

Neutrino Mass Generation

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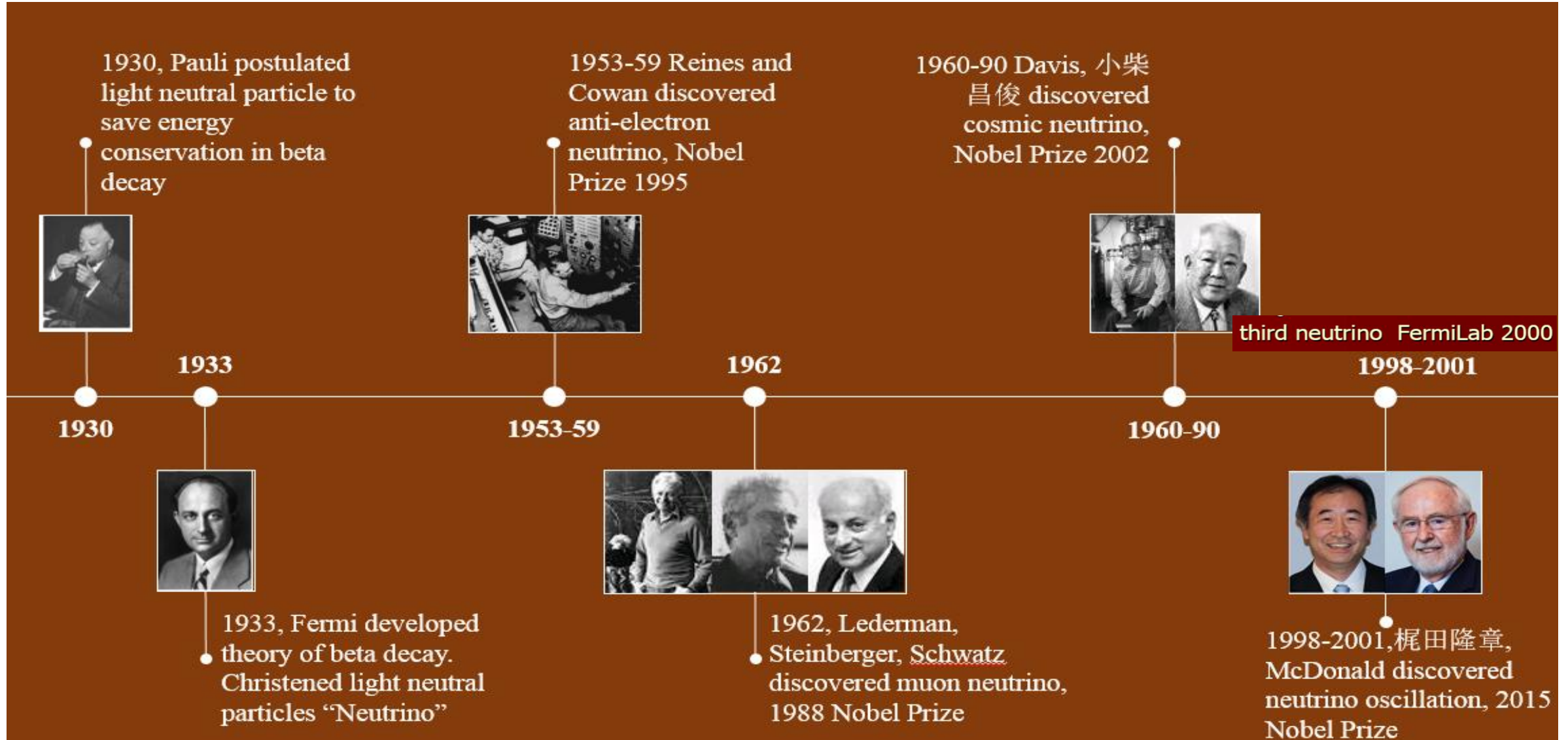
1. Neutrino mass and neutrino oscillations

2. Neutrino mass generation mechanism

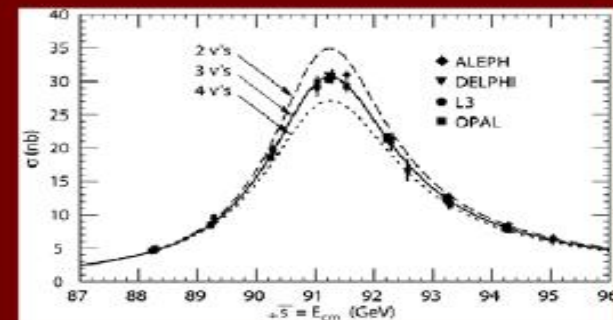
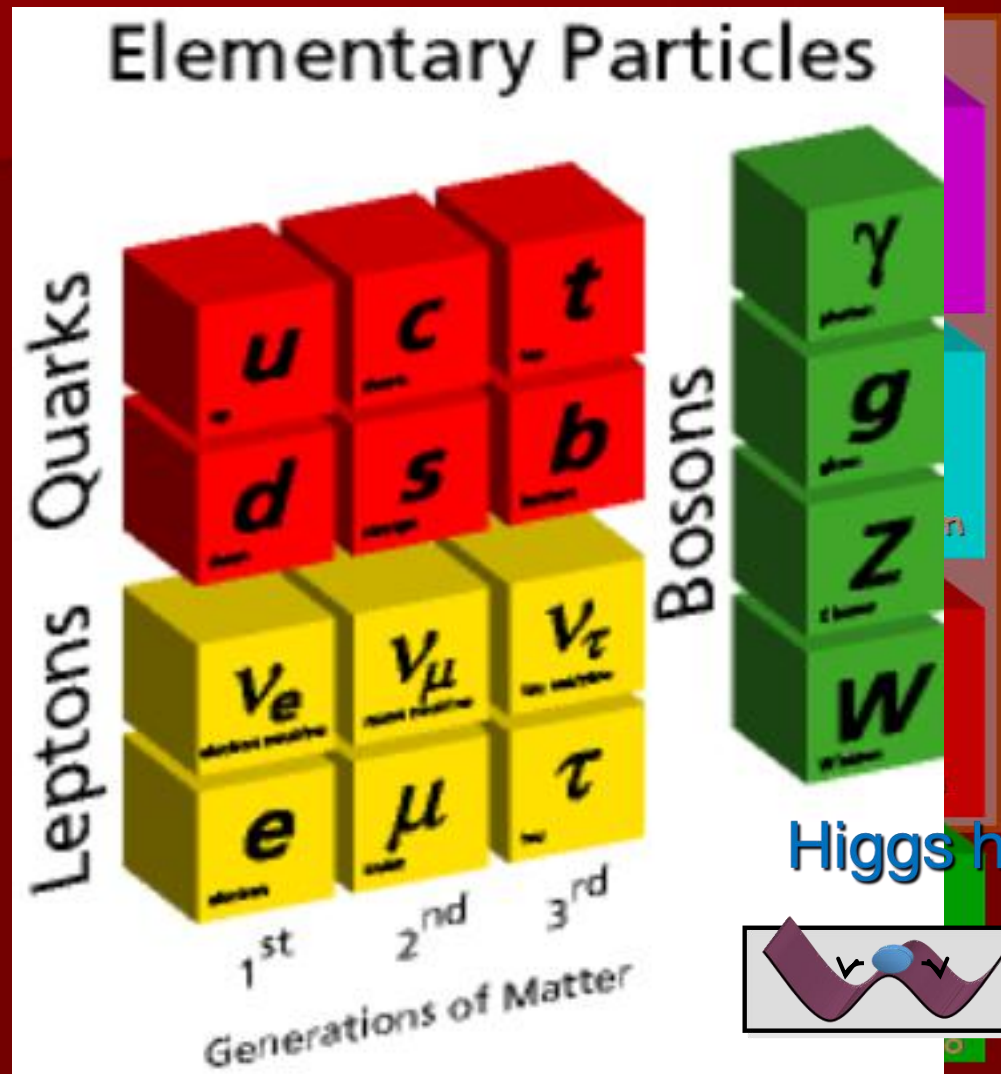
3. Neutrinos In Cosmology And Astrophysics

1. Neutrino Mass and Neutrino Oscillations

A brief neutrino history



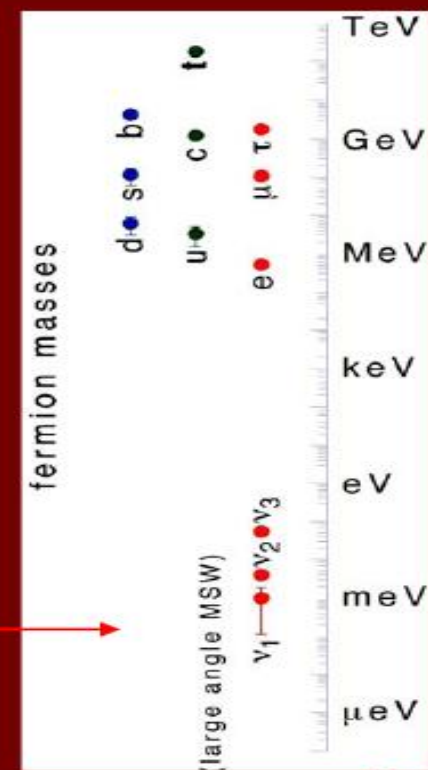
Nature's building blocks








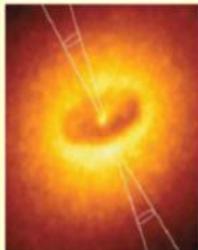

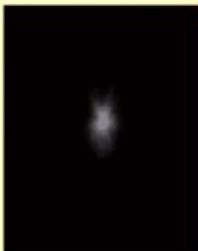
Three flavors or generations, and no more, and we do not know why.

