Equation of state of dense matter from multi-messenger observations of neutron stars

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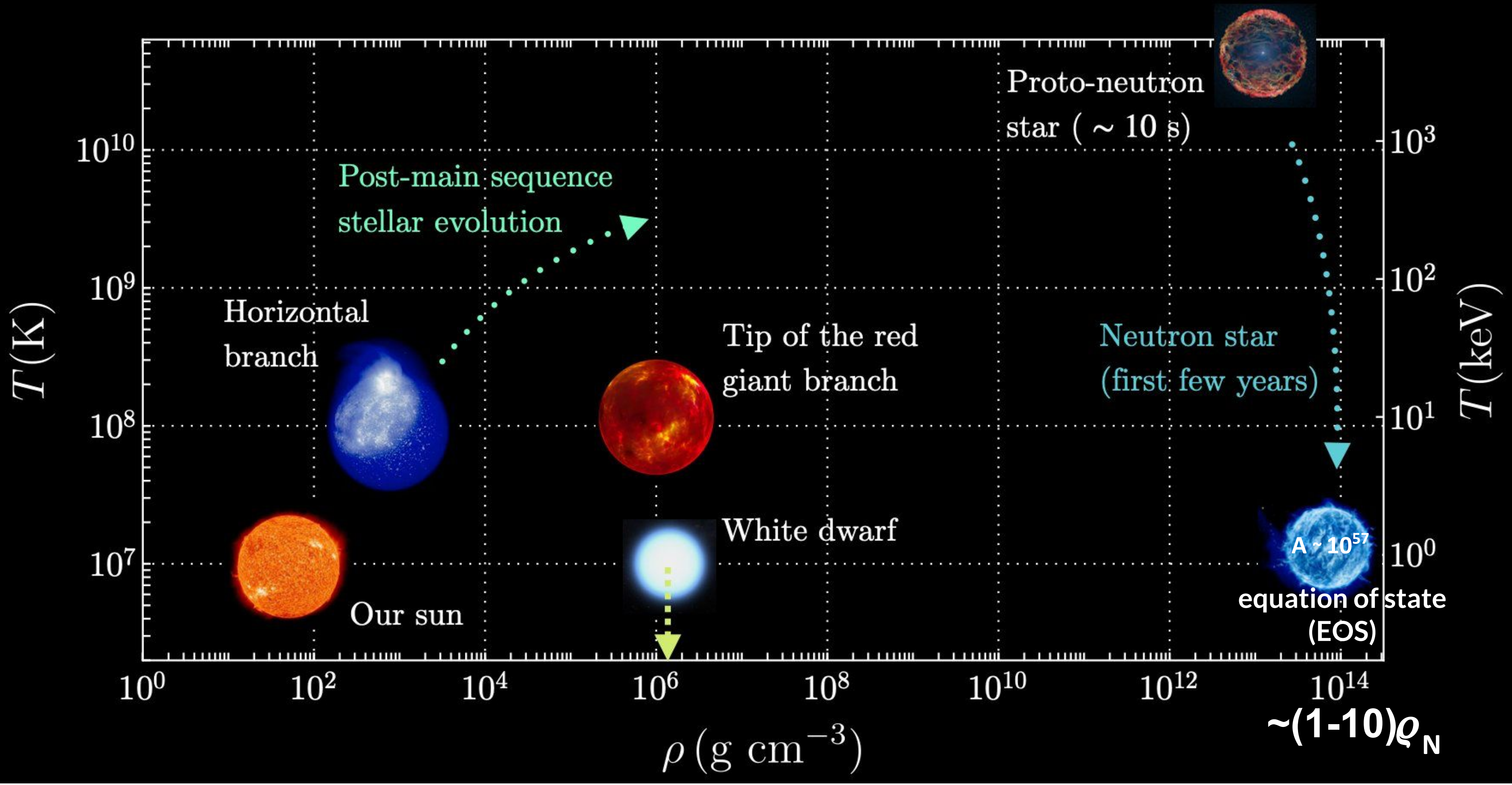
Many thanks to Organizers!



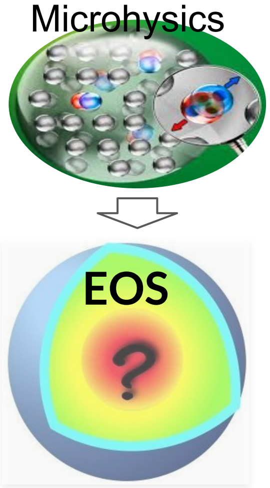
equation of state (EOS)

~(1-10)ρ_N

Outline

- **Basic** for neutron star structure and the EOS
- **Recent works** towards the determination of the nuclear force and NS properties from multimessenger astronomy
- **Take-home messages**

some neutron star EM & GW observables have an intrinsic correlation with microphysical EOS



Hydrostatic equlibrium

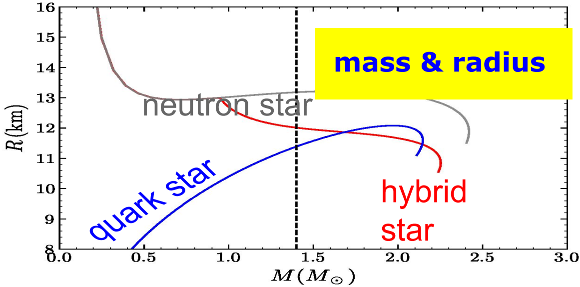
(GR version) TOV eq.

Perturbed hydrostatic equilibrium

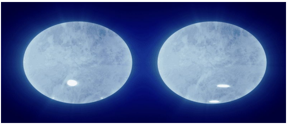
(Newtonian version)

Equations governing stellar structure :

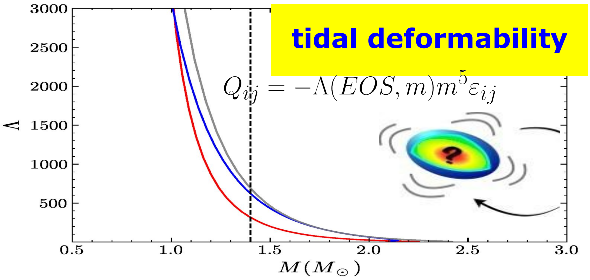
1. $\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$: hydrostatic pressure balance
2. ~~$\frac{dL}{dr} = 4\pi r^2 \rho[\epsilon_n(r) - \epsilon_\nu]$: energy generation~~
3. ~~$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L(r)}{4\pi r^2}$: energy transport~~
4. $P = P(\rho, T)$: equation of state
5. $\frac{dM}{dr} = 4\pi r^2 \rho$: mass-radius relation



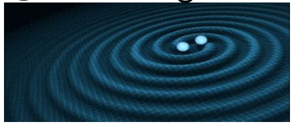
@NICER, ... STROBE-X, eXTP;



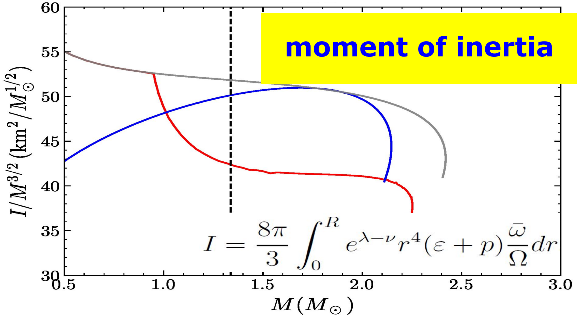
PSR J0030+0415
PSR J0740+6620



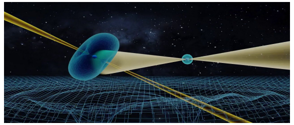
@LIGO/Virgo/KAGRA, ... CE, ET;



GW170817
GW190425



@平方公里陣列射電望遠鏡(SKA)



PSR J0737-3039

To probe the EOS at different density regimes with comprehensive analysis of multi-messenger, multi-wavelength data

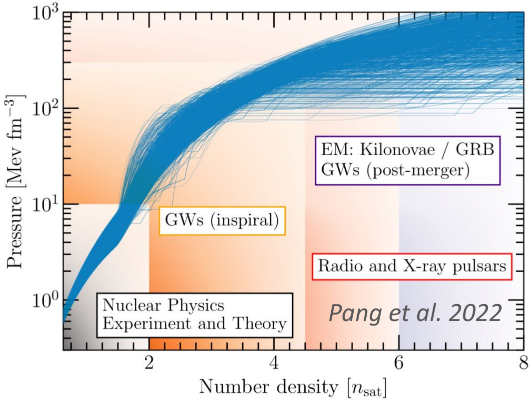
Data

GW event of GW170817(+GW190425) & **kilonova** light curve of AT2017gfo,

NICER×XMM-Newton's measurement of mass and radius of 2 PSRs,

(Mocked) **SKA**'s moment of inertia measurement on PSR J0737-3039,

Neutron-skin from PREX-II, CREX and the ab initio predictions on ^{208}Pb, ^{48}Ca



