Systematic matching LEFT to ChPT for new physics studies

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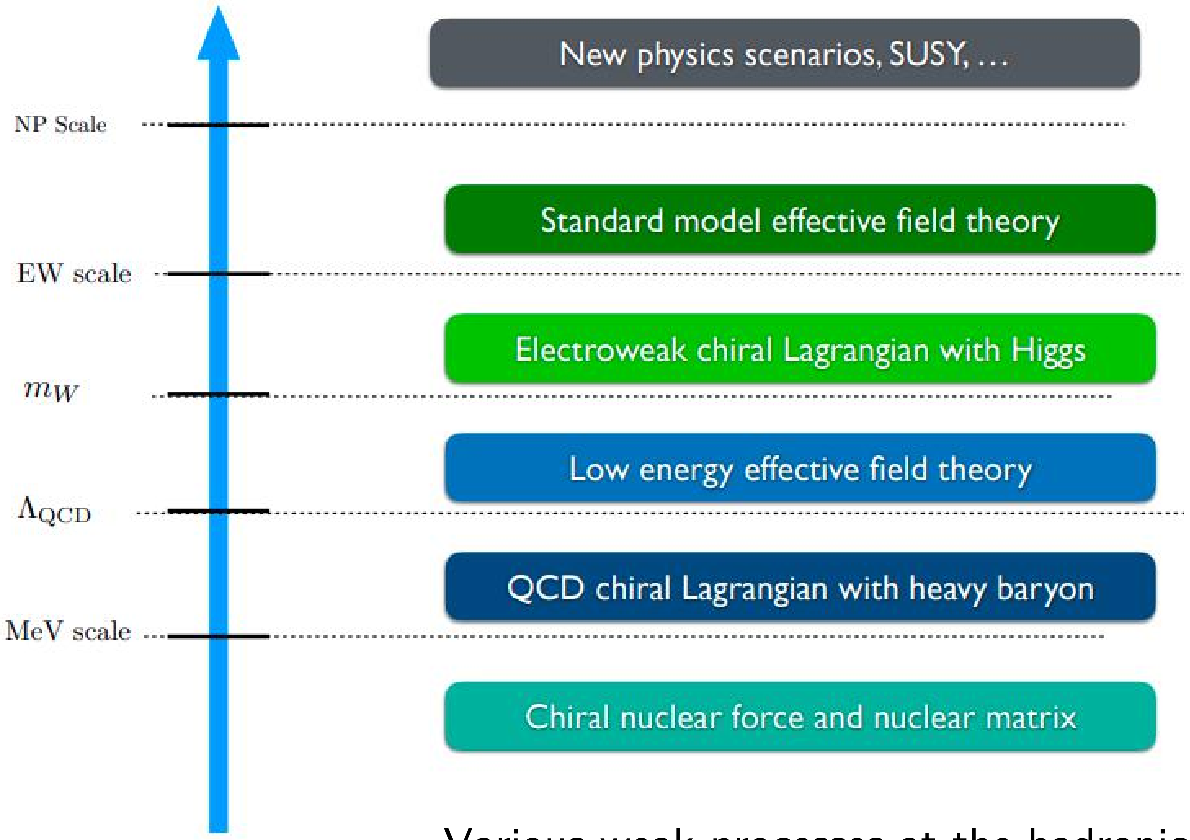
Effective Field Theory

Bottom-up approach to new physics:

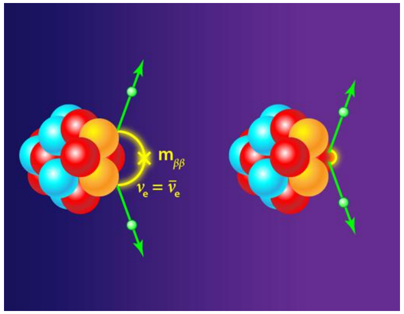


credit: Jiang-Hao Yu

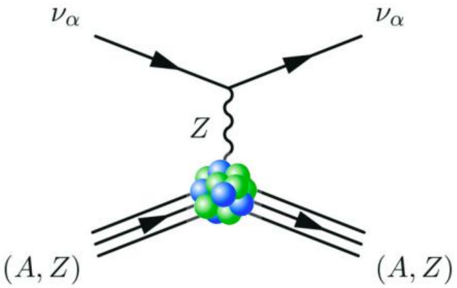
Various weak processes at the hadronic scale have been utilized to search for new physics at high energy scale

Effective Field Theory

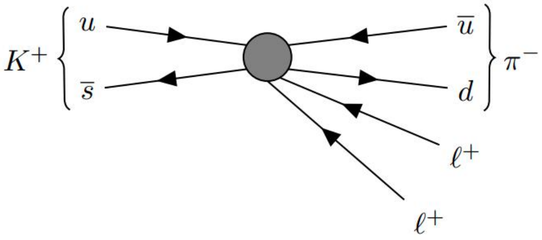
Bottom-up approach to new physics:



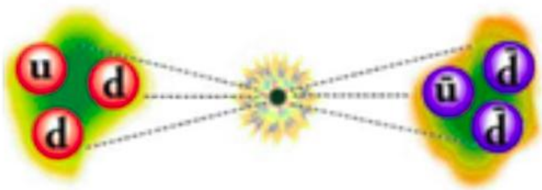
$0\nu\beta\beta$ decay



CEvNS



rare kaon decay



$n - \overline{n}$ oscillation

Effective Field Theory

Three tasks for particle physicists:

		operator	matching	RGE
	SMEFT	✔		✔
~ 100 GeV			✔	
	LEFT	✔		✔
~ 1 GeV			**Y/N**	
	ChPT	**Y/N**		

Y for SM interactions
N for BSM interactions

Outline

- ChPT and external source method
- Conventional and systematic spurion methods
- Matching for LEFT operators with dim ≤ 9
- Summary

External source method

Quark-level interactions:

$$\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \mathcal{L}_{\rm ext}$$

J. Gasser, H. Leutwyler, Annals Phys. 158 (1984) 142; Nucl. Phys. B 250 (1985) 465–516

$$\mathcal{L}_{\rm ext} = \bar{q}_L\gamma^\mu \ell_\mu q_L + \bar{q}_R\gamma^\mu r_\mu q_R - [\bar{q}_L(s-ip)q_R + \text{ h.c. }]$$

under chiral symmetry $SU(2)_L \times SU(2)_R$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad q_L \to Lq_L, \quad q_R \to Rq_R$$

$$U(x) = \exp\left(i\frac{\vec{\phi}\cdot\vec{\tau}}{F_0}\right) \qquad U \to RUL^\dagger$$

guiding rules of constructing chiral Lagrangian:

① **Real (Hermiticity)** ② **Flavor neutral (Trace)** ③ **Scalar (Parity even)**

④ **Chiral transformation** ⑤ **Proper Lorentz transformations**

⑥ **Charge conjugation C** ⑦ **Parity P** ⑧ **Time reversal T**

External source method

Matching from quark to hadronic operators:

covariant derivate:

J. Gasser, H. Leutwyler, Annals Phys. 158 (1984) 142; Nucl. Phys. B 250 (1985) 465–516

$$D_\mu U = \partial_\mu U - i r_\mu U + i U \ell_\mu$$

local chiral symmetry

ingredients:

$$\begin{aligned} \chi &= 2B(s + m_q + ip) \\ F_L^{\mu\nu} &= \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i[\ell^\mu, \ell^\nu] \\ F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu] \end{aligned}$$

Building blocks:

$$(U, U^\dagger, \chi, \chi^\dagger, F_L^{\mu\nu}, F_R^{\mu\nu}, D_\mu)$$

p^2 order:

$$\mathcal{L}_2 = \frac{F_0^2}{4} \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger\right] + \frac{F_0^2}{4} \mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)$$

External source method

Matching from quark to hadronic operators:

p^4 order:

J. Gasser, H. Leutwyler, Annals Phys. 158 (1984) 142; Nucl. Phys. B 250 (1985) 465–516

$$\begin{aligned}\mathcal{L}_4 =& \frac{l_1}{4}\left\{\mathrm{Tr}\left[D_\mu U(D^\mu U)^\dagger\right]\right\}^2 + \frac{l_2}{4}\mathrm{Tr}\left[D_\mu U(D_\nu U)^\dagger\right]\mathrm{Tr}\left[D^\mu U(D^\nu U)^\dagger\right] \\ &+ \frac{l_3}{16}\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\mathrm{Tr}\left[D_\mu U(D^\mu \chi)^\dagger + D_\mu \chi(D^\mu U)^\dagger\right] \\ &+ l_5\left[\mathrm{Tr}\left(F_{R\mu\nu}UF_L^{\mu\nu}U^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(F_{L\mu\nu}F_L^{\mu\nu} + F_{R\mu\nu}F_R^{\mu\nu}\right)\right] \\ &+ i\frac{l_6}{2}\mathrm{Tr}\left[F_{R\mu\nu}D^\mu U(D^\nu U)^\dagger + F_{L\mu\nu}(D^\mu U)^\dagger D^\nu U\right] \\ &- \frac{l_7}{16}\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1+h_3}{16}\mathrm{Tr}\left(U\chi^\dagger U\chi^\dagger + \chi U^\dagger \chi U^\dagger\right) \\ &- \frac{h_1-h_3}{16}\left\{\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} \\ &- 2h_2\,\mathrm{Tr}\left(F_{L\mu\nu}F_L^{\mu\nu} + F_{R\mu\nu}F_R^{\mu\nu}\right)\end{aligned}$$

LR basis

External source method

Matching from quark to hadronic operators:

$$u(x) = \sqrt{U} = \exp\left(i \frac{\vec{\phi}\cdot\vec{\tau}}{2F_0} \right)$$

Ecker, Gasser, Pich, de Rafael, Nucl.Phys.B 321 (1989) 311

under chiral symmetry $SU(2)_L \times SU(2)_R$

$$u \to RuK^\dagger = KuL^\dagger, \quad u^\dagger \to Lu^\dagger K^\dagger = Ku^\dagger R^\dagger$$

ingredients:

$$\begin{aligned} \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\ u_\mu &= i\left[u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right] = i u^\dagger D_\mu U u^\dagger \\ f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u, \end{aligned}$$

$$\boxed{X \to KXK^\dagger, \quad K \in SU(2)_V} \qquad X = \chi_\pm, u^\mu, f_\pm^{\mu\nu}$$

External source method

Matching from quark to hadronic operators:

covariant derivate:

$$\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X] \qquad \Gamma_\mu \equiv \frac{1}{2}\left[u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\right]$$

Building blocks:

$(u^\mu, \chi_\pm, f_\pm^{\mu\nu}, \nabla^\mu)$

p^2 order:

$$\mathcal{L}_2' = \frac{F_0^2}{4}\langle u^\mu u_\mu + \chi_+\rangle$$

p^4 order:

$$\begin{aligned}\mathcal{L}_4' = & L_1 \langle u_\mu u^\mu\rangle \langle u_\nu u^\nu\rangle + L_2 \langle u_\mu u_\nu\rangle \langle u^\mu u^\nu\rangle + L_3 \langle \chi_+\rangle \langle u^\mu u_\mu\rangle \\ & + L_4 \left\langle f_+^{\mu\nu} u_\mu u_\nu\right\rangle + L_5 \left\langle f_+^{\mu\nu} f_{+\mu\nu}\right\rangle + L_6 \left\langle f_+^{\mu\nu} f_{+\mu\nu}\right\rangle \\ & + L_7 \langle \chi_+ \chi_+\rangle + L_8 \langle \chi_- \chi_-\rangle + L_9 \langle \chi_+\rangle \langle \chi_+\rangle + L_{10} \langle \chi_-\rangle \langle \chi_-\rangle\end{aligned}$$

u basis

External source method

LR basis vs u basis:

- two bases are equivalent
- construction of chiral Lagrangian in the u basis is simpler in cases:

✓ at higher orders of p

$\mathcal{O}(p^6)$: **Bijnens, Colangelo, Ecker, JHEP 02 (1999) 020**

$\mathcal{O}(p^8)$: **Bijnens, Hermansson-Truedsson, Wang, JHEP 01 (2019) 102**

✓ tensor interactions

$$\mathcal{L}_{\text{ext}} \supset \bar{q}\sigma_{\mu\nu}\bar{t}^{\mu\nu}q = \bar{q}_L\sigma_{\mu\nu}t^{\mu\nu\dagger}q_R + \bar{q}_R\sigma_{\mu\nu}t^{\mu\nu}q_L$$

$$t_+^{\mu\nu} = u^\dagger t^{\mu\nu}u^\dagger + ut^{\mu\nu\dagger}u$$

Cata, Mateu, JHEP 09 (2007) 078

External source method

Marriage with Young tensor technique: **H.-L. Li, et al., 2005.00008 (PRD), 2007.07899 (PRD), 2201.04639 (PRD)**

$$\mathcal{L}_{\text{ext}} = \bar{q}\gamma^{\mu}\left(v_{\mu} + \gamma^{5}a_{\mu}\right)q - \bar{q}\left(s - i\gamma^{5}p\right)q$$

C and P symmetries:

Fermionic building blocks							
	1	γ^5	γ^μ	$\gamma^5\gamma^\mu$	$\sigma^{\mu\nu}$	$\epsilon_{\mu\nu\rho\lambda}$	$\overleftrightarrow{D}^\mu$
C	$+$	$+$	$-$	$+$	$-$	$+$	$-$
P	$+$	$-$	$+$	$-$	$+$	$-$	$+$
$\mathcal{O}(p)$	0	1	0	0	0	0	0

Bosonic building blocks					
fields	u_μ	f_+	f_-	χ_+	χ_-
C	u_μ^T	$-f_+^T$	f_-^T	χ_+	χ_-
P	$-u_\mu$	f_+	$-f_-$	χ_+	$-\chi_-$
$\mathcal{O}(p)$	1	2	2	2	2

External source method

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Matching in the CP eigenstates

interactions in the intrinsic CP eigenstates → Y.T. matching →

$$P : X \xrightarrow{P} \eta_X X$$

$$C : X \xrightarrow{C} \xi_X X^T$$

u basis

$$X = \chi_{\pm}, u^{\mu}, f_{\pm}^{\mu\nu}$$

All reduandancies are eliminated, leading to complete and independent basis of chiral Lagrangian at p^8 order

X.-H. Li, H. Sun, F.-J. Tang, J.-H. Yu, 2404.14152 (JHEP)

External source method

Merits:

- In all, the external source framework is convenient and has been widely adopted in new physics studies

✓ long-distance kaon decay **Y. Liao, X.-D. Ma, H.-L. Wang, 2001.07378 (JHEP)**

✓ $\mu \to e\gamma$ **Dekens, Jenkins, Manohar, Stoffer, 1810.05675 (JHEP)**

✓ neutrino-nucleus scattering **F.-Z. Chen, M.-D. Zheng, H.-H. Zhang 2206.13122 (JHEP)**

✓ dark matter-nucleus scattering **J.-H. Liang, Y. Liao, X.-D. Ma, H.-L. Wang, 2401.05005 (CPC)**

External source method

Limitations:

- The external source method is limited by the interactions (V/A, S/P, T). How to deal with more complicated Lorentz structure?

- The external source method is only applicable to single quark bilinear. How about two or more quark bilinears?