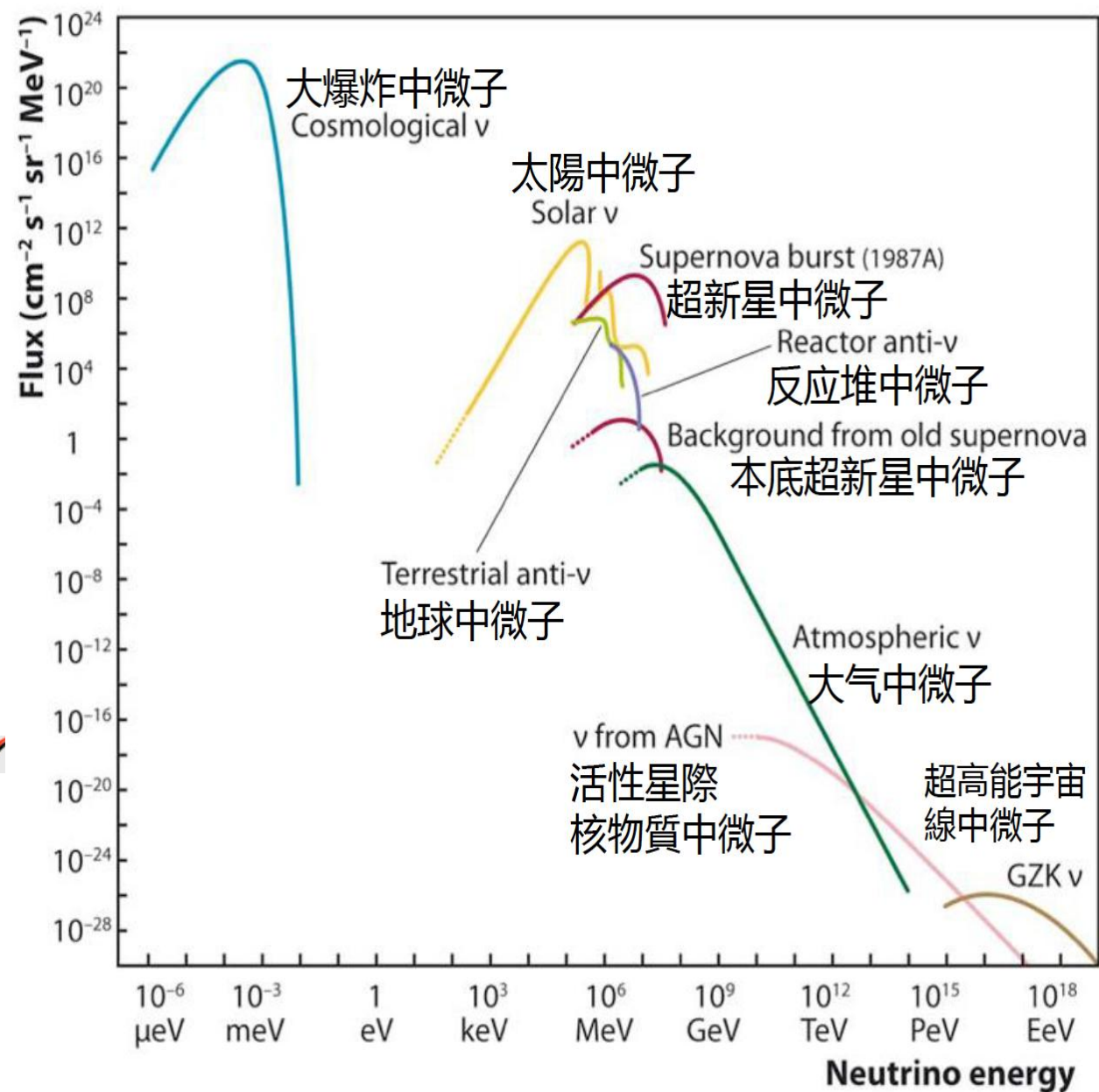
Cosmology BBN also implies 3 generations of neutrinos

Some mass, but curiously little.



Where neutrinos come from ?

✓ Nuclear Reactors (power stations, ships)			Sun
✓ Particle Accelerator			Supernovae (star collapse) SN 1987A ✓
✓ Earth's Atmosphere (Cosmic Rays)			Astrophysical Accelerators
✓ Earth's Crust (Natural Radioactivity)			Big Bang (here 330 v/cm ³) Indirect Evidence



Although weakly interacting, can be seen almost everywhere!

Neutrino Mixing

B. Pontecorvo (1957). "Mesonium and anti-mesonium". *Zh. Eksp. Teor. Fiz.* 33: 549–551.

Oscillation between neutrino and anti-neutrino.

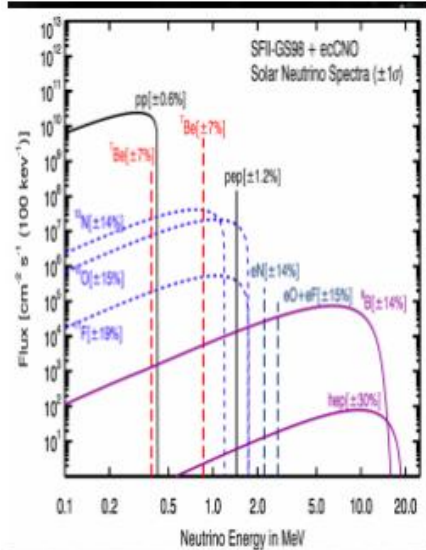
Z. Maki, M. Nakagawa, and S. Sakata (1962).
"Remarks on the Unified Model of Elementary Particles".
Progress of Theoretical Physics 28 (5): 870.

B. Pontecorvo (1967).
"Neutrino Experiments and the Problem of Conservation
of Leptonic Charge". *Zh. Eksp. Teor. Fiz.* 53: 1717.

Oscillations between different flavors.



Experimental Discovery Of Nuetrino Mixing

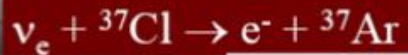


Homestake Gold Mine

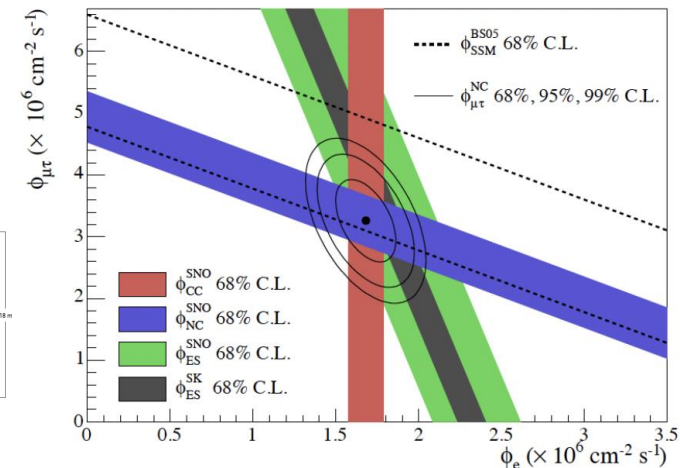
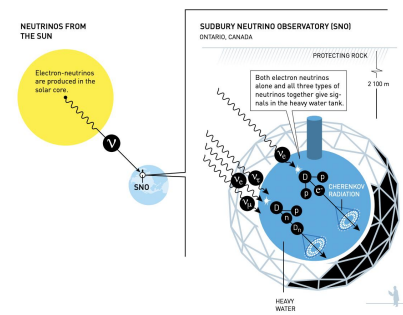
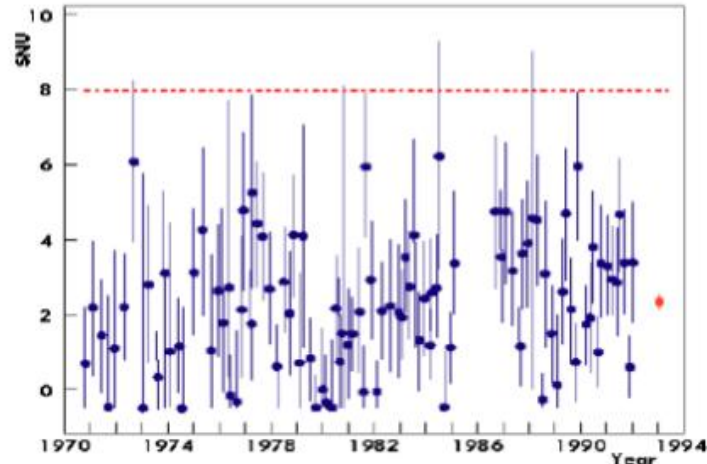
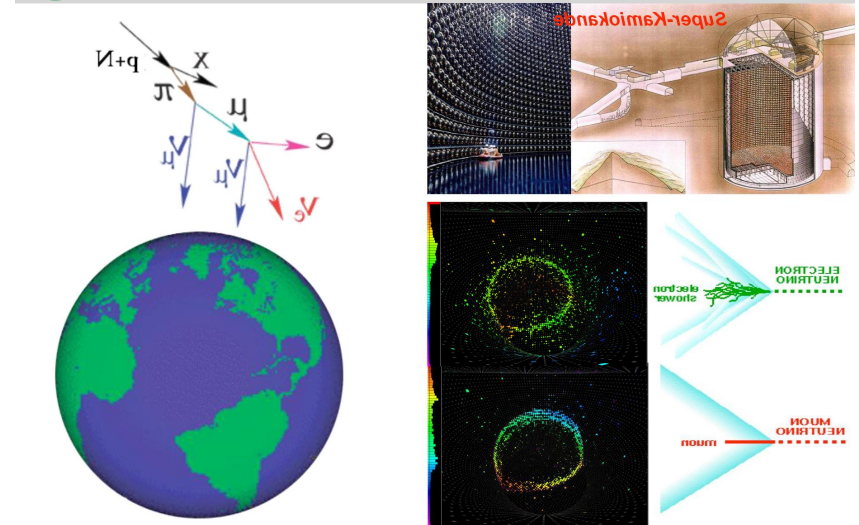
100,000 gallons of cleaning fluid C_2Cl_4

Expected 1.5 interactions per day
Measured 0.5 interactions per day

Sensitive to ^8B solar neutrinos only



1968!



Mixing and Non-zero Neutrino Masses

$$\begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}$$

At $t=0$, neutrino $\nu_1(0)$ produced, at t , the state becomes

$$|\nu_1(0)\rangle = \cos \theta e^{-i(Et - p_1 L)} |\nu_{m1}\rangle - \sin \theta e^{-i(Et - p_2 L)} |\nu_{m2}\rangle.$$

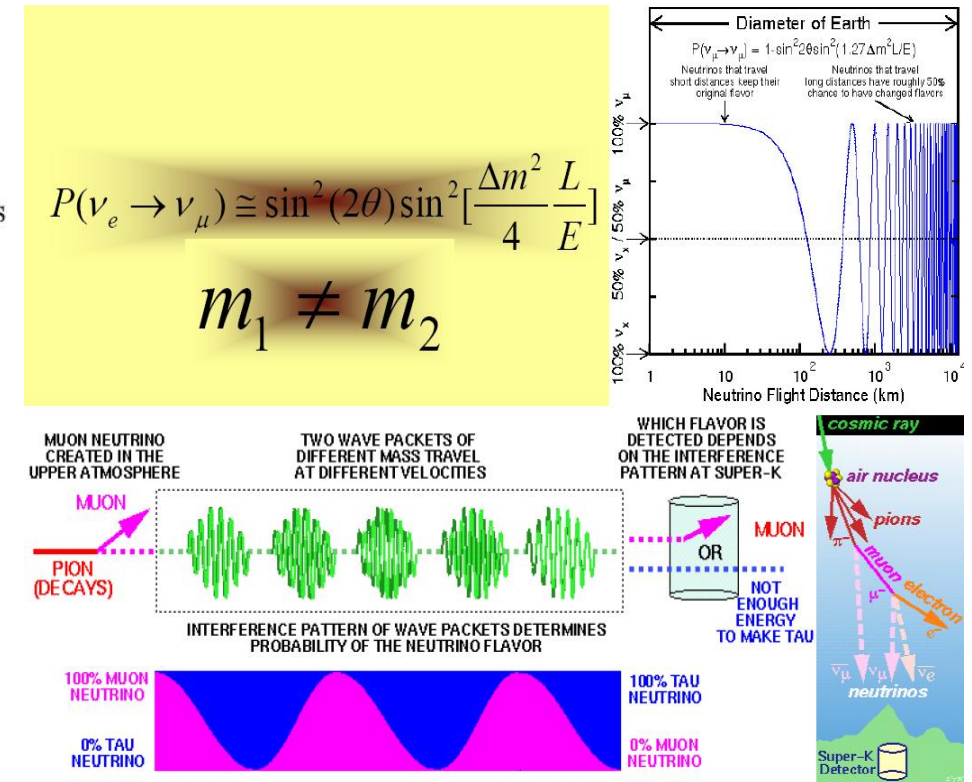
Using $t \approx L$ and $E - P_i = E - \sqrt{E^2 - m_i^2} \approx m_i^2/2E$, the probability amplitude of find $\nu_2(0)$ at time t is given by

$$\langle \nu_2(0) | \nu_1(t) \rangle = \cos \theta \sin \theta (e^{-im_1^2 L/2E} - e^{-im_2^2 L/2E})$$

which leads to the probability of finding $\nu_2(0)$ is

$$P(\nu_1 \rightarrow \nu_2) = |\langle \nu_1(0) | \nu_2(t) \rangle|^2 = \sin^2(2\theta) \sin^2(\Delta m_{21}^2 L/4E),$$

$$\Delta m_{21}^2 = m_2^2 - m_1^2.$$



Need mixing and non-zero mass to have oscillation!

