大统一理论 与 中微子实验

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2025-07-24

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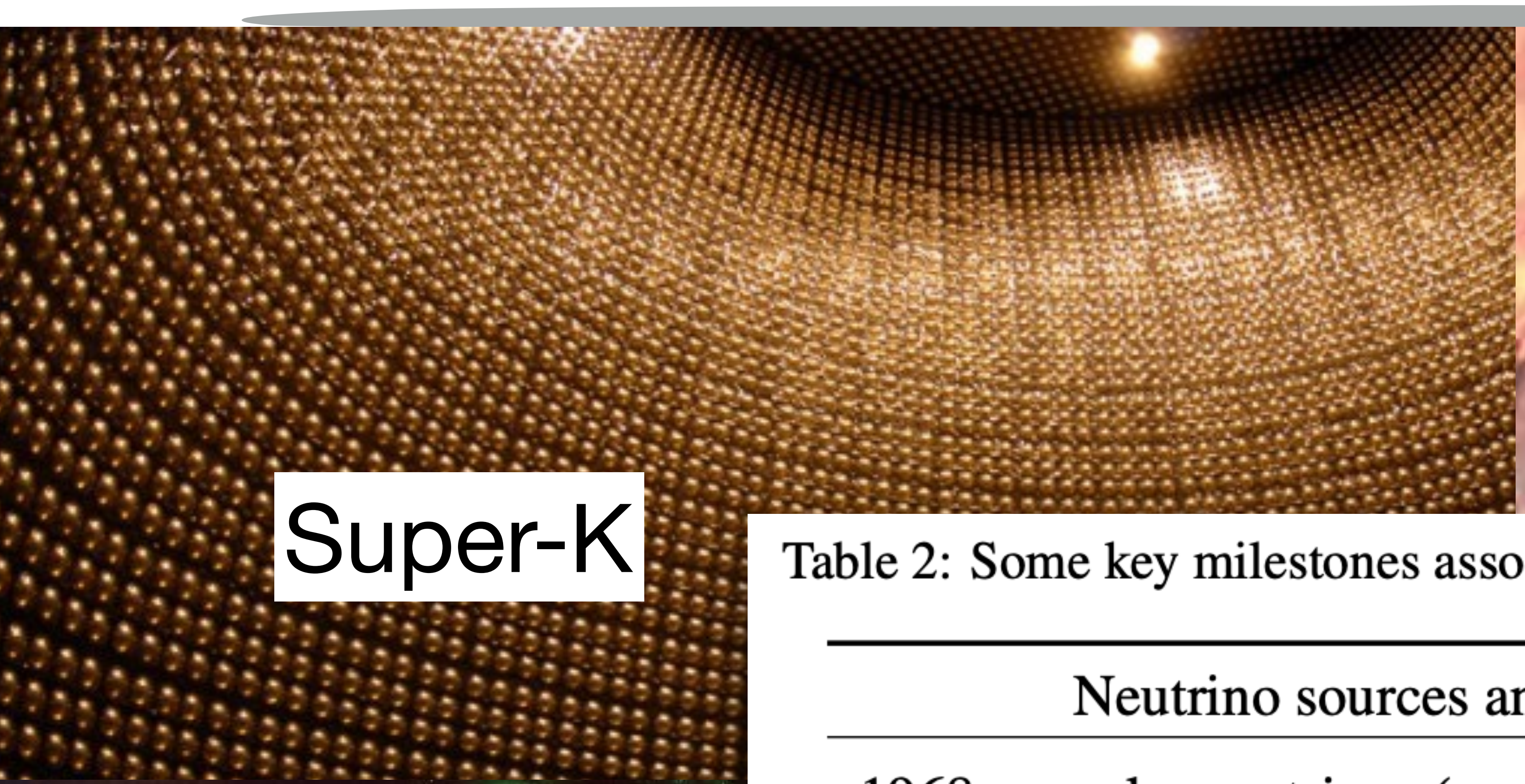
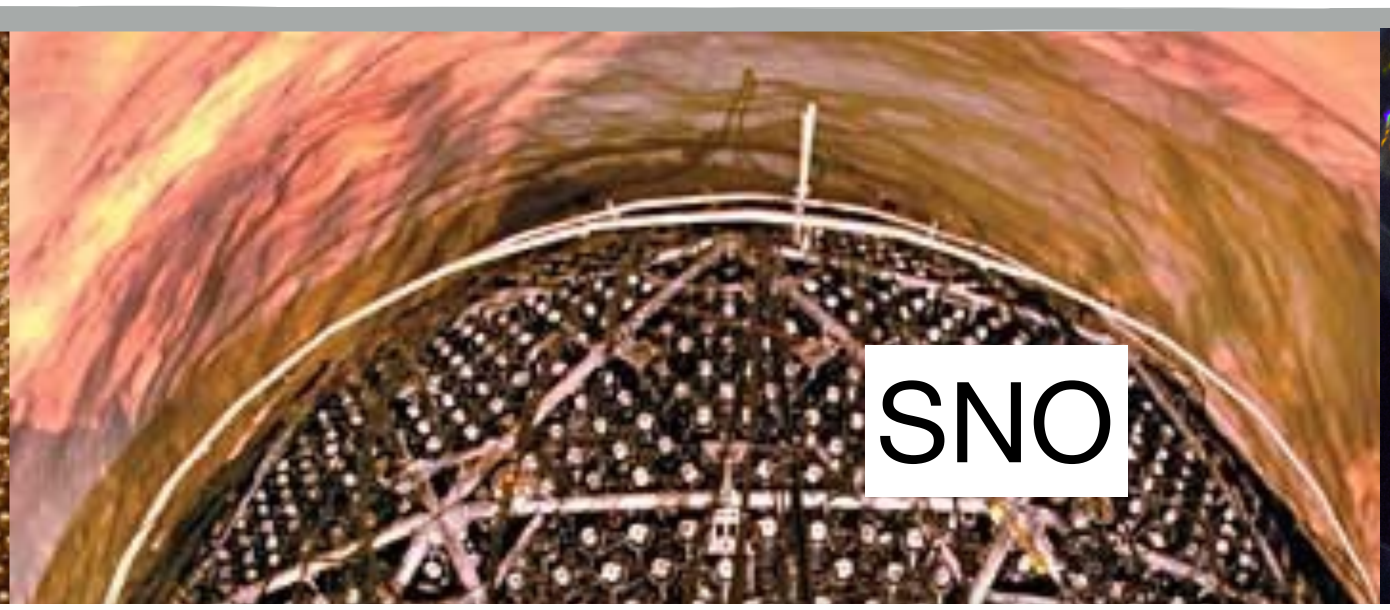
Introduction

Introduction





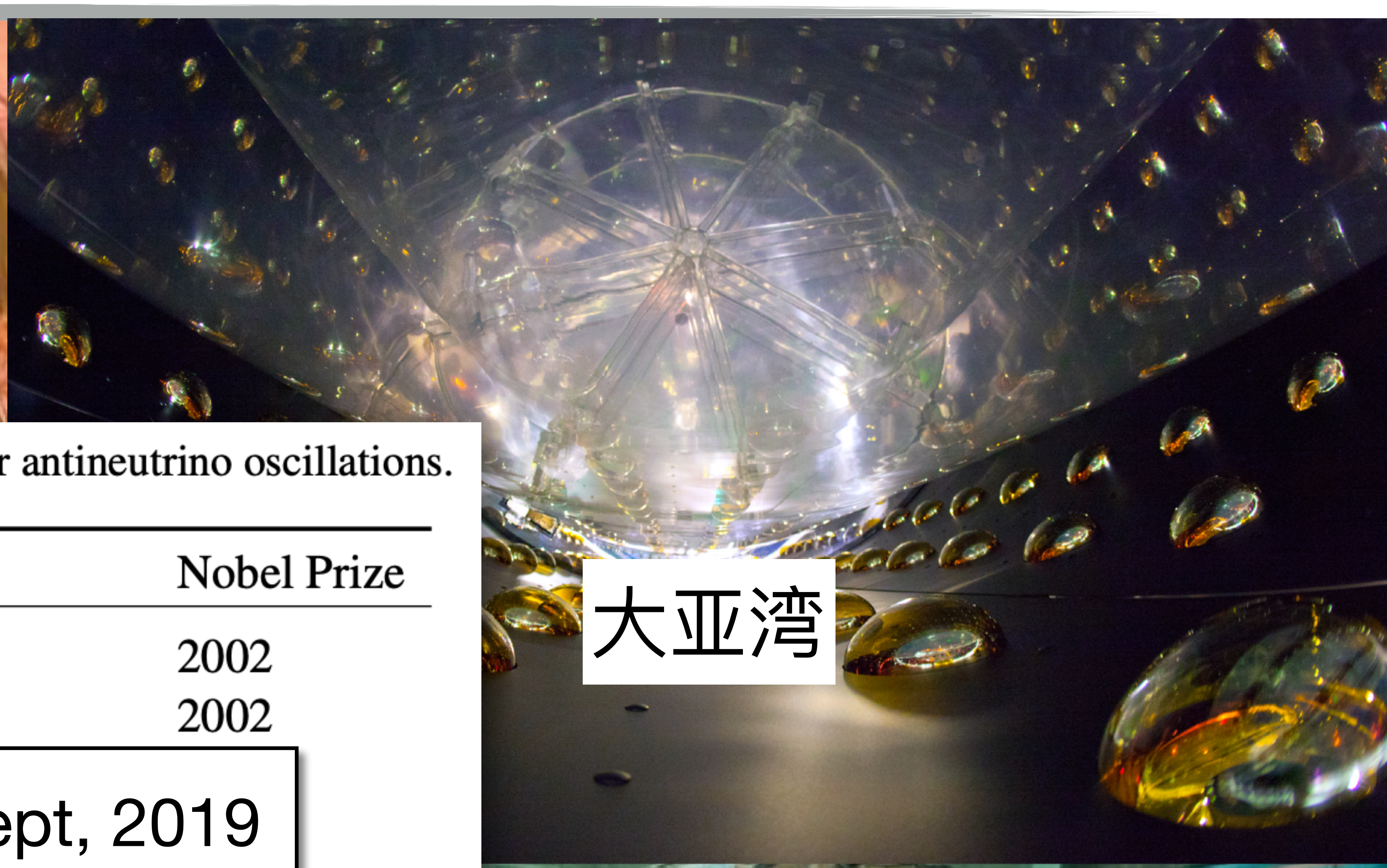
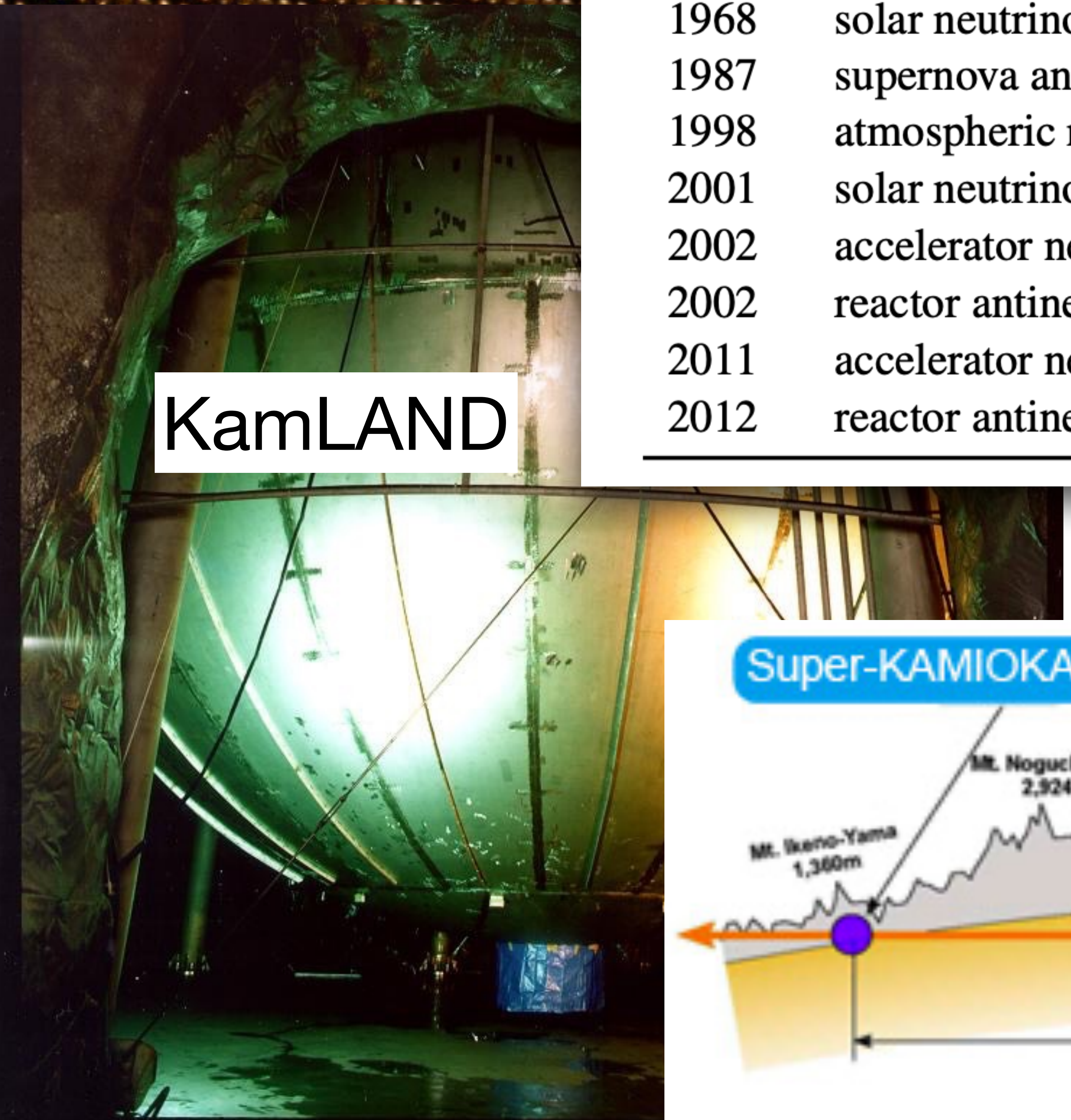
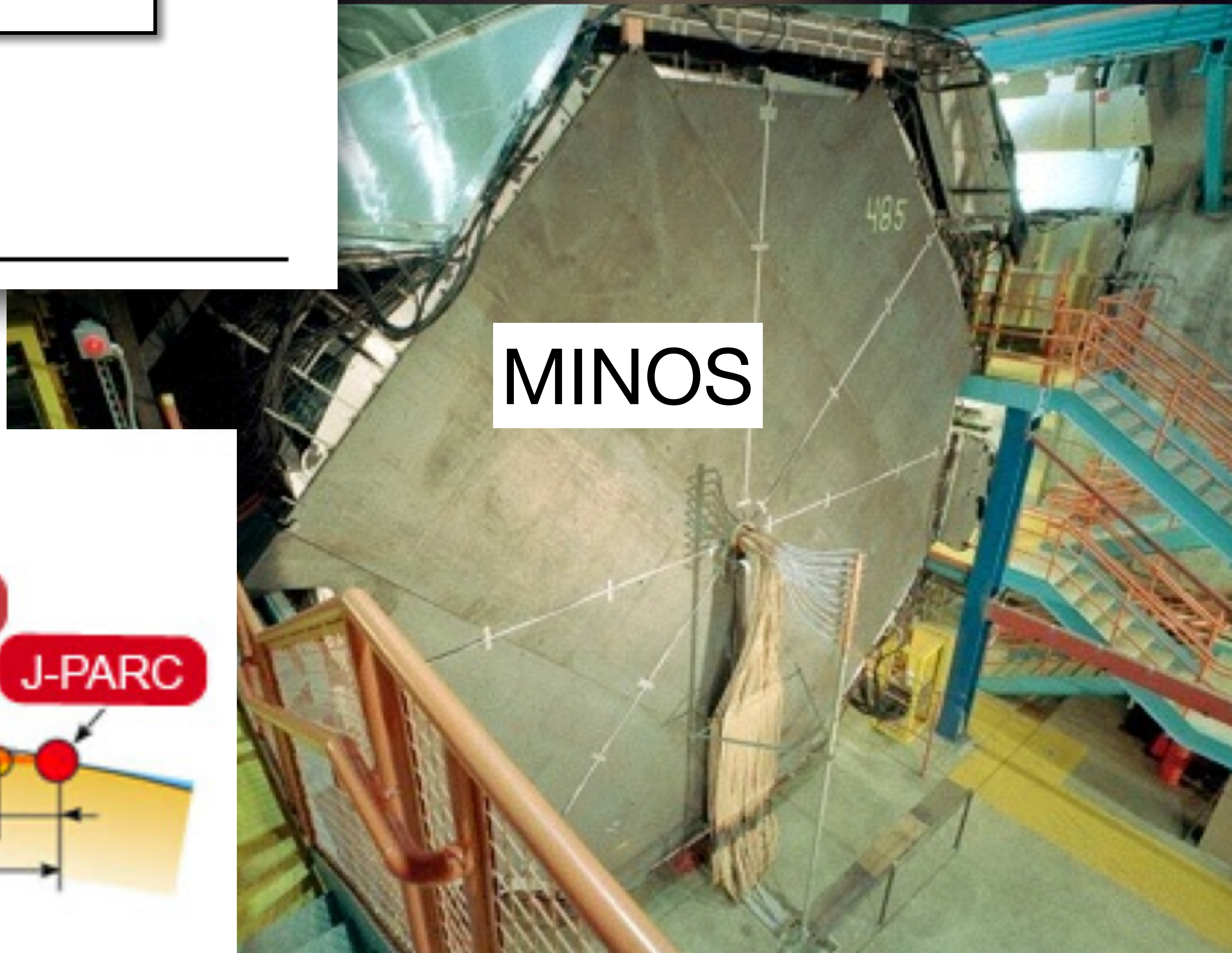


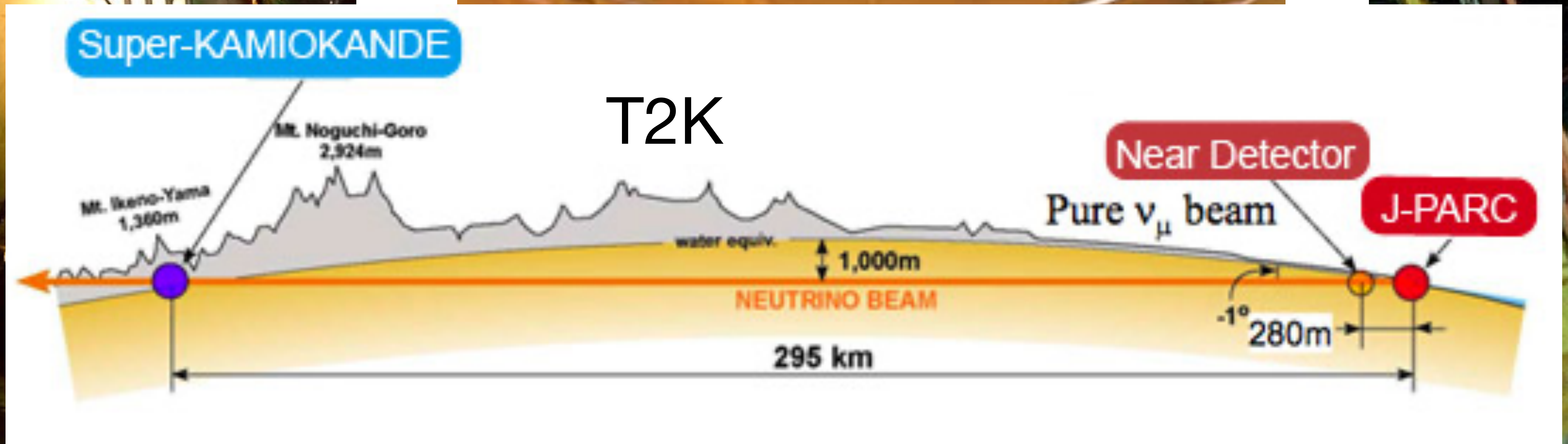
Table 2: Some key milestones associated with the experimental discoveries of neutrino or antineutrino oscillations.

	Neutrino sources and (or) oscillations	Main discoverers	Nobel Prize
1968	solar neutrinos ($\nu_e \to \nu_e$) [59]	R. Davis	2002
1987	supernova antineutrinos ($\overline{\nu}_e$) [62, 63]	M. Koshiba	2002
1998	atmospheric neutrinos ($\nu_\mu \to \nu_\mu$) [64]		
2001	solar neutrinos ($\nu_e \to \nu_e, \nu_\mu, \nu_\tau$) [65, 66, 67]		
2002	accelerator neutrinos ($\nu_\mu \to \nu_\mu$) [68]	K. Nishikawa	
2002	reactor antineutrinos ($\overline{\nu}_e \to \overline{\nu}_e$) [69]	A. Suzuki	
2011	accelerator neutrinos ($\nu_\mu \to \nu_e$) [70, 71]	K. Nishikawa	
2012	reactor antineutrinos ($\overline{\nu}_e \to \overline{\nu}_e$) [72]	K. B. Luk, Y. Wang	

