S factor of radiative α capture on ^{12}C in effective field theory

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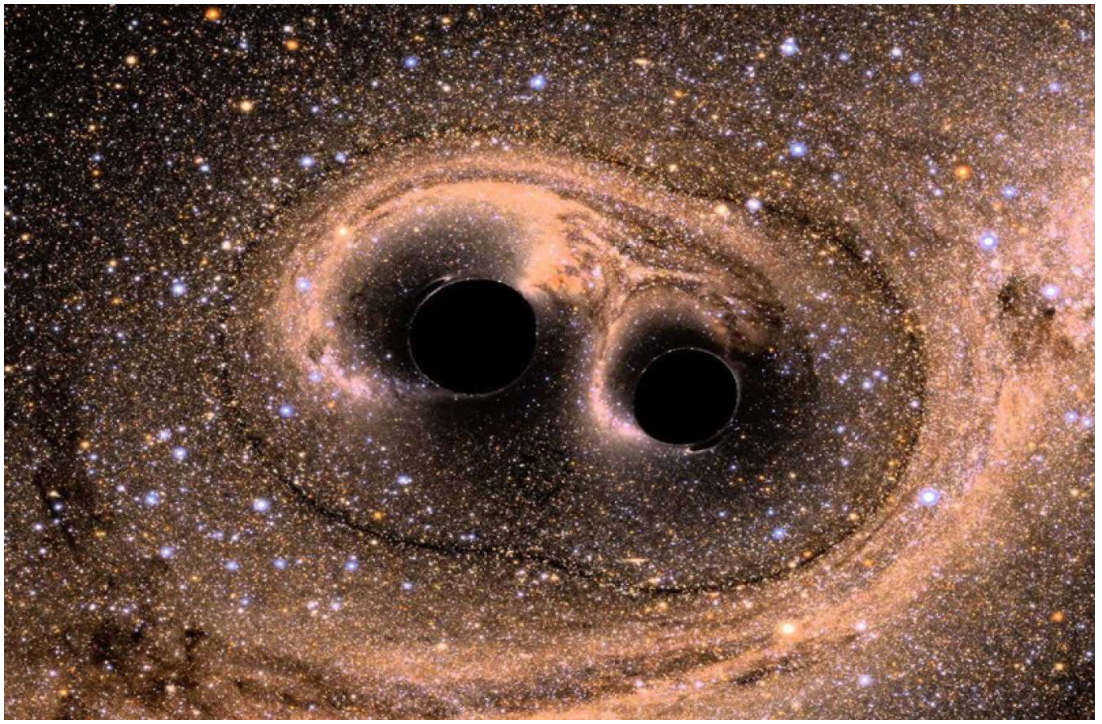
OECE2026@BUAA, Beijing, China, November 1-6, 2026

Outline of the talk

- Gravitational waves, Black Hole mass gap, and $^{12}C(\alpha,\gamma)^{16}O$
- Cluster EFT for the α-^{12}C system
- Elastic α-^{12}C scattering at low energies
- β delayed α emission from ^{16}N
- S factors for E1 and E2 transitions of $^{12}C(\alpha,\gamma)^{16}O$ at $E_G = 0.3$ MeV
- Results and discussion

Gravitational waves

- LIGO-Virgo-KAGRA (LVK) collaboration reported over 350 total compact mergers
- Roughly 90-95%, Binary Black Hole (BBH) systems



BH mass spectrum

The Astrophysical Journal, 937:112 (16pp), 2022 October 1

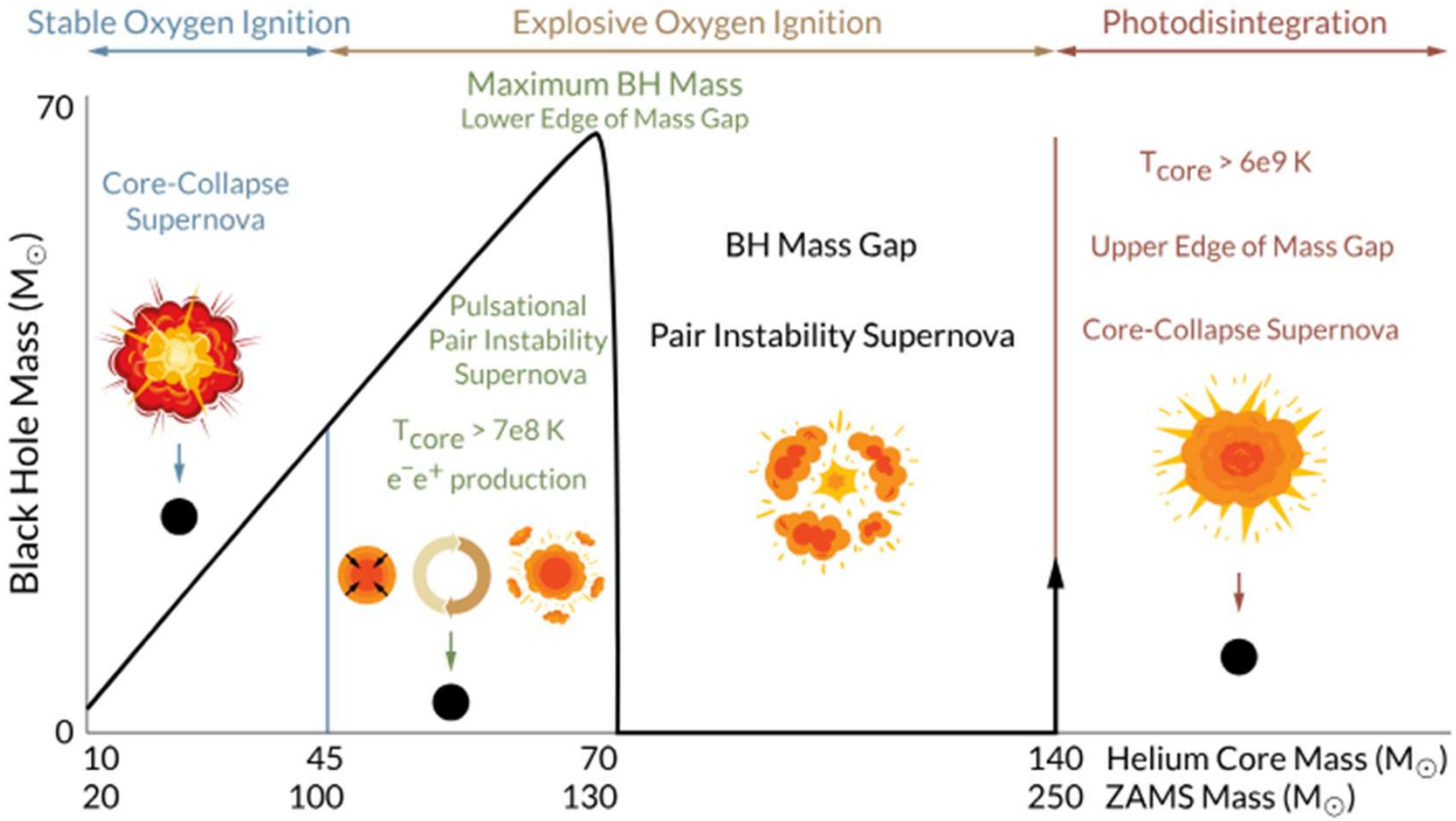


Figure 1. Illustration of a single-star BH mass spectrum.

