Nuclear matrix elements for neutrinoless double-beta decay

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Nuclear matrix elements (NMEs)

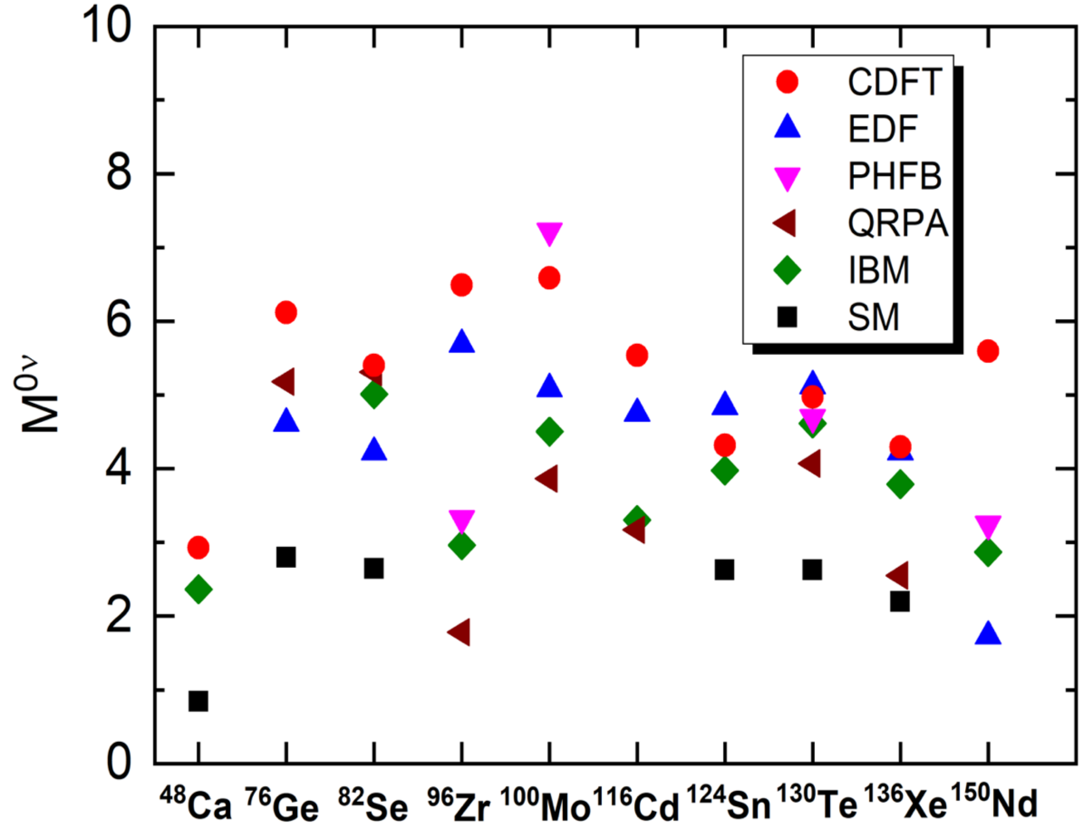
Nuclear matrix elements encode the impact of the nuclear structure on the **decay half-life**, crucial to interpreting the $0\nu\beta\beta$ experimental limits on the **effective neutrino mass**.

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu}\,|M^{0\nu}|^2\,\langle m_{\beta\beta}\rangle^2$$

Nuclear matrix element

$$M^{0\nu} = \langle \Psi_f | \hat{O}^{0\nu} | \Psi_i \rangle$$

- Different **nuclear models** for $\Psi_{f,i}$
- Large uncertainty ….



中国学科发展战略,《无中微子双贝塔衰变实验》2020

Decay operator $\hat{O}^{0\nu}$?

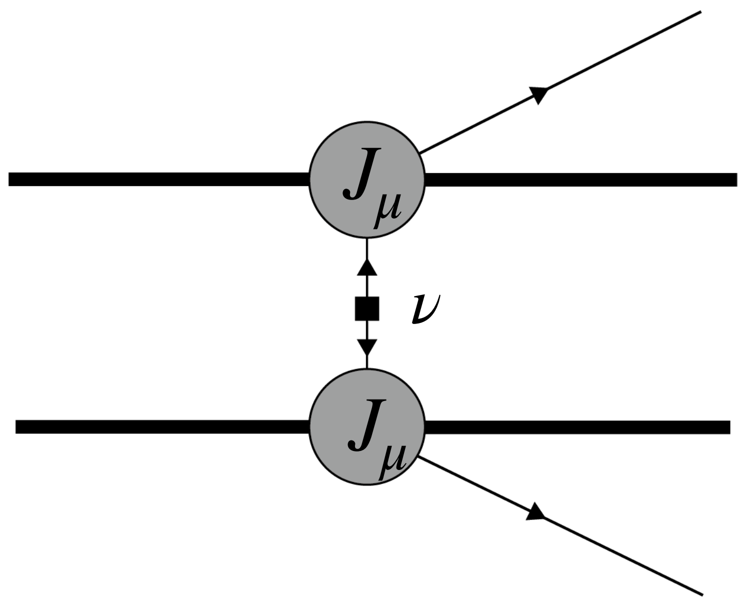
Outline

- Decay operators
- Nuclear many-body wavefunctions
- Summary

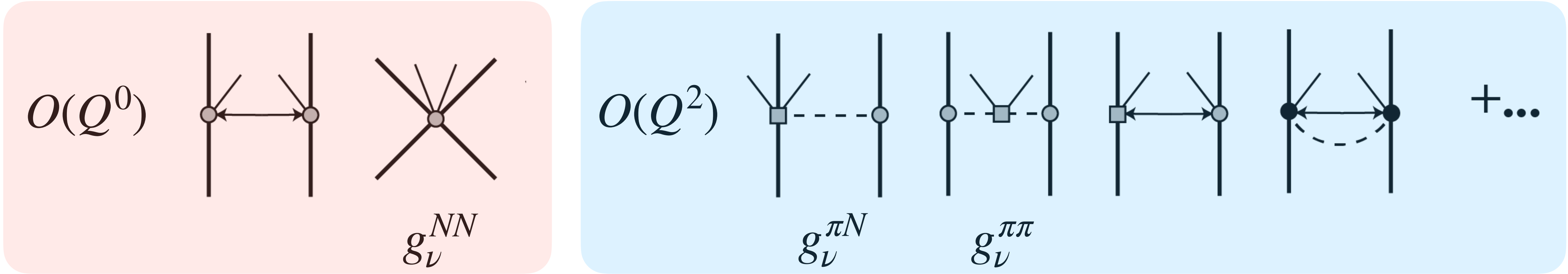
Decay operators for standard mechanism

- Impulse approximation +
  phenomenological nucleon currents

  Avignone, Elliott, and Engel, Rev. Mod. Phys. 80, 481 (2008)



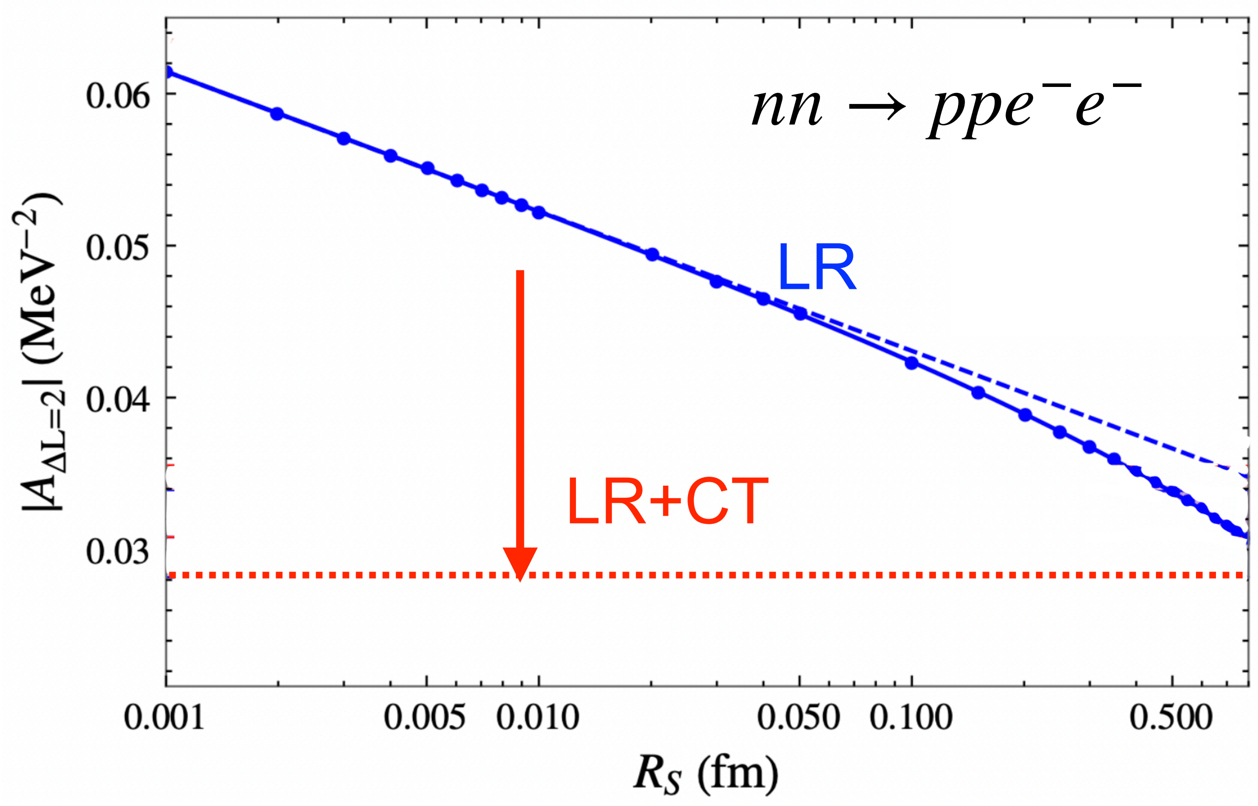
- **Chiral EFT framework**
  - Maps the quark-gluon level Lagrangian to chiral EFT Lagrangian
  - Perform low-energy expansion according to a power counting
  - Matching the low-energy constants (LECs) to Lattice QCD amplitudes