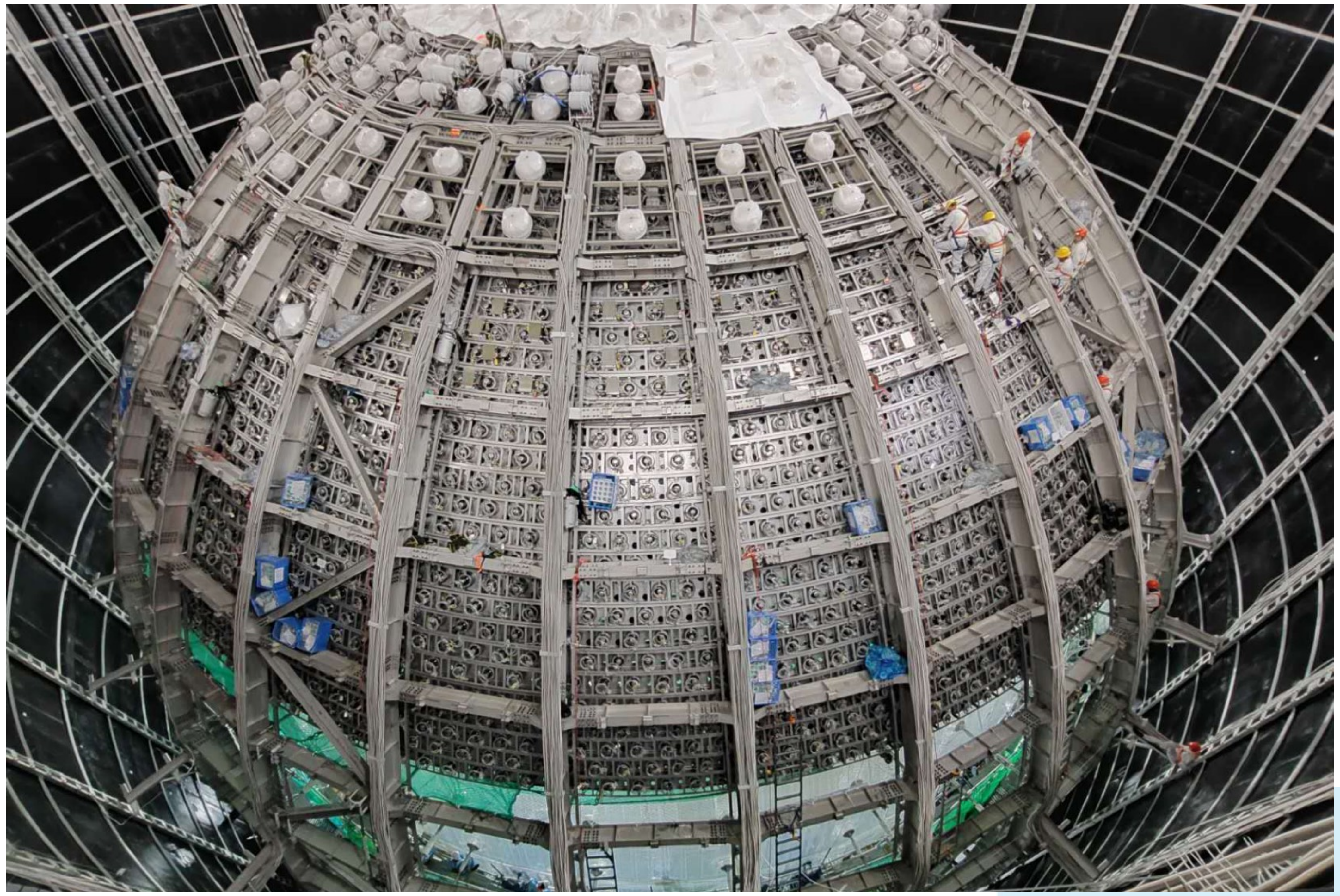
Xing, Phys.Rept, 2019



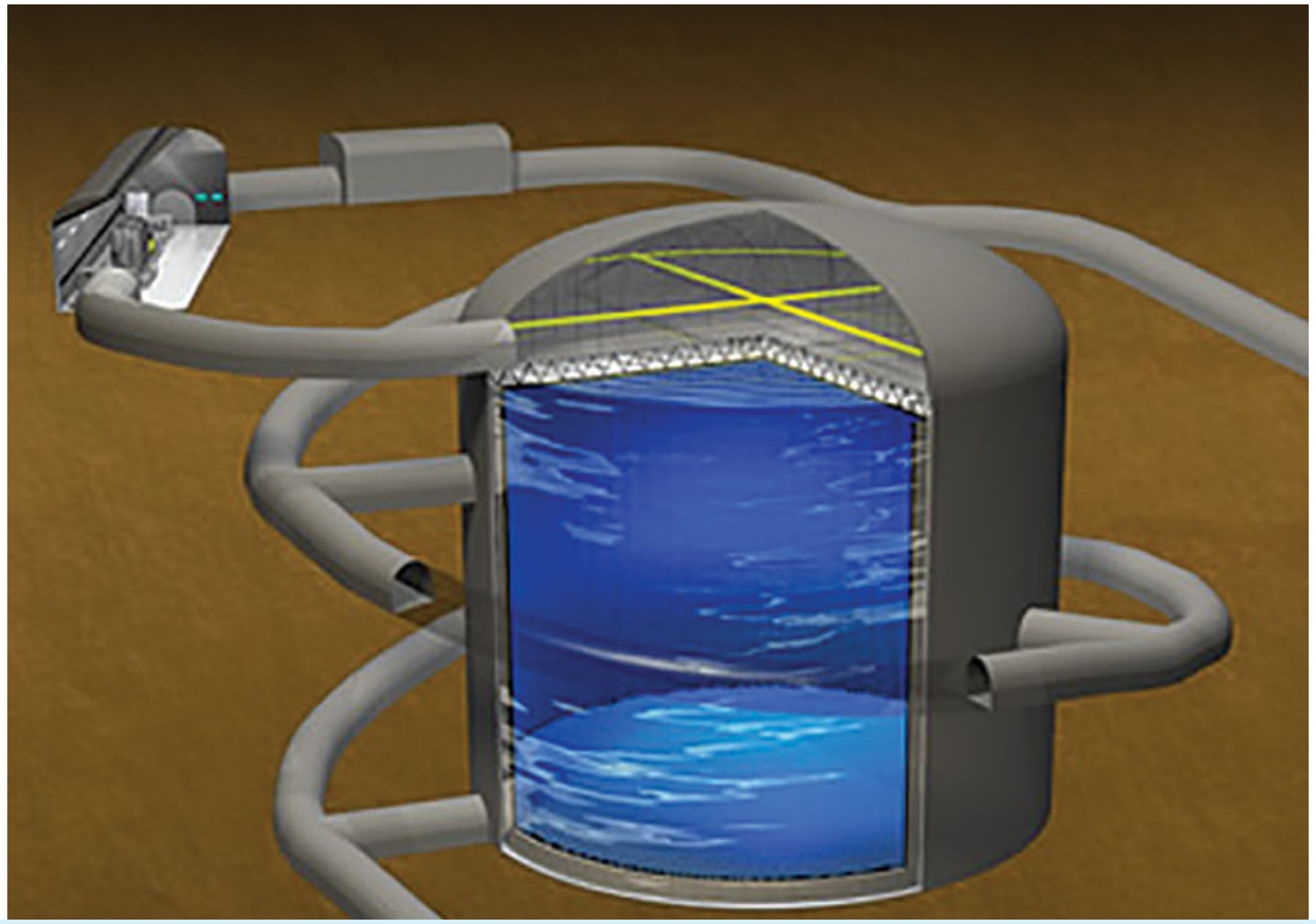




Upcoming large-scale neutrino experiments

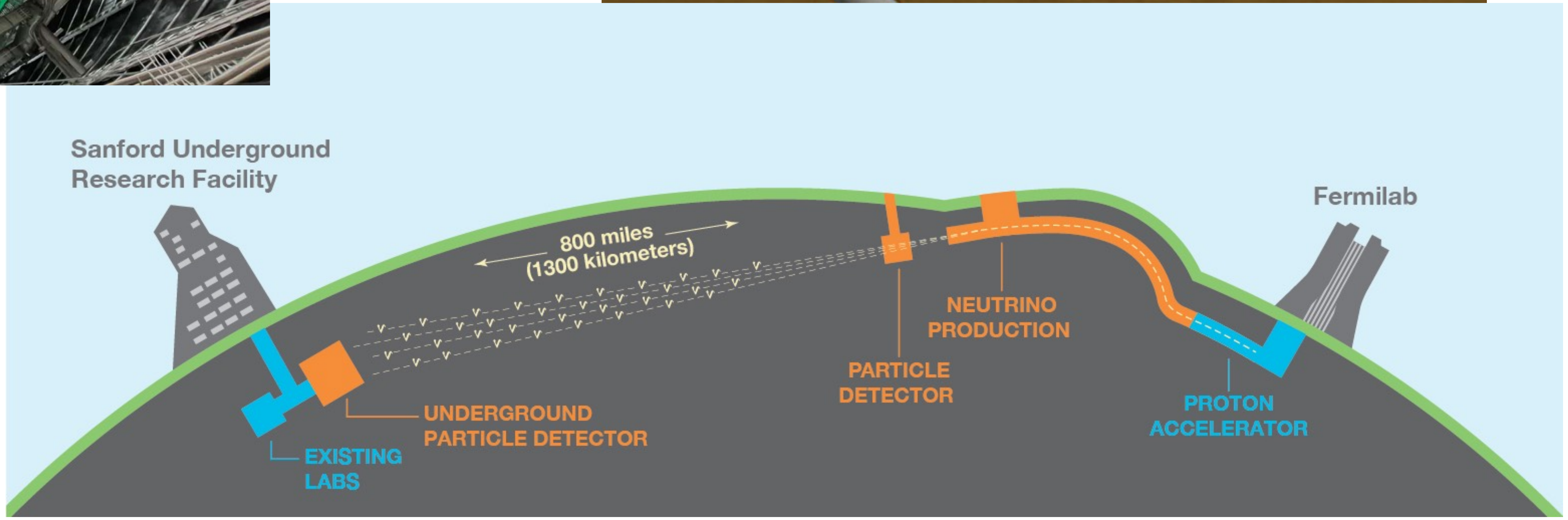


JUNO, taking data next month



Hyper-K

expected to run in 2027



DUNE

supposed to run in 2031?

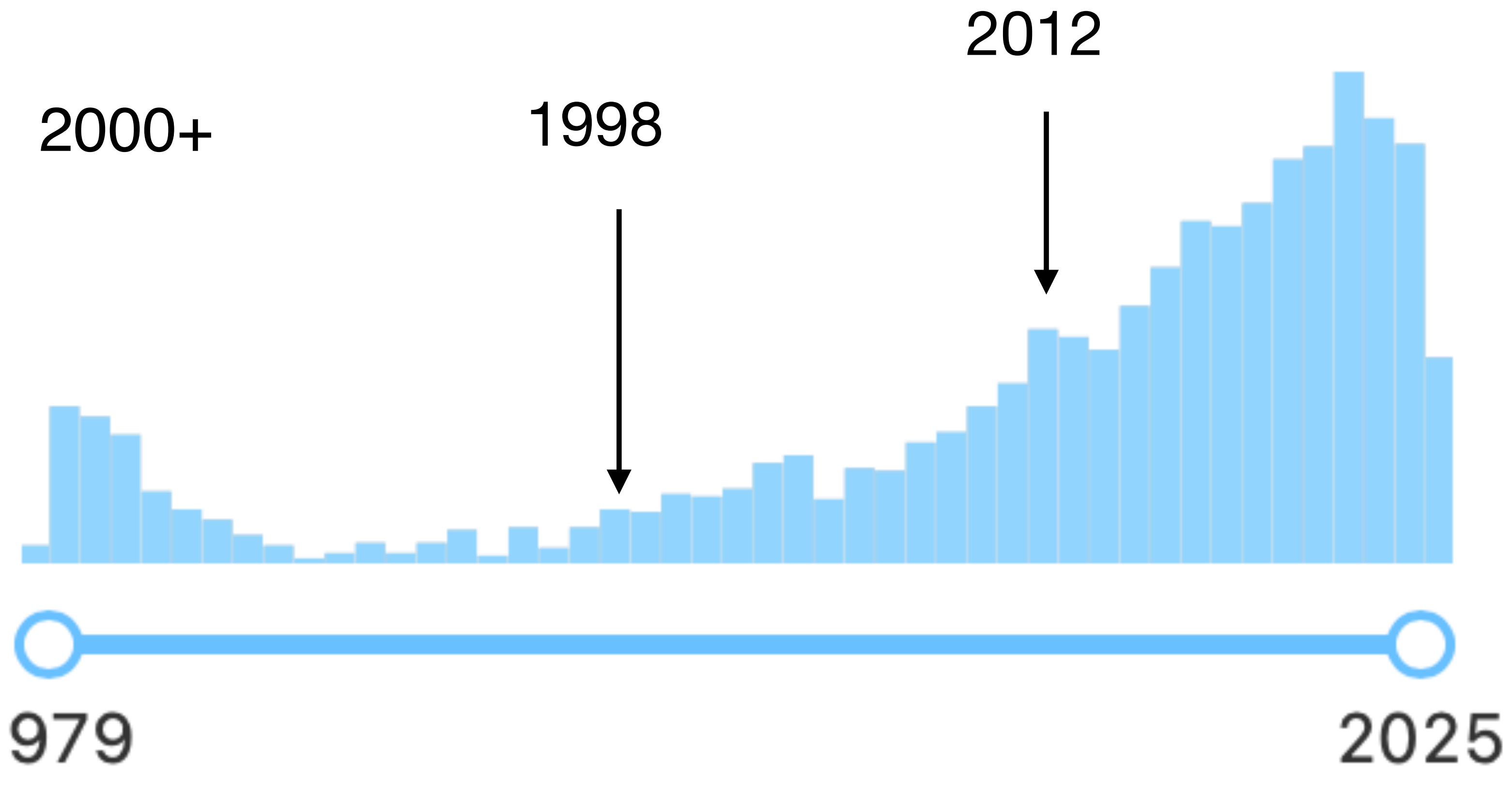
Neutrino precision measurements are coming!

Experiments motivate Theories

Baryon and Lepton Nonconserving Processes

Steven Weinberg (Harvard U.) (1979)

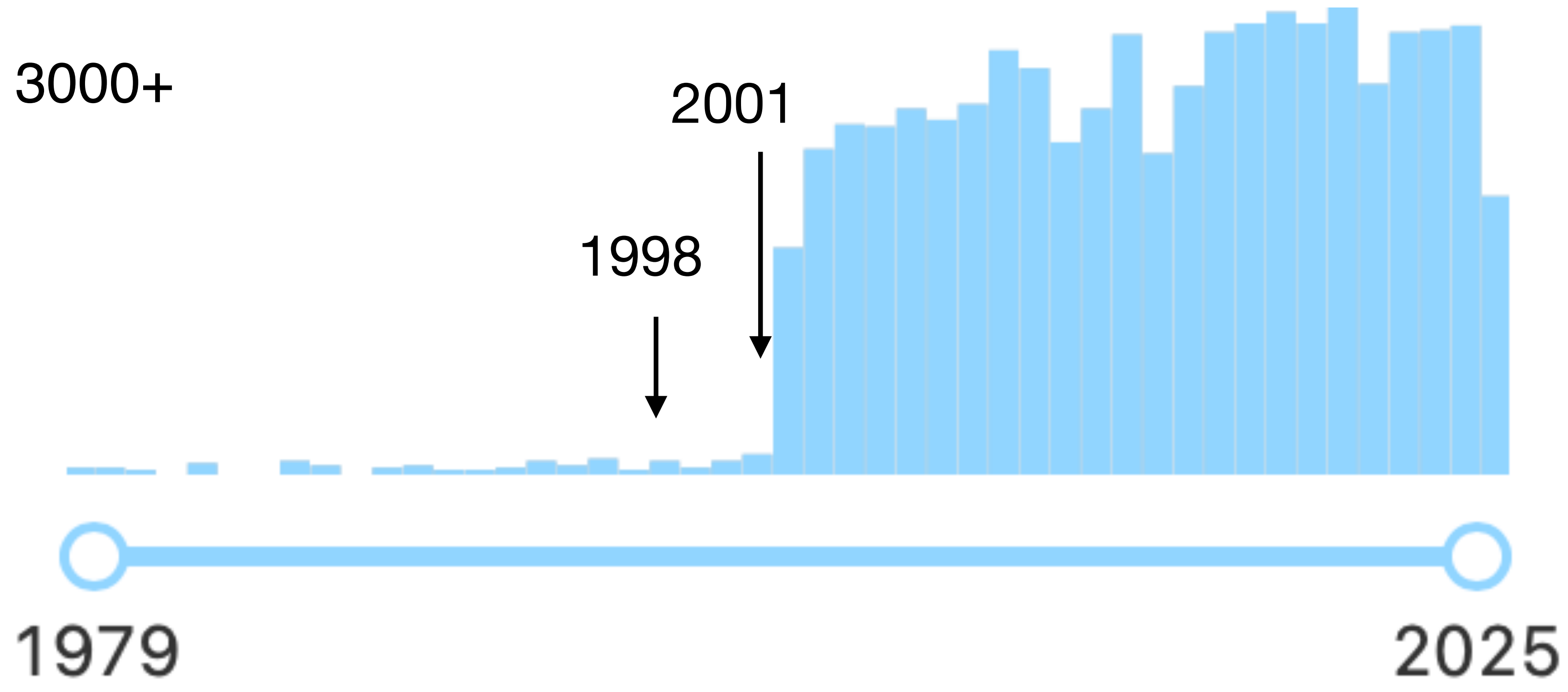
Published in: *Phys.Rev.Lett.* 43 (1979) 1566-1570



Complex Spinors and Unified Theories

Murray Gell-Mann (CERN), Pierre Ramond (Caltech), Richard Slansky (Los Alamos) (1979)

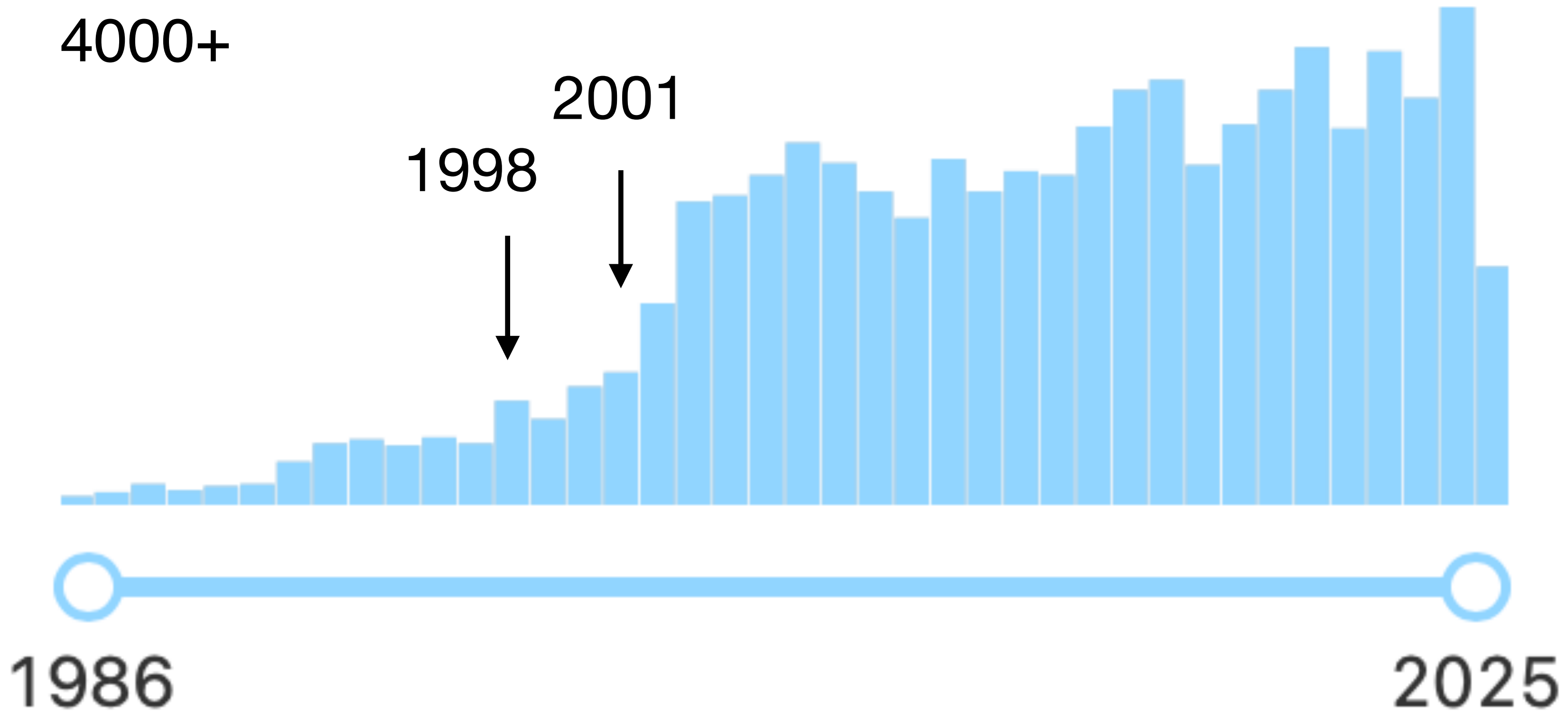
Published in: *Conf.Proc.C* 790927 (1979) 315-321 • Contribution to: Supergravity Workshop
[hep-th]



Baryogenesis Without Grand Unification

M. Fukugita (Kyoto U., Yukawa Inst., Kyoto), T. Yanagida (Tohoku U.) (Jan

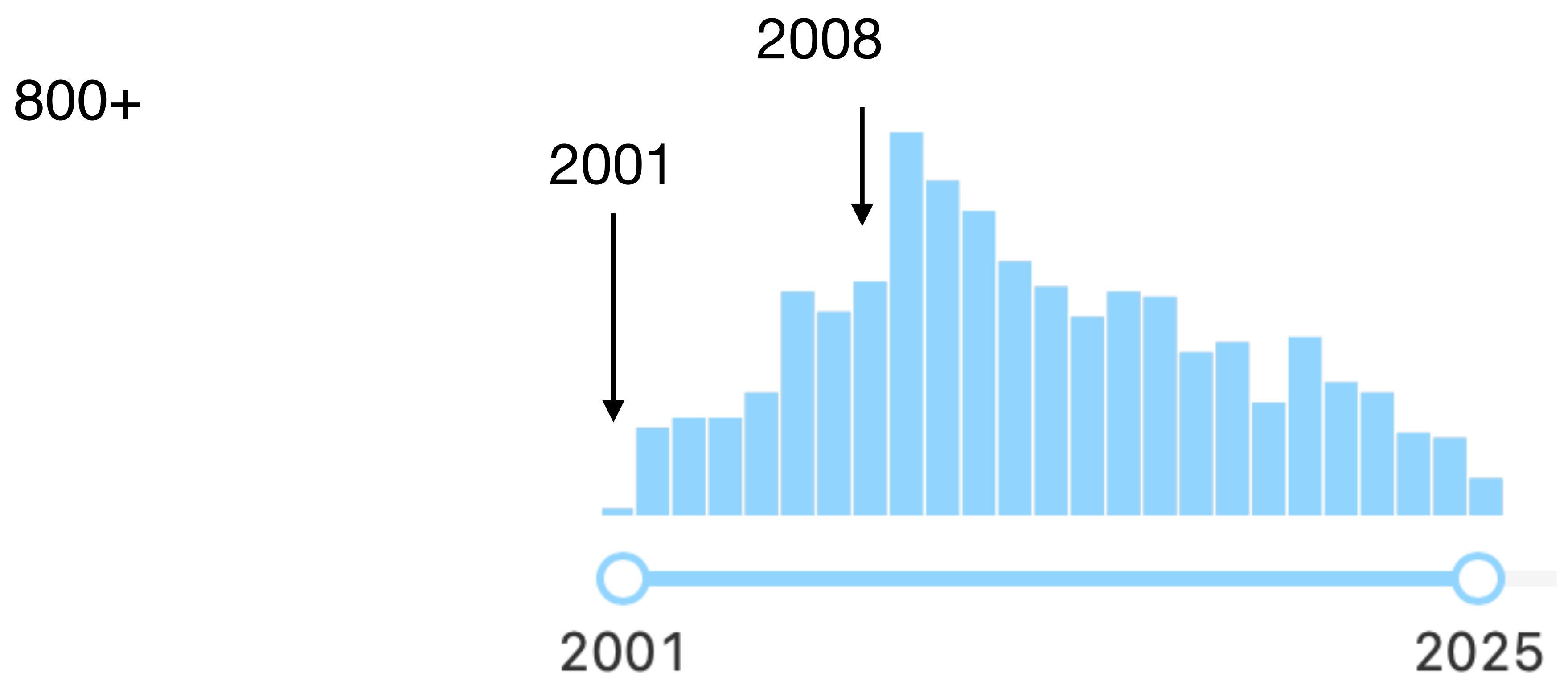
Published in: *Phys.Lett.B* 174 (1986) 45-47



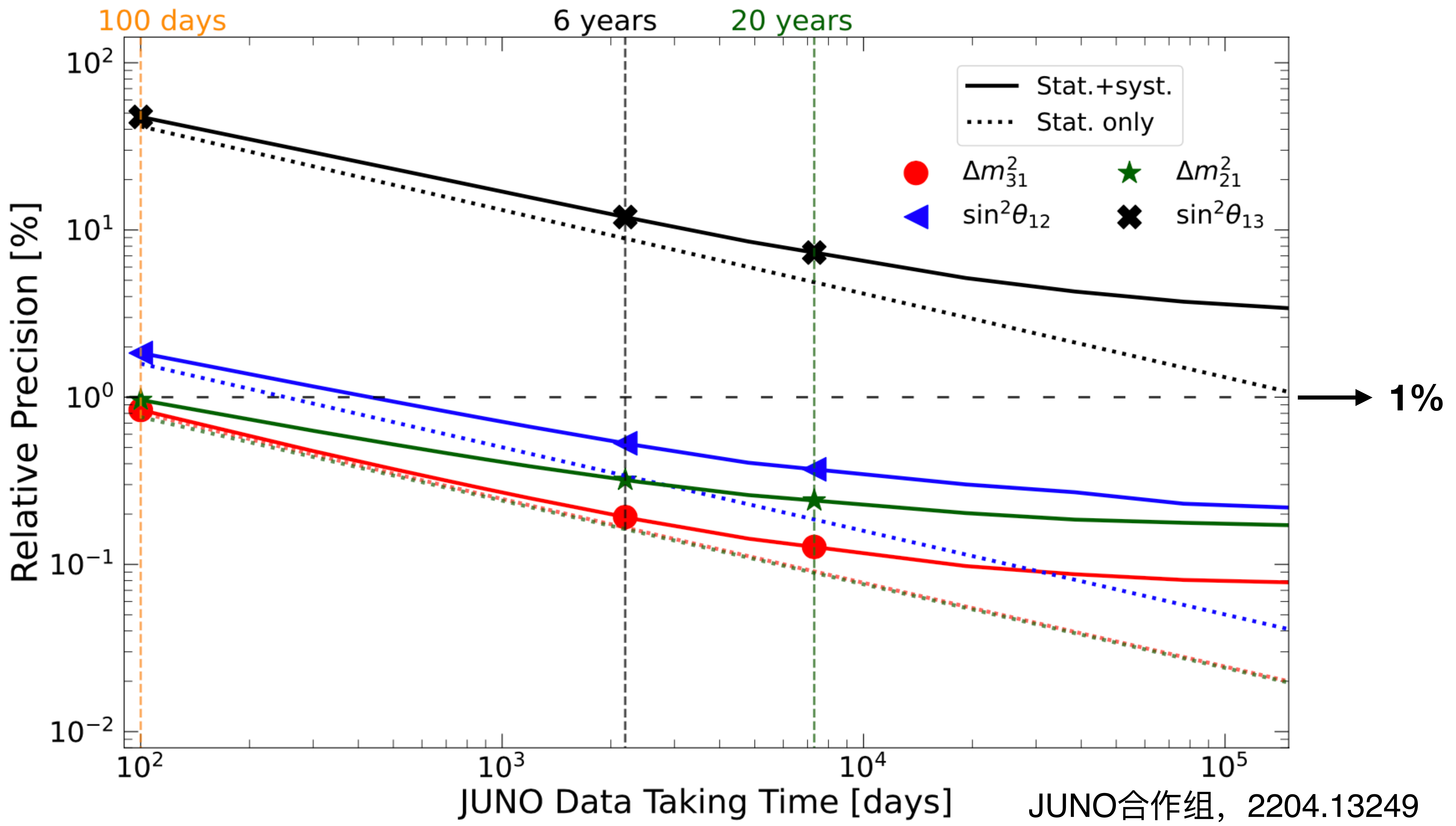
Softly broken A(4) symmetry for nearly degenerate neutrino masses