Pair-instability mass gap

- A key observable: Pair-Instability mass gap

  For the stars with a helium core about 34-64 $M_{\odot}$

  1. Pair production ($\gamma \rightarrow e^{-}e^{+}$) and softening, core collapse
  2. Explosive carbon burning (with a high fraction of ^{12}C)

     -Pulsation Pair instability (PPISN) → small BH

     -Pair Instability Supernova (PISN) → no BH in 50-130$M_{\odot}$

- The lower edge of the gap is sensitive to the C/O ratio of the carbon/oxygen core due to the reaction rate of $^{12}C(\alpha,\gamma)^{16}O$

The S factor and the mass gap

- Values of S factor (Gemini)
  -- Small 100 to 120 keV b (with 40 to 45 $M_{\odot}$)
  -- Standard 140 to 160 keV b (with 45 to 50 $M_{\odot}$)
  -- Large 180 to 220 keV b (with 55 to 60 $M_{\odot}$)
- Recently, the S factor is deduced from the LOGO-Virgo-KAGRA's fourth Gravitational-Wave Transient Catalog (GWTC-4) with a lower boundary of 44+5-4 $M_{\odot}$ as

S = 260+190-108 keV b

Reported values: S factor of $^{12}C(\alpha,\gamma)^{16}O$

deBore et al, RMP(2017)

Ref.	S(300 keV) keV b $E1$	$E2$	Cascades	Total	Model
Cluster Models					
Descouvemont *et al.* (1984)	300	90			GCM
Langanke and Koonin (1985)	160-280	70	<10[c]	230-350	H*R*&PM
Funck *et al.* (1985)		100			PM
Redder *et al.* (1987)	140^{+120}_{-80}	80±25	7±3[c] $1.3^{+0.5d}_{-1.0}$		*R*&PM
Descouvemont and Baye (1987)	160	70			GCM
Ouellet *et al.* (1992)	1^{+6}_{-1}	40±7			*R*&PM
Descouvemont (1993)		90			GCM
Ouellet *et al.* (1996)	79±16	36±6		120±40	*R*,*K*,PM
Dufour and Descouvemont (2008)		42±2			GCM
Katsuma (2012)	≈ 3	150^{+41}_{-17}	18.0±4.5[e]	171^{+46}_{-22}	PM
Xu *et al.* (2013) (NACRE2)	80±18	61±19	$6.5^{+4.7e}_{-2.2}$	148±27	PM
Phenomenological Fits					
Burbidge *et al.* (1957)	340			340	BW
Barker (1971)	50-330			50-330	*R*
Koonin *et al.* (1974)	80^{+50}_{-40}			80^{+50}_{-40}	H*R*
Dyer and Barnes (1974)	140^{+140}_{-40}			140^{+140}_{-40}	*R*&H*R*
Weisser *et al.* (1974)	170			170	*R*
Humblet *et al.* (1976)	80^{+140}_{-70}			80^{+140}_{-70}	*K*
Kettner *et al.* (1982)	250	180	12(2)[c,d]	420^{+160}_{-120}	BW
Langanke and Koonin (1983)	150 or 340	<4% of $E1$		150 or 340	H*R*
Barker (1987)	150^{+140}_{-60}	30^{+50}_{-30}			*R*
Kremer *et al.* (1988)	0-140				*R*&H*R*
Filippone *et al.* (1989)	0-170	5-28		0-170	*K*
Barker and Kajino (1991)	150^{+170}_{-70} or 260^{+140}_{-160}	120^{+60}_{-70}	10[c] 1-2[d]	280^{+230}_{-140} or 390^{+200}_{-230}	*R*
Humblet *et al.* (1991)	43^{+20}_{-16}	7^{+24}_{-5}		50^{+30}_{-20}	*K*
Humblet *et al.* (1993)	45^{+5}_{-6}				*K*
Azuma *et al.* (1994)	79±21 or 82±26				*R*&*K*
Buchmann *et al.* (1996)	79±21	70±70	16±16[a,c,d]	165±75	*R*&*K*
Hale (1997)	20				*R*
Trautvetter *et al.* (1997)	79	14.5			BW
Brune *et al.* (1999)	101±17	42^{+16}_{-23}			*R*
Roters *et al.* (1999)	79±21				*R*
Angulo and Descouvemont (2000)		190-220			*R*
Gialanella *et al.* (2001)	82±16 or 2.4±1.0				*R*
Kunz *et al.* (2001)	76±20	85±30	4±4[e]	165±50	*R*
Tischhauser *et al.* (2002)		53^{+13}_{-18}			*R*
Hammer *et al.* (2005b)	77±17	81±22		162±39	*R*
Buchmann and Barnes (2006)			$5^{+7}_{-4.5}$ [a] 7^{+13c}_{-4}		*R*
Matei *et al.* (2006)			25^{+16a}_{-15}		*R*
Matei *et al.* (2008)			7.1±1.6[c]		*R*
Tang *et al.* (2010)	86±22				*R*
Schürmann *et al.* (2011)			<1[a]		*R*
Schürmann *et al.* (2012)	83.4	73.4	4.4[e]	$161\pm19_{(\mathrm{stat})}{}^{+8}_{-2(\mathrm{syst})}$	*R*
Oulebsir *et al.* (2012)	100±28	50±19		175^{+63}_{-62}	*R*
Sayre *et al.* (2012)		62^{+9}_{-6}			*R*
Avila *et al.* (2015)			1.96±0.30 or 4.36±0.45[a] 0.12±0.04 or 1.44±0.12[b]		*R*
An *et al.* (2015)	98.0±7.0	56±4.1	8.7±1.8[e]	162.7±7.3	*R*
this work	86.3	45.3	7[e]	$140\pm21_{(\mathrm{MC})}{}^{+18}_{-11(\mathrm{model})}$	*R*

Nuclear theories

- Nuclear structure
  - -- Mean field theories
  - -- Ab initio calculations (with chiral nuclear forces)
- Nuclear reaction
  - -- Breit-Wigner formula
  - -- R-matrix theory
  - -- Potential models
  - -- Effective field theories

EFT for $^{12}C(\alpha,\gamma)^{16}O$ at $E_G = 0.3$ MeV

- A targe energy, $E_G = 0.3$ MeV

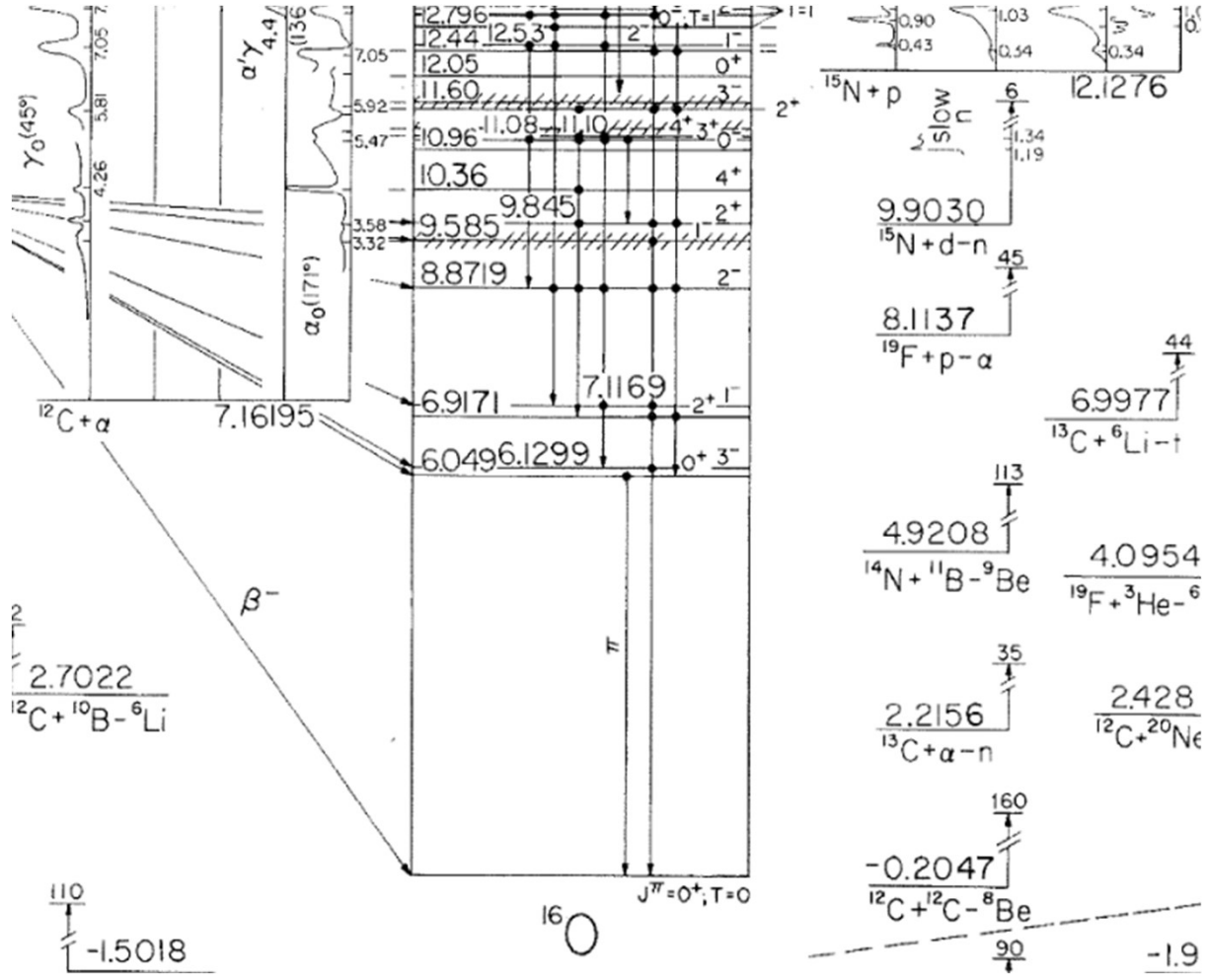
  its momentum scale, $Q = \sqrt{2\mu E_G} = 40$ MeV