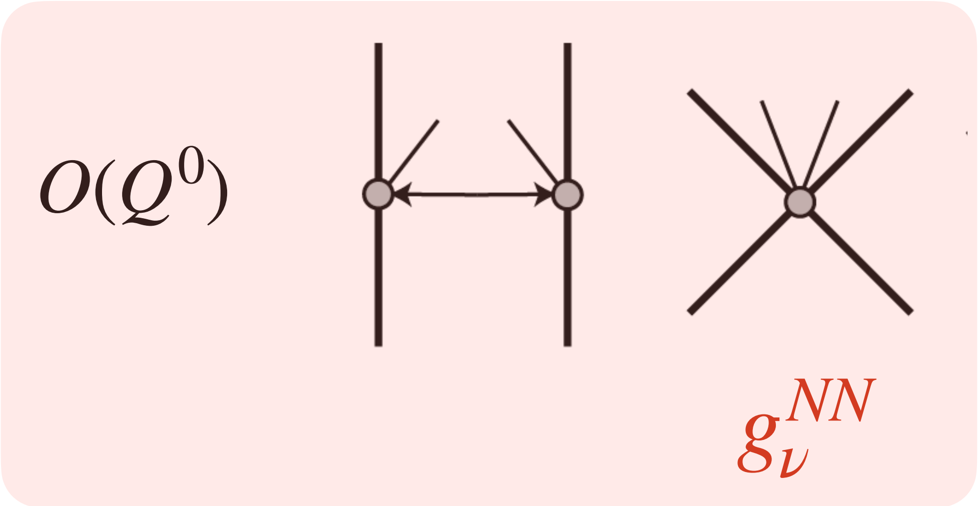
Cirigliano, Detmold, Nicholson, and Shanahan, Prog. Part. Nucl. Phys. 112, 103771 (2020)

Determine the LO decay operator

- The LO decay operator from chiral EFT includes already a short-range operator with **unknown** LEC g_ν^{NN}.



Cirigliano et. al., PRL 120, 202001 (2018)



How to determine its size ?

Determine the LO decay operator

- The LO decay operator from chiral EFT includes already a short-range operator with **unknown** LEC g_ν^{NN}.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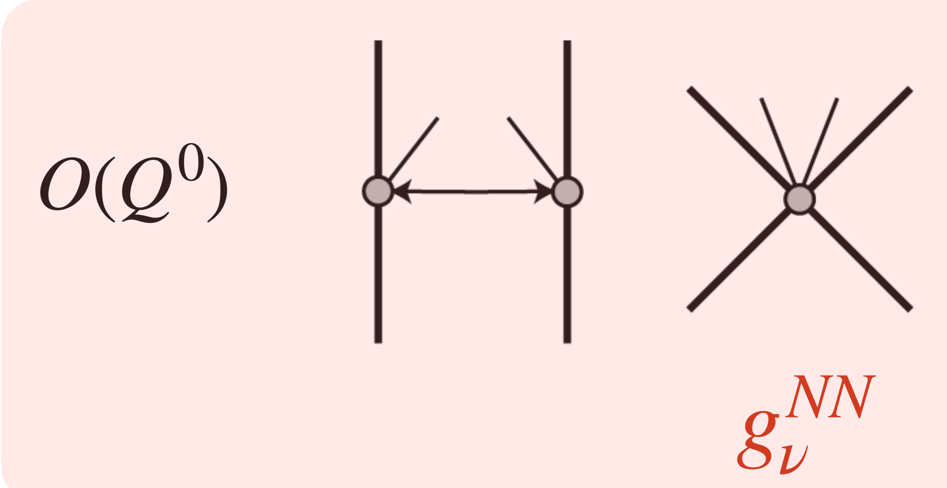
SYNOPSIS

A Missing Piece in the Neutrinoless Beta-Decay Puzzle

May 16, 2018 • *Physics* 11, s58

The inclusion of short-range interactions in models of neutrinoless double-beta decay could impact the interpretation of experimental searches for the elusive decay.

Cirigliano et. al., PRL 120, 202001 (2018)



How to determine its size ?

Matching to Cottingham model

- A generalized Cottingham model for $nn \to ppee$ amplitude

  Cirigliano, Dekens, de Vries, Hoferichter, and Mereghetti, PRL 126, 172002 (2021); JHEP 05, 289 (2021)

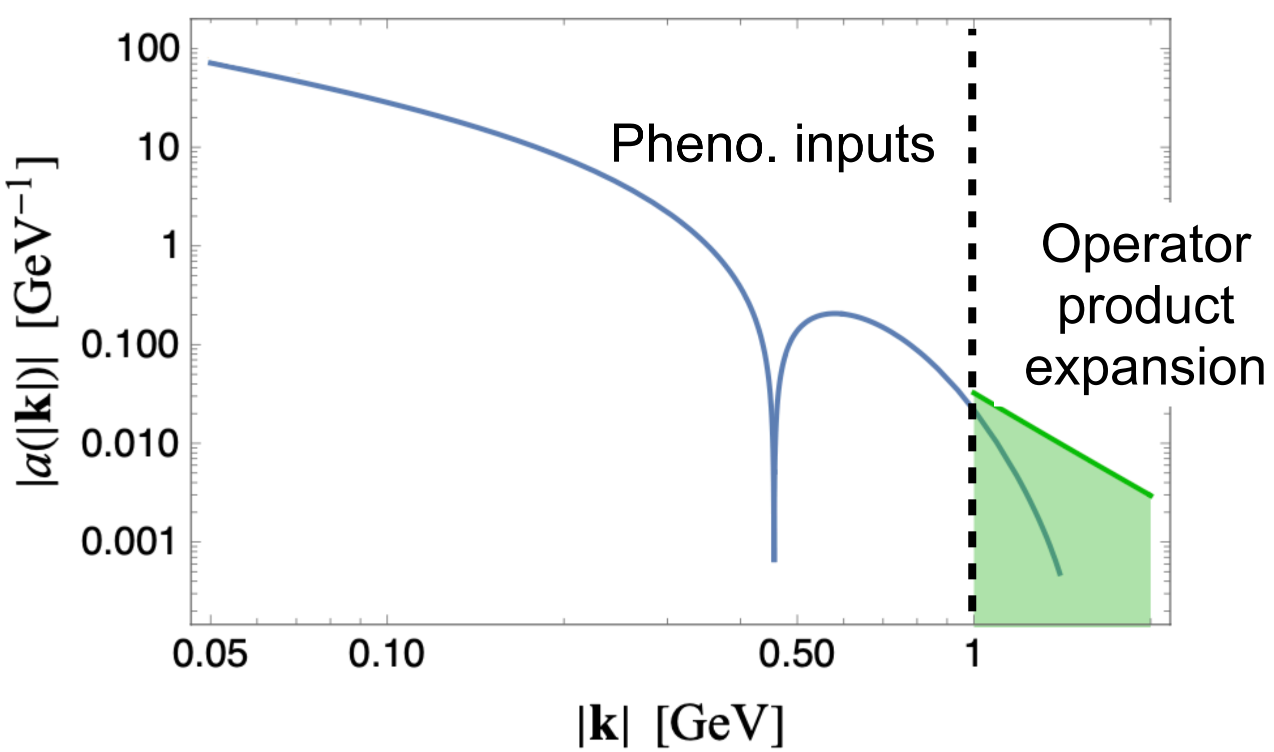
$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4} \frac{g_{\alpha\beta}}{k^2 + i\epsilon} \int d^4x \, e^{ik\cdot x} \langle pp | T\{ j_{\mathrm{w}}^{\alpha}(x) j_{\mathrm{w}}^{\beta}(0) \} | nn \rangle$$

$$= \int_0^{\Lambda} d|\mathbf{k}| \; a_<(|\mathbf{k}|) \; + \; \int_{\Lambda}^{\infty} d|\mathbf{k}| \; a_>(|\mathbf{k}|) \,,$$