Ernest Ma (UC, Riverside), G. Rajasekaran (IMSc, Chennai) (Jun, 2001)

Published in: *Phys.Rev.D* 64 (2001) 113012 • e-Print: hep-ph/0106291 [hep-ph]



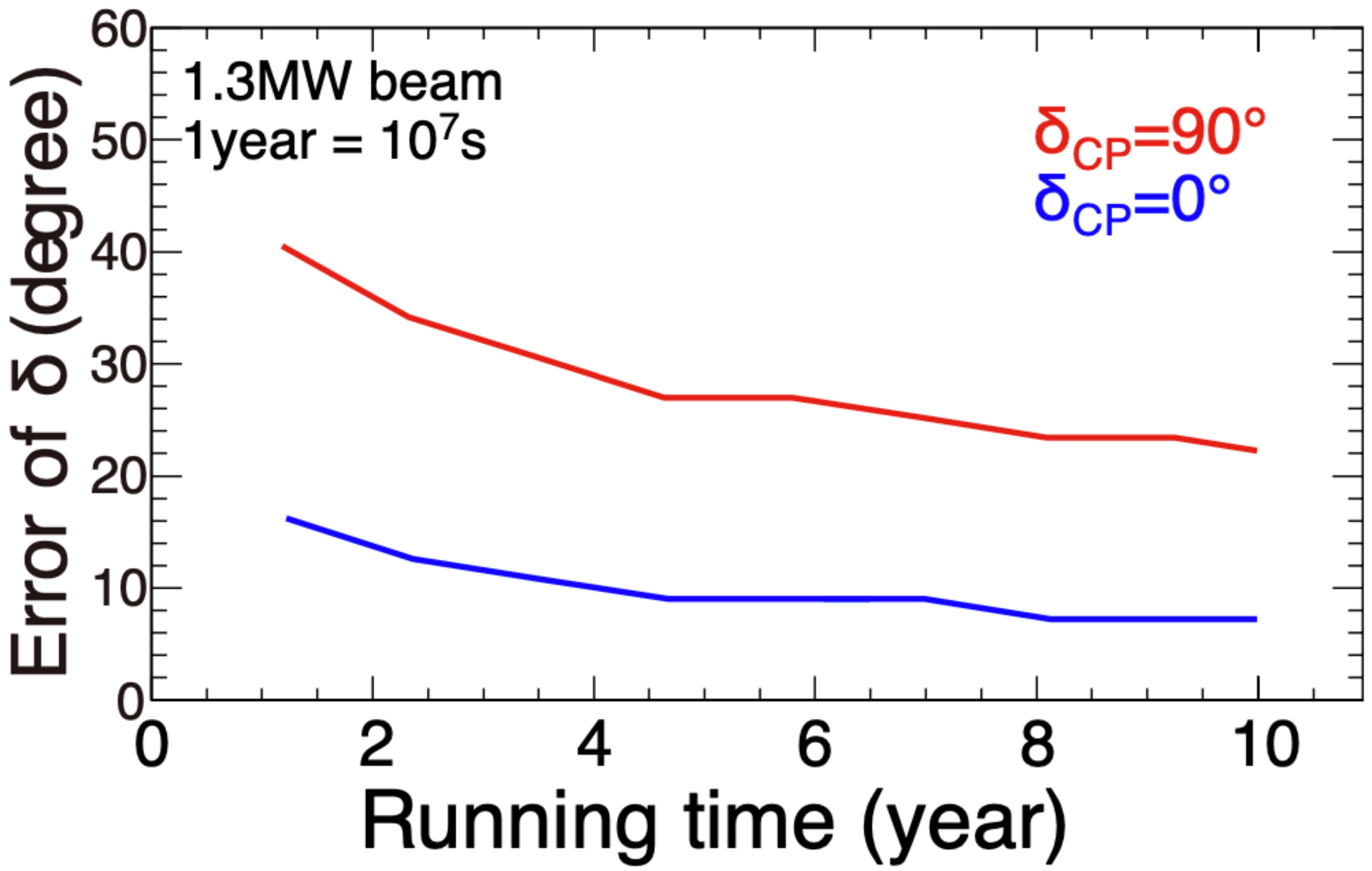
Precision measurements in JUNO



JUNO合作组，2204.13249

Hyper-K & DUNE

Hyper-K
1805.04163



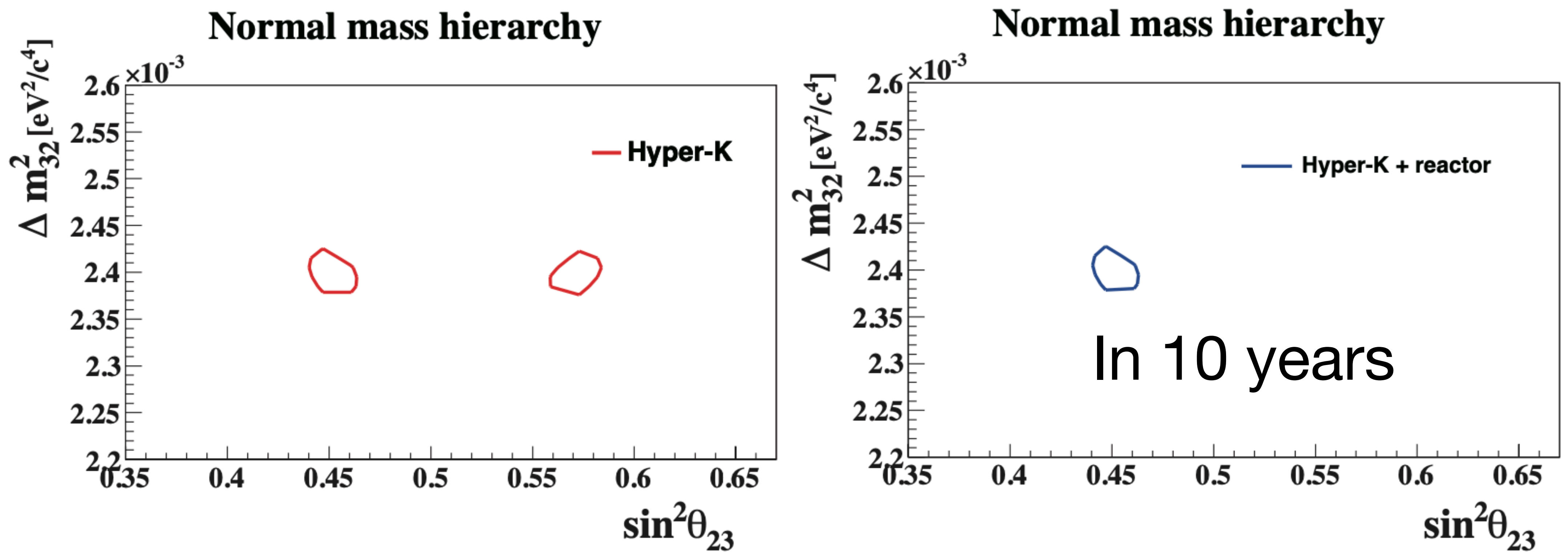
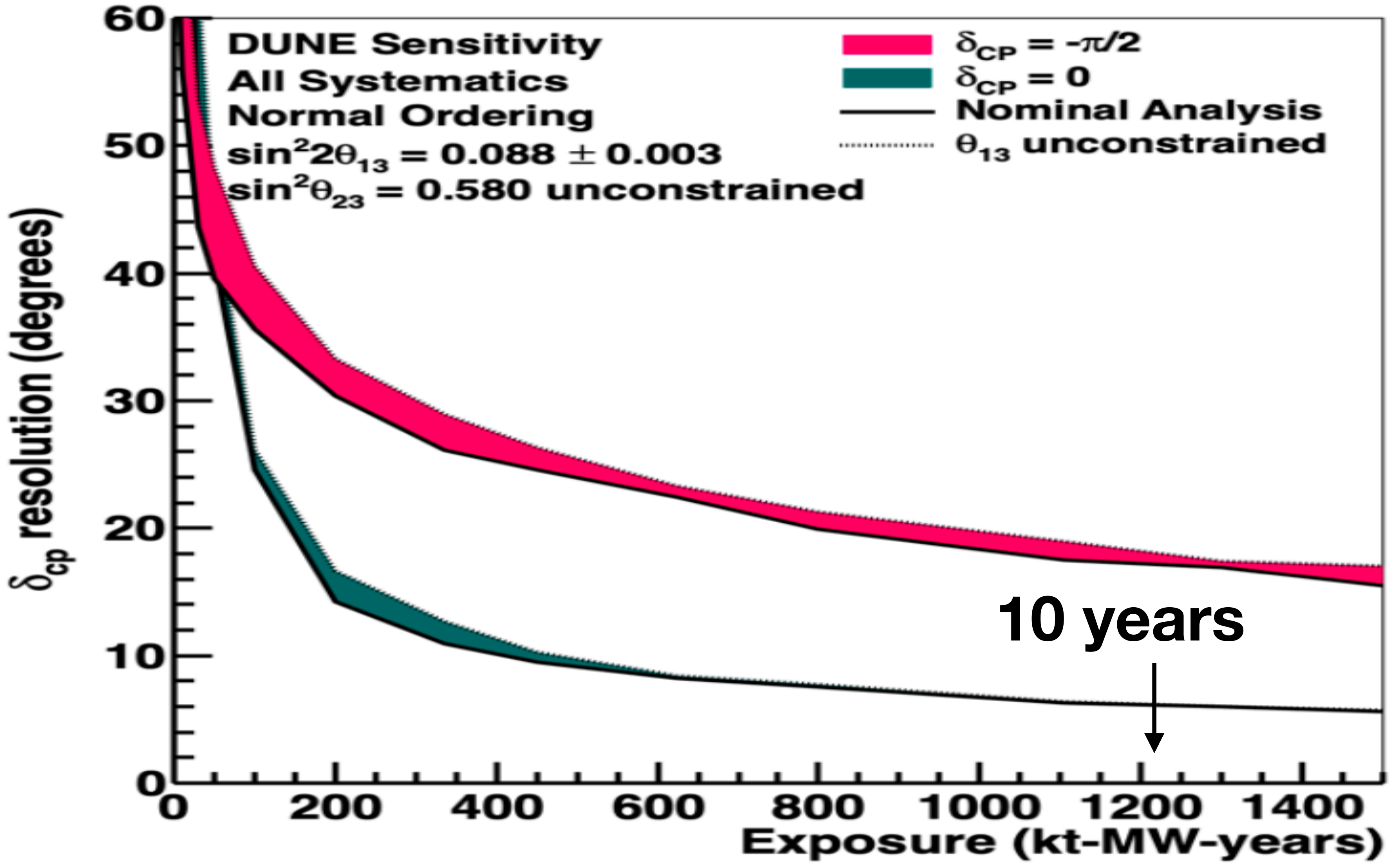
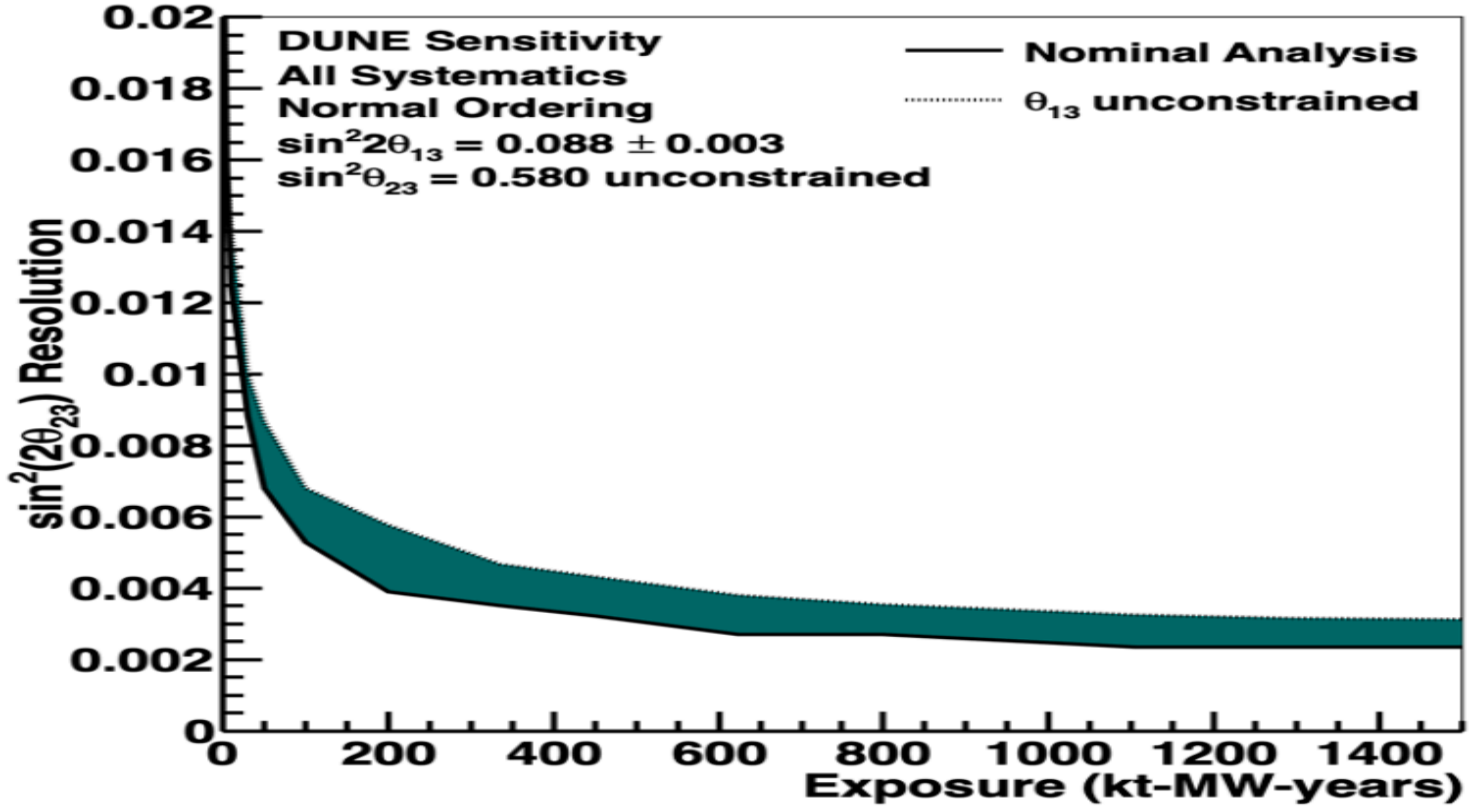


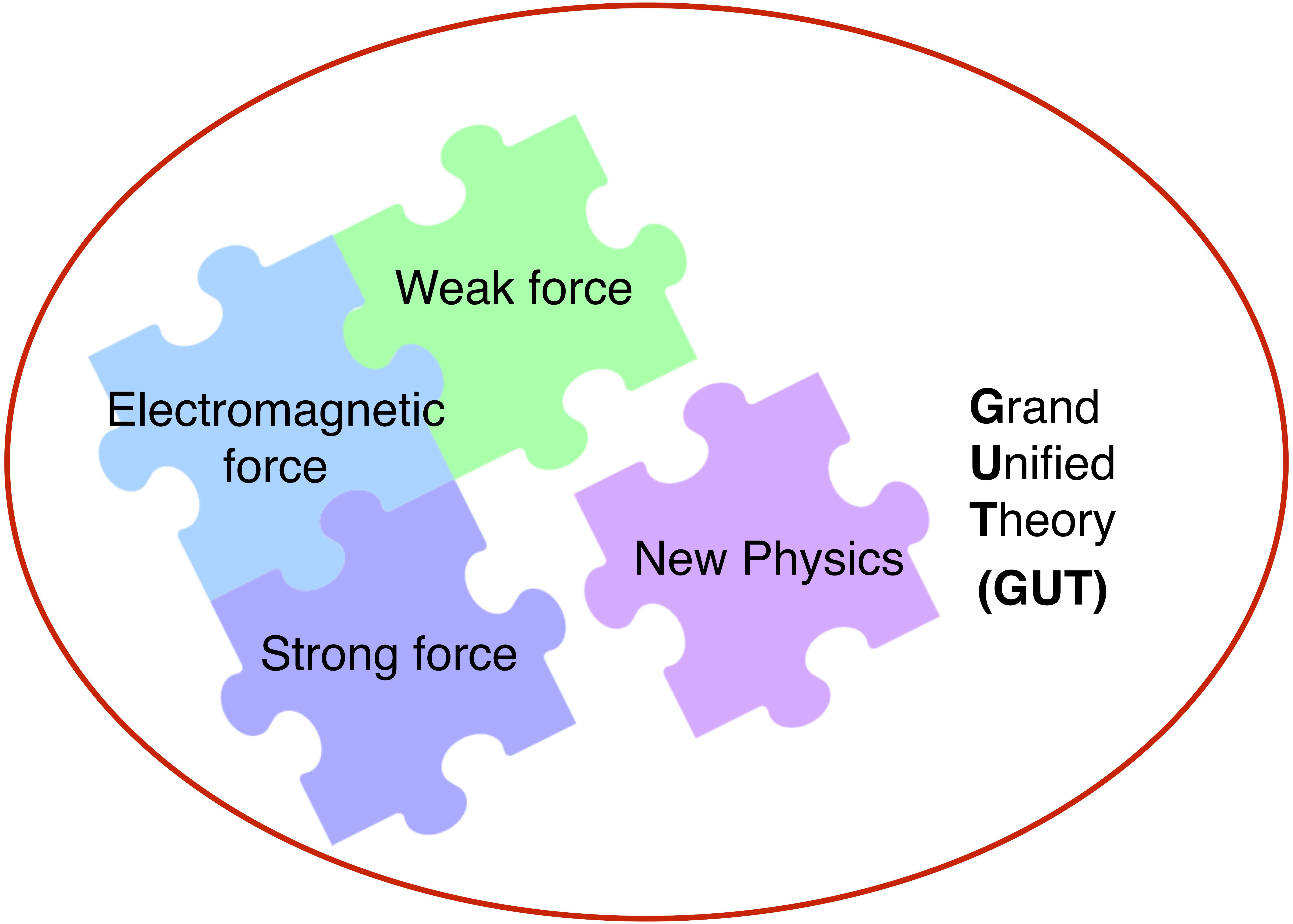
FIG. 144. 90% CL allowed regions in the $\sin^2\theta_{23}$–Δm^2_{32} plane. The true values are $\sin^2\theta_{23} = 0.45$ and $\Delta m^2_{32} = 2.4 \times 10^{-3}$ eV^2. Effect of systematic uncertainties is included. Left: Hyper-K only. Right: With a reactor constraint.

DUNE
2002.03005





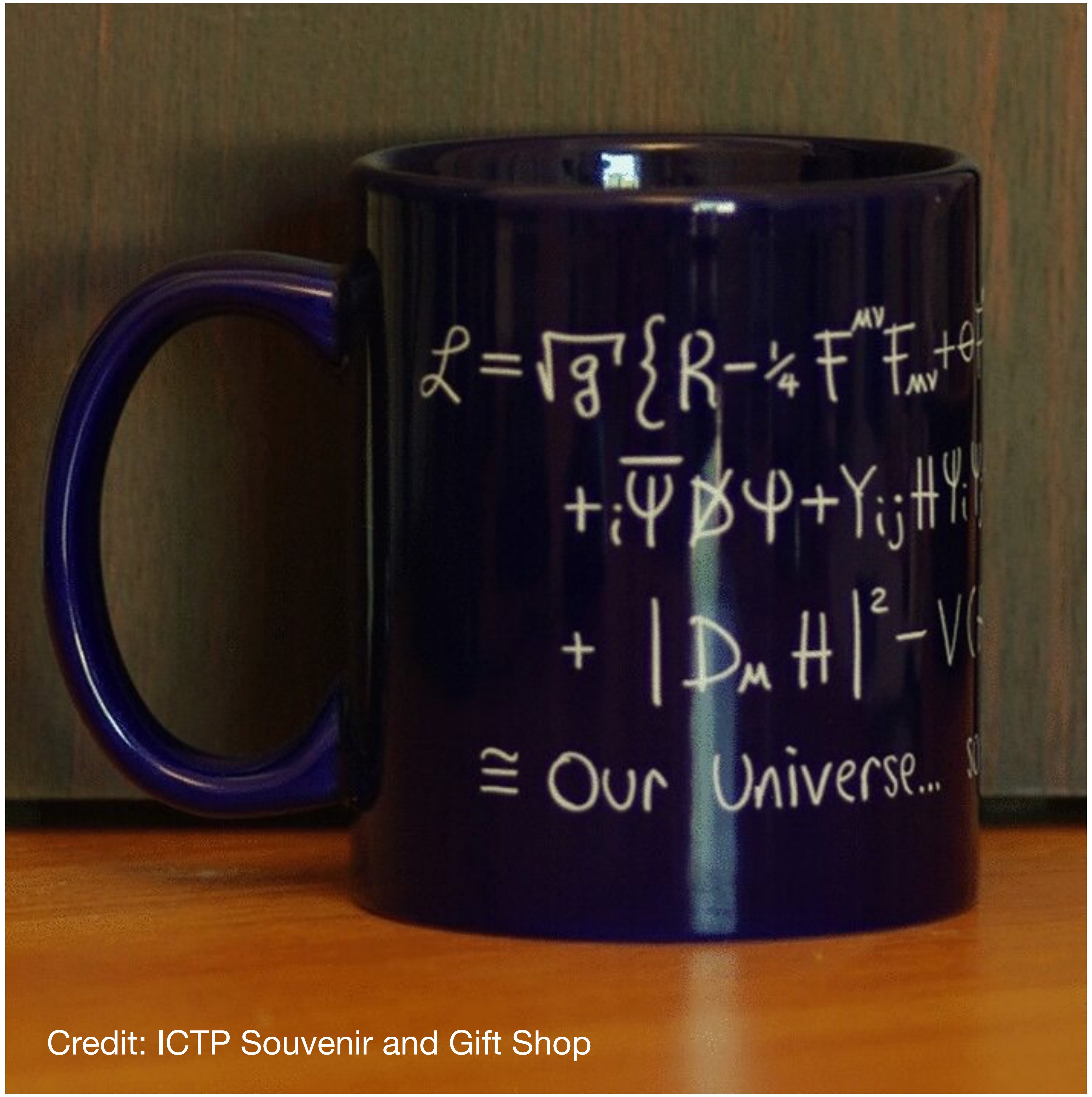
Framework of GUTs



Grand
Unified
Theory
(GUT)

Gravity… not included

What we know so far··· about particle physics



Credit: ICTP Souvenir and Gift Shop

The Standard Model of Particle Physics

- Gauge theories in

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

Strong force — Weak & EM force

- Particle content

ν_τ	τ	t	b	γ	$W^\pm$	H
ν_μ	μ	c	s	g	Z^0	
ν_e	e	u	d			

- Yukawa couplings
- Higgs mechanism

And neutrinos have masses…

How to get a GUT?