External source method

Limitations:

- The external source method is limited by the interactions (V/A, S/P, T). How to deal with more complicated Lorentz structure?

  derivative operators: nucleon EDMs

  Akdag, Kubis, Wirzba, 2212.07794 (JHEP)

- The external source method is only applicable to single quark bilinear. How about two or more quark bilinears?

  four-quark operators: $0\nu\beta\beta$ decay

  M. L. Graesser, 1606.04549 (JHEP)

  short-distance kaon decay

  Y. Liao, X.-D. Ma, H.-L. Wang, 1909.06272 (JHEP)

Conventional spurion method

Spurion field:

- External sources can be regarded as spurions

J. Gasser, H. Leutwyler, Annals Phys. 158 (1984) 142

$$(v_\mu - a_\mu)' = L(v_\mu - a_\mu + i\partial_\mu)L^\dagger$$
$$(v_\mu + a_\mu)' = R(v_\mu + a_\mu + i\partial_\mu)R^\dagger$$
$$(s - ip)' = L(s - ip)R^\dagger$$

[single quark bilinear]

- Non-leptonic weak decays

Manohar, Georgi, Nucl.Phys.B 234 (1984) 189-212; Donoghue, Grinstein, Rey, Wise Phys.Rev.D 33 (1986) 1495

$$T^{bd}_{ac}\,(\bar{q}^a_L \gamma^\mu q_{Lb})(\bar{q}^c_L \gamma_\mu q_{Ld})$$

[two quark bilinears]

$$T^{ab}_{cd} \to T^{\alpha\beta}_{\rho\sigma} L^\rho_c L^\sigma_d L^{\dagger a}_\alpha L^{\dagger b}_\beta$$

a, b, c, d are $SU(3)$ or $SU(2)$ flavor indices

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix} \qquad P_{L/R} \equiv (1 \mp \gamma^5)/2$$

Conventional spurion method

Map each quark field to u field:

$$\begin{cases} q_L \to L q_L, \quad q_R \to R q_R \\ u \to R u K^\dagger = K u L^\dagger, \quad u^\dagger \to L u^\dagger K^\dagger = K u^\dagger R^\dagger \end{cases}$$

chiral basis

LO: $q_L \to u^\dagger, \quad q_R \to u, \quad \bar{q}_L \to u, \quad \bar{q}_R \to u^\dagger$

NLO: $q_L \to (D_\mu u)^\dagger, \quad q_R \to \left(D_\mu u^\dagger\right)^\dagger, \quad \bar{q}_L \to D_\mu u, \quad \bar{q}_R \to D_\mu u^\dagger$

$$D_\mu = \partial_\mu - i\mathcal{V}_\mu, \quad \mathcal{V}_\mu = \frac{i}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right)$$

global chiral symmetry

M. L. Graesser, 1606.04549 (JHEP); Y. Liao, X.-D. Ma, H.-L. Wang, 1909.06272 (JHEP)

Conventional spurion method

Construct the chiral Lagrangian:

- Map each quark field to u field in the chiral basis
- Spurions remain invariant during the matching
- Hadronic operators have the same chiral transformations as the quark operators
- Reduce the redundancies:

$$
\begin{aligned}
(D_\mu u)u^\dagger &= -u(D_\mu u)^\dagger, \qquad & u^\dagger (D_\mu u) &= -(D_\mu u)^\dagger u \\
\left(uD_\mu u^\dagger\right)^\dagger &= -uD_\mu u^\dagger, \qquad & \left(u^\dagger D_\mu u\right)^\dagger &= -u^\dagger D_\mu u
\end{aligned}
$$

- Convert from the u-parameterization to U-parameterization

$$
\begin{aligned}
uD_\mu u^\dagger &= U\partial_\mu U^\dagger/2, \qquad & u^\dagger D_\mu u &= U^\dagger \partial_\mu U/2 \\
u^\dagger D_\mu u^\dagger &= \partial_\mu U^\dagger/2, \qquad & uD_\mu u &= \partial_\mu U/2
\end{aligned}
$$

Conventional spurion method

Challenges:

- While straightforward, this method is plagued by redundancies:

  ✓ Different insertions of covariant derivative (eg: $p^{4,6,\cdots}$ order, derivative interactions)

  ✓ Hadronic operators with wrong CP symmetries (eg: tensor interactions)

  ✓ More spurions are needed for LEFT at higher dimension

  eg. six-quark operators:

  $n-\overline{n}$ oscillation **J. Bijnens and E. Kofoed, 1710.04383 (EPJC)**

  dinucelon decay **X.-G. He, X.-D. Ma, 2102.02562 (JHEP)**

Systematic spurion method

Marriage with Young tensor technique:

Matching in the CP eigenstates

interactions in the intrinsic CP eigenstates

Y.T.

matching

$$P : X \xrightarrow{P} \eta_X X$$

$$C : X \xrightarrow{C} \xi_X X^T$$

u basis

- LEFT operators in the CP eigenstates
- new building blocks X

Systematic spurion method

Classify the LEFT operators in the CP eigenstates

Quark bilinears in the chiral basis:

$$(\bar{q}_L \Gamma \Sigma_L q_L), \quad (\bar{q}_R \Gamma \Sigma_R q_R), \quad (\bar{q}_R \Gamma \Sigma q_L), \quad (\bar{q}_L \Gamma \Sigma^\dagger q_R)$$

Their CP eigenstates in the q basis:

$$\bar{q}\gamma_\mu (\Sigma_R P_R \pm \Sigma_L P_L) q$$

$$\bar{q}\left(\Sigma^\dagger P_R \pm \Sigma P_L\right) q$$

$$\bar{q}\sigma_{\mu\nu}(\Sigma^\dagger P_R + \Sigma P_L) q$$

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

$$\Sigma^\dagger = \Sigma = \Sigma_R = \Sigma_L$$

$$= \begin{cases} \tau^{p/n} \equiv (1 \pm \tau^3)/2 & \text{neutral current} \\ \tau^{\pm} \equiv (\tau^1 \pm i\tau^2)/2 & \text{charged current} \end{cases}$$

Systematic spurion method

Classify the LEFT operators in the CP eigenstates

dimension-6:

$$\mathcal{O}_9^{(6)} = (\bar{\nu}_L\gamma^\mu\nu_L)\left[\bar{q}\gamma_\mu\left(\Sigma_R P_R + \Sigma_L P_L\right)q\right] \qquad \mathcal{O}_{13}^{(6)} = (\bar{\nu}_L e_R)\left[\bar{q}\left(\Sigma^\dagger P_R + \Sigma P_L\right)q\right]$$

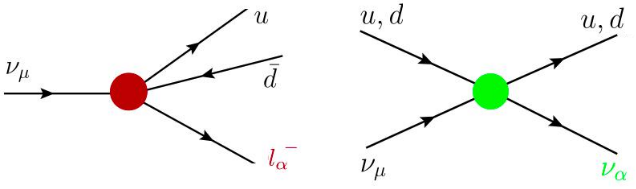
$$\mathcal{O}_{10}^{(6)} = (\bar{\nu}_L\gamma^\mu\nu_L)\left[\bar{q}\gamma_\mu\left(\Sigma_R P_R - \Sigma_L P_L\right)q\right] \qquad \mathcal{O}_{14}^{(6)} = (\bar{\nu}_L e_R)\left[\bar{q}\left(\Sigma^\dagger P_R - \Sigma P_L\right)q\right]$$

dimension-7:

$$\mathcal{O}_9^{(7)} = (\bar{\nu}_L\gamma^\mu\nu_L)[\bar{q}i\overleftrightarrow{D}_\mu\left(\Sigma^\dagger P_R + \Sigma P_L\right)q] \qquad \mathcal{O}_{13}^{(7)} = (\bar{\nu}_L\gamma^\mu e_L)[\bar{q}i\overleftrightarrow{D}_\mu\left(\Sigma^\dagger P_R + \Sigma P_L\right)q]$$

$$\mathcal{O}_{10}^{(7)} = (\bar{\nu}_L\gamma^\mu\nu_L)[\bar{q}i\overleftrightarrow{D}_\mu\left(\Sigma^\dagger P_R - \Sigma P_L\right)q] \qquad \mathcal{O}_{14}^{(7)} = (\bar{\nu}_L\gamma^\mu e_L)[\bar{q}i\overleftrightarrow{D}_\mu\left(\Sigma^\dagger P_R - \Sigma P_L\right)q]$$

neutrino non-standard interactions



Systematic spurion method

Spurions

Chiral transformation under global $SU(2)_L \times SU(2)_R$:

$$\begin{pmatrix} \hat{\chi} \\ \hat{\chi}^\dagger \\ \Sigma \\ \Sigma^\dagger \\ \Sigma_L \\ \Sigma_R \end{pmatrix} \to \begin{pmatrix} R\hat{\chi}L^\dagger \\ L\hat{\chi}^\dagger R^\dagger \\ R\Sigma L^\dagger \\ L\Sigma^\dagger R^\dagger \\ L\Sigma_L L^\dagger \\ R\Sigma_R R^\dagger \end{pmatrix}$$

