

2025北大加速器

加速器的反馈控制

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前馈控制：射箭——设定初始参数和初值



反馈控制：导弹——不断根据与目标的偏离调整状态

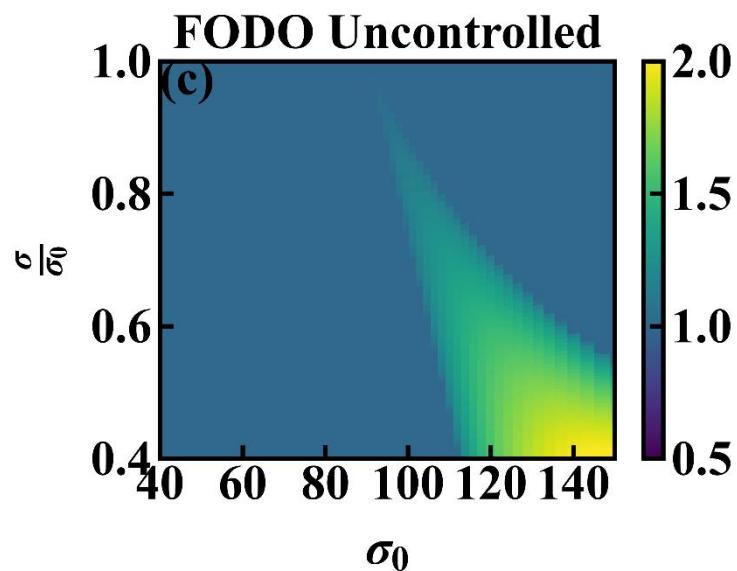
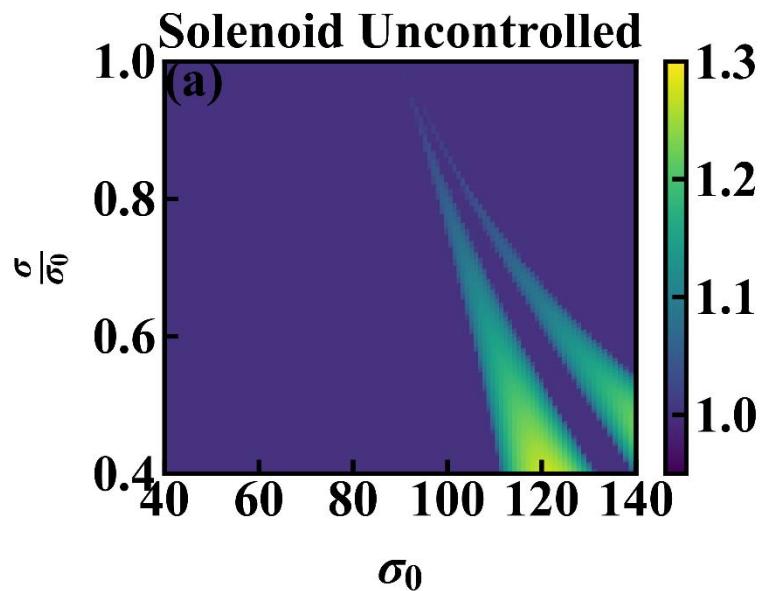


前馈控制：现阶段的加速器

束流包络动力学 Kapchinsky-Vladimirsky 分布理论

$$\frac{d^2}{ds^2} r_x(s) + k_x(s) r_x(s) - \frac{2Q}{r_x(s) + r_y(s)} - \frac{\varepsilon_x^2}{r_x^3(s)} = 0$$

$$\frac{d^2}{ds^2} r_y(s) + k_y(s) r_y(s) - \frac{2Q}{r_x(s) + r_y(s)} - \frac{\varepsilon_y^2}{r_y^3(s)} = 0$$



工程控制论中的极点配置方法

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$u(t) = -Kx(t) + r(t)$$

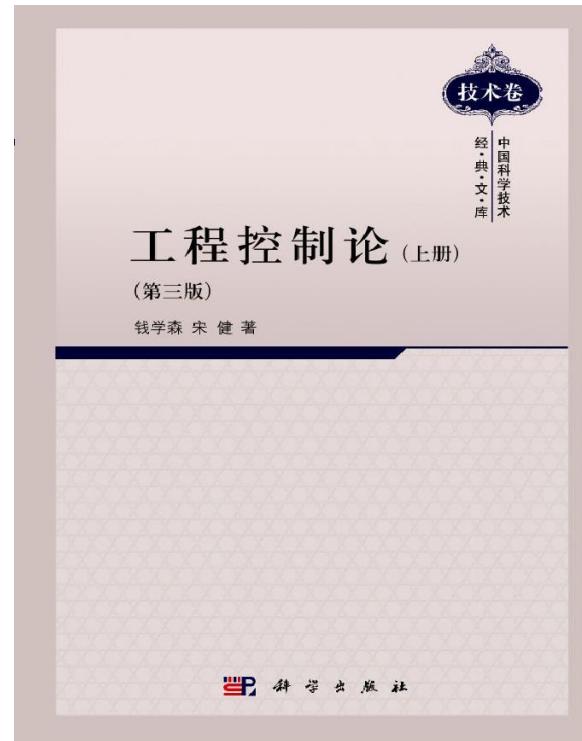
闭环系统

$$\dot{x}(t) = (A - BK)x(t) + Br(t)$$

$A - BK$ 的特征值决定稳定性

极点配置方法的目标是通过选择适当的反馈增益矩阵 K ，使得系统的特征值位于期望的闭环极点位置

极点配置方法广泛应用于机器人控制、飞行控制、航天器轨迹控制、汽车稳定性控制等领域。



反馈控制用于加速器的两个目标：

- (1) 保证稳定参数区在扰动或参数漂移下稳态，或束流弥散度更小
- (2) 不稳定区稳定化，拓广应用参数区

动力学系统的周期运动与混沌运动

以Lorenz系统为例

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = rx - y - xz \\ \dot{z} = xy - bz \end{cases}$$

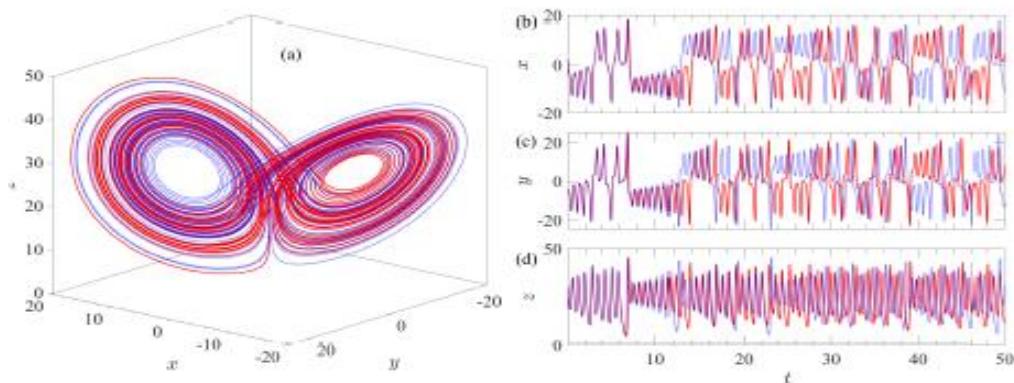
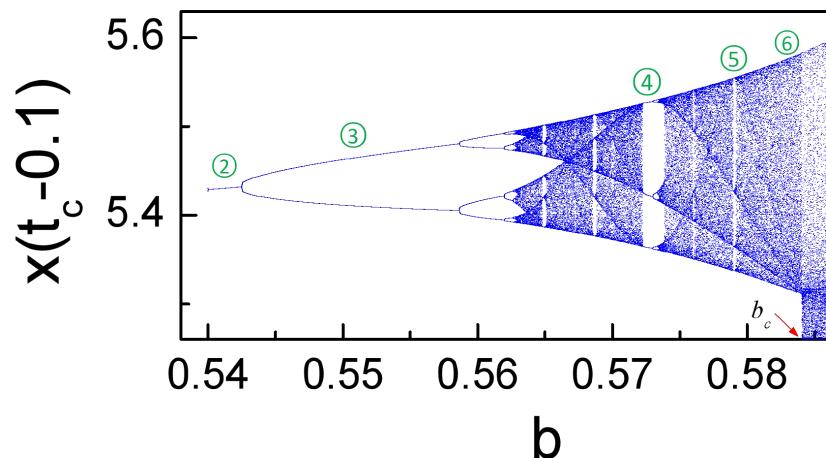