- Unification of symmetries $G_{\rm GUT} \supset G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$

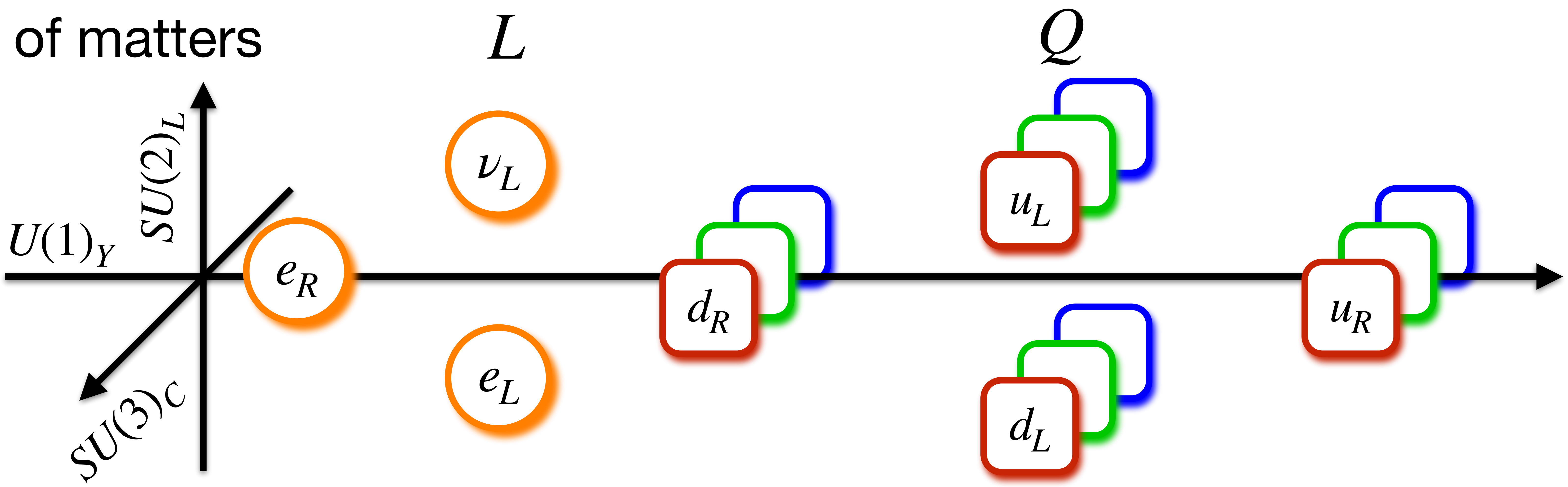
EW

- Unification of couplings $g_3 = g_2 = g_1$

(up to loop correction factors, and if $G_{\rm GUT}$ is a simple Lie group)

The scale where three gauge couplings are unified, denoted as $M_{\rm GUT}$ in this talk

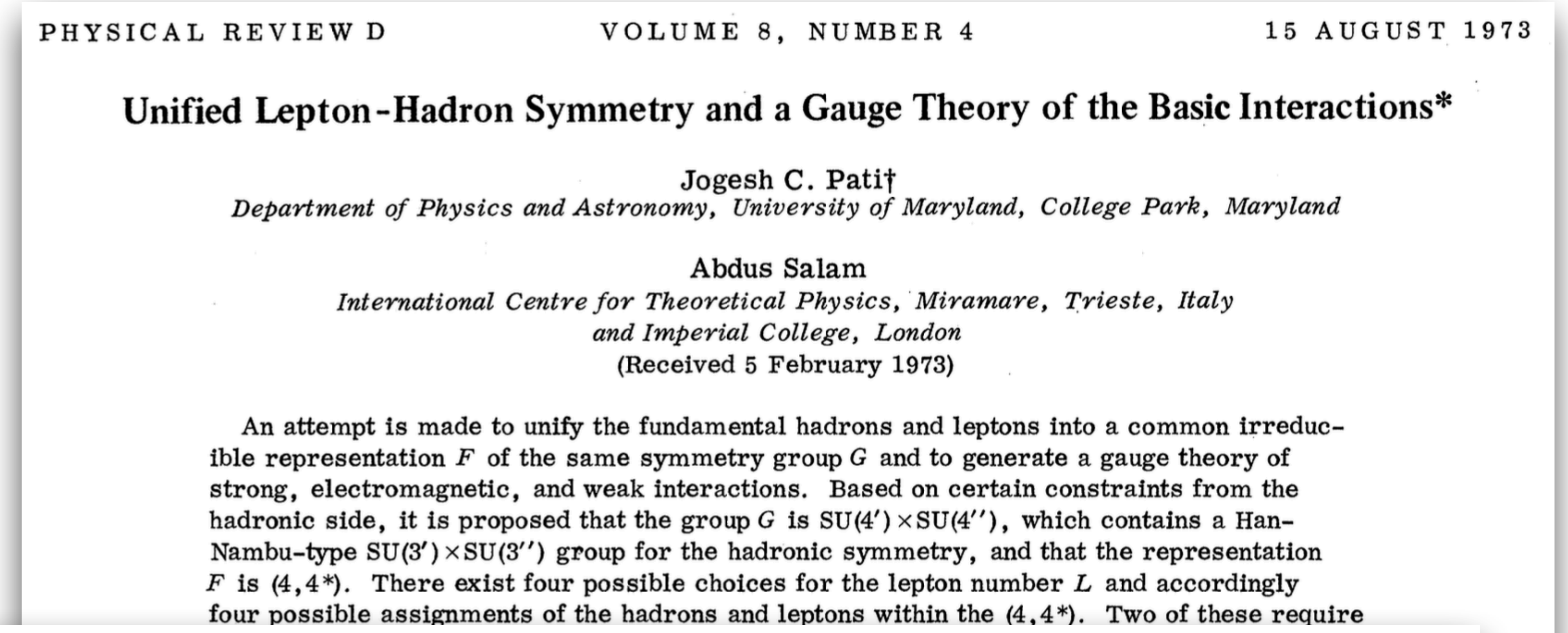
- Unification of matters



Weak hypercharge: $Y = -1$ $\quad Y = -\frac{1}{2}$ $\quad Y = -\frac{1}{3}$ $\quad Y = \frac{1}{6}$ $\quad Y = \frac{2}{3}$

GUT models

Pati-Salam (1973, 1974) $SU(4)_c \times SU(2)_L \times SU(2)_R := G_{422}$



PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

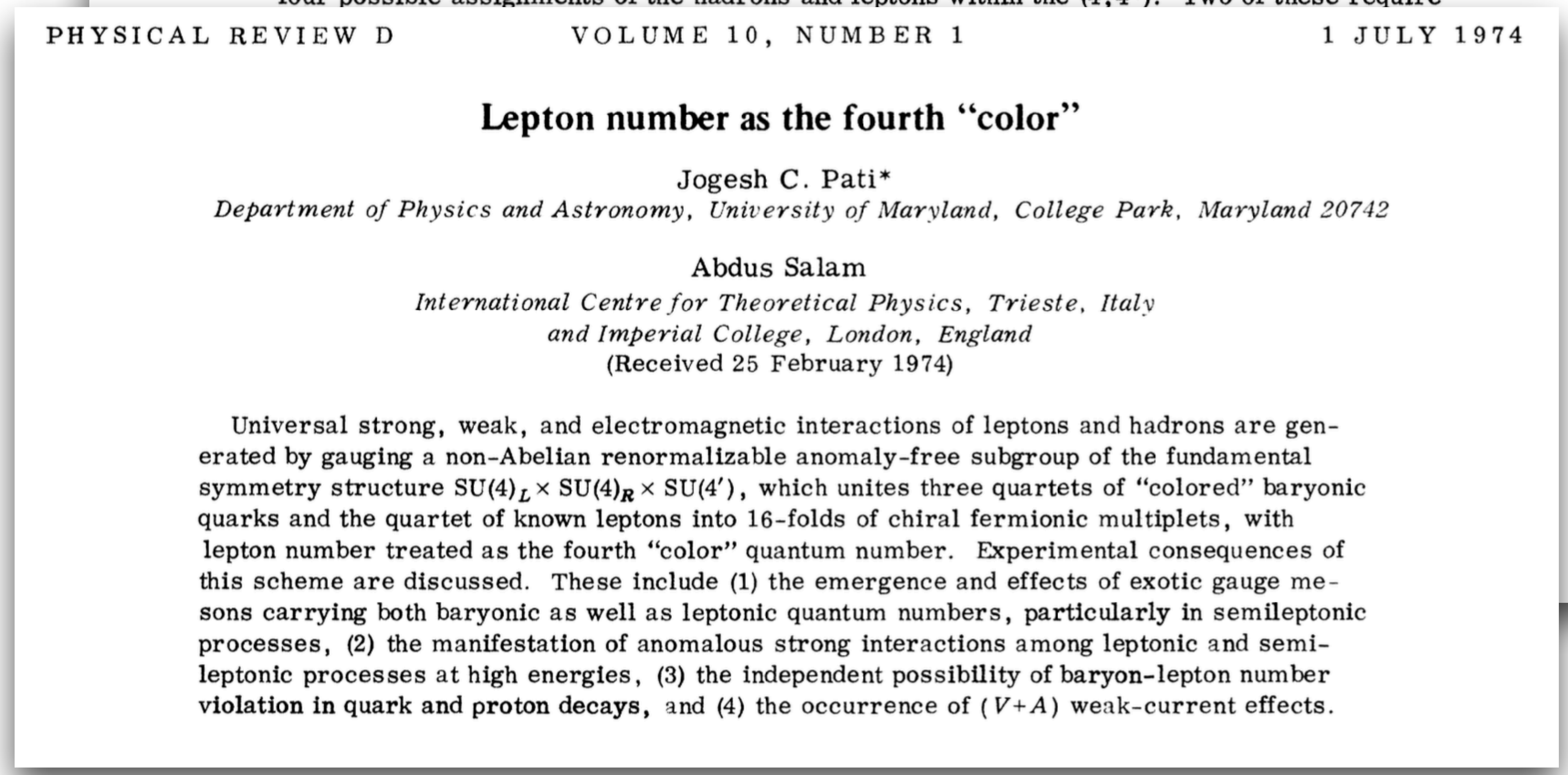
Unified Lepton-Hadron Symmetry and a Gauge Theory of the Basic Interactions*

Jogesh C. Pati†
Department of Physics and Astronomy, University of Maryland, College Park, Maryland

Abdus Salam
International Centre for Theoretical Physics, Miramare, Trieste, Italy and Imperial College, London

(Received 5 February 1973)

An attempt is made to unify the fundamental hadrons and leptons into a common irreducible representation F of the same symmetry group G and to generate a gauge theory of strong, electromagnetic, and weak interactions. Based on certain constraints from the hadronic side, it is proposed that the group G is SU(4′)×SU(4″), which contains a Han-Nambu-type SU(3′)×SU(3″) group for the hadronic symmetry, and that the representation F is (4,4*). There exist four possible choices for the lepton number L and accordingly four possible assignments of the hadrons and leptons within the (4,4*). Two of these require



PHYSICAL REVIEW D VOLUME 10, NUMBER 1 1 JULY 1974

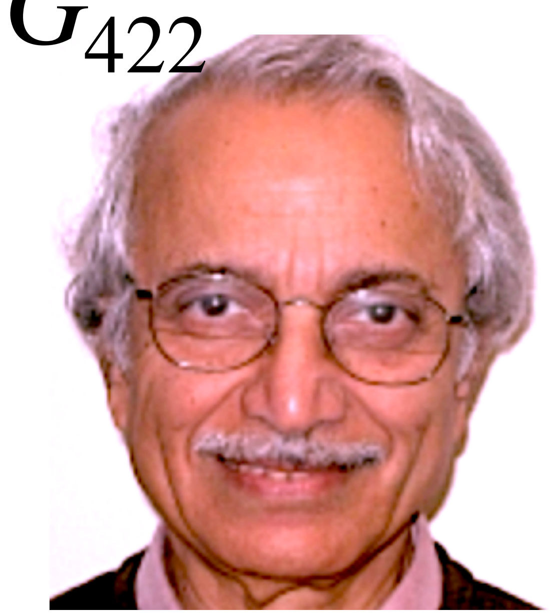
Lepton number as the fourth "color"

Jogesh C. Pati*
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

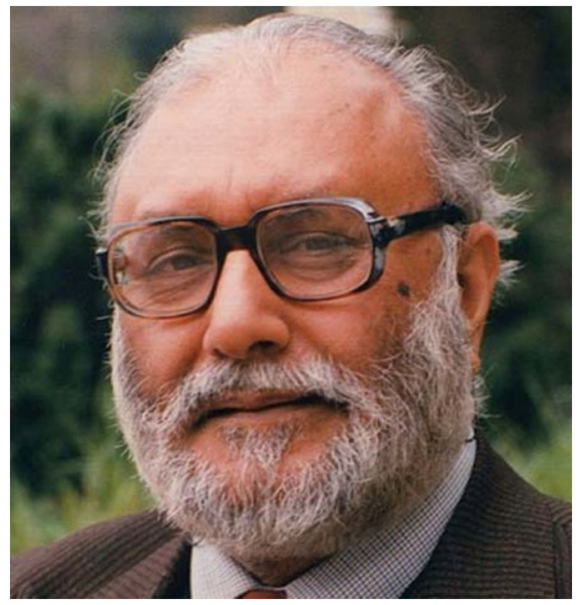
Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy and Imperial College, London, England

(Received 25 February 1974)

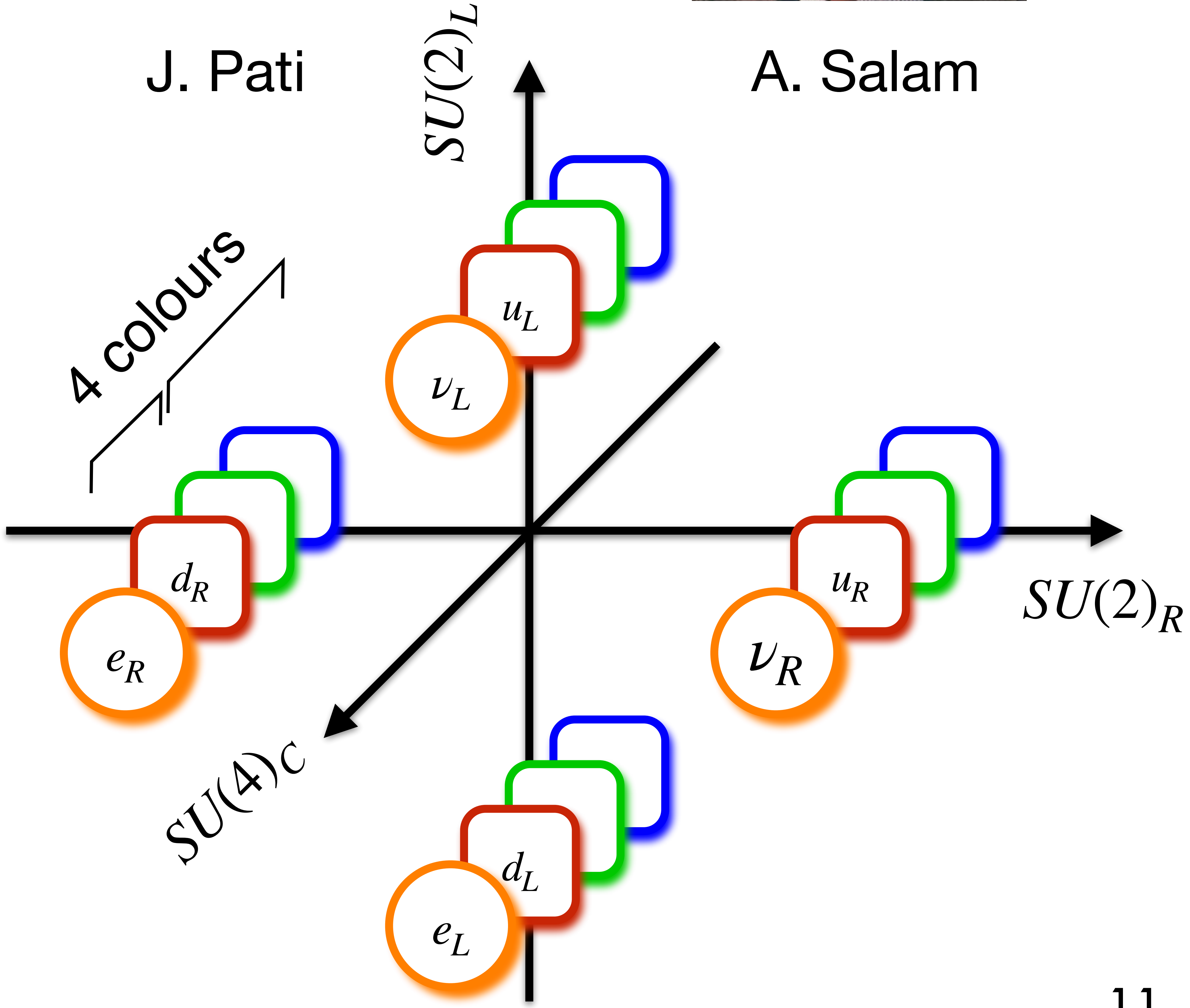
Universal strong, weak, and electromagnetic interactions of leptons and hadrons are generated by gauging a non-Abelian renormalizable anomaly-free subgroup of the fundamental symmetry structure $SU(4)_L \times SU(4)_R \times SU(4')$, which unites three quartets of "colored" baryonic quarks and the quartet of known leptons into 16-folds of chiral fermionic multiplets, with lepton number treated as the fourth "color" quantum number. Experimental consequences of this scheme are discussed. These include (1) the emergence and effects of exotic gauge mesons carrying both baryonic as well as leptonic quantum numbers, particularly in semileptonic processes, (2) the manifestation of anomalous strong interactions among leptonic and semi-leptonic processes at high energies, (3) the independent possibility of baryon-lepton number violation in quark and proton decays, and (4) the occurrence of $(V+A)$ weak-current effects.



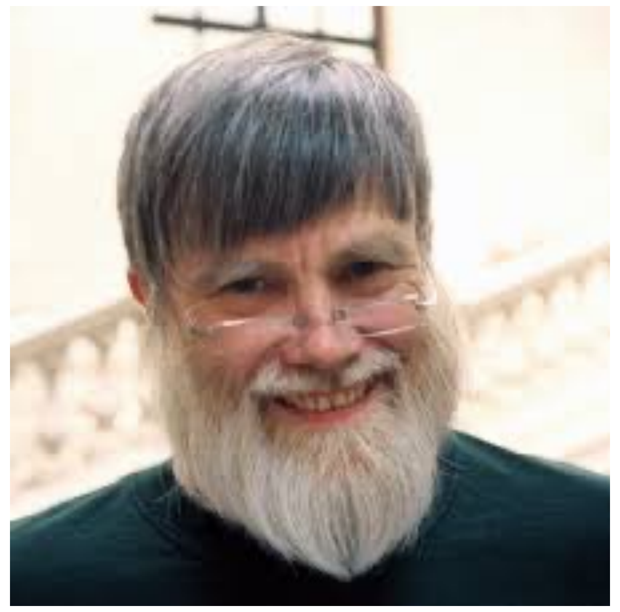
J. Pati



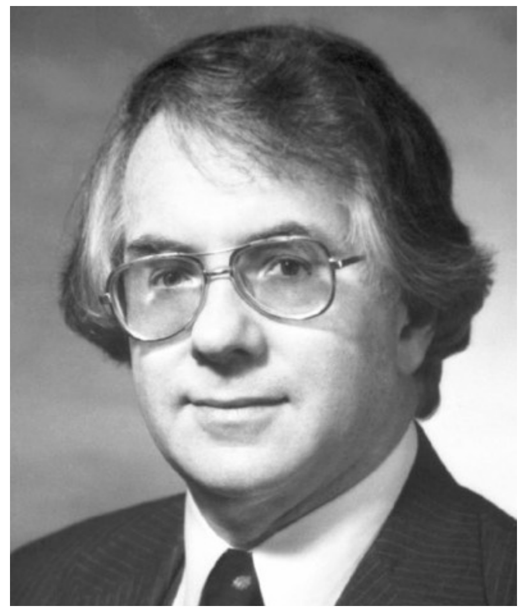
A. Salam



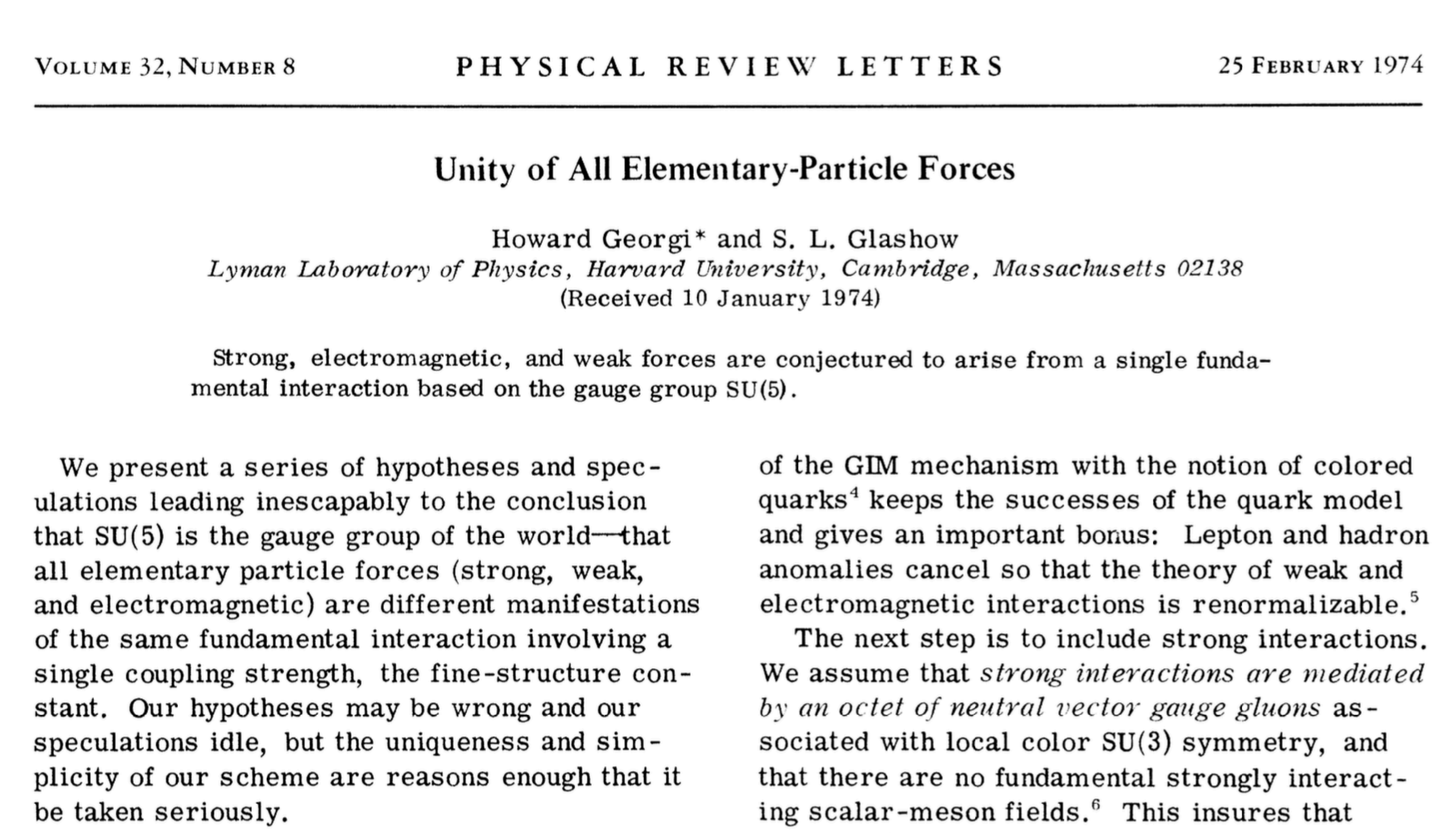
GUT models



H. Georgi



S. Glashow



VOLUME 32, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1974

Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world—that all elementary particle forces (strong, weak, and electromagnetic) are different manifestations of the same fundamental interaction involving a single coupling strength, the fine-structure constant. Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously.

of the GIM mechanism with the notion of colored quarks[4] keeps the successes of the quark model and gives an important bonus: Lepton and hadron anomalies cancel so that the theory of weak and electromagnetic interactions is renormalizable.[5]

The next step is to include strong interactions. We assume that *strong interactions are mediated by an octet of neutral vector gauge gluons* associated with local color SU(3) symmetry, and that there are no fundamental strongly interacting scalar-meson fields.[6] This insures that

- Matter: $\psi_a \sim \bar{\mathbf{5}}, \;\; \psi^{ab} \sim \mathbf{10}$

$$\begin{pmatrix} (d_R^1)^c \\ (d_R^2)^c \\ (d_R^3)^c \\ e_L \\ -\nu_L \end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & (u_R^3)^c & -(u_R^2)^c & -u_L^1 & -d_L^1 \\ -(u_R^3)^c & 0 & (u_R^1)^c & -u_L^2 & -d_L^2 \\ (d_R^2)^c & -(d_R^1)^c & 0 & -u_L^3 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 & 0 & (e_R)^c \\ d_L^1 & d_L^2 & d_L^3 & -(e_R)^c & 0 \end{pmatrix}$$