For more convenient power countings, we isolate the mass term:

$$\hat{\chi} = 2Bm_q$$

Systematic spurion method

Building blocks

- Using the matching in the conventional spurion method in the q basis as a guide, we can address the spurions with u and $u^\dagger$ to define the building blocks:

$$\Sigma_\pm = u\Sigma^\dagger u \pm u^\dagger \Sigma u^\dagger \qquad \hat{\chi}_\pm = u\hat{\chi}^\dagger u \pm u^\dagger \hat{\chi} u^\dagger$$

$$Q_\pm = u^\dagger \Sigma_R u \pm u \Sigma_L u^\dagger \qquad \hat{u}_\mu = i\left(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger\right)$$

- Chiral transformations

$$\boxed{X \to K X K^\dagger, \quad K \in SU(2)_V}$$

$$X = \Sigma_\pm, Q_\pm, \hat{\chi}_\pm, \hat{u}^\mu \qquad \nabla_\mu X = \partial_\mu X + [\hat{\Gamma}_\mu, X]$$

$$\hat{\Gamma}_\mu \equiv \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right)$$

Systematic spurion method

From quark to hardonic operators:

- The matching for S/P, V/A interactions fully agree with those in the external source method
- The matching for derivative operators and four-quark operators are simpler than those in the conventional spurion method
- The matching for tensor interactions is noteworthy, and shown in the three methods

Systematic spurion method

V/A, S/P interactions:

LEFT operator	χPT operators	
	$\mathcal{O}(p^2)$	$\mathcal{O}(p^4)$
$\mathcal{O}_9^{(6)} = (\bar{\nu}_L\gamma^\mu\nu_L)[\bar{q}\gamma_\mu(\Sigma_R P_R + \Sigma_L P_L)q]$	$(\bar{\nu}_L\gamma^\mu\nu_L)\langle Q_-\hat{u}_\mu\rangle$	$(\bar{\nu}_L\gamma^\mu\nu_L)\langle Q_-\hat{u}_\mu\rangle\langle\hat{u}^\nu\hat{u}_\nu\rangle$ $(\bar{\nu}_L\gamma^\mu\nu_L)\langle Q_-\hat{u}_\nu\rangle\langle\hat{u}^\mu\hat{u}^\nu\rangle$ $(\bar{\nu}_L\gamma^\mu\nu_L)\langle Q_-\hat{u}_\mu\rangle\langle\hat{\chi}_+\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_+[\hat{u}_\mu, \hat{\chi}_-]\rangle$
$\mathcal{O}_{10}^{(6)} = (\bar{\nu}_L\gamma^\mu\nu_L)[\bar{q}\gamma_\mu(\Sigma_R P_R - \Sigma_L P_L)q]$	$(\bar{\nu}_L\gamma^\mu\nu_L)\langle Q_+\hat{u}_\mu\rangle$	$(\bar{\nu}_L\gamma^\mu\nu_L)\langle Q_+\hat{u}_\mu\rangle\langle\hat{u}^\nu\hat{u}_\nu\rangle$ $(\bar{\nu}_L\gamma^\mu\nu_L)\langle Q_+\hat{u}_\nu\rangle\langle\hat{u}^\mu\hat{u}^\nu\rangle$ $(\bar{\nu}_L\gamma^\mu\nu_L)\langle Q_+\hat{u}_\mu\rangle\langle\hat{\chi}_+\rangle$ $(\bar{e}\gamma^\mu e)\langle Q_-[\hat{u}_\mu, \hat{\chi}_-]\rangle$
$\mathcal{O}_{13}^{(6)} = (\bar{\nu}_L e_R)[\bar{q}(\Sigma^\dagger P_R + \Sigma P_L)q]$	$(\bar{\nu}_L e_R)\langle\Sigma_+\rangle$	$(\bar{\nu}_L e_R)\langle\Sigma_+\rangle\langle\hat{u}_\mu\hat{u}^\mu\rangle$ $(\bar{\nu}_L e_R)\langle\Sigma_+\rangle\langle\hat{\chi}_+\rangle$, $(\bar{\nu}_L e_R)\langle\Sigma_+\hat{\chi}_+\rangle$ $(\bar{\nu}_L e_R)\langle\Sigma_-\rangle\langle\hat{\chi}_-\rangle$, $(\bar{\nu}_L e_R)\langle\Sigma_-\hat{\chi}_-\rangle$
$\mathcal{O}_{14}^{(6)} = (\bar{\nu}_L e_R)[\bar{q}(\Sigma^\dagger P_R - \Sigma P_L)q]$	$(\bar{\nu}_L e_R)\langle\Sigma_-\rangle$	$(\bar{\nu}_L e_R)\langle\Sigma_-\rangle\langle\hat{u}_\mu\hat{u}^\mu\rangle$ $(\bar{\nu}_L e_R)\langle\Sigma_-\rangle\langle\hat{\chi}_+\rangle$, $(\bar{\nu}_L e_R)\langle\Sigma_-\hat{\chi}_+\rangle$ $(\bar{\nu}_L e_R)\langle\Sigma_+\rangle\langle\hat{\chi}_-\rangle$, $(\bar{\nu}_L e_R)\langle\Sigma_+\hat{\chi}_-\rangle$

Fully agree with the results using the external source method

Systematic spurion method

Tensor interactions:

$$\mathcal{L}^{(6)}_{\text{eff}, T} = j^{\mu\nu}_{\ell} \left[\bar{q}\sigma_{\mu\nu}(\Sigma^{\dagger}P_R + \Sigma P_L)q\right] + \text{h. c.}$$

The current

$$j^{\mu\nu}_{\ell} = \alpha_{\ell\ell'} \left\{\mathcal{C}^{(6)}_{11}\bar{e}\sigma^{\mu\nu}e, \mathcal{C}^{(6)}_{12}\bar{e}i\gamma^5\sigma^{\mu\nu}e, \mathcal{C}^{(6)}_{17}\bar{\nu}_L\sigma^{\mu\nu}e_R\right\} \qquad \alpha_{\ell\ell'} = \begin{cases} \frac{1}{2} & \text{for } \ell = \ell' \\ 1 & \text{for } \ell \neq \ell' \end{cases}$$

