Nucleon and pion structure in Minkowski space

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Institute of Modern Physics, Chinese Academy of Sciences, Huizhei

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Introduction

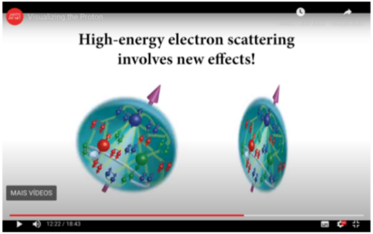
“Relativistic Few-body Systems in Minkowski space”

Nucleon

Pion

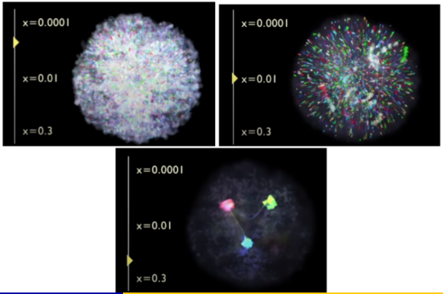
NUCLEON

- How do we see?



High-energy electron scattering
involves new effects!

Figure: https://www.nanotechnologyworld.org/post/visualizing-the-proton-through-animation-and-film

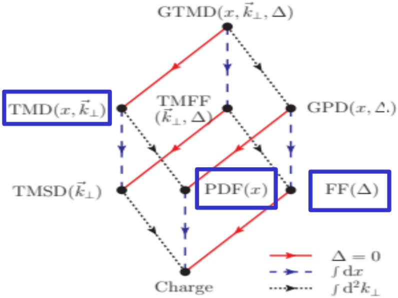


High resolution
short-distances

GTMD

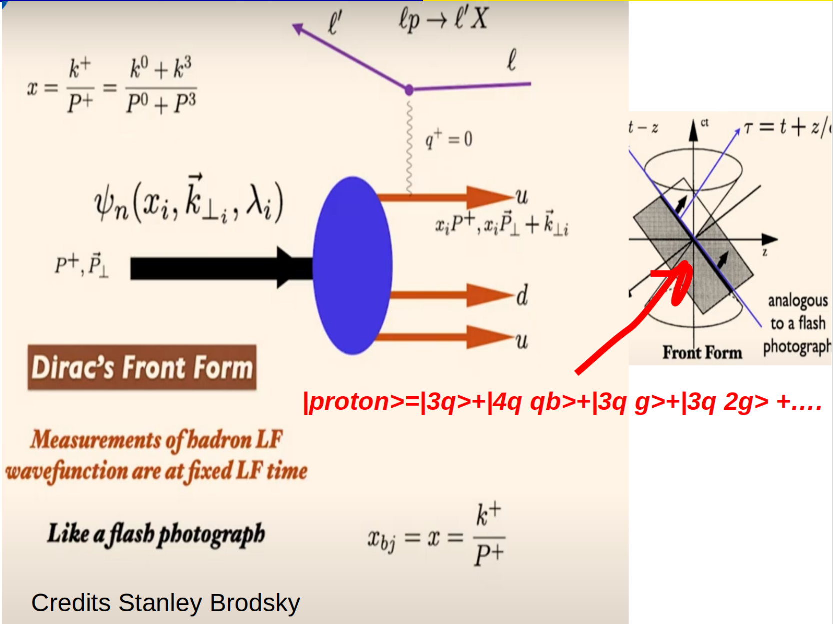
How to get the details?

Observables associated with the hadron structure



Lorcé, Pasquini, Vanderhaeghen JHEP05(2011)041

- SL form factor, PDF, TMD & 3D image



$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

Dirac's Front Form

analogous to a flash photograph

|proton>=|3q>+|4q qb>+|3q g>+|3q 2g> +….

Measurements of hadron LF wavefunction are at fixed LF time

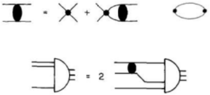
Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Credits Stanley Brodsky

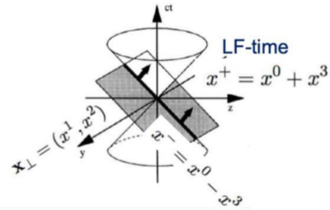
Nucleon Structure in Minkowski space

& LF Fock space decomposition of the nucleon state



$$v(q,p) = 2iF(M_{12}) \int \frac{d^4k}{(2\pi)^4} \frac{i}{[k^2 - m^2 + i\epsilon]} \times \frac{i}{[(p-q-k)^2 - m^2 + i\epsilon]} v(k,p).$$

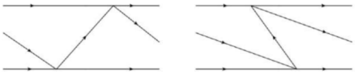
T. Frederico, Phys. Lett. B 282 (1992) 409.



|proton>=|3q>+|4q qb>+|5q 2qb > +....

E. Ydrefors et al. / Physics Letters B 770 (2017) 131–137

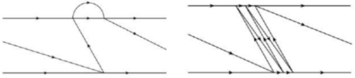
|3q>



|4q qb>

Fig. 2. The three-body LF graphs obtained by time-ordering of the Feynman graph shown in right panel of Fig. 1.

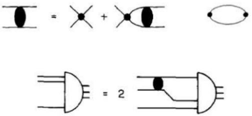
|4q qb>



....

Fig. 3. Examples of many-body intermediate state contributions to the LF three-body forces.

Three-body vertex function in Minkowski space



$$v(q,p) = 2iF(M_{12}) \int \frac{d^4k}{(2\pi)^4} \frac{i}{[k^2 - m^2 + i\epsilon]} \times \frac{i}{[(p-q-k)^2 - m^2 + i\epsilon]} v(k,p).$$

E. Ydrefors et al. / Physics Letters B 791 (2019) 276–280

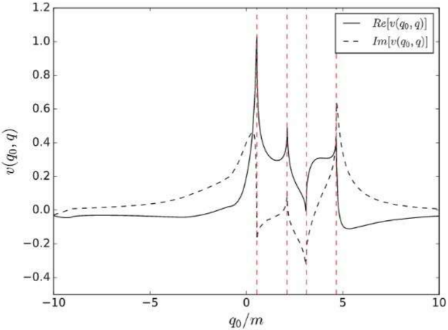


Fig. 1. The vertex function, $v(q_0, q_v = 0.5m)$ with respect to q_0 for the input parameters $am = -1.5$ and $B_3/m = 0.395$. The analytical positions of the peaks, given in Eq. (13), are shown with dashed-red lines.

