

LUND UNIVERSITY

Neutron reactions from Hamiltonian solutions in deformed nuclei

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DREB 2026
Shenzhen, 2 November 2026

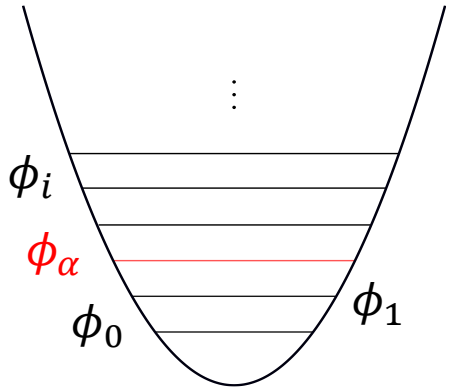
A tale of two spectroscopic factors

components of many-body states

$$S = \left|\left\langle \psi_0^A \middle| c_\alpha \middle| \psi_n^{A+1} \right\rangle\right|^2$$

α s.p. (basis) component of
A+1 manybody state (e.g. shell model)

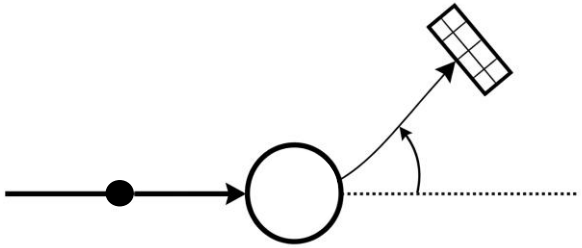
HO basis

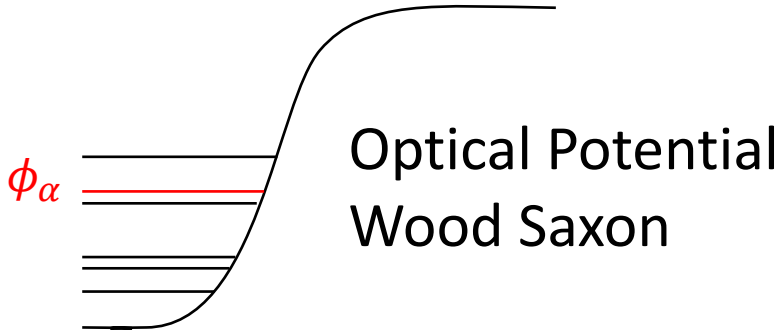


single particle states + "rest"

$$(C^2)S = \frac{\sigma^{exp}}{\sigma^{Indip.\,Part.\,Model}}$$

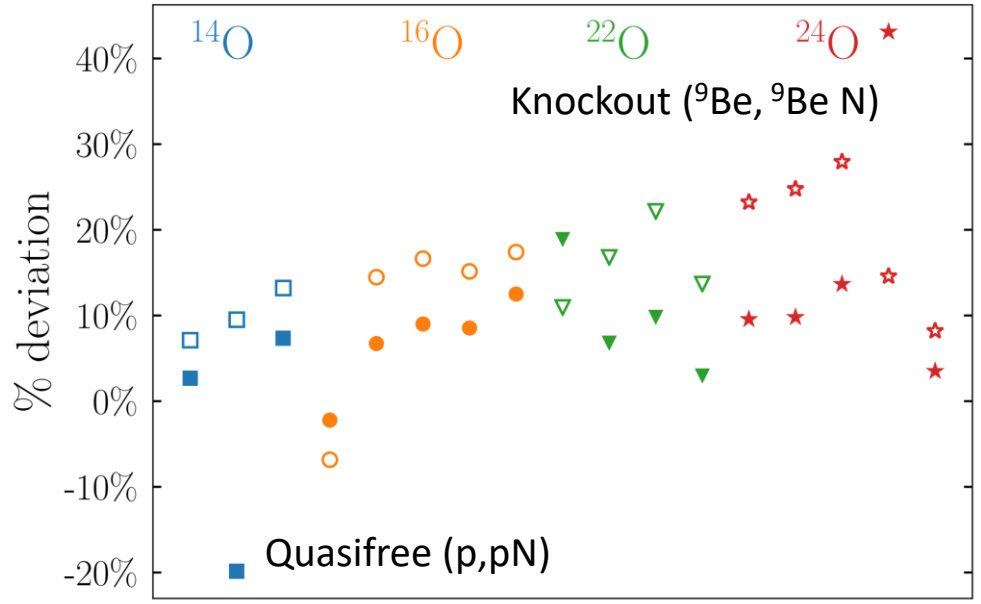
α independent particle state





Different reactions probe the correlated wavefunction

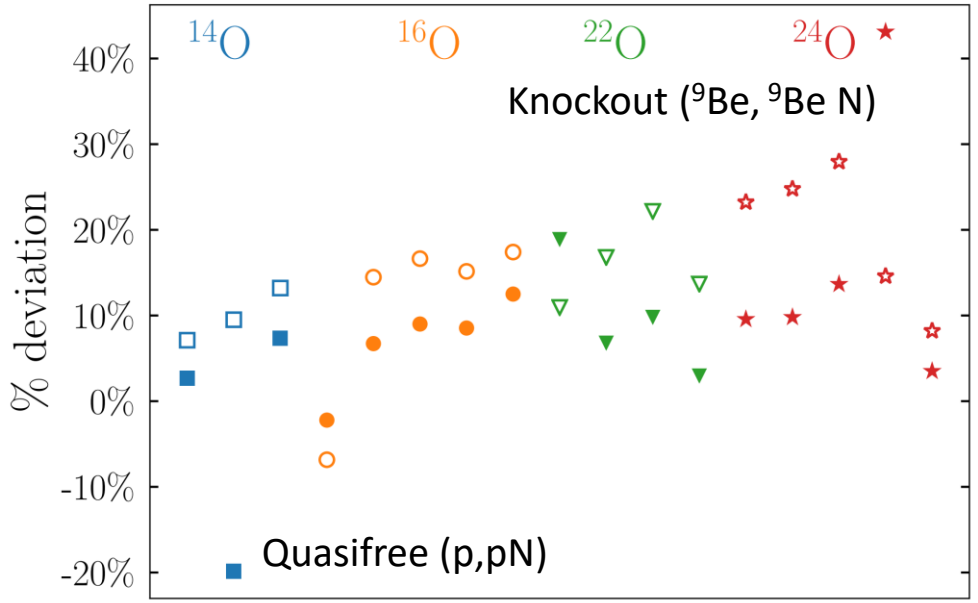
% Deviation of σ between analysis with standard Optical potential vs. Manybody Green's function optical model with Chiral 3B interaction *("ab initio")*



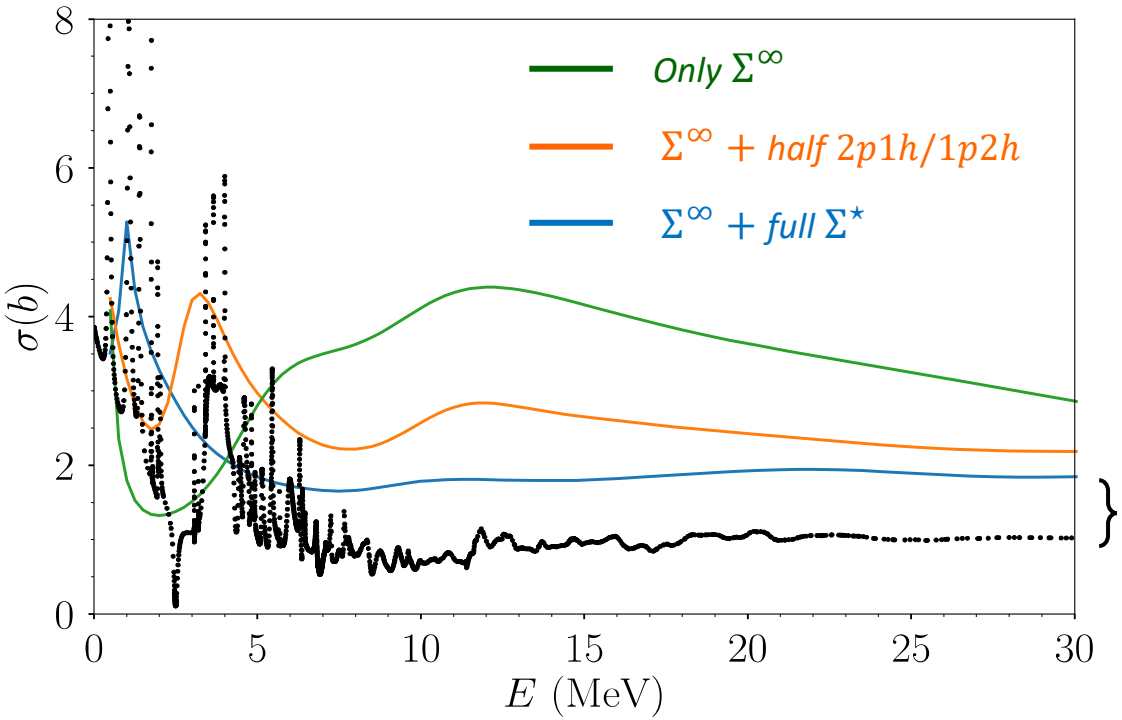
Bertulani, AI, Barbieri, PRC 104, L061602 (2021)

Different reactions probe the correlated wavefunction

% Deviation of σ between analysis with standard Optical potential vs. Manybody Green's function optical model with Chiral 3B interaction *("ab initio")*



Neutron elastic scattering, ^{16}O+n



This calculation is still
~ twice the experiment

AI, Barbieri, Navratil, PRL 123, 092501 (2019)
Bertulani, AI, Barbieri, PRC 104, L061602 (2021)

Generator Coordinate Method

Wavefunction as linear combination of HFB states (shapes, pairing, cranking)
Projected to good number of particles and angular momentum

$$|\Psi\rangle = P^N P^Z P^I_{MK}\left(h_1|\Phi_1(\,)\rangle + h_2|\Phi_2(\,)\rangle + \underbrace{h_3|\Phi_3(\,)\rangle}_{\text{Basis states}} + \right.$$
$$\left. + \underbrace{h_4}_{\text{Coefficients}}|\Phi_4(\,)\rangle + \cdots\right)$$

