Theory Overview: Short-Ranged Correlations are here to stay and we don’t care what people say

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What are short-ranged correlations (SRC)

Why SRC must exist

How SRC show up in matrix elements

Why they can be observed

Discovery versus precision in nuclear physics:
A tale of three scales
G A Miller, PRC 102,055206 (2020)

What are short-range correlations?

Two body density $\rho(\mathbf{r},\mathbf{r}') = \langle\Psi|\sum_{j<k}[\delta(\mathbf{r}-\mathbf{r}_j)\delta(\mathbf{r}'-\mathbf{r}_k)+\delta(\mathbf{r}-\mathbf{r}_k)\delta(\mathbf{r}'-\mathbf{r}_j)]|\Psi\rangle$

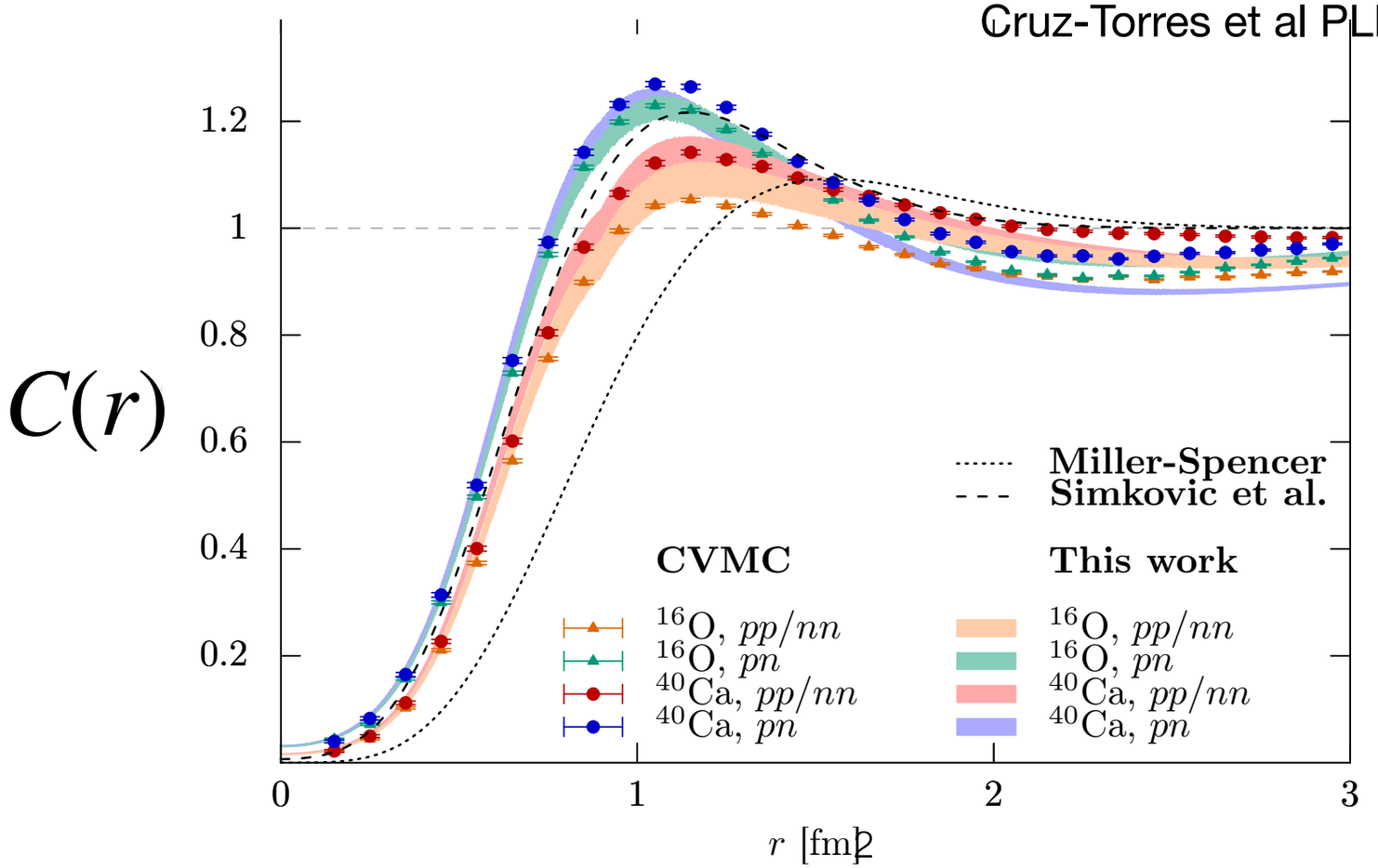
Probability of finding one particle at j and another at k

$$\rho(\mathbf{r},\mathbf{r}') = (1 + C(|\mathbf{r}-\mathbf{r}'|))\rho(\mathbf{r})\rho(\mathbf{r}')$$

Measure in momentum space

Two nucleon wave function is not a product! C is the correlation function

Cruz-Torres et al PLB 785, 304



SRC is C(r) at small r

Implications of short-range correlations

C(r) is used in computing the following and more:

- Neutrino-less double beta decay matrix elements Kortelainen:2007rh,Kortelainen:2007mn,Menendez:2008jp,Simkovic:2009pp, Benhar:2014cka,Cruz-Torres:2017sjy,Wang:2019hjy
- Nuclear charge radii: Miller:2018mfb
- Nuclear symmetry energy and neutron star properties: Li:2018lpy
- Internal structure of nucleons in nuclei Hen:2013oha,Hen:2016kwk,Schmookler:2019nvf

Intent here: explain from very beginning

What holds the nucleus together?

"What holds the nucleus of the atom together? In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem - probably more man-hours than have been given to any other scientific question in the history of mankind." – Hans Bethe, Scientific American 1953.