  its length scale, $Q^{-1} = 4.8$ fm
- A separation scale, the energy difference of p-^{15}N and α-^{12}C

  $\Delta E = 12.13 - 7.16 = 4.97$ MeV

  its momentum scale, $\Lambda_H = \sqrt{2\mu\Delta E} = 160$ MeV
- The perturbative expansion parameter, $Q/\Lambda_H = 1/4$
- Construct the effective Lagrangian and calculate reaction amplitudes including Coulomb interaction non-perturbatively

Level diagram of ^{16}O



S matrices of the elastic scattering

- S matrices for I = 0,1,2,3,4,5,6,
- Accurate measurement reported by Tischhauser (2009)
- Bound and resonant states of ^{16}O

l	(Bound states)	2.6 MeV $\leq E_\alpha \leq$ 6.62 MeV,	6.62 MeV $< E_\alpha$
0	0_2^+	0_3^+	0_4^+
1	1_1^-	1_2^-	1_3^-
2	2_1^+	$2_2^+, 2_3^+$	2_4^+
3	3_1^-	3_2^-	3_3^-
4	–	$4_1^+, 4_2^+$	4_3^+
5	–	–	5_1^-
6	(bg)	–	6_1^+

- Effective range expansion

$$p\cot\delta_0 = -\frac{1}{a_0} + \frac{1}{2}r_0p^2 + \cdots,$$

S matrices and effective range expansion of the Just function

$$S_l - 1 = e^{2i\delta_l} - 1 = \frac{2ip}{p\cot\delta_l - ip}$$

$$S_l = 1 + (S_l - 1) = 1 + \frac{2ip}{p\cot\delta_l - ip} = \frac{p\cot\delta_l + ip}{p\cot\delta_l - ip}$$

$$= \frac{f_l(p)^*}{f_l(p)} = \frac{f_l(-p)}{f_l(p)} = \frac{-\frac{1}{a_l} + \frac{1}{2}r_l p^2 + ip}{-\frac{1}{a_l} + \frac{1}{2}r_l p^2 - ip}$$

$f_l(p)$ Jost function

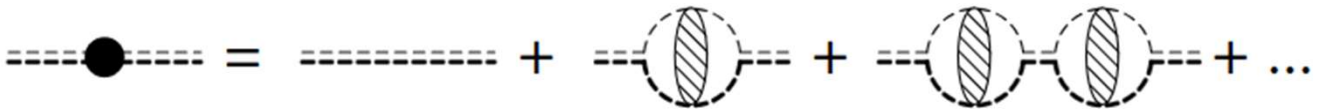
- We assume

$$S_l \longrightarrow S_l = \prod_i S_{l(i)}$$

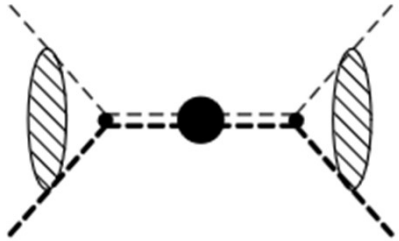
↑ S factors for i-th states for l which have different resonant energies.

Elastic α-^{12}C scattering at low energies

- Vertex functions and propagators from effective Lagrangian
- Feynman diagrams
  -- Dressed ^{16}O propagators



  -- Elastic scattering amplitudes



S matrices and scattering amplitudes

- Scattering amplitudes (bound states)

$$\tilde{A}_l^{(bs)} = \frac{C_\eta^2 W_l(p)}{K_l(p) - 2\kappa H_l(p)},$$

$$C_\eta^2 = \frac{2\pi\eta}{\exp(2\pi\eta) - 1},$$

$$W_l(p) = \left(\frac{\kappa^2}{l^2} + p^2\right) W_{l-1}(p), \quad W_0(p) = 1,$$

$$H_l(p) = W_l(p) H(\eta), \quad H(\eta) = \psi(i\eta) + \frac{1}{2i\eta} - \log(i\eta),$$

$$K_l(p) = -\frac{1}{a_l} + \frac{1}{2} r_l p^2 - \frac{1}{4} P_l p^4 + Q_l p^6 - R_l p^8,$$

S matrices and scattering amplitudes

- Scattering amplitudes (resonant states)

$$\tilde{A}_l^{(rsN)} = -\frac{1}{p}\frac{\frac{1}{2}\Gamma_{(li)}(E)}{E - E_{R(li)} + R_{(li)}(E) + i\frac{1}{2}\Gamma_{(li)}(E)},$$

$$\Gamma_{(li)}(E) = \Gamma_{R(li)}\frac{pC_\eta^2 W_l(p)}{p_r C_{\eta_r}^2 W_l(p_r)},$$

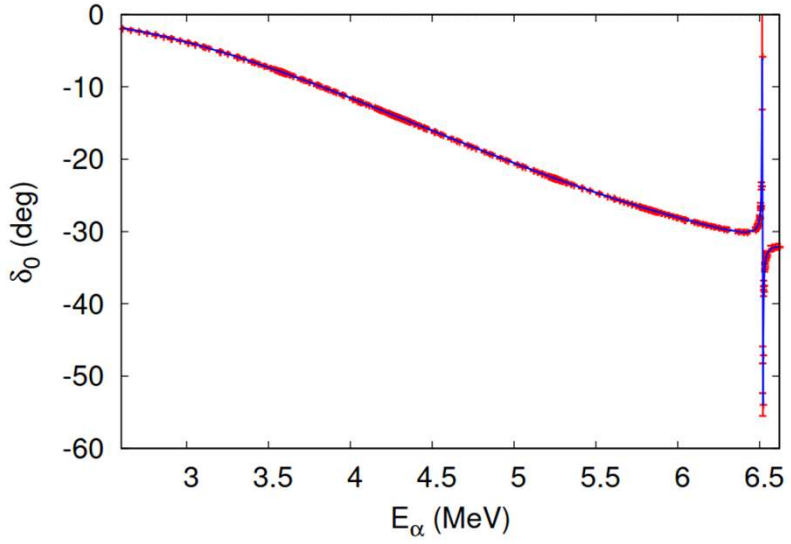
$$R_{(li)}(E) = a_{(li)}(E - E_{R(li)})^2 + b_{(li)}(E - E_{R(li)})^3,$$