Active neutrinos mix with each other and have non-zero masses!
Must go beyond SM to have neutrino masses!

2. Neutrion mass generation mechanisms

Neutrino mass, what mass?

Gravitational mass or inertia mass? Equivalence principle, they are the same.

$E = mc^2$ hadrons due to color confinement, not all masses of hadrons are due to quark masses. For neutrinos, no confinement, all masses are due to masses in the primary SM Lagrangian.

Masses due to Higgs mechanism, may be “bare” masses?

Neutrinos can have “bare” masses which is not due to the usual mechanism like those other SM particles, quarks, W and Z from the VEV of a Higgs doublet.

Neutrinos are special due to neutral electric charge, it can have masses not due to the usual Higgs mechanism, due to Higgs doublet non-zero VEV. Can have Dirac mass or Majorana mass!

In the minimal SM neutrino mass is zero

In the minimal SM: Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$G(8, 1) (0), W(1, 3) (0), B(1, 1)(0) ,$$

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} (3, 2)(1/6) , \quad U_R (3, 1)(2/3) , \quad D_R (3, 1)(-1/3) ,$$

$$L_l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} (1, 2)(-1/2) , \quad E_R (1, 1)(-1) ,$$

$$H = \begin{pmatrix} h^+ \\ (v + h^0)/\sqrt{2} \end{pmatrix} (1, 2, 1/2) , \quad v - \text{vev of Higgs} .$$

Quark and charged lepton masses are from the following Yukawa couplings

$$\bar{Q}_L \tilde{H} U_R , \quad \bar{Q}_L H D_R , \quad \bar{L}_L H E_R .$$

Nothing to pair up with $L_L(\nu_L)$. In minimal SM, neutrinos are massless!

Extensions needed: Give neutrino masses and small ones!

Including higher dimension operator:

$\frac{H H L L c}{\Lambda}$ the Weinberg operator $\Rightarrow (v^2/\Lambda) \nu_L \nu_L^c$ Majorana mass

This term is not renormalizable!! Can one construct a renormalizable one???

How To Generate Neutrino Masses?

Introduce right handed neutrinos ν_R : (1,1)(0) to pair up with L to form Dirac mass: Dirac neutrino mass term

$$L = -\bar{L}_L Y_\nu \tilde{H} \nu_R + H.C. \rightarrow -\bar{\nu}_L m_\nu \nu_R \rightarrow m_\nu = \frac{v}{\sqrt{2}} Y_\nu$$
$$m_{\nu_e} < 0.3 \text{ eV}, \rightarrow Y_{\nu_e}/Y_e < 10^{-5}, \text{ very much fine tuned!}$$

Neutrino is its own anti-particle, particle and anti-particle to pair up for mass term: $\bar{\nu}_R^c M_R \nu_R / 2$ gauge invariant should be included.

One can also make the above a Higgs-like generation of mass. Introduce a singlet scalar S couple to $\bar{\nu}_R^c M_R \nu_R / 2$ S/vs. When S develops a VEV vs, generate the ν_R mass

This term is not directly related to the usual Higgs mechanism, no need to have Higgs H to give mass. It is a “bare” mass.

Non-zero M_R is called Majorana mass.



How to generation small neutrino masses?

Dirac neutrino mass term

$$L = -\bar{L}_L Y_\nu \tilde{H} \nu_R + H.C, \rightarrow -\bar{\nu}_L m_\nu \nu_R, \rightarrow m_\nu = \frac{v}{\sqrt{2}} Y_\nu$$

$$m_{\nu_e} < 0.3 \text{ eV}, \rightarrow Y_{\nu_e}/Y_e < 10^{-5}, \text{ very much fine tuned!}$$

Fine tune Yukawa couplings
Not natural

The Seesaw Mechanism $L = \bar{\nu}_L (Y_\nu v / \sqrt{2}) \nu_R + \bar{\nu}_R^c M_R \nu_R / 2$

The Seesaw mechanism refers to the neutrino mass matrix of the form

$$L_m = -\frac{1}{2} (\nu_L^c, \nu_R) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}.$$

For one generation, if $M_R \gg m_D$, the eigenmasses are

$$m_\nu \approx -m_D M_R^{-1} m_D^T, \quad m_N \approx M_R$$



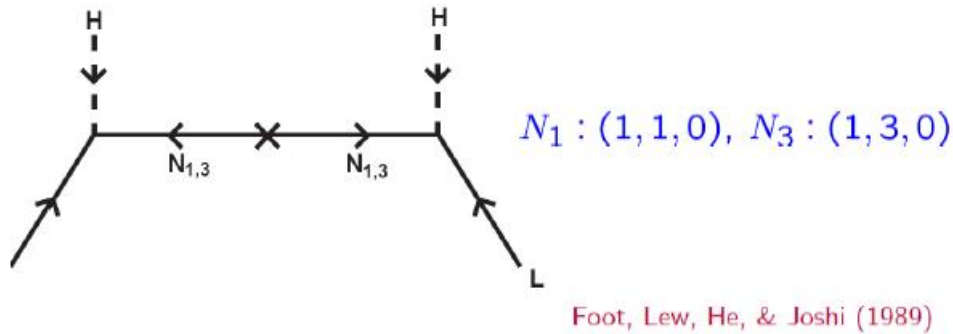
A very nice way to explain why light neutrino masses are so much lighter than their charged lepton partners.

(Minkowski (1977); Gell-Mann, Ramond, and Slansky (1979); Yanagida (1979); Glashow (1980); Mohapatra and Senjanovic(1980))

Majorana Neutrinos and Seesaw Mechanism

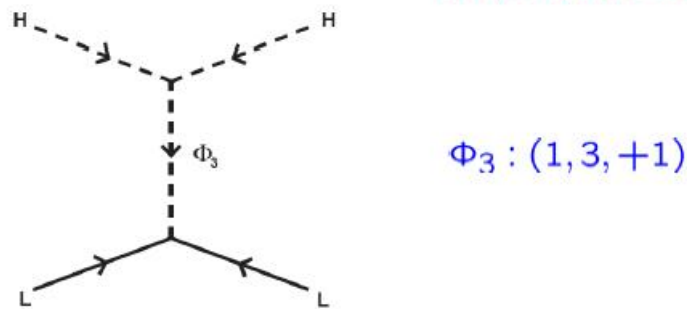
Type (I,III) seesaw

Minkowski (1977)
Yanagida (1979)
Gell-Mann, Ramond, & Slansky (1980)
Mohapatra & Senjanovic (1980)



Type II seesaw

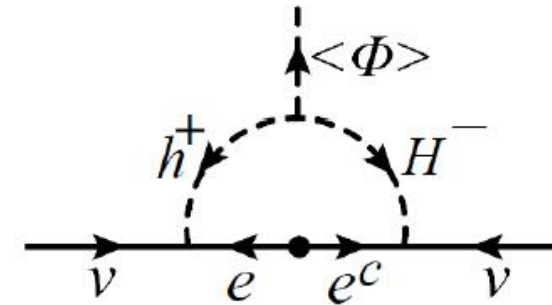
Mohapatra & Senjanovic (1980)
Schechter & Valle (1980)
Lazarides, Shafi, & Wetterich (1981)



$$\mathcal{L}_{\text{eff}} = \frac{LLHH}{M} \Rightarrow m_\nu \sim \frac{v^2}{M}$$

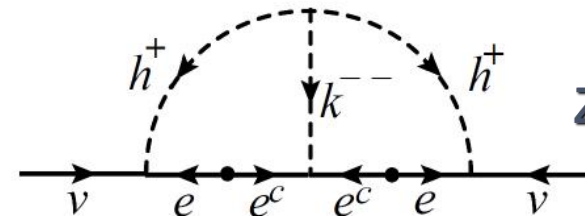
Loop generation neutrino masses

Zee, 1980



Simplest model ruled out
XG He, EPJC34(2004)371

Zee, 1985, Babu 1988



Provide understanding why neutrino masses are small
but no information about mixing!

Three Generations and Their Mixing

Quark mixing the Cabibbo -Kobayashi-Maskawa (CKM) matrix V_{CKM} ,
 lepton mixing the Pontecorvo -Maki-Nakawaga-Sakata (PMNS) matrix U_{PMNS}

$$L = -\frac{g}{\sqrt{2}}\bar{U}_L\gamma^\mu V_{\text{CKM}}D_LW_\mu^+ - \frac{g}{\sqrt{2}}\bar{E}_L\gamma^\mu U_{\text{PMNS}}N_LW_\mu^- + H.C. ,$$

$$U_L = (u_L, c_L, t_L, \dots)^T, D_L = (d_L, s_L, b_L, \dots)^T, E_L = (e_L, \mu_L, \tau_L, \dots)^T, \text{ and } N_L = (\nu_1, \nu_2, \nu_3, \dots)^T$$

For n-generations, $V = V_{\text{CKM}}$ or U_{PMNS} is an $n \times n$ unitary matrix.