Theory: Discovery vs Precision phases

Discovery- there is a puzzle to be solved, main mechanisms to be identified.

1953 -discovery of binding based on nucleon-nucleon interaction in the medium

1975 - main pieces identified, corrections needed

Precision - 1980's relativistic effects put in

late 1990's -2000 chiral eft

2000- now soft interactions, similarity renormalization

Sometimes pursuit of precision obscures the basic elements and leads to confusion

Experiment re short-range correlations -still in discovery phase, but moving rapidly

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Short distance-high momentum relation

Bethe (PhysRev.103.1353) "Indeed, it is well established that the forces between two nucleons are of short range, and of very great strength" and "there are strong arguments to show that the two-body forces continue to exist inside a complex nucleus ".

1955

Brueckner, Eden, & Francis, (PhysRev.98.1445) argued that nuclear wave function contains nucleons with a significant probability to have high momentum: The (p, d) pick up reaction with 95 MeV protons. The neutron in the nucleus must have high momentum comparable to that of the proton, about 420 MeV/c, so that combination with the incident proton allows the deuteron to emerge from the nucleus. The only way a bound neutron could acquire such momentum is via interactions with another nearby nucleon.

If p has high momentum, n in nucleus must have high momentum to make high-momentum deuteron

pp scattering at high energy- strong repulsive core

Symmetric about 90 deg-identity of particles flat
except for forward peak due to coulomb

Jastrow PR 81, 165(1951)

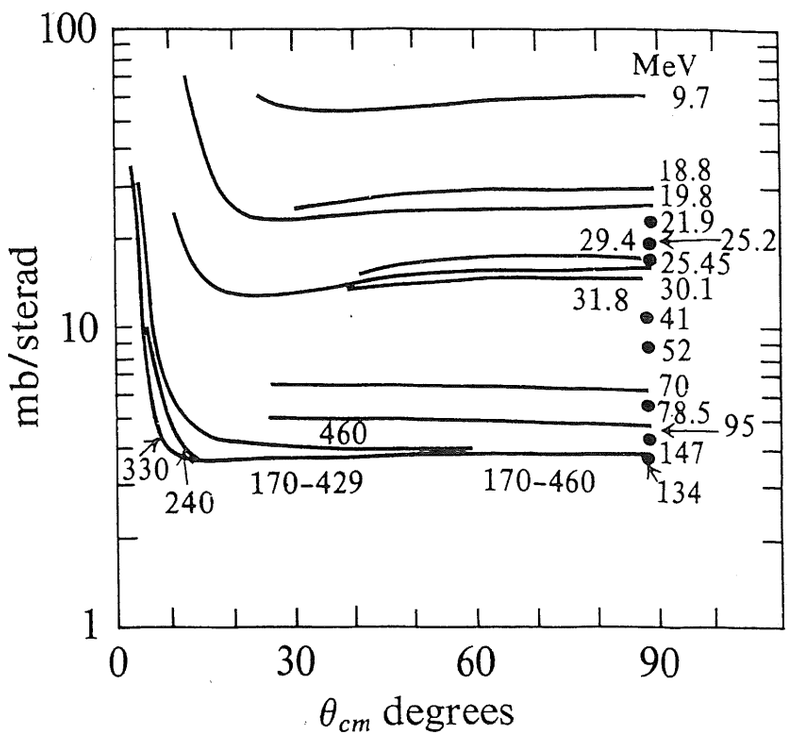
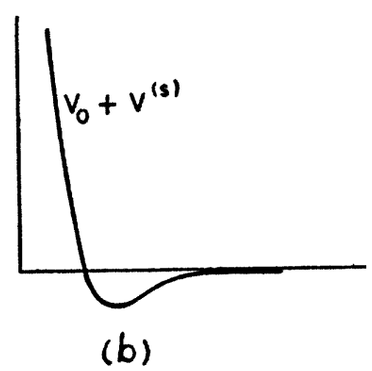
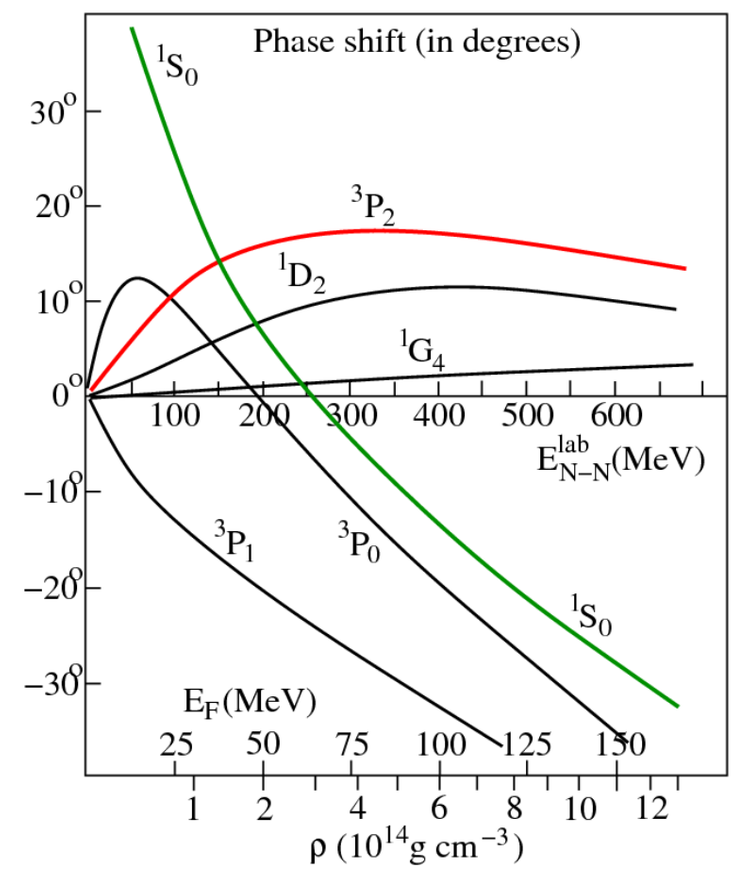


Fig. 38.2 Experimental *p-p* differential cross section in the center-of-momentum system at various laboratory energies (in MeV). The forward peak is due to coulomb scattering.