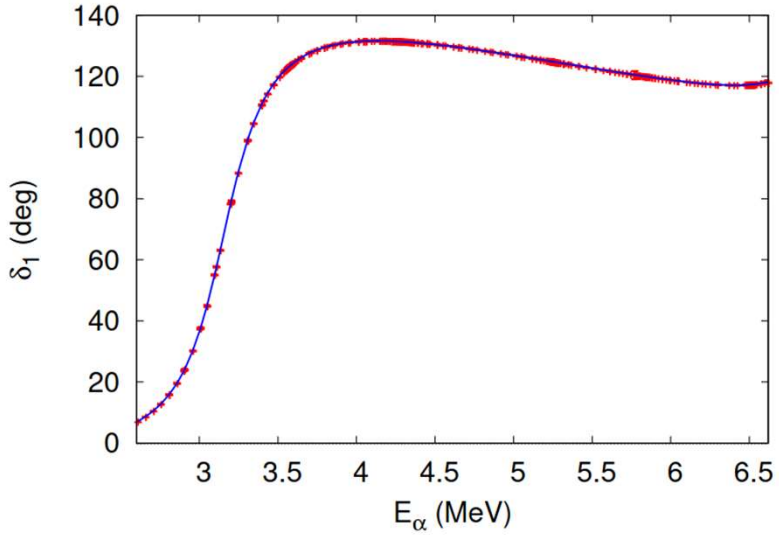
- S matrices

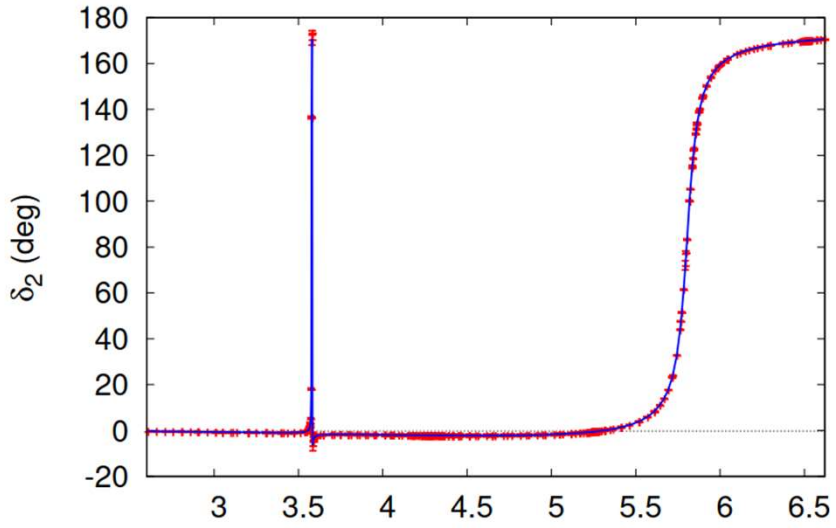
$$e^{2i\delta_l} = \frac{K_l(p) - 2\kappa ReH_l(p) + ipC_\eta^2 W_l(p)}{K_l(p) - 2\kappa ReH_l(p) - ipC_\eta^2 W_l(p)} \prod_i \frac{E - E_{R(li)} + R_{(li)}(E) - i\frac{1}{2}\Gamma_{(li)}(E)}{E - E_{R(li)} + R_{(li)}(E) + i\frac{1}{2}\Gamma_{(li)}(E)},$$

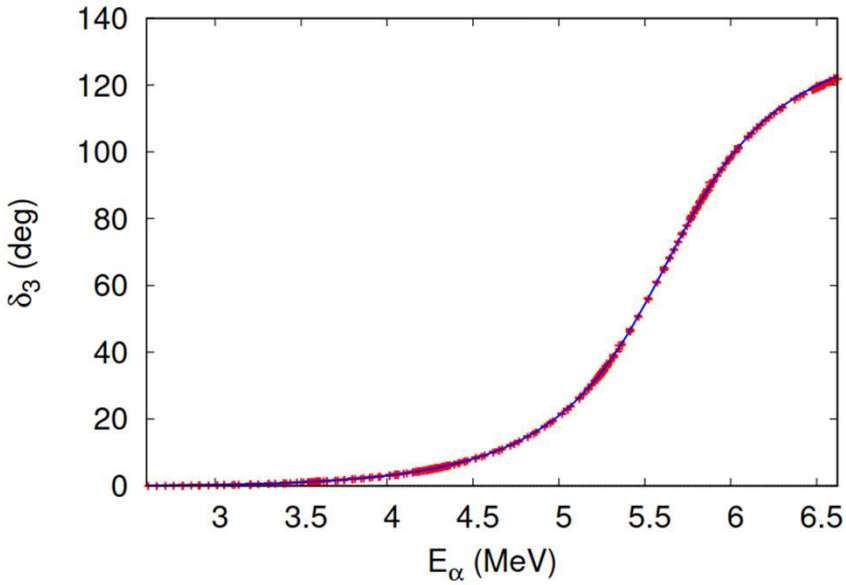
Formalism of the elastic scattering

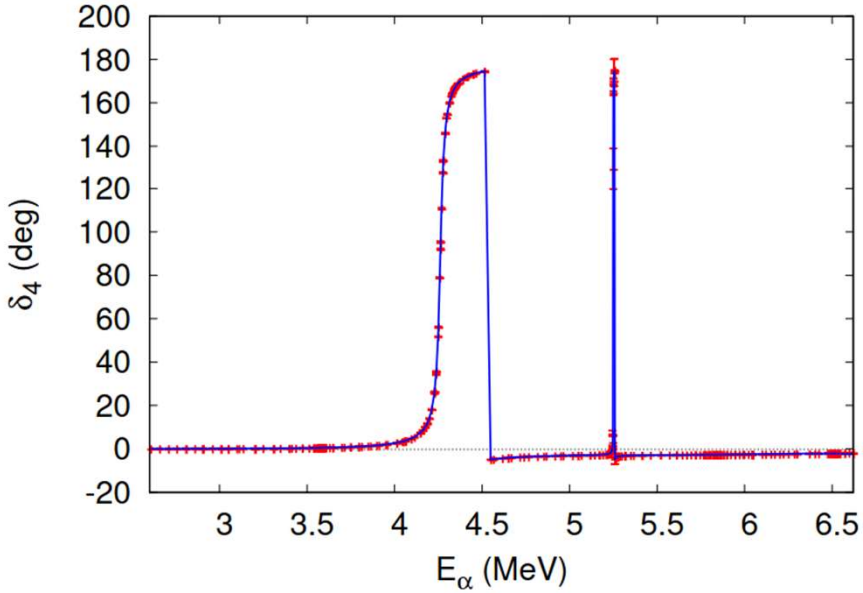
- S matrices for the elastic scattering for l=0,1,2,3,4,5,6
- 4 parameters for each bound and resonant states of ^{16}O
- Parameters fitted to the accurate phase shift data up to the p-^{15}N breakup reported by Tishchhauser et al. (2009).