Massive PSRs (radio)
+ GW (static tide)
+ X-ray (NICER)
2103.15119

+mocked MoI (radio)
2107.07979
+dark matter
2204.05560

+n skin
2305.16058
+hypernuclei
2205.10631
+kilonova (optical+)
2211.02007

+GW (dynamic tide)
2305.08401
+X-ray (NICER×XMM-Newton)
2402.02799

Neutron star group @XMU

arXiv:2408.15022
arXiv:2402.02799 PRD
arXiv:2312.17102 ApJ
arXiv:2312.12185 ApJ
arXiv:2312.04305 MNRAS
arXiv:2305.16058 PRC
arXiv:2305.08401 ApJ
arXiv:2304.12050 PRD
arXiv:2211.04978 PRD
arXiv:2211.02007 ApJ
arXiv:2205.10631 ApJ
arXiv:2204.05560 ApJ
arXiv:2203.04798 PRD
arXiv:2201.12053 PRC
arXiv:2108.00560 ApJ
arXiv:2107.13997 ApJL
arXiv:2107.07979 MNRAS
arXiv:2103.15119 ApJ
arXiv:2011.11934 ApJ
arXiv:2009.12571 MNRAS
arXiv:2007.05116 JHEAp (review)
arXiv:2006.00839 ApJ
arXiv:2005.12875 ApJS
arXiv:2005.02677 PRD
arXiv:2001.03859 PRC

Nucl_Astrophys_xmu (厦大天文核天体物理小组)

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Many-body theory;
Merger simulation
Numerical relativity

Graduated in 2021;
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NS oscillation
Hybrid star. ;
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Superfluidity;
Neutron star cooling;
Nuclear pinning force

Wenli Yuan 苑文莉

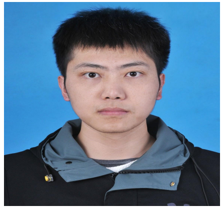
Quark star;

Graduated in 2023;
postdoc in PKU

Peng Liu 刘鹏

Glitch;
Pulsar observation
PSRs J1048-5832, J1028−5819, J1420−6048, J1509−5850, J1709−4429, J1718−3825

Xiangdong Sun 孙向东

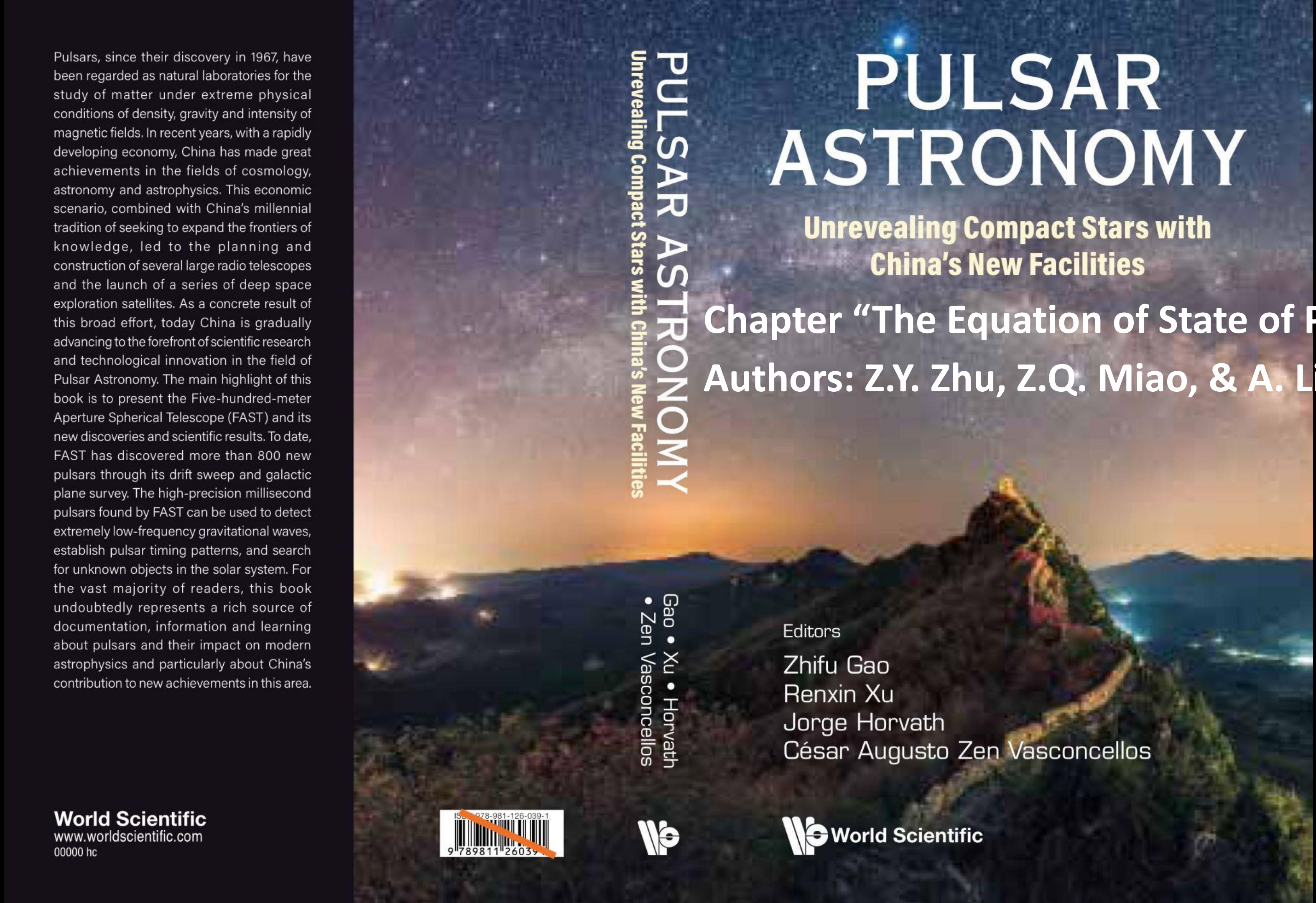


Nuclear matter;
Hyperon matter;
Many-body theory

Shuochong Han 韩烁冲

Many-body theory;
Nuclear transport

with 4 undergraduate students



Pulsars, since their discovery in 1967, have been regarded as natural laboratories for the study of matter under extreme physical conditions of density, gravity and intensity of magnetic fields. In recent years, with a rapidly developing economy, China has made great achievements in the fields of cosmology, astronomy and astrophysics. This economic scenario, combined with China's millennial tradition of seeking to expand the frontiers of knowledge, led to the planning and construction of several large radio telescopes and the launch of a series of deep space exploration satellites. As a concrete result of this broad effort, today China is gradually advancing to the forefront of scientific research and technological innovation in the field of Pulsar Astronomy. The main highlight of this book is to present the Five-hundred-meter Aperture Spherical Telescope (FAST) and its new discoveries and scientific results. To date, FAST has discovered more than 800 new pulsars through its drift sweep and galactic plane survey. The high-precision millisecond pulsars found by FAST can be used to detect extremely low-frequency gravitational waves, establish pulsar timing patterns, and search for unknown objects in the solar system. For the vast majority of readers, this book undoubtedly represents a rich source of documentation, information and learning about pulsars and their impact on modern astrophysics and particularly about China's contribution to new achievements in this area.