Too challenging numerically!

LF dynamical model: valence proton wave function

- We consider a Light-front dynamical three-body model for the proton valence wave function (ultimately including the full BS amplitude):

$$\longrightarrow \quad \Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}) = \frac{\Gamma(x_1,\vec{k}_{1\perp})+\Gamma(x_2,\vec{k}_{2\perp})+\Gamma(x_3,\vec{k}_{3\perp})}{\sqrt{x_1x_2x_3}(M_N^2 - M_0^2(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}))},$$

$M_0^2(x_1,\vec{k}_{1\perp}...) = \sum_{i=1}^3(k_{i\perp}^2+m^2)/x_i$

- Fock basis truncated to valence order and spin degree-of-freedom not included.
- Valence three-body regularized LF equation:

$$\longrightarrow \quad \Gamma(x,k_\perp) = \frac{\mathcal{F}(M_{12}^2)}{(2\pi)^3}\int_0^{1-x}\frac{dx'}{x'(1-x-x')}\quad\int_0^\infty d^2k'_\perp\left[\frac{1}{M_0^2-M_N^2}-\frac{1}{M_0^2+\mu^2}\right]\Gamma(x',k'_\perp)$$

where μ is a cut-off, $k_\perp$ transverse momentum, x momentum fraction of spectator and

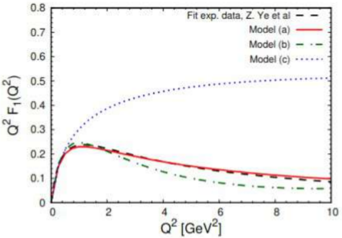
$$M_0^2 = (k_\perp'^2+m^2)/x' + (k_\perp^2+m^2)/x + ((\vec{k}'_\perp+\vec{k}_\perp)^2+m^2)/(1-x-x')$$

- The cut-off μ avoids the unphysical solution $M_3^2 < 0$, and enhances the IR with respect to UV.
- The quark-quark transition amplitude has a pole representing the s-wave diquark introduced through a zero-range effective interaction between two constituent quarks.

E. Ydrefors and TF, PRD 104 (2021) 114012; PLB 838 (2023) 137732

TF and G.Salme, Few Body Syst. 49 (2011) 163 "Projecting the Bethe-Salpeter Equation onto the Light-Front and back: A Short Review"

Electromagnetic form factor



Model	m [MeV]	a $[m^{-1}]$	μ/m	M_{dq} [MeV]
(a)	366	2.70	1	644
(b)	362	3.60	∞	682
(c)	317	-1.84	∞	-

Valence Proton PDF @ initial and experimental scale

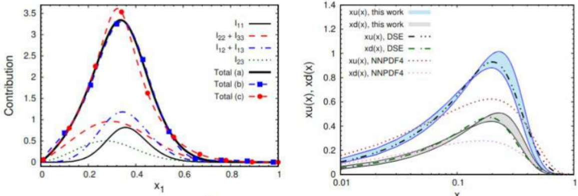
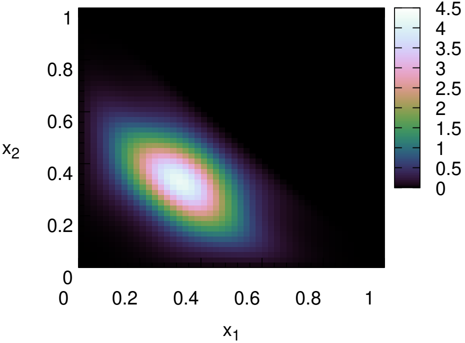


Figure: [Left] proton PDF at initial scale $\int_0^1 dx_1 f(x_1) = 1$. [Right] Valence u-quark and d-quark PDFs evolved to $Q = 3.097\,\text{GeV}$, compared with the DSE results of Lu et al (2203.00753 [hep-ph]) and the results of the NNPDF4 global fit. The shaded areas indicate the uncertainty with respect to the initial scale $Q_0 = 0.33 \pm 0.03\,\text{GeV}$.

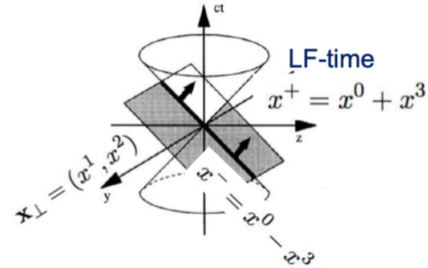
Valence Proton Double PDF

$$D_3(x_1, x_2; \vec{\eta}_\perp) = \frac{1}{(2\pi)^6} \int d^2k_{1\perp} d^2k_{2\perp} \Psi_3^\dagger(x_1, \vec{k}_{1\perp} + \vec{\eta}_\perp; x_2, \vec{k}_{2\perp} - \vec{\eta}_\perp; x_3, \vec{k}_{3\perp}) \Psi_3(x_1, \vec{k}_{1\perp}; x_2, \vec{k}_{2\perp}; x_3, \vec{k}_{3\perp}).$$

- Fourier transform in $\vec{\eta}_\perp$: probability of quarks 1 and 2 for x_1 and x_2 with a separation in the transverse direction $\vec{y}_\perp$.
- $D_3 = 0$ for $x_1 + x_2 > 1$ - momentum conservation. (Below $\vec{y}_\perp = \vec{0}_\perp$)