- Mediator of forces: $A_a^b \sim \mathbf{24}$

$$\begin{pmatrix} G_1^1 - 2B' & G_2^1 & G_3^1 & \bar{X}^1 & \bar{Y}^1 \\ G_1^2 & G_2^2 - 2B' & G_3^2 & \bar{X}^2 & \bar{Y}^2 \\ G_1^3 & G_2^3 & G_3^3 - 2B' & \bar{X}^3 & \bar{Y}^3 \\ X^1 & X^2 & X^3 & W'^3 + 3B' & W^+ \\ Y^1 & Y^2 & Y^3 & W^- & -W'^3 + 3B' \end{pmatrix}$$

$$B' = B/\sqrt{30} \qquad\qquad W' = W/\sqrt{2}$$

- Higgs particles: $\mathbf{5}, \mathbf{45}, \mathbf{24}.$

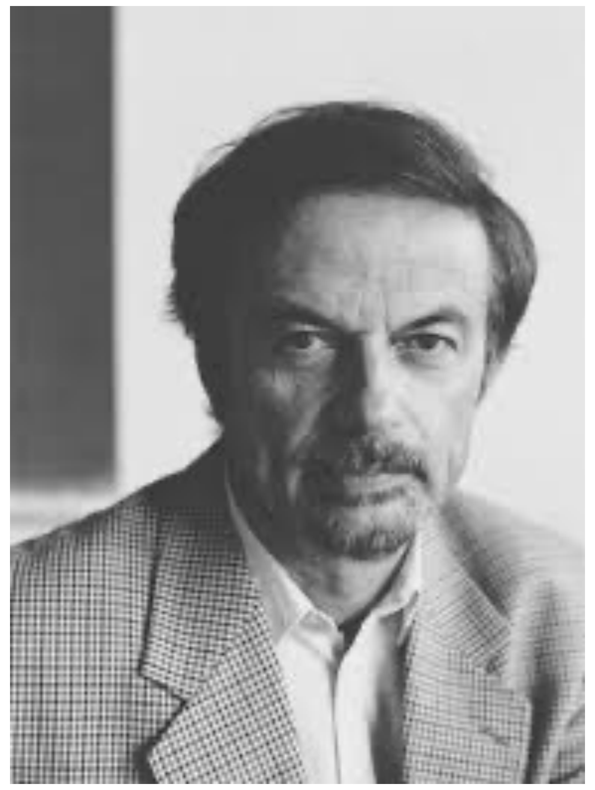
GUT models

- $SO(10)$ GUTs Fritzsch, Minkowski (1975)

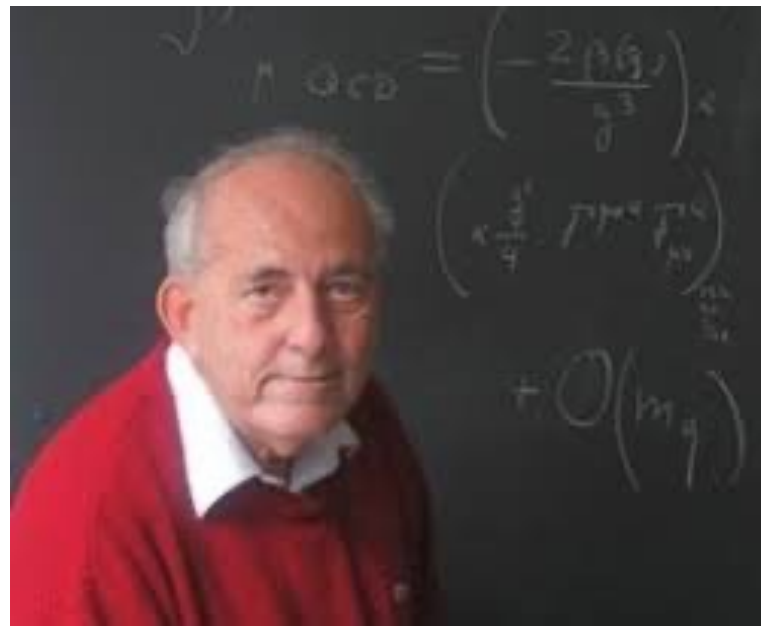
$$\mathbf{16} = \bar{\mathbf{5}} + \mathbf{10} + \mathbf{1} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}, \mathbf{1}, \mathbf{2})$$

$SO(10)$ $SU(5)$ $SU(4)_c \times SU(2)_L \times SU(2)_R$

Contains SU(5) and Pati-Salam, and more …



H. Fritzsch



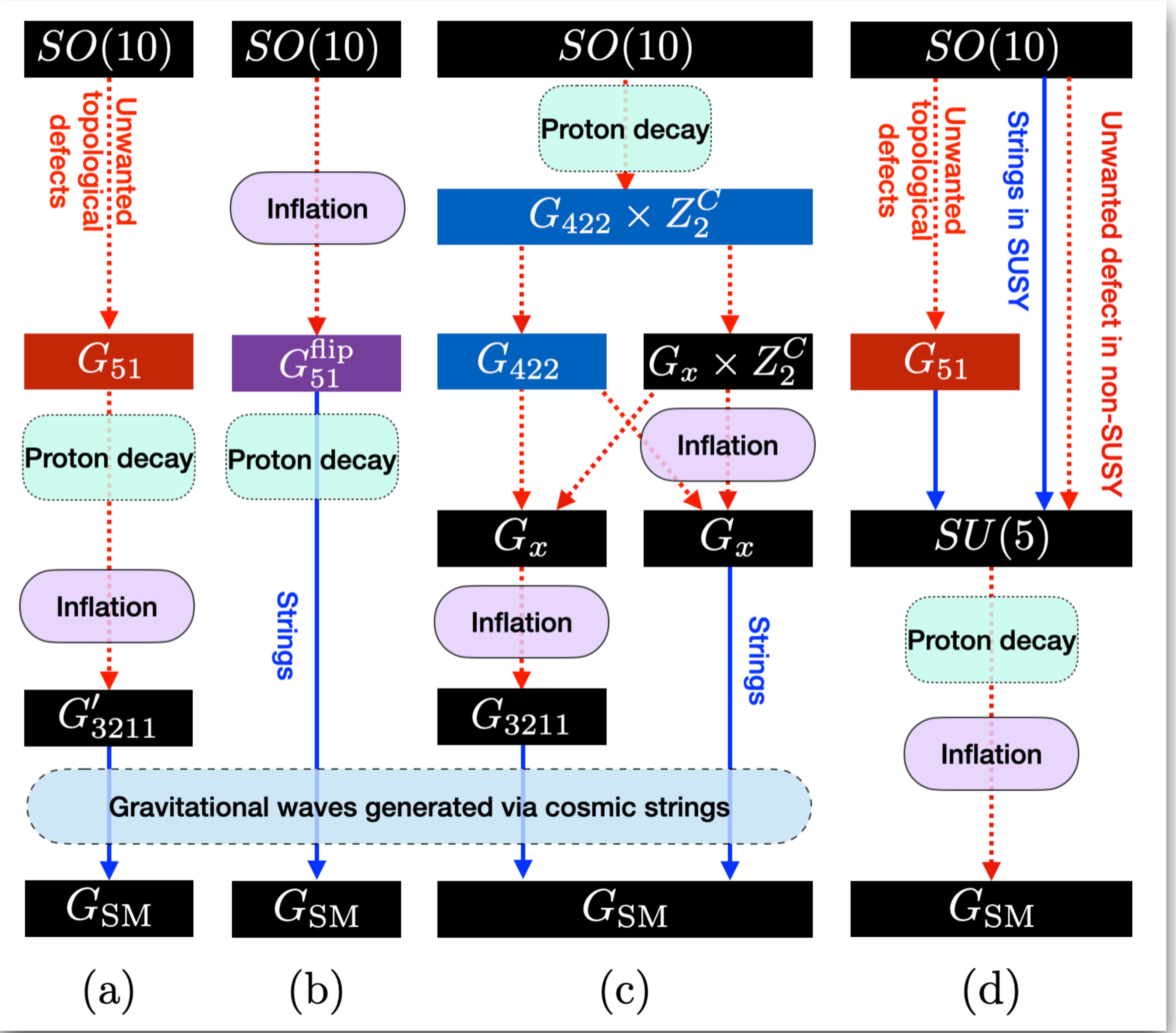
P. Minkowski

- Not minimal but realistic $SU(5)$ e.g., with extra $\mathbf{15}_H$, $\mathbf{24}_F$
- $SU(5) \times U(1)_{B-L} := G_{51}$ $\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}$, $\nu_{\mathrm{R}} \sim \mathbf{1}$
- Flipped $SU(5) \times U(1)_X := G_{51}^{\text{flip}}$ $u \leftrightarrow d$, $\nu \leftrightarrow e$

Rujula, Georgi, Glashow (1980); Barr,(1982); Derendinger, Kim, Nanopoulos (1984); Antoniadis, Ellis, Hagelin, Nanopoulos (1989)

GUT phenos

Fermion masses and mixing



Unwanted topological defects: monopoles and domain walls

In any breaking chains, inflation has to been introduced to inflate unwanted defects

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$G_{51} = SU(5) \times U(1)_X$$

$$G_{51}^{\text{flip}} = SU(5)_{\text{flip}} \times U(1)_{\text{flip}}$$

$$Z_2^C: \quad \psi_L \leftrightarrow \psi_R^c$$

$$G_x = G_{421} \text{ or } G_{3221}$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$

$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

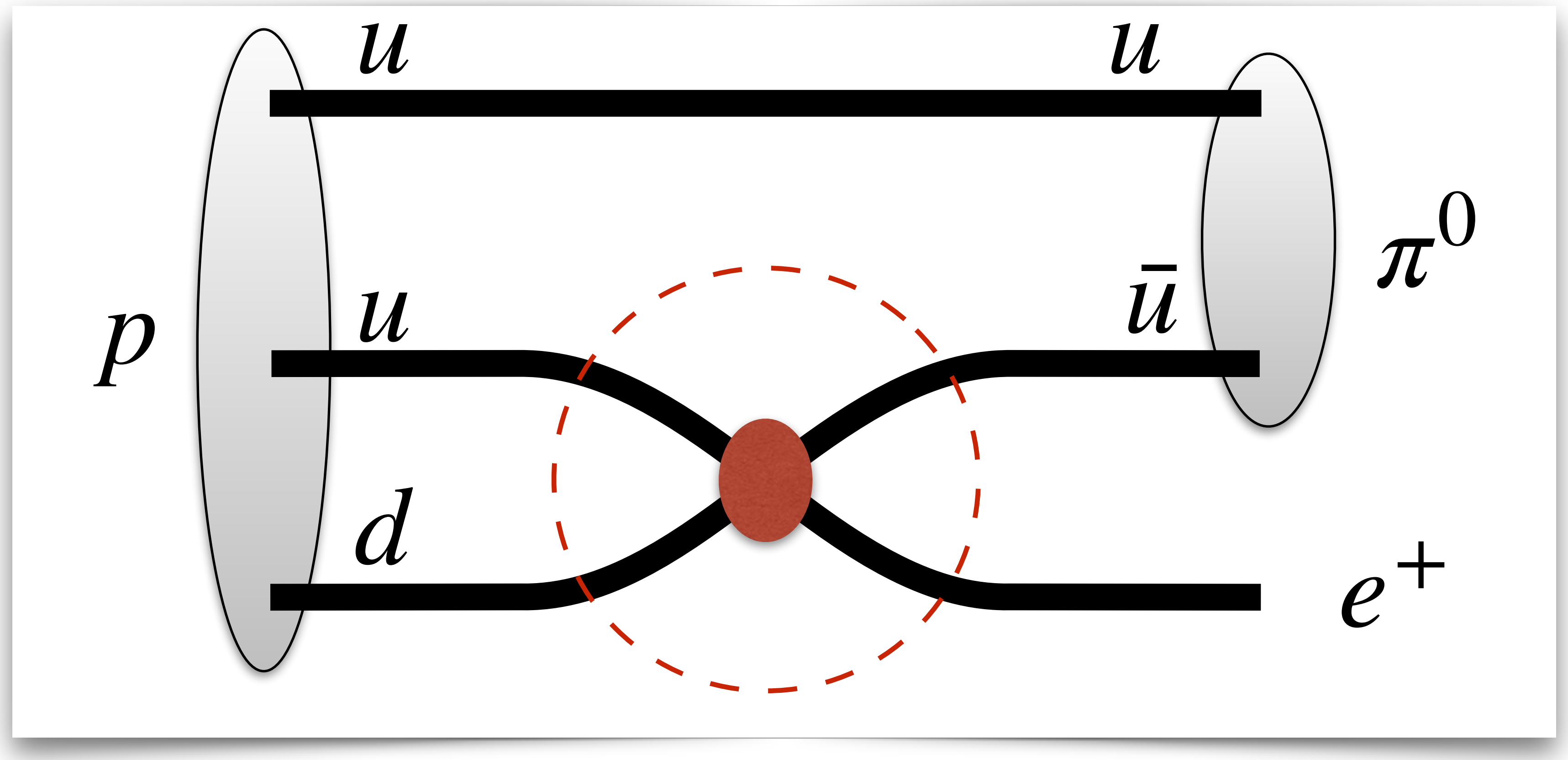
$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

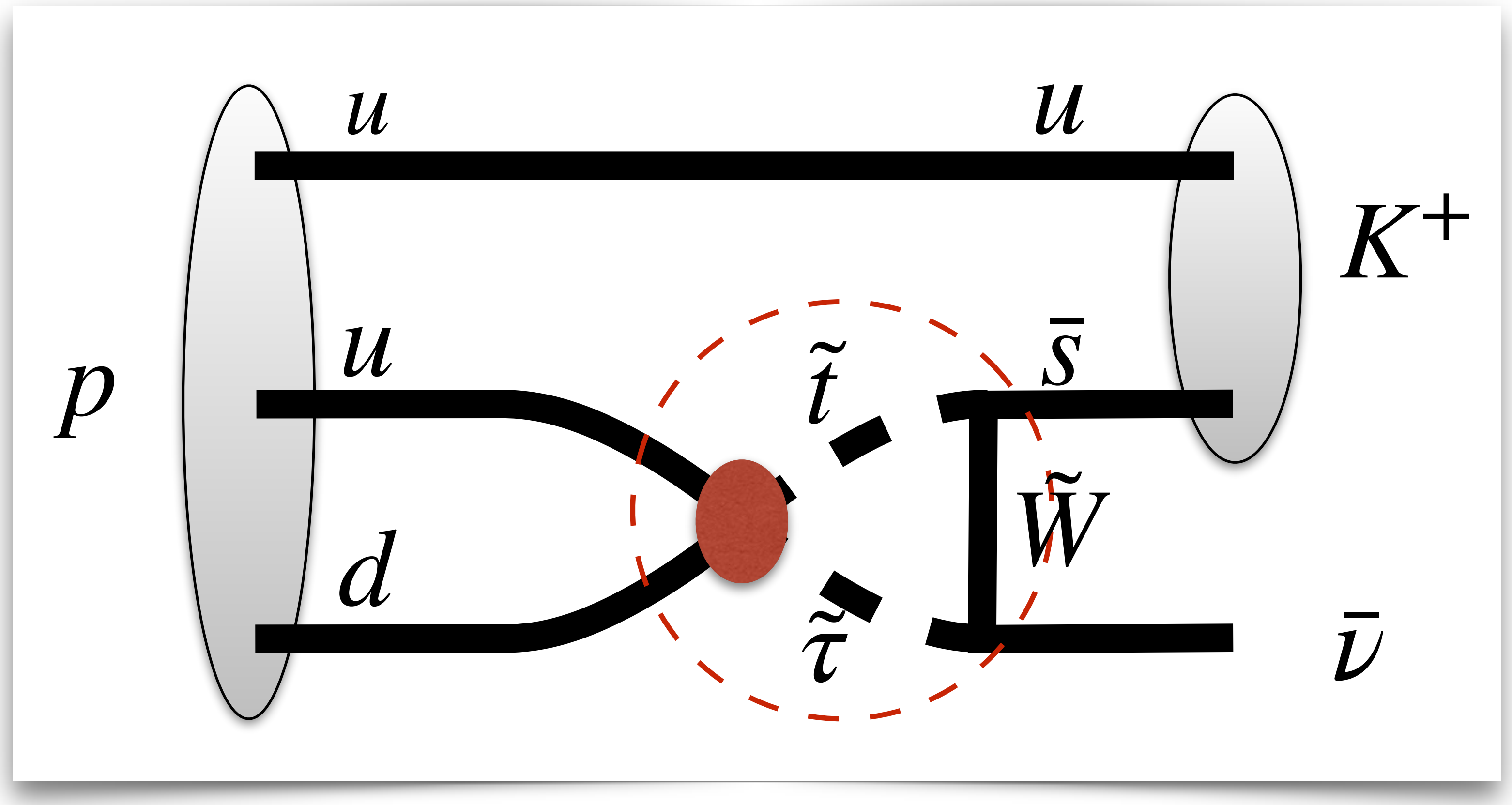
$$G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

King, Pascoli, Turner, **YLZ**, 2005.13549

Smoking gun of GUTs —— proton decay

Typical channels $p \to \pi^0 e^+$ $p \to K^+\bar{\nu}$





$$\tau_{\pi^0 e^+} \sim 10^{35}\,\text{years} \times \left(\frac{M_{\rm GUT}}{10^{16}\,\text{GeV}}\right)^4$$

$$\tau(p \to K^+\bar{\nu}) \propto M_{\rm GUT}^2\,\Lambda_{\rm SUSY}^2$$

Why neutrino experiments?

20 kton water $\simeq 2 \times 10^{10}$ g $\times$ $(2+8)/18$ mol/g $\times$ N_A /mol $\simeq 7 \times 10^{33}$ proton

Experiments of proton decay vs neutrino oscillation

KamiokaNDE [edit] https://en.wikipedia.org/wiki/Kamioka_Observatory#KamiokaNDE

The first of the Kamioka experiments was named KamiokaNDE for **Kamioka Nucleon Decay Experiment**. It was a large water Čerenkov detector designed to search for proton decay. To observe the decay of a particle with a lifetime as long as a proton an experiment must run for a long time and observe an enormous number of protons. This can be done most cost effectively if the target (the source of the protons) and the detector itself are made of the same material. Water is an ideal candidate because … production of Čeren… e background from cosmic ray muons in such a large detector located on the surface of the Earth would be far too large. The muon rate in the KamiokaNDE experiment was about 0.4 events per second, roughly five orders of magnitude smaller than what it would have been if the detector had been located at the surface.[4]

KamiokaNDE for **Kamioka Nucleon Decay Experiment**.

A model of KamiokaNDE

The distinct pattern produced by Čerenkov radiation allows for particle identification, an important tool both understanding the potential proton decay signal and for rejecting backgrounds. The ID is possible because the sharpness of the edge of the ring depends on the particle producing the radiation. Electrons (and therefore also gamma rays) produce fuzzy rings due to the multiple scattering of the low mass electrons. Minimum ionizing muons, in contrast produce very sharp rings as their heavier mass allows them to propagate directly.

Construction of Kamioka Underground Observatory (the predecessor of the present Kamioka Observatory, Institute for Cosmic Ray Research, University of Tokyo) began in 1982 and was completed in April, 1983. The detector was a cylindrical tank which contained 3,000 tons of pure water and had about 1,000 50 cm diameter photomultiplier tubes (PMTs) attached to the inner surface. The size of the outer detector was 16.0 m in height and 15.6 m in diameter. The detector failed to observe proton decay, but set what was then the world's best limit on the lifetime of the proton.

KamiokaNDE → Kamiokande-II → Super-Kamiokande → …

神冈

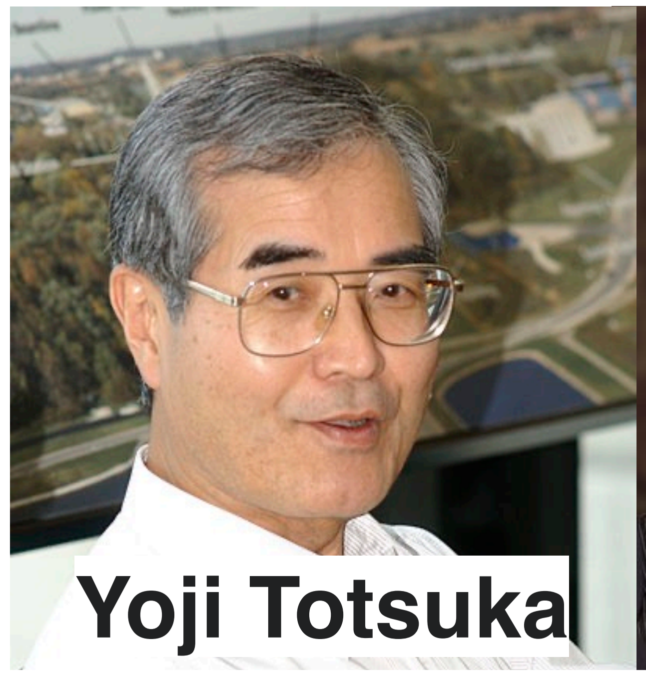
神冈II

Masatoshi Koshiba

Neutrino astro from SN 1987A

超级神冈

Neutrno oscillation in 1998



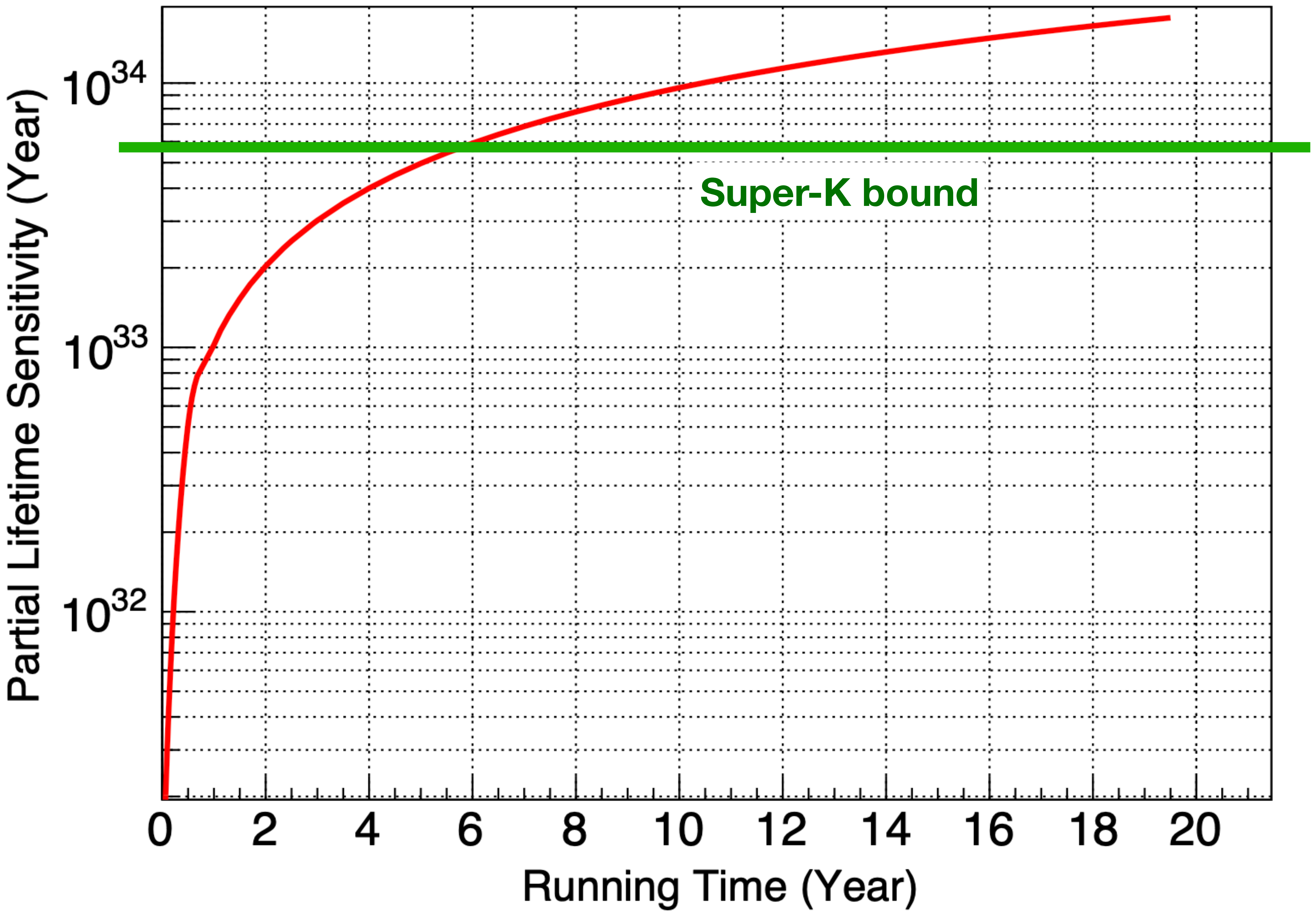
Capability of JUNO

Efficiency $\epsilon_{\text{JUNO}} = 36.9\,\%$

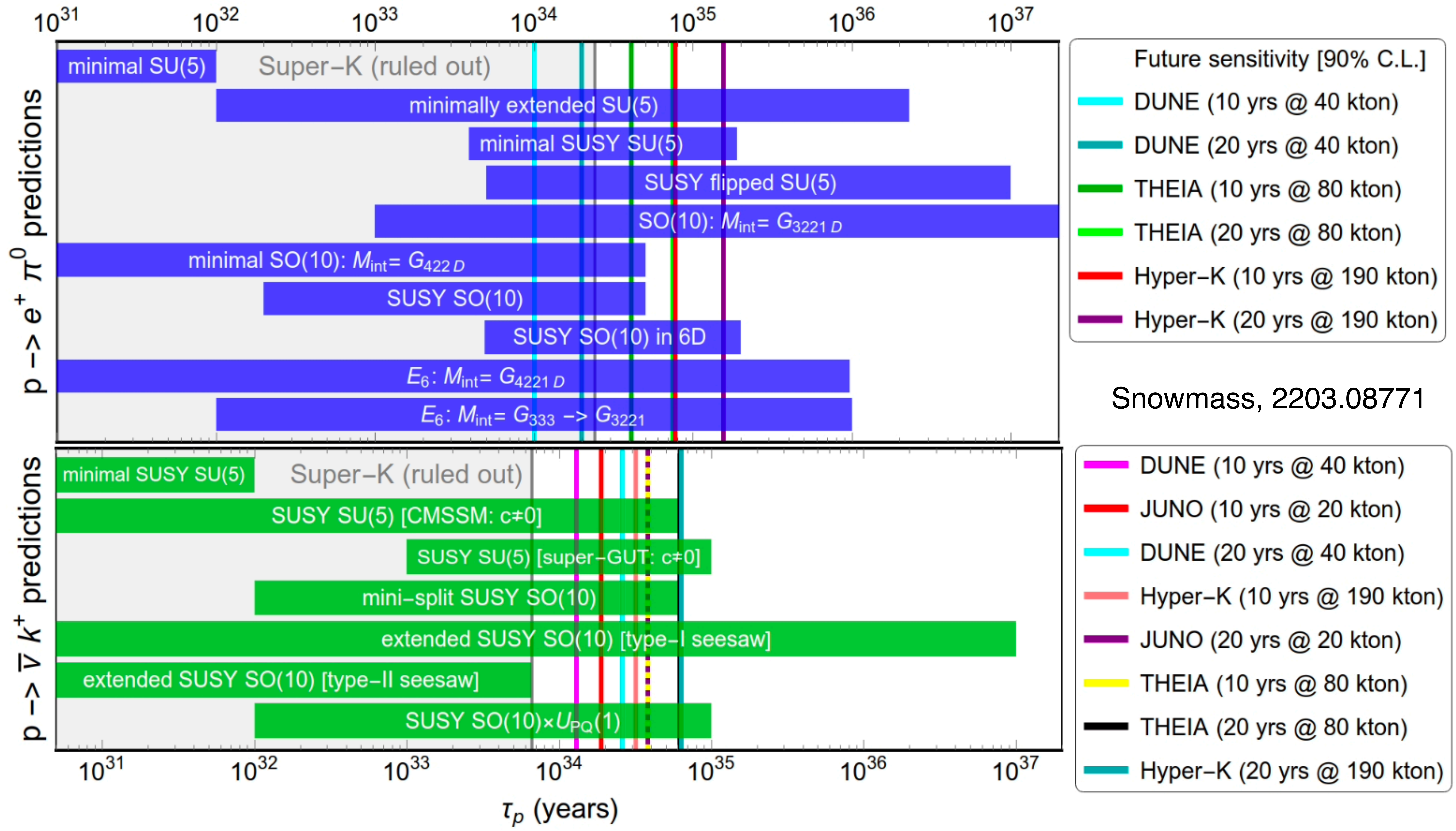
$$\tau / B(p \to \bar{\nu} K^+) = \frac{N_p T \epsilon}{n_{90}}$$