Coefficients are found to “exactly” solve a given Hamiltonian:

$$H = T + V + W$$

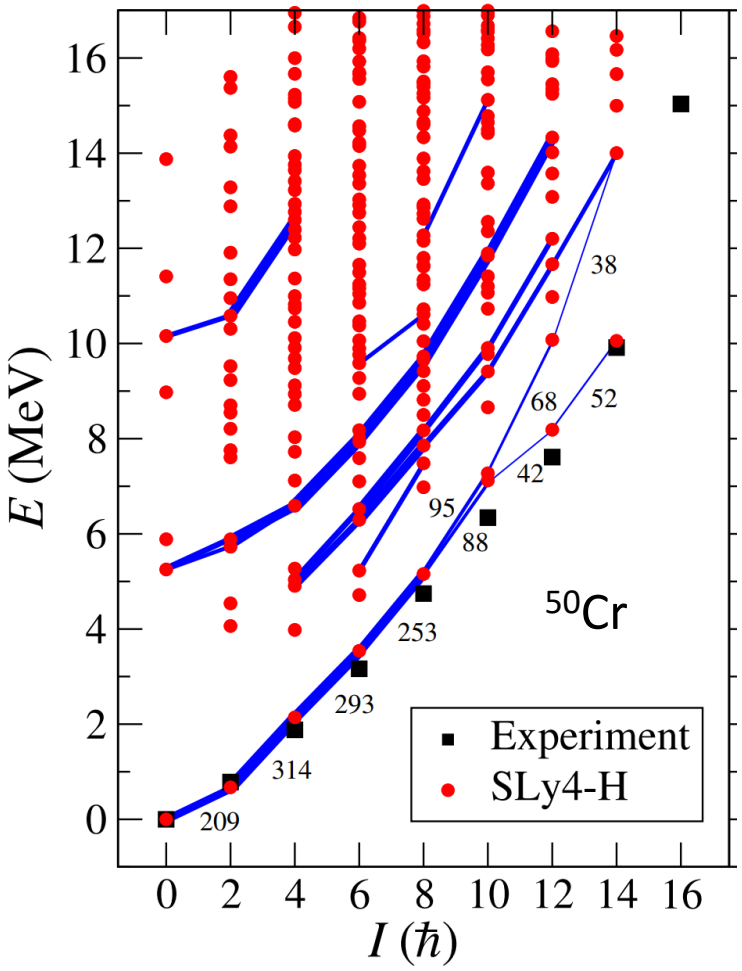
We write effectively in a separable expansion:

$$H = H_0 + PP + \sum QQ + QQQ$$

Carlsson et al., *in preparation*
Ljungberg et al., PRC 106, 014314 (2022)

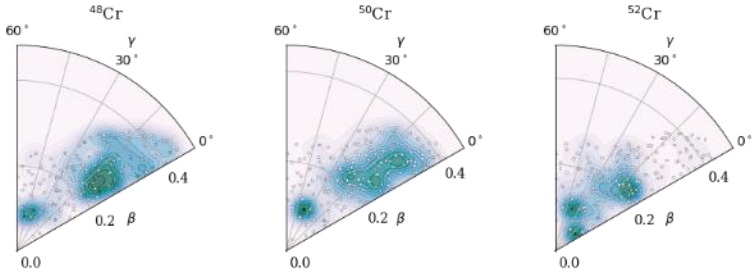
It works!

Even and odd nuclei



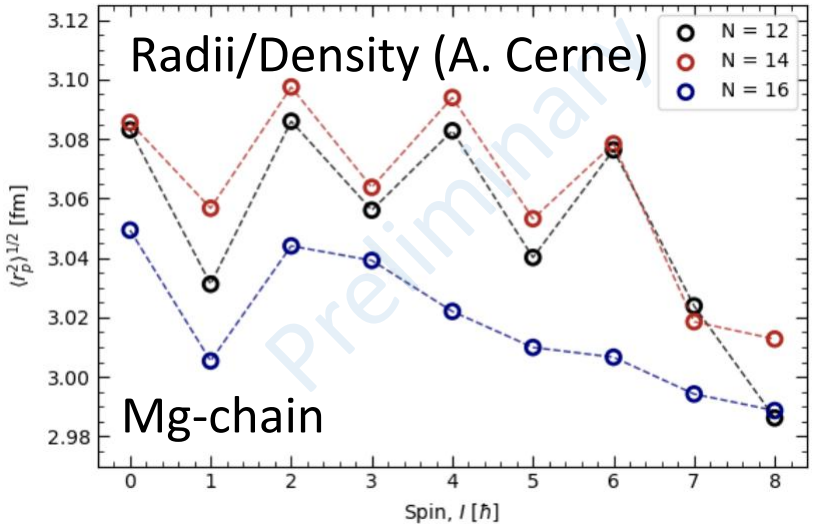
Ljungberg et al., PRC 106, 014314 (2022)

Collective Wavefunctions

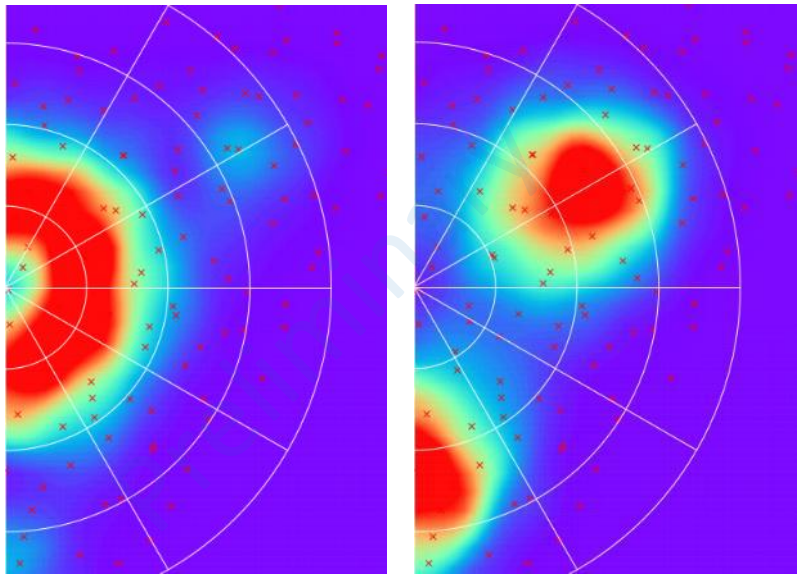


AI et al. N. Cim. 47C, 31 (2024)

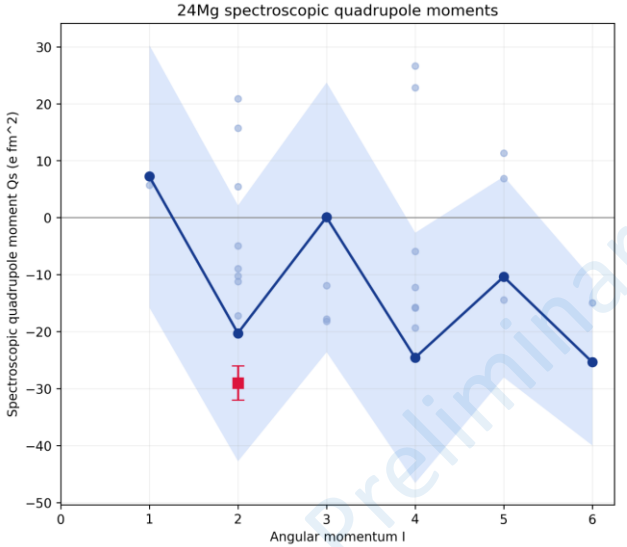
Radii/Density (A. Cerne)



^{110}Sn vibrational and rotational 2$^+$



Electromagnetic Transitions Quadrupole moments



Construct the optical potential

We have spectroscopic amplitudes and energies for A, $A \pm 1$ states: ingredients of the Green's function

$$G^{J\pi}_{\alpha,\beta}(E) = \lim_{\eta \to 0^+} \sum_{i,s=\pm} \frac{\sigma^{s*}_{J,i,\alpha} \sigma^{s}_{J,i,\beta}}{E - (\bar{E}^s)^{J\pi}_i(\eta)}$$

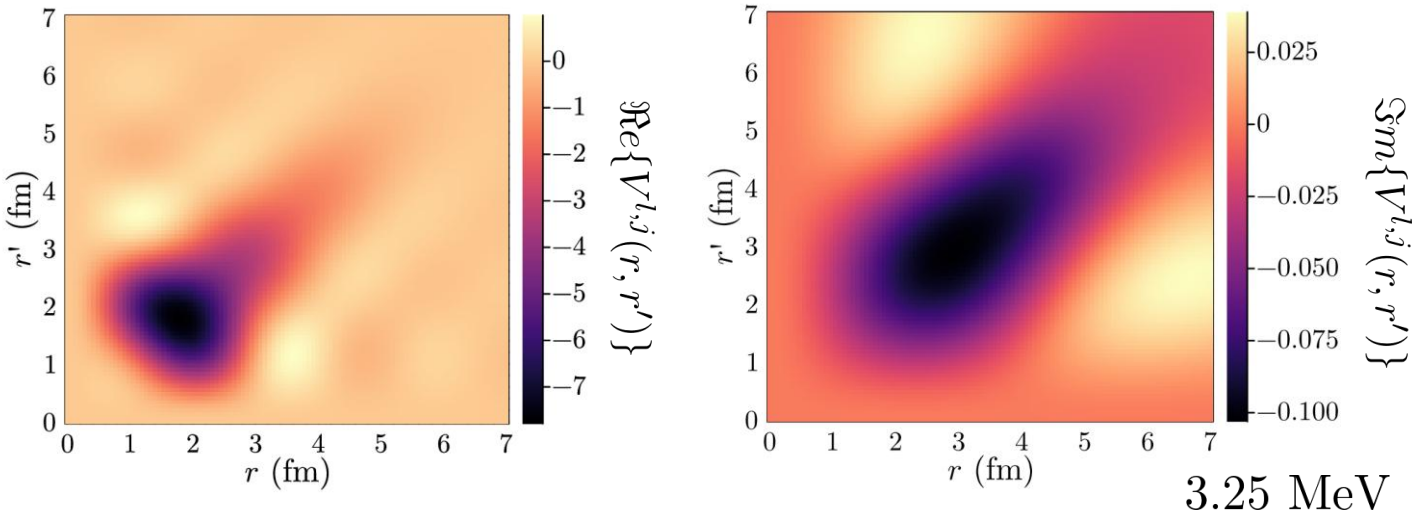
We know how a neutron particle (hole) moves in the correlated A particle nucleus, we can construct the potential it feels.
Inversion of Dyson Equation:

$$\Sigma = G_0^{-1} - G^{-1}$$

We get the Optical Potential:

$$V^{l,j}(r, r'; E) = V_0 + \Sigma^{l,j}(r, r'; E)$$

Nonlocal, partial wave, energy dependent optical potential obtained directly from Hamiltonian solutions for deformed nuclei.
No tuning or data from the isotope