3D Hadron Image on the null plane

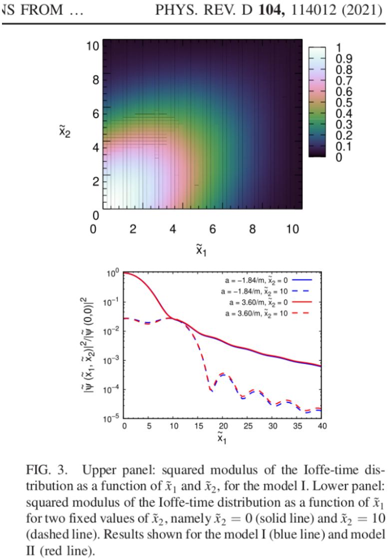


The Ioffe-time is useful for studying the relative importance of short and long light-like distances. It is defined as:

$$\tilde{z} = x \cdot P_{target} = x^- P^+_{target}/2 \quad \text{on the hyperplane } x^+ = 0$$

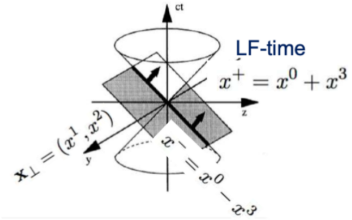
Miller & Brodsky, PRC 102, 022201 (2020)

Ioffe-time image - valence state



NS FROM … PHYS. REV. D **104,** 114012 (2021)

FIG. 3. Upper panel: squared modulus of the Ioffe-time distribution as a function of $\tilde x_1$ and $\tilde x_2$, for the model I. Lower panel: squared modulus of the Ioffe-time distribution as a function of $\tilde x_1$ for two fixed values of $\tilde x_2$, namely $\tilde x_2 = 0$ (solid line) and $\tilde x_2 = 10$ (dashed line). Results shown for the model I (blue line) and model II (red line).



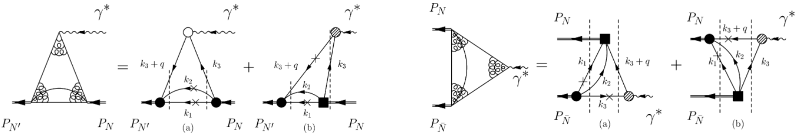
$$\Phi(\tilde x_1,\tilde x_2) \equiv \tilde\Psi_3(\tilde x_1,\vec 0_\perp,\tilde x_2,\vec 0_\perp) = \int_0^1 dx_1 e^{i\tilde x_1 x_1}\int_0^{1-x_1} dx_2 \int_0^1 dx_3 \times \delta(1-x_1-x_2-x_3) e^{i\tilde x_2 x_2}\phi(x_1,x_2,x_3).$$

Time-like and space-like nucleon EM factors

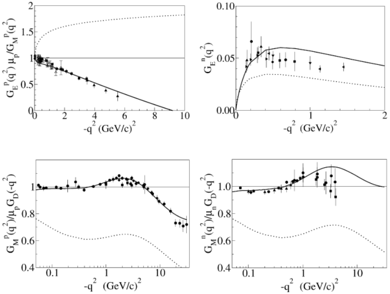
de Melo, TF, Pace, Pisano, Salmè, PLB 671 (2009) 153

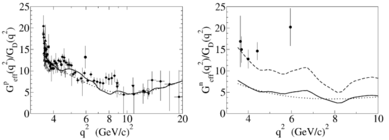
TL nucleon EM FF

J.P.B.C. de Melo et al. / Physics Letters B 671 (2009) 153–157



J.P.B.C. de Melo et al. / Physics Letters B 671 (2009) 153–157



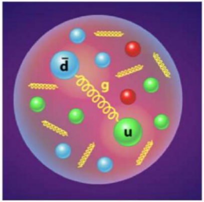


Pion - Interesting?

Falando: Pat k Barry

Pions

- Pion is the Goldstone boson associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry
- Lightest hadron
- Made up of q and $\bar{q}$ constituents



Light-front hypersurface

Credits to Patrick Barry

barryp@jlab.org

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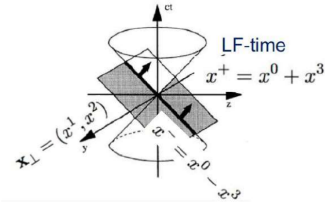
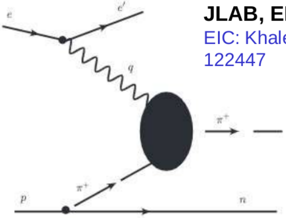


$$|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}\,2g\rangle + \cdots$$

de Melo TALK on monday

How to look?



JLAB, EIC...

EIC: Khalek et al. NPA 1026 (2022) 122447

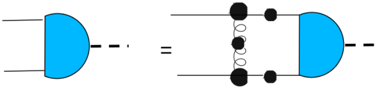
FIG. 1. Sullivan process: $ep \to e'\pi^+n$ scattering. The black blob represents the half-on-mass shell photo absorption amplitude. Diagrammatic representation of the pion pole amplitude for $p(e,e')\pi^+n$ process.

off-shell pion EM FF: Choi, TF, Ji, de Melo, PRD 100, 116020 (2019)

Leão, de Melo, TF, Choi, Ji, PRD 110, 074035 (2024)

How we model:

BSE quark-antiquark & pion model



Ladder approximation (L): suppression of XL for Nc=3 in a bosonic system
[A. Nogueira, CR Ji, Ydrefors, TF, PLB777(2017) 207]

- dressed quark propagator (mass =255MeV) $S(P) = \frac{i}{\not{P} - m + i\epsilon}$
- dressed gluon propagator (mass =637MeV) $i\mathcal{K}_V^{(Ld)\mu\nu}(k,k') = -ig^2 \frac{g^{\mu\nu}}{(k-k')^2 - \mu^2 + i\epsilon}$
- dressed quark-gluon vertex (306 MeV) $\lambda_1 \gamma_\mu \;\; F(q) = \frac{\mu^2 - \Lambda^2}{q^2 - \Lambda^2 + i\epsilon}$

- Model parameters: quark and gluon masses & quark-gluon vertex

SOLUTION IN MINKOWSKI SPACE [pion mass → g]

Dressed quarks/gluons?