$\mathcal{O}^{(6)}_{11} = (\bar{e}\sigma^{\mu\nu}e)[\bar{q}\sigma_{\mu\nu}(\Sigma^{\dagger}P_R + \Sigma P_L)q]$	$(\bar{e}\sigma^{\mu\nu}e)\langle\Sigma_+[\hat{u}_\mu, \hat{u}_\nu]\rangle$ $(\bar{e}\gamma^5\sigma^{\mu\nu}e)\langle\Sigma_-[\hat{u}_\mu, \hat{u}_\nu]\rangle$
$\mathcal{O}^{(6)}_{12} = (\bar{e}i\gamma^5\sigma^{\mu\nu}e)[\bar{q}\sigma_{\mu\nu}(\Sigma^{\dagger}P_R + \Sigma P_L)q]$	$(\bar{e}i\gamma^5\sigma^{\mu\nu}e)\langle\Sigma_+[\hat{u}_\mu, \hat{u}_\nu]\rangle$ $(\bar{e}i\sigma^{\mu\nu}e)\langle\Sigma_-[\hat{u}_\mu, \hat{u}_\nu]\rangle$
$\mathcal{O}^{(6)}_{17} = (\bar{\nu}_L\sigma^{\mu\nu}e_R)[\bar{q}\sigma_{\mu\nu}(\Sigma^{\dagger}P_R + \Sigma P_L)q]$	$(\bar{\nu}_L\sigma^{\mu\nu}e_R)\langle\Sigma_+[\hat{u}_\mu, \hat{u}_\nu]\rangle$ $(\bar{\nu}_L\sigma^{\mu\nu}e_R)\langle\Sigma_-[\hat{u}_\mu, \hat{u}_\nu]\rangle$

similar for DM tensor interactions

Different in the independence of LECs for tensor interactions compared to the external source method

Systematic spurion method

Derivative interactions:

LEFT operator	χPT operators at $\mathcal{O}(p^4)$
$\mathcal{O}_9^{(7)} \sim (\bar{\nu}_L\gamma^\mu\nu_L)[\bar{q}i\overleftrightarrow{\partial}_\mu(\Sigma^\dagger P_R + \Sigma P_L)q]$	$i(\bar{\nu}_L\gamma^\mu\nu_L)\langle\Sigma_+[\nabla_\mu\hat{u}_\nu,\hat{u}^\nu]\rangle$ $i(\bar{\nu}_L\gamma^\mu\nu_L)\langle\nabla^\nu\Sigma_+[\hat{u}_\mu,\hat{u}_\nu]\rangle$ $(\bar{\nu}_L\gamma^\mu\nu_L)\langle\hat{u}_\mu[\Sigma_+,\hat{\chi}_-]\rangle$ $(\bar{\nu}_L\gamma^\mu\nu_L)\langle\hat{u}_\mu[\Sigma_-,\hat{\chi}_+]\rangle$
$\mathcal{O}_{10}^{(7)} \sim (\bar{\nu}_L\gamma^\mu\nu_L)[\bar{q}i\overleftrightarrow{\partial}_\mu(\Sigma^\dagger P_R - \Sigma P_L)q]$	$i(\bar{\nu}_L\gamma^\mu\nu_L)\langle\Sigma_-[\nabla_\mu\hat{u}_\nu,\hat{u}^\nu]\rangle$ $i(\bar{\nu}_L\gamma^\mu\nu_L)\langle\nabla^\nu\Sigma_-[\hat{u}_\mu,\hat{u}_\nu]\rangle$ $(\bar{\nu}_L\gamma^\mu\nu_L)\langle\hat{u}_\mu[\Sigma_-,\hat{\chi}_-]\rangle$ $(\bar{\nu}_L\gamma^\mu\nu_L)\langle\hat{u}_\mu[\Sigma_+,\hat{\chi}_+]\rangle$
$\mathcal{O}_{13}^{(7)} \sim (\bar{\nu}_L\gamma^\mu e_L)[\bar{q}i\overleftrightarrow{\partial}_\mu(\Sigma^\dagger P_R + \Sigma P_L)q]$	$i(\bar{\nu}_L\gamma^\mu e_L)\langle\Sigma_+[\nabla_\mu\hat{u}_\nu,\hat{u}^\nu]\rangle$ $i(\bar{\nu}_L\gamma^\mu e_L)\langle\nabla^\nu\Sigma_+[\hat{u}_\mu,\hat{u}_\nu]\rangle$ $(\bar{\nu}_L\gamma^\mu e_L)\langle\hat{u}_\mu[\Sigma_+,\hat{\chi}_-]\rangle$ $(\bar{\nu}_L\gamma^\mu e_L)\langle\hat{u}_\mu[\Sigma_-,\hat{\chi}_+]\rangle$
$\mathcal{O}_{14}^{(7)} \sim (\bar{\nu}_L\gamma^\mu e_L)[\bar{q}i\overleftrightarrow{\partial}_\mu(\Sigma^\dagger P_R - \Sigma P_L)q]$	$i(\bar{\nu}_L\gamma^\mu e_L)\langle\Sigma_-[\nabla_\mu\hat{u}_\nu,\hat{u}^\nu]\rangle$ $i(\bar{\nu}_L\gamma^\mu e_L)\langle\nabla^\nu\Sigma_-[\hat{u}_\mu,\hat{u}_\nu]\rangle$ $(\bar{\nu}_L\gamma^\mu e_L)\langle\hat{u}_\mu[\Sigma_-,\hat{\chi}_-]\rangle$ $(\bar{\nu}_L\gamma^\mu e_L)\langle\hat{u}_\mu[\Sigma_+,\hat{\chi}_+]\rangle$

Derivative in the matching: $\overleftrightarrow{\partial}_\mu \to \nabla_\mu$ new result

Systematic spurion method

Four-quark operators:

$$\mathcal{L}^{(9)}_{\Delta L=2} = \frac{1}{v^5}\sum_i \left[\left(C^{(9)}_{i\,\mathrm{R}}\bar{e}_R e^c_R + C^{(9)}_{i\,\mathrm{L}}\bar{e}_L e^c_L\right)O^{(9)}_i + C^{(9)}_i \bar{e}\gamma^\mu\gamma^5 e^c O^{\mu(9)}_i\right] + \mathrm{h.c.}$$

$$O^{(9)}_1 = \bar{q}^\alpha_L\gamma_\mu\tau^+ q^\alpha_L\ \bar{q}^\beta_L\gamma^\mu\tau^+ q^\beta_L\ , \qquad O^{(9)\prime}_1 = \bar{q}^\alpha_R\gamma_\mu\tau^+ q^\alpha_R\ \bar{q}^\beta_R\gamma^\mu\tau^+ q^\beta_R\ ,$$

$$O^{(9)}_2 = \bar{q}^\alpha_R\tau^+ q^\alpha_L\ \bar{q}^\beta_R\tau^+ q^\beta_L\ , \qquad O^{(9)\prime}_2 = \bar{q}^\alpha_L\tau^+ q^\alpha_R\ \bar{q}^\beta_L\tau^+ q^\beta_R\ ,$$

$$O^{(9)}_3 = \bar{q}^\alpha_R\tau^+ q^\beta_L\ \bar{q}^\beta_R\tau^+ q^\alpha_L\ , \qquad O^{(9)\prime}_3 = \bar{q}^\alpha_L\tau^+ q^\beta_R\ \bar{q}^\beta_L\tau^+ q^\alpha_R\ ,$$

$$O^{(9)}_4 = \bar{q}^\alpha_L\gamma_\mu\tau^+ q^\alpha_L\ \bar{q}^\beta_R\gamma^\mu\tau^+ q^\beta_R\ ,$$