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ISBN 978-981-126-039-1

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Unrevealing Compact Stars with China's New Facilities
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PULSAR ASTRONOMY

Unrevealing Compact Stars with China's New Facilities

Editors
Zhifu Gao
Renxin Xu
Jorge Horvath
César Augusto Zen Vasconcellos

World Scientific

Chapter "The Equation of State of Pulsars"

Authors: Z.Y. Zhu, Z.Q. Miao, & A. Li

Outline

- **Basic** for neutron star structure and the EOS
- **Recent works** towards the determination of the nuclear force and NS properties from #multimessenger/multiwavelength astronomy

(Biased selected results; Highlighing work done by our group)

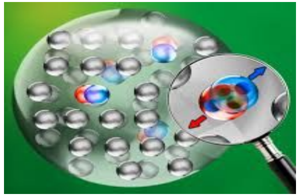
- **Take-home messages**

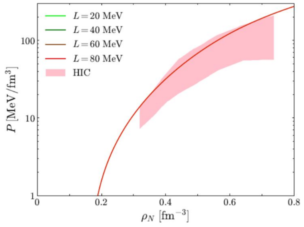
From nuclear force to multimessenger/multiwavelength astronomy

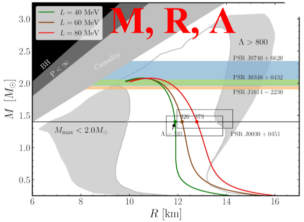
$$\hat{\mathcal{H}}\Psi = E\Psi$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

1. Model for interaction between particles → **2. The EOS p(ρ)** → **3. NS observations on #global properties (GW, photons, neutrinos)**





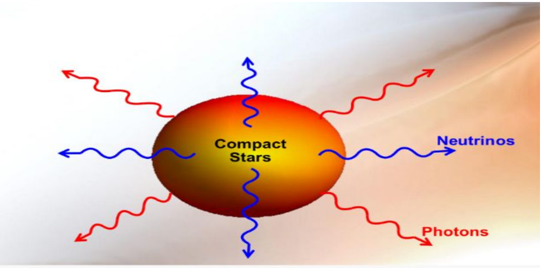


M, R, Λ

plus

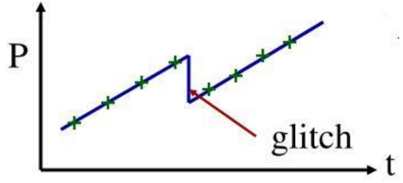
-s. p. properties (e.g., neutron superfluidity);

-thermal conductivity; specific heat capacity; bulk/shear viscosity; neutrino emissivity;...



NS dynamiccs:

Pulsar glitch (this talk)
NS cooling
NS oscillation
Binary NS merger
…



Solving nuclear many-body problem for the EOS

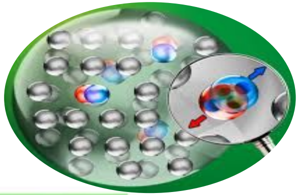
$$\hat{H}\Psi = E\Psi \qquad \Psi = \Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A; s_1,s_2,\ldots,s_A; t_1,t_2,\ldots,t_A)$$

3A nucleon coordinates in r-space

nucleon spins: ±1/2

nucleon isospins (p or n): ±1/2

A ~ 10^{57}

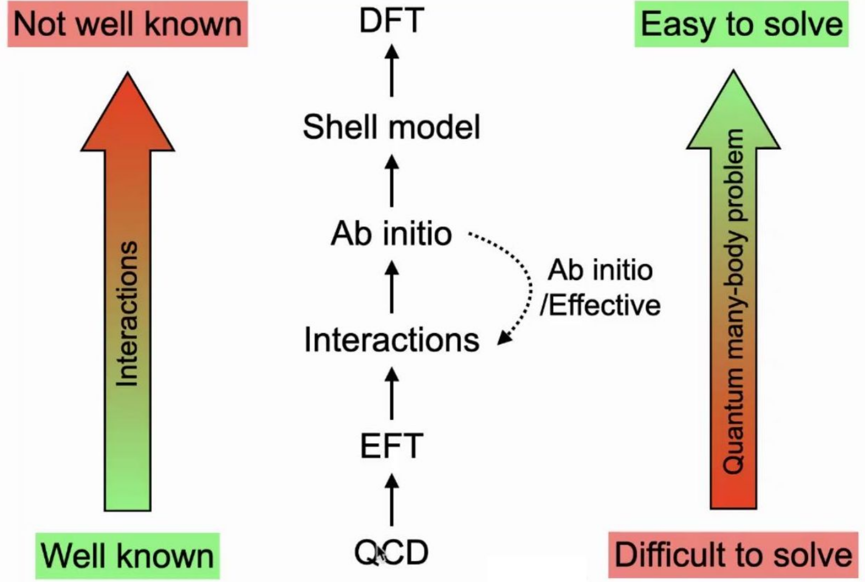


micro

- Green's Function Monte Carlo
- Chiral Perturbation Theory (ChPT)
- $V_{\mathrm{low}k}$ + Renormalization Group
- Variational Many-Body (VMB)
- ***<u>Brueckner-Hartree-Fock</u>*** (BHF)
- Dirac-Brueckner-Hartree-Fock (DBHF)

pheno

- Quark mean-field (QMF)
- Quark Meson Coupling (QMC)
- Relativistic mean-field (RMF)
- Skyrme energy density functional
- …



Not well known

Interactions

Well known

DFT

Shell model

Ab initio

Interactions

EFT

QCD

Ab initio /Effective

Easy to solve

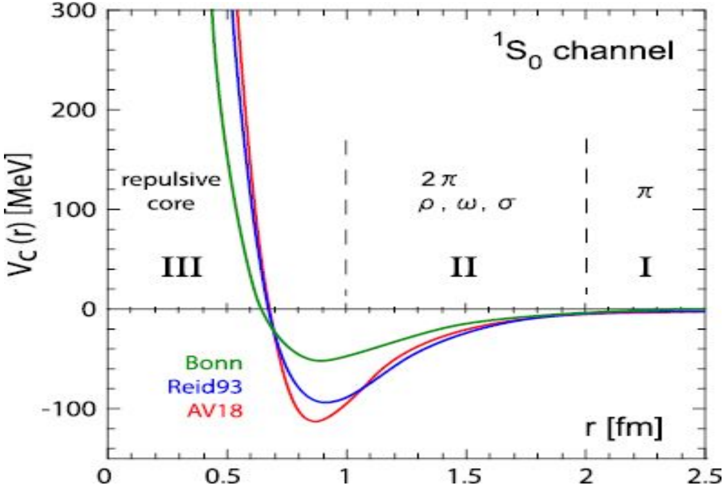
Quantum many-body problem

Difficult to solve

幾乎無法求解量子色動力學（Quantum chromodynamics, QCD）得到EOS！

Brueckner-Hartree-Fock (BHF) (58-present)

- A theory based on independent nucleon pair, for handling the repulsive core of nuclear force;
- Input: Bare NN interaction (AV18, Bonn,...) and many-body forces;
- To solve the Bethe–Goldstone Eq. and s.p. Eqs. self-consistently:



$$
\begin{aligned}
V_{NN} = {} & V_0(r) + V_\sigma(r)\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_\tau(r)\vec{\tau}_1\cdot\vec{\tau}_2 + V_{\sigma\tau}(r)(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2) \\
& + V_{LS}(r)\vec{L}\cdot\vec{S} + + V_{LS\tau}(r)(\vec{L}\cdot\vec{S})(\vec{\tau}_1\cdot\vec{\tau}_2) \\
& + (V_T(r) + V_{T\tau}(r)\vec{\tau}_1\cdot\vec{\tau}_2)\{3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2\} \\
& + (V_Q(r) + V_{Q\tau}(r)\vec{\tau}_1\cdot\vec{\tau}_2)\frac{1}{2}\{(\vec{\sigma}_1\cdot\vec{L})(\vec{\sigma}_2\cdot\vec{L}) + (\vec{\sigma}_2\cdot\vec{L})(\vec{\sigma}_1\cdot\vec{L})\} \\
& + (V_{pp}(r) + V_{pp\tau}(r)\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{p})(\vec{\sigma}_2\cdot\vec{p})
\end{aligned}
$$

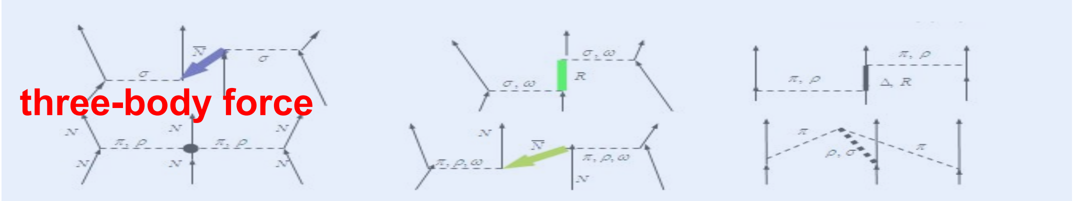
central

spin-orbit

tensor

two-body force
(P, I, J symmetry)

+



three-body force

$$Q(k_a, k_b) = [1 - n(k_a)][1 - n(k_b)]$$

$$e(k) = \frac{\hbar^2 k^2}{2m} + \sum_{k'} n(k')\,\mathrm{Re}\langle kk'|G[e(k)+e(k')]|kk'\rangle_A$$

$$G(\rho,\beta;\omega) = v + v\sum_{k_a k_b}\frac{|k_a k_b\rangle Q(k_a,k_b)\langle k_a k_b|}{\omega - e(k_a) - e(k_b) + i\eta}G(\rho,\beta;\omega)$$