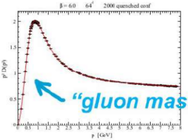
Haag theorem!!!!

INPUTS FROM LQCD in Landau gauge: SL momenta

Gluon propagator

$$D^{ab}_{\mu\nu}(q) = -i\,\delta^{ab}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) D(q^2)$$

Dudal, Oliveira, Silva, Ann. Phys. **397**, 351 (2018)

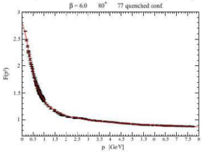


"gluon mass" ~ 500 MeV

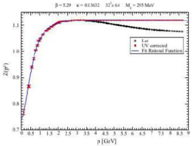
Ghost propagator

$$D_{gh}(p^2) = \frac{F(p^2)}{p^2}$$

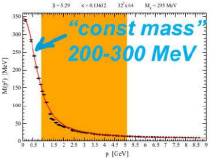
Duarte, Oliveira, Silva, PRD 94 (2016) 014502



Quark propagator Oliveira, Silva, Skullerud and Sternbeck, PRD 99 (2019) 094506



$$i\,Z(p^2)\,\frac{\not p + M(p^2)}{p^2 - M^2(p^2)}$$



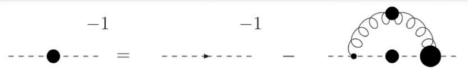
"const mass" 200-300 MeV

Parametrizations summarized in Oliveira, de Paula, Frederico, de Melo, EPJ C 79 (2019) 116

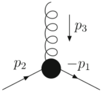
The Quark-Gap Equation and the Quark-Gluon Vertex

Spontaneous Chiral symmetry breaking & pion as a Goldstone boson
(origin of the nucleon mass – "constituent quarks", Roberts, Maris, Tandy, Cloet, Maris...)

Schwinger-Dyson eq.
Quark propagator



Quark-gluon vertex



$$\Gamma^a_\mu(p_1, p_2, p_3) = g\, t^a\, \Gamma_\mu(p_1, p_2, p_3)$$

$$\Gamma_\mu(p_1, p_2, p_3) = \Gamma^{(L)}_\mu(p_1, p_2, p_3) + \cancel{\Gamma^{(T)}_\mu(p_1, p_2, p_3)}$$

Longitudinal component

$$\Gamma^{\rm L}_\mu(p_1,p_2,p_3) = -i\left(\lambda_1\,\gamma_\mu \;+\; \lambda_2\,(\not{p}_1 - \not{p}_2)\,(p_1 - p_2)_\mu \right.$$
$$\left. +\; \lambda_3\,(p_1 - p_2)_\mu \;+\; \lambda_4\,\sigma_{\mu\nu}\,(p_1 - p_2)^\nu \right)$$

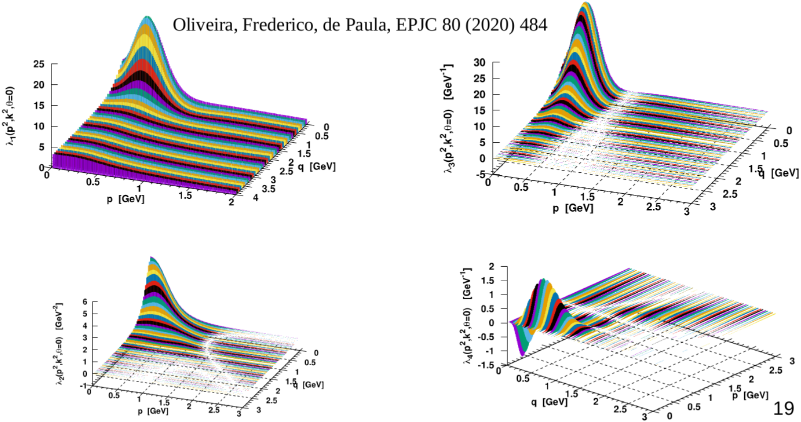
Rojas, de Melo, El-Bennich, Oliveira, Frederico, JHEP 1310 (2013) 193;
Oliveira, Paula, Frederico, de Melo EPJC **78**(7), 553 (2018) & EPJC 79 (2019) 116
Oliveira, Frederico, de Paula, EPJC 80 (2020) 484

quark-gluon vertex from factors

- Schwinger-Dyson eq. quark self-energy
- Longitudinal components quark-gluon vertex
- Slanov-Taylor identity & Quark-Ghost Kernel
- Padé approximants
- Error minimization ~ 2-4%

$\alpha_s = 0.22$ and all propagators renormalised at $\mu = 4.3\,\mathrm{GeV}$

Oliveira, Frederico, de Paula, EPJC 80 (2020) 484



How to solve the Pion Bethe-Salpeter eq. in Minkowski space?