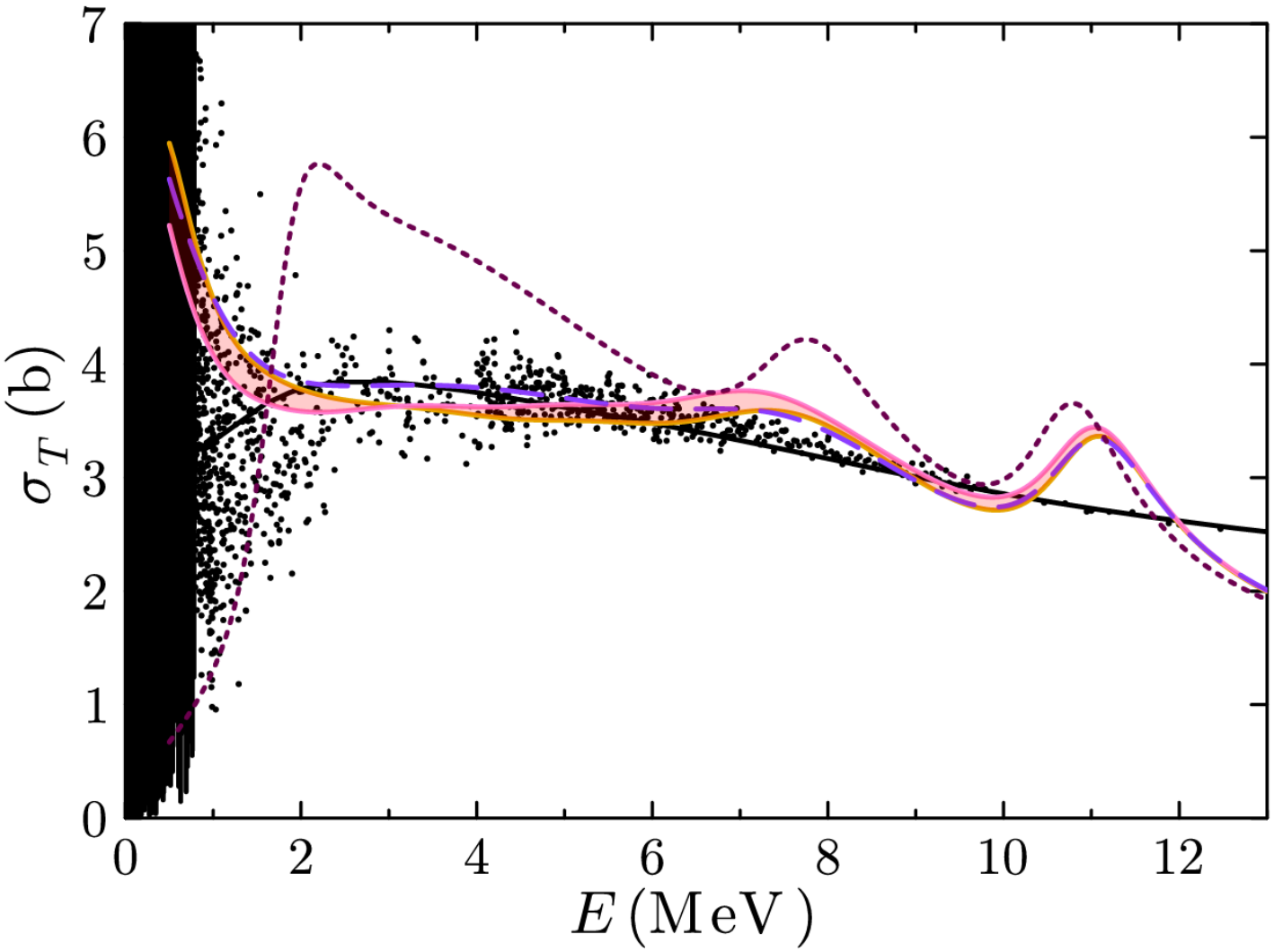


Boström, Carlsson, AI arXiv:2604.0103 (2026)

Results: n + ^{50}Cr

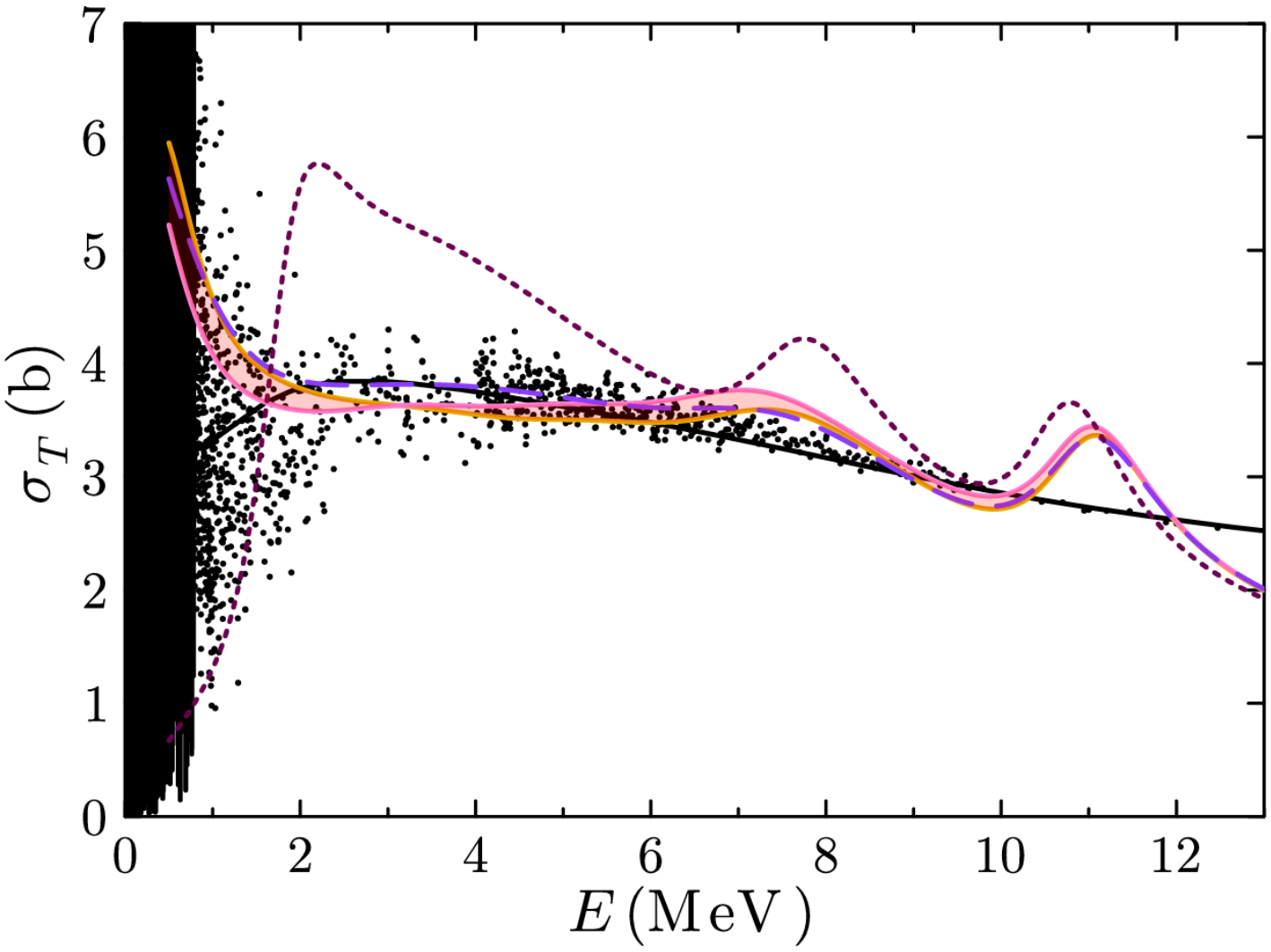
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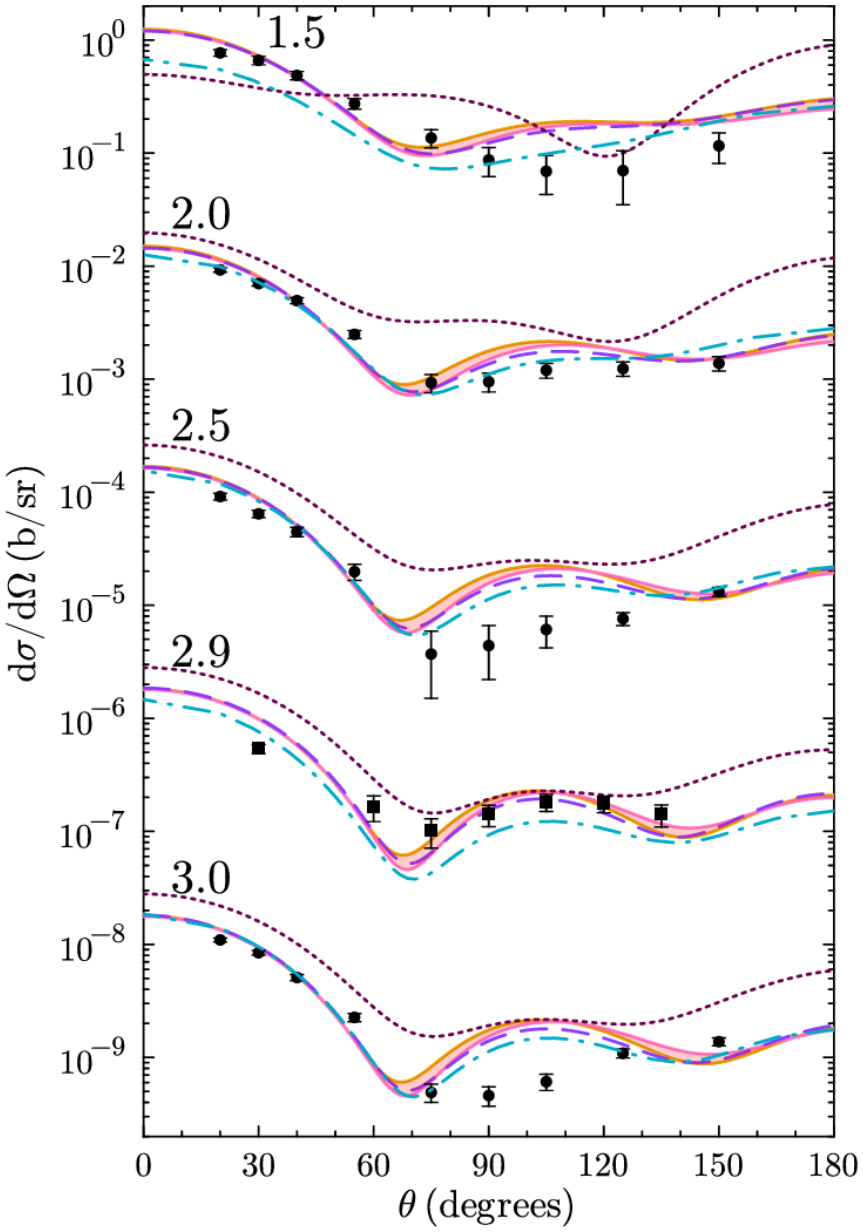
Results: n + ^{50}Cr



Boström, Carlsson, AI arXiv:2604.0103 (2026)