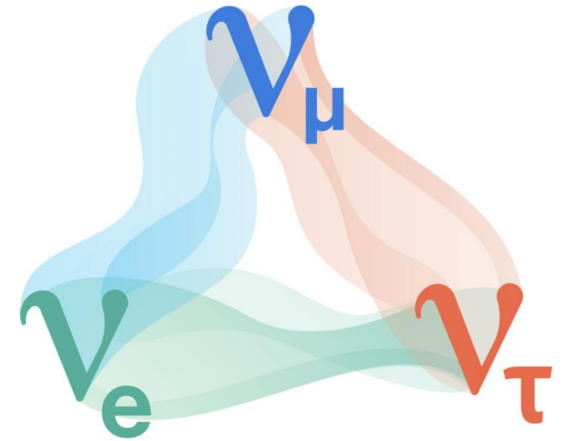
A commonly used form of mixing matrix for three generations of fermions is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal

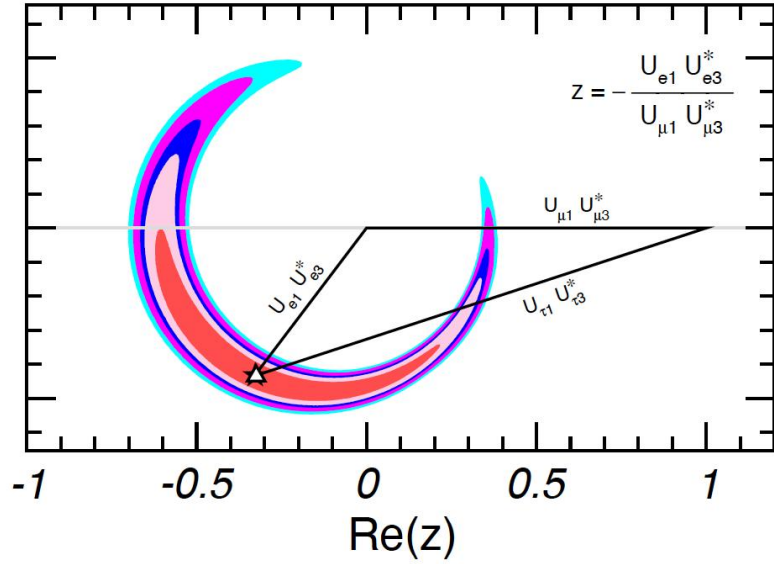
matrix with two Majorana phases $\text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.



Neutrino Oscillation With Three Generations

$$\begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{\text{CP}}} \\ -s_{12} c_{23} - c_{12} s_{13} s_{23} e^{i\delta_{\text{CP}}} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta_{\text{CP}}} & c_{13} s_{23} \\ s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta_{\text{CP}}} & -c_{12} s_{23} - s_{12} s_{13} c_{23} e^{i\delta_{\text{CP}}} & c_{13} c_{23} \end{pmatrix}$$

Table 14.7: 3ν oscillation parameters obtained from different global analyses of neutrino data. In all cases, the numbers labeled as NO (IO) are obtained assuming NO (IO), *i.e.*, relative to the respective local minimum. SK-ATM makes reference to the tabulated χ^2 map from the Super-Kamiokande analysis of their data in Ref. [184].



	Ref. [181] w/o SK-ATM		Ref. [181] w SK-ATM		Ref. [182] w SK-ATM		Ref. [183] w SK-ATM	
NO	Best Fit Ordering		Best Fit Ordering		Best Fit Ordering		Best Fit Ordering	
Param	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2 \theta_{12}$	$3.03^{+0.12}_{-0.11}$	$2.70 \rightarrow 3.41$	$3.03^{+0.12}_{-0.12}$	$2.70 \rightarrow 3.41$	$3.03^{+0.13}_{-0.13}$	$2.63 \rightarrow 3.45$	$3.18^{+0.16}_{-0.16}$	$2.71 \rightarrow 3.69$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \rightarrow 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \rightarrow 35.74$	$33.40^{+0.80}_{-0.82}$	$30.85 \rightarrow 35.97$	$34.3^{+1.0}_{-1.0}$	$31.4 \rightarrow 37.4$
$\sin^2 \theta_{23}$	$5.72^{+0.18}_{-0.23}$	$4.06 \rightarrow 6.20$	$4.51^{+0.19}_{-0.16}$	$4.08 \rightarrow 6.03$	$4.55^{+0.18}_{-0.15}$	$4.16 \rightarrow 5.99$	$5.74^{+0.14}_{-0.14}$	$4.34 \rightarrow 6.10$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \rightarrow 51.9$	$42.2^{+1.1}_{-0.9}$	$39.7 \rightarrow 51.0$	$42.4^{+1.0}_{-0.9}$	$40.2 \rightarrow 50.7$	$49.3^{+0.8}_{-0.8}$	$41.2 \rightarrow 51.3$
$\sin^2 \theta_{13}$	$2.203^{+0.056}_{-0.059}$	$2.029 \rightarrow 2.391$	$2.225^{+0.056}_{-0.059}$	$2.052 \rightarrow 2.398$	$2.23^{+0.07}_{-0.06}$	$2.04 \rightarrow 2.44$	$2.200^{+0.069}_{-0.062}$	$2.00 \rightarrow 2.405$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \rightarrow 8.89$	$8.58^{+0.11}_{-0.11}$	$8.23 \rightarrow 8.91$	$8.59^{+0.13}_{-0.12}$	$8.21 \rightarrow 8.99$	$8.53^{+0.13}_{-0.12}$	$8.13 \rightarrow 8.92$
$\delta_{\text{CP}}/^\circ$	197^{+42}_{-25}	$108 \rightarrow 404$	232^{+36}_{-26}	$144 \rightarrow 350$	223^{+32}_{-23}	$139 \rightarrow 355$	194^{+24}_{-22}	$128 \rightarrow 359$
Δm_{21}^2	$7.41^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.03$	$7.36^{+0.16}_{-0.15}$	$6.93 \rightarrow 7.93$	$7.50^{+0.22}_{-0.20}$	$6.94 \rightarrow 8.14$
Δm_{32}^2	$2.437^{+0.028}_{-0.027}$	$2.354 \rightarrow 2.523$	$2.433^{+0.026}_{-0.027}$	$2.353 \rightarrow 2.516$	$2.448^{+0.023}_{-0.031}$	$2.367 \rightarrow 2.521$	$2.47^{+0.02}_{-0.03}$	$2.40 \rightarrow 2.46$
IO	$\Delta\chi^2 = 2.3$		$\Delta\chi^2 = 6.4$		$\Delta\chi^2 = 6.5$		$\Delta\chi^2 = 6.4$	
$\sin^2 \theta_{12}$	$3.03^{+0.12}_{-0.11}$	$2.70 \rightarrow 3.41$	$3.03^{+0.12}_{-0.11}$	$2.70 \rightarrow 3.41$	$3.03^{+0.13}_{-0.13}$	$2.63 \rightarrow 3.45$	$3.18^{+0.16}_{-0.16}$	$2.71 \rightarrow 3.69$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \rightarrow 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \rightarrow 35.74$	$33.40^{+0.80}_{-0.82}$	$30.85 \rightarrow 35.97$	$34.3^{+1.0}_{-1.0}$	$31.4 \rightarrow 37.4$
$\sin^2 \theta_{23}$	$5.78^{+0.16}_{-0.21}$	$4.12 \rightarrow 6.23$	$5.69^{+0.16}_{-0.21}$	$4.12 \rightarrow 6.13$	$5.69^{+0.13}_{-0.21}$	$4.17 \rightarrow 6.06$	$5.78^{+0.10}_{-0.17}$	$4.33 \rightarrow 6.08$
$\theta_{23}/^\circ$	$49.5^{+0.9}_{-1.2}$	$39.9 \rightarrow 52.1$	$49.0^{+1.0}_{-1.2}$	$39.9 \rightarrow 51.5$	$49.0^{+0.7}_{-1.4}$	$40.2 \rightarrow 51.1$	$49.5^{+0.6}_{-1.0}$	$41.2 \rightarrow 51.2$
$\sin^2 \theta_{13}$	$2.219^{+0.060}_{-0.057}$	$2.047 \rightarrow 2.396$	$2.223^{+0.058}_{-0.058}$	$2.048 \rightarrow 2.416$	$2.23^{+0.06}_{-0.06}$	$2.03 \rightarrow 2.45$	$2.225^{+0.064}_{-0.070}$	$2.02 \rightarrow 2.42$
$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.11}$	$8.23 \rightarrow 8.90$	$8.57^{+0.11}_{-0.11}$	$8.23 \rightarrow 8.94$	$8.59^{+0.13}_{-0.12}$	$8.19 \rightarrow 9.00$	$8.58^{+0.12}_{-0.14}$	$8.17 \rightarrow 8.96$
$\delta_{\text{CP}}/^\circ$	286^{+27}_{-32}	$192 \rightarrow 360$	276^{+22}_{-29}	$194 \rightarrow 344$	274^{+25}_{-27}	$193 \rightarrow 342$	284^{+26}_{-28}	$200 \rightarrow 353$
Δm_{21}^2	$7.41^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.03$	$7.36^{+0.16}_{-0.15}$	$6.93 \rightarrow 7.93$	$7.50^{+0.22}_{-0.20}$	$6.94 \rightarrow 8.14$
Δm_{32}^2	$-2.498^{+0.032}_{-0.025}$	$-2.581 \rightarrow -2.408$	$-2.486^{+0.028}_{-0.025}$	$-2.570 \rightarrow -2.406$	$-2.492^{+0.025}_{-0.030}$	$-2.578 \rightarrow -2.413$	$-2.52 \pm^{+0.03}_{-0.02}$	$-2.60 \rightarrow -2.44$

Problems related to neutrino masses

Mixing pattern: Very different quark and lepton mixing patterns

Mixing pattern in quark sector

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

Mixing pattern in lepton sector

$$|U| = \begin{pmatrix} 0.801 \rightarrow 0.845 & 0.514 \rightarrow 0.580 & 0.137 \rightarrow 0.158 \\ 0.225 \rightarrow 0.517 & 0.441 \rightarrow 0.699 & 0.614 \rightarrow 0.793 \\ 0.246 \rightarrow 0.529 & 0.464 \rightarrow 0.713 & 0.590 \rightarrow 0.776 \end{pmatrix},$$

$$\theta_{12} = 13.021^\circ \pm 0.039^\circ, \quad \theta_{23}^Q = 2.350^\circ \pm 0.052^\circ,$$

$$\theta_{13} = 0.199^\circ \pm 0.008^\circ, \quad \delta^Q = 68.9^\circ$$

More precise data needed to determine mixing pattern

T2K, NOvA, JUNO, DUNE ...

Stay tuned!

Theory before and after Daya-Bay/Reno results

Before: popular mixing -The Tribimaximal Mixing

Harrison, Perkins, Scott (2002), Z-Z. Xing (2002), He& Zee (2003)

The mixing pattern is consistent, within 2σ , with the tri-bimaximal mixing

$$V_{\text{tri-bi}} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

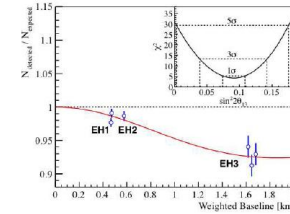
A4 a promising model (Ma&Ranjasekara, 2001) and realizations (Altarelli&Feruglio 2005, Babu&He 2005). Later many realizations: S4, D3, S3,D4, D7,A5,T',S4, $\Delta(27, 96)$, $\text{PSL}_2(7)$... discrete groups Altarelli&Feruglio for review. (H. Lam; Mohapatra et al), T. Mahanthappa&M-C. Chen; Frampton&Kephart; Y-L Wu,

After: Need to have a nonzero θ_{13}

Modification to tri-bimaximal mixing pattern need to be made. (Keum&He&Volkas; He&Zee, 2006).