Goal: BS amplitude→ observables

Pion BS amplitude

$$\Phi(k,p) = S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4$$

$$S_1 = \gamma_5 \quad S_2 = \frac{1}{M}\not{p}\gamma_5 \quad S_3 = \frac{k\cdot p}{M^3}\not{p}\gamma_5 - \frac{1}{M}\not{k}\gamma_5 \quad S_4 = \frac{i}{M^2}\sigma_{\mu\nu}p^\mu k^\nu\gamma_5$$

Main Tool: Nakanishi Integral Representation (NIR)

(Nakanishi 1962)

Each BS amplitude component:

$$\Phi_i(k,p) = \int_{-1}^{1} dz' \int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{(\gamma' + \kappa^2 - k^2 - p.kz' - i\epsilon)^3}$$

$$\kappa^2 = m^2 - \frac{M^2}{4}$$

Bosons: Kusaka and Williams, PRD 51 (1995) 7026;

Light-front projection: integration in k^-

Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11;

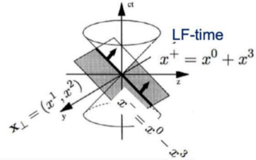
TF, Salme, Viviani PRD89(2014) 016010,…

Fermions (0^-): Carbonell and Karmanov EPJA 46 (2010) 387;
de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;
de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

Projecting BSE onto the LF hyper-plane x⁺=0

LF amplitudes

$$\psi_i(\gamma,\xi) = \int \frac{dk^-}{2\pi}\, \phi_i(k,p) = -\frac{i}{M}\int_0^\infty d\gamma' \frac{g_i(\gamma',z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2]^2}$$



$$\int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2]^2} = iMg^2 \sum_j \int_0^\infty d\gamma' \int_{-1}^1 dz' \mathcal{L}_{ij}(\gamma,z;\gamma'z') g_j(\gamma,z')$$

Generalized Stietjes transform: inversible Carbonell, TF, Karmanov PLB769 (2017) 418

The coupled equations are formally equivalent to BSE, once NIR is applied, and the validity of NIR is assessed by the existence of unique solutions to the GEVP!

Kernel contains singular contributions!

- Kernel of the LF projected pion BSE with NIR

  de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;
  de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

- end-point singularities in the k^- integration (zero-modes)

T.M. Yan , Phys. Rev. **D 7**, 1780 (1973)

$$\mathcal{I}(\beta, y) = \int_{-\infty}^{\infty} \frac{dx}{\left[\beta x - y \mp i\epsilon\right]^2} = \pm \frac{2\pi i \ \delta(\beta)}{\left[-y \mp i\epsilon\right]}$$

Kernel with delta and its derivative!

End-point singularities–more intuitive: can be treated by the pole-dislocation method
de Melo et al. NPA631 (1998) 574C, PLB708 (2012) 87

Results

- *Pion valence wf, f_π and DA*
- *Pion image*
- *EM FF*
- *PDF*
- *TMDs*

BS norm, valence wave function, decay constant

Paula, Ydrefors, Alvarenga Nogueira, TF and Salme PRD 103 014002 (2021).

Normalization: $$i\,N_c \int \frac{d^4k}{(2\pi)^4}\,\left[\phi_1\phi_1 + \phi_2\phi_2 + b\phi_3\phi_3 + b\phi_4\phi_4 - 4\ b\phi_1\phi_4 - 4\frac{m}{M}\phi_2\phi_1\right] = -1$$

$$\gamma = k_\perp^2 \text{ and } z = 2\xi - 1$$

Valence wf:
$$\left\{\begin{aligned} \boxed{\psi_{\uparrow\downarrow}(\gamma, z)} &= -i\frac{M}{4p^+}\int \frac{dk^-}{2\pi}\mathrm{Tr}[\gamma^+\gamma_5\Phi(k;p)] \\ &= \psi_2(\gamma,z) + \frac{z}{2}\psi_3(\gamma,z) + \int_0^\infty \frac{d\gamma'}{M^3}\frac{\partial g_3(\gamma',z)/\partial z}{[\gamma+\gamma'+z^2m^2+(1-z^2)\kappa^2]} \\ \boxed{\psi_{\uparrow\uparrow}(\gamma, z)} &= \frac{\sqrt{\gamma}M}{4ip^+}\int \frac{dk^-}{2\pi}\mathrm{Tr}[\sigma^{+i}\gamma_5\Phi(k;p)] = \frac{\sqrt{\gamma}}{M}\psi_4(\gamma,z) \end{aligned}\right.$$

Aligned spin component of Purely relativistic nature!

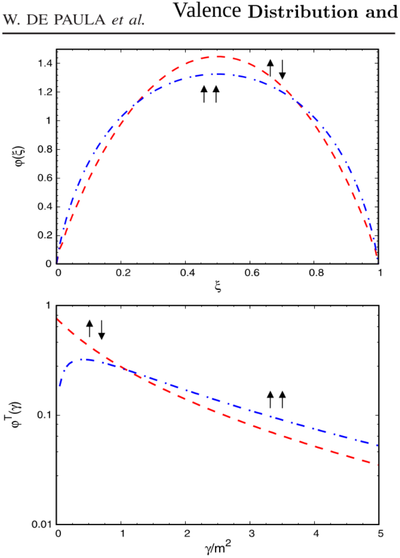
Valence probability: $$\boxed{P_{\text{val}} =} \frac{N_c}{16\,\pi^2}\int_{-1}^{1} dz \int_0^\infty d\gamma\ \left[|\psi^{\uparrow\downarrow}(\gamma,z)|^2 + |\psi^{\uparrow\downarrow}(\gamma,z)|^2\right]$$

Decay constant: $$\boxed{f_\pi =} -i\,\frac{N_c}{p^+}\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\,\gamma^+\,\gamma^5\,\Phi(p,k)] = \frac{2\,N_c}{M}\int \frac{d^2k_\perp}{(2\pi)^2}\frac{dk^+}{2\pi}\ \psi_{\uparrow\downarrow}(\gamma,z)$$

=130 MeV

The experimental value of f_π is 130.50± 0.017 MeV

Valence Distribution and transverse amplitudes



Prob_val=0.7
Prob_antialigned=0.57
Prob_aligned=0.13

$$\varphi_{\uparrow\downarrow}(\xi) = \frac{\int_0^\infty d\gamma\, \psi_{\uparrow\downarrow}(\gamma, z)}{\int_0^1 d\xi \int_0^\infty d\gamma \psi_{\uparrow\downarrow}(\gamma, z)},$$

$$\varphi_{\uparrow\uparrow}(\xi) = \frac{\int_0^\infty d\gamma \psi_{\uparrow\uparrow}(\gamma, z)}{\int_0^1 d\xi \int_0^\infty d\gamma \psi_{\uparrow\uparrow}(\gamma, z)}.$$

$$\varphi^T_{\uparrow\downarrow}(\gamma) = \frac{\int_0^1 d\xi\, \psi_{\uparrow\downarrow}(\gamma, z)}{\int_0^1 d\xi \int_0^\infty d\gamma\, \psi_{\uparrow\downarrow}(\gamma, z)},$$