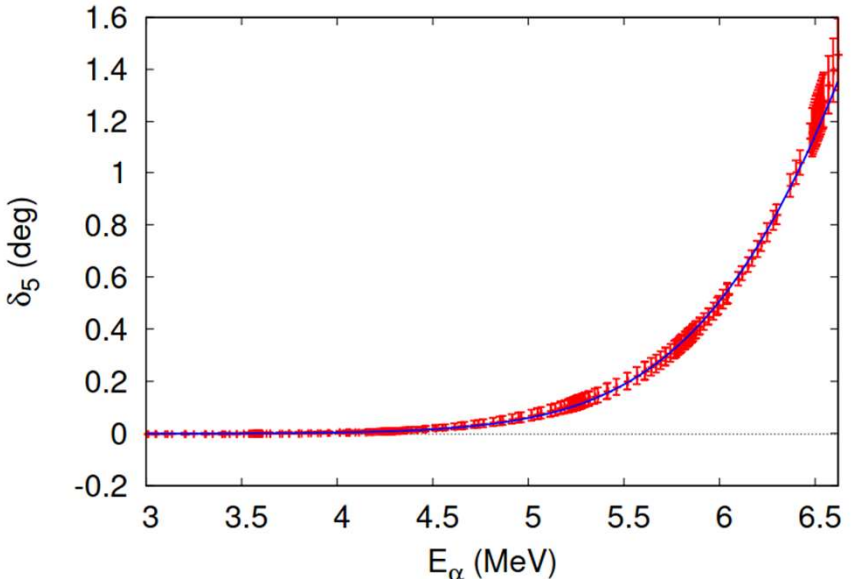





δ2 (deg)
180
160
140
120
100
80
60
40
20
0
-20
3
3.5
4
4.5
5
5.5
6
6.5

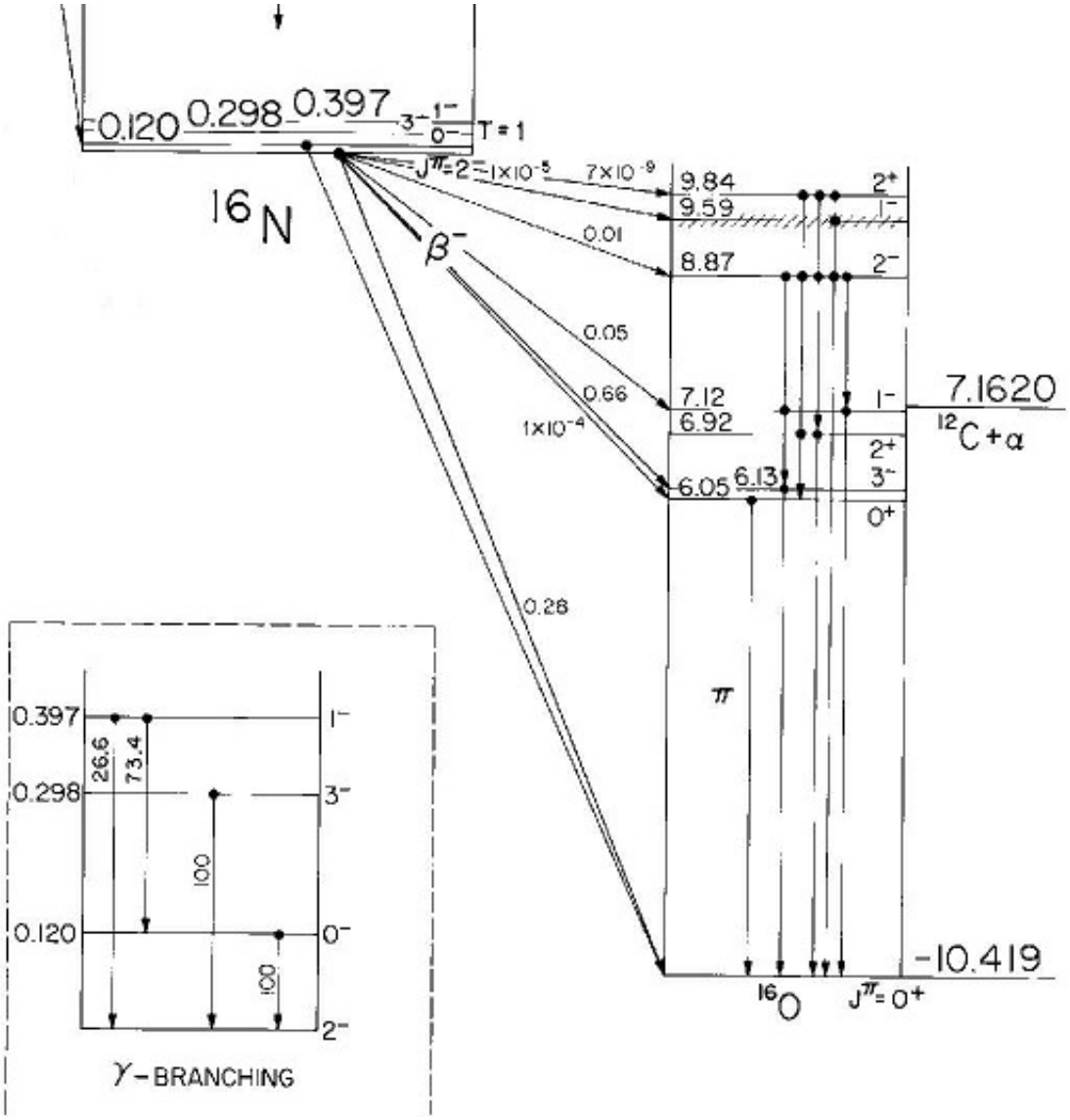

δ3 (deg)
140
120
100
80
60
40
20
0
3
3.5
4
4.5
5
5.5
6
6.5
Eα (MeV)


δ4 (deg)
200
180
160
140
120
100
80
60
40
20
0
-20
3
3.5
4
4.5
5
5.5
6
6.5
Eα (MeV)


δ5 (deg)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
3
3.5
4
4.5
5
5.5
6
6.5
Eα (MeV)

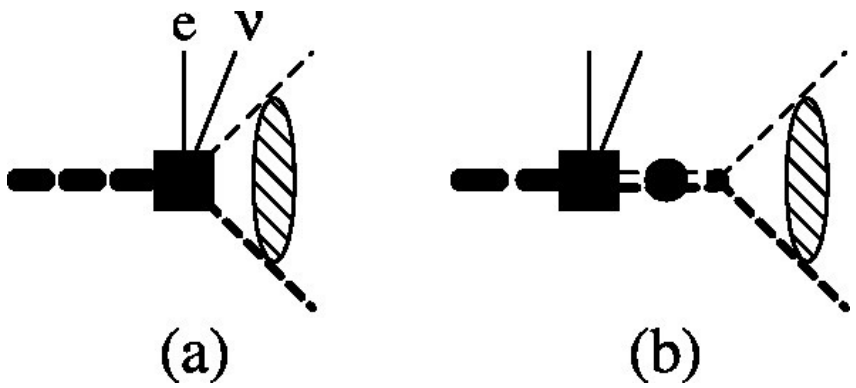
β delayed α emission from ^{16}N

- Beta decay from nitrogen-16
  Nitrogen-14,15: stable nuclei
  Nitrogen-16 decays through Gamow-Teller transition
  from 2^- ground state to some states of oxygen-16.
  half-lifetime 7.13 ± 0.02 sec
- Beta delayed alpha emission from nitrogen-16
  $^{16}N \rightarrow e^- + \nu + \alpha + {}^{12}C$
  branching ratio, $(1.59 \pm 0.06) \times 10^{-5}$
  p- and f-waves dominant for alpha and carbon-12 system


0.120
0.298
0.397
3⁻
1⁻
0⁻
T = 1
16N
β⁻
Jπ=2⁻
1×10⁻⁵
7×10⁻⁹
9.84
9.59
2⁺
1⁻
0.01
8.87
2⁻
0.05
0.66
1×10⁻⁴
7.12
6.92
1⁻
7.1620
12C+α
2⁺
3⁻
6.05
6.13
0⁺
0.28
π
-10.419
16O
Jπ= 0⁺
0.397
0.298
0.120
1⁻
3⁻
0⁻
2⁻
26.6
73.4
100
100
γ-BRANCHING

