_\perp)$:

$$\mathcal{E}(r_\perp) = M \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} \left\{ \left(1 + \frac{q_\perp^2}{4M^2} A(q_\perp^2)\right) + \frac{q_\perp^2}{4M^2} D(q_\perp^2) \right\}$$

- Energy density $\mathcal{E}(r_\perp)$ is positive
- The high energy peeks of radial excitation states appear at large $r_\perp$

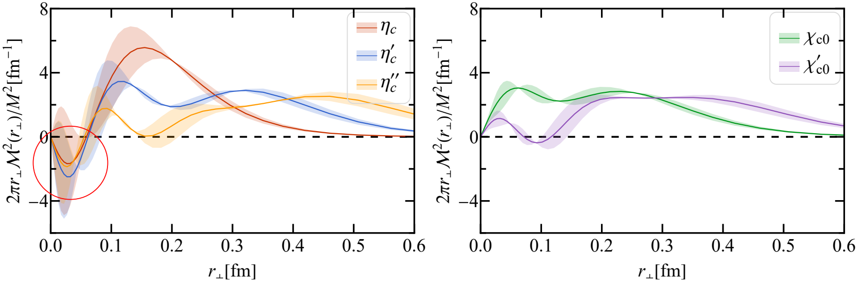


Invariant mass squared density $\mathcal{M}^2(r_\perp)$

Invariant mass squared density $\mathcal{M}^2(r_\perp)$:

$$\mathcal{M}^2(r_\perp) = M^2 \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} \left\{ \left(1 + \frac{q_\perp^2}{4M^2} A(q_\perp^2)\right) + \frac{q_\perp^2}{2M^2} D(q_\perp^2) \right\} = M\left[\mathcal{E}(r_\perp) - \frac{3}{2}\mathcal{P}(r_\perp)\right]$$

- More peek values for higher radial excitation states
- Invariant mass squared density $\mathcal{M}^2(r_\perp)$ of η_c becomes negative at small $r_\perp$: tachyonic core within charmonium?

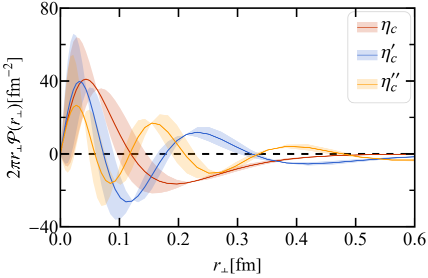


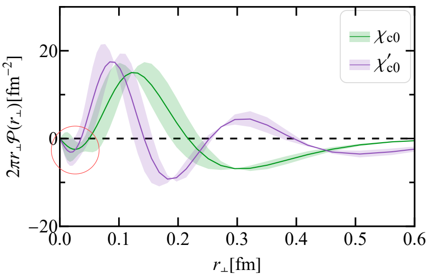
Stress within charmonium

Pressure $\mathcal{P}(r_\perp)$:

$$\mathcal{P}(r_\perp) = -\frac{1}{6M}\int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} q_\perp^2 D(q_\perp^2)$$

It is speculted that a mechanically stable system should have a repulsive core with an attractive periphery.





- All particles have an attractive periphery, but χ_{c0} and χ_{c0}' also have an attractive core
- Particles with higher radial excitation have a more complicated mechanical structure

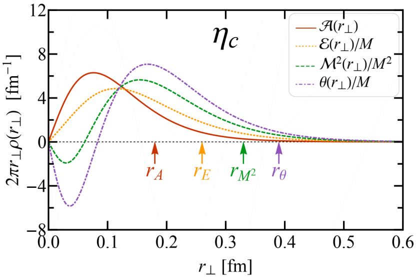
Physical densities

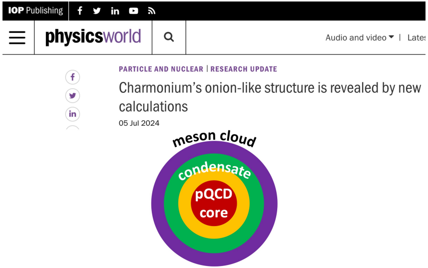
- Different physical densities: matter density $\mathcal{A}(r_\perp)$, energy density $\mathcal{E}(r_\perp)$, invariant mass squared density $\mathcal{M}^2(r_\perp)$ and scalar density $\theta(r_\perp) = \mathcal{T}^\alpha_{\ \alpha}(r_\perp) = \mathcal{E}(r_\perp) - 3\mathcal{P}(r_\perp)$
- Because of $D < 0$, there is a chain of inequalities about their root mean square radii

$$r_A < r_E < r_{M^2} < r_\theta$$

where,

$$r_A^2 = -6A'(0),\ r_E^2 = r_A^2 - \frac{3}{2}\lambda_C^2(1+D),\ r_{M^2}^2 = r_A^2 - \frac{3}{2}\lambda_C^2(1+2D),\ r_\theta^2 = r_A^2 - \frac{3}{2}\lambda_C^2(1+3D)$$





Summary

- Hadronic energy-momentum tensor encodes the energy-stress information of hadron

- We compute gravitational form factors of spin-0 charmonium and extract corresponding physical densities

- From physical distributions we find novel behaviors in charmonium: the tachyonic core in η_c, the attractive core in χ_{c0} and the multi-layer structure of charmonium

- Our methods can be applied to other systems, such as the more complicated nucleon and the high spin J/ψ

Thank you!