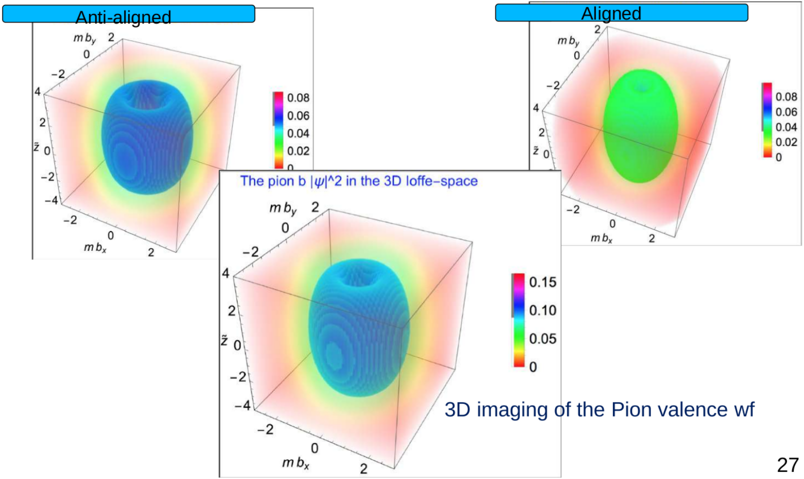
$$\varphi^T_{\uparrow\uparrow}(\gamma) = \frac{\int_0^1 d\xi\, \psi_{\uparrow\uparrow}(\gamma, z)}{\int_0^1 d\xi \int_0^\infty d\gamma\, \psi_{\uparrow\uparrow}(\gamma, z)},$$

Transverse amplitude can be computed directly in Euclidean and Minkowski spaces!

Gutierrez et al PLB 759 (2016) 131

3D Pion image on the null-plane: Spin configurations

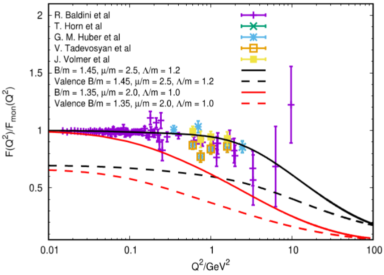
Space-time structure of the pion in terms o $\tilde{z} = x^- p^+/2$ and transverse coord. {b_x, b_y}

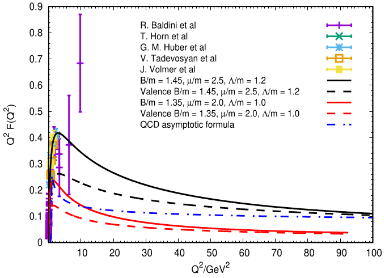


3D imaging of the Pion valence wf

Pion EM Form Factor

Alvarenga Nogueira, de Paula, TF, Ydrefors, Salmè, PLB *820, 136494 (2021)*





$Q^2 F_{\rm asymp}(Q^2) = 8\pi\alpha_s(Q^2) f_\pi^2$ G. Lepage, S. J. Brodsky Phys. Lett. B 87 (1979) 359

Decomposition of the pion EM form factor

$$\boxed{F_\pi(Q^2) = \sum_n F_n(Q^2) = F_{val}(Q^2) + F_{nval}(Q^2)}$$

qq+gluons

$$r_\pi^2 = P_{val}\, r_{val}^2 + (1 - P_{val})\, r_{nval}^2$$

r_π (fm)	r_{val} (fm)	r_{nval} (fm)
0.663	0.710	0.538

0.657 ± 0.003 fm B. Ananthanarayan, I. Caprini, D. Das, Phys. Rev. Lett. 119 (2017) 132002

higher Fock-components → large virtuality → more compact

Kharzeev, “Mass radius of the proton” PRD104, 054015 (2021)

$R_m = 0.55 \pm 0.03$ fm $R_C = 0.8409 \pm 0.0004$ fm

Pion PDF

de Paula, et al PRD 105, L071505 (2022)

Comparison with experimental data

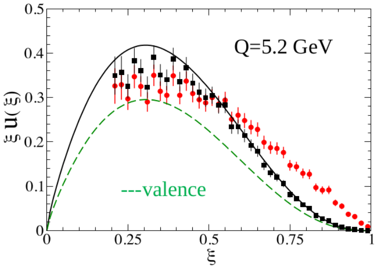


FIG. 2. (Color online). The distribution function $\xi\ u(\xi)$ in a pion. Solid line: full calculation (see Eqs. (7) and (8)), obtained from the BS amplitude solution of the BSE with $m = 255$ MeV, $\mu = 637.5$ MeV and $\Lambda = 306$ MeV, and evolved from the initial scale $Q_0 = 0.360$ GeV to $Q = 5.2$ GeV (see text). Dashed line: the evolved LF valence component, Eq. (9). Full dots: experimental data from Ref. [32]. Full squares: reanalyzed data by using the ratio between the fit 3 of Ref. [33], evolved to 5.2 GeV, and the experimental data [32], at each data point, so that the resummation effects (see text) are accounted for.

[32] J. Conway *et al.*, Experimental Study of Muon Pairs Produced by 252-GeV Pions on Tungsten, Phys. Rev. D **39**, 92 (1989).

[33] M. Aicher, A. Schäfer, and W. Vogelsang, Soft-gluon resummation and the valence parton distribution function of the pion, Phys. Rev. Lett. **105**, 252003 (2010), arXiv:1009.2481 [hep-ph].