- Model assumptions and inputs：

  1. *Neglect inelastic intermediate states*
  2. *Phenomenological off-shell NN amplitudes*
  3. *Phenomenological weak form factors*
  4. *Separation of low- and high-energy region*
  5. *Unknown matrix element $\bar{g}_1^{NN}(\mu)$*

Low ← Λ → High



➡ $\tilde{C}_1(\mu_\chi = M_\pi) \simeq 1.32(50)_{\mathrm{inel}}(20)_{V_S}(5)_{\mathrm{par}}$

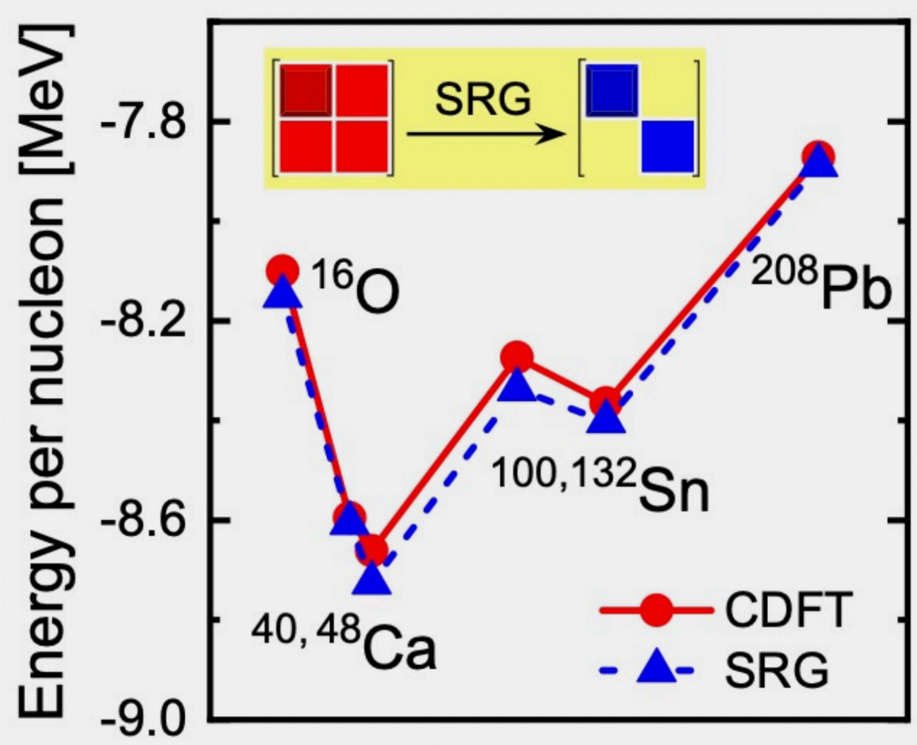
A model-free prediction without the unknown contact term?

Relativistic effects in nuclear systems

Bridge the Rel. and Nonrel. DFTs

$$4\pi r^2\rho_v(r) = \rho_0 + \frac{d}{dr}\left[\frac{1}{4\tilde{M}^2}\frac{\kappa}{r}\rho_0\right] + \frac{d^2}{dr^2}\left[\frac{1}{8\tilde{M}^2}\rho_0\right] + O(\tilde{M}^{-3}).$$

Rel. Nonrel. (including high-order terms …)



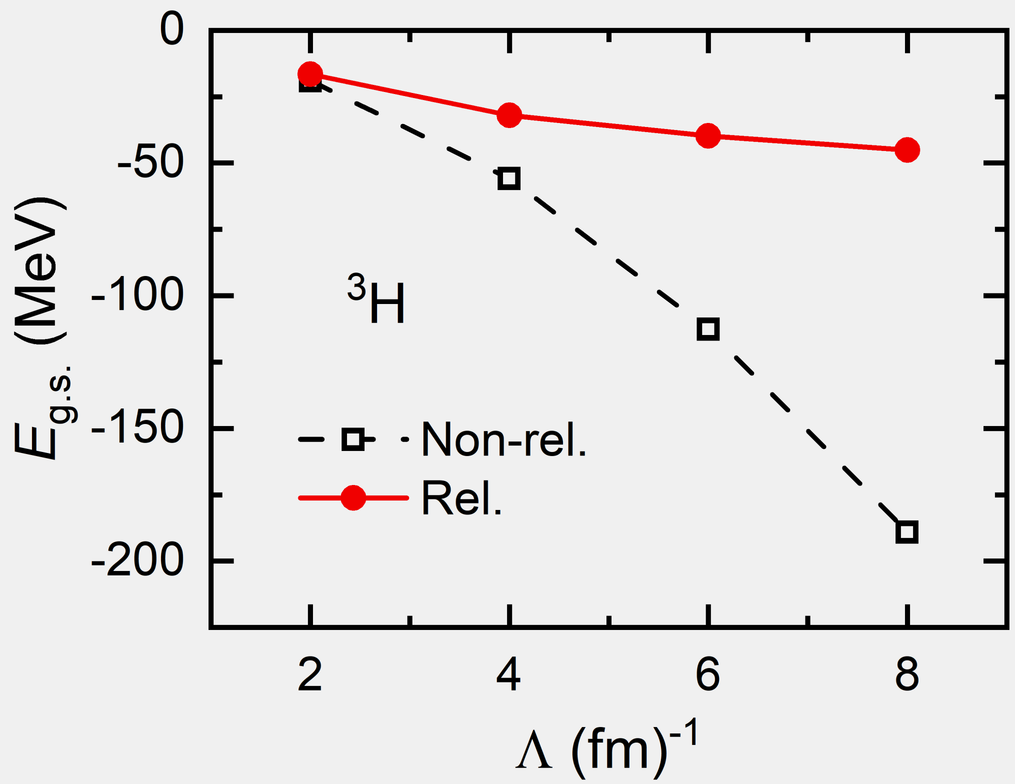
Ren and PWZ, PRC 102, 021301(R) (2020)
Editors' Suggestion

Rel. and Nonrel. VMC with LO forces

$$\left[\sum_{i=1}^{A} K_i + \sum_{i<} v(\boldsymbol{r}_{ij})\left(1 + v_t(\boldsymbol{r}_{ij},\hat{\boldsymbol{p}}_{ij}^2) + v_b(\boldsymbol{r}_{ij},\hat{\boldsymbol{P}}_{ij}^2)\right)\right]\Psi(\boldsymbol{R}) = E\Psi(\boldsymbol{R})$$

Nonrel Rel. corrections

Thomas collapse avoided !



Yang, PWZ, PLB 835, 137587 (2022)

The relativity brings high-order effects …

Relativistic framework for $0\nu\beta\beta$

- Manifestly Lorentz-invariant effective Lagrangian

Free — Strong — Weak

$$\mathscr{L}_{\Delta L=0} = \underbrace{\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}^2 + \overline{\Psi}(\mathrm{i}\partial\!\!\!/ - M_N)\Psi}_{\text{Free}} + \underbrace{\frac{g_A}{2f_\pi}\overline{\Psi}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi + \sum_\alpha C_\alpha(\overline{\Psi}\Gamma\Psi)^2}_{\text{Strong}}$$
$$+ \underbrace{\frac{1}{2}\mathrm{tr}(l_\mu\vec{\tau})\cdot\partial_\mu\vec{\pi} + \frac{1}{2}\overline{\Psi}\gamma^\mu l_\mu\Psi - \frac{g_A}{2}\overline{\Psi}\gamma^\mu\gamma_5 l_\mu\Psi}_{\text{Weak}} + \cdots$$

Machleidt and Entem, Phys. Rep. 503, 1 (2011)

- Standard mechanism of $0\nu\beta\beta$: electron-neutrino Majorana mass

$$\mathscr{L}_{\Delta L=2} = -\frac{m_{\beta\beta}}{2}\nu_{eL}^T C\nu_{eL}, \quad C = \mathrm{i}\gamma_2\gamma_0$$

- Relativistic Kadyshevsky equation：

Kadyshevsky, NPB 6, 125 (1968)

$$T(\boldsymbol{p}',\boldsymbol{p};E) = V(\boldsymbol{p}',\boldsymbol{p}) + \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi^3)}V(\boldsymbol{p}',\boldsymbol{k})\frac{M^2}{\omega_k^2}\frac{1}{E-2\omega_k+\mathrm{i}0^+}T(\boldsymbol{k},\boldsymbol{p};E) \qquad \omega_k = \sqrt{M^2+\boldsymbol{k}^2}$$

For the interaction $V(\boldsymbol{p}',\boldsymbol{p})$, (1) neglect anti-nucleon d.o.f (2) only include the leading term in Dirac spinor (3) neglect retardation effects.