Statistics (90%CL)

JUNO, 2212.08502

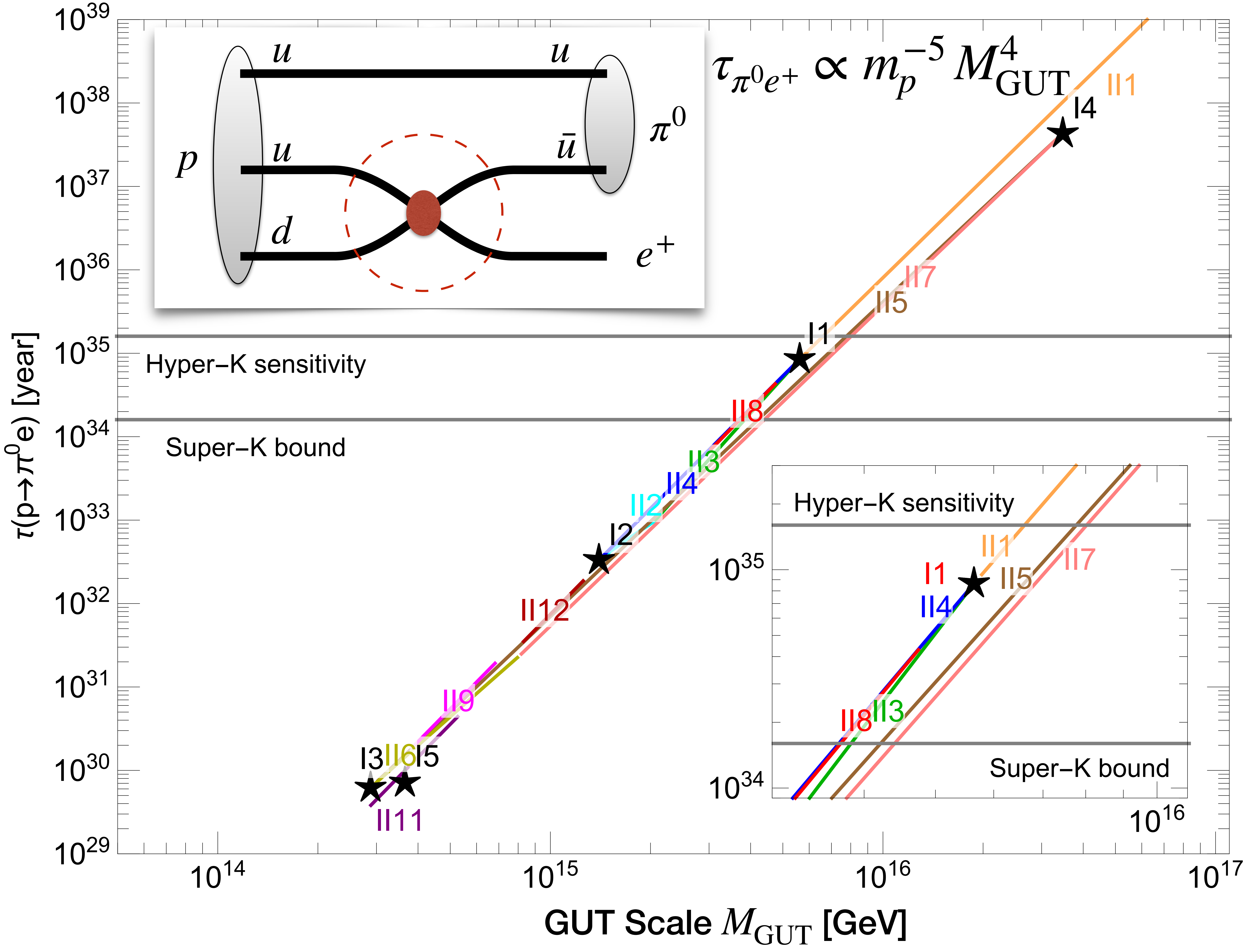


Potential in upcoming neutrino experiments



Snowmass, 2203.08771

Proton decay in SO(10) GUTs



$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	$G_{\rm SM}$
II1:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$	
II2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$	
II3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$	
II4:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$	
II5:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$	
II6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$	
II7:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{126,45}]{\text{m}}$	
II8:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$	
II9:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$	
II10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$	
II11:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$	
II12:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$	

King, Pascoli, Turner, **YLZ**,
2106.15634

Discussions

I. Sources at high scales: LNV and BNV operators

II. Mapping at the hadron level

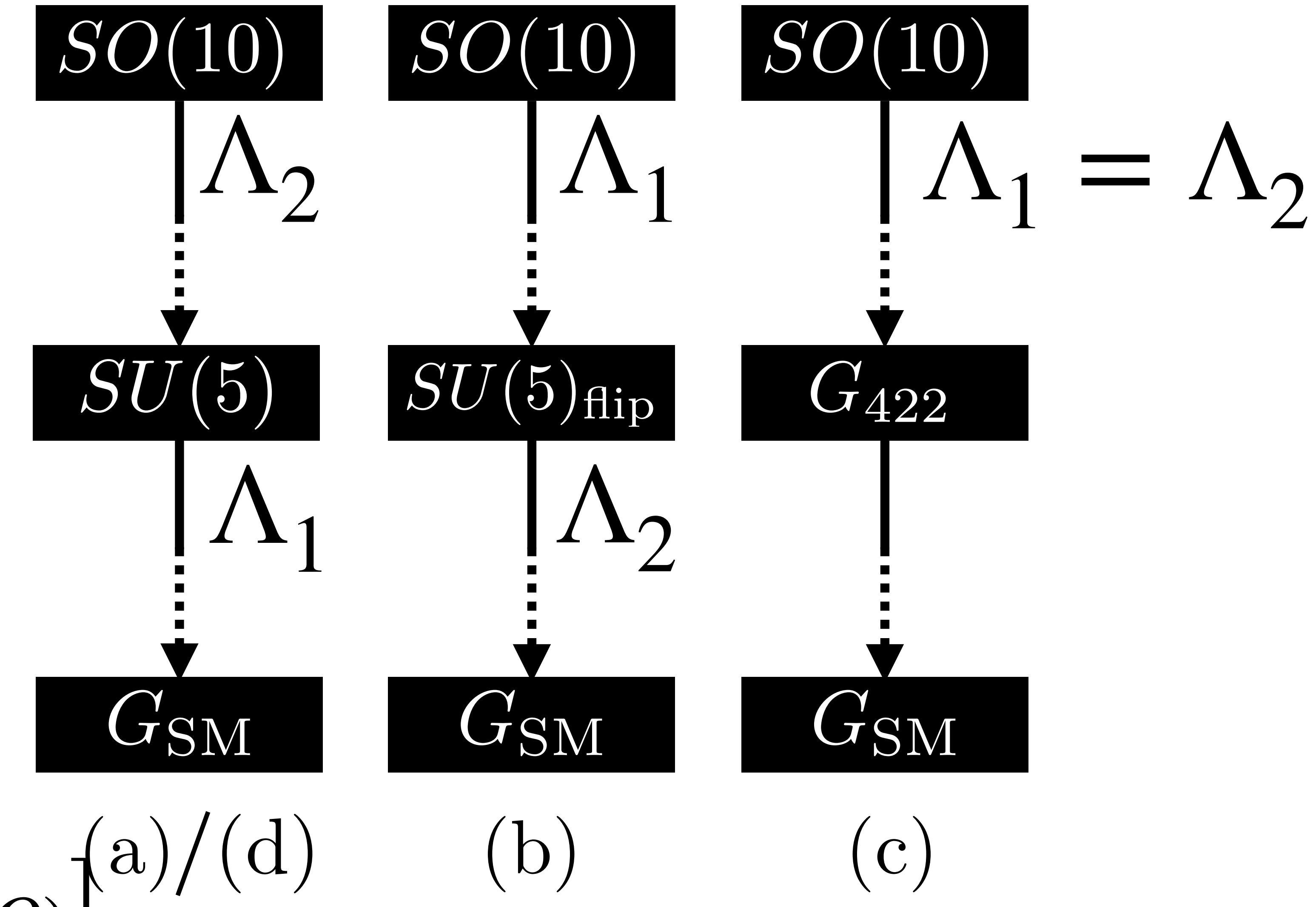
III. Mass correlations between quarks and leptons

IV. Mapping between low and high scales: RG running

Discussion I —— LNV and BNV operators: (B-L)-preserving ones

- As GUTs broken to SM, heavy particles integrated out, higher-dimensional baryon-number-violating (BNV) operators appear

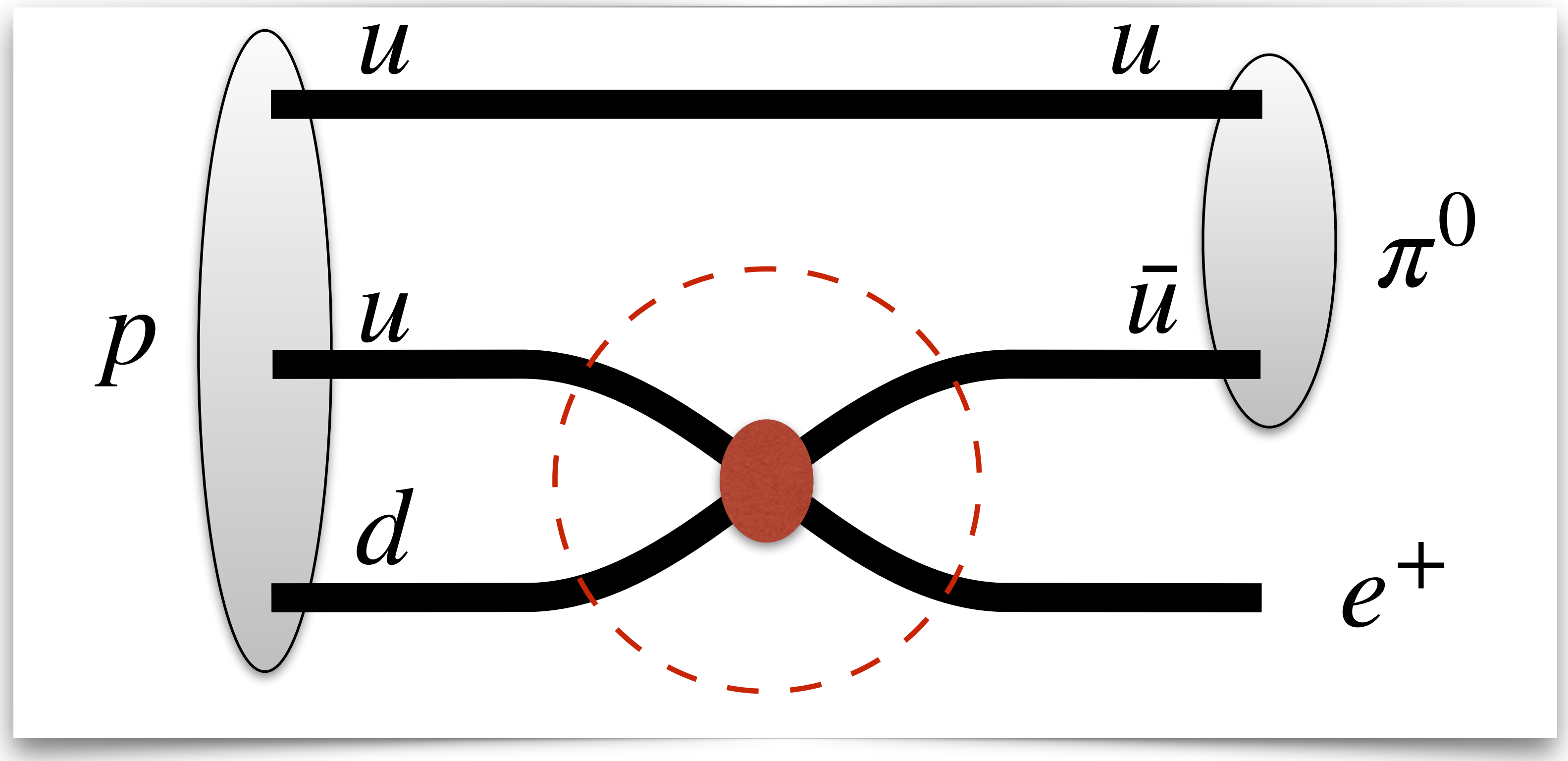
- D6 BNV operators via gauge mediators



$$\begin{pmatrix} G^1_1 - 2B' & G^1_2 & G^1_3 & \bar{X}^1 & \bar{Y}^1 \\ G^2_1 & G^2_2 - 2B' & G^2_3 & \bar{X}^2 & \bar{Y}^2 \\ G^3_1 & G^3_2 & G^3_3 - 2B' & \bar{X}^3 & \bar{Y}^3 \\ X^1 & X^2 & X^3 & W'^3 + 3B' & W^+ \\ Y^1 & Y^2 & Y^3 & W^- & -W'^3 + 3B' \end{pmatrix}$$

$$\frac{1}{\Lambda_1^2}\left[(\overline{u^c_R}\gamma^\mu Q)(\overline{d^c_R}\gamma_\mu L) + (\overline{u^c_R}\gamma^\mu Q)(\overline{e^c_R}\gamma_\mu Q)\right] + \frac{1}{\Lambda_2^2}\left[(\overline{d^c_R}\gamma^\mu Q)(\overline{u^c_R}\gamma_\mu L) + (\overline{d^c_R}\gamma^\mu Q)(\overline{\nu^c_R}\gamma_\mu Q)\right]$$

- Representative channel $p \to \pi^0 e^+$



$$\tau_{\pi^0 e^+} \simeq 8 \times 10^{34}\,\text{years} \times \left(\frac{\Lambda_1}{10^{16}\,\text{GeV}}\right)^4$$

or $$\simeq 7 \times 10^{35}\,\text{years} \times \left(\frac{\Lambda_2}{10^{16}\,\text{GeV}}\right)^4$$

Discussion I —— LNV and BNV operators: (B-L)-preserving ones

- In SUSY, additional sources (e.g., Higgs superfield) may enhance proton decay.

- D5 operators between 2 fermions and 2 sfermions are induced by Higgs mediation

$$\frac{c_1}{M_T}(\overline{Q^c}Q)(\tilde{L}\tilde{Q}) + \frac{c_2}{M_T}(\overline{u^c_R}d_R)(\tilde{e}_R\tilde{u}_R) + \cdots$$

c_1, c_2 are model-dependent,

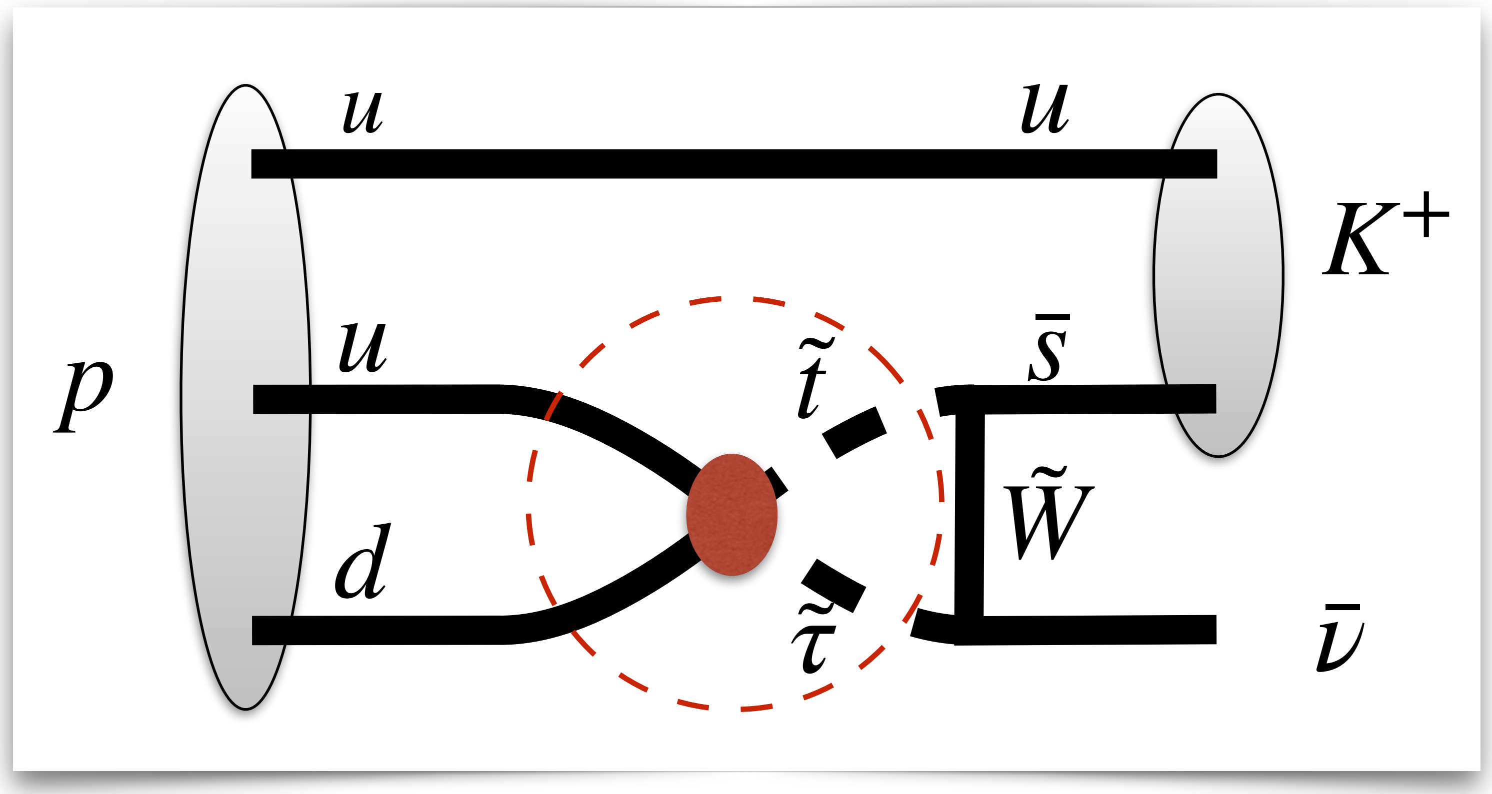
M_T is the heavy colour-triplet Higgs mass close to GUT scale

D5 operators dressed by gluinos, charginos or neutralinos generates D6 operators suppressed by $M_T\Lambda_{\rm SUSY}$

- Representative channel $p \to K^+\bar{\nu}$

$$\tau_{K^+\bar{\nu}} \propto (M_T\Lambda_{\rm SUSY})^2$$

weaker connection to proton decay scale



Discussion I —— LNV and BNV operators: (B-L)-breaking ones

- $U(1)_{B-L} \subset SO(10)$, B-L can only be broken spontaneously.