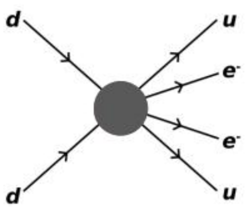
$$O^{(9)}_5 = \bar{q}^\alpha_L\gamma_\mu\tau^+ q^\beta_L\ \bar{q}^\beta_R\gamma^\mu\tau^+ q^\alpha_R\ ,$$

$$O^{\mu(9)}_6 = \left(\bar{q}_L\tau^+\gamma^\mu q_L\right)\left(\bar{q}_L\tau^+ q_R\right)\ , \qquad O^{\mu(9)\prime}_6 = \left(\bar{q}_R\tau^+\gamma^\mu q_R\right)\left(\bar{q}_R\tau^+ q_L\right)\ ,$$

$$O^{\mu(9)}_7 = \left(\bar{q}_L T^A\tau^+\gamma^\mu q_L\right)\left(\bar{q}_L T^A\tau^+ q_R\right)\ , \qquad O^{\mu(9)\prime}_7 = \left(\bar{q}_R T^A\tau^+\gamma^\mu q_R\right)\left(\bar{q}_R T^A\tau^+ q_L\right)\ ,$$

$$O^{\mu(9)}_8 = \left(\bar{q}_L\tau^+\gamma^\mu q_L\right)\left(\bar{q}_R\tau^+ q_L\right)\ , \qquad O^{\mu(9)\prime}_8 = \left(\bar{q}_R\tau^+\gamma^\mu q_R\right)\left(\bar{q}_L\tau^+ q_R\right)\ ,$$

$$O^{\mu(9)}_9 = \left(\bar{q}_L T^A\tau^+\gamma^\mu q_L\right)\left(\bar{q}_R T^A\tau^+ q_L\right)\ , \qquad O^{\mu(9)\prime}_9 = \left(\bar{q}_R T^A\tau^+\gamma^\mu q_R\right)\left(\bar{q}_L T^A\tau^+ q_R\right)\ ,$$



$0\nu\beta\beta$ decay

V. Cirigliano, et al, 1806.02780 (JHEP)

Systematic spurion method

- CP eigenstates:

basis trans. →

$$O_{XY}=\left(\bar{q}X\tau^{+}q\right)\left(\bar{q}Y\tau^{+}q\right),\quad X,Y=V,A,S,P$$

$$V=\gamma^{\mu},A=\gamma^{\mu}\gamma^{5},S=1,P=\gamma^{5}$$

$$\begin{pmatrix}O_1^{(9)}\\O_1^{(9)\prime}\\O_4^{(9)}\end{pmatrix}=\rho_1\begin{pmatrix}O_{VV}\\O_{AA}\\O_{VA}\end{pmatrix},\quad\begin{pmatrix}O_2^{(9)}\\O_2^{(9)\prime}\end{pmatrix}=\rho_2\begin{pmatrix}O_{SS}\\O_{PP}\\O_{SP}\end{pmatrix},\quad\begin{pmatrix}O_6^{\mu(9)}\\O_6^{\mu(9)\prime}\\O_8^{\mu(9)}\\O_8^{\mu(9)\prime}\end{pmatrix}=\rho_3\begin{pmatrix}O_{VS}\\O_{AP}\\O_{VP}\\O_{AS}\end{pmatrix}$$

correlation matrices:

$$\rho_1=\frac{1}{4}\begin{pmatrix}1&1&-2\\1&1&2\\1&-1&0\end{pmatrix},\quad\rho_2=\frac{1}{4}\begin{pmatrix}1&1&-2\\1&1&2\end{pmatrix},\quad\rho_3=\frac{1}{4}\begin{pmatrix}1&-1&1&-1\\1&-1&-1&1\\1&1&-1&-1\\1&1&1&1\end{pmatrix}$$

Systematic spurion method

- Building blocks:

$$\Sigma_{\pm} = u\Sigma^{\dagger}u \pm u^{\dagger}\Sigma u^{\dagger}$$
$$Q_{\pm} = u^{\dagger}\Sigma_R u \pm u\Sigma_L u^{\dagger}$$
$$\hat{u}_{\mu} = i\left(u^{\dagger}\partial_{\mu}u - u\partial_{\mu}u^{\dagger}\right)$$

same as those for single quark bilinears

- LO matching:

CP property	four-quark operator	chiral operator
$C+P+$	O_{VV}	$\langle Q_- Q_- \rangle$
$C+P+$	O_{AA}	$\langle Q_+ Q_+ \rangle$
$C-P-$	O_{VA}	$\langle Q_- Q_+ \rangle$

$$\langle Q_+ Q_+ \rangle = 2\left\langle U^{\dagger}\tau^{+}U\tau^{+}\right\rangle$$
$$\langle Q_- Q_- \rangle = -2\left\langle U^{\dagger}\tau^{+}U\tau^{+}\right\rangle$$
$$\langle Q_- Q_+ \rangle = \left\langle U^{\dagger}\tau^{+}U\tau^{+}\right\rangle - \left\langle U\tau^{+}U^{\dagger}\tau^{+}\right\rangle = 0$$

p^0 order: $O_1^{(9)} \to 0$

Systematic spurion method

- NLO matching:

CP property	four-quark operator	chiral operator
$C+P+$	O_{VV}	$\langle Q_-\hat{u}^\mu Q_-\hat{u}_\mu\rangle$
$C+P+$	O_{AA}	$\langle Q_+\hat{u}^\mu Q_+\hat{u}_\mu\rangle$
$C-P-$	O_{VA}	$\langle Q_-\hat{u}^\mu Q_+\hat{u}_\mu\rangle$

p^2 order:

$$\begin{aligned} O_1^{(9)} &\rightarrow \langle Q_-\hat{u}^\mu Q_-\hat{u}_\mu\rangle + \langle Q_+\hat{u}^\mu Q_+\hat{u}_\mu\rangle - 2\,\langle Q_-\hat{u}^\mu Q_+\hat{u}_\mu\rangle \\ &= -4\,\langle \tau^+ u D^\mu u^\dagger \tau^+ u D_\mu u^\dagger\rangle \\ &= -4\,\langle U^\dagger \partial^\mu U \tau^+ U^\dagger \partial_\mu U \tau^+\rangle \end{aligned}$$

Fully agree with the results using the conventional spurion method

Summary

- Comparison of three methods:

method	param.	bases	building blocks
external	U	chiral/LR	U , $U^\dagger$, χ , $\chi^\dagger$, $F_L^{\mu\nu}$, $F_R^{\mu\nu}$, D_μ
external	u	q/u	u^μ , $\chi_\pm$, $f_\pm^{\mu\nu}$, $t_\pm^{\mu\nu}$, ∇_μ
conventional	u or U	chiral/LR	u, $u^\dagger$, D_μ, (λ, $\lambda^\dagger$, λ_L, λ_R, $\lambda^{\mu\nu}$, $\lambda^{\mu\nu\dagger}$)
systematic	u	q/u	$\hat{u}^\mu$, ∇_μ, ($\hat{\chi}_\pm$, $\Sigma_\pm$, $Q_\pm$), j_ℓ, j_ℓ^μ, $j_\ell^{\mu\nu}$, A^μ

- They are equivalent for V/A, S/P, T* interactions

Summary

- For the systematic spurion method, we identify a minimal set of building blocks and establish a one-to-one correspondence between LEFT and ChPT.
- This method
  - ✓ discards external sources and irreducible decomposition
  - ✓ get rids of redundancies (eg. derivative interations)
  - ✓ avoids new spurions for higher dimensional LEFT operators (eg. four-quark operators)

Thank you

Conventional spurion method

Vector/axial-vector interactions:

$$\mathcal{L}^q_{V,A} = \bar{q}_R \gamma^\mu \lambda_R q_R + \bar{q}_L \gamma^\mu \lambda_L q_L$$

At p^2 order:

$$\begin{aligned}\mathcal{L}^{(1)}_{V,A} &= \mathrm{Tr}\left[u^\dagger \lambda_R (D^\mu u^\dagger)^\dagger + u \lambda_L (D^\mu u)^\dagger\right] g^{(1)}_{V,1} \\ &\quad + \mathrm{Tr}\left[D^\mu u^\dagger \lambda_R u + D^\mu u \lambda_L u^\dagger\right] g^{(1)}_{V,2} \\ &= \mathrm{Tr}\left[\lambda_R u D^\mu u^\dagger + \lambda_L u^\dagger D^\mu u\right] \tilde{g}^{(1)}_V \qquad \tilde{g}^{(1)}_V \equiv g^{(1)}_{V,2} - g^{(1)}_{V,1}\end{aligned}$$