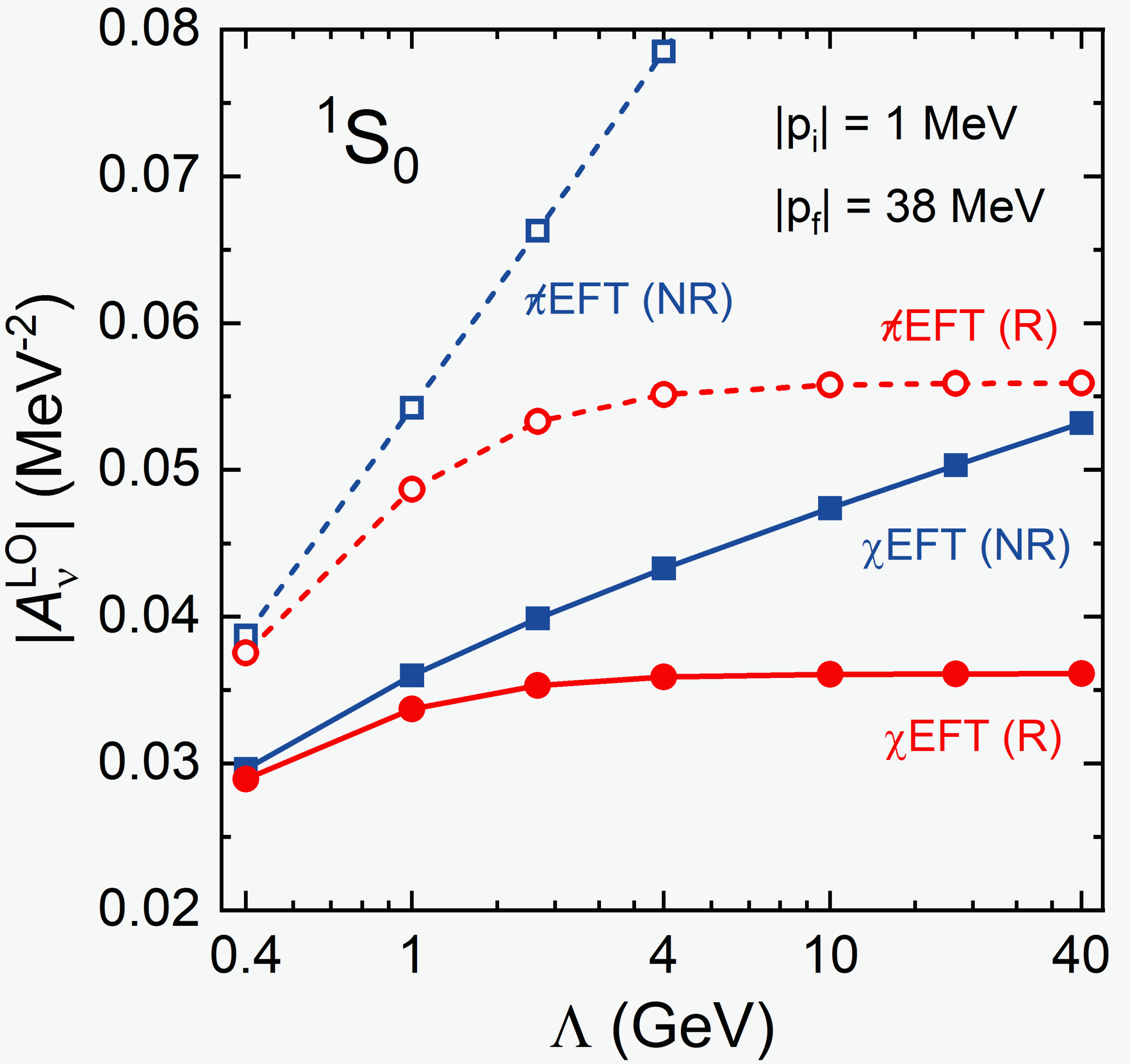
Modified Weinberg's approach in NN scattering: Epelbaum and Gegelia, PLB 716, 338 (2012)

Relativistic LO operator

- In contrast to nonrelativistic case, the unknown short-range operator is **NOT** needed at LO in the relativistic chiral EFT.

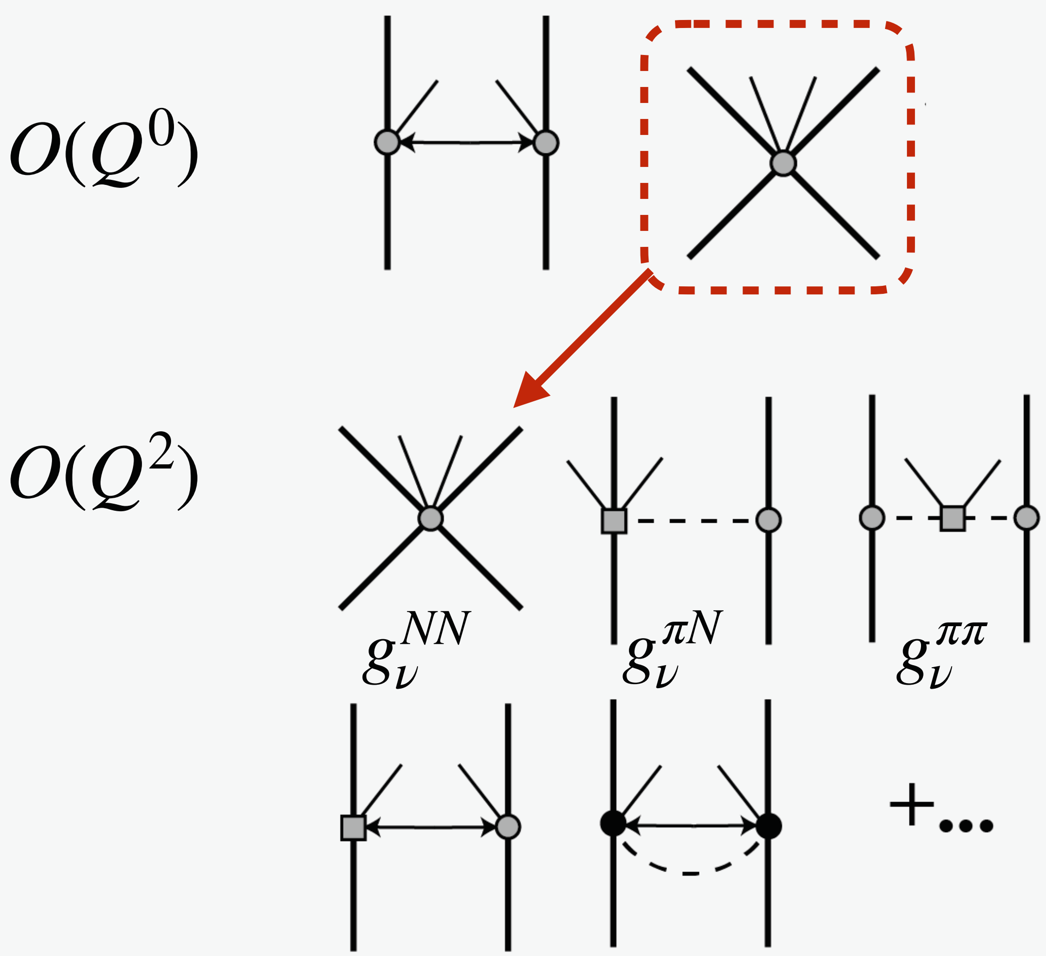
Yang and PWZ, PLB 855, 138782 (2024)