In-medium effective Interaction G matrix

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- Input: Bare NN interaction (AV18, Bonn,...) and many-body forces;
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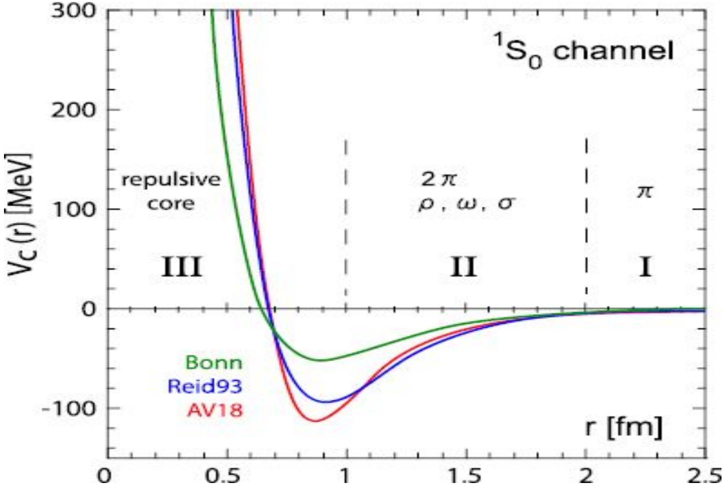
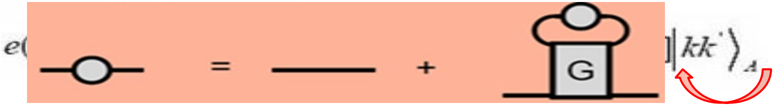


$$Q(k_a,k_b)=[1-n(k_a)][1-n(k_b)]$$



$$\begin{aligned}V_{NN} = {} & V_0(r)+V_\sigma(r)\vec{\sigma}_1\cdot\vec{\sigma}_2+V_\tau(r)\vec{\tau}_1\cdot\vec{\tau}_2+V_{\sigma\tau}(r)(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)\\ & +V_{LS}(r)\vec{L}\cdot\vec{S}++V_{LS\tau}(r)(\vec{L}\cdot\vec{S})(\vec{\tau}_1\cdot\vec{\tau}_2)\\ & +(V_T(r)+V_{T\tau}(r)\vec{\tau}_1\cdot\vec{\tau}_2)\{3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})-\vec{\sigma}_1\cdot\vec{\sigma}_2\}\\ & +(V_Q(r)+V_{Q\tau}(r)\vec{\tau}_1\cdot\vec{\tau}_2)\frac{1}{2}\{(\vec{\sigma}_1\cdot\vec{L})(\vec{\sigma}_2\cdot\vec{L})+(\vec{\sigma}_2\cdot\vec{L})(\vec{\sigma}_1\cdot\vec{L})\}\\ & +(V_{pp}(r)+V_{pp\tau}(r)\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{p})(\vec{\sigma}_2\cdot\vec{p})\end{aligned}$$

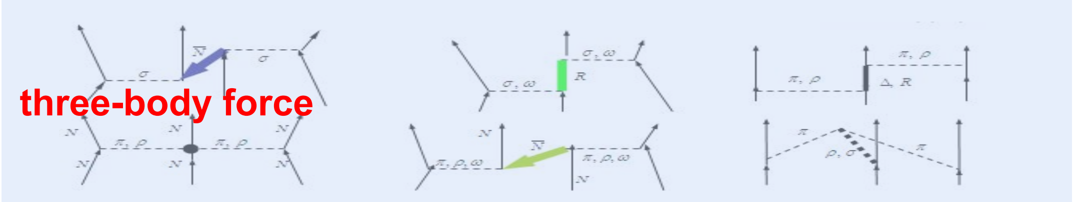
central

spin-orbit

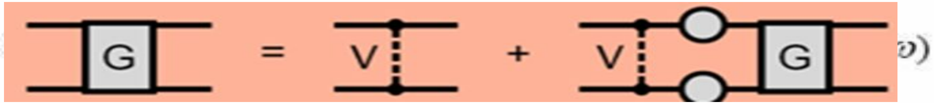
tensor

two-body force
(P, I, J symmetry)

+



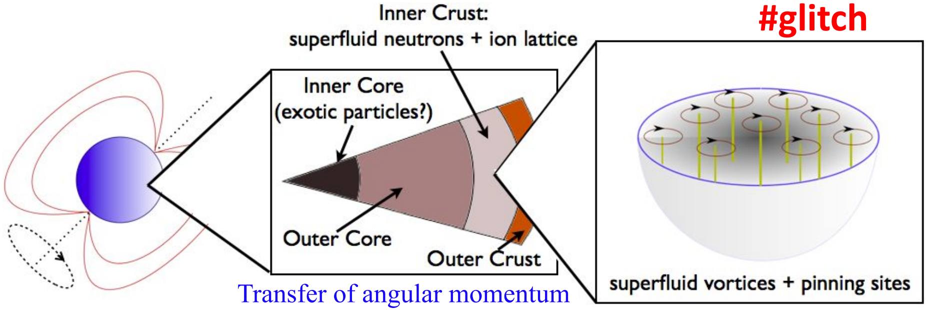
three-body force



In-medium effective Interaction G matrix

A realistic calculation of NS dynamics must go beyond the EOS relation!

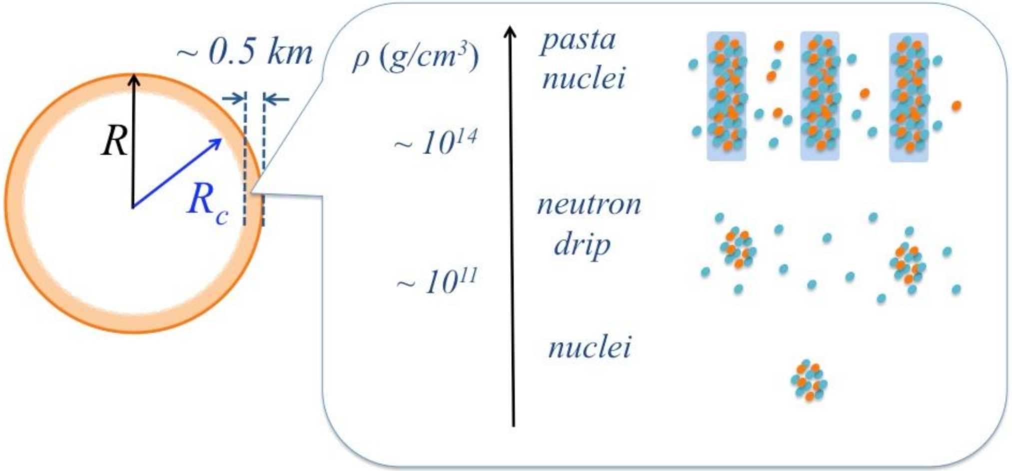
- Although the EOS, i.e., ε = ε(p) is the only relation required from the thermodynamics to solve the TOV eq., it is NOT sufficient to describe the complete thermodynamical state of NS matter;



Transfer of angular momentum from the neutron superfluid to the rest of the star

- Idealy, all stellar matter should be described with SAME nuclear interaction, e.g., unified EOS:

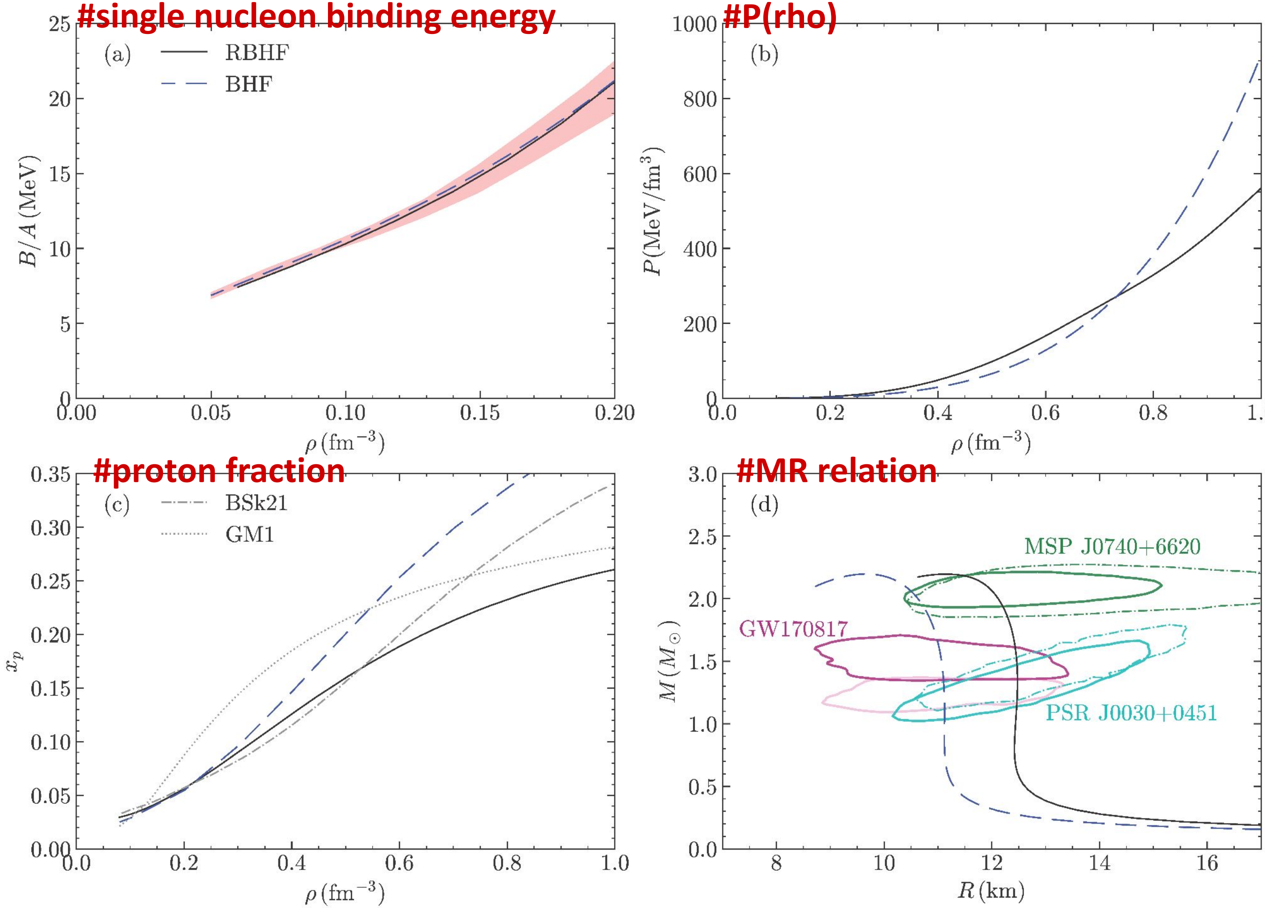
  A bulk part obtained from the BHF calculations for core **uniform** nuclear matter,
  PLUS
  +the phenomenological **surface** part,
  +the **Coulomb** part,
  +the **spin-orbit** part,
  +the **pairing**
  for **non-uniform** nuclear matter at crust.



Realistic NS model: Bulk (EOS) + composition + s.p. properties

State-of-art calculation of the thermodynamics of dense nuclear matter provides NS properties consistent with astrophysical observations on e.g., mass, radius, tidal deformability.

2108.00560



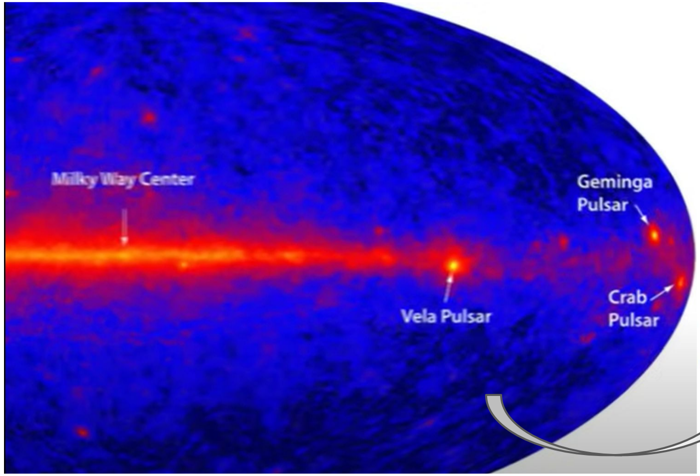
#single nucleon binding energy

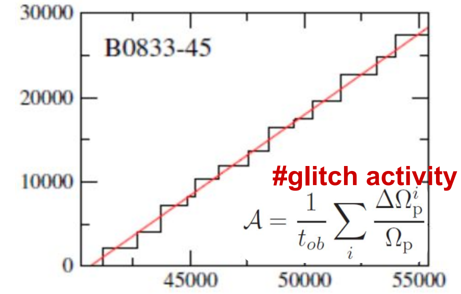
#P(rho)

#proton fraction

#MR relation

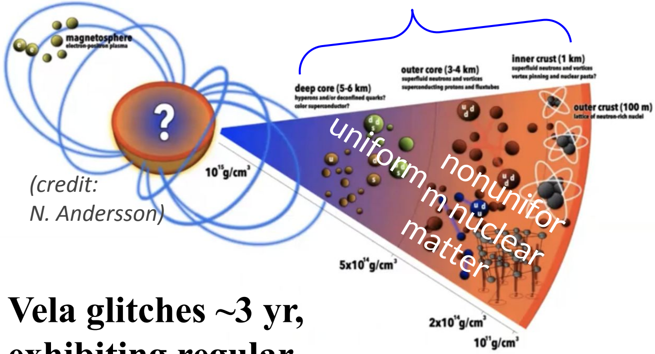
e.g., structure of Vela pulsar (spin period 89.33 ms) from unified BHF EOS





$$\mathcal{A} = \frac{1}{t_{ob}} \sum_i \frac{\Delta\Omega_p^i}{\Omega_p}$$

The accumulated $\sum_i \Delta\Omega_p^i/\Omega_p$ ($\times 10^{-9}$) as a function of the modified Julian date



(credit: N. Andersson)

Vela glitches ~3 yr, exhibiting regular, GIANT glitches.

1512.00340

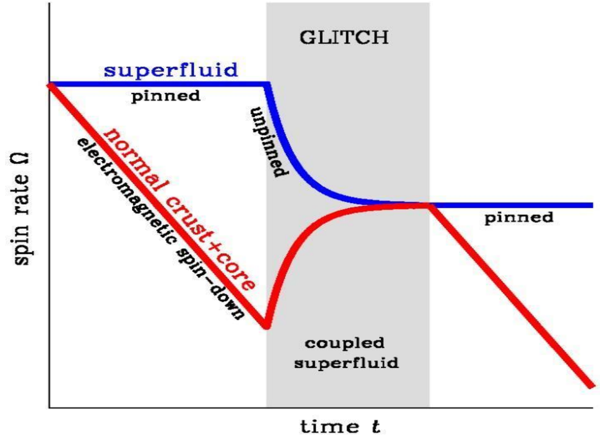
Mass	Cent.	Mass			Radius				Moment of Inertia	
		Core 核心	icrust 内壳层	ocrust 外壳层	Total	Core 核心	icrust 内壳层	ocrust 外壳层	Total	Fraction
1.0	0.403	0.97	0.026	4.59	11.79	10.33	0.81	0.64	0.894	5.33
1.1	0.427	1.08	0.024	4.15	11.80	10.50	0.73	0.57	1.029	4.51
1.2	0.452	1.18	0.022	3.72	11.80	10.64	0.66	0.51	1.170	3.84
1.3	0.480	1.28	0.020	3.37	11.79	10.75	0.59	0.45	1.318	3.29
1.4	0.508	1.38	0.019	3.05	11.78	10.84	0.53	0.41	1.474	2.82
1.5	0.536	1.48	0.017	2.73	11.76	10.92	0.48	0.36	1.638	2.41
1.6	0.567	1.58	0.016	2.46	11.73	10.97	0.43	0.32	1.809	2.06
1.7	0.602	1.69	0.014	2.18	11.67	10.99	0.39	0.29	1.987	1.76
1.8	0.643	1.79	0.013	1.94	11.58	10.98	0.35	0.26	2.170	1.49
1.9	0.696	1.89	0.011	1.67	11.45	10.92	0.31	0.22	2.358	1.24
2.0	0.764	1.99	0.0093	1.39	11.26	10.81	0.26	0.19	2.552	1.00