Taking $\lambda_{R/L} = v_\mu \pm a_\mu$

$$\mathcal{L}^{(1)}_V = \frac{1}{2} \mathrm{Tr}\left(v_\mu [U, \partial^\mu U^\dagger]\right) \tilde{g}^{(1)}_V$$

$$\mathcal{L}^{(1)}_A = \frac{1}{2} \mathrm{Tr}\left(a_\mu \{U, \partial^\mu U^\dagger\}\right) \tilde{g}^{(1)}_V$$

Conventional spurion method

Scalar/pseudo-scalar interactions:

$$\mathcal{L}^{q}_{S,P} = \bar{q}_L \lambda^\dagger q_R + \bar{q}_R \lambda q_L$$

At p^2 order:

$$\mathcal{L}^{(0)}_{S,P} = \mathbf{Tr}\left[u\lambda^\dagger u + u^\dagger \lambda u^\dagger\right] g_S^{(0)}$$

Taking $\lambda = -(s + ip)$

$$\mathcal{L}^{(0)}_{S} = -\mathbf{Tr}\left(sU^\dagger + Us\right) g_S^{(0)}$$

$$\mathcal{L}^{(0)}_{P} = i\mathbf{Tr}\left(pU^\dagger - Up\right) g_S^{(0)}$$

Conventional spurion method

In the q basis

$$\mathcal{L}^q_{S,P} = \bar{q}\lambda_S q + \bar{q}\gamma^5\lambda_P q$$

Since λ_S is Hermitian and λ_P is anti-Hermitian, rewrite

$$\bar{q}\lambda_S q = \bar{q}_L\lambda_S q_R + \bar{q}_R\lambda_S^\dagger q_L \qquad \lambda_{S/P} \to L\lambda_{S/P}R^\dagger$$
$$\bar{q}\gamma^5\lambda_P q = \bar{q}_L\lambda_P q_R + \bar{q}_R\lambda_P^\dagger q_L \qquad \lambda_{S/P}^\dagger \to R\lambda_{S/P}^\dagger L^\dagger$$

Matching in the q/u bases:

$$\bar{q}\lambda_S q \to \mathbf{Tr}\left[u\lambda_S u + u^\dagger\lambda_S^\dagger u^\dagger\right] \qquad \lambda_S = \lambda_S^\dagger = (\lambda^\dagger + \lambda)/2$$
$$\bar{q}\gamma^5\lambda_P q \to \mathbf{Tr}\left[u\lambda_P u + u^\dagger\lambda_P^\dagger u^\dagger\right] \qquad \lambda_P = -\lambda_P^\dagger = (\lambda^\dagger - \lambda)/2$$

similar for V/A interactions

Conventional spurion method

Four-quark operator:

$$T_{ac}^{bd}(\bar{q}_L^a \gamma^\mu q_{Lb})(\bar{q}_L^c \gamma_\mu q_{Ld})$$

a, b, c, d are $SU(3)$ or $SU(2)$ flavor indices

Irreducible decomposition:

$SU(3)$: $\quad T \in (\mathbf{8}_L \otimes \mathbf{8}_L) \otimes \mathbf{1}_R = (\mathbf{27}_L \oplus \mathbf{10}_L \oplus \mathbf{10}_L \oplus \mathbf{8}_L \oplus \mathbf{8}_L \oplus \mathbf{1}_L) \otimes \mathbf{1}_R$

$SU(2)$: $\quad T \in (\mathbf{3}_L \otimes \mathbf{3}_L) \otimes \mathbf{1}_R = (\mathbf{5}_L \oplus \mathbf{3}_L \oplus \mathbf{1}_L) \otimes \mathbf{1}_R$

Transformation of the spurion T depends on the process

$K^\pm \to \pi^\mp l^\pm l^\pm$	$\mathbf{27}_L \otimes \mathbf{1}_R$	**Y. Liao, X.-D. Ma, H.-L. Wang, 1909.06272 (JHEP)**
$0\nu\beta\beta$ decay	$\mathbf{5}_L \otimes \mathbf{1}_R$	**Prezeau, Ramsey-Musolf, Vogel, Phys.Rev.D 68 (2003) 034016**

Conventional spurion method

Four-quark operator:

$$T^{bd}_{ac}(\bar{q}^a_L \gamma^\mu q_{Lb})(\bar{q}^c_L \gamma_\mu q_{Ld})$$

a, b, c, d are $SU(3)$ or $SU(2)$ flavor indices

chiral operator

$$T^{ab}_{cd} u^{\dagger i}_a u^{\dagger j}_b u^c_k u^d_l$$

i, j, k, l are isospin indices

For $0\nu\beta\beta$ decay:

M. L. Graesser, 1606.04549 (JHEP)

$$T^{ab}_{cd} = (\tau^+)_c{}^a (\tau^+)_d{}^b$$

$$\mathrm{Tr}(u\tau^+ u^\dagger)\,\mathrm{Tr}(u\tau^+ u^\dagger) = 0$$

$$\mathrm{Tr}(u\tau^+ u^\dagger u\tau^+ u^\dagger) = 0$$

chira Lagrangain vanishes at the LO

Tensor interactions

- External source method

$$\mathcal{L}_{\text{ext},T} = \bar{q}\sigma_{\mu\nu}\bar{t}^{\mu\nu}q$$

$\bar{q}\sigma_{\mu\nu}\gamma^5 q$ is redundant since

$$\sigma^{\mu\nu}\gamma^5 = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$$

Chiral projectors:

$$P_R^{\mu\nu\lambda\rho} = \frac{1}{4}\left(g^{\mu\lambda}g^{\nu\rho} - g^{\nu\lambda}g^{\mu\rho} + i\varepsilon^{\mu\nu\lambda\rho}\right) = \left(P_L^{\mu\nu\lambda\rho}\right)^\dagger$$

$$\bar{t}^{\mu\nu} = P_L^{\mu\nu\lambda\rho}t_{\lambda\rho} + P_R^{\mu\nu\lambda\rho}t_{\lambda\rho}^\dagger$$

$$t^{\mu\nu} = P_L^{\mu\nu\lambda\rho}\bar{t}_{\lambda\rho}$$

$$\mathcal{L}_{\text{ext},T} = \bar{q}_L\sigma_{\mu\nu}t^{\mu\nu\dagger}q_R + \bar{q}_R\sigma_{\mu\nu}t^{\mu\nu}q_L$$

Mesonic chiral Lagrangian:

Cata, Mateu, JHEP 09 (2007) 078

$$\Delta\mathcal{L}'_{4,\pi} = -i\Lambda_2\left\langle t_+^{\mu\nu}\hat{u}_\mu\hat{u}_\nu\right\rangle$$

Tensor interactions

- External source method

$$\mathcal{L}_{\text{ext},T} = \bar{q}\sigma_{\mu\nu}\bar{t}^{\mu\nu}q$$

Remarks:

- by definition, the external tensor sources are anti-symmetric under $\mu \leftrightarrow \nu$
- In the matching, one cannot use

$$\mathcal{L}_{\text{ext},T} = \bar{q}_L\sigma_{\mu\nu}\bar{t}^{\mu\nu}q_R + \bar{q}_R\sigma_{\mu\nu}\bar{t}^{\mu\nu}q_L$$

- The ambiguity in the definition of chiral projectors $P_{R/L}^{\mu\nu\lambda\rho}$
- The interaction is C-odd, such that

$$\Delta\mathcal{L}'_{4,\pi} = -\frac{i}{2}\Lambda_2\langle(u^\dagger t^{\mu\nu}u^\dagger + ut^{\mu\nu\dagger}u)[\hat{u}_\mu, \hat{u}_\nu]\rangle$$