Isotropy due to only s-wave scattering ruled out, high energy
Interference between S&D gives flatness IF
Potential is hard repulsive core at short distance &
long range attraction. As energy increases
sign of s-wave changes from + to -.

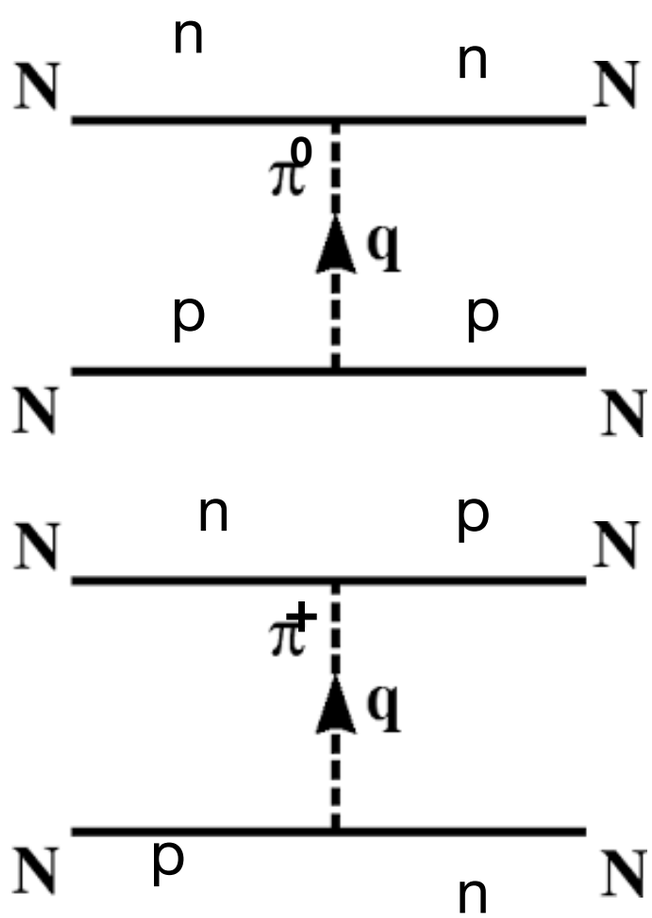




One pion exchange at higher energies

np scattering

Symmetry about 90



Forward scattering looks like backward scattering

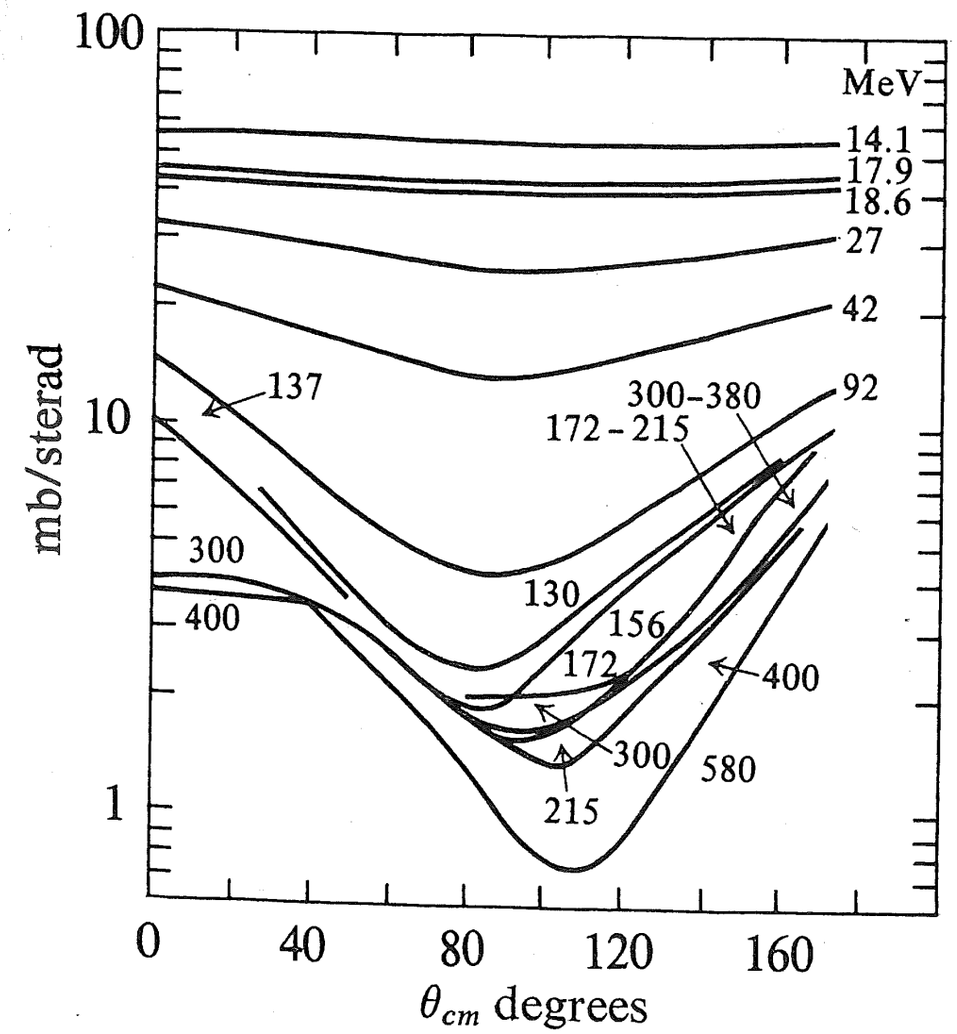
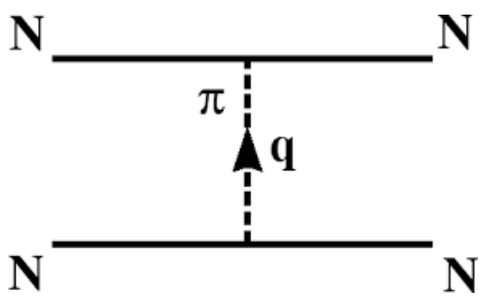