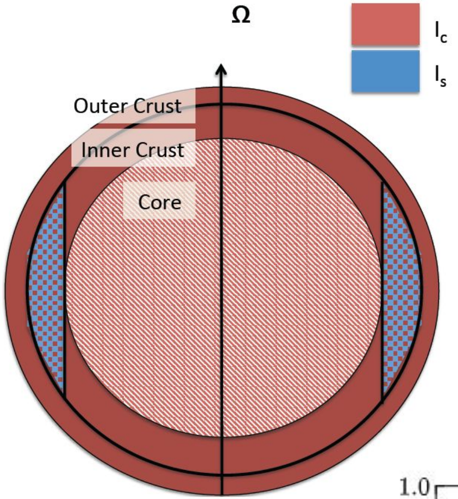
$\times 10^{-5}$

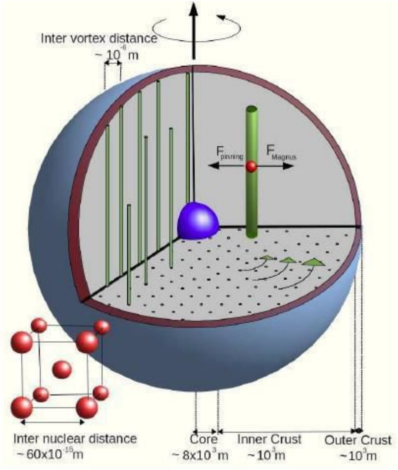
$\times 10^{45}$ g/cm^2

Microphysical state of the matter and the glitch(週期躍變)

#the two-components model

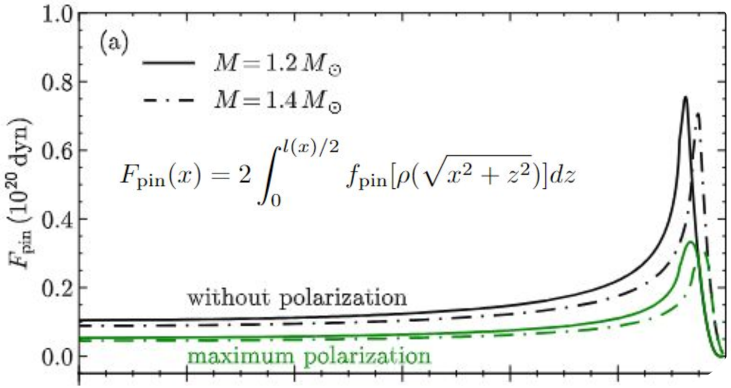






Decouple/Recouple/Decouple/Recouple…

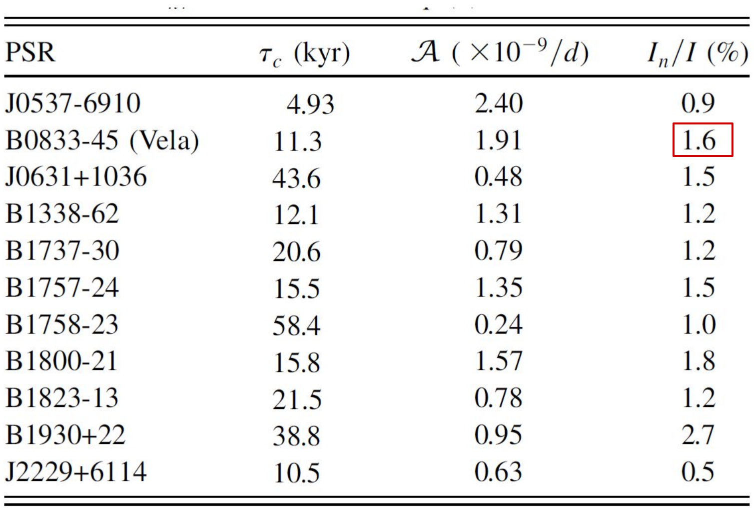
- Superfuid near the surface, from neutron drip density (4×10^{11} g/cm^3) to nuclear saturation density (2.8×10^{14}g/cm^3) in $^1S_0$ state: microphysical state of the matter needed;
- **Pinned**/Unpinned/Pinned/Unpinned…



Glitch crisis? Is there enough superfluid reservoir?

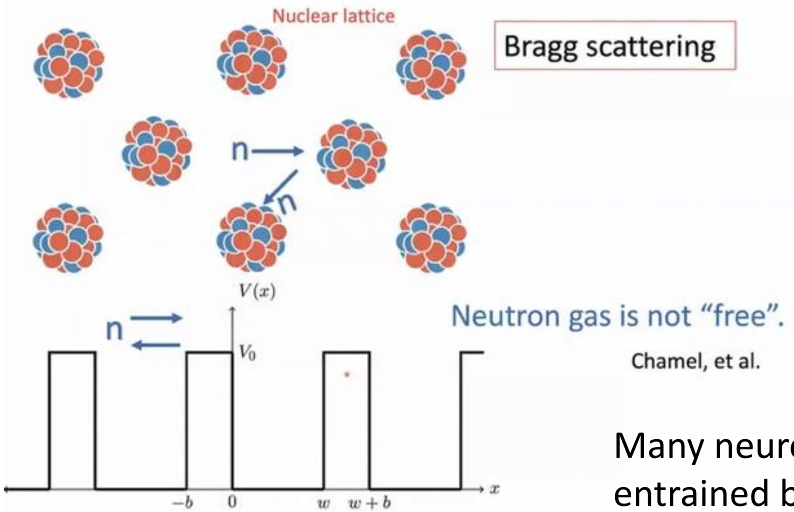
Require enough angular momentum transferred to trigger big Vela-like glitches:

$$2\tau_c A_g \lesssim \frac{I_n}{I_p}$$



PSR	τ_c (kyr)	$\mathcal{A}$ ($\times 10^{-9}/d$)	I_n/I (%)
J0537-6910	4.93	2.40	0.9
B0833-45 (Vela)	11.3	1.91	1.6
J0631+1036	43.6	0.48	1.5
B1338-62	12.1	1.31	1.2
B1737-30	20.6	0.79	1.2
B1757-24	15.5	1.35	1.5
B1758-23	58.4	0.24	1.0
B1800-21	15.8	1.57	1.8
B1823-13	21.5	0.78	1.2
B1930+22	38.8	0.95	2.7
J2229+6114	10.5	0.63	0.5

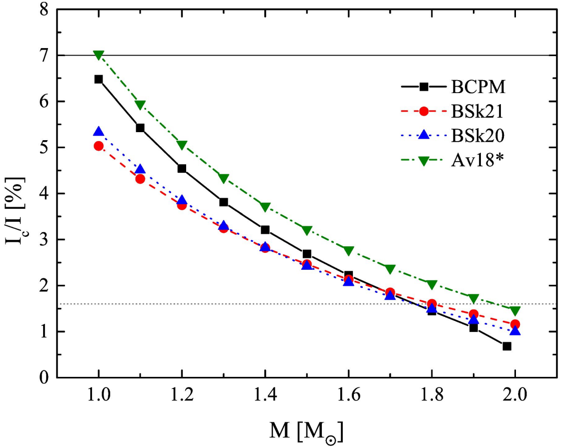
Andersson et al. 2012



Bragg scattering

Neutron gas is not “free”.

Chamel, et al.



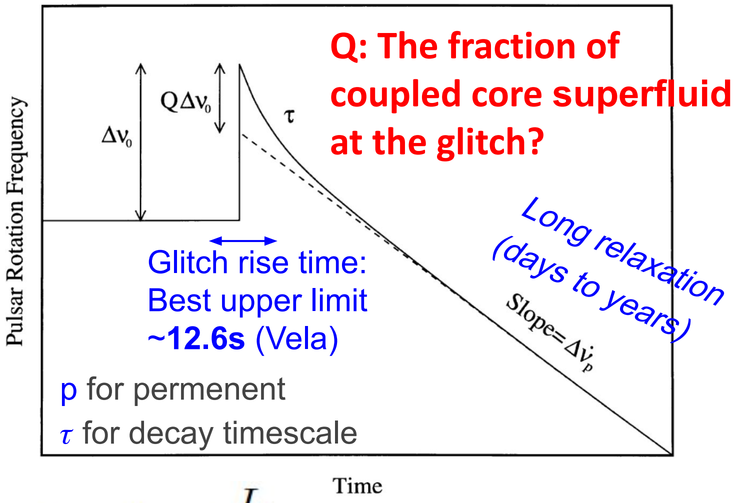
Many neurons are entrained by crust;

Entrainment reduce I_n by factor of ~5

1512.00340

Testing the standard superfluid glitch theory go beyond the two-component model

2108.00560



Q: The fraction of coupled core superfluid at the glitch?