FIG. 1. $\sigma = 10$, $r = 28$, and $\beta = 8/3$



混沌控制

动力学映射：

$$x_{n+1} = F(x_n, p)$$

$$J = \frac{\partial F(x_n, p)}{\partial x_n} \Big|_{x_*}$$

稳定性：J有绝对值大于1的本征值，则不稳定

OGY混沌控制理论 (1990)：

令 $p_n = p_0 + \varepsilon \cdot (x_n - x_*)$

$$J = \frac{\partial F(x_n, \varepsilon \cdot (x_n - x_*))}{\partial x_n} \Big|_{x_*}$$

适当选择 ε 使得新的雅可比矩阵特征值绝对值都小于1

因此传统的混沌控制理论等价于极点配置方法

我们发展的混沌控制理论 (1996) :

令 $p_{n+1} = p_0 + \varepsilon \cdot (x_{n+1} - x_n) + \varepsilon_{N+1}(p_n - p_0)$

$$J = \begin{bmatrix} \frac{\partial F(x_n, p_n)}{\partial x_n} & \frac{\partial p_{n+1}}{\partial x_n} \\ \frac{\partial F(x_n, p_n)}{\partial p_n} & \frac{\partial p_{n+1}}{\partial p_n} \end{bmatrix} (x_*, p_0)$$

H. Zhao, J. Yan, J. Wang and Y.H.Wang, General method of controlling chaos, Phys. Rev. E 53, 299(1996).

H. Zhao, Y.H. Wang and Z.B.Zhang, Extended pole placement technique and its applications for targeting unstable periodic orbit, Phys. Rev. E 57, 5358(1998).

H. Zhao, Y. H. Wang and J. Yan, Stabilizing unstable orbits, Phys.Lett. A 236,53(1997)

Y.W. Liu, H. Zhao and Y.H. Wang, Controlling the Unstable State in an Optical Bistable System, ACTA Physica Sinica 48, 198(1999)

控制函数扩展为非线性控制函数

$$\begin{aligned}\mathbf{z}_{i+1} &= \mathbf{f}(\mathbf{z}_i, p_{i-r}, p_{i-r+1}, \dots, p_{i-1}, p_i), \\ p_{i-r+1} &= p_{i-r+1}, \\ &\vdots\end{aligned}\tag{2.1}$$

$$p_{i-1} = p_{i-1},$$

$$p_i = p_i,$$

$$p_{i+1} = g(\mathbf{z}_i, p_{i-r}, p_{i-r+1}, \dots, p_{i-1}, p_i),$$

when the parameter p is activated. The Jacobian matrix of this map is

$$\mathbf{T} = \begin{pmatrix} \mathbf{D}_z \mathbf{f} & \mathbf{D}_{p_{i-r}} \mathbf{f} & \mathbf{D}_{p_{i-r+1}} \mathbf{f} & \dots & \mathbf{D}_{p_{i-1}} \mathbf{f} & \mathbf{D}_{p_i} \mathbf{f} \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ \mathbf{D}_z g & \frac{\partial g}{\partial p_{i-r}} & \frac{\partial g}{\partial p_{i-r+1}} & \dots & \frac{\partial g}{\partial p_{i-1}} & \frac{\partial g}{\partial p_i} \end{pmatrix}, \tag{2.2}$$

求解：我们推广的极点配置方法求

控制函数扩展为动力系统方程未知的时间序列控制

时间序列相空间重构控制（一维序列为例）

$$x_{n+1} = f(x_n, x_{n-1}, \dots, p_n, p_{n-1}, \dots)$$

$$p_{n+1} = \varepsilon_1(x_{n+1} - x_n) + \varepsilon_1(x_n - x_{n-1}) + \dots + \varepsilon_{2d+1}(p_n - p_0) + \varepsilon_{2d+2}(p_{n-1} - p_0) + \dots$$

同样可以按照我们推广的极点配置方法求出控制参数

加速器束流控制

K-V分布下，束流包络方程：

$$\frac{d^2}{ds^2} r_x(s) + k_x(s) r_x(s) - \frac{2Q}{r_x(s) + r_y(s)} - \frac{\varepsilon_x^2}{r_x^3(s)} = 0$$

$$\frac{d^2}{ds^2} r_y(s) + k_y(s) r_y(s) - \frac{2Q}{r_x(s) + r_y(s)} - \frac{\varepsilon_y^2}{r_y^3(s)} = 0$$

转换成映射： $\mathbf{R}_{i+1} = f(\mathbf{R}_i, K(s), \Gamma)$

$$\mathbf{R}_* = f(\mathbf{R}_*, K, \Gamma)$$

加速器情景下的四种控制函数

$$K_{n+1} = K_0 + a * (R_{n+1} - R_0)$$

$$K_{n+1} = K_0 + a * (R_{n+1} - R_n) + b(K_n - K_0)$$

$$K_{n+1} = K_0 + a * (R_n - R_0) + b(K_n - K_0)$$

$$K_{n+1} = K_0 + a * (R_n - R_{n-1}) + b(K_{n-1} - K_0) + c(K_n - K_0)$$

推广的极点配置方法求解

$$\lambda^m + c_{m-1}\lambda^{m-1} + \cdots + c_1\lambda + c_0 = 0 \quad (19)$$

记我们要配置到的极点为 $\lambda_{target,i}$, 则有

$$\lambda^m + c_{target,m-1}\lambda^{m-1} + \cdots + c_{target,1}\lambda + c_{target,0} = \prod_{i=1}^m (\lambda - \lambda_{target,i}) \quad (20)$$

特征多项式系数对控制参数的灵敏度通过灵敏度矩阵描述:

$$S = \frac{\partial \mathbf{c}}{\partial \mathbf{p}} = \begin{bmatrix} \frac{\partial c_{m-1}}{\partial p_1} & \cdots & \frac{\partial c_{m-1}}{\partial p_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial c_0}{\partial p_1} & \cdots & \frac{\partial c_0}{\partial p_n} \end{bmatrix} \quad (21)$$

对于 EFRC 模式, 控制参数 $\mathbf{p} = [a_1, a_2, a_3, a_4, b]^T$, 灵敏度矩阵 $S \in \mathbb{R}^{5 \times 5}$ 。

由于未知参数只出现在矩阵的最后一行, 因此特征方程系数 c 对于 p 的依赖是线性的, 因此有:

$$S \cdot \Delta \mathbf{p} = \Delta \mathbf{c} \quad (22)$$

其中 $\Delta \mathbf{c} = \mathbf{c}_{target} - \mathbf{c}_{current}$ 为特征多项式系数偏差。

采用正则化最小二乘法求解控制参数:

$$\min_{\mathbf{p}} \|S\mathbf{p} - (\mathbf{c}_{target} - \mathbf{c}_0)\|^2 + \lambda \|\mathbf{p}\|^2 \quad (23)$$