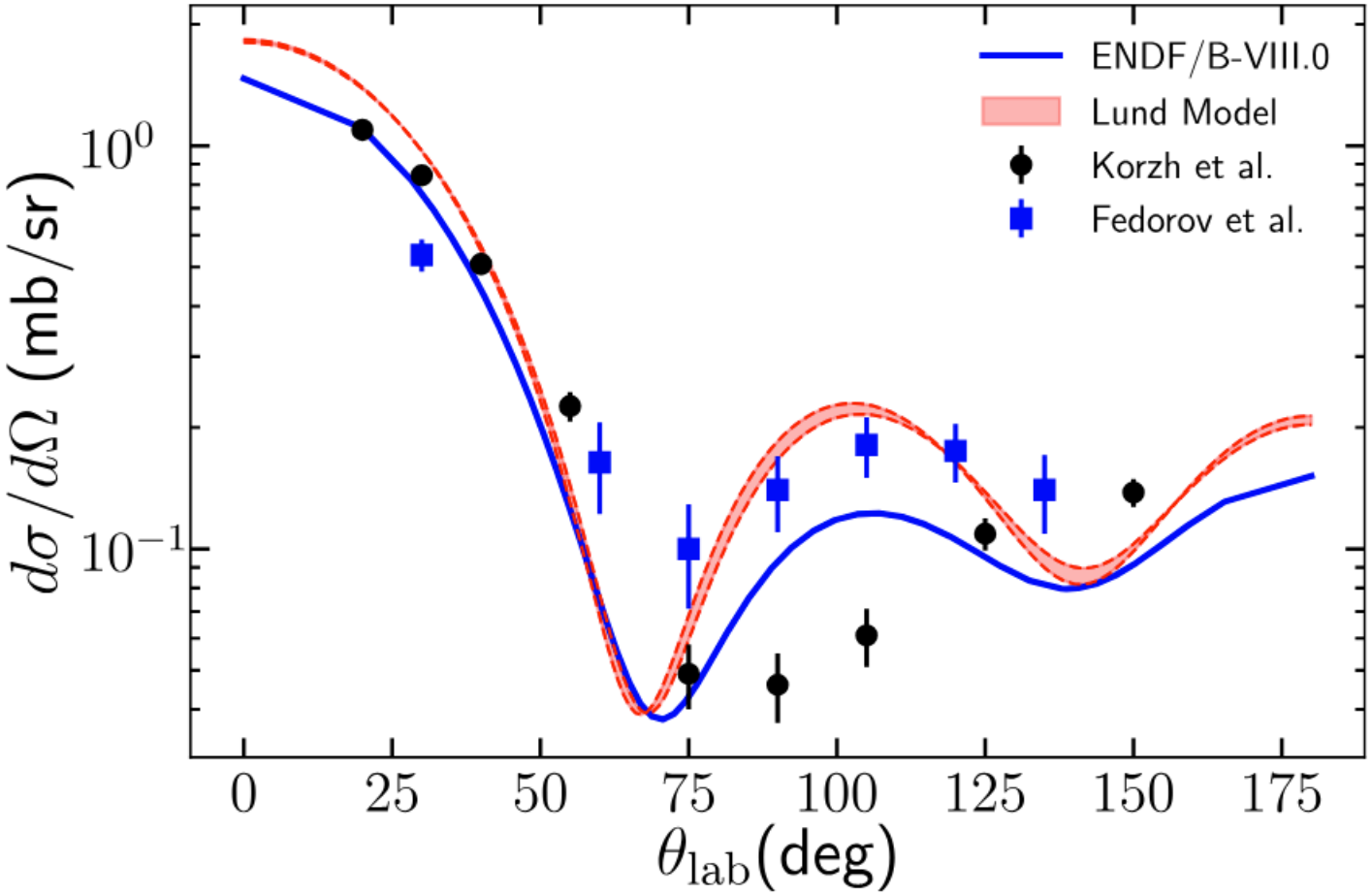
Results: n + ^{50}Cr

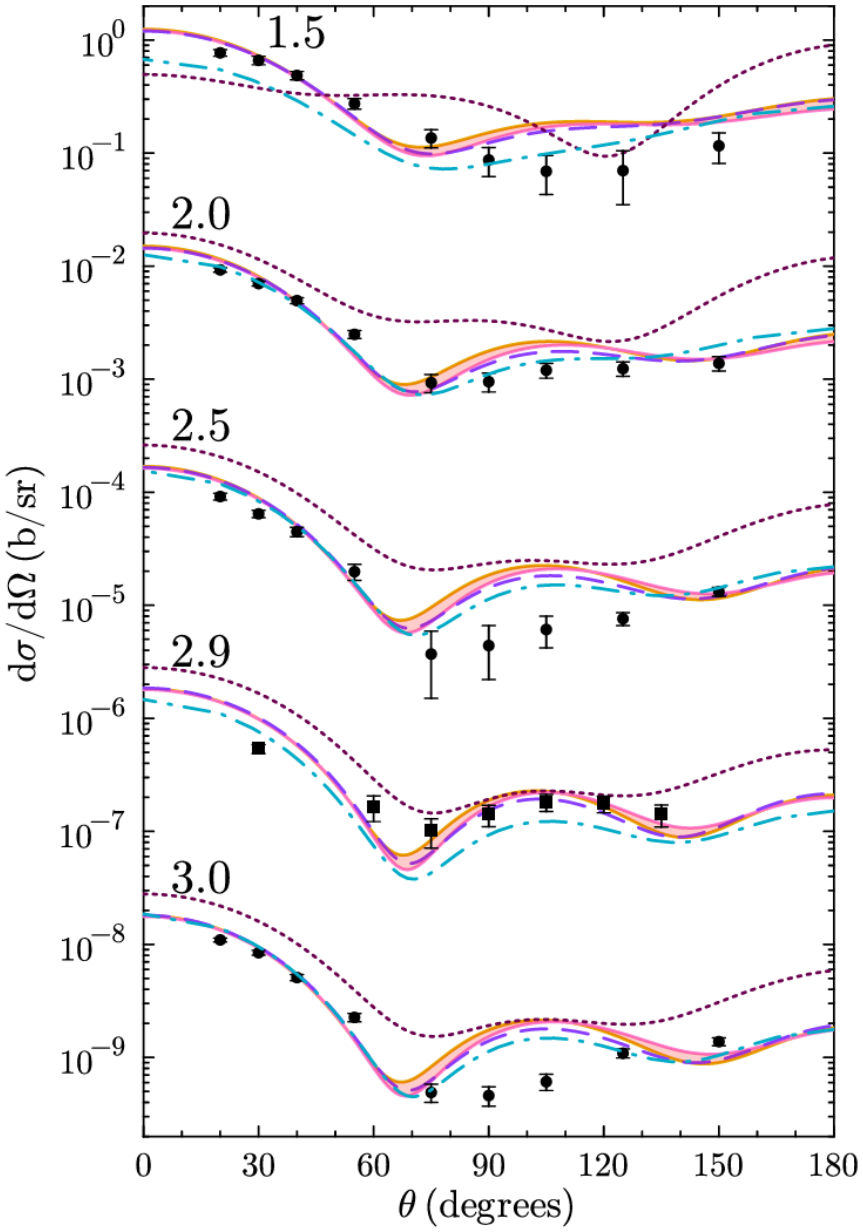




Boström, Carlsson, AI arXiv:2604.0103 (2026)

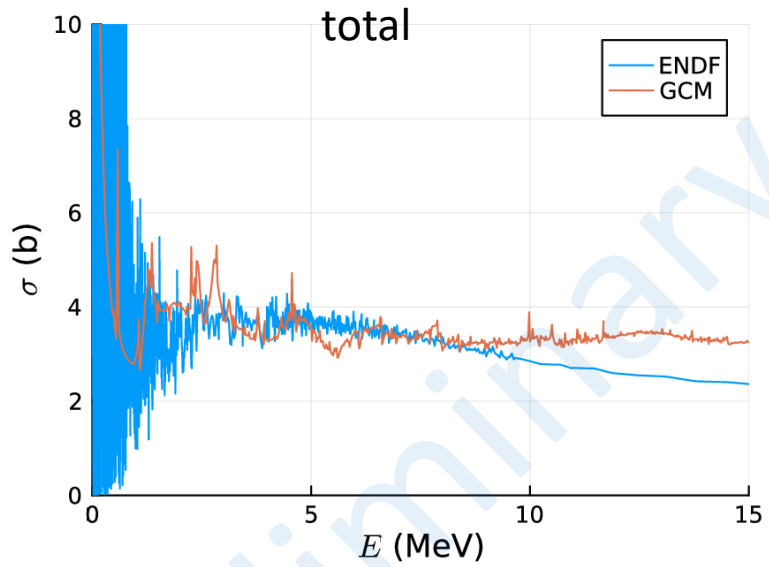
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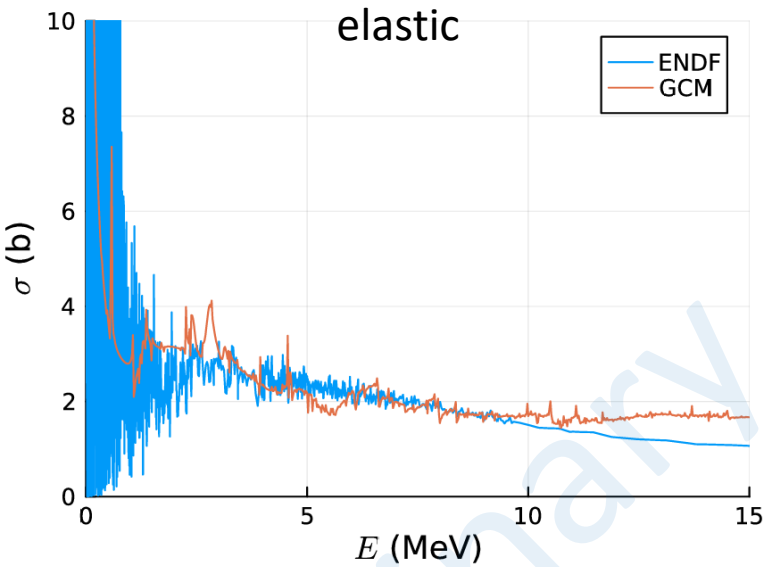