Fig. 38.1 Experimental *n-p* differential cross section in the center-of-momentum system at various laboratory energies (in MeV). (From

Deuteron and One Pion Exchange Potential



Longest ranged component of NN force

$$V_{\mathrm{OPE}}(\mathbf{r}) = \frac{1}{3}\frac{f^2}{4\pi} m_\pi \tau_1 \cdot \tau_2 \frac{e^{-m_\pi r}}{m_\pi r}[(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2})S_{12} + \sigma_1 \cdot \sigma_2]$$

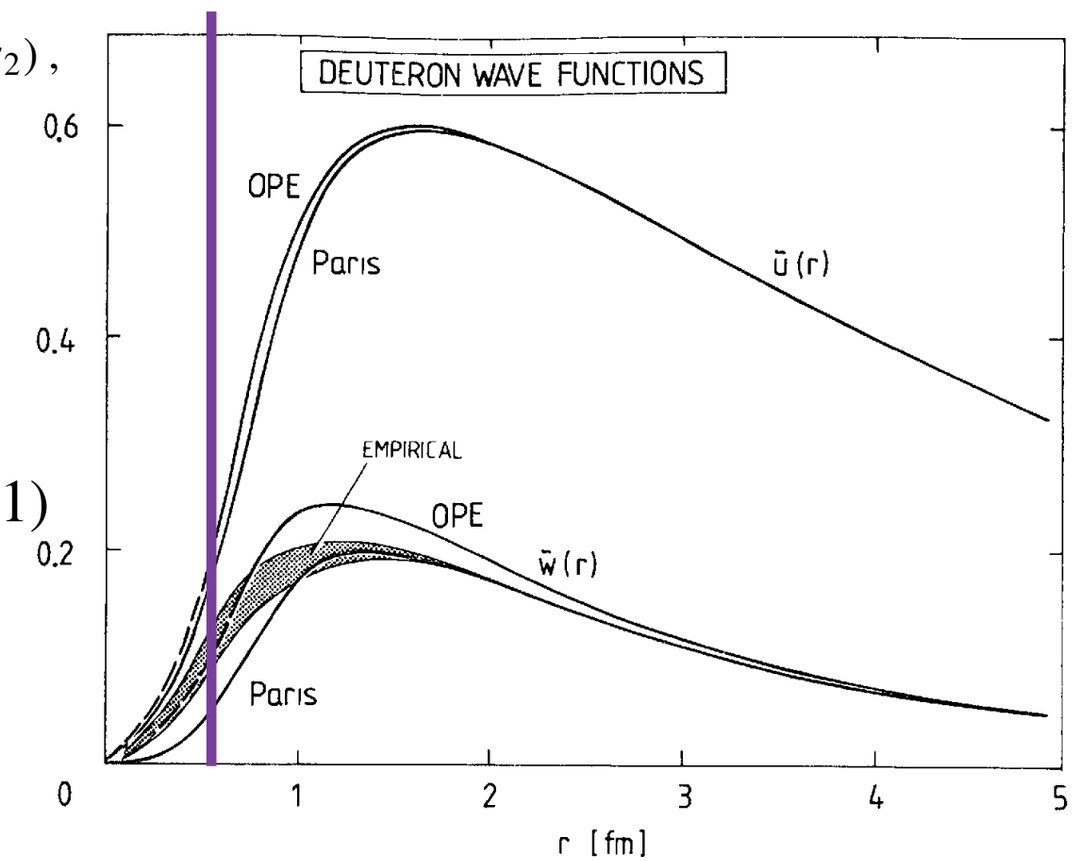
$m_\pi \approx 0.7\mathrm{fm}^{-1}$, $f^2/4\pi \approx 1$

Ericson and Rosa-Clot ARNPS 35,271 (1985)

$$S_{12} = \frac{3}{r^2}(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\,,$$

Small r :OPE ~1/r³
Singular

$$\tau_1 \cdot \tau_2 = -3(T=0) \text{ or } +1(T=1)$$



OPEP explains deuteron wave function for distances down to about 0.5 fm

Gives np dominance

Summary of NN scattering-basic

- Tensor force due to OPE very important for deuteron and np scattering

- pp scattering can be described by hard core plus longer-ranged attractive force

- Implication-these pair-wise forces bind nuclei -there must be nucleon-nucleon correlations- nucleons do not move independently in the nucleus

NN forces-Life is way more complicated

Origin of Core

- 1950-1980 Exchange of omega (vector) meson gives repulsion like photon exchange between like charges

- 1980-1990 Pauli principle limits number of quarks at the same place

- 1990- Effective field theory - mechanism is high energy, unknowable use a low-energy constant

Opinion and one theme of this talk-
giving up knowledge of the short-distance physics is a mistake, to understand nuclei one needs to know all scales

First nuclear phenomenology: Independent particle (Shell model)