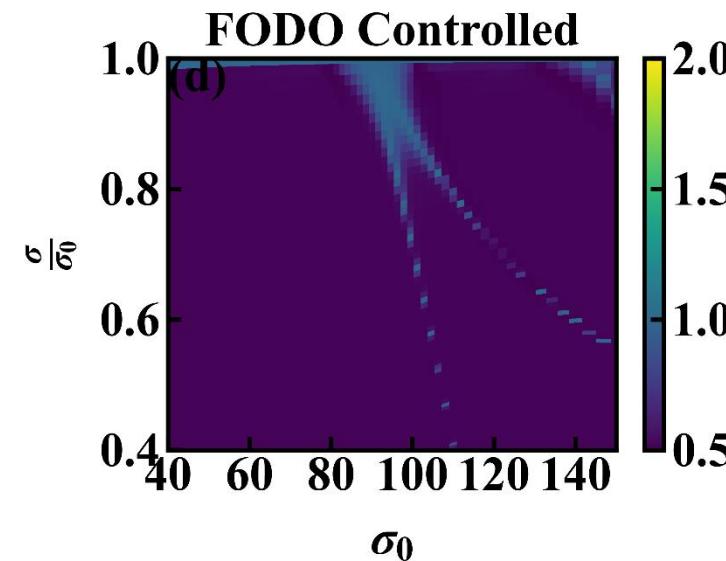
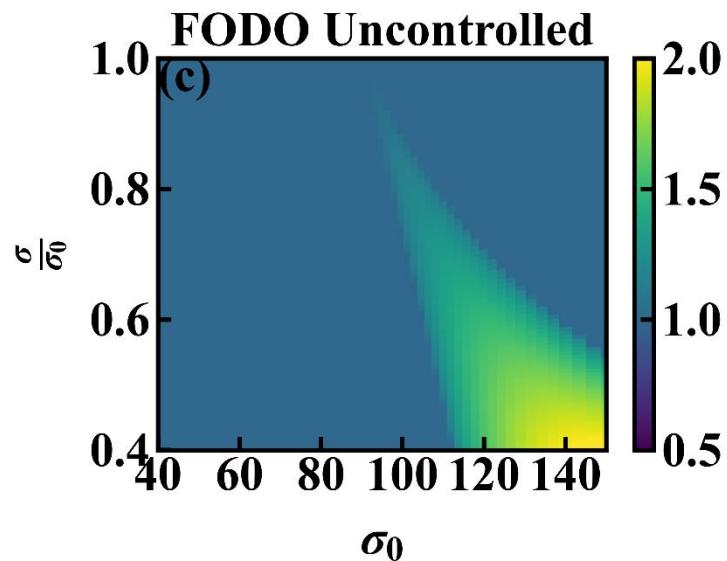
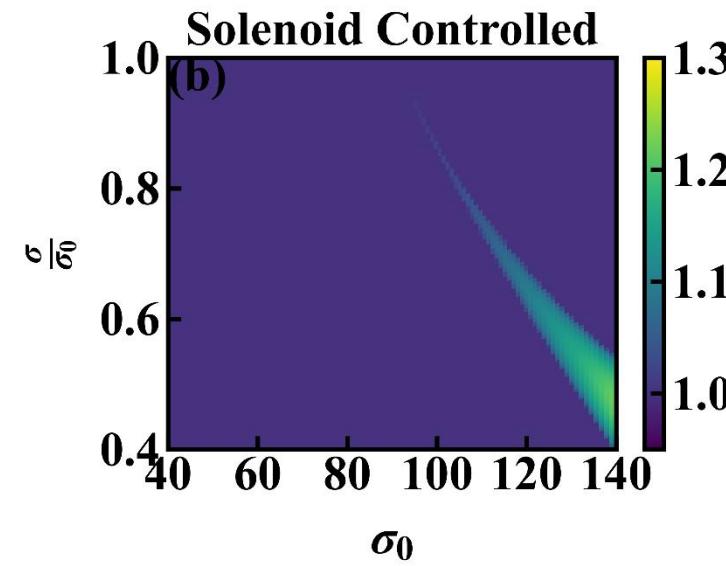
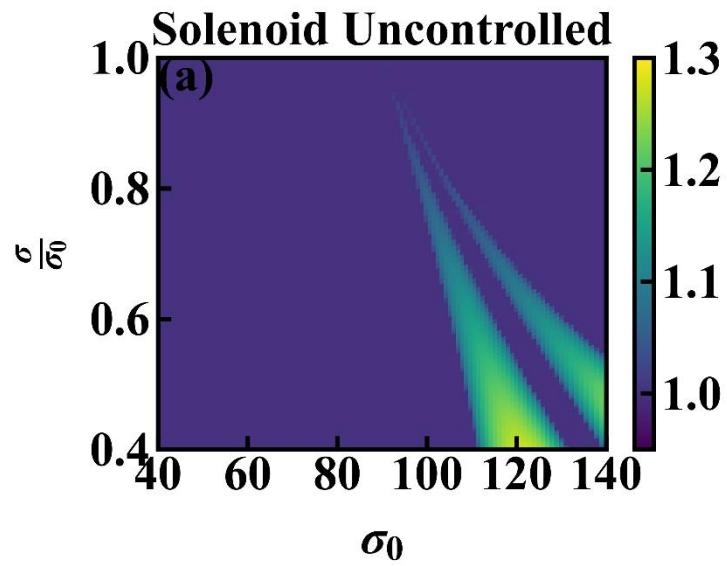
其中 $\lambda > 0$ 为正则化参数, \mathbf{c}_0 为参数取值均为 0 的情况下, 特征多项式的系数。

优化问题的解析解为:

$$\mathbf{p}^* = (S^T S + \lambda I)^{-1} S^T (\mathbf{c}_{target} - \mathbf{c}_0) \quad (24)$$

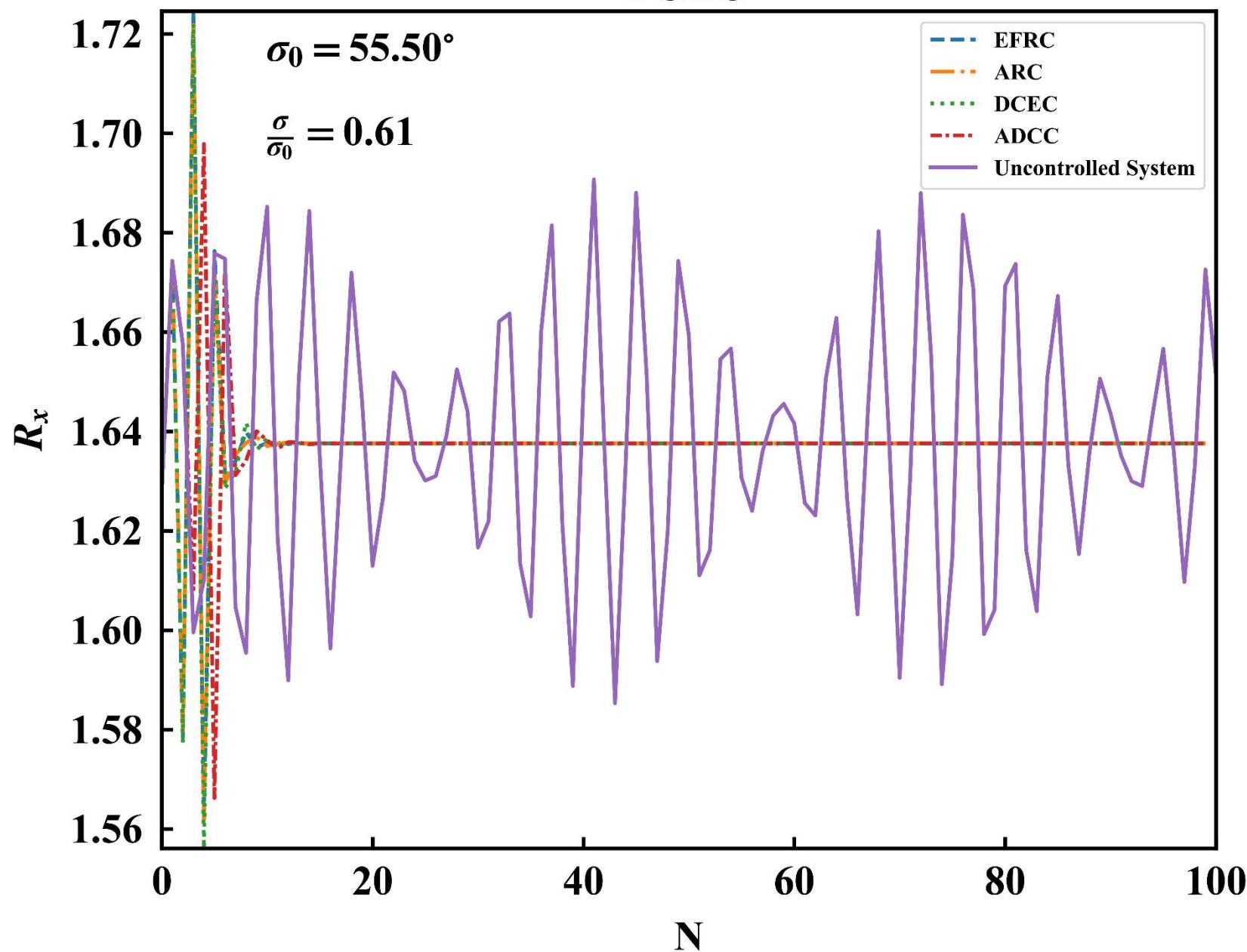
该方法通过系统的极点配置和正则化优化, 为四种控制模式提供了可靠的控制参数整定策略, 确保闭环系统在满足稳定性的同时具有良好的动态性能。