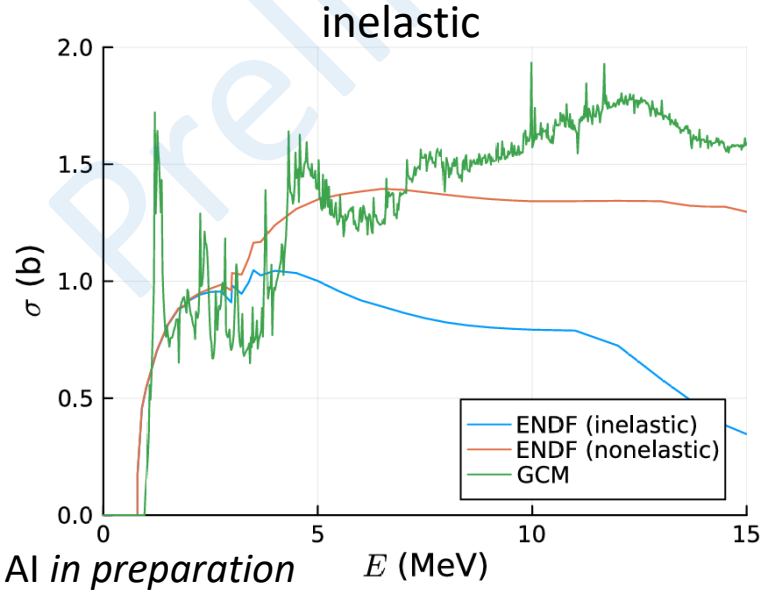


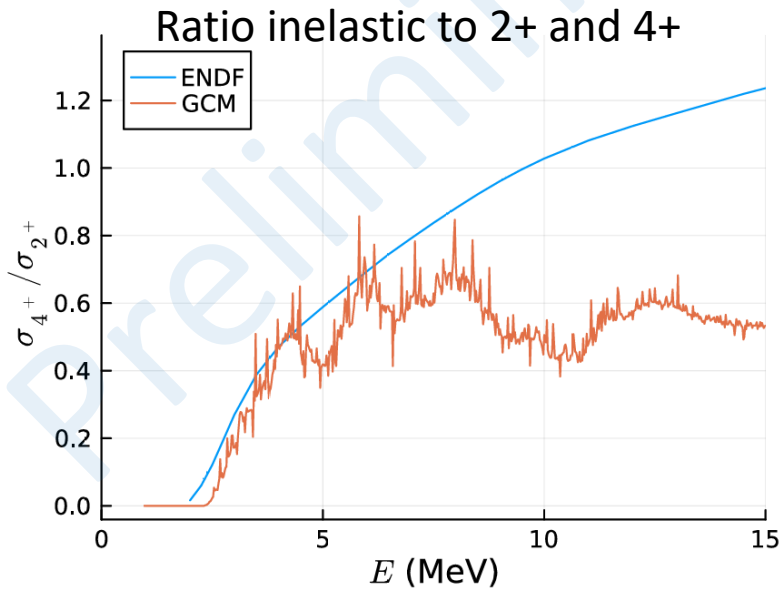
Boström, Carlsson, AI arXiv:2604.0103 (2026)

Preliminary results: n + ^{50}Cr with J-matrix method







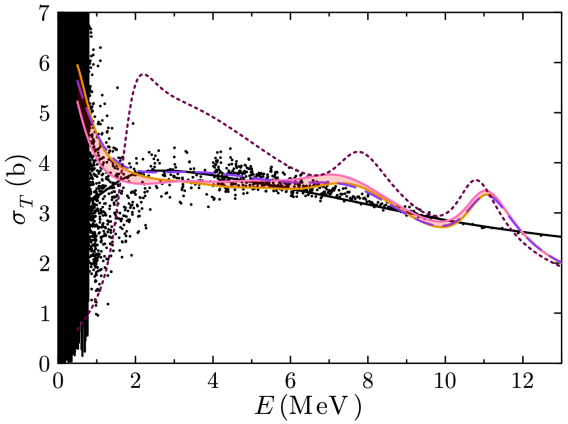


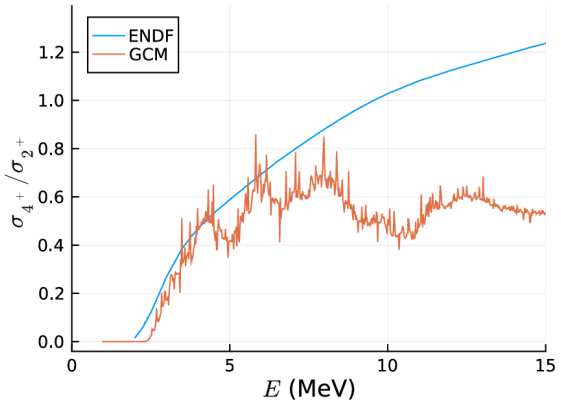
Boström, Carlsson, AI *in preparation*

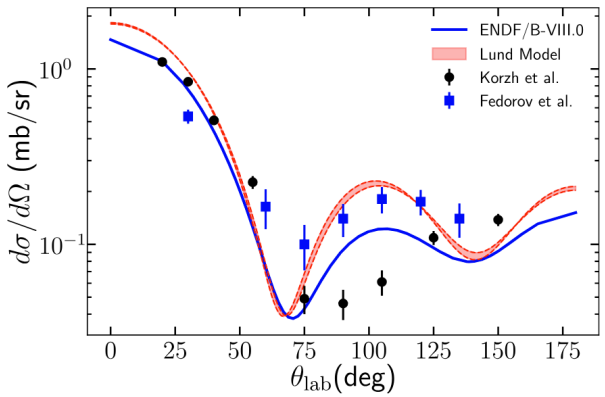
Preliminary

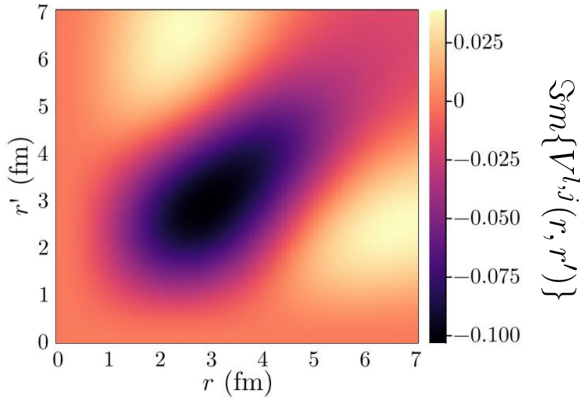
Conclusions & Outlook

- GCM can describe nuclear states within a consistent framework accounting for collective many-body effects.
- This was used to construct the first optical potential based on exploiting symmetry breaking, without any tuning, with exceptional results. *Potentially enabling a consistent parameter-free analysis.*
- Working towards:
  - Generalizing the effective Hamiltonian
  - Systematic studies: uncertainty analysis and isotopic chains.
  - Additional reactions









[Ljungberg et al., Phys. Rev. C 106, 014314 (2022)]
[Boström, Rotureau, Carlsson, AI, PRC 112, L051602 (2025)]
[Boström, Carlsson, AI arXiv:2604.0103 (2026)]

Challenge 1:
Spectroscopic amplitude for projected states

$$\sigma^{+}_{J,i,\alpha} = \left\langle \Psi^{+\,I,M}_{\ i} \middle| a^{\dagger}_{\alpha} \middle| \Psi^{0,0}_{0} \right\rangle$$

$$= \sum_{a,a',K} \left(h^{I}_{ia,K}\right)^{*} h^{0}_{0a',0} \left\langle \phi_a \middle| \beta_\gamma \hat{P}^{A+1} \hat{P}^{I}_{KM} a^{\dagger}_{\alpha} \hat{P}^{0}_{00} \hat{P}^{A} \middle| \phi_{a'} \right\rangle .$$

Brute force: evaluate an operator 100k for each quasiparticle α times

Solution: Modify overlap formula from [Carlsson, Rotureau, PRL126, 172501 (2021)]

$$\langle \phi | \beta_\gamma \beta_\delta | \phi' \rangle = (-1)^{n/2} \,\mathrm{pf} \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^{\mathrm{T}} & \mathcal{C} \end{pmatrix} = (-1)^{n/2} \,\mathrm{pf}\left(\mathcal{A}\right) \mathrm{pf}\left(\mathcal{C} + \mathcal{B}^{\mathrm{T}} \mathcal{A}^{-1} \mathcal{B}\right)$$

An expensive part representing the even state, to evaluate once

A 2x2 matrix that depends on the blocked quasiparticles

Obtained a formula to calculate spectroscopic factors of Bogoliubov states efficiently.

Boström, Carlsson, AI arXiv:2604.0103 (2026)
Boström, AI, et al. J. Phys. Conf. 2586, 012080 (2023)

Challenge 2:
Not a complete set of states

$$\delta_{\alpha,\beta} = \left\langle \Psi_0 \middle| \left\{ a_\alpha, a_\beta^\dagger \right\} \middle| \Psi_0 \right\rangle = \sum_{i,s=\pm} \sigma^{s*}_{J,i,\alpha} \sigma^{s}_{J,i,\beta}.$$

Sum rule 1: Sum of spectroscopic factors (probabilities) should be 1.
Problem: it isn't: they are separate $A, A \pm 1$ calculations. We would need the whole Fock space!

Solution: Derive new sum rule

$$\left\langle \Psi_0 \middle| \left\{ a_\alpha, \left[\hat{H}, a_\beta^\dagger \right] \right\} \middle| \Psi_0 \right\rangle = \sum_i \sigma^{+*}_{J,i,\alpha} (E^{+}{}^{J}_{i} - E_0) \sigma^{+}_{J,i,\beta} - \sum_i \sigma^{-*}_{J,i,\alpha} (E^{-}{}^{J}_{i} - E_0) \sigma^{-}_{J,i,\beta},$$

many-body single-particle field (generalized mean field)

Use it to "complete" the spectroscopic states on the basis of the generalized mean field

$$\sigma^\dagger \sigma + c^\dagger c = \mathrm{I}$$

$$\sigma^\dagger \epsilon \sigma + c^\dagger \epsilon_c c = H_A.$$

Boström, Carlsson, AI arXiv:2604.0103 (2026)
Boström, AI, et al. J. Phys. Conf. 2586, 012080 (2023)