Pion:Quark unpolarized transverse-momentum distribution functions

de Paula, TF, Salmè, EPJC 83 (2023) 985

T-even uTMD leading-twist from the quark-quark correlator
Mulders & Tangerman NPB461, 197 (1996)

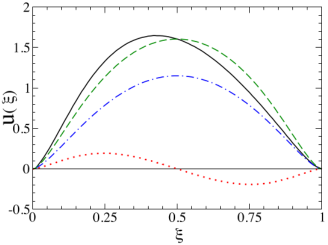
$$f_1^q(\gamma,\xi) = \frac{N_c}{4}\int d\phi_{\hat{\mathbf{k}}_\perp}\int_{-\infty}^{\infty}\frac{dy^-\, d\mathbf{y}_\perp}{2(2\pi)^3}$$

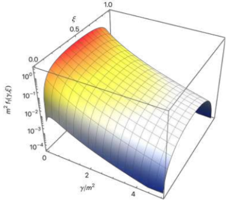
$$\times\, e^{i[\xi P^+ \frac{y^-}{2} - \mathbf{k}_\perp\cdot\mathbf{y}_\perp]}\langle P|\bar{\psi}_q(-\tfrac{y}{2})\gamma^+\psi_q(\tfrac{y}{2})|P\rangle\Big|_{y^+=0}$$

$$\gamma = |\mathbf{k}_\perp|^2$$

$$f_1^q(\gamma,\xi) = \frac{N_c}{4(2\pi)^3}\int_{-\infty}^{\infty}\frac{dk^+}{2(2\pi)}\delta\Big(k^+ + \frac{P^+}{2} - \xi P^+\Big)$$

$$\times\int_{-\infty}^{\infty}dk^-\int_0^{2\pi}d\phi_{\hat{\mathbf{k}}_\perp}\mathrm{Tr}\left[S^{-1}(-p_{\bar{q}})\bar{\Phi}(k,P)\,\gamma^+\,\Phi(k,P)\right]$$



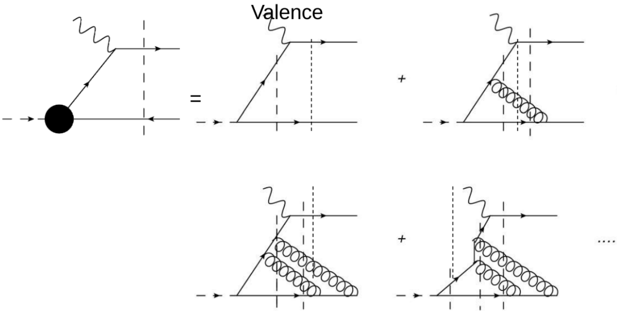


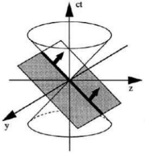
$$\langle \xi_q \rangle = \int_0^1 d\xi \int_0^\infty d\gamma\ \xi\ f_1^q(\gamma,\xi) = 0.471$$

$$|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}\,2g\rangle + \cdots$$

Bethe-Salpeter amplitude: beyond the valence states

Light-front projection





- **higher Fock-components** $|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}\,2g\rangle + \cdots$
- **gluon radiation from initial state interaction (ISI)**
-

Sales, TF, Carlson,Sauer, PRC 63, 064003 (2001);
Marinho, TF, Pace,Salme,Sauer, PRD 77, 116010(2008)

Gluon momentum in the pion

$$|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}\,2g\rangle + \cdots$$

quark momentum distribution

$$u^q(\xi) = \sum_{n=2}^{\infty}\left\{\prod_i^n \int \frac{d^2k_{i\perp}}{(2\pi)^2}\int_0^1 d\xi_i\right\}$$
$$\times\delta\left(\xi-\xi_1\right)\delta\left(1-\sum_{i=1}^{n}\xi_i\right)\,\delta\left(\sum_{i=1}^{n}\mathbf{k}_{i\perp}\right)$$
$$\times\left|\Psi_n(\xi_1,\mathbf{k}_{1\perp},\xi_2,\mathbf{k}_{2\perp},...)\right|^2,$$

first-moment

$$\langle\xi_q\rangle = P_{val}\;\langle\xi_q\rangle_{val} + \sum_{n>2} P_n\;\langle\xi_q\rangle_n$$

0.471 0.5

$$= P_{val}\;\langle\xi_q\rangle_{val} + (1-P_{val})\;\langle\xi_q\rangle_{HFS}$$

0.4

P_{val}=0.7

momentum sum-rule in the HFS

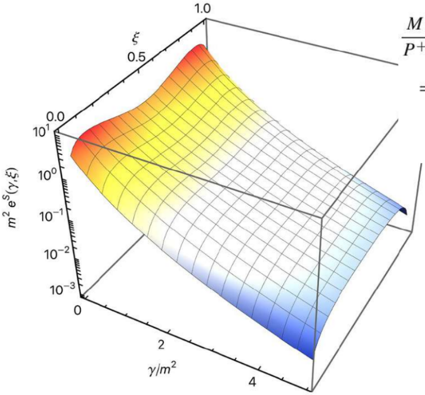
$$\langle\xi_q\rangle_{HFS} = 1 - \langle\xi_{\bar{q}}\rangle_{HFS} - \langle\xi_g\rangle$$

0.2

Gluons carry 6% of the longitudinal momentum of the pion!