控制效果：稳定区进一步增加稳定性，不稳定区出了本征不可控的螺线管的一个模式，其他区域都稳定化了



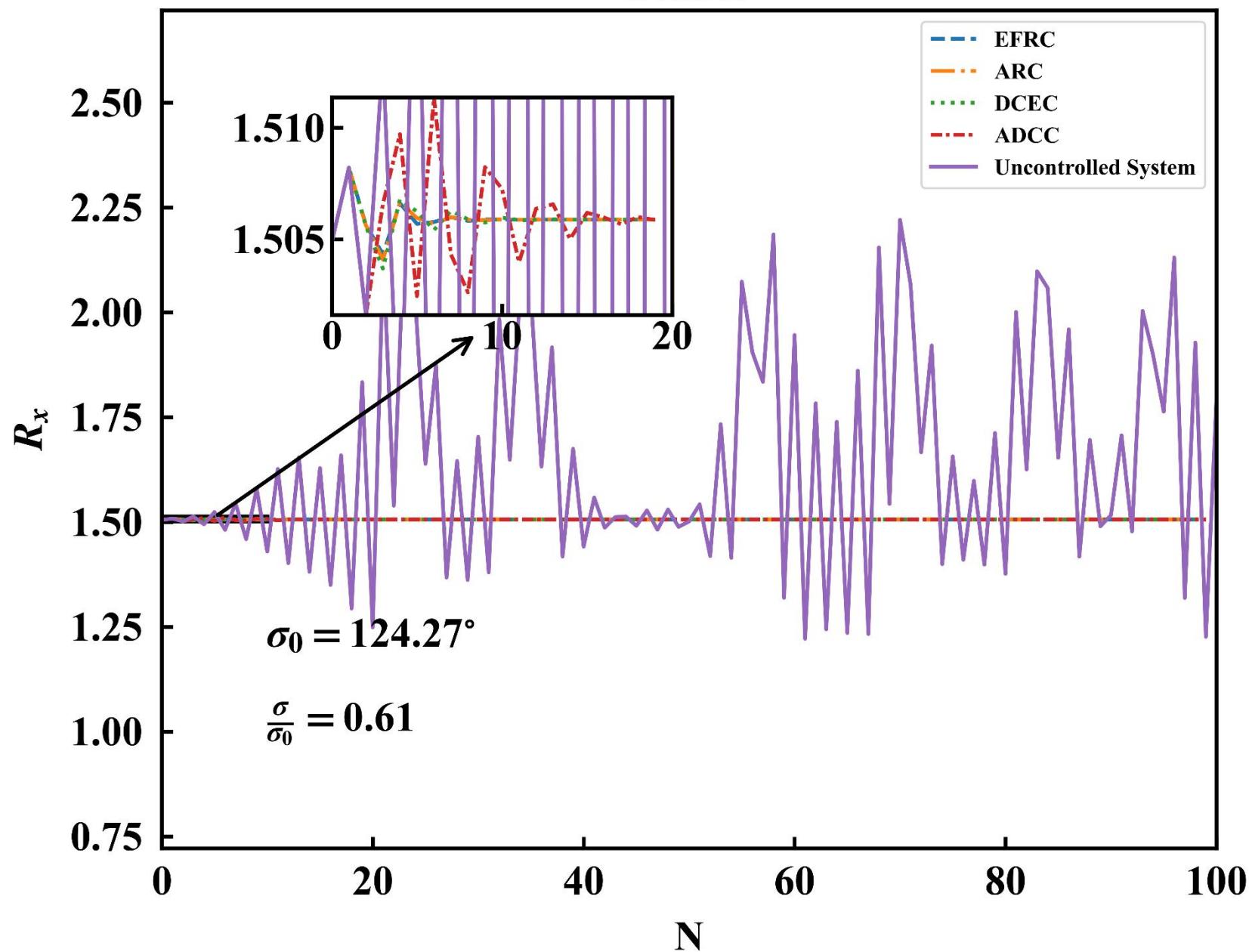
控制效果：

FODO