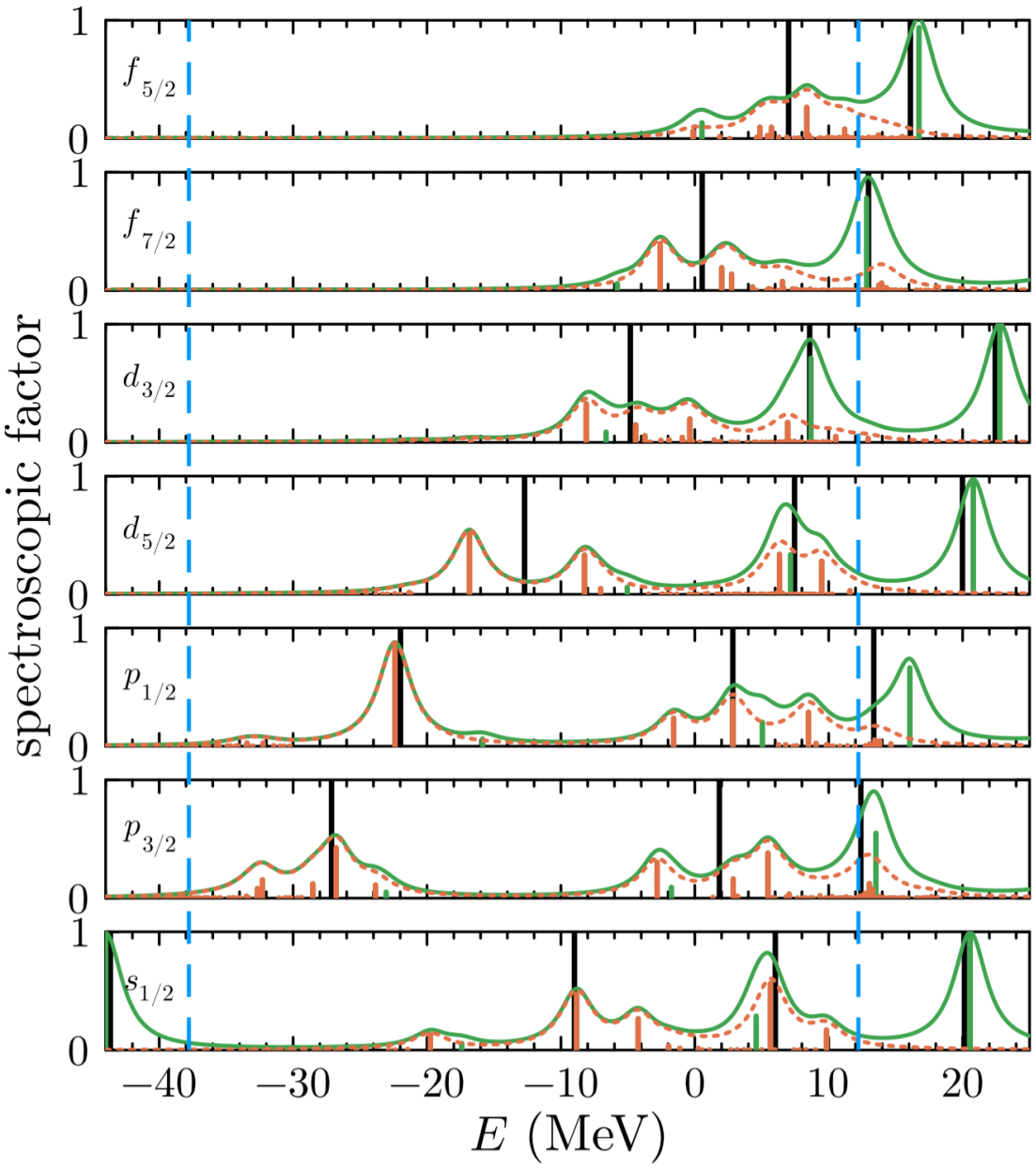
Challenge 2:
Not a complete set

— H_A

— GCM

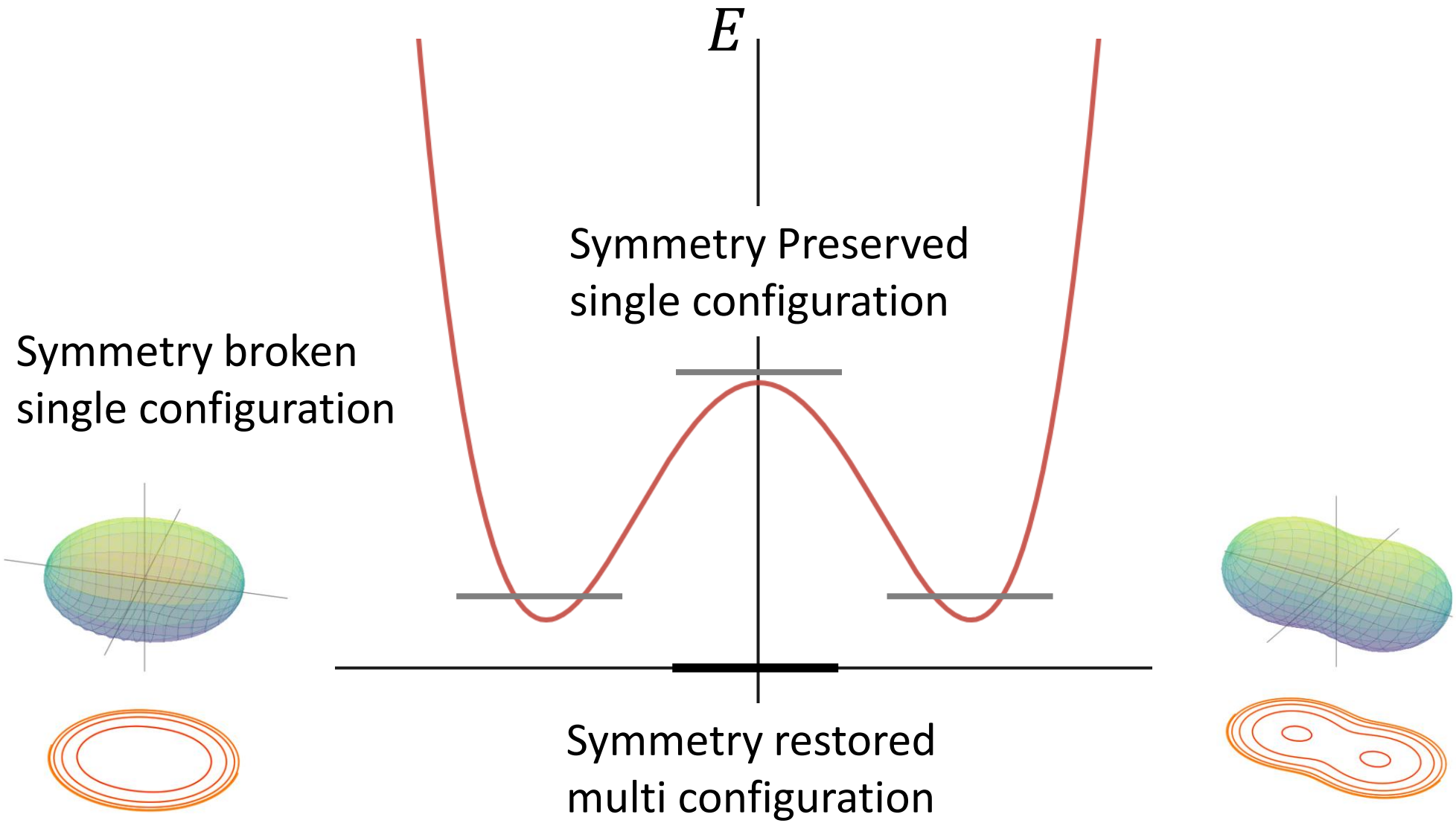
— c (+GCM)

GCM is covering a good range of the Fock space reproducing most low-energy single particle excitations



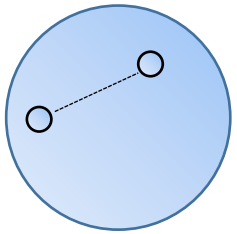
Boström, Carlsson, AI arXiv:2604.0103 (2026)