RG invariance at LO



Relativistic power counting

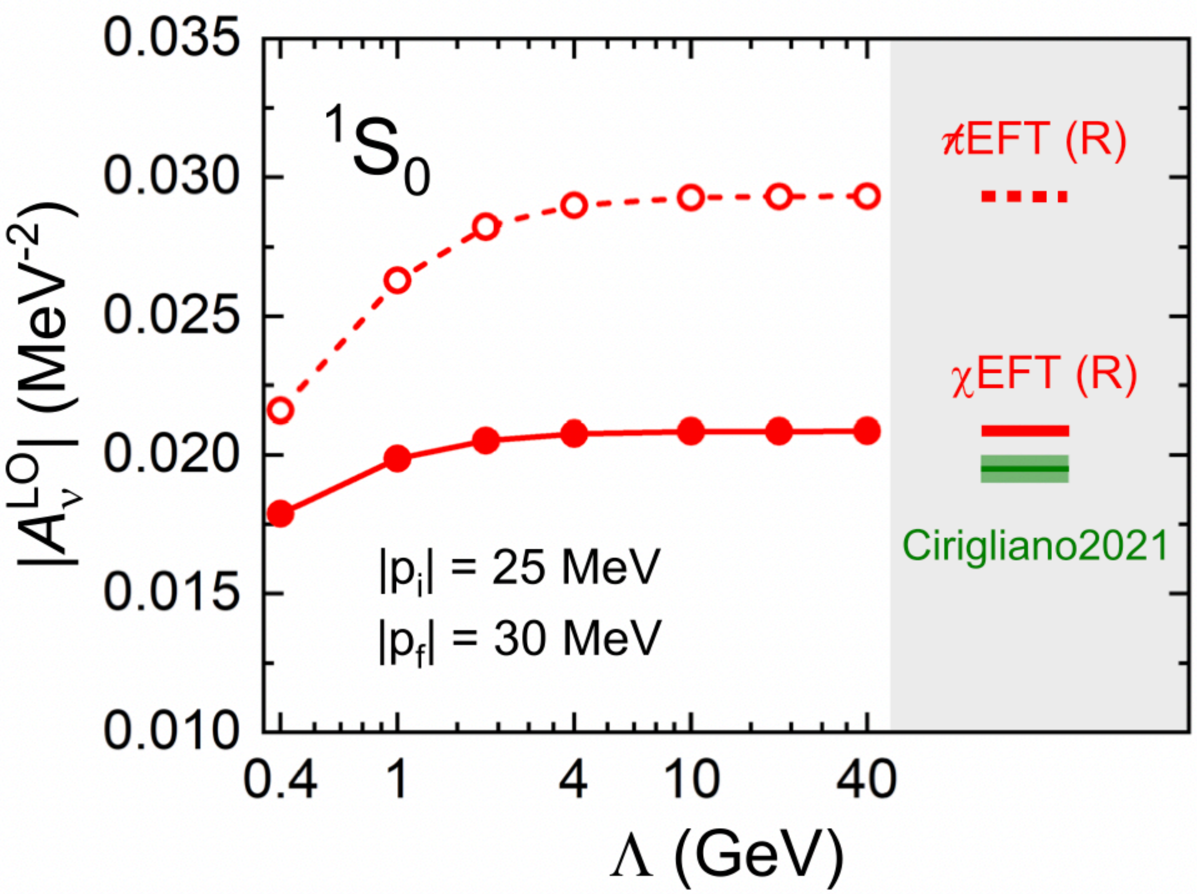
No Free Parameter at LO !

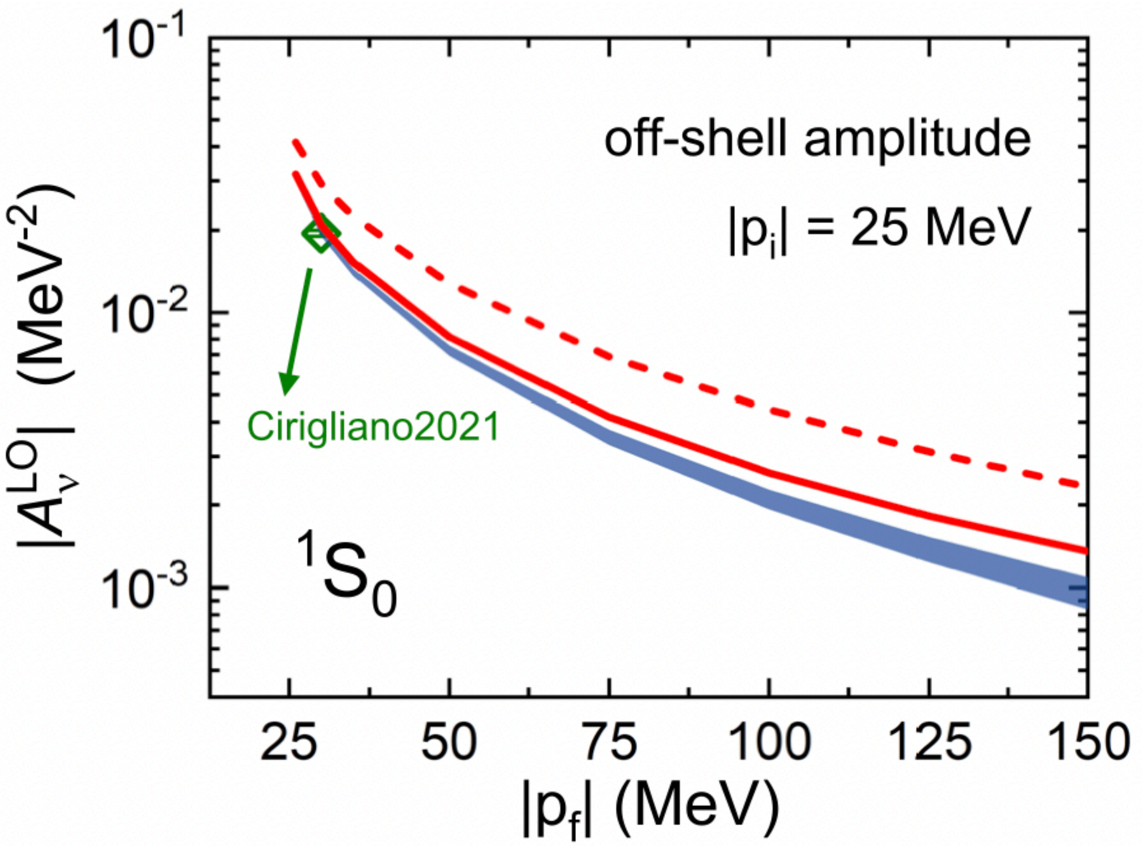


Comparison with existing results

- The $nn \to ppee$ amplitude obtained from the Cottingham model and the LO relativistic chiral EFT is consistent within 10%~40% .

Yang and PWZ, PLB 855, 138782 (2024)
Cirigliano et al., PRL 126, 172002 (2021)





Benchmark with LQCD: LO at $m_\pi = 806$ MeV

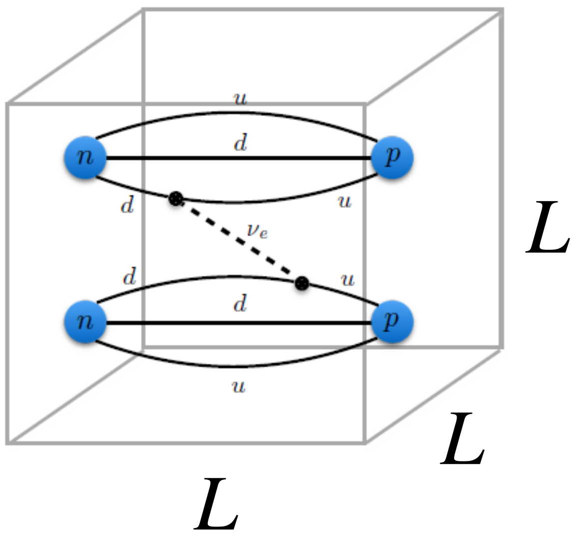
The prediction from LO relativistic pionless EFT is encouraging!