Long relaxation (days to years)

Glitch rise time: Best upper limit **~12.6s** (Vela)

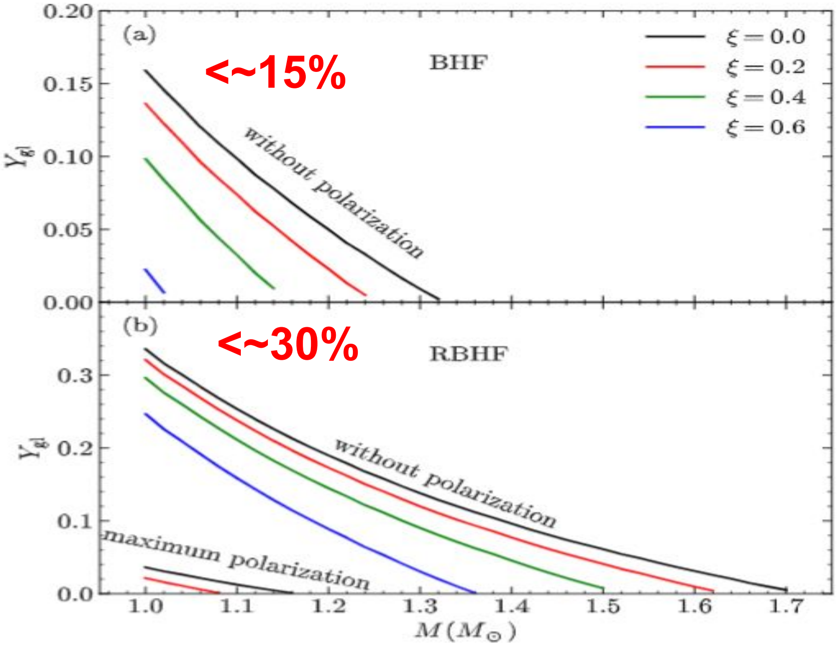
p for permenent

τ for decay timescale

$$2\tau_\mathrm{c} A_\mathrm{g} \lesssim \frac{I_\mathrm{n}}{\boxed{I_\mathrm{p}}}$$

- Entrainment reduce I_n by factor of ~5;
- I_p reduced by factor of 2~1000, since core superfluid coupling on timescales larger than glitch rise time.

NO crisis even with entrainment!



<~15%

<~30%

Is it possible to fit both the glitch size and the short-term relaxation from the 2000 Vela glitch?


THE ASTROPHYSICAL JOURNAL SUPPLEMENT SERIES, 223:16 (8pp), 2016 March doi:10.3847/0067-0049/223/1/16
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STRUCTURES OF THE VELA PULSAR AND THE GLITCH CRISIS FROM THE BRUECKNER THEORY

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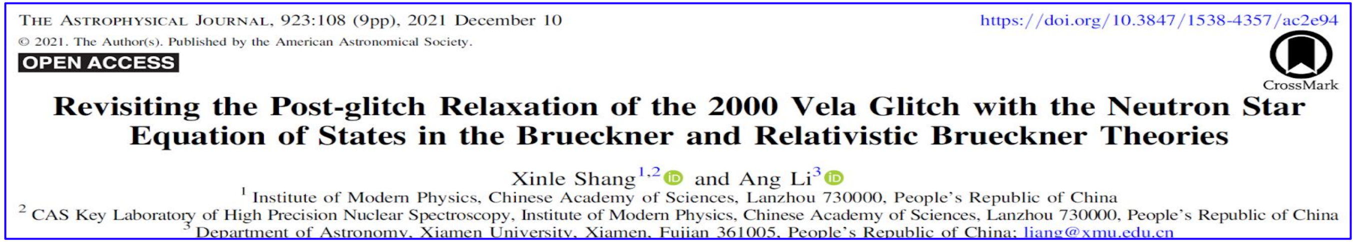
THE ASTROPHYSICAL JOURNAL, 923:108 (9pp), 2021 December 10 https://doi.org/10.3847/1538-4357/ac2e94
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Revisiting the Post-glitch Relaxation of the 2000 Vela Glitch with the Neutron Star Equation of States in the Brueckner and Relativistic Brueckner Theories

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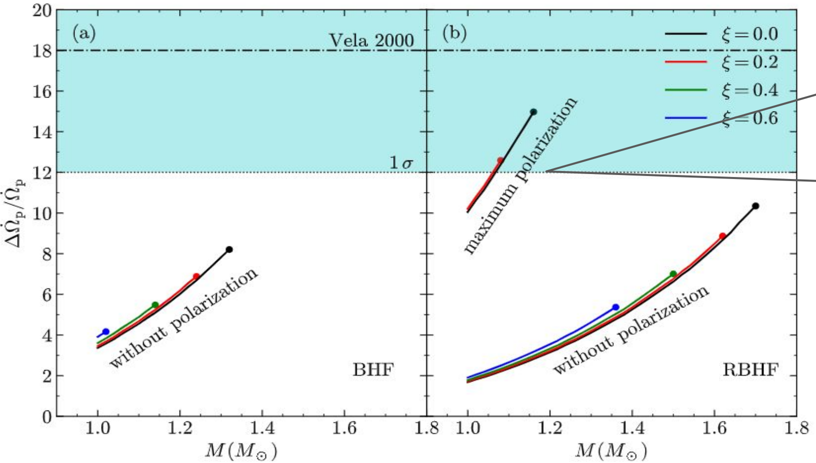
Glitch size
+
short-time relaxation

$$\frac{\Delta\dot{\Omega}_{\rm p}}{\dot{\Omega}_{\rm p}} = \frac{Q(1-Y_{\rm gl})}{1-Q(1-Y_{\rm gl})}$$



possible if

- a small fraction (less than~30%) of pinned neutron vorticity in the stellar core;
- a strong suppression/reduction of the **pairing gap in the nuclear medium**;
- a stiff **EOS** (resulting in a typical stellar radius larger than~12.5 km).

New frontier: Multiwavelength study of the glitch and post-glitch relaxation

Five Hundred Metre Aperture Radio Telescope (FAST)

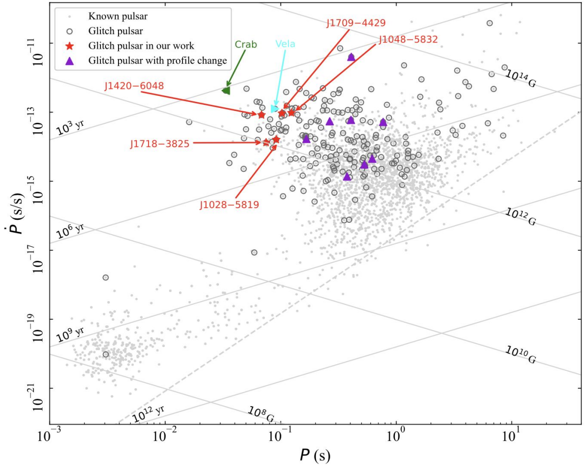
Tian Ma 65-m Radio Telescope

NanShan 25-m Radio Telescope

Fermi Gamma-ray Space Telescope

Parkes 64-m Radio Telescope

MeerKAT radio telescope

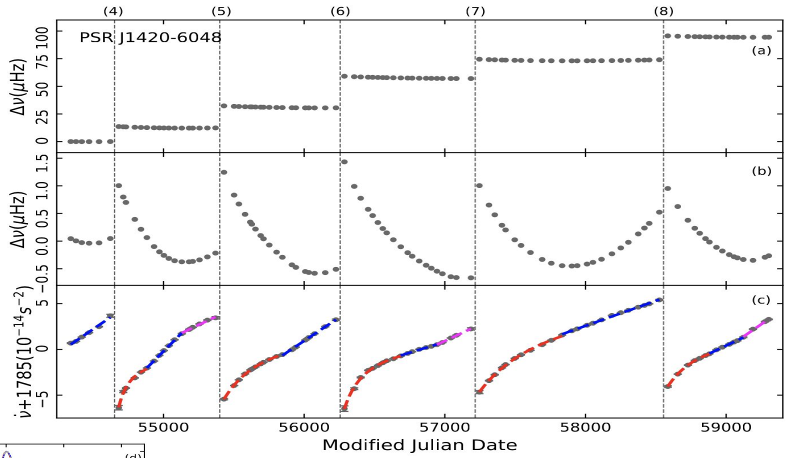


To date ~679 glitches in ~226 objects reported in the Jodrell Bank catalogue, http://www.jb.man.ac.uk/pulsar/glitches/gTable.html