In fact, more generically, A_4 symmetry leads to

$$V = \begin{pmatrix} \frac{2c}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \end{pmatrix}$$



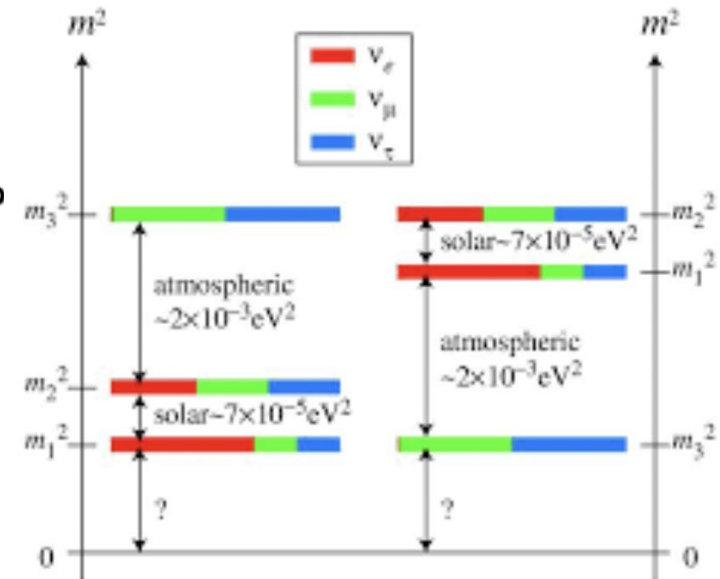
ArXiv:1203.1669 (hep-ex)

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$$

$$\text{Read as } \theta_{13} = 8.8^\circ \text{ with significance } 5.2\sigma$$

Neutrino mass hierarchy

Normal or Inverted



Mixing pattern very different than their quark counterparts. Mass hierarchy not yet determined!

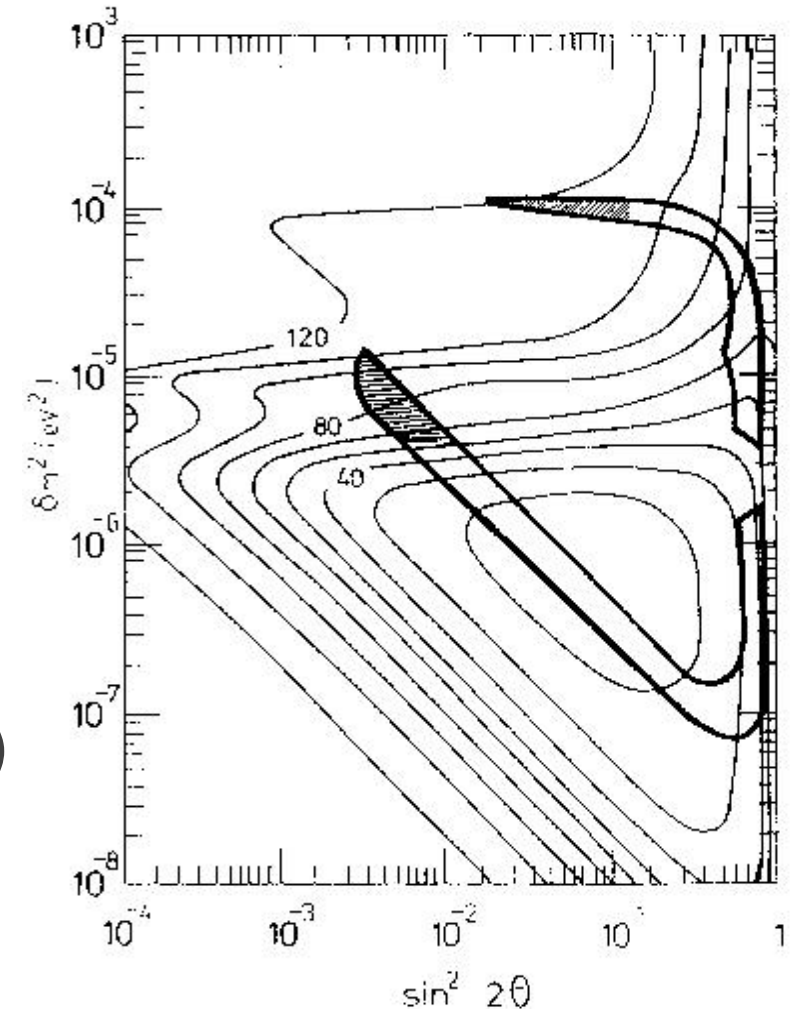
Mass hierarchy soon be answered by JUNO? Stay tuned! CP V, DUNE, HyperK ... within 10 years?

Neutrino Mixing Pattern

Early days, expecting neutrino mixing might be following a similar pattern as quarks, mixing angles are small.

For example, 1992 people are trying to produce mixing on the right

(Davies and He, PRD46, 3208)



But both solar and atmospheric show large mixing angles!

Model building for $\theta_{23} = \pi/4$ and $\delta = +(-)\pi/2$

X-G He, Chin. J. Phys. 53(2015) 100101 E Ma, PRD92(2015)051301; G-N Li, X-G He, PLB750(2015)620

In the charged lepton mass eigenstate basis, $m_\nu = V_{PMNS} \hat{m}_\nu V_{PMNS}^T$

$$\delta = -\pi/2 \text{ and } \theta_{23} = \pi/4,$$

For $\delta = +\pi/2$, $C \leftrightarrow C^*$ and $D \leftrightarrow D^*$

$$m_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}$$

$\mu - \tau$ conjugate symmetry

W. Grimus and L. Lavoura, PLB579(2004)113;
Z.-z Xing and Y. L. Zhou, PLB693(2010)584.

A A4 model to achieve this:

under A4

$$\nu_R = (\nu_R^1, \nu_R^2, \nu_R^3) \quad l_L = (l_L^1, l_L^2, l_L^3), \quad (l_R^1, l_R^2, l_R^3), \quad 3, \quad (1, 1'', 1') \text{ and } 3$$

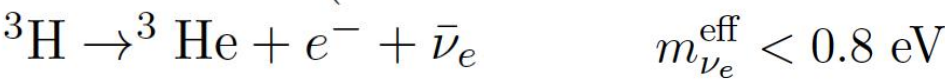
$$\Phi = (\Phi_1, \Phi_2, \Phi_3) \text{ (SM doublet)}, \quad \phi \text{ (SM doublet)} \quad \chi = (\chi_1, \chi_2, \chi_3) \text{ (SM singlet)}$$

Φ and χ both transform as 3, and ϕ as 1

$$\langle \Phi_{1,2,3} \rangle = v_\Phi, \quad \langle \chi_{1,3} \rangle = 0, \quad \langle \chi_2 \rangle = v_\chi, \quad \text{and} \quad \langle \phi \rangle = v_\phi,$$

Absolute Neutrino masses and Neutrinos Are Dirac Or Majorana Particle?

Absolute mass measurement
Tritium decay spectrum Katrin, Mainz, Troitsk..



$$\frac{dN}{dE} = C p E (Q - T) \sqrt{(Q - T)^2 - (m_{\nu_e}^{\text{eff}})^2} F(E) \equiv R(E) \sqrt{(E_0 - E)^2 - (m_{\nu_e}^{\text{eff}})^2}$$

$m_{\nu_\mu}^{\text{eff}} < 190 \text{ keV (90\% CL)}$	from	$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$
$m_{\nu_\tau}^{\text{eff}} < 18.2 \text{ MeV (95\% CL)}$	from	$\tau^- \rightarrow n\pi + \nu_\tau$

Neutrinoless double beta decay
Test Majorana mass nature

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^- \qquad (T_{1/2}^{0\nu})^{-1} = G^{0\nu} |M^{0\nu}|^2 \left(\frac{m_{ee}}{m_e}\right)^2$$

KamLAND-Zen (Xenon)	$T_{1/2}^{0\nu} > 2.3 \times 10^{26} \text{ yr,}$	$m_{ee} < 36 - 156 \text{ meV}$
---------------------	---	---------------------------------

GERDA (Germanium)	$T_{1/2}^{0\nu} > 1.8 \times 10^{26} \text{ yr, at 90\% CL}$	$m_{ee} < 79 - 180 \text{ meV}$
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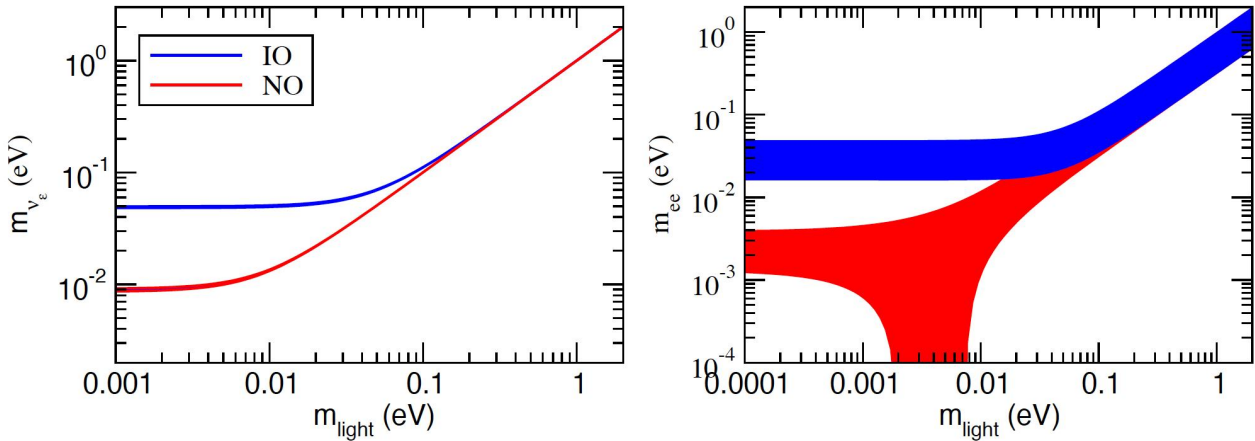
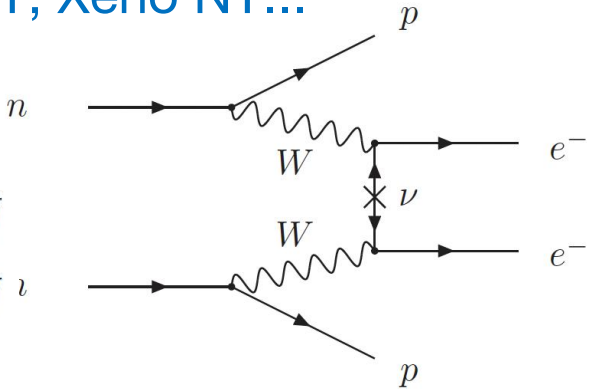


Figure 14.11: Allowed 95% CL ranges (1 dof) for the neutrino mass observable determined in ^3H beta decay (left panel) and in $0\nu\beta\beta$ (right panel) in the framework of 3ν mixing as a function of the lightest neutrino mass. The ranges are obtained by projecting the results of the global analysis of oscillation data (w/o SK-atm) in Ref. [184]. The region for each ordering is defined with respect to its local minimum.