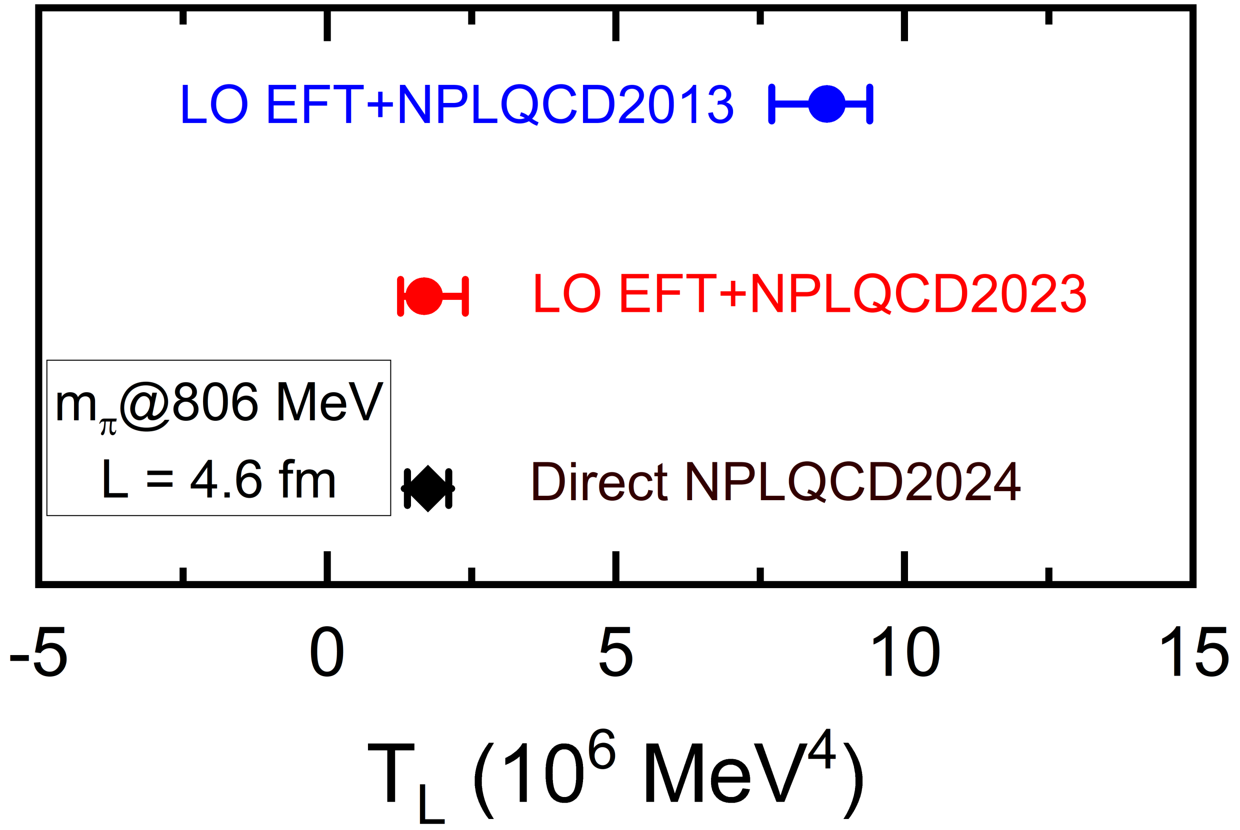
Yang and PWZ, PRD 111, 014507 (2025)

$$\mathcal{T}_L^{(M)}(E_f, E_i) = \int \mathrm{d}z_0 \int_L \mathrm{d}^3 z [\langle E_f, L | T[\mathcal{J}(z_0, \boldsymbol{z}) S_\nu(z_0, \boldsymbol{z}) \mathcal{J}(0)] | E_i, L\rangle]_L$$

LQCD: $|E, L\rangle$ is made from quarks in a box

REFT: $|E, L\rangle$ is made of two neutrons in a box





Inputs of EFT

$g_A = 1.27$

$m_\pi = 0.81$ GeV

$M_N = 1.64$ GeV

$E_{nn} = -17(4)$ MeV

$E_{nn} = -3.3(7)$ MeV

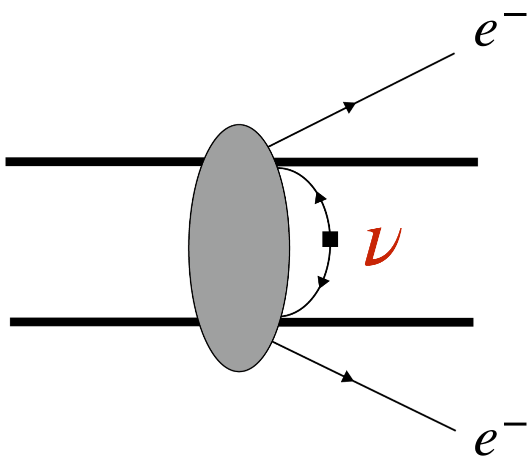
Beane et al. (NPLQCD), PRD 87, 034506 (2013)
Amarasinghe et al. (NPLQCD), PRD107,094508 (2023)
Davoudi et al. (NPLQCD), PRD 109, 114514 (2024)

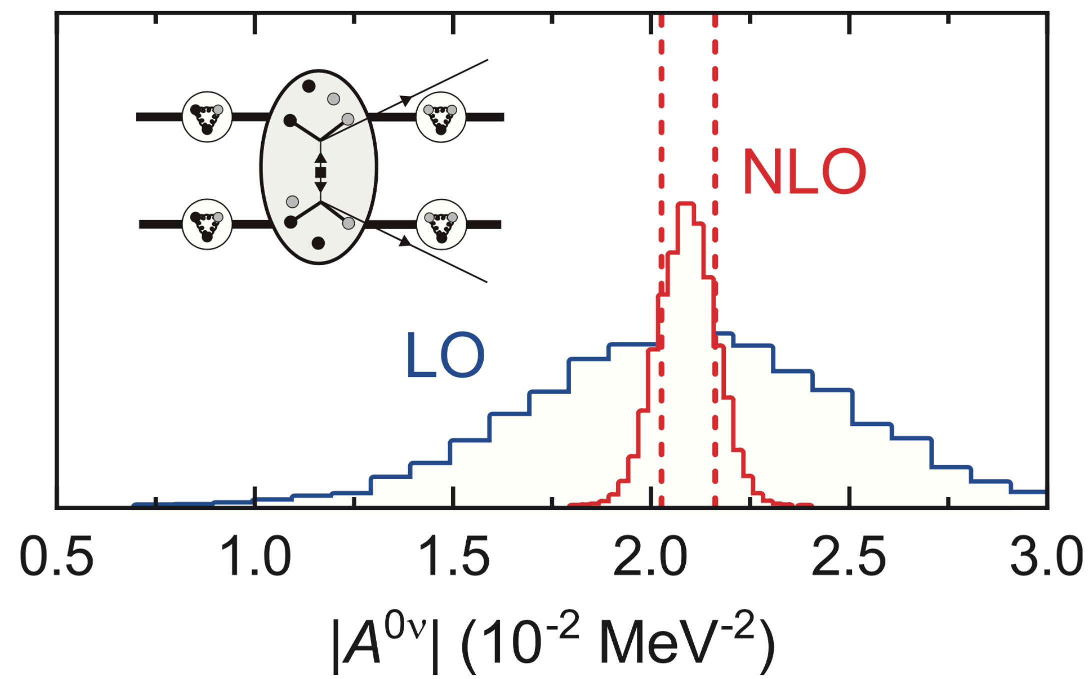
The first NLO prediction

The NLO (Q^1) effective-range corrections included; relativistic results reproduce the CSB and CIB contributions to the *NN* scattering amplitude.

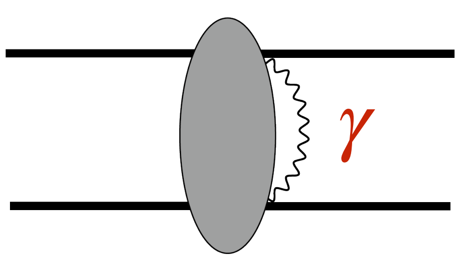
Yang and PWZ, PRL 134, 242502 (2025)

$nn \to ppee$



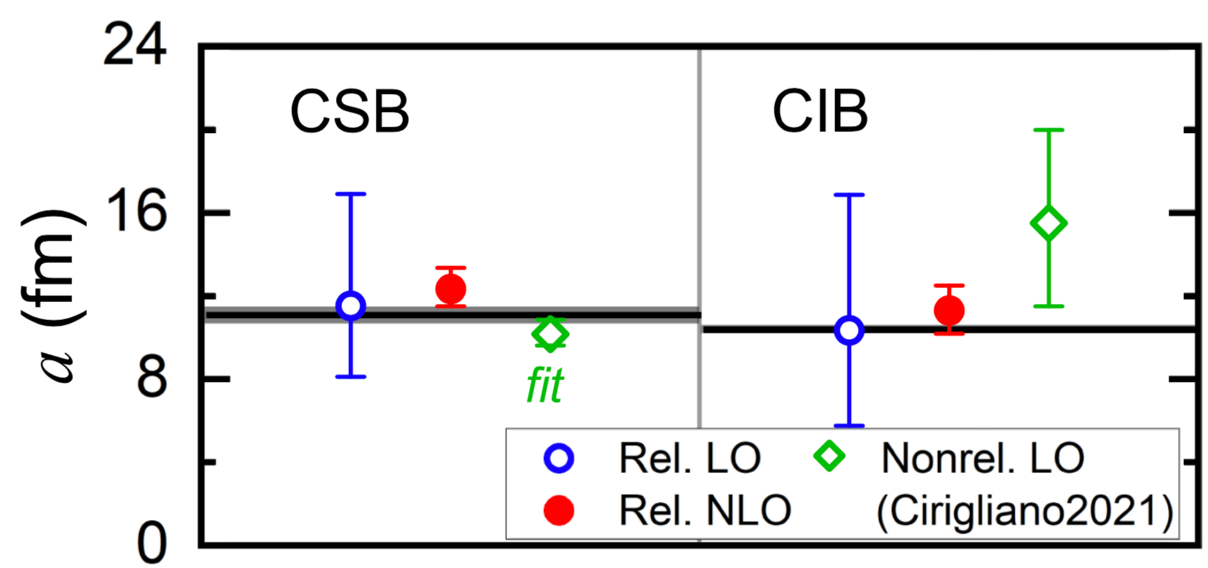


$NN \to NN$



Charge-Symmetry/
Independence Breaking

Prediction with NO free parameters!