Nucleon moves in average potential provided by other nucleons

Schematic

$$U(r) \sim \int d^3r' V_{NN}(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')$$

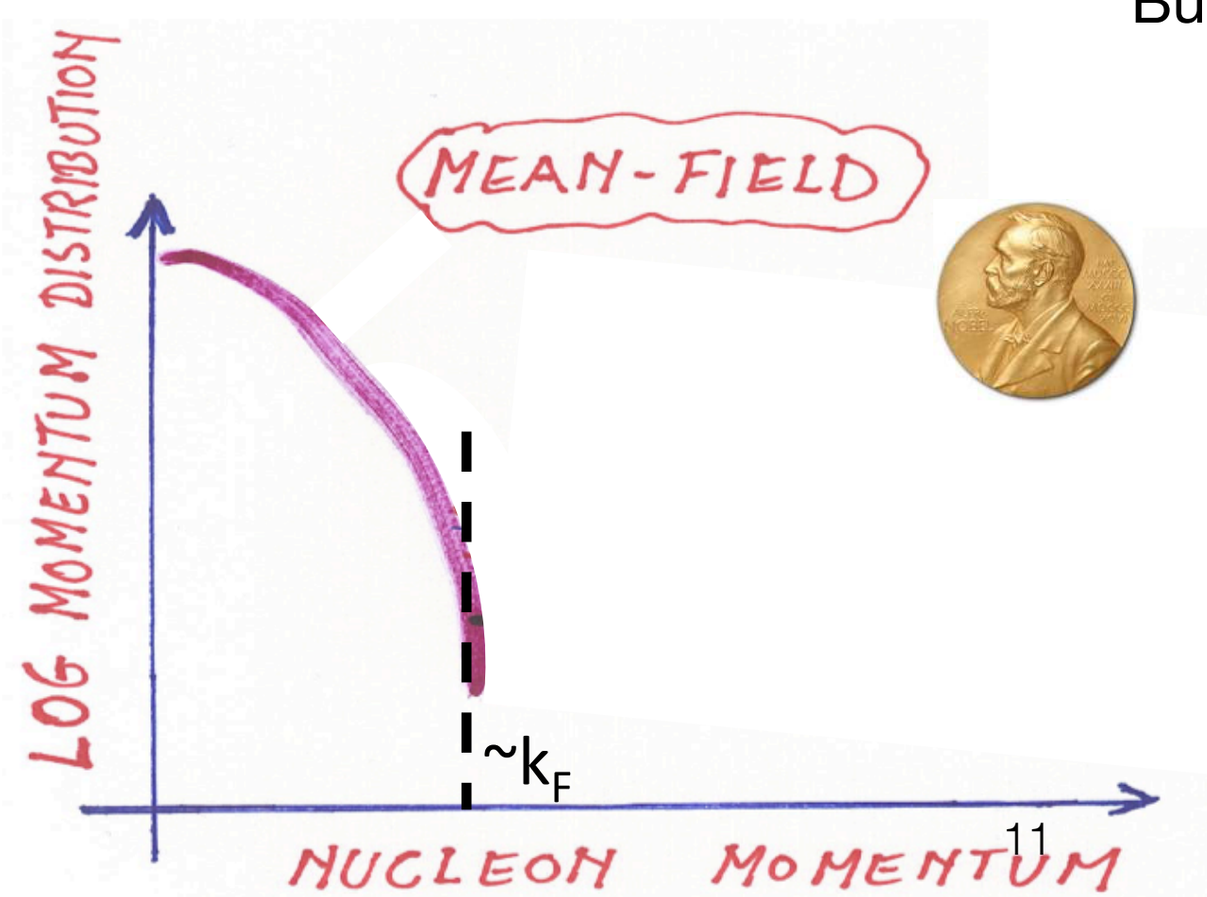
Single particle potential

Nuclear density

Nucleon-nucleon potential

Nucleon momentum distribution

But no $p + A \rightarrow d + X$



First nuclear phenomenology: Independent particle (Shell model)

Problems

- There is no fundamental central potential as in atomic physics- potential is strong and short-ranged. Why does shell model work?
- U is first-order in V, V is strong. How can divergent series give physical results?
- ….

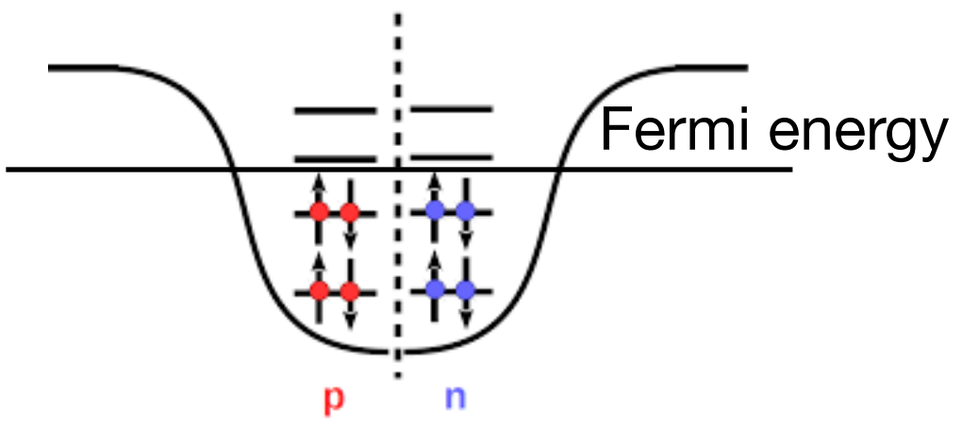
How to fix the problems

Scattering of 2 bound nucleons

$$T(E) = V + V\frac{1}{E - H_0 + i\epsilon}T(E), \quad E > 0$$

Free

Bound



In medium E becomes -B, <0
Scattering -is Pauli Blocked
Resulting matrix is called G (gasp)