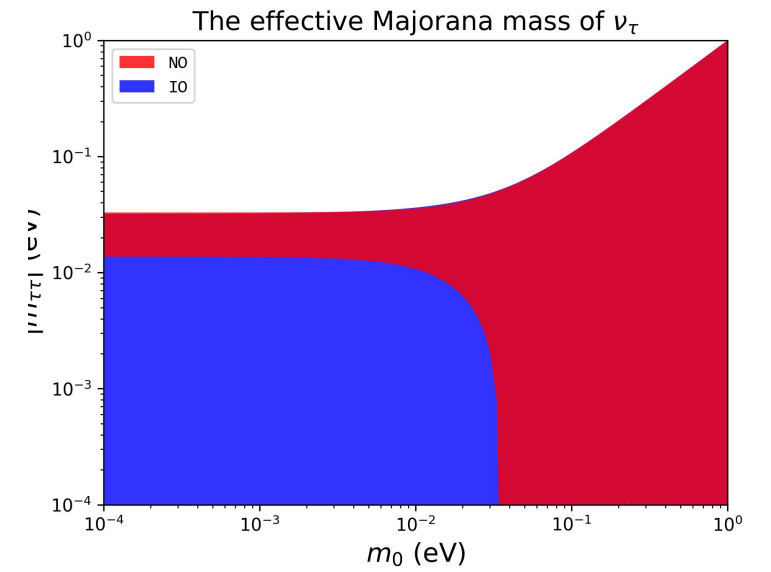
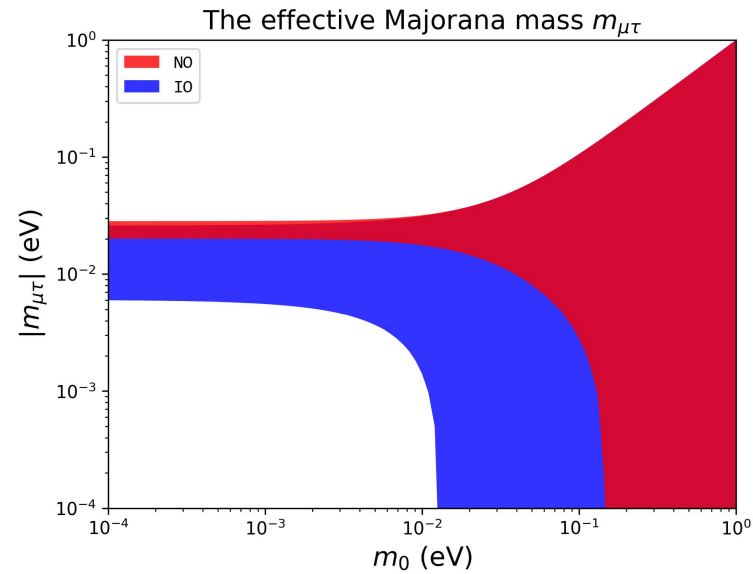
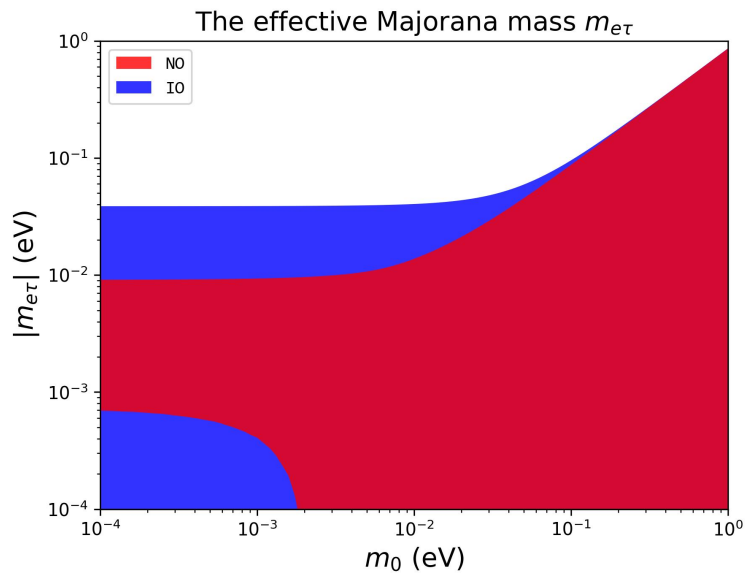
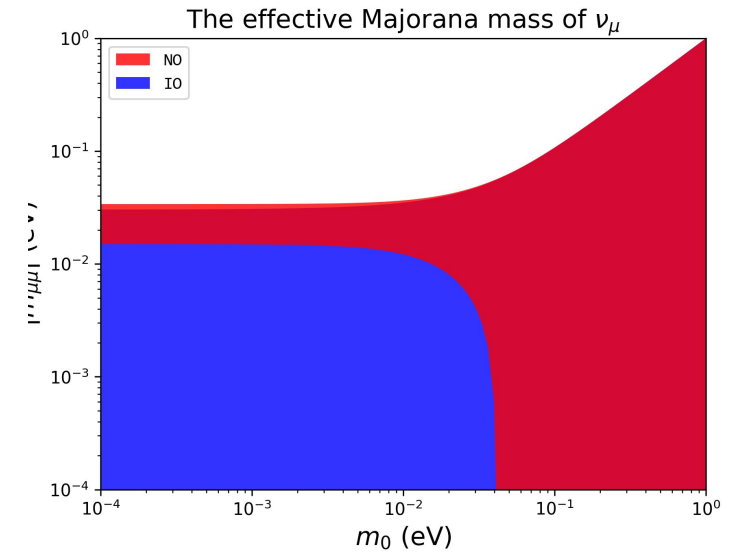
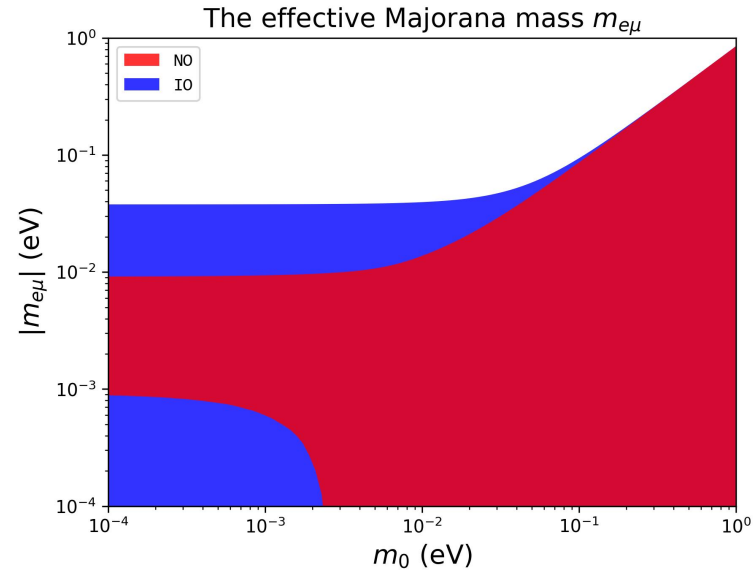
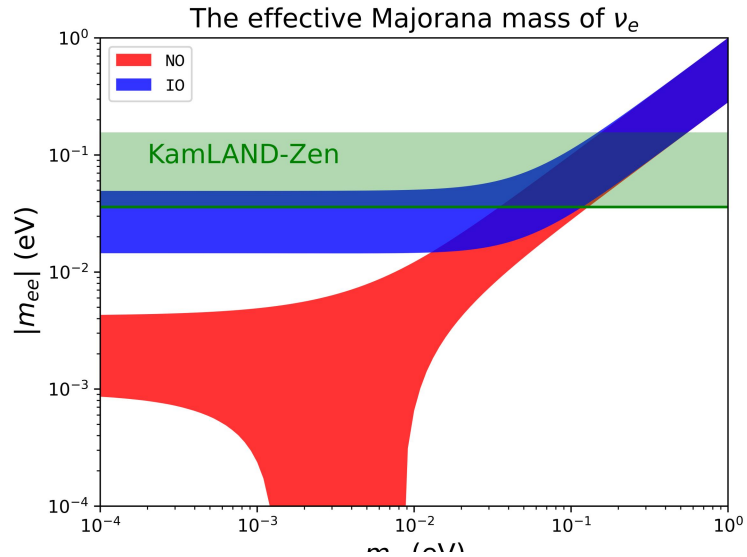
KamLAND-Zen, LEGEND, CUORE, GERDA...
and more to come, PandaX NT, Xeno NT...



Other propertise of neutrions: dipole moment... Are there light right handed neutrions

Constraints on Majorana neutrino mass matrix

$$m_\nu = (m_{ij}) = (V_{ij}m_iV_{lj})$$

$$m_\nu = V_{PMNS}\hat{m}_\nu V_{PMNS}^T$$


Grand Unification Theory SO(10) Predictions

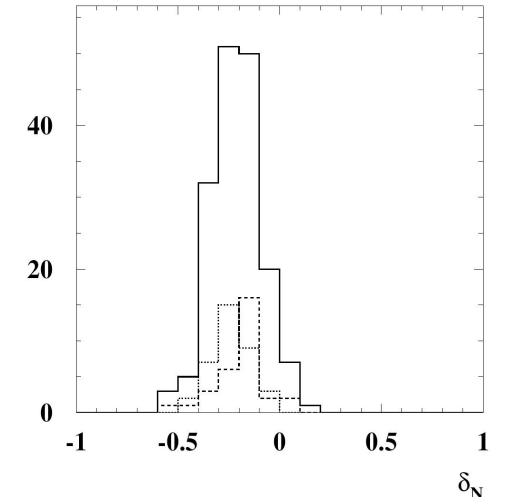
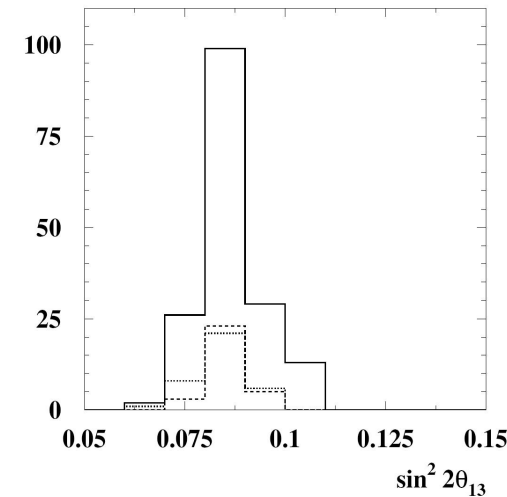
Minimal SO(10) Model without 120

$$16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H + Y_{120}120_H)16_F$$

$\mathcal{L}_{\text{Yukawa}} = Y_{10} 16 16 10_H + Y_{126} 16 16 \overline{126}_H$ Good prediction for $\theta_{13}\delta$ Away from $-\pi/2$!