Symmetry Breaking



Nuclear Correlations

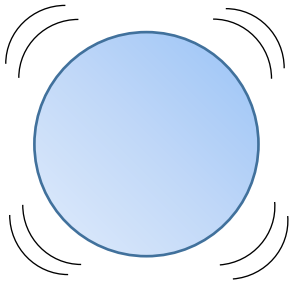
Pairing



Bogoliubov basis

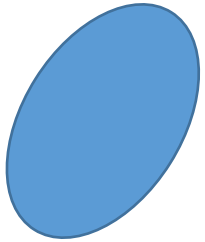
Particle number symmetry breaking

Vibrations



RPA

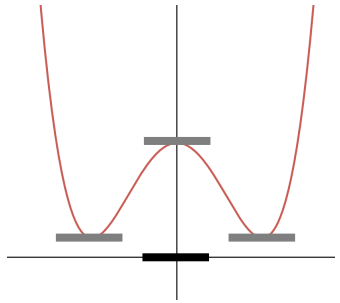
Rotations

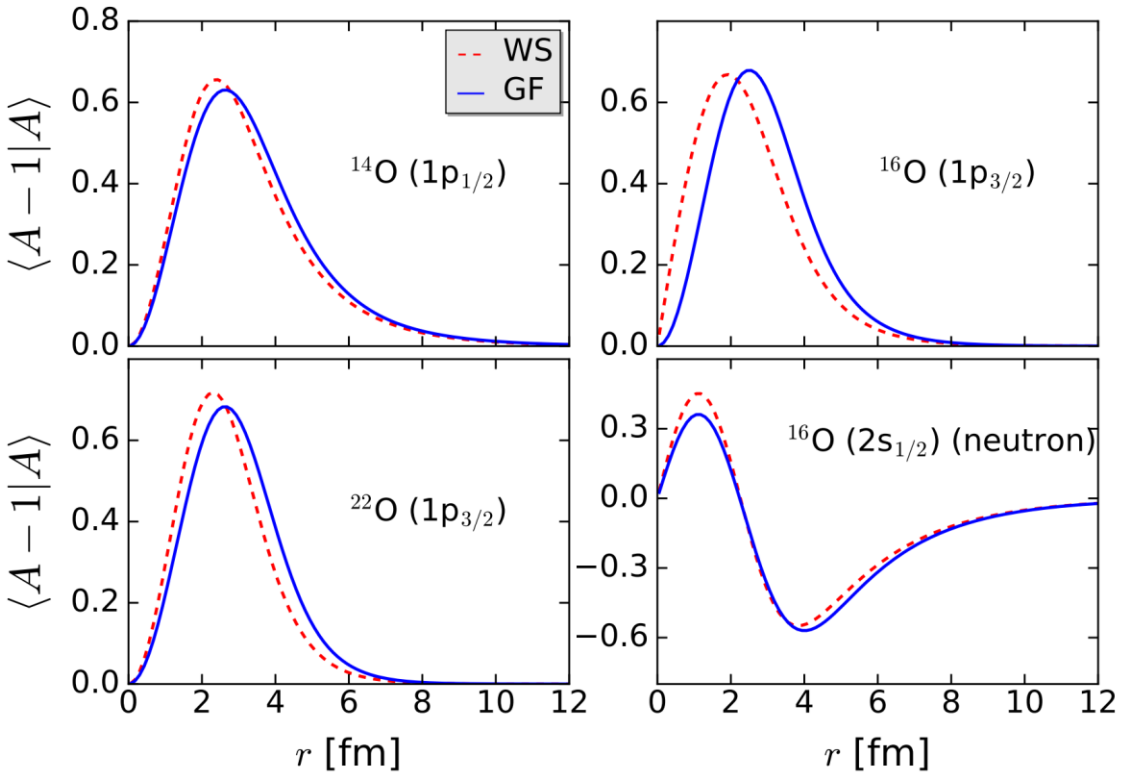


Constrained variation

Dynamic and static deformation symmetry breaking

Projection to restore “good” symmetries of the Hamiltonian

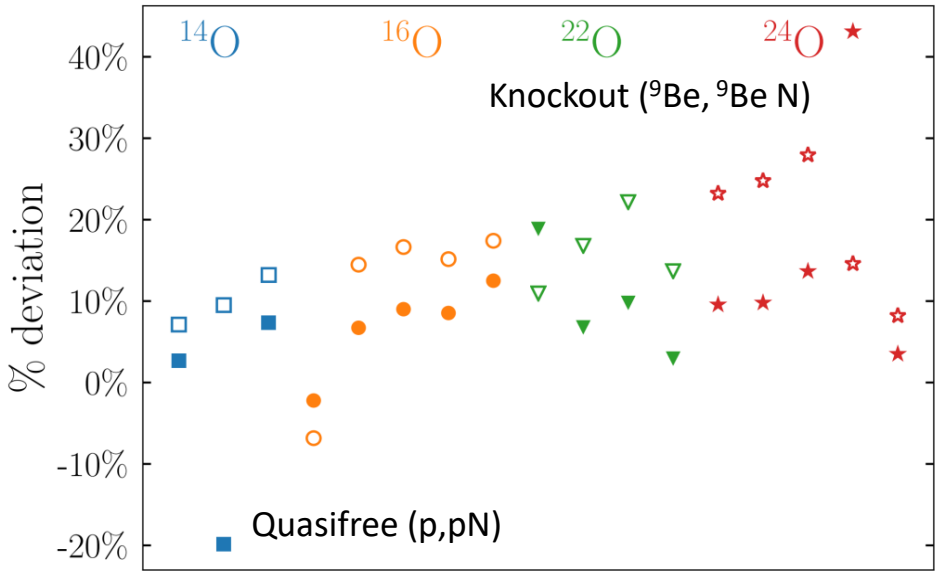


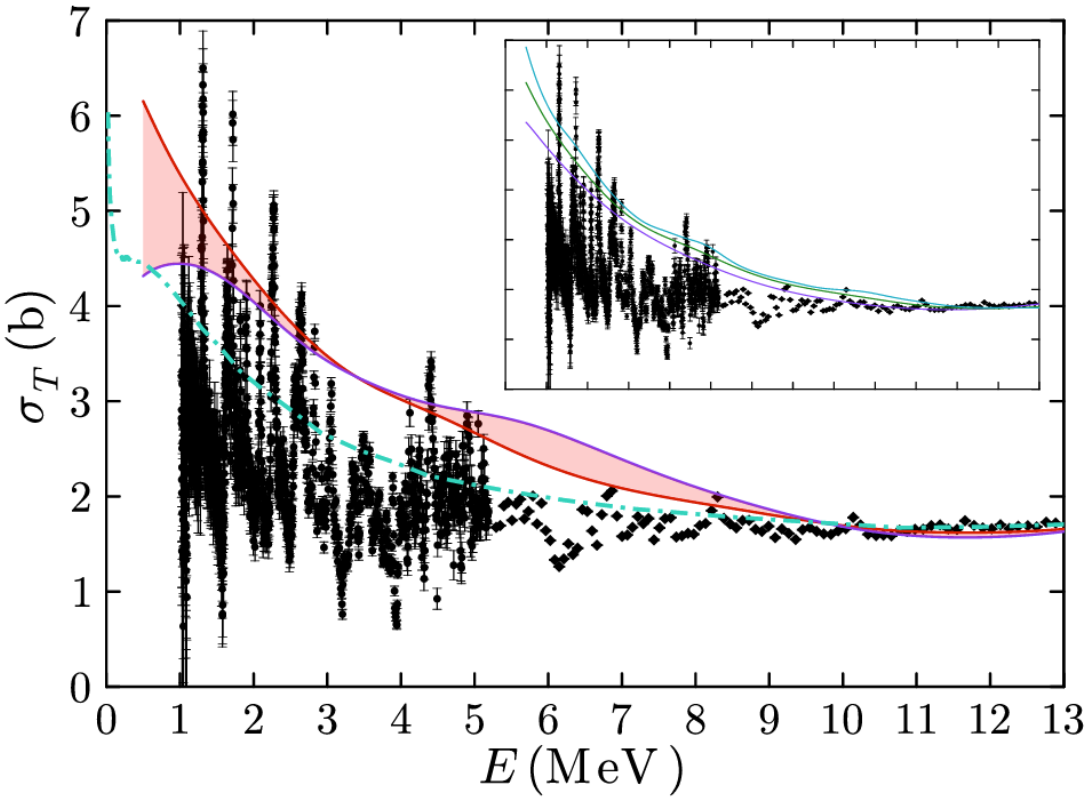


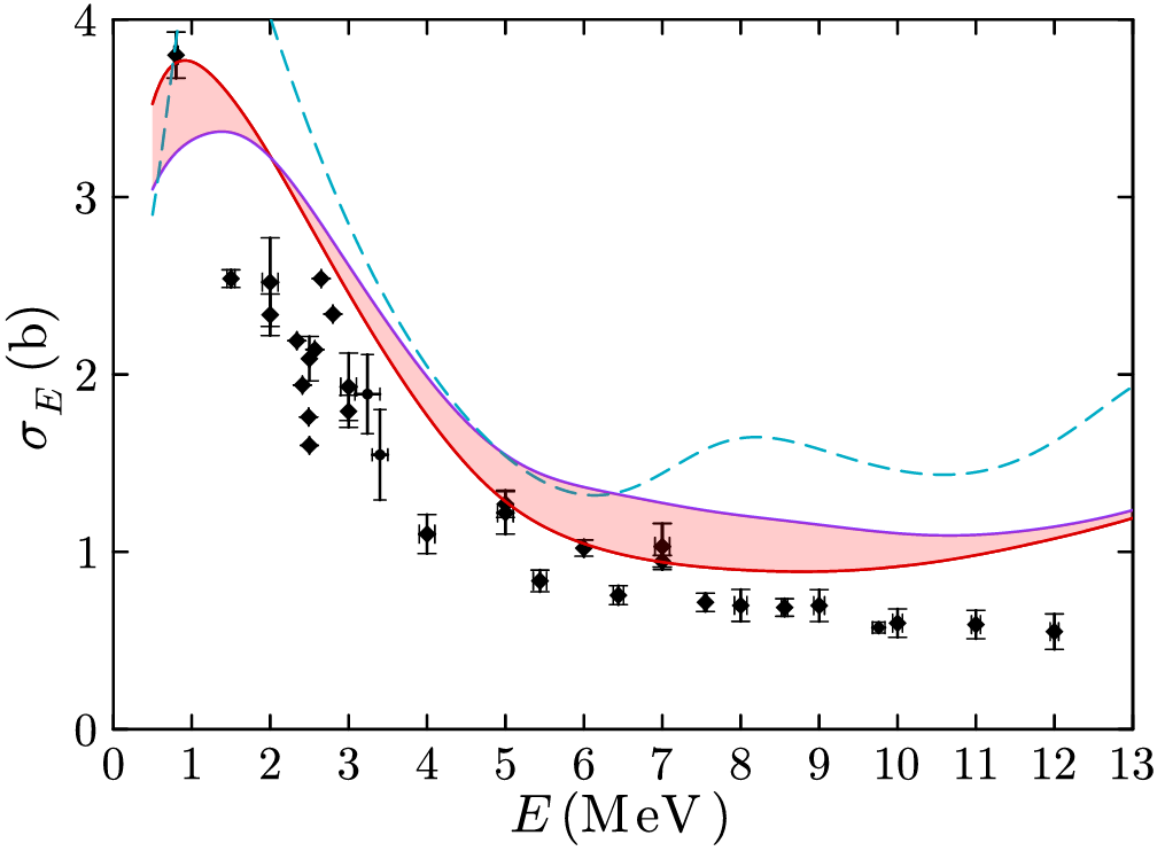
$$\sigma = S(lj)\frac{2\pi}{2j+1}\sum_m \left\langle \frac{d\sigma_{pN}}{d\Omega} \right\rangle_{o.s.} |\mathcal{C}_{lm}|^2$$

$$\times \int_0^\infty db\, b |\langle \mathcal{S}(b)\rangle_{o.s.}|^2 \int_{-\infty}^{\infty} dz \left| \frac{u_{lj}(r)}{r} P_{lm}(b,z) \right|^2 .$$

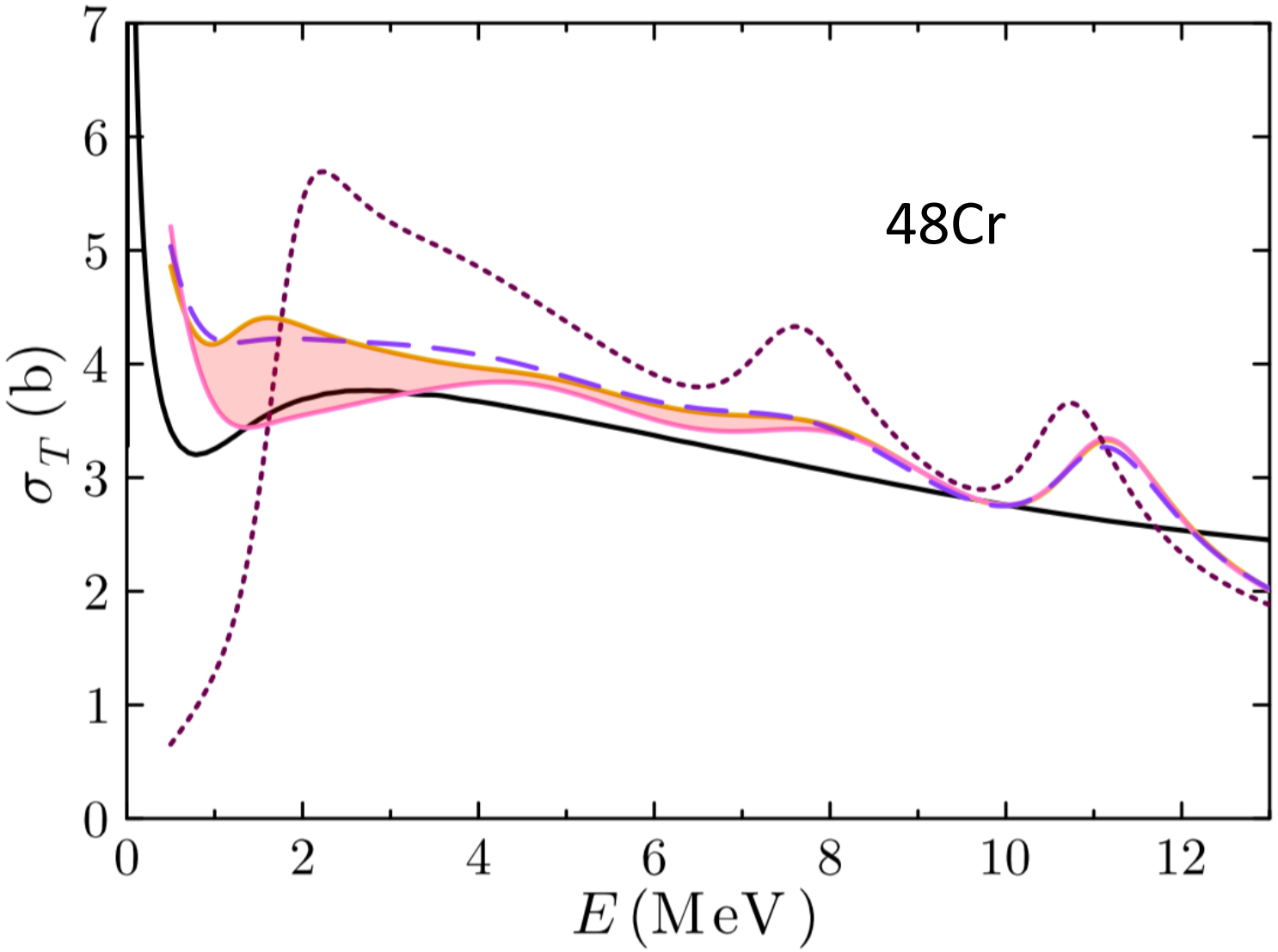
Overlap function for
Correlated or independent particle