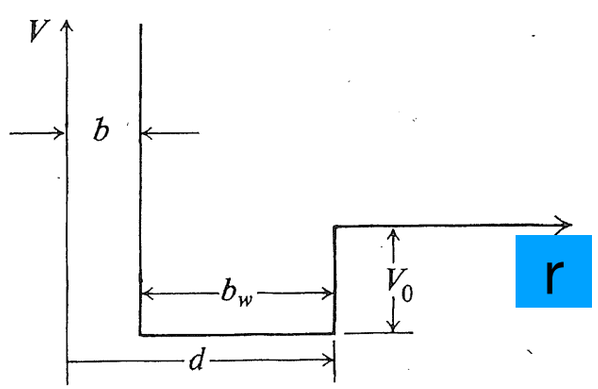
$$G(B) = V + V\frac{Q}{-(B + H_0)}G(B), \quad B > 0$$

$$G(B)\phi = V\psi, \text{no plane wave term}$$

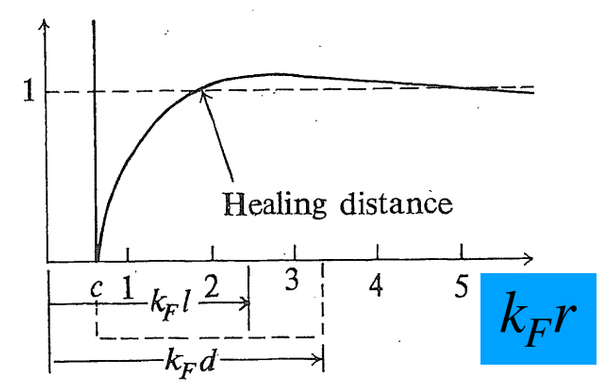
Denominator is negative definite asymptotic behavior of wave function $\sim e^{-\sqrt{B}r}/r$

V is singular, G(B) is not singular

Example Fetter & Walecka $^1S_0$ potential



$r\psi(r)$



Derived Shell Model From Strong Int's

Single particle potential
Mean field

$$U(r) \sim \int d^3r' V_{NN}(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')$$ Non-convergent

$$U(r) \sim \int d^3r' G_{NN}(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')$$ Convergent

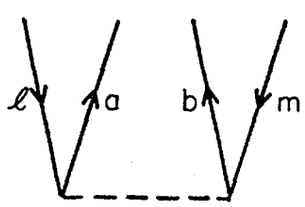
First approximation to the nuclear wave function $|\Phi_A\rangle$
= anti-symmetrized product of single particle (sp) wave functions defined by the potential U

Full nuclear wave function:

$$|\Psi\rangle_A = |\Phi\rangle_A + \frac{Q_A}{E_A - H_0}\sum_{i<j} G_{ij}|\Phi\rangle_A$$

NN correlations
Products of sp wave functions not sufficient

Two nucleons below Fermi sea kicked above the Fermi sea 2p-2h excitations



G

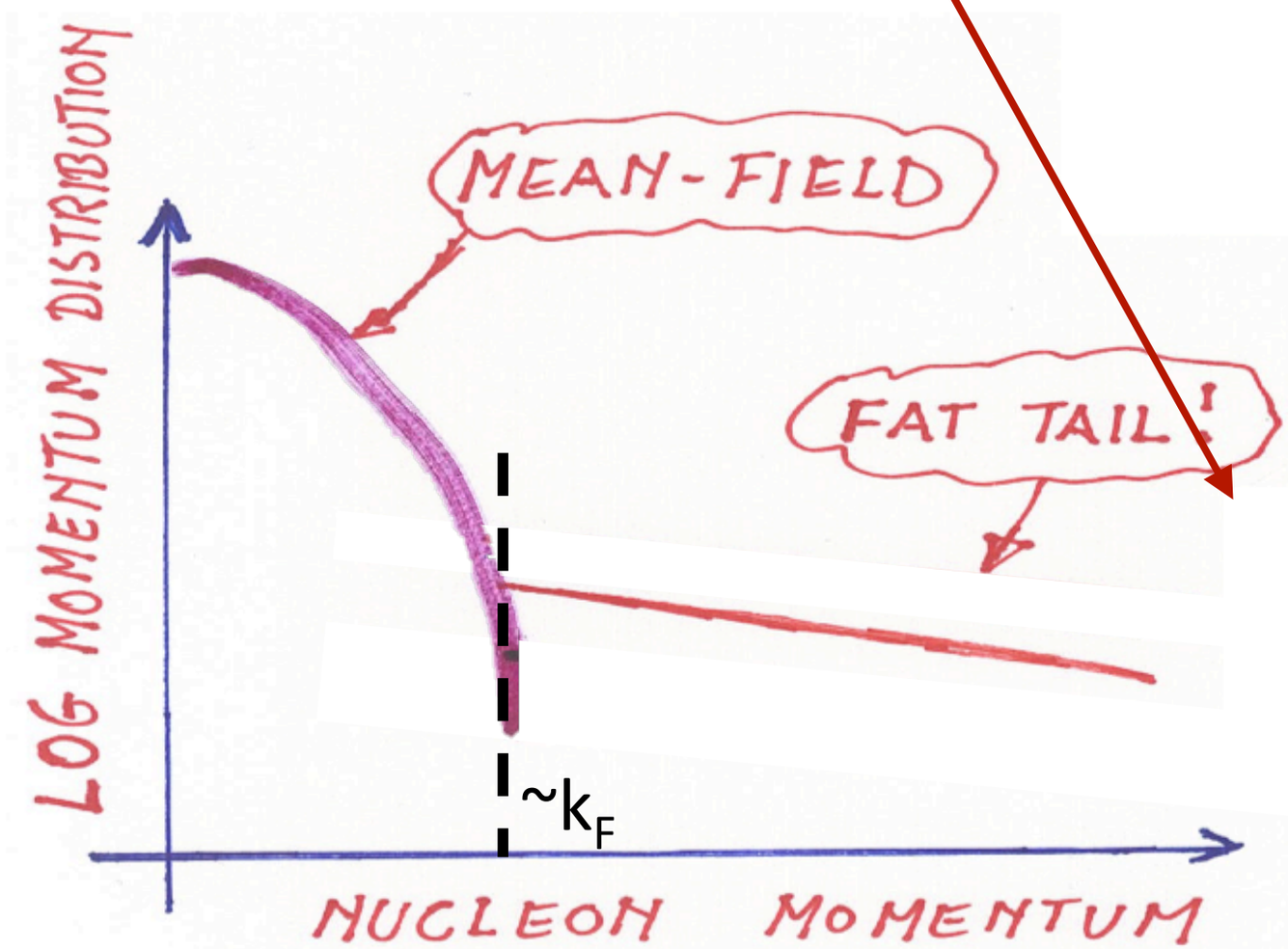
l,m are single particle states below Fermi sea (holes)
a,b are single-particle states above Fermi sea (particles)