$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned} \quad \begin{aligned} M_{\nu R} &= \langle \Delta_R \rangle Y_{126} \\ M_{\nu L} &= \langle \Delta_L \rangle Y_{126} \end{aligned}$$

Model has only 11 real parameters plus 7 phases



Loop generated Dirac mass models also work!
Eax:Babu, He, Su &Thapa, JHEP 08 (2022) 140

More data will help to finally determine mixing pattern, whether there is CPV!

3. Neutrinos In Cosmology And Astrophysics

In early universe, all energy forms existed in form of elementary particles, or
Temperature is high and were in thermal equilibrium

Criteria for thermal equilibrium: particle interaction length $1/\Gamma$ (Γ interaction rate) is smaller than Hubble length $1/H_0$

???Planck mass $T \sim 10^{19}\text{GeV}$

Inflation

Big Bang $\sim T > 10^{16} \text{ GeV}$

(Not in thermal equilibrium by SM for particle physics)

Grand Unification $\sim 10^{16} \text{ GeV}$

EW symmetry breaking 300GeV

Color confinement $\sim 300 \text{ MeV}$

BBN $\sim 1 \text{ MeV}$

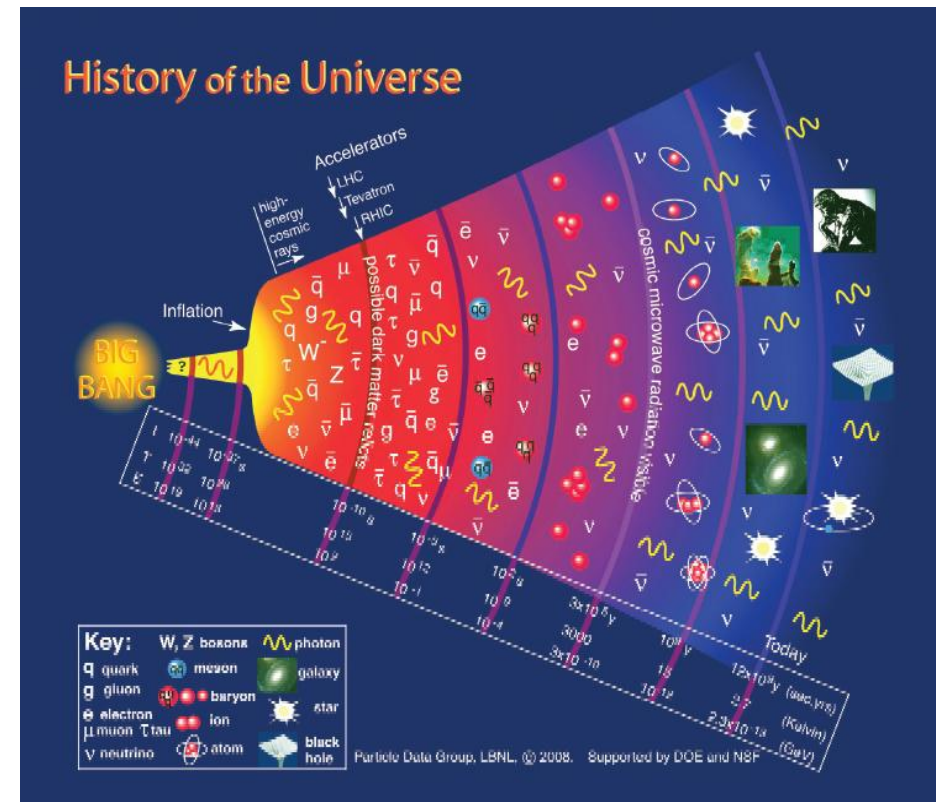
CvB $\sim 1 \text{ MeV}$

CMB $\sim 0.3 \text{ eV}$

Large structure formation

...

Today $\sim 2.7\text{K}$



Relic neutrino density:

$$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8} N_{\text{eff}} \left(\frac{4}{11} \right)^{4/3}$$

Big-Bang Nucleosynthesis:

$N_{\text{eff}} \sim 3$ light neutrinos,
consistent with Z decay width.

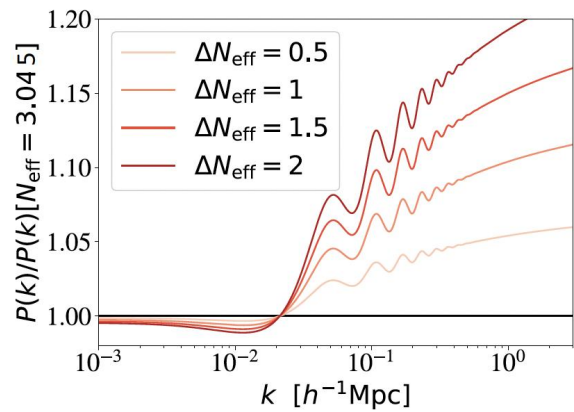


Table 26.1: Summary of N_{eff} constraints.

	Model	95%CL	Ref.
CMB alone			
Pl18[TT,TE,EE+lowE]	$\Lambda\text{CDM}+N_{\text{eff}}$	$2.92^{+0.36}_{-0.37}$	[22]
CMB + background evolution + LSS			
Pl18[TT,TE,EE+lowE+lensing] + BAO	$\Lambda\text{CDM}+N_{\text{eff}}$	$2.99^{+0.34}_{-0.33}$	[22]
" + BAO + R21	$\Lambda\text{CDM}+N_{\text{eff}}$	3.34 ± 0.14 (68%CL)	[11]
"	" +5-params.	2.85 ± 0.23 (68%CL)	[23]

Neutrino back ground temperature: $T_\nu/T_\chi = (4/11)^{1/3} = 1.95 \text{ K}$

$$n_\chi = 440/\text{cm}^3, \quad n_\nu = 339/\text{cm}^3$$

Energy density from neutrinos:

$$\Omega_\nu = \frac{\rho_\nu^0}{\rho_{\text{crit}}^0} = \frac{\sum m_\nu}{93.14 h^2 \text{ eV}}$$

Neutrino Mass On CMB And Matter Spectrum

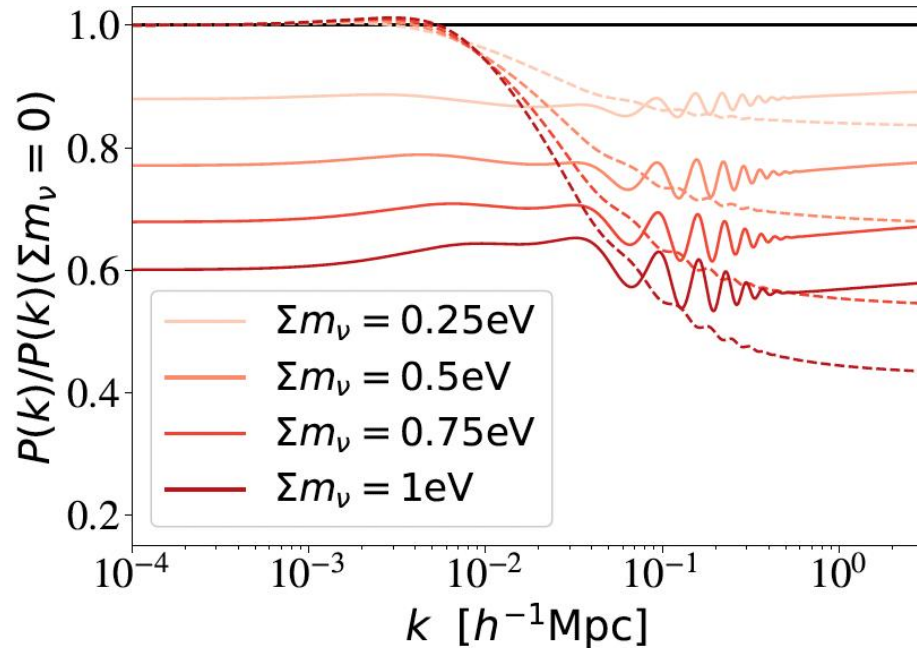


Table 26.2: Summary of $\sum m_\nu$ constraints.

	Model	95% CL (eV)
CMB alone		
Pl18[TT+lowE]	Λ CDM+ $\sum m_\nu$	< 0.54
Pl18[TT,TE,EE+lowE]	Λ CDM+ $\sum m_\nu$	< 0.26
CMB + probes of background evolution		
Pl18[TT+lowE] + BAO	Λ CDM+ $\sum m_\nu$	< 0.13
Pl18[TT,TE,EE+lowE]+BAO	Λ CDM+ $\sum m_\nu$ +5 params.	< 0.515
CMB + LSS		
Pl18[TT+lowE+lensing]	Λ CDM+ $\sum m_\nu$	< 0.44
Pl18[TT,TE,EE+lowE+lensing]	Λ CDM+ $\sum m_\nu$	< 0.24
CMB + probes of background evolution + LSS		
Pl18[TT,TE,EE+lowE] + BAO + RSD	Λ CDM+ $\sum m_\nu$	< 0.10
Pl18[TT+lowE+lensing] + BAO + Lyman- α	Λ CDM+ $\sum m_\nu$	< 0.087
Pl18[TT,TE,EE+lowE] + BAO + RSD + Pantheon + DES	Λ CDM+ $\sum m_\nu$	< 0.13

The sum of the neutrino masses should be less than 0.6 eV.

Future, reach 0.06 eV reaching the allowed range by neutrino oscillation!
DESI, Euclid, LSST, SPHEREx, SKA.

Neutrinos play important role in BBN!

Cosmology constraint their masses compareable to neutrino oscillation data!