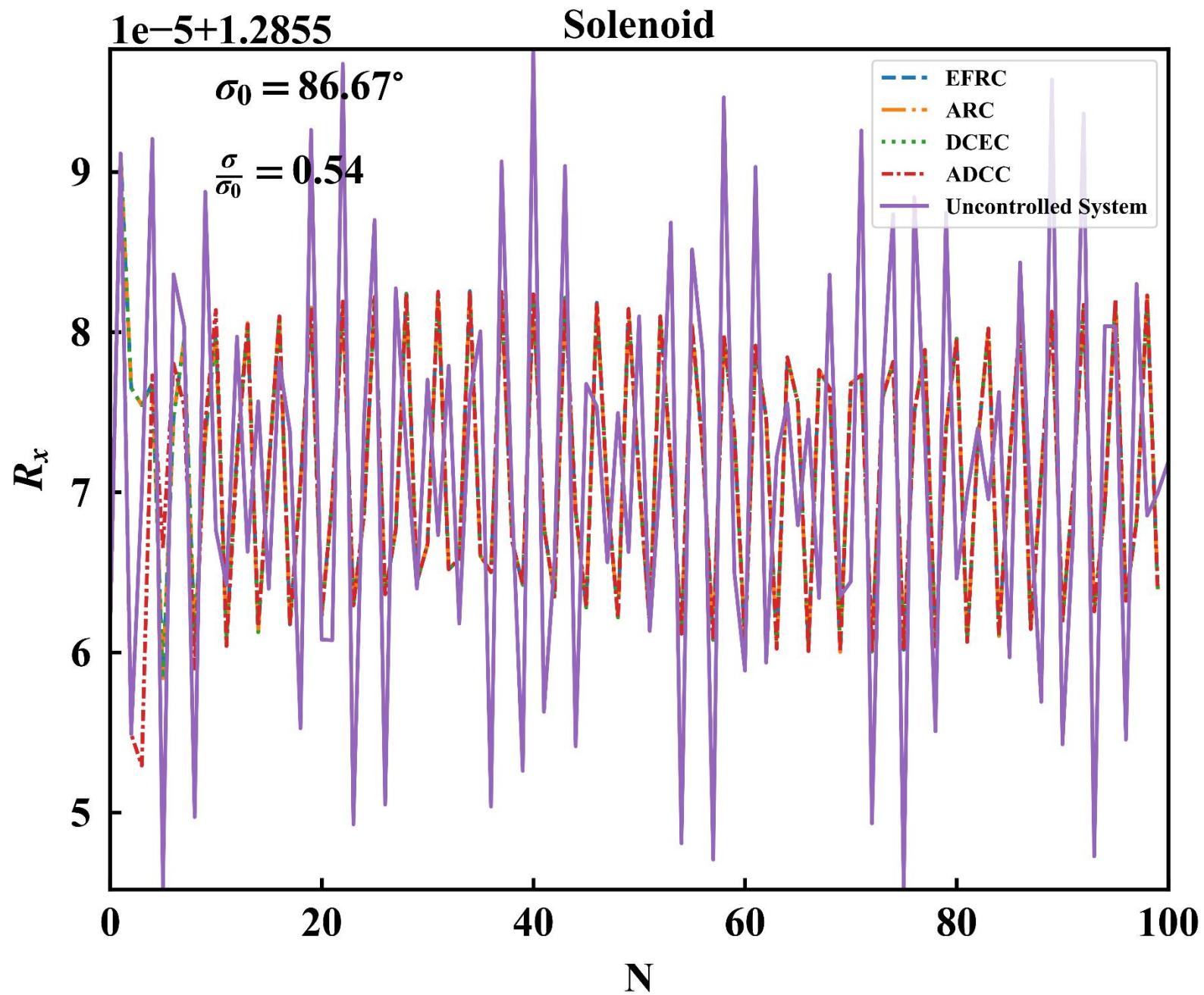


控制效果：

FODO

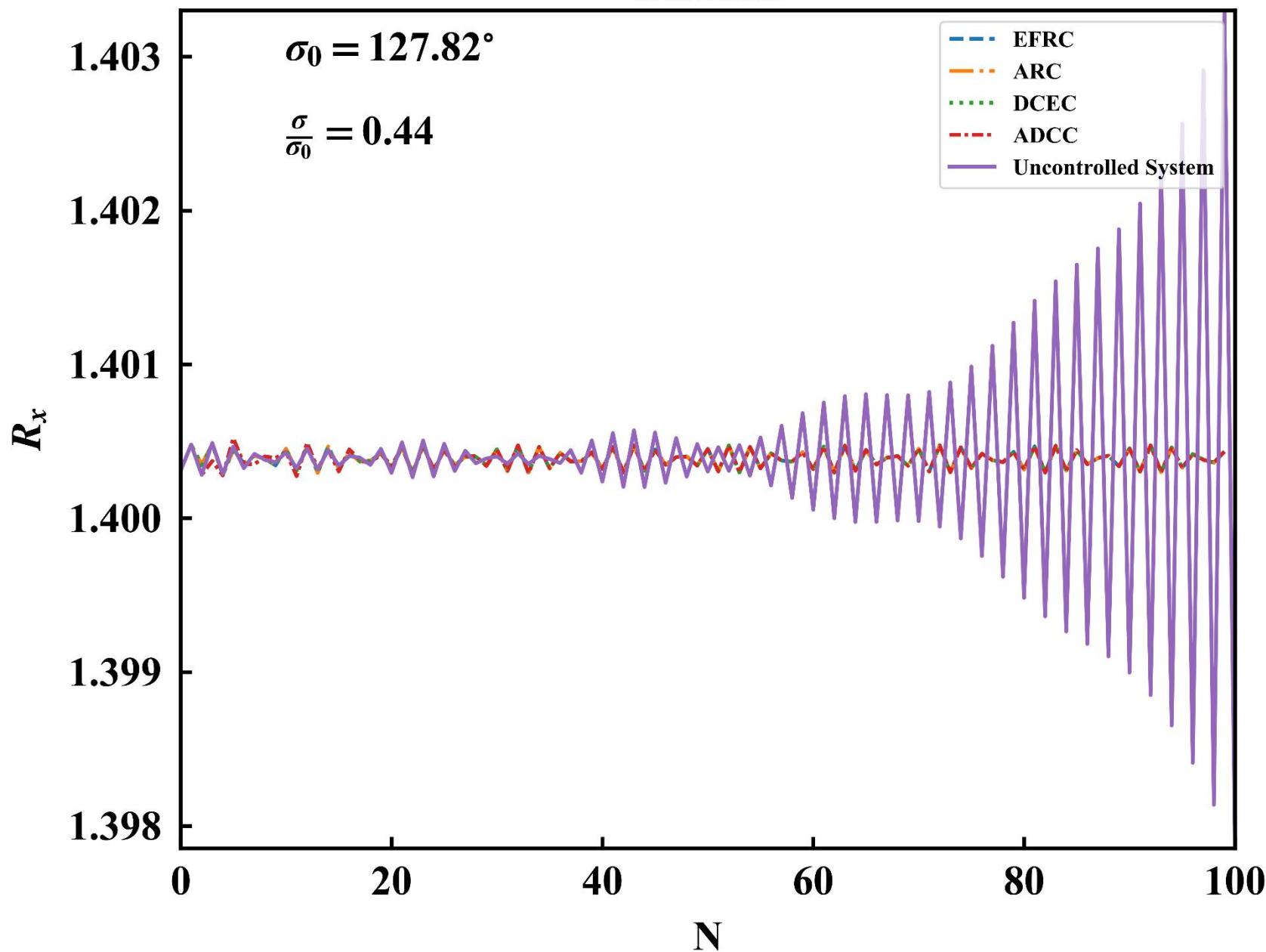


控制效果：



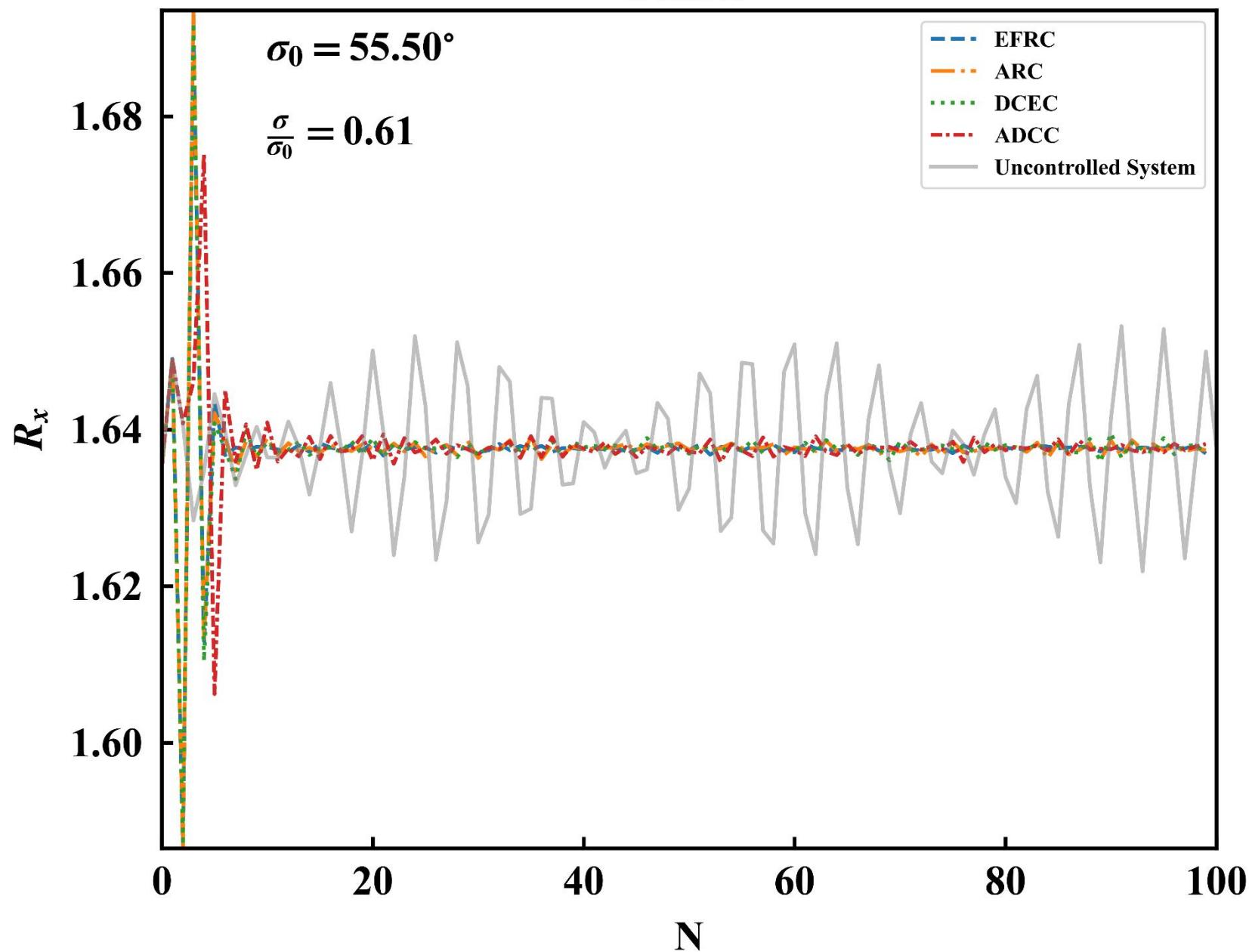