Formalism

- Diagrams



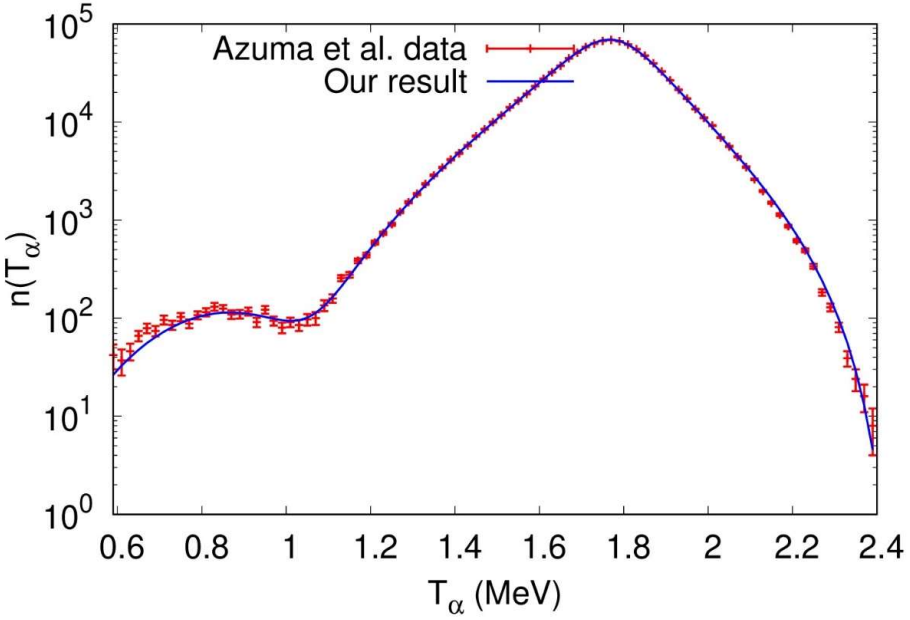
- Reduced amplitudes

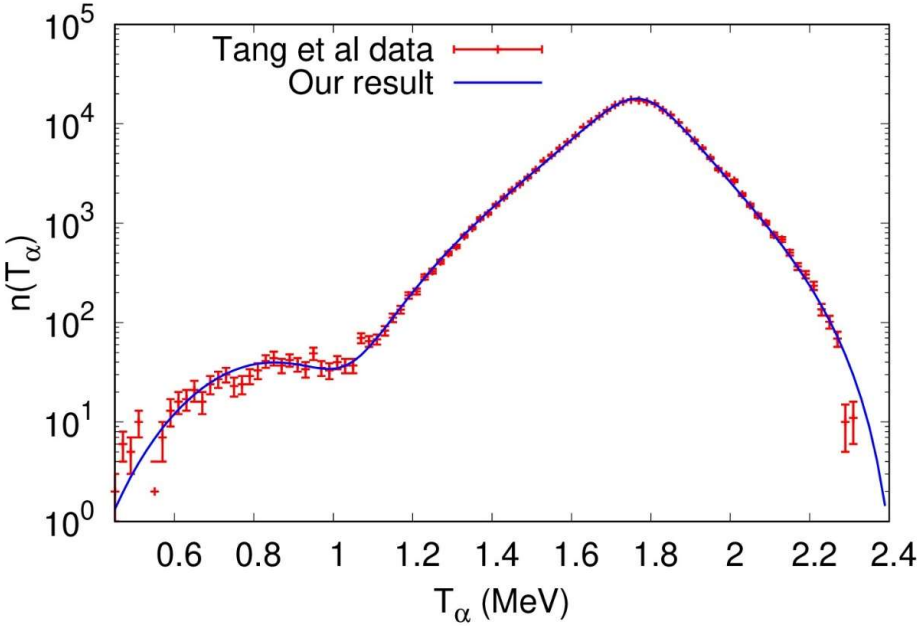
$$\tilde{A}_1 = C_a^{(l=1)} + D_a^{(l=1)} \frac{p^2}{\mu^2} + \frac{C_b^{(l=1)} + D_b^{(l=1)} \frac{p^2}{m_O^2}}{K_1(p) - 2\kappa H_1(p)},$$

$$\tilde{A}_3 = C_a^{(l=3)} + \frac{C_b^{(l=3)}}{K_3(p) - 2\kappa H_3(p)},$$

Numerical results

- Beta delayed alpha energy distributions from nitrogen-16





- Fitted parameters

Exp. data set	Azuma *et al.*	Tang *et al.*
C (MeV^{-6})	$7.2(4) \times 10^{6}$	$4.22(7) \times 10^{6}$
$C_a^{(l=1)}$ (MeV^{-2})	$-6.9(2) \times 10^{-3}$	$-9.46(5) \times 10^{-3}$
$D_a^{(l=1)}$ (MeV^{-2})	$2.61(9)$	$3.36(3)$
$D_b^{(l=1)}$ (MeV)	$-2.55(2) \times 10^{5}$	$-2.297(4) \times 10^{5}$
$C_b^{(l=3)}$ (MeV^{3})	$-2.65(5) \times 10^{-7}$	$-2.46(1) \times 10^{-7}$
χ^2/N (N)	4.06 (91)	3.56 (93)

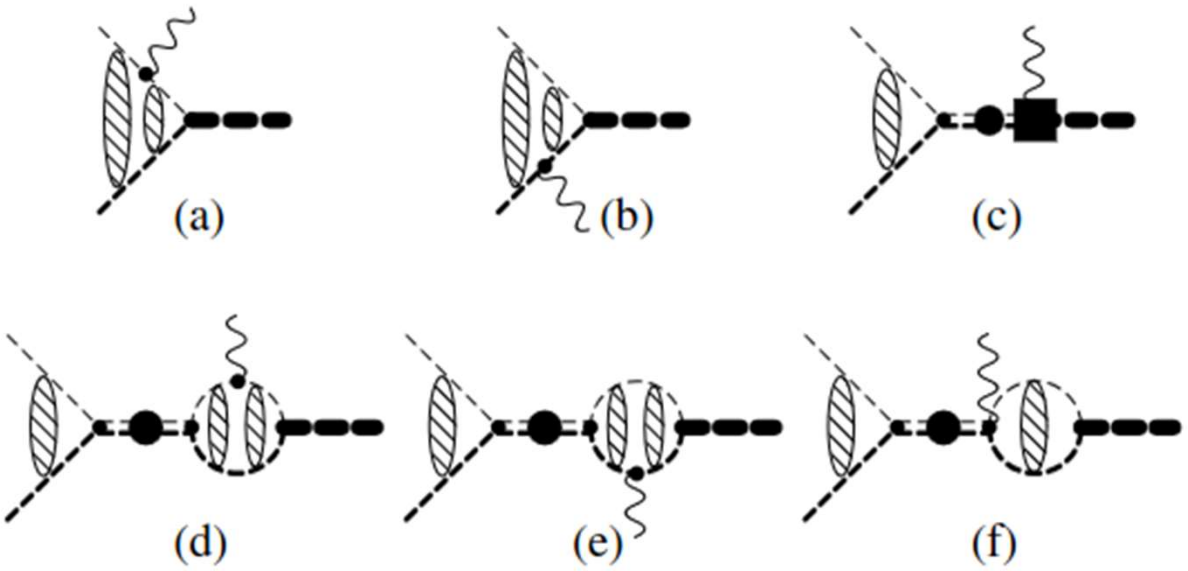
where $C_b^{(l=1)}$ and $C_b^{(l=3)}$ are fixed by branching ratio of the beta decay.

E1 and E2 transitions of $^{12}C(\alpha,\gamma)^{16}O$

- Some issues:
- E1 transition is suppressed between the iso-singlet states of initial and final states.
- Asymptotic normalization coefficient of the sub-threshold 2_1^+ state deduced from the effective range parameters of elastic scattering is significantly smaller than those deduced the transfer reactions.
- Large cancellations between the counter terms and loop contributions.

E1 transition of $^{12}C(\alpha,\gamma)^{16}O$

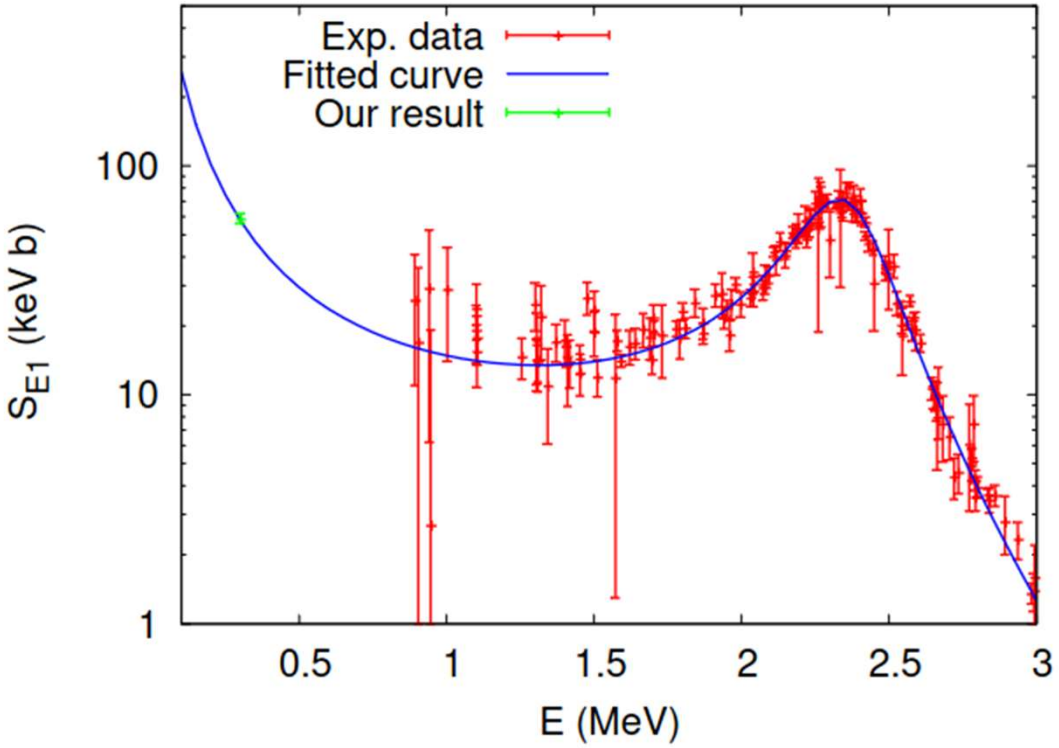
- Feynman diagrams
  -- Five parameters
  ERPs r_1, P_1, Q_1,
  an ANC $y^{(0)}$,
  counter term $h_R^{(1)}$
  -- $y^{(0)}$ and $h_R^{(1)}$ are fitted to the data of S_{E1} factor