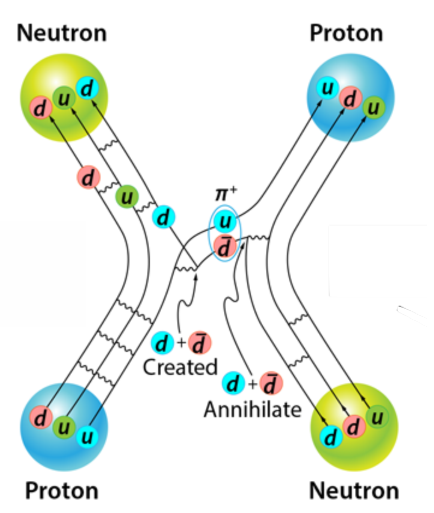
Outline

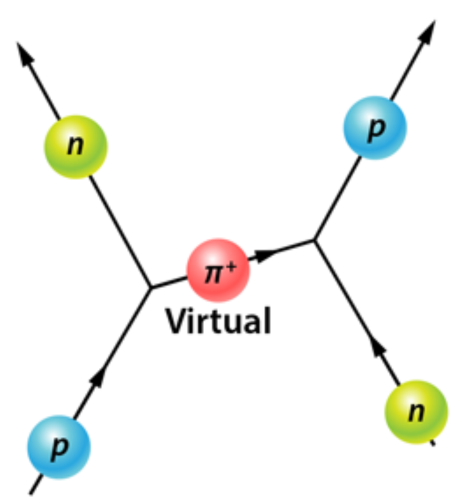
- Decay operators
- Nuclear many-body wavefunctions
- Summary

Nuclear many-body problem

Nuclear force + Many-body approach ⇒ Wavefunctions

- **Nuclear force is complicated and not known precisely**
  - Hard to derive from QCD, non-perturbative at low energies
  - Strong repulsive core, two- and three-body forces …
- **Nuclear many-body problem is hard to solve**
  - A-body Schrödinger equation with tens or hundreds nucleons
  - Spin-isospin degrees of freedom, pairing correlations …





Nuclear force

Bare nuclear forces

- Accurate description of free-space scattering and light nuclei
- Intractable to solve for medium-mass and heavy nuclei

Effective nuclear forces

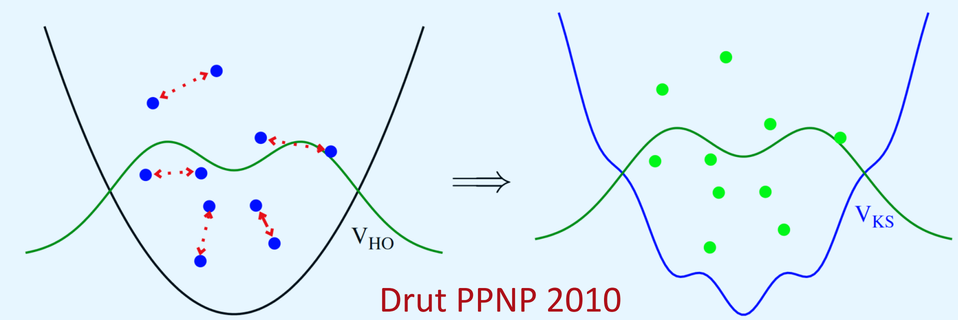
- Not suitable to describe free-space scattering
- Tractable to solve for medium-mass and heavy nuclei
- Well description of nuclear properties

Softened/SRG evolved nuclear forces

- Moderate description of free-space scattering and light nuclei
- Tractable to solve for medium-mass nuclei
- Moderate description of nuclear properties

Nuclear many-body approaches

Nuclear Density Functional Theory (DFT)

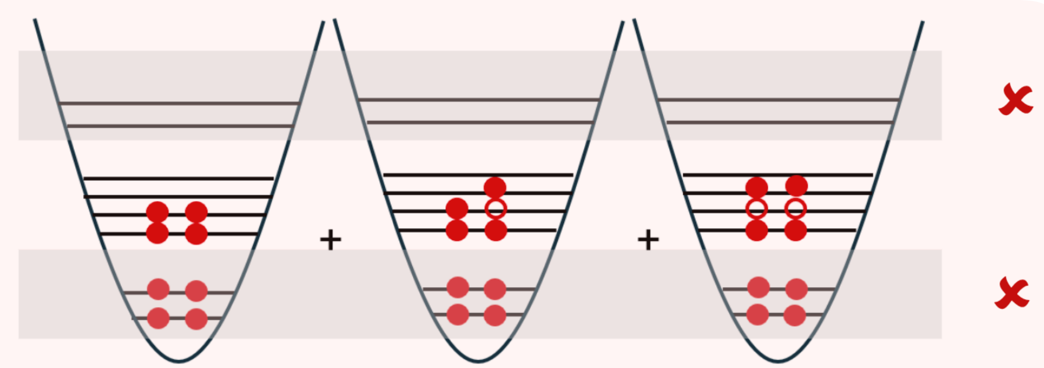


The exact ground-state energy is a **universal functional** of local densities

$\Rightarrow$ *full model space, limited correlations*

$$E[\rho] \Rightarrow \hat{h} = \frac{\partial E[\rho]}{\partial \rho}, \hat{h}\phi_i = \varepsilon_i \phi_i \Rightarrow \rho = \sum_i |\phi_i|^2$$

Nuclear Shell Model (SM)

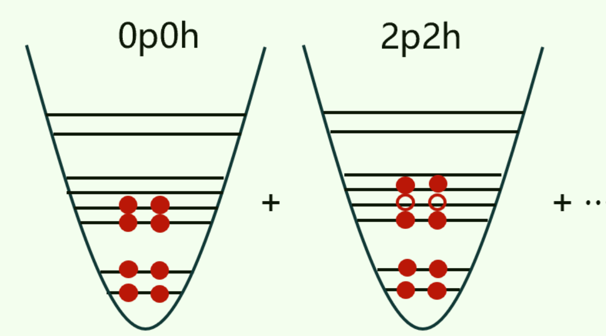


The full nuclear Hamiltonian in the complete model space is replaced by **an effective Hamiltonian in a limited model space**

$\Rightarrow$ *limited model space, sufficient correlations*

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \quad \Rightarrow \quad \hat{H}_{\text{eff}}|\tilde{\Psi}\rangle = E_{\text{eff}}|\tilde{\Psi}\rangle$$

Random Phase Approximation (RPA)



A particle-hole theory with **ground-state correlations** (based on DFT or Bonn potential)

$\Rightarrow$ *larger model space and less correlations compared to the Shell Model*

$$|\text{RPA}\rangle = N_0 \exp(\hat{Z})|\text{HF}\rangle$$

$$\hat{Z} = \frac{1}{2}\sum_{minj} Z_{minj}\hat{a}_m^\dagger \hat{a}_i \hat{a}_n^\dagger \hat{a}_j$$

DFT vs Shell Model

DFT

- ✓ Universal density functionals

  Symmetry broken
  Single config. fruitful physics
  No Configuration mixing

- ✓ Applicable for almost all nuclei
- ✗ No spectroscopic properties

Shell Model

- ✗ Non-universal effective interactions

  No symmetry broken
  Single config. little physics
  Configuration mixing

- ✗ intractable for deformed heavy nuclei
- ✓ spectroscopy from multi config.