控制效果：

Solenoid

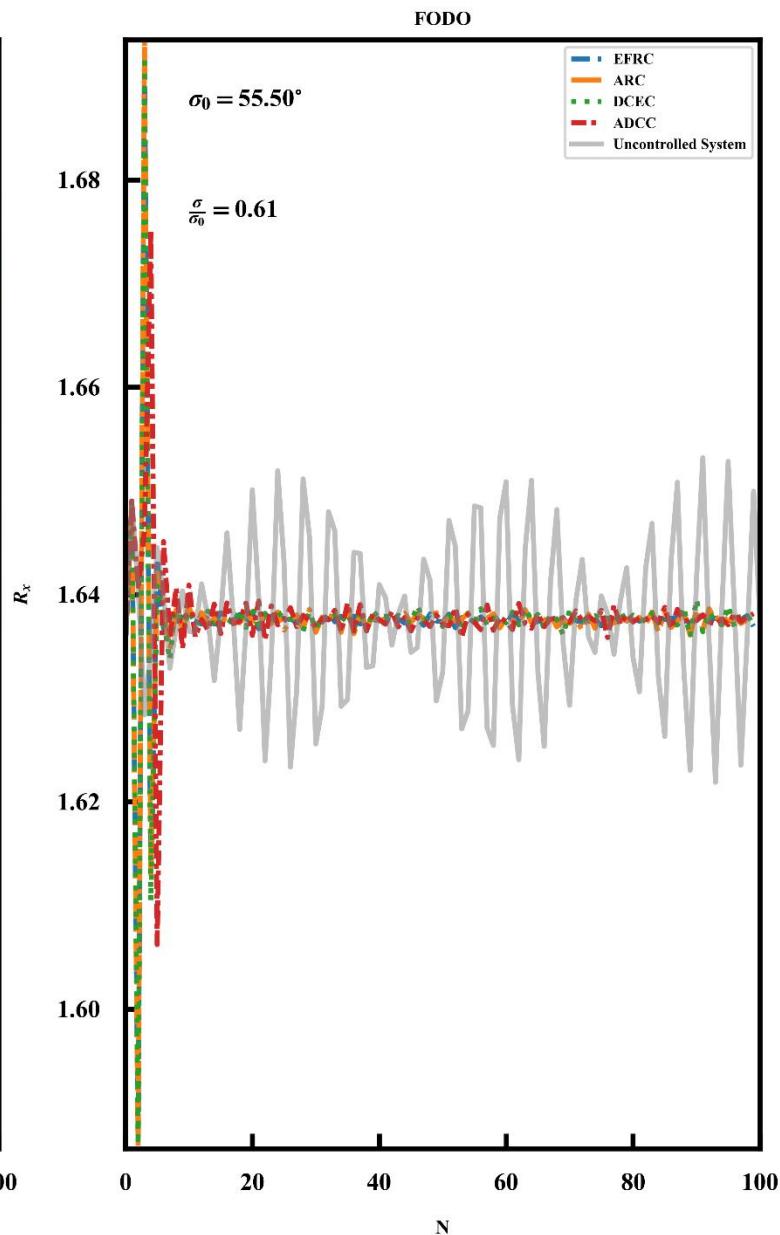
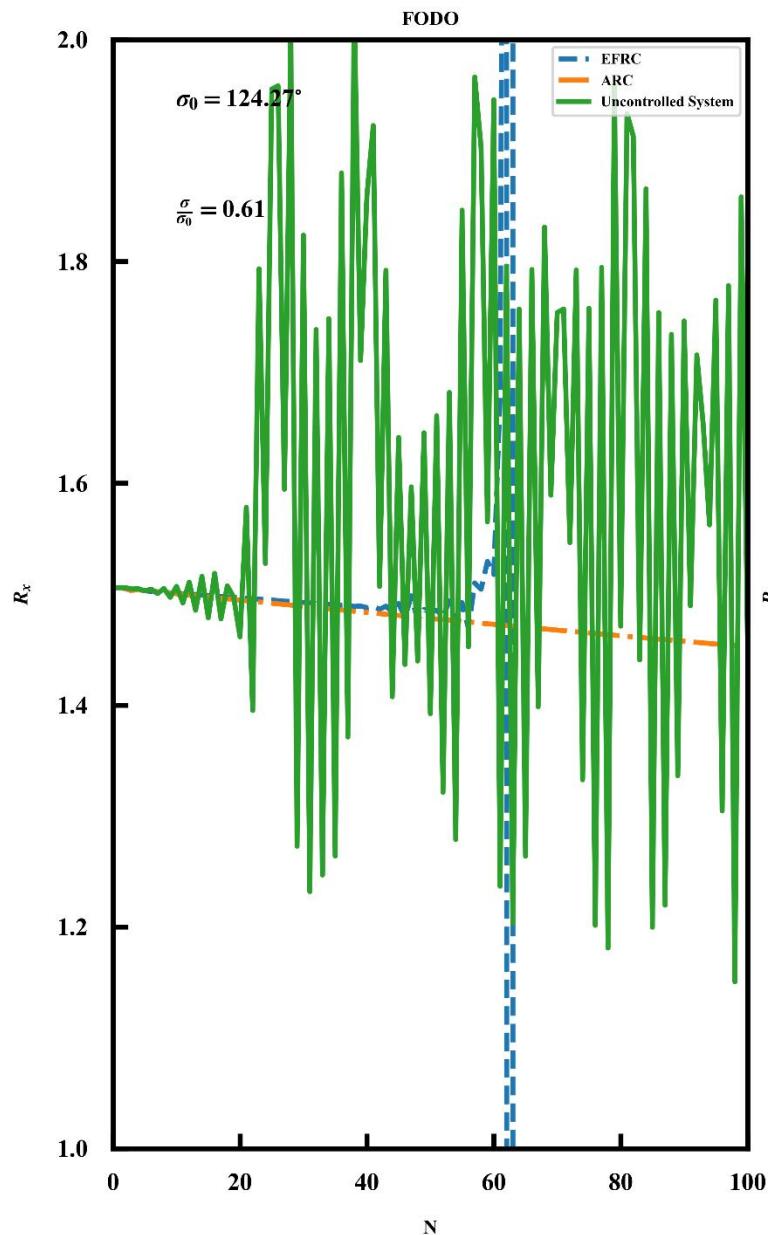