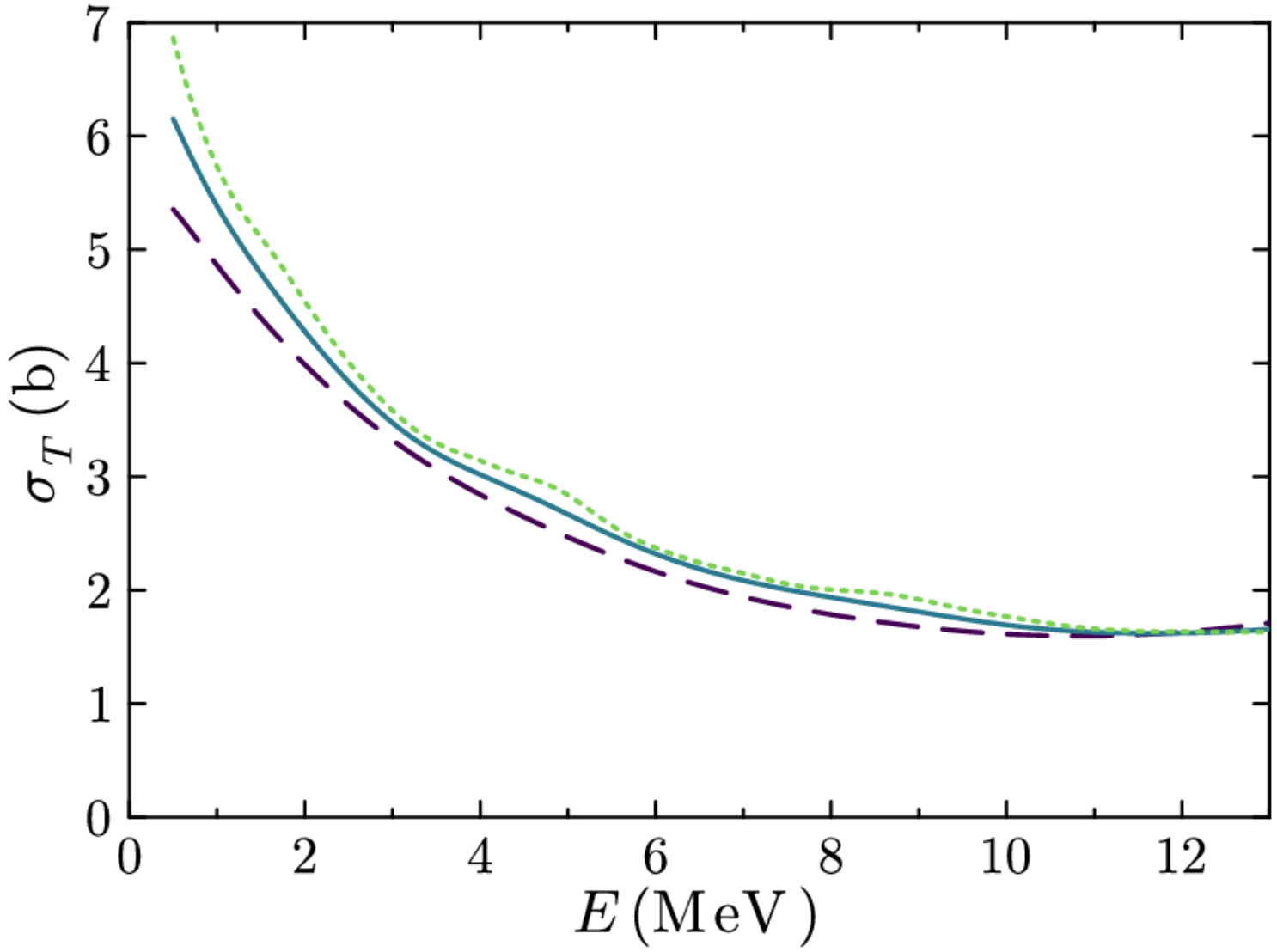




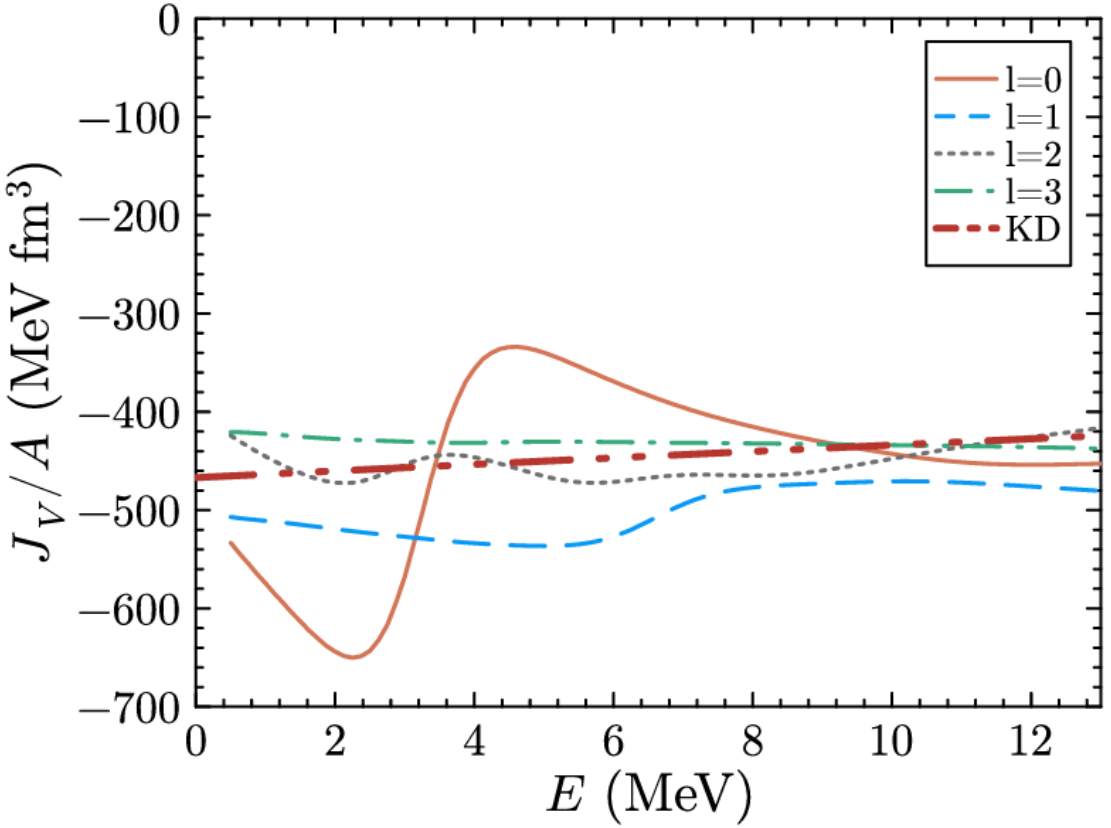


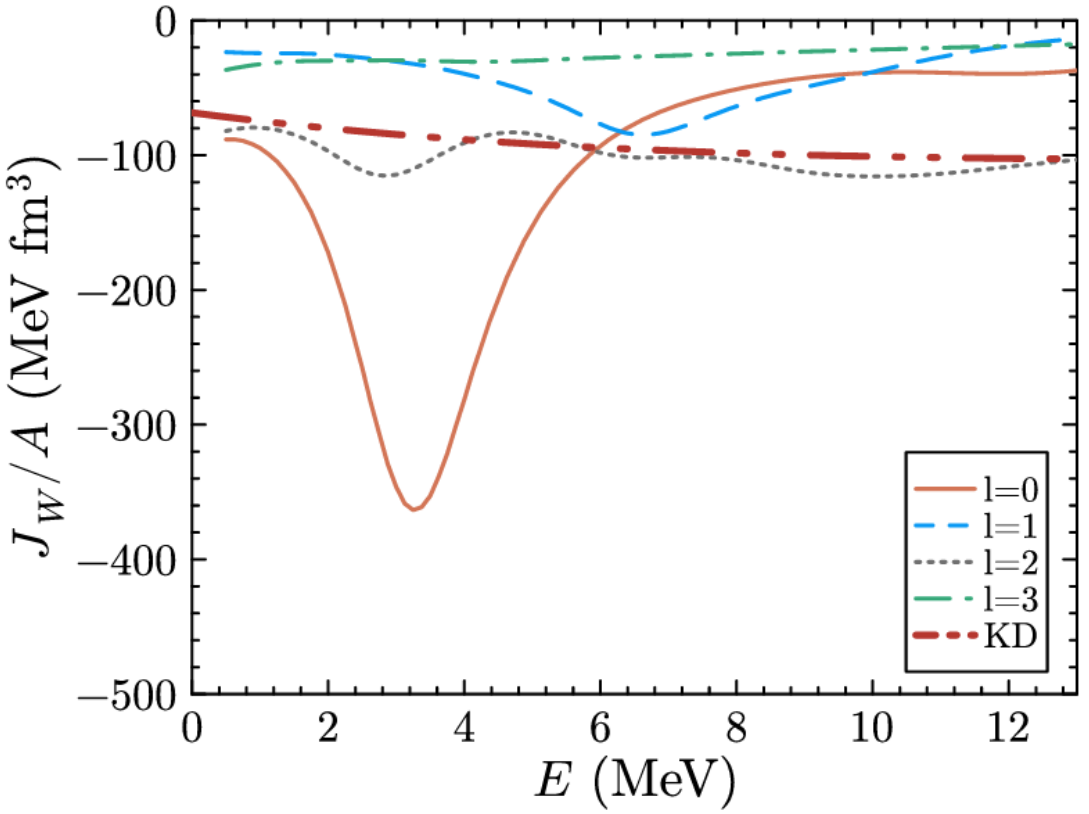
Boström, Rotureau, Carlsson, AI, PRC 112, L051602 (2025)


48Cr
σ_T (b)
E (MeV)
0
1
2
3
4
5
6
7
0
2
4
6
8
10
12



Mg24, Ncut=10, eta=6,12,24 MeV





Volume integrals, 50Cr compared to global Koning Delaroche optical potential

BSkG-H3, With 3 Body, 3/11/2026 24Mg calculated and experimental electromagnetic structure