E1 transition amplitudes

$$
\begin{aligned}
X^{(l=1)}_{(a+b)} &= 2y^{(0)}e^{i\sigma_1}\Gamma(1+\kappa/\gamma_0) \\
&\quad \times \int_0^\infty dr r W_{-\kappa/\gamma_0,\frac{1}{2}}(2\gamma_0 r)\left[\frac{Z_\alpha \mu}{m_\alpha} j_0\left(\frac{\mu}{m_\alpha}k' r\right) - \frac{Z_C \mu}{m_C} j_0\left(\frac{\mu}{m_C}k' r\right)\right] \\
&\quad \times \left\{\frac{\partial}{\partial r}\left[\frac{F_1(\eta, pr)}{pr}\right] + 2\frac{F_1(\eta, pr)}{pr^2}\right\}, \\
X^{(l=1)}_{(c)} &= +y^{(0)}h^{(1)R}\frac{6\pi Z_O}{\mu m_O}\frac{e^{i\sigma_1}p\sqrt{1+\eta^2}C_\eta}{K_1(p)-2\kappa H_1(p)}, \\
X^{(l=1)}_{(d+e)} &= +i\frac{2}{3}y^{(0)}\frac{e^{i\sigma_1}p^2\sqrt{1+\eta^2}C_\eta}{K_1(p)-2\kappa H_1(p)}\Gamma(1+\kappa/\gamma_0)\Gamma(2+i\eta) \\
&\quad \times \int_{r_C}^\infty dr r W_{-\kappa/\gamma_0,\frac{1}{2}}(2\gamma_0 r)\left[\frac{Z_\alpha \mu}{m_\alpha} j_0\left(\frac{\mu}{m_\alpha}k' r\right) - \frac{Z_C \mu}{m_C} j_0\left(\frac{\mu}{m_C}k' r\right)\right] \\
&\quad \times \left\{\frac{\partial}{\partial r}\left[\frac{W_{-i\eta,\frac{3}{2}}(-2ipr)}{r}\right] + 2\frac{W_{-i\eta,\frac{3}{2}}(-2ipr)}{r^2}\right\}, \\
X^{(l=1)}_{(f)} &= -3y^{(0)}\mu\left[-2\kappa H(\eta_{b0})\right]\left(\frac{Z_\alpha}{m_\alpha} - \frac{Z_C}{m_C}\right)\frac{e^{i\sigma_1}p\sqrt{1+\eta^2}C_\eta}{K_1(p)-2\kappa H_1(p)},
\end{aligned}
$$

S_{E1} factor of $^{12}C(\alpha,\gamma)^{16}O$ at E_G



$h_R^{(1)} = 2.67 \times 10^4\ MeV^3$

$y^{(0)} = 0.260\ MeV^{-1/2}$

$\chi^2/N = 1.73$

$S_{E1} = 59 \pm 2\ keVb$ at E_G

ANC of subthreshold 2_1^+ state of ^{16}O

- A problem
  -- Effective range parameters from elastic scattering for l = 2

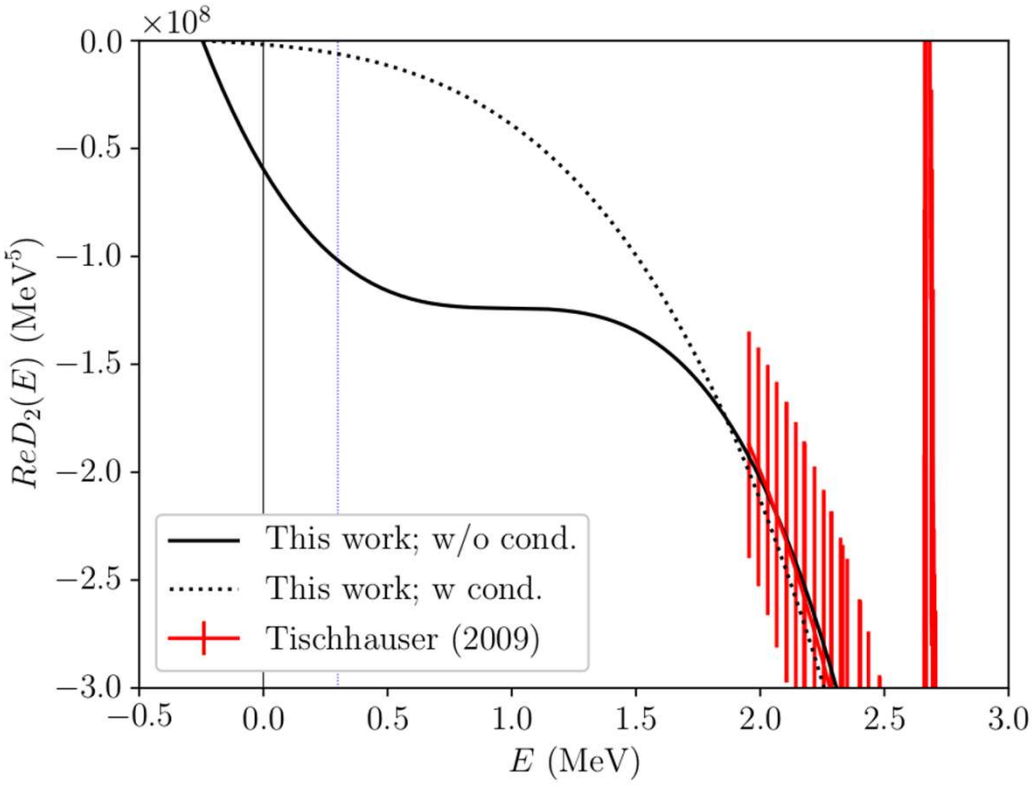
  $|C_b|_2 = (2.41 \pm 0.38) \times 10^4\ fm^{-1/2}$

  -- Deduced from the α-transfer reactions, $^{12}C(^6Li, d)^{16}O$

  $|C_b|_2 = (11.1 \pm 1.1) \times 10^4\ fm^{-1/2}$

  -- About a factor of five difference

Inverse of dressed ^{16}O propagator, $D_2(E)$



-- Two ANCs

$|C_b|_2 = 3.24 \times 10^4\ fm^{-1/2}$

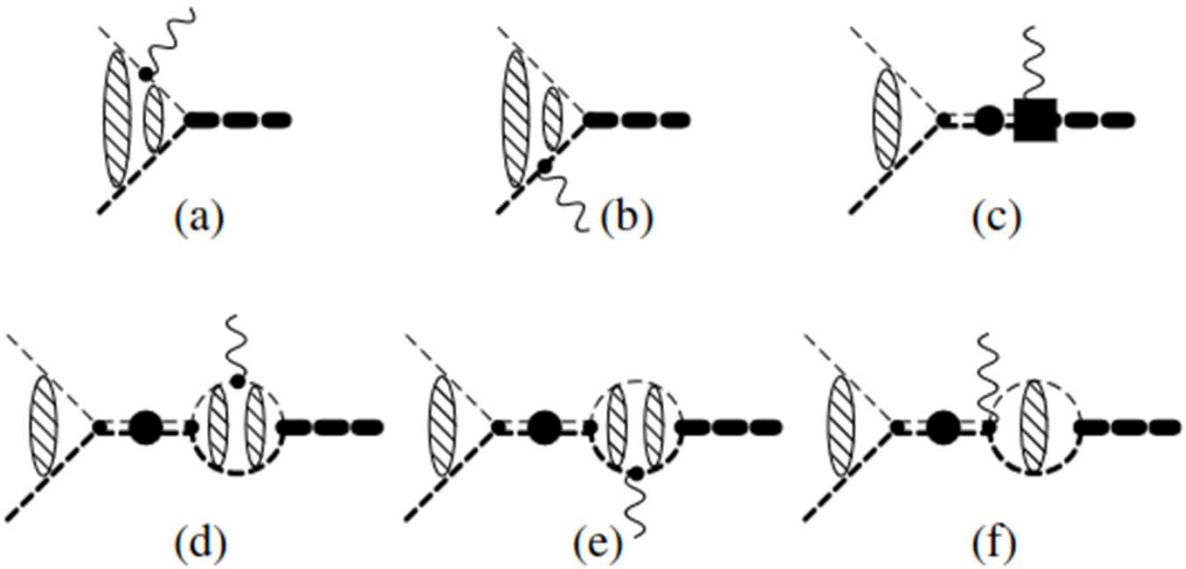
$|C_b|_2 = 22.8 \times 10^4\ fm^{-1/2}$

-- Both the lines reproduce the phase data for I=2 well.