Explains why nucleons not completely in shell-model orbitals

Consequences of

$$|\Psi\rangle_A = |\Phi\rangle_A + \frac{Q_A}{E_A - H_0} \sum G_{ij} |\Phi\rangle_A$$

NN correlations
Products of sp wave
ufficient



Responsible for production of deuterons!

Basics solved, end of discovery phase but issues remaine

Understanding G and nuclear binding -Chiral dynamics of Kaiser et al nucl-th/0105057

Previously we saw OPEP dominates the deuteron wave function
Kaiser et al showed that OPEP dominates nuclear binding

$$G(B) \approx V_{\mathrm{OPEP}} + V_{\mathrm{OPEP}} \frac{Q}{-(B + H_0)} V_{\mathrm{OPEP}}$$

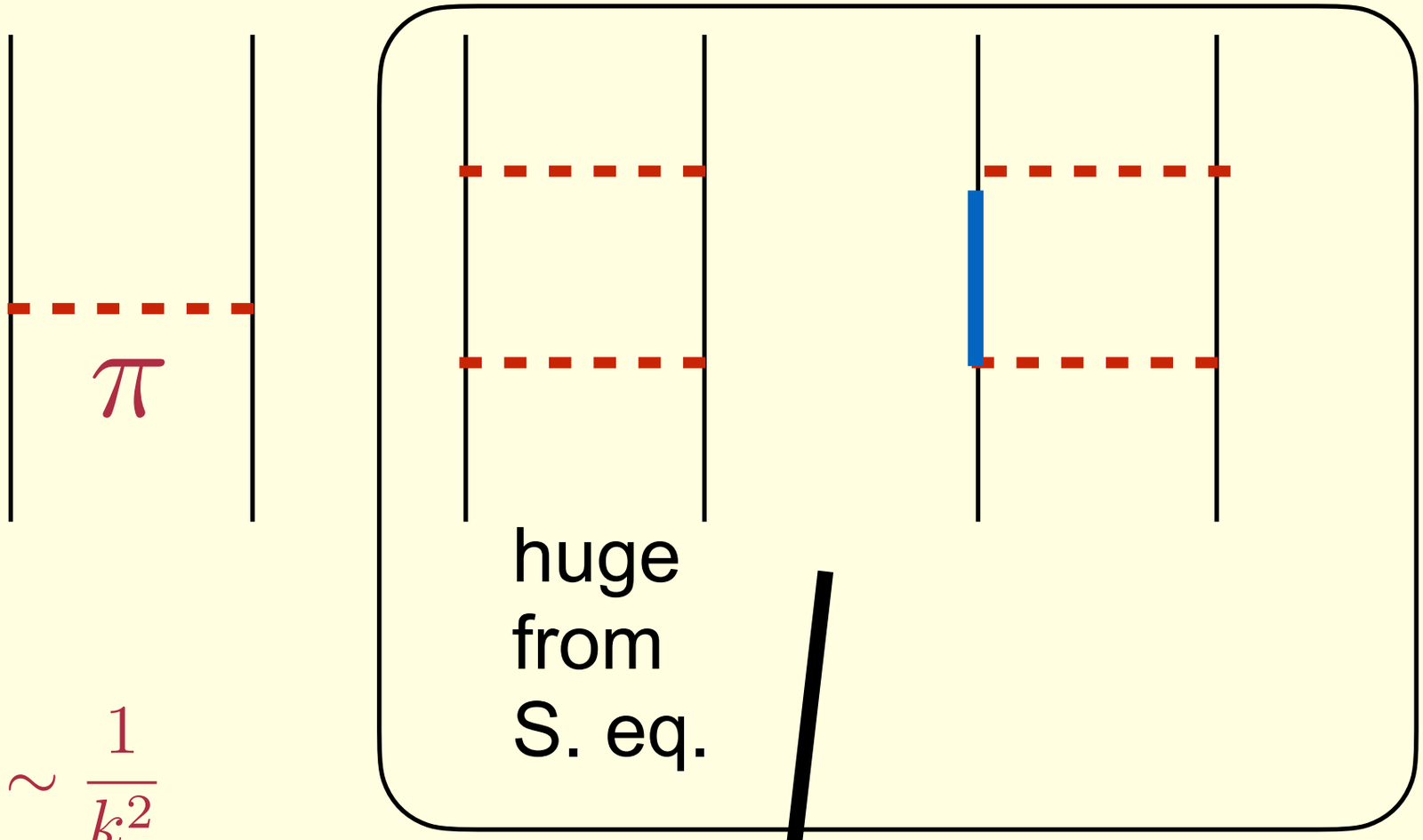
Dominance of tensor force V_T. See also Appendix of Hen et al RMP arXiv:1611.09748