控制效果：

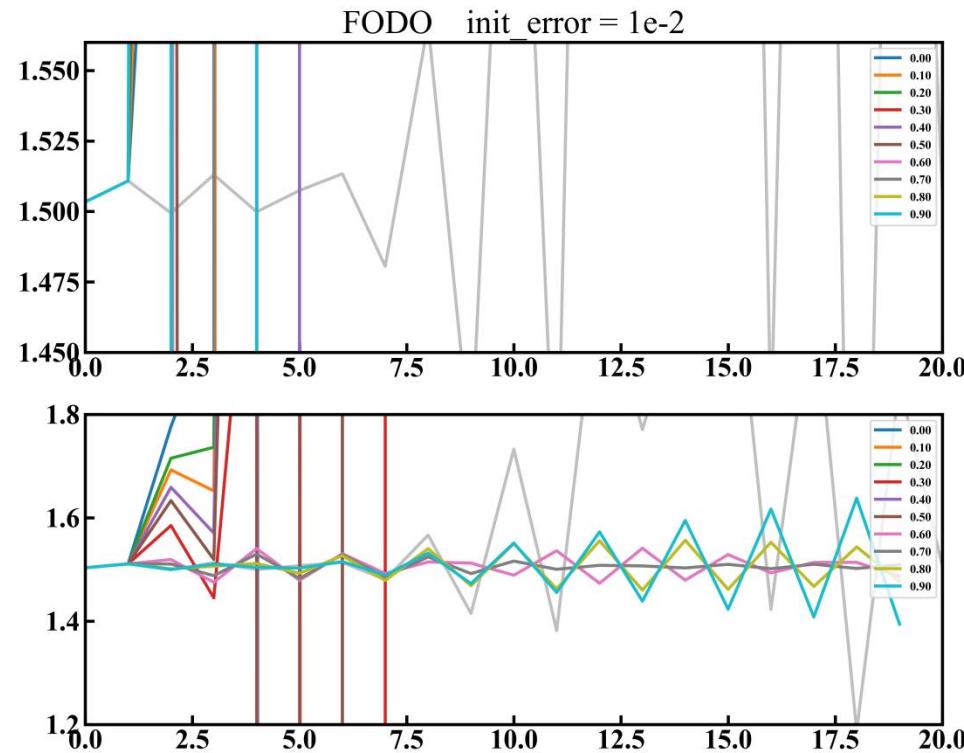
FODO

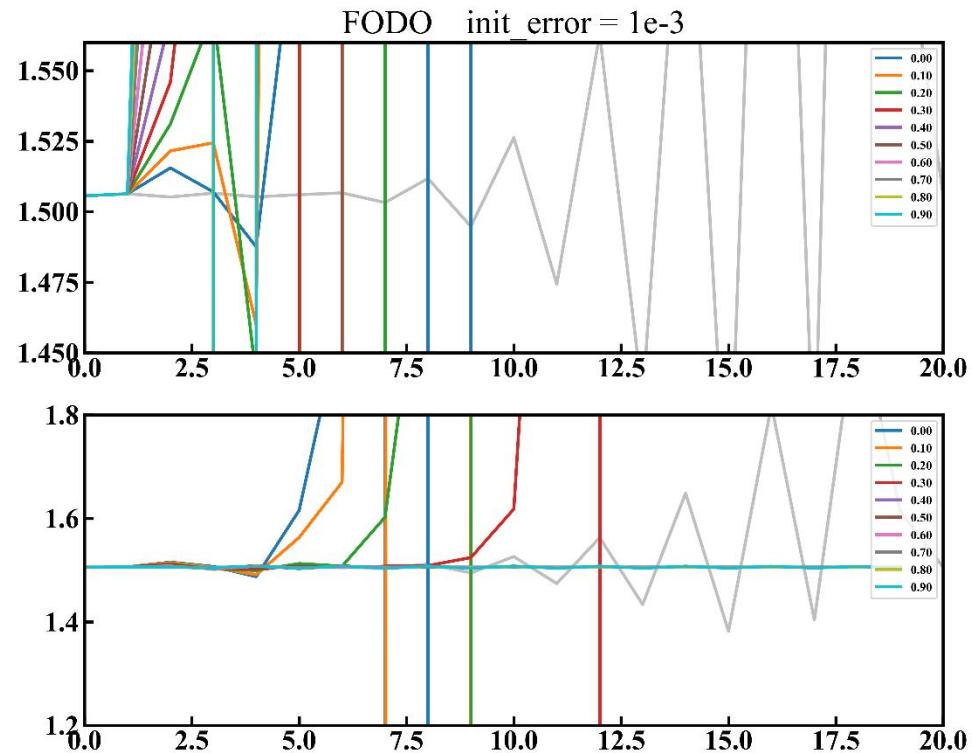


控制效果：隐含控制可跟踪参数漂移

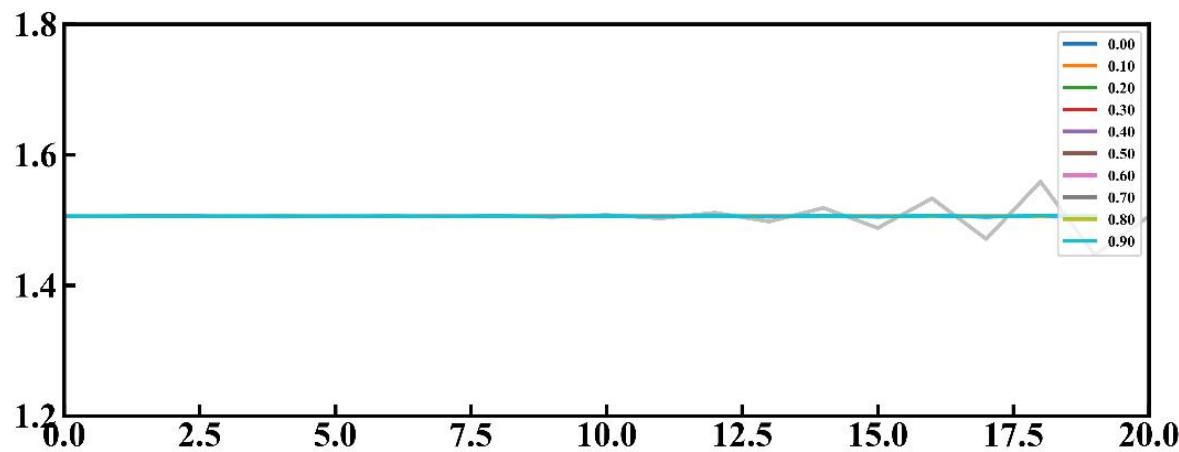
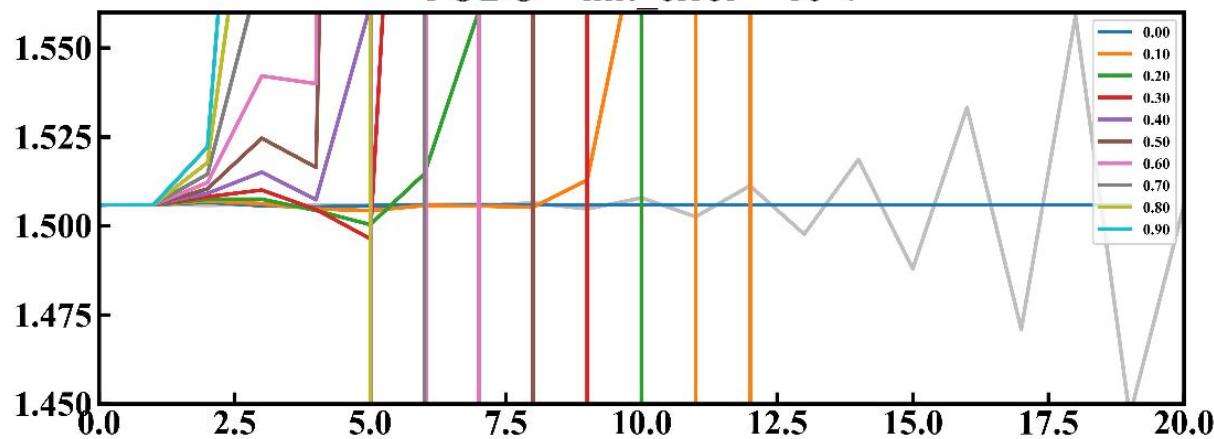


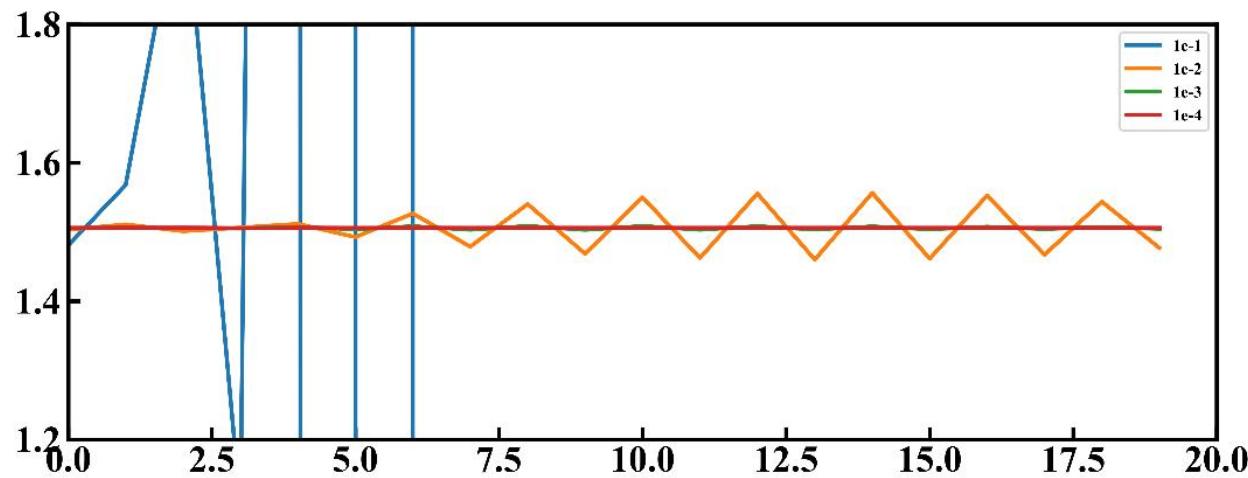
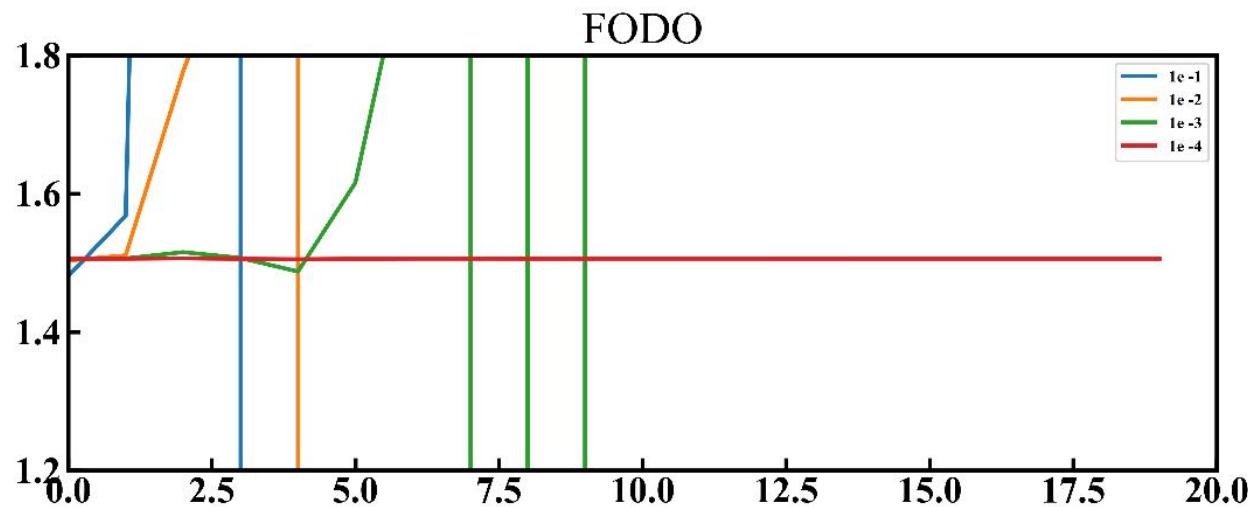
另一个重要的发现：随机极点配置具有非常好的控制性能，
提高了一个量级