@ the pion scale

Subleading-twist 3 uTMDs



$$\frac{M}{P^+}\, e^q(\gamma,\xi) = \frac{N_c}{4}\int_0^{2\pi} d\phi_{\hat{\mathbf{k}}_\perp} \int_{-\infty}^{\infty} \frac{dy^- d\mathbf{y}_\perp}{2(2\pi)^3} \times e^{i[\xi P^+ \frac{y^-}{2} - \mathbf{k}_\perp\cdot\mathbf{y}_\perp]} \langle P|\bar{\psi}_q(-\tfrac{y}{2})\,\mathbb{1}\,\psi_q(\tfrac{y}{2})|P\rangle\Big|_{y^+=0}$$

de Paula, TF, Salmè, EPJC 83 (2023) 985

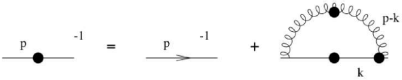
Fig. 7 Pion unpolarized transverse-momentum distribution $e^S(\gamma,\xi)$, Eq. (44), at the initial scale

Towards Dynamical Chiral symmetry breaking in Minkowski

Dynamically Dressed Quarks

Dressing the Quark: Schwinger-Dyson equation

The model: Bare vertices, massive vector boson, Pauli-Villars regulator



Credits to Wayne de Paula

The rainbow ladder Schwinger-Dyson equation in Minkowski space is:

$$S_q^{-1}(k) = \not{k} - m_B + ig^2 \int \frac{d^4q}{(2\pi)^4} \Gamma_\mu(q,k) S_q(k-q) \gamma_\nu D^{\mu\nu}(q)\,,$$

where m_B is the quark bare mass and g is the coupling constant.
The massive gauge boson is given by

$$D^{\mu\nu}(q) = \frac{1}{q^2 - m_g^2 + \imath\epsilon} \left[g^{\mu\nu} - \frac{(1-\xi) q^\mu q^\nu}{q^2 - \xi m_g^2 + \imath\epsilon} \right] ,$$

where we have introduce an effective gluon mass m_g, as suggested by LQCD calculations (*see Dudal, Oliveira and Silva, PRD 89 (2014) 014010*).
The dressed fermion propagator is

$$S_q(k) = \left[\not{k} A\left(k^2\right) - B\left(k^2\right) + i\epsilon \right]^{-1} .$$

Duarte, TF, de Paula, Ydrefors, PRD105, 114055 (2022)

Schwinger-Dyson equation in Rainbow ladder truncation

The vector and scalar self-energies are given by the NIR, respectively as:

$$
\begin{aligned}
A\left(k^2\right) &= 1+\int_0^\infty ds \frac{\rho_A\left(s\right)}{k^2-s+i\epsilon}\,, \\
B\left(k^2\right) &= m_B+\int_0^\infty ds \frac{\rho_B\left(s\right)}{k^2-s+i\epsilon}\,.
\end{aligned}
$$

The quark propagator can also be written as:

$$
S_q(k) = R\frac{\not{k}+\overline{m}_0}{k^2-\overline{m}_0^2+i\epsilon}+\not{k}\int_0^\infty ds \frac{\rho_v\left(s\right)}{k^2-s+i\epsilon}+\int_0^\infty ds \frac{\rho_s\left(s\right)}{k^2-s+i\epsilon}\,,
$$

where $\overline{m}_0$ is the renormalized mass.

Gauge fixing

$$
\begin{aligned}
\not{k}A(k^2)-B(k^2) = {} & ig^2\int\frac{d^4q}{(2\pi)^4}\frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2-m_g^2+i\epsilon}\left[g^{\mu\nu}-\frac{(1-\xi)\,q^\mu q^\nu}{q^2-\xi m_g^2+i\epsilon}\right] \\
& -i\,g^2\int\frac{d^4q}{(2\pi)^4}\frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2-\Lambda^2+i\epsilon}\left[g^{\mu\nu}-\frac{(1-\xi)q^\mu q^\nu}{q^2-\xi\Lambda^2+i\epsilon}\right]
\end{aligned}
$$

Pauli-Villars
regulator

Duarte et al. PRD105 (2022) 114055

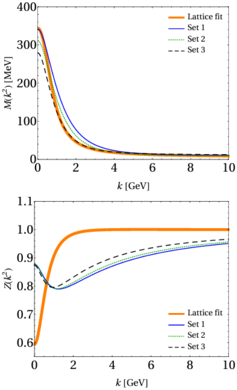


FIG. 1. Landau gauge results for the running mass $M(k^2)$ and quark wave function $Z(k^2)$ as functions of spacelike momentum k, using the sets of parameters given in Table I. Solid thick curves are the fit of LQCD calculations for the mass function and wave function renormalization given in [4].

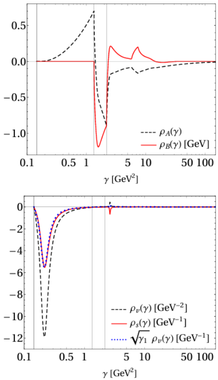


FIG. 2. Spectral densities for the self-energy (upper panel) and for the propagator (lower panel) as functions of γ, computed for set 2 from Table I. The vertical lines are $\overline{m}_0^2$, $(\overline{m}_0+m_g)^2$ and $(\overline{m}_0+\Lambda)^2$ from the thresholds of the driving terms in Eqs. (12) and (13). ($\gamma_1 = 0.216$ GeV^2, see text).

Set	$\overline{m}_0$ (GeV)	m_g (GeV)	Λ (GeV)	α
1	0.42	0.84	1.20	19.70
2	0.38	0.78	1.10	20.30
3	0.35	0.60	1.00	13.25

Set	(outputs)	m_B (MeV)	R
1		9.29	2.22
2		8.78	2.09
3		11.92	2.64

Summary and Prospects

- BSE in Minkowski space: proton and pion
    - PDFs, EM FF, TMDs, Ioffe-time Image
- Quark Dressing in Minkowski space

Prospects:

- K, D, B, ρ, Nucleon (spin)...
- T-odd TMDS, GTMDs (SL & TL), GFF...
- dressed constituents in BS equation - **Castro et al PLB845 (2023) 138159**
- Gluon exchange & dressed vertices
- Integral representation to solve the FBS equation
- Expand the applicability of Quantum algorithm for solving the pion BS & FBS equations **Fornetti, et al, PRD 110 (2024) 056012**
- Confinement?

THANK YOU!