- Momentum-space cutoff Λ regularizes divergent parts of chiral two-pion exchange
- Isospin symmetric and asymmetric nuclear matter given in terms of one parameter Λ
- Λ characterizes short-distance physics
- Pion exchange plus Λ gives basics of nuclear physics

Gives np dominance

Reason for np dominance

one pion exchange between n and p. Strong in S=1,T=0



$$\psi(k) \sim \frac{1}{k^2}$$

$$300\,\mathrm{MeV/c} < k < 500\,\mathrm{MeV/c}$$

Supports high momentum transfer

Not effective range

Two nucleons are stuck/struck together

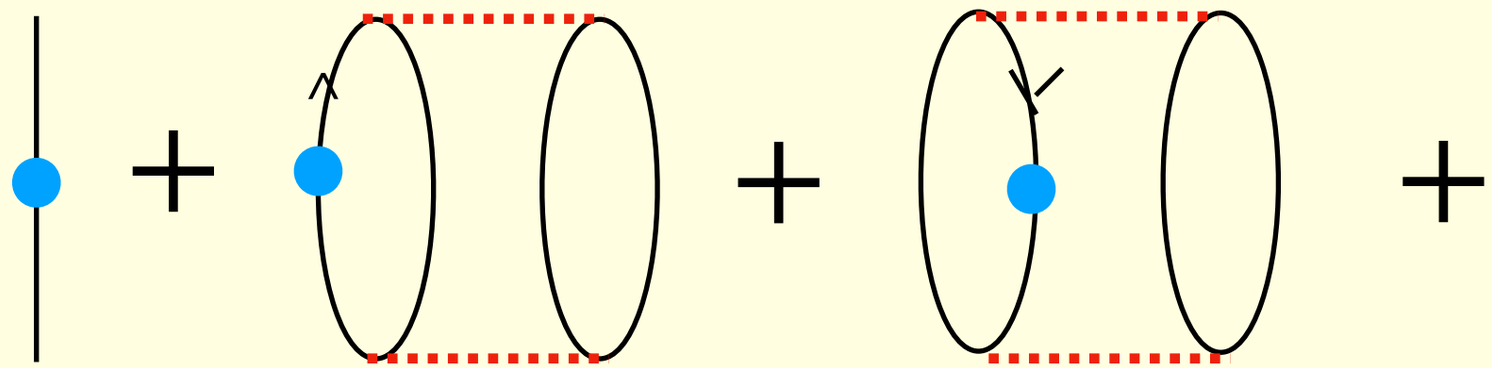
Same mechanism as Kaiser et al that holds nucleus together

Long range effects are influenced by short-range interactions

PLB 793,360 (2019)

Nuclear charge radii $\hat{r}^2 =$ 

$\langle A | \hat{r}^2 | A \rangle =$



single particle correlations correlations

Can SRC be measured?

- Why ask? Everyone knows wave functions can't be measured and src are part of wave function
  Matrix elements are measured. Normal procedure:
  use wave functions to compute measured matrix elements and
  verify wave functions.

- Furnstahl & Schwenk J. Phys. G37,064004(2010) -"systematic framework needed to address questions such as whether short-range correlations are important for nuclear structure

- Many examples show that momentum-space wave functions are closely connected to cross sections: photoabsoprtion cross section on hydrogen proportional to square of wave function. Modern version: Angle resolved photoemission spectroscopy, gives electron wave functions in solids RMP75,473.

  So experience teaches us that wave functions can be determined.
  Perhaps the only difference between atomic physics and nuclear physics is
  that the interaction is known in the former case.

  In nuclear physics the interaction is well described in terms of OPEP and one
  parameter to account for short-distance effects.

My opinion- SRC can be measured

More than opinion is needed

Theorists have softened the interaction to simplify calculations of nuclear energies this is called $V_{\text{low k}}$ or using Similarity Renormalization Group (SRG), or in -medium SRG. These transformations can be thought of as unitary transformations that eliminate matrix elements of the potential between low and high momentum states, which do not change the scattering phase shifts:

$$\langle \text{low}\, k | V_{\text{SRG}} | \text{high}\, k \rangle = 0$$

These transformations can be expressed as unitary transformations that leave matrix elements invariant

$$\langle \Psi | \mathcal{O} | \Psi \rangle = \langle \Psi\, U^\dagger | U \mathcal{O} U^\dagger | U \Psi \rangle$$

The extracted wave functions depend on whether the interaction operator is $\mathcal{O}$ or $U\mathcal{O}U^\dagger$. Accepting this ambiguity means agreeing that wave functions cannot be studied or constrained by experiments.

This ambiguity is not acceptable for the hydrogen atom. Who would say textbook wave functions are wrong?

G A Miller, PRC 102,055206 (2020)

Why should not use $U\mathcal{O}U^\dagger$ instead of $\mathcal{O}$

- $\mathcal{O}$ is simple $U\mathcal{O}U^\dagger$ is not
- To find stuff out do experiment at kinematic conditions where you know $\mathcal{O}$ is correct
- Basically impulse approximation

Summary

- basic elements of nuclear saturation identified
- Nuclear shell-model is an approximation, there are two-nucleon correlations
- Short range correlations that produce high-momentum nucleons are inevitable
- Effects of SRC are measurable
- Short range correlations have been measured- other talks
- SRC are here to stay, we don’t care what people say