Table 8: Decomposition of the representation $\mathbf{16}$

$(4, 2_L, 2_R)$	$(3_C, 2_L, 2_R, 1_{B-L})$	$(3_C, 2_L, 1_R, 1_{B-L})$	$(3_C, 2_L, 1_Y)$
$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$(\mathbf{3}, \mathbf{2}, \mathbf{1}; \frac{1}{6})$ $(\mathbf{1}, \mathbf{2}, \mathbf{1}; -\frac{1}{2})$	$(\mathbf{3}, \mathbf{2}; 0, \frac{1}{6})$ $(\mathbf{1}, \mathbf{2}; 0, -\frac{1}{2})$	$(\mathbf{3}, \mathbf{2}; \frac{1}{6})$ $(\mathbf{1}, \mathbf{2}; -\frac{1}{2})$
$(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	$(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2}; -\frac{1}{6})$ $(\mathbf{1}, \mathbf{1}, \mathbf{2}; \frac{1}{2})$	$(\overline{\mathbf{3}}, \mathbf{1}; \frac{1}{2}, -\frac{1}{6})$ $(\overline{\mathbf{3}}, \mathbf{1}; -\frac{1}{2}, -\frac{1}{6})$ $(\mathbf{1}, \mathbf{1}; \frac{1}{2}, \frac{1}{2})$ $(\mathbf{1}, \mathbf{1}; -\frac{1}{2}, \frac{1}{2})$	$(\overline{\mathbf{3}}, \mathbf{1}; \frac{1}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}; -\frac{2}{3})$ $(\mathbf{1}, \mathbf{1}; 1)$ $(\mathbf{1}, \mathbf{1}; 0)$

Table 12: Decomposition of the representation $\overline{\mathbf{126}}$

$(4, 2_L, 2_R)$	$(3_C, 2_L, 2_R, 1_{B-L})$	$(3_C, 2_L, 1_R, 1_{B-L})$	$(3_C, 2_L, 1_Y)$
$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$(\mathbf{3}, \mathbf{1}, \mathbf{1}; -\frac{1}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}; \frac{1}{3})$	$(\mathbf{3}, \mathbf{1}; 0, -\frac{1}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}; 0, \frac{1}{3})$	$(\mathbf{3}, \mathbf{1}; -\frac{1}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}; \frac{1}{3})$
$(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})$	$(\mathbf{1}, \mathbf{3}, \mathbf{1}; 1)$ $(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3}; \frac{1}{3})$ $(\overline{\mathbf{6}}, \mathbf{1}, \mathbf{3}; -\frac{1}{3})$	$(\mathbf{1}, \mathbf{3}; 0, 1)$ $(\overline{\mathbf{3}}, \mathbf{3}; 0, \frac{1}{3})$ $(\overline{\mathbf{6}}, \mathbf{3}; 0, -\frac{1}{3})$	$(\mathbf{1}, \mathbf{3}; 1)$ $(\overline{\mathbf{3}}, \mathbf{3}; \frac{1}{3})$ $(\overline{\mathbf{6}}, \mathbf{3}; -\frac{1}{3})$
$(\mathbf{10}, \mathbf{1}, \mathbf{3})$	$(\mathbf{1}, \mathbf{1}, \mathbf{3}; -1)$	$(\mathbf{1}, \mathbf{1}; 1, -1)$ $(\mathbf{1}, \mathbf{1}; 0, -1)$ $(\mathbf{1}, \mathbf{1}; -1, -1)$	$(\mathbf{1}, \mathbf{1}; 0)$ $(\mathbf{1}, \mathbf{1}; -1)$ $(\mathbf{1}, \mathbf{1}; -2)$

Fukuyama, et al, hep-ph/0405300

注：这里B-L的荷与通常的约定相差2倍

$$y_{126} \mathbf{16}_F \overline{\mathbf{126}}_H \mathbf{16}_F \Rightarrow M_N \overline{N} N^c + M_L \overline{\nu^c} \nu$$

$$M_N = y_{126} v_{B-L} \qquad M_L = y_{126} v_\Delta \sim y_{126} \frac{v_{\rm EW}^2}{M_\Delta}$$

Type I + II $\qquad M_\nu = -\frac{M_D^2}{M_N} + M_L$

B-L破坏 ⇒ 中微子的Majorana质量 ⇒ $0\nu 2\beta$ 衰变

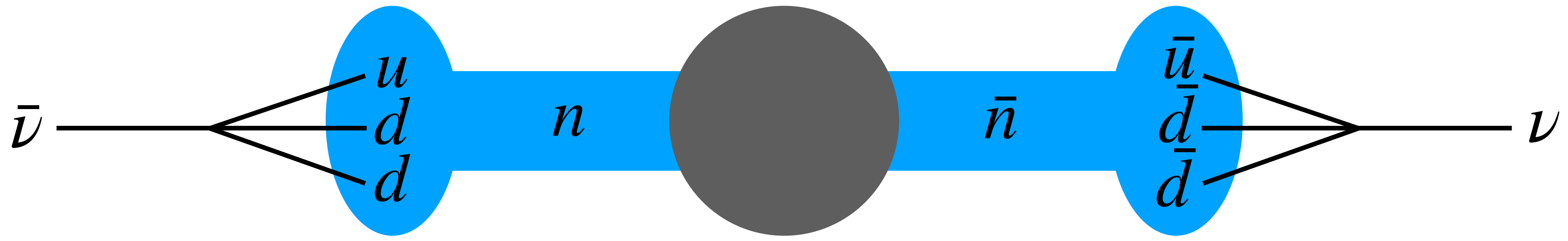
Discussion I —— LNV and BNV operators: (B-L)-breaking ones

- 轻子数破坏但是B-L守恒的过程（比如质子衰变）跟中微子的Majorana质量无直接关系

质子衰变 $\not\Rightarrow$ 中微子的Majorana质量

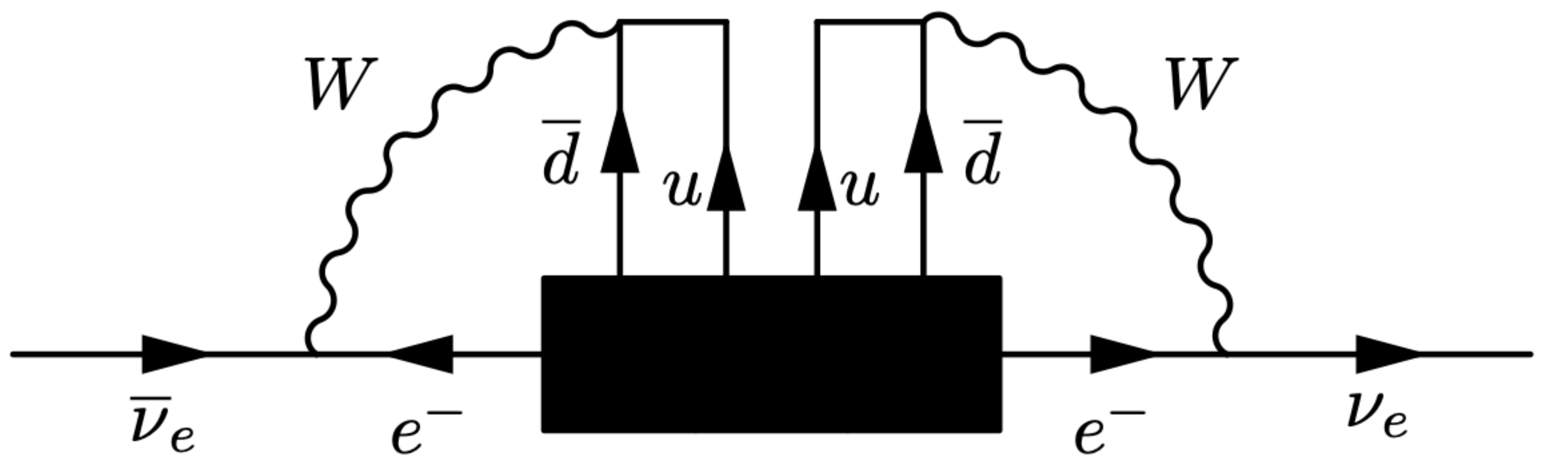
- 中微子的Majorana质量的本质是B-L破坏

中子-反中子振荡 在大统一框架下可以诱导出中微子的Majorana质量



大家常说的轻子数破坏过程会导致Majorana质量，是在默认重子数没有破坏的前提下说的

- Schechter-Valle 定理 [PRD25 (1982) 2951]



$0\nu 2\beta \Rightarrow$ Majorana mass J.H Yu’s talk

Discussion II —— Mapping at the hadron level

相空间	长程 RGE	短程 RGE	新物理能标和味结构	核子矩阵元 (NME)

$$\Gamma(p \to \pi^0 e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right)^2 A_{\text{long}}^2 \times \left\{ A_{\text{short},R}^2 \frac{1+V_{ud}^2}{\Lambda_1^2} |\langle \pi | (ud)_R u_L | p\rangle|^2 + A_{\text{short},L}^2 \left(\frac{1}{\Lambda_1^2} + \frac{V_{ud}^2}{\Lambda_2^2}\right) |\langle \pi | (ud)_L u_L | p\rangle|^2 \right\}$$

$A_{\text{long}}^2 = 1.247$

$$\Rightarrow \langle M | \mathcal{O}_{L,R} | N\rangle = P_{L,R}\left(W_0(q^2) - \frac{i\not{q}}{m_N} W_1(q^2)\right) u_N(q)$$

q: 出射轻子的动量

Y. Aoki et al, hep-lat/9911026; 1705.01338; 2111.01608

$q^2 = 0$ (upper row of each pair) ⟷ $q^2 = -m_\mu^2$ (lower row of each pair)

	W_0 [GeV^2] 24ID	32ID	cont.
$\langle \pi^+ \| (ud)_L d_L \| p\rangle$	0.1032(86)(26)	0.1252(48)(50)	0.151(14)(8)(26)
	0.1050(87)(36)	0.1271(49)(50)	0.153(14)(7)(26)
$\langle \pi^+ \| (ud)_L d_R \| p\rangle$	−0.1125(78)(41)	−0.134(5)(11)	−0.159(15)(20)(25)
	−0.1139(78)(45)	−0.136(5)(12)	−0.161(15)(20)(26)
$\langle K^0 \| (us)_L u_L \| p\rangle$	0.0395(22)(36)	0.0411(13)(25)	0.0430(38)(12)(19)
	0.0397(22)(36)	0.0411(13)(25)	0.0427(37)(12)(16)
$\langle K^0 \| (us)_L u_R \| p\rangle$	0.0688(37)(19)	0.0764(17)(36)	0.0854(57)(55)(90)
	0.0693(36)(20)	0.0769(17)(36)	0.0860(56)(55)(91)
$\langle K^+ \| (us)_L d_L \| p\rangle$	0.0263(19)(6)	0.0273(9)(11)	0.0284(30)(17)(12)
	0.0266(19)(6)	0.0278(9)(11)	0.0293(30)(18)(15)
$\langle K^+ \| (us)_L d_R \| p\rangle$	−0.0301(21)(10)	−0.0345(9)(14)	−0.0398(31)(20)(52)
	−0.0307(21)(10)	−0.0351(8)(15)	−0.0403(31)(20)(52)
$\langle K^+ \| (ud)_L s_L \| p\rangle$	0.0923(48)(35)	0.0961(26)(46)	0.1006(80)(60)(46)
	0.0932(47)(37)	0.0972(26)(48)	0.1019(79)(60)(47)
$\langle K^+ \| (ud)_L s_R \| p\rangle$	−0.0835(58)(3)	−0.0954(32)(39)	−0.109(10)(8)(14)
	−0.0846(58)(6)	−0.0964(32)(40)	−0.110(10)(8)(14)
$\langle K^+ \| (ds)_L u_L \| p\rangle$	−0.0651(33)(26)	−0.0681(18)(33)	−0.0717(54)(41)(35)
	−0.0658(32)(28)	−0.0686(18)(34)	−0.0720(53)(40)(34)
$\langle K^+ \| (ds)_L u_R \| p\rangle$	−0.0394(22)(20)	−0.0417(11)(23)	−0.0443(35)(26)(27)
	−0.0393(21)(21)	−0.0416(11)(23)	−0.0444(35)(26)(27)

	W_1 [GeV^2] 24ID	32ID	cont.
$\langle \pi^+ \| (ud)_L d_L \| p\rangle$	−0.130(10)(17)	−0.1316(67)(82)	−0.134(18)(2)(2)
	−0.132(10)(17)	−0.1335(67)(81)	−0.136(19)(3)(2)
$\langle \pi^+ \| (ud)_L d_R \| p\rangle$	0.116(8)(11)	0.140(5)(14)	0.169(14)(18)(29)
	0.118(8)(12)	0.142(5)(15)	0.170(14)(18)(28)
$\langle K^0 \| (us)_L u_L \| p\rangle$	0.0256(29)(4)	0.0264(18)(22)	0.0275(50)(53)(10)
	0.0254(29)(4)	0.0265(19)(22)	0.0278(52)(52)(13)
$\langle K^0 \| (us)_L u_R \| p\rangle$	−0.0250(27)(30)	−0.0253(9)(18)	−0.0258(38)(3)(4)
	−0.0254(28)(31)	−0.0256(9)(19)	−0.0259(38)(4)(2)
$\langle K^+ \| (us)_L d_L \| p\rangle$	−0.0448(30)(13)	−0.0467(17)(27)	−0.0489(51)(44)(22)
	−0.0453(30)(16)	−0.0472(16)(28)	−0.0496(51)(43)(23)
$\langle K^+ \| (us)_L d_R \| p\rangle$	0.0452(31)(23)	0.0487(10)(25)	0.0529(45)(28)(42)
	0.0458(31)(23)	0.0492(10)(26)	0.0532(45)(29)(40)
$\langle K^+ \| (ud)_L s_L \| p\rangle$	−0.0638(54)(24)	−0.0691(23)(52)	−0.0754(82)(86)(63)
	−0.0653(54)(32)	−0.0701(23)(55)	−0.0757(80)(82)(57)
$\langle K^+ \| (ud)_L s_R \| p\rangle$	0.0588(50)(11)	0.0687(28)(43)	0.080(9)(8)(12)
	0.0605(50)(15)	0.0693(28)(43)	0.080(9)(8)(10)
$\langle K^+ \| (ds)_L u_L \| p\rangle$	0.0192(31)(15)	0.0213(13)(16)	0.0239(46)(18)(26)
	0.0201(31)(19)	0.0217(13)(17)	0.0237(46)(15)(19)
$\langle K^+ \| (ds)_L u_R \| p\rangle$	−0.0203(31)(5)	−0.0231(9)(12)	−0.0265(42)(32)(34)
	−0.0204(31)(7)	−0.0233(9)(12)	−0.0269(42)(33)(35)

$L_t = 64$ $L_x = 24$ $L_x = 32$

$\langle \pi^0 | \mathcal{O}_{L,R} | N\rangle = \sqrt{2}\langle \pi^+ | \mathcal{O}_{L,R} | N\rangle$ due to isospin symmetry

Discussion II —— Mapping at the hadron level

χEFT in $SU(3)_L \times SU(3)_R$ (味对称性) [Claudson, Wise, Hall, NPB 1982]

$$M = \begin{bmatrix} \sqrt{\tfrac{1}{2}}\pi^0 + \sqrt{\tfrac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\tfrac{1}{2}}\pi^0 + \sqrt{\tfrac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\tfrac{2}{3}}\eta \end{bmatrix}, \quad B = \begin{bmatrix} \sqrt{\tfrac{1}{2}}\Sigma^0 + \sqrt{\tfrac{1}{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\sqrt{\tfrac{1}{2}}\Sigma^0 + \sqrt{\tfrac{1}{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\tfrac{2}{3}}\Lambda^0 \end{bmatrix}.$$

$$\Sigma = e^{2iM/f}, \xi = e^{iM/f} \qquad \Sigma \to U_L \Sigma U_R^\dagger, \xi \to U_L \xi U_B^\dagger = U_B \xi U_R^\dagger, B \to U_B B U_B^\dagger$$

$$\begin{aligned} \mathcal{L}_0 = &\tfrac{1}{8}f^2 \, \mathrm{Tr}\,(\partial_\mu \Sigma)(\partial^\mu \Sigma^+) + \mathrm{Tr}\,\bar{B}(i\not{\partial} - M_B)B \\ &+ \tfrac{1}{2}i\,\mathrm{Tr}\,\bar{B}\gamma^\mu[\xi\partial_\mu\xi^+ + \xi^+\partial_\mu\xi]B \\ &+ \tfrac{1}{2}i\,\mathrm{Tr}\,\bar{B}\gamma^\mu B[(\partial_\mu\xi)\xi^+ + (\partial_\mu\xi^+)\xi] \\ &- \tfrac{1}{2}i(D-F)\,\mathrm{Tr}\,\bar{B}\gamma^\mu\gamma_5 B[(\partial_\mu\xi)\xi^+ - (\partial_\mu\xi^+)\xi] \\ &+ \tfrac{1}{2}i(D+F)\,\mathrm{Tr}\,\bar{B}\gamma^\mu\gamma_5[\xi\partial_\mu\xi^+ - \xi^+\partial_\mu\xi]B \end{aligned}$$

$$\begin{aligned} \mathcal{L}_1 = &v^3\,\mathrm{Tr}\,(\Sigma^+ m + m\Sigma) + a_1\,\mathrm{Tr}\,\bar{B}(\xi^+ m\xi^+ + \xi m\xi)B \\ &+ a_2\,\mathrm{Tr}\,\bar{B}B(\xi^+ m\xi^+ + \xi m\xi) + b_1\,\mathrm{Tr}\,\bar{B}\gamma_5(\xi^+ m\xi^+ - \xi m\xi)B \\ &+ b_2\,\mathrm{Tr}\,\bar{B}\gamma_5 B(\xi^+ m\xi^+ - \xi m\xi) \end{aligned}$$

Soft breaking $\quad m = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix}.$

$$\mathcal{L}^{|\Delta B|=1} = \sum_i C_i \mathcal{O}^{(i)}$$

$$\begin{aligned} O^{(1)}_{abcd} &= (d_{\alpha a R} u_{\beta b R})(q_{i\gamma c L} l_{jdL})\varepsilon_{\alpha\beta\gamma}\varepsilon_{ij}, \\ O^{(2)}_{abcd} &= (q_{i\alpha a L} q_{j\beta b L})(u_{\gamma c R} l_{dR})\varepsilon_{\alpha\beta\gamma}\varepsilon_{ij}, \\ O^{(3)}_{abcd} &= (q_{i\alpha a L} q_{j\beta b L})(q_{k\gamma c L} l_{ldL})\varepsilon_{\alpha\beta\gamma}\varepsilon_{il}\varepsilon_{jk} \\ O^{(4)}_{abcd} &= (d_{\alpha a R} u_{\beta b R})(u_{\gamma c R} l_{dR})\varepsilon_{\alpha\beta\gamma}. \end{aligned}$$

$$\mathcal{L}^{|\Delta B|=1} = \alpha\left[C_1 l_L \mathrm{Tr}[O_{qq'}\xi B_L \xi] + C_2 l_R \mathrm{Tr}[O_{qq'}\xi^\dagger B_R \xi^\dagger]\right] + \beta\left[C_3 l_L \mathrm{Tr}[O_{qq'}\xi B_L \xi^\dagger] + C_4 l_R \mathrm{Tr}[O_{qq'}\xi^\dagger B_R \xi]\right]$$

$$v \approx \frac{3f^2 m_\eta^2}{4(m_u + m_d + 4m_s)}, \quad a_1 \approx (M_\Sigma - M_\Xi)/2m_s \approx -0.4, \quad a_2 \approx (M_\Sigma - M_N)/2m_s \approx 0.85$$

$b_{1,2}$不贡献

$$\langle M|\mathcal{O}_{L,R}|N\rangle = P_{L,R}\left(W_0(q^2) - \frac{i\not{q}}{m_N}W_1(q^2)\right)u_N(q)$$

$$\begin{aligned} \langle\pi^+|(ud)_L u_L|p\rangle &= \frac{\beta}{f}(1+D+F), \quad \color{red}{+\mathcal{O}(q^2)} \\ \langle\pi^+|(ud)_L u_R|p\rangle &= \frac{\alpha}{f}(1+D+F), \\ \langle K^0|(us)_L u_L|p\rangle &= \frac{\beta}{f}\left(1-(D-F)\frac{m_N}{m_B}\right), \\ \langle K^0|(us)_L u_R|p\rangle &= -\frac{\alpha}{f}\left(1+(D-F)\frac{m_N}{m_B}\right), \\ \langle K^+|(us)_L d_L|p\rangle &= \frac{\beta}{f}\left(\frac{2D}{3}\frac{m_N}{m_B}\right) \\ \langle K^+|(us)_L d_R|p\rangle &= \frac{\alpha}{f}\left(\frac{2D}{3}\frac{m_N}{m_B}\right) \\ \langle K^+|(ud)_L s_L|p\rangle &= \frac{\beta}{f}\left(1+\left(\frac{D}{3}+F\right)\frac{m_N}{m_B}\right), \\ \langle K^+|(ud)_L s_R|p\rangle &= \frac{\alpha}{f}\left(1+\left(\frac{D}{3}+F\right)\frac{m_N}{m_B}\right), \\ \langle K^+|(ds)_L u_L|p\rangle &= -\frac{\beta}{f}\left(1-\left(\frac{D}{3}-F\right)\frac{m_N}{m_B}\right), \\ \langle K^+|(ds)_L u_R|p\rangle &= \frac{\alpha}{f}\left(1+\left(\frac{D}{3}-F\right)\frac{m_N}{m_B}\right), \end{aligned}$$

Discussion II —— Mapping at the hadron level

- 我们是否可以在核子层面定量计算其他衰变道?

比如，$p \to \eta^0 e^+, \rho^0 \mu^+, n\pi + e^+, n\pi + \bar{\nu}$

- 是否有必要在原子核层面下进行计算?

比如，碳原子核和氧原子核

- 进一步扩展到原子/分子层面?

Derived bound on hydrogen 2-body decay			
Mode	Γ^{-1}(yr)	Mode	Γ^{-1}(yr)
$\mathrm{H} \to \gamma\gamma$	7.6×10^{44}	$\mathrm{H} \to \pi^0\pi^0$	1.1×10^{48}
$\mathrm{H} \to e^+e^-$	1.5×10^{45}	$\mathrm{H} \to \pi^0\eta$	9.6×10^{46}
$\mathrm{H} \to e^-\mu^+$	1.5×10^{45}	$\mathrm{H} \to \pi^+\pi^-$	6.0×10^{47}
$\mathrm{H} \to e^+\mu^-$	✗(LEFT@dim9)	$\mathrm{H} \to \pi^0 K^0$	4.5×10^{46}
$\mathrm{H} \to \mu^+\mu^-$	✗(QED@loop)	$\mathrm{H} \to \pi^- K^+$	5.0×10^{46}
$\mathrm{H} \to \nu_i\bar{\nu}_j$	✗(m_ν)	$\mathrm{H} \to \pi^+ K^-$	✗(Weak)
$\mathrm{H} \to \nu_e\nu_e$	9.1×10^{55}	$\mathrm{H} \to \pi^0 \bar{K}^0$	✗(Weak)
$\mathrm{H} \to \nu_e\nu_{\mu,\tau}$	1.8×10^{56}		
$\mathrm{H} \to \bar{\nu}_i\bar{\nu}_j$	✗(LEFT@dim9)		
$\mathrm{H} \to \pi^0\gamma$	–	$\mathrm{H} \to K^0\gamma$	–
$\mathrm{H} \to \eta\gamma$	–	$\mathrm{H} \to \bar{K}^0\gamma$	–

TABLE I. BNV two-body decay modes of hydrogen atom and derived bounds on inverse decay widths Γ^{-1}. The modes marked with a '✗' indicate suppression compared to other modes in the same type of final states, and the entries with a '–' are forbidden by Lorentz and gauge symmetries.

Fan, Liao, Ma, Wang, 2412.20774

Discussion III —— Mass correlations between quarks and leptons

- Field arrangements: fermion $\sim \mathbf{16}$, $\mathbf{16}\times\mathbf{16} = \mathbf{10}_S + \mathbf{120}_A + \mathbf{126}_S$

  $\Rightarrow$ scalars $\sim \mathbf{10}, \overline{\mathbf{126}}, \mathbf{120}$

- Yukawa couplings $\mathscr{L}_Y = \Psi_{\mathbf{16}} \left[Y_{\mathbf{10}} H_{\mathbf{10}} + Y_{\mathbf{120}} H_{\mathbf{120}} + Y_{\overline{\mathbf{126}}} H_{\overline{\mathbf{126}}} \right] \Psi_{\mathbf{16}}$

  $(+\Psi_{\mathbf{16}} \left[Y_{\mathbf{10}} H^*_{\mathbf{10}} + Y^{ij}_{\mathbf{120}} H^*_{\mathbf{120}} \right] \Psi_{\mathbf{16}})$ can be forbidden by the Peccei-Quinn symm $U(1)_{PQ}$

- Correlations of Yukawa matrices of quarks and leptons — Dutta, Mimura, Mohapatra, 0412105

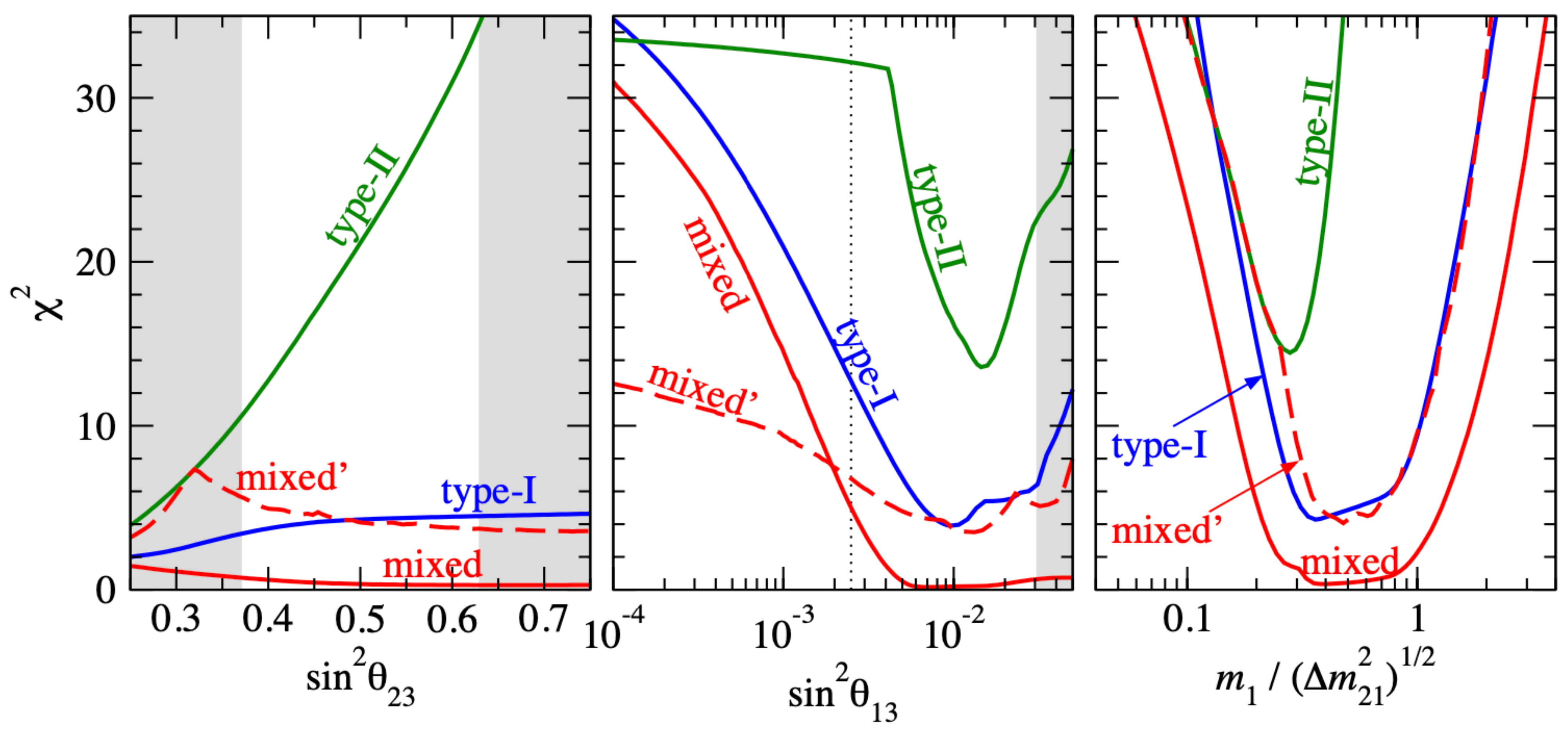
$$Y_d = r_1(h + f + i\,h') \qquad Y_u = h + r_2 f + i\,r_3 h'$$
$$Y_e = r_1(h - 3f + i\,c_e h') \qquad Y_\nu = h - 3r_2 f + i\,c_\nu h'$$
$$M_{\nu_R} = m_{\nu_R} f$$