Neutrino And Our Matter Universe

In our Universe, matter dominates over anti-matter

- Why this is so is the problem of Baryon Asymmetry of our Universe (BAU)

In cosmological terms, the problem is as follows

If initially, the universe is matter and anti-matter symmetric

$$n_B/n_\gamma = n_{\bar{B}}/n_\gamma \sim 10^{-20}$$

$n_B(n_{\bar{B}})$ - baryon (anti-baryon) number density, n_γ - photon number density

However observation, BBN and CMB, show that

$$\eta = (n_B - n_{\bar{B}})/n_\gamma \sim 6 \times 10^{-10}$$

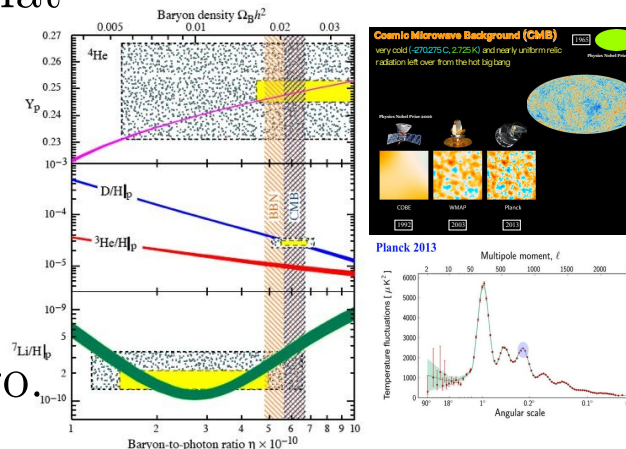
There is a 10^{10} order of magnitude difference.

Initially, there is a baryon asymmetry?

But inflation will dilute any asymmetry to zero.

Possible to generate a η which fits observation

from an, initially, matter anti-matter symmetry universe?



Sakharov Conditions (1967)

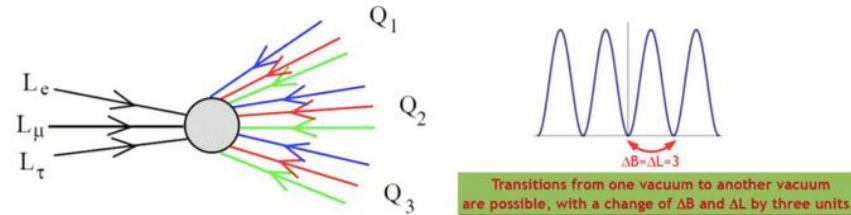
Baryon Number B Violation

C and CP Violation

Interactions Out Of Thermal Equilibrium

Standard Model Has All Ingredients, But Too Small

Baryon number violation: Sphaleron effects-tunneling effects from different vacuum states with non-zero baryon number differences. Violated $B+L$, but conserves $B-L$.



C and CP violation: Electroweak interaction violates C, and phase in Kobayashi-Maskawa mixing matrix violates CP.

Out of thermal equilibrium: Electroweak symmetry breaking

But, CP violation rate too small, out of thermal equilibrium too weak. Not enough to generate a large enough Baryon Asymmetry.

If Higgs mass is less than 70 GeV, second order phase transition at electroweak symmetry breaking, too weak.

$\eta \sim 10^{-20}$ Too small. Needs to go beyond SM!

Electroweak baryogenesis, Leptogenesis, Gut baryogenesis....

Leptogenesis

Fukugita and Yanagida, PLB174, 45(1986)

Translate lepton number asymmetry generated in the early universe to baryon number asymmetry!

Requires lepton asymmetry generated before Sphelaron effects to be in effective ($T \sim 10^{12} - \text{a few TeV}$). Initial $a_L(i)=a$, $a_B(i)=0$.

Sphelaron effect: Conserve B-L, but violates B+L

After: $a_L(f)+a_B(f) = 0$, $a_L(i) - a_B(i) = a_L(f) - a_B(f)$

$$a_L(f) = a/2; \quad a_B(f) = -a/2$$

half of initial lepton asymmetry will be translated into baryon asymmetry if complete.

SM Sphelaron effect: $a_B = - (28/79)a_L$

Seesaw Model Plays The Right Role

$$L_M = -\bar{L}_L Y_e \tilde{H} E_R - \bar{L}_L Y_\nu H \nu_R - \frac{1}{2} (\bar{\nu}_L, \bar{\nu}_R^c) M^\nu \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + H.C.$$

The last term violates lepton number L by two units!

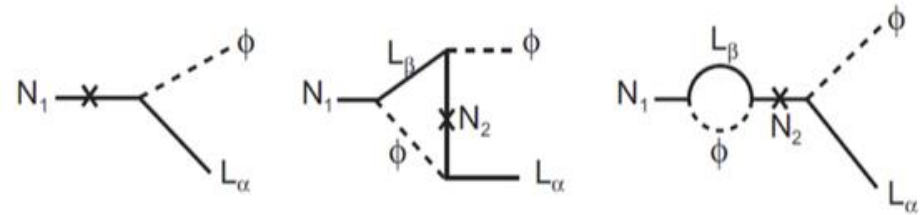
Out of thermal equilibrium decay, new CP violation in $N \rightarrow L h(\phi)$

N decays into L and anti-L differently

$$a_i \approx -\frac{1}{8\pi} \frac{1}{[\hat{Y}_\nu \hat{Y}_\nu^\dagger]_{ii}} \sum_j \text{Im}\{[\hat{Y}_\nu \hat{Y}_\nu^\dagger]_{ij}^2\} f\left(\frac{M_j^2}{M_i^2}\right)$$

$$a_i = \frac{\Gamma(N_i \rightarrow L\phi) - \bar{\Gamma}(N_i \rightarrow \bar{L}\phi)}{\Gamma(N_i \rightarrow L\phi) + \bar{\Gamma}(N_i \rightarrow \bar{L}\phi)}$$

$$f(x) = \sqrt{x} \left(\frac{2}{x-1} + \ln \frac{1+x}{x} \right).$$



M_N and m_ν masses are correlated to obtain the right number for η ,
 m_ν of order 0.05 eV, $M_N \sim 1000$ GeV.

Neutrino Seesaw model is a viable model for Baryon Asymmetry of our Universe

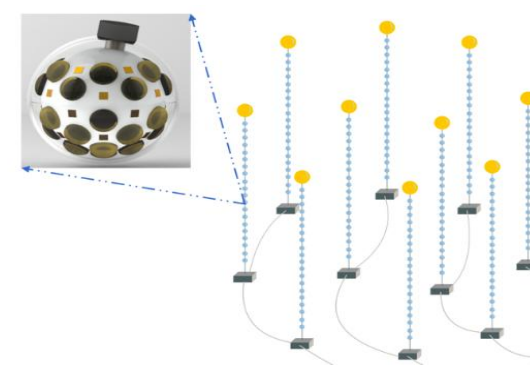
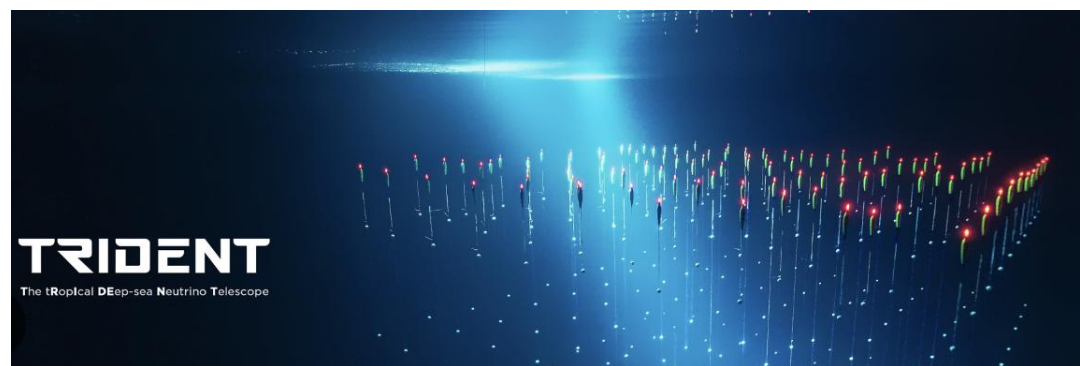
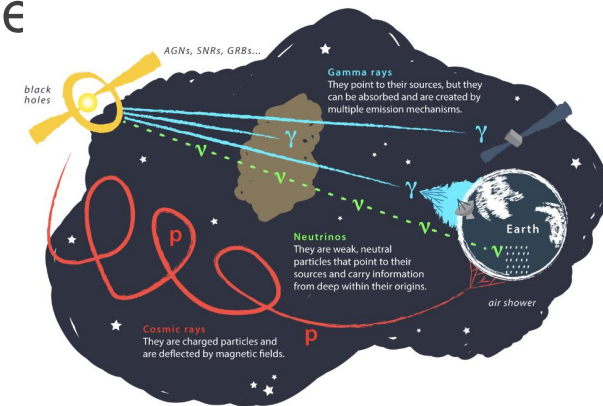
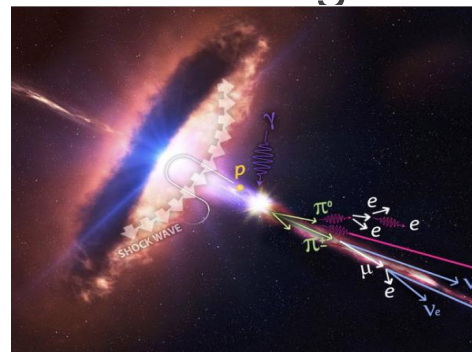
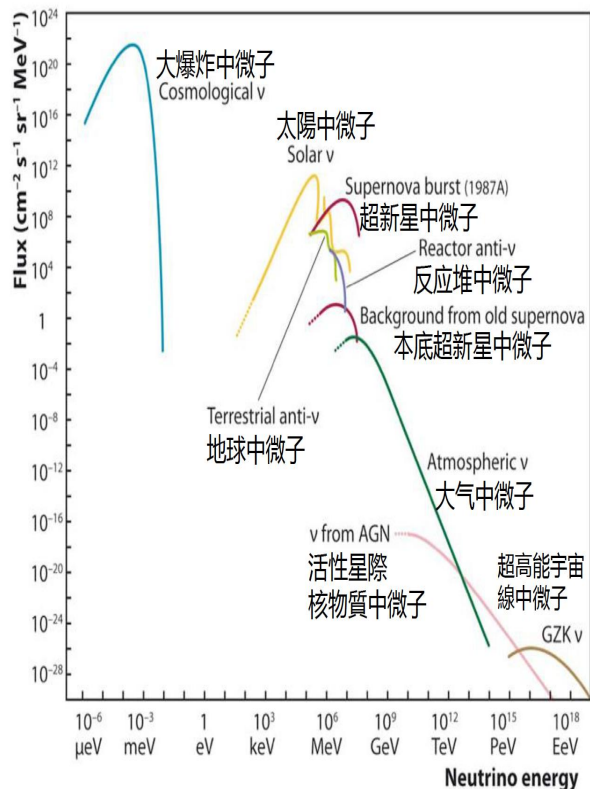
Cosmic Neutrino Ray As Messenger For Our Universe

Where the very high energy neutrino ray come from? How they are acceleratted to have such high energie

Any use of the neutrino cosmic ray?

Like we use neutrino beam on earth to probe new interactions, human being can use cosmic neutrino beams as messengers to probe our universe with much less disturbances compared with hadrons, photons!

Extremely low energy: CvB and super high energy neutrino rays.



Messenger for our universe! Icecube, KM3net and future:Trident and Hunt projects are initiatives for this!

Thank you for your attentions!