-- The phase shift data cannot determine the ANC of 2_1^+ state of ^{16}O

E2 transition of $^{12}C(\alpha,\gamma)^{16}O$

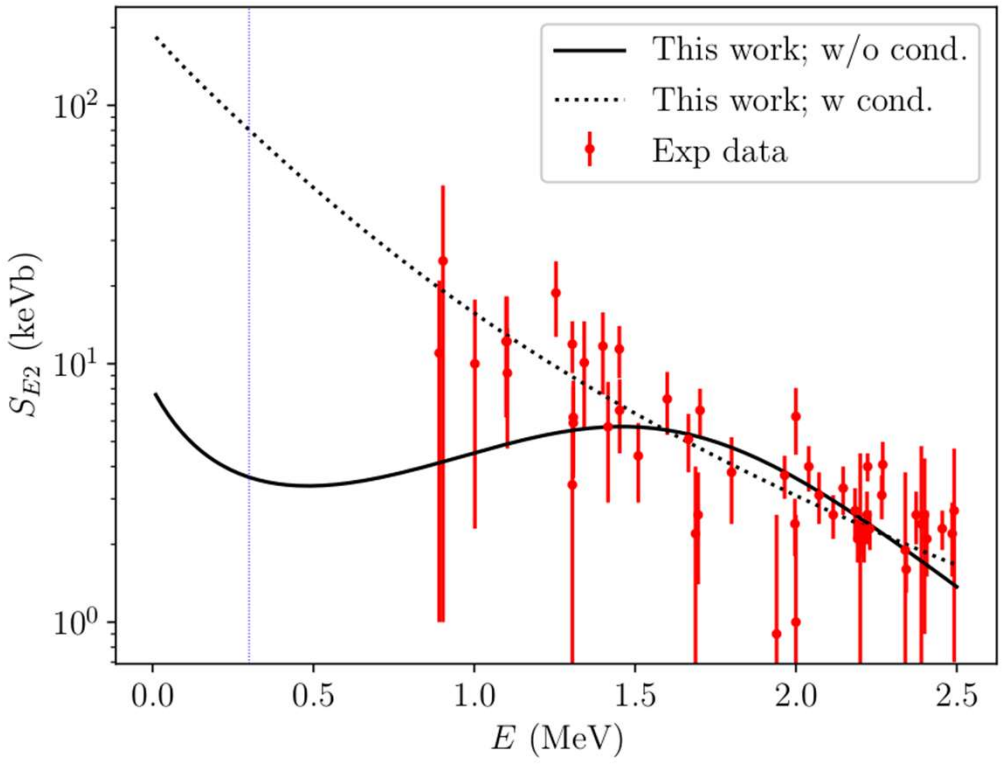
- Feynman diagrams
  -- Five parameters
  ERPs r_2, P_2, Q_2,
  an ANC $y^{(0)}$,
  counter term $h_R^{(2)}$
  -- $y^{(0)}$ and $h_R^{(2)}$ are fitted to the data of S_{E2} factor



E2 transition amplitudes

$$
\begin{aligned}
X^{(l=2)}_{(a+b)} &= -6y^{(0)}e^{i\sigma_2}\Gamma(1+\kappa/\gamma_0) \\
&\times \int_0^\infty dr r W_{-\kappa/\gamma_0,\frac{1}{2}}(2\gamma_0 r)\left[\frac{Z_\alpha \mu}{m_\alpha} j_1\left(\frac{\mu}{m_\alpha}k' r\right) + \frac{Z_C \mu}{m_C} j_1\left(\frac{\mu}{m_C}k' r\right)\right] \\
&\times \left(\frac{\partial}{\partial r}+\frac{3}{r}\right)\frac{F_2(\eta, pr)}{pr}, \qquad (51) \\
X^{(l=2)}_{(c)} &= +y^{(0)}\left\{-h^{(2)}_R + \frac{3\kappa\mu^3 m_O^2}{2\pi Z_O}\left(\frac{Z_\alpha}{m_\alpha^2}+\frac{Z_C}{m_C^2}\right)\left[\frac{4}{225}\ln\left(\frac{\mu_{DR}}{2}r_C\right) - \ln\left(\frac{\mu_{DR}}{\kappa}\right)\right]\right\} \\
&\times \frac{5\pi Z_O}{\mu m_O^2}\frac{e^{i\sigma_2}k' p^2\sqrt{(1+\eta^2)(4+\eta^2)}C_\eta}{K_2(p)-2\kappa H_2(p)}, \qquad (52) \\
X^{(l=2)}_{(d+e)} &= +\frac{1}{5}y^{(0)}\frac{e^{i\sigma_2}p^4\sqrt{(1+\eta^2)(4+\eta^2)}C_\eta}{K_2(p)-2\kappa H_2(p)}\Gamma(1+\kappa/\gamma_0)\Gamma(3+i\eta) \\
&\times \int_{r_C}^\infty dr r W_{-\kappa/\gamma_0,\frac{1}{2}}(2\gamma_0 r)\left[\frac{Z_\alpha \mu}{m_\alpha} j_1\left(\frac{\mu}{m_\alpha}k' r\right) + \frac{Z_C \mu}{m_C} j_1\left(\frac{\mu}{m_C}k' r\right)\right] \\
&\times \left(\frac{\partial}{\partial r}+\frac{3}{r}\right)\frac{W_{-i\eta,\frac{5}{2}}(-2ipr)}{r}, \qquad (53) \\
X^{(l=2)}_{(f)} &= -\frac{15}{4}y^{(0)}\mu^2\left(\frac{Z_\alpha}{m_\alpha^2}+\frac{Z_C}{m_C^2}\right)[-2\kappa H(\eta_{0b})]\frac{e^{i\sigma_2}k' p^2\sqrt{(1+\eta^2)(4+\eta^2)}C_\eta}{K_2(p)-2\kappa H_2(p)}, \qquad (54)
\end{aligned}
$$

S_{E2} factor of $^{12}C(\alpha,\gamma)^{16}O$ at E_G



-- Small ANC

$S_{E2} = 3.6^{+0.3}_{-0.6}\ keVb$ at E_G

$\chi^2/N = 1.55$

-- Large ANC

$S_{E2} = 81^{+19}_{-9}\ keVb$ at E_G

$\chi^2/N = 1.25$

-- ANC fixed by Transfer reactions

$S_{E2} = 42^{+14}_{-13}\ keVb$ at E_G

$\chi^2/N = 1.18$

Results and discussion

- We obtained the S factor at E_G, $S_{E1+E2} = 101 \pm 14$ keV b.
- In the fit, a small error bar and a large χ^2 value for S_{E1}, and a large error bar and a small χ^2 value for S_{E2}
- Different fitted values of $y^{(0)}$ in the E1 and E2 amplitudes, the ANC of the ground state of ^{16}O, the αCO vertex function
- The large cancelations between the counter terms, $h_R^{(1)}$ and $h_R^{(2)}$, and the contributions from the loops are observed.
- New data will be taken by JUNA in the future.