FODO init_error = 1e-4





整体计划：发展一套可实用的加速器系统控制理论

- (1) 理论框架——线性理论（这个报告）
- (2) 机器学习实现——非线性控制
- (2) 模拟加速器实现
- (3) 实际加速器实现

Machine Learning for Online Control of Particle Accelerators

Xiaolong Chen^{1,2,3,4}, Zhijun Wang^{1,3,4*}, Yuan He^{1,3,4*}, Hong Zhao⁵,
Chunguang Su^{1,3,4}, Shuhui Liu^{1,3,4}, Weilong Chen^{1,3,4}, Xiaoying Zhao^{1,3,4},
Xin Qi^{1,3,4}, Kunxiang Sun^{1,3,4}, Chao Jin^{1,3,4}, Yimeng Chu^{1,3,4}, and
Hongwei Zhao^{1,3,4}

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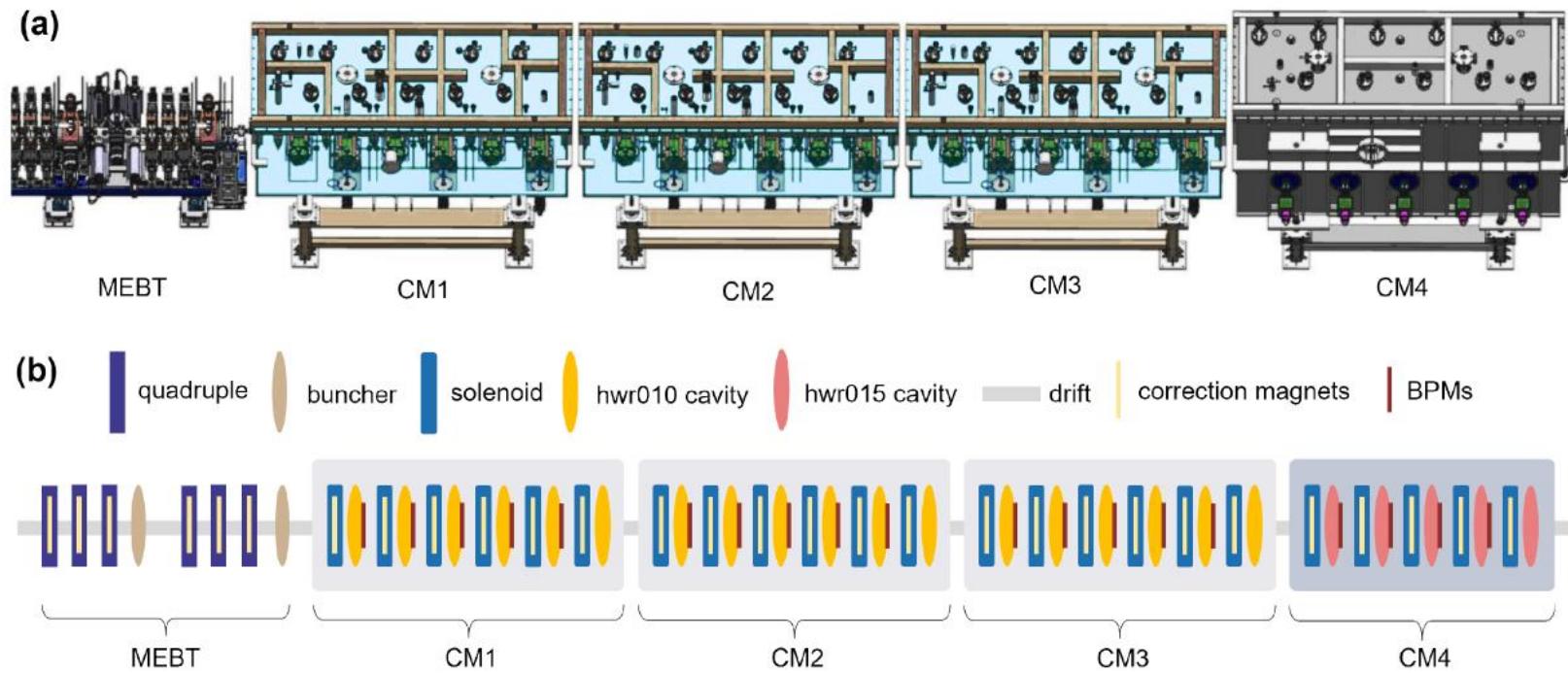


Figure 1 (a) Overview of the MEBT and the superconducting section in CAFE II. (b) The detailed structure of the MEBT and the superconducting section of CAFE II. Within the superconducting section (CM1 to CM4), there are a total of 19 groups of beam position monitors and 46 correction magnets, of which 16 groups of beam position monitors and 42 correction magnets are utilized in our research.

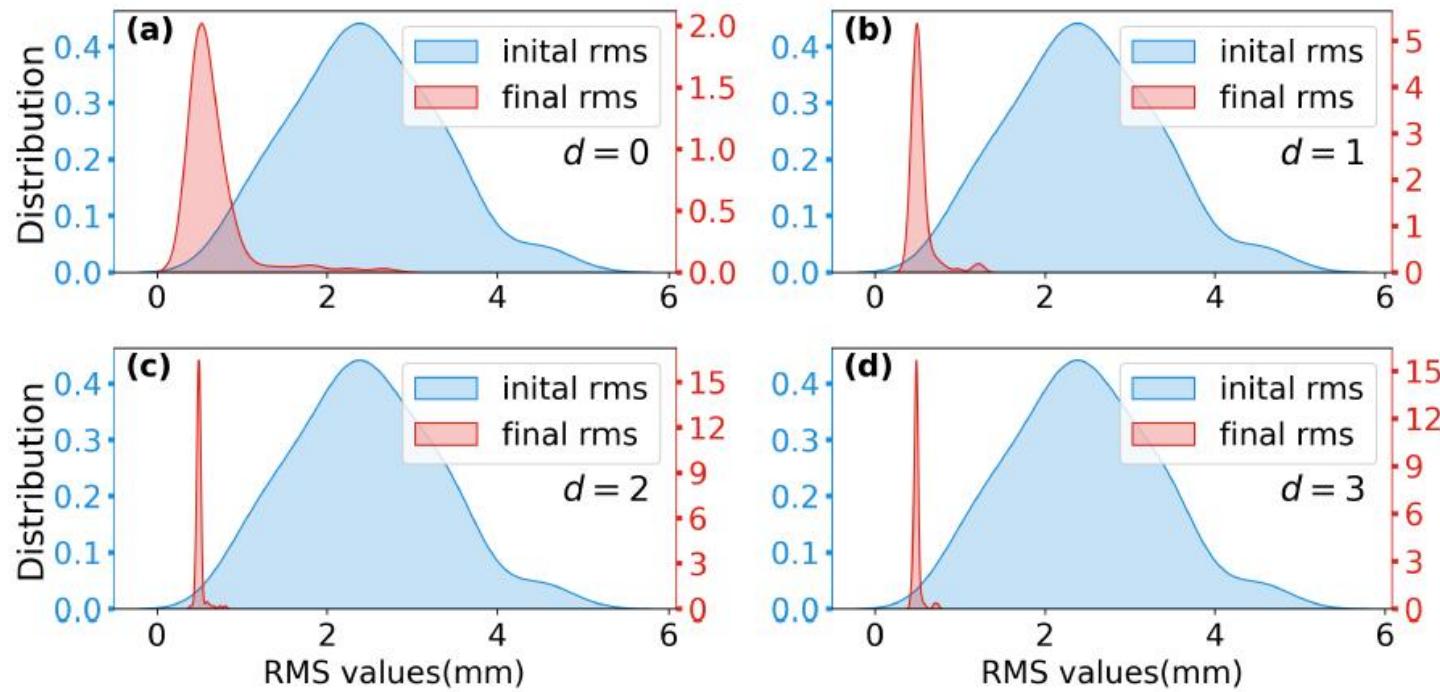


Figure 2 The distributions of the rms of beam positions before (in blue) and after (in red) correction across 100 episodes for CM1 with $d = 0$ (a), $d = 1$ (b), $d = 2$ (c), and $d = 3$ (d), respectively.

敬请指正！