Towards a thorough understanding of various observed glitch behaviors

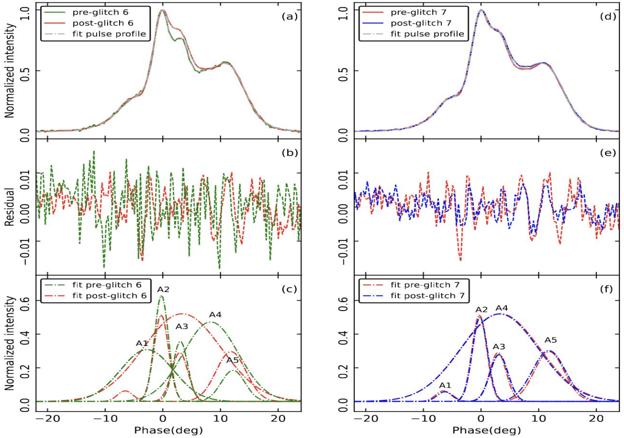
#NEW glitch event

Pulsar Name (PSR)	Gl. No.	Epoch (MJD)	$\Delta\nu/\nu$ (10^{-9})	$\Delta\dot{\nu}/\dot{\nu}$ (10^{-3})	NEW? (Y/P)	Q	τ_d (d)	RMS (μs)	Data span (MJD)
J1028−5819	1	57904(6)	2284.1(5)	20(2)	P	0.0061(2)	62(7)	624	57359 – 58243
J1420−6048	4	54653(19)	940(3)	6.5(12)	P	0.0122(2)	49(27)	28	54879 – 54562
	5	55400(9)	1366(3)	5.1(8)	P	0.016(1)	212(148)	605	55035 – 55765
	6	56256(1)	1974(4)	15(3)	P	0.011(2)	20(4)	1619	55747 – 56731
	7	57216(12)	1206(3)	5.0(4)	P	0.016(2)	93(15)	813	56853 – 57752
	8	58555(2)	1481(2)	6.3(5)	P	0.0097(1)	69(17)	332	58136 – 58842
J1709−4429	4	54691(2)	2777(4)	63(15)	P	0.0103(5)	72(4)	819	55415 – 55167
						0.006(3)	4(1)		
	5	56339(2)	2962(2)	9.7(5)	P	0.0079(4)	60(5)	833	56025 – 56663
	6	58175(2)	2438(2)	12(1)	P	0.0066(4)	33(5)	1202	57980 – 58523
J1718−3825	1	54952(44)	1.8(1)	-0.47(3)	P	–	–	186	54498 – 55400
	2	57950(14)	7.7(1)	-0.17(3)	P	–	–	148	57309 – 58410
	3	59121(8)	2.0(1)	-0.32(4)	Y	–	–	138	58578 – 59962

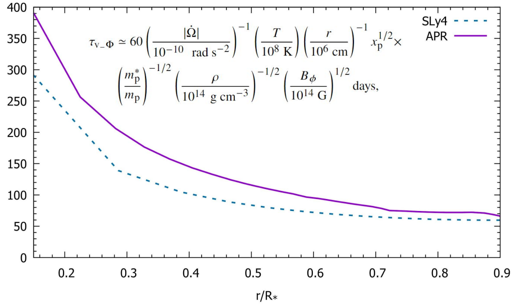


#NEW relaxation behaviour

#NEW correlation between pre- and post-glitch



Exponential decay timescale



$$\tau_{\mathrm{v\_\Phi}} \simeq 60\left(\frac{|\dot{\Omega}|}{10^{-10}\ \mathrm{rad\ s^{-2}}}\right)^{-1}\left(\frac{T}{10^{8}\ \mathrm{K}}\right)\left(\frac{r}{10^{6}\ \mathrm{cm}}\right)^{-1} x_{\mathrm{p}}^{1/2}\times \left(\frac{m_{\mathrm{p}}^{*}}{m_{\mathrm{p}}}\right)^{-1/2}\left(\frac{\rho}{10^{14}\ \mathrm{g\ cm^{-3}}}\right)^{-1/2}\left(\frac{B_{\phi}}{10^{14}\ \mathrm{G}}\right)^{1/2}\ \mathrm{days},$$

#NEW connection with EOS microphysics

Glitch No	$t_{\mathrm{ig,obs}}$ (days)	t_{g} (days)	I_{A}/I (10^{-3})	I_{B}/I (10^{-3})	I_{cs}/I (10^{-2})
6	2261(19)	2290(69)	2.48(1)	16.6(7)	3.13(7)
7	2456(26)	2661(405)	2.05(22)	15.6(25)	2.60(25)

Physics motivated model related to the dynamics of superfluid vortex

Two different regimes of dynamical response to the glitch:

Regions with weaker pinning energies will response **linearly** and those with stronger pinning energies will respond **nonlinearly**;

Relaxation time:

Linear regime:

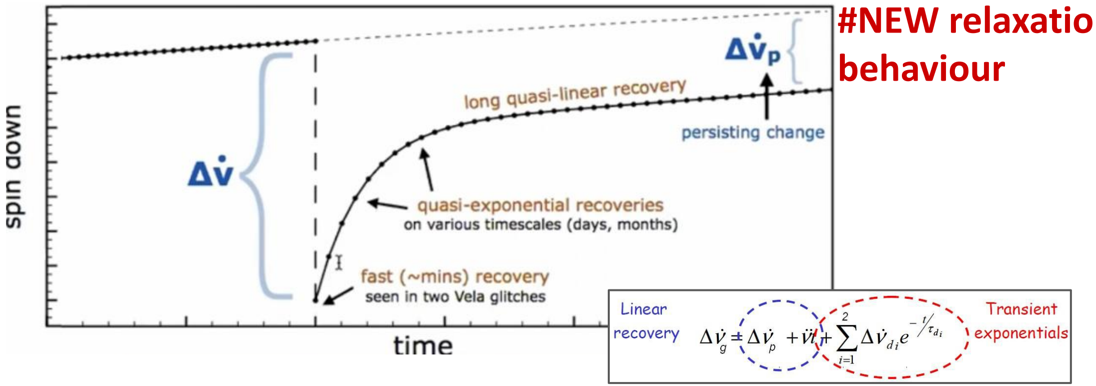
$$\tau_l = \frac{kT}{E_p}\frac{\omega_{cr} r}{4\Omega_c v_o} e^{E_p/kT}$$

Nonlinear regime:

$$\tau_n = \frac{kT}{E_p}\frac{\omega_{cr}}{|\dot{\Omega}|}$$

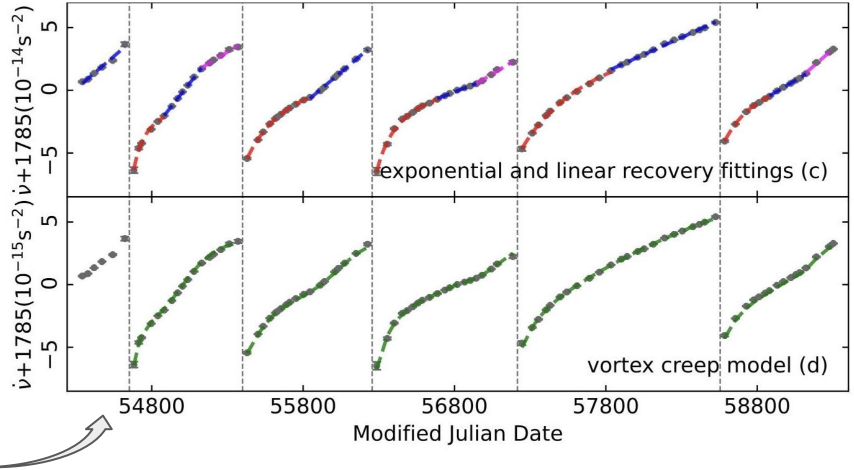
pinning energy

Preliminary recovery fit from the vortex-creep model describes the observations reasonably well



$$\Delta\dot{\nu}_g = \Delta\dot{\nu}_p + \ddot{\nu}t + \sum_{i=1}^{2}\Delta\dot{\nu}_{d_i} e^{-t/\tau_{d_i}}$$

#NEW relaxation behaviour



Take-home message

- The multi-messenger/multi-wavelength era of EOS study!
- We recently apply **complete** thermodynamical state of dense matter from bare NN+NNN force to the study of pulsar spin evolution, focusing on the glitch, and find various constraints on the **nuclear force in medium**, as well as the star properties;
- Glitch provides **unique** insights into the internal structure of neutron stars;
- Many exciting ways to combine various fields: glitch-induced GW; laboratory counterpart (e.g., ultra-cold atom), TD pinning dynamics, etc.

Thank you
Welcome to XMU