Yukawa/mass matrices in SO(10)

3×3 matrices $\quad h \propto Y_{\mathbf{10}} \quad f \propto Y_{\overline{\mathbf{126}}} \quad h' \propto Y_{\mathbf{120}}$

Discussion III —— Mass correlations between quarks and leptons

- $10_C + \overline{126}_C$



Bertolini, Schwetz, Malinsky, hep-ph/0605006

X.G. He's talk

SUSY version, Babu, Bajc, Saad, 1805.10631

- $10_R + 120_R + \overline{126}_C$ Babu, Bajc, Saad, 1612.04329

Most recently see: Babu, Di Bari, Fong, Saad, 2409.03840

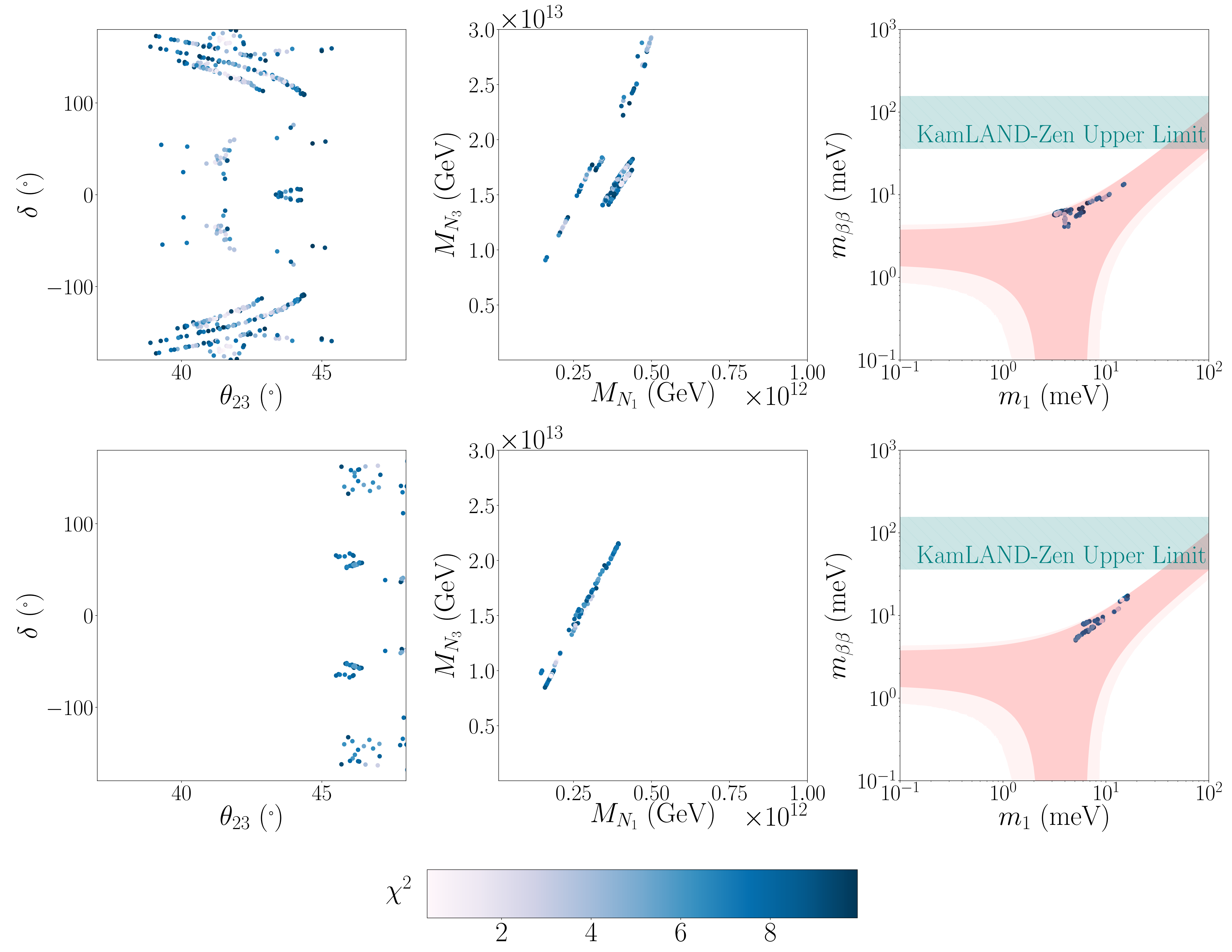
$$\text{NO} : (M_1, M_2, M_3) = \left(6.57 \times 10^4, 2.08 \times 10^{12}, 8.10 \times 10^{14}\right) \text{ GeV},$$

Large hierarchy between RH neutrino masses $\Rightarrow$ Leptogenesis works?

See also in Joshipura, Patel, 1102.5148; Dueck, Rodejohann, 1306.4468; Ohlsson, Pernow, 1804.04560;1903.08241; Saad, Shafi, 2506.11806 …

Discussion III —— Mass correlations between quarks and leptons

- $\mathbf{10}_\mathrm{C} + \mathbf{120}_\mathrm{C} + \overline{\mathbf{126}}_\mathrm{C} + \mathrm{CP} + U(1)_{PQ}$



Take-away message:

$M_1 \sim (2,\ 5) \times 10^{11}$ GeV

$M_3 \sim (1,\ 3) \times 10^{13}$ GeV

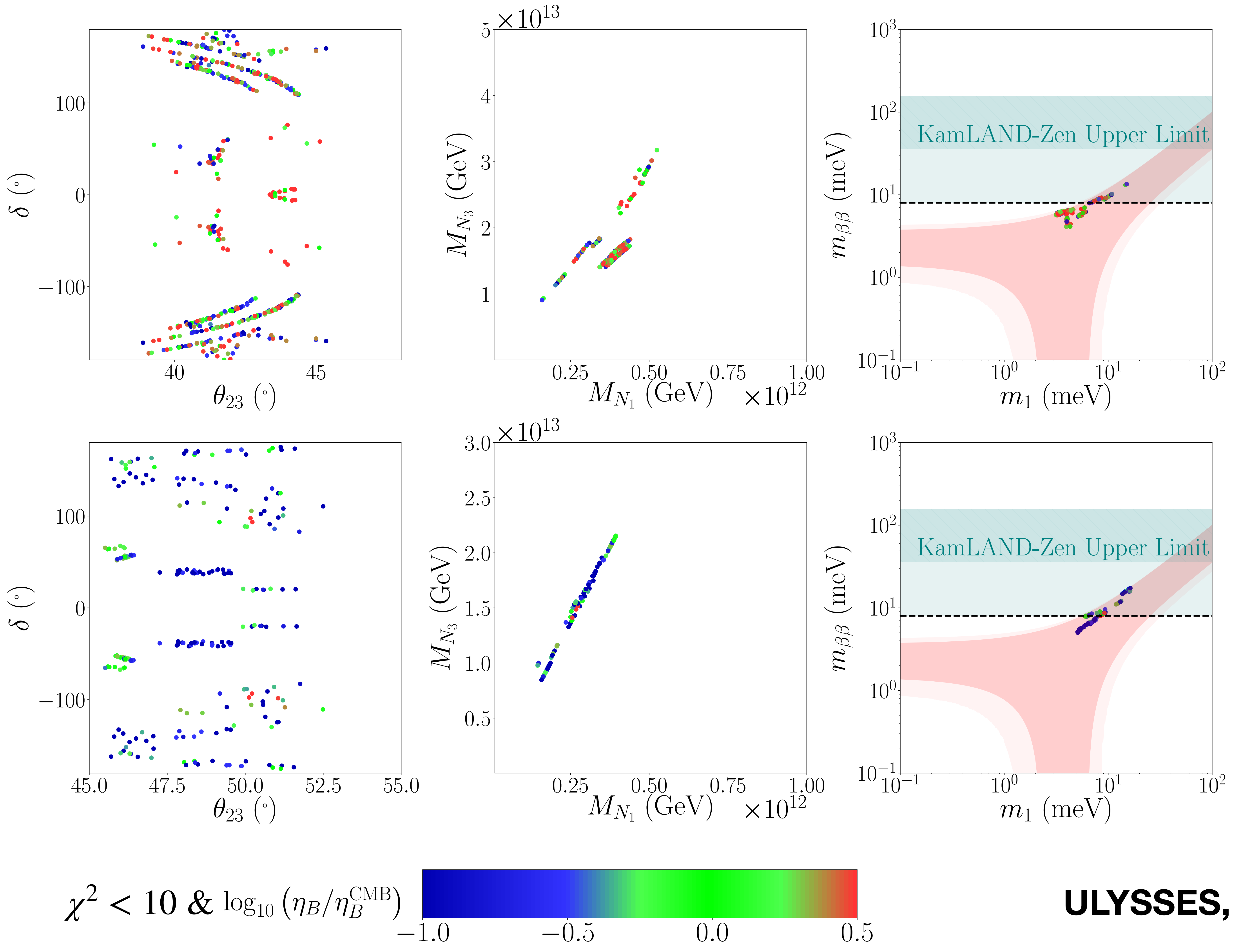
$m_{\beta\beta} \sim 10^{-2}$ eV

And normal ordering preferred

Fu, King, Marsili, Pascoli, Turner, **YLZ**, 2209.00021

Discussion III —— Mass correlations between quarks and leptons

- $\mathbf{10}_C + \mathbf{120}_C + \overline{\mathbf{126}}_C + \mathrm{CP} + U(1)_{PQ}$



Data of quark masses, CKM mixing, lepton masses, PMNS mixing

⇓

Heavy neutrino masses and Dirac ν Yukawa couplings

⇓

CP violation in heavy neutrino decay

⇓

Thermal leptogenesis

Fu, King, Marsili, Pascoli, Turner, **YLZ**, 2209.00021

ULYSSES, [Granelli et al, 2007.09150]

Discussion IV —— Mapping between low and high scales: RG running

Given $\alpha_i = \frac{g_i^2}{4\pi}$ for gauge coupling g_i of group G_i $\qquad G_i = SU(3)_c,\ SU(2)_L,\ U(1)_Y, \cdots$

$$\frac{\mathrm{d}\alpha_i}{\mathrm{d}t} = \beta_i$$

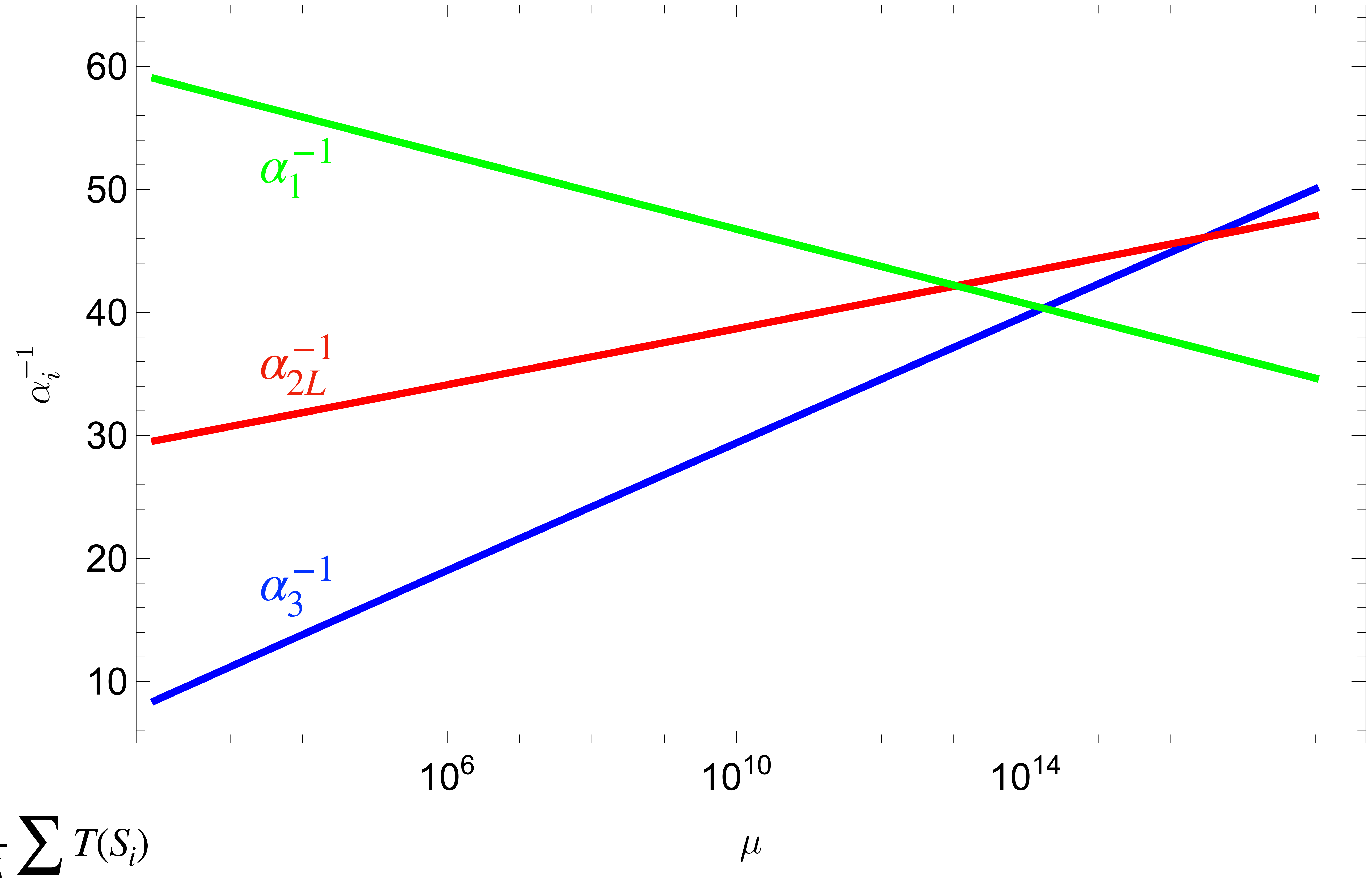
$$t = \log\frac{\mu}{\mu_0}$$

β function at leading order

$$\beta_i = b_i \frac{\alpha_i^2}{2\pi} + \cdots$$

$$b_i = -\frac{11}{3}C_2(G_i) + \frac{4}{3}\sum_F T(F_i) + \frac{1}{6}\sum_S T(S_i)$$

$M_Z \xrightarrow{G_{\mathrm{SM}}}$ Unification scale?



$b_3 = -7$

$b_{2L} = -\frac{19}{6}$

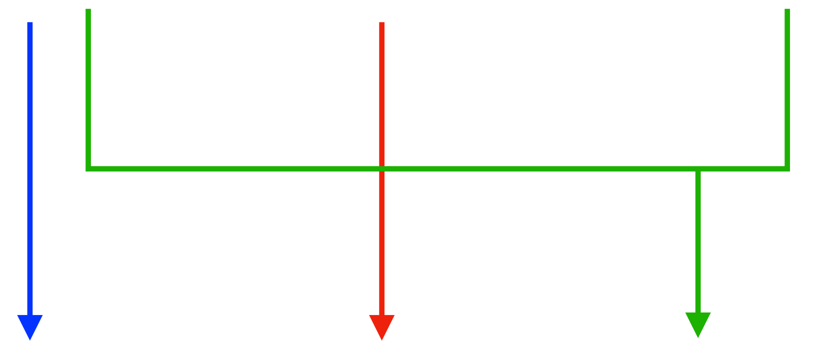
$b_1 = \frac{41}{10}$

Discussion IV —— Mapping between low and high scales: RG running

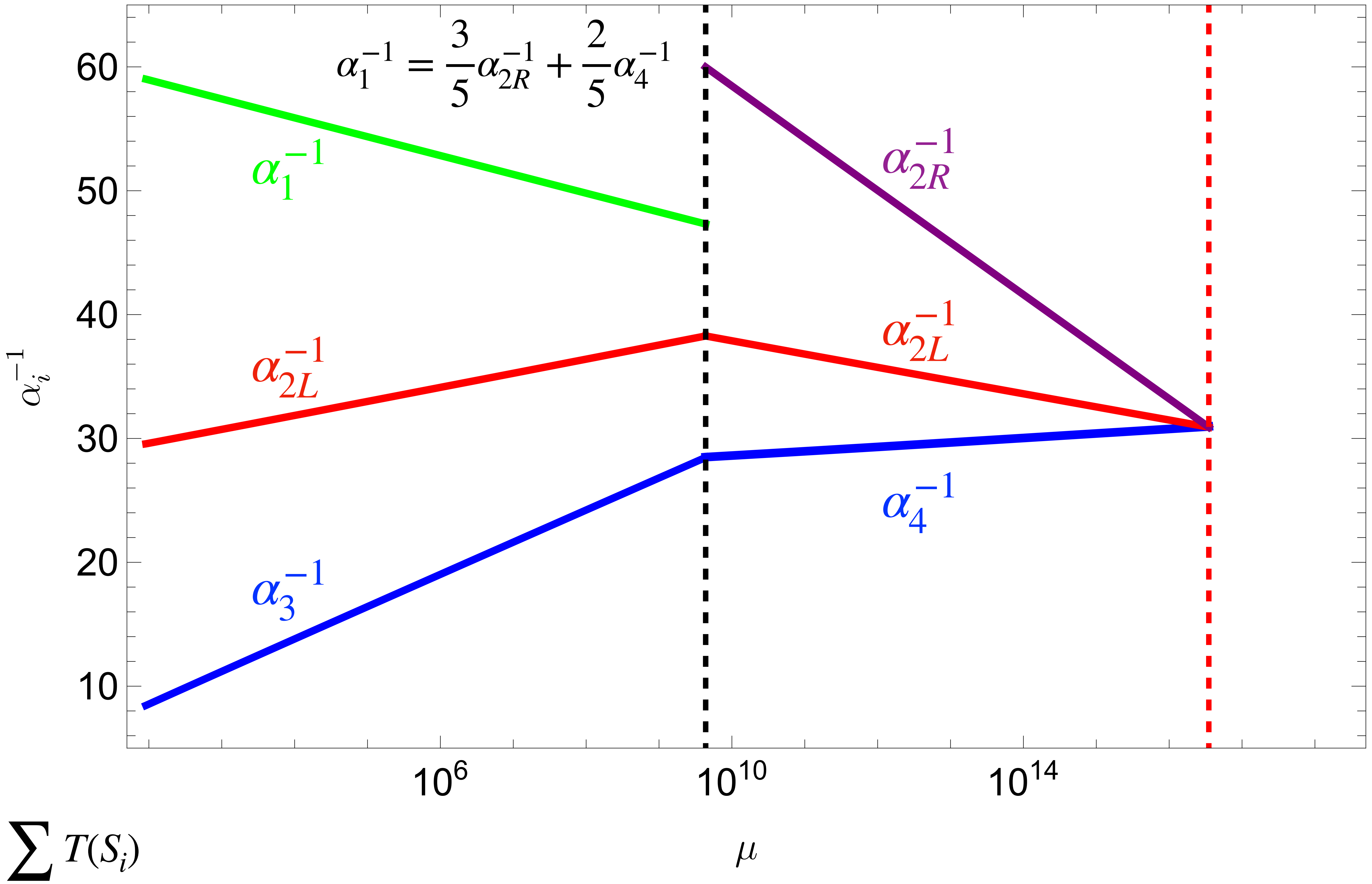
Given $\alpha_i = \dfrac{g_i^2}{4\pi}$ for gauge coupling g_i of group G_i $\qquad G_i = SU(4)_c, SU(2)_L, SU(2)_R, \cdots$

$$M_Z \xrightarrow{G_{\rm SM}} M_{\rm PS} \xrightarrow{G_{422}} M_X \text{ (GUT scale)}$$

$$G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$$



$$G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$



$b_4 = -\dfrac{7}{3}$

$b_{2L} = 2$

$b_{2R} = \dfrac{28}{3}$

$$b_i = -\frac{11}{3}C_2(G_i) + \frac{4}{3}\sum_F T(F_i) + \frac{1}{6}\sum_S T(S_i)$$

Discussion IV —— Mapping between low and high scales: RG running

- RGE of gauge couplings above the GUT scale

take SO(10) as an example, $\alpha_{10} = \frac{g_{10}^2}{4\pi}$

$$2\pi \frac{d\alpha_{10}}{dt} = b_{10}\alpha_{10}^2$$

$$\alpha_{10}(\mu > M_X) = \frac{\alpha_{10}(M_X)}{1 - \alpha_{10}(M_X)\frac{b_{10}}{2\pi}\log(\frac{\mu}{M_X})}$$

b_{10}: depending on fermion and Higgs particle contents of the model

Higgs contents	b_{10}
$(\mathbf{10}_c, \overline{\mathbf{126}}, \mathbf{45}_c)$	$-\frac{32}{3}$
$(\mathbf{10}_r, \mathbf{120}_r, \overline{\mathbf{126}}, \mathbf{45}_r)$	$-\frac{15}{2}$
$(\mathbf{10}_c, \mathbf{120}_c, \overline{\mathbf{126}}, \mathbf{45}_c)$	$-\frac{4}{3}$
$(\mathbf{10}_c, \mathbf{120}_c, \overline{\mathbf{126}}, \mathbf{45}_c, \mathbf{54}_c)$	$\frac{8}{3}$
$(\mathbf{10}_c, \mathbf{120}_c, \overline{\mathbf{126}}, \mathbf{210}_c)$	$\frac{44}{3}$

→ $\alpha_{10} \to \infty$ for $b_{10} > 0$

Landau pole at $\mu = M_X \exp\left(\frac{2\pi}{\alpha_{10} b_{10}}\right)$

→ $\alpha_{10} \to 0$ for $b_{10} < 0$

asymptotically approach to $\frac{2\pi}{-b_{10}\log(\mu/M_X)}$

Discussion IV —— Mapping between low and high scales: RG running

- General RGE of Yukawa couplings $Y^a_{ij}\psi_i\psi_j\phi^a$

$$16\pi^2\frac{\mathrm{d}Y^a}{\mathrm{d}t}=\frac{1}{2}\left[\sum_{b=1}^{N_\phi}Y^bY^{\dagger b}Y^a+Y^a\sum_{b=1}^{N_\phi}Y^{\dagger b}Y^b\right]+2Y^bY^{\dagger a}Y^b+2\kappa\frac{1}{2}Y^b\mathrm{Tr}(Y^{\dagger b}Y^a+Y^{\dagger a}Y^b)-3g^2[C_2(F)Y^a+Y^aC_2(F)]\,,$$

- Above the EW scale but below New Physics scale

SM — Machacek, Vaughn, NPB236 (1984) 221

SM + neutrino masses — Babu, Leung, Pantaleone, hep-ph/9309223; Chankowski, Pluciennik, hep-ph/9306333; Antusch, Drees, Kersten, Lindner, Ratz, hep-ph/0108005

Updated values of running quark and lepton masses at three loops

Xing, Zhang, S. Zhou, 0712.1419; 1112.3112

SMEFT — Liao, Yu, S. Zhou's series of works, Yu and S. Zhou's talks

- Above the New Physics scale and until the GUT scale? G.X. Fang, YLZ, in progress

- Running behaviour of Higgs quartic coupling? Jarkovska, Malinsky, Susic, 2304.14227

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Yifang Wang — IHEP, CAS

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Ke Han	SJTU
Yu-Feng Li	IHEP, CAS
Xianguo Lu	Warwick U
Danny Marfatia	Hawaii U
Igor V. Ostrovskiy	IHEP, CAS
Thomas Schwetz	KIT, Karlsruhe
Ke-Pan Xie	Beihang U
Zhao-Huan Yu	SYSU, Guangzhou
Ye-Ling Zhou	HIAS, UCAS

Discussions led by

Liang-Jian Wen	IHEP, CAS
Zhi-zhong Xing	IHEP, CAS
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