Tensor interactions

- Conventional spurion method

$$\mathcal{L}_T^q = \bar{q}_L \sigma^{\mu\nu} \lambda_{\mu\nu}^\dagger q_R + \bar{q}_R \sigma^{\mu\nu} \lambda_{\mu\nu} q_L$$

The matching at $O(p^4)$ gives

$$\mathcal{L}_T^{(2)} \supset \mathrm{Tr}\left[D^\nu u \lambda_{\mu\nu}^\dagger (D^\mu u^\dagger)^\dagger + D^\nu u^\dagger \lambda_{\mu\nu} (D^\mu u)^\dagger\right] g_T^{(2)}$$

$$= -\frac{1}{4}\mathrm{Tr}\left[(u\lambda_{\mu\nu}^\dagger u + u^\dagger \lambda_{\mu\nu} u^\dagger)\hat{u}^\mu \hat{u}^\nu\right] g_T^{(2)}$$

Relations:

$$u D^\mu u^\dagger = \frac{i}{2} u \hat{u}^\mu u^\dagger, \qquad u^\dagger D^\mu u = -\frac{i}{2} u^\dagger \hat{u}^\mu u,$$

$$u^\dagger D^\mu u^\dagger = \frac{i}{2} u^\dagger \hat{u}^\mu u^\dagger, \qquad u D^\mu u = -\frac{i}{2} u \hat{u}^\mu u.$$

Tensor interactions

- Conventional spurion method

$$\mathcal{L}_T^q = \bar{q}_L \sigma^{\mu\nu} \lambda_{\mu\nu}^\dagger q_R + \bar{q}_R \sigma^{\mu\nu} \lambda_{\mu\nu} q_L$$

Under C transformation:

$$\mathcal{L}_T^q \xrightarrow{C} -\left(\bar{q}_L \sigma^{\mu\nu} \lambda_{\mu\nu}^{\dagger c} q_R + \bar{q}_R \sigma^{\mu\nu} \lambda_{\mu\nu}^{c} q_L\right)$$

C-odd

$$\mathrm{T}^c \equiv C \mathrm{T}^T C^{-1}, \quad \text{for } \mathrm{T} = \lambda_{\mu\nu}, \lambda_{\mu\nu}^\dagger$$

$$(u \lambda_{\mu\nu}^\dagger u + u^\dagger \lambda_{\mu\nu} u^\dagger) \xrightarrow{C} (u^T \lambda_{\mu\nu}^{\dagger c} u^T + u^{\dagger T} \lambda_{\mu\nu}^{c} u^{\dagger T})$$

$$\hat{u}^\mu \hat{u}^\nu \xrightarrow{C} \hat{u}^{\mu T} \hat{u}^{\nu T}$$

wrong C symmetry

Tensor interactions

- Conventional spurion method

$$\mathcal{L}_T^q = \bar{q}_L \sigma^{\mu\nu} \lambda_{\mu\nu}^\dagger q_R + \bar{q}_R \sigma^{\mu\nu} \lambda_{\mu\nu} q_L$$

Rewrite

$$\mathcal{L}_T^q = -\left(\bar{q}_L \sigma^{\nu\mu} \lambda_{\mu\nu}^\dagger q_R + \bar{q}_R \sigma^{\nu\mu} \lambda_{\mu\nu} q_L\right)$$

The matching at $O(p^4)$ also gives

$$\mathcal{L}_T^{(2)} \supset -\mathbf{Tr}\left[D^\mu u \lambda_{\mu\nu}^\dagger (D^\nu u^\dagger)^\dagger + D^\mu u^\dagger \lambda_{\mu\nu} (D^\nu u)^\dagger\right] \tilde{g}_T^{(2)}$$

$$= \frac{1}{4} \mathbf{Tr}\left[(u \lambda_{\mu\nu}^\dagger u + u^\dagger \lambda_{\mu\nu} u^\dagger) \hat{u}^\nu \hat{u}^\mu\right] \tilde{g}_T^{(2)}$$

To preserve the C property in the matching, $\tilde{g}_T^{(2)} = g_T^{(2)}$

Tensor interactions

- Conventional spurion method

The sum of two terms is

$$\mathcal{L}_T^{(2)} \supset -\frac{1}{4}\mathbf{Tr}\left[(u\lambda_{\mu\nu}^{\dagger}u + u^{\dagger}\lambda_{\mu\nu}u^{\dagger})[\hat{u}^{\mu},\hat{u}^{\nu}]\right]g_T^{(2)}$$ C-odd

$$[\hat{u}^{\mu},\hat{u}^{\nu}] \xrightarrow{C} [\hat{u}^{\mu T},\hat{u}^{\nu T}]$$

$$\mathbf{Tr}\left[(u\lambda_{\mu\nu}^{\dagger}u + u^{\dagger}\lambda_{\mu\nu}u^{\dagger})[\hat{u}^{\mu},\hat{u}^{\nu}]\right] \xrightarrow{C} -\mathbf{Tr}\left[(u\lambda_{\mu\nu}^{\dagger c}u + u^{\dagger}\lambda_{\mu\nu}^{c}u^{\dagger})[\hat{u}^{\mu},\hat{u}^{\nu}]\right]$$

However, this is not the final result, since

$$\sigma^{\mu\nu}\gamma^5 = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$$

Tensor interactions

- Conventional spurion method

The interaction can also be rewritten as

$$\mathcal{L}_T^q = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}(\bar{q}_L\sigma_{\alpha\beta}\lambda_{\mu\nu}^\dagger q_R - \bar{q}_R\sigma_{\alpha\beta}\lambda_{\mu\nu}q_L)$$

The matching is

$$\mathcal{L}_T^{(2)} \supset -\frac{i}{16}\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\left[(u\lambda_{\mu\nu}^\dagger u - u^\dagger\lambda_{\mu\nu}u^\dagger)[\hat{u}_\alpha, \hat{u}_\beta]\right]g_T^{(2)\prime}$$

In total, the chiral Lagrangian at $O(p^4)$ is

$$\begin{aligned}\mathcal{L}_T^{(2)} = &-\frac{1}{8}\mathrm{Tr}\left[(u\lambda_{\mu\nu}^\dagger u + u^\dagger\lambda_{\mu\nu}u^\dagger)[\hat{u}^\mu, \hat{u}^\nu]\right]g_T^{(2)} \\ &-\frac{i}{16}\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\left[(u\lambda_{\mu\nu}^\dagger u - u^\dagger\lambda_{\mu\nu}u^\dagger)[\hat{u}_\alpha, \hat{u}_\beta]\right]g_T^{(2)\prime}\end{aligned}$$

two independent LECs

Tensor interactions

- Conventional spurion method

Assuming the LECs $g_T^{(2)}$ and $g_T^{(2)\prime}$ are equal

$$\mathcal{L}_T^{(2)} = -\frac{1}{4}\mathrm{Tr}\left[(uX_{\mu\nu}^{\dagger}u + u^{\dagger}X_{\mu\nu}u^{\dagger})[\hat{u}^{\mu}, \hat{u}^{\nu}]\right] g_T^{(2)}$$

Definition:

$$X_{\mu\nu}^{\dagger} \equiv \frac{1}{2}\lambda_{\mu\nu}^{\dagger} + \frac{i}{4}\varepsilon_{\mu\nu\alpha\beta}\lambda^{\alpha\beta\dagger}$$
$$X_{\mu\nu} \equiv \frac{1}{2}\lambda_{\mu\nu} - \frac{i}{4}\varepsilon_{\mu\nu\alpha\beta}\lambda^{\alpha\beta}$$

The result agrees with that using external source method by taking

$$X_{\mu\nu} = t_{\mu\nu}\,, \quad X_{\mu\nu}^{\dagger} = t_{\mu\nu}^{\dagger}$$