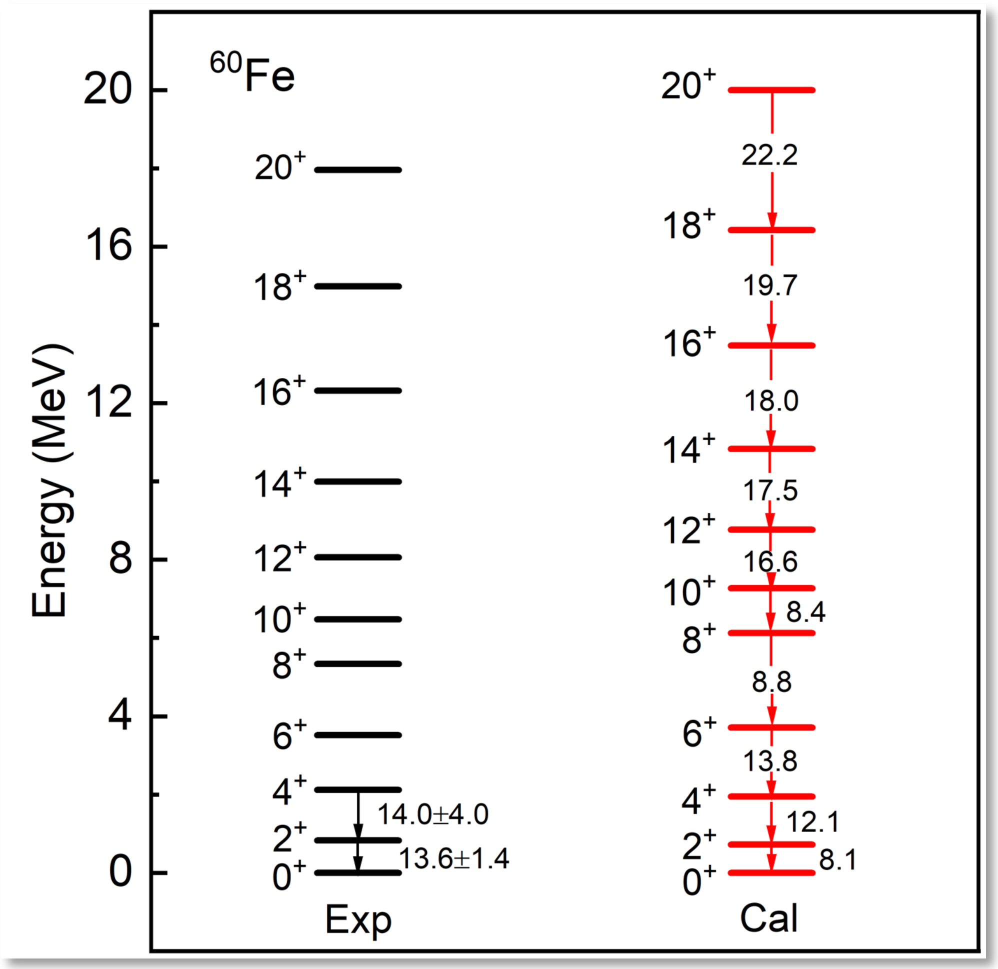
a theory combining the advantages from both approaches?

The ReCD method

Relativistic **C**onfiguration-interaction **D**ensity functional theory

a theory combining the advantages from both Shell model and DFT

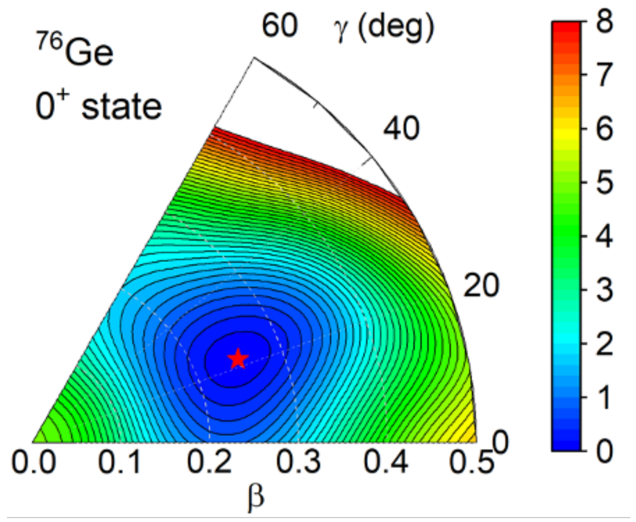
1. **Covariant Density Functional Theory**
   a minimum of the energy surface
2. **Configuration space**
   multi-quasiparticle states
3. **Angular momentum projection**
   rotational symmetry restoration
4. **Shell model calculation**
   configuration mixing / interaction from CDFT

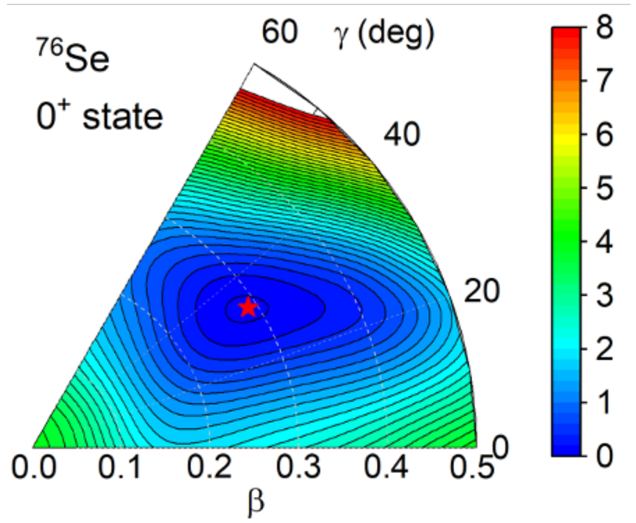


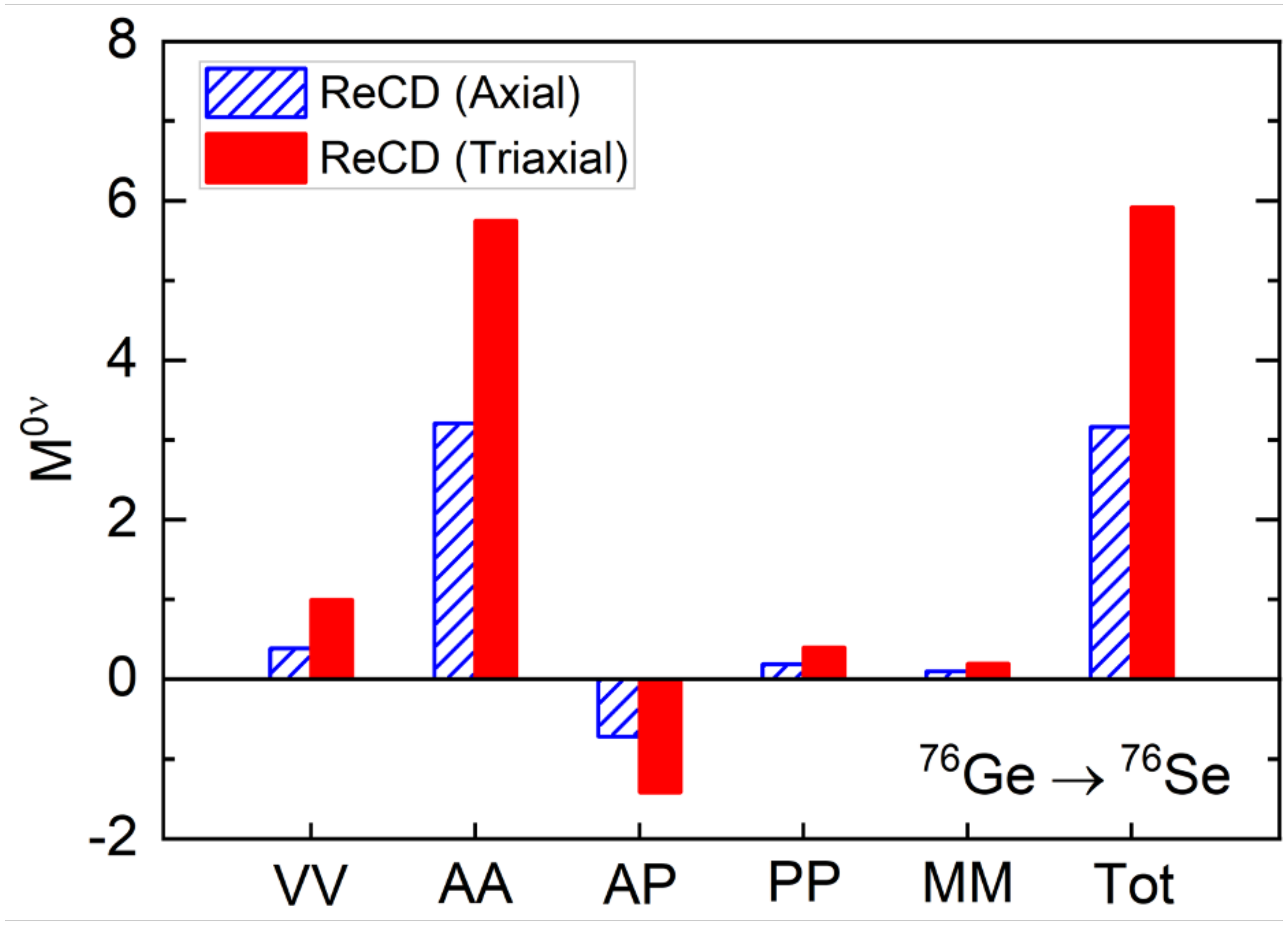
PWZ, Ring, Meng, PRC 94 (2016) 041301(R)
Wang, PWZ, Meng, PRC 105 (2022) 054311

ReCD method for $0\nu\beta\beta$

Both axial and triaxial degrees of freedom are included in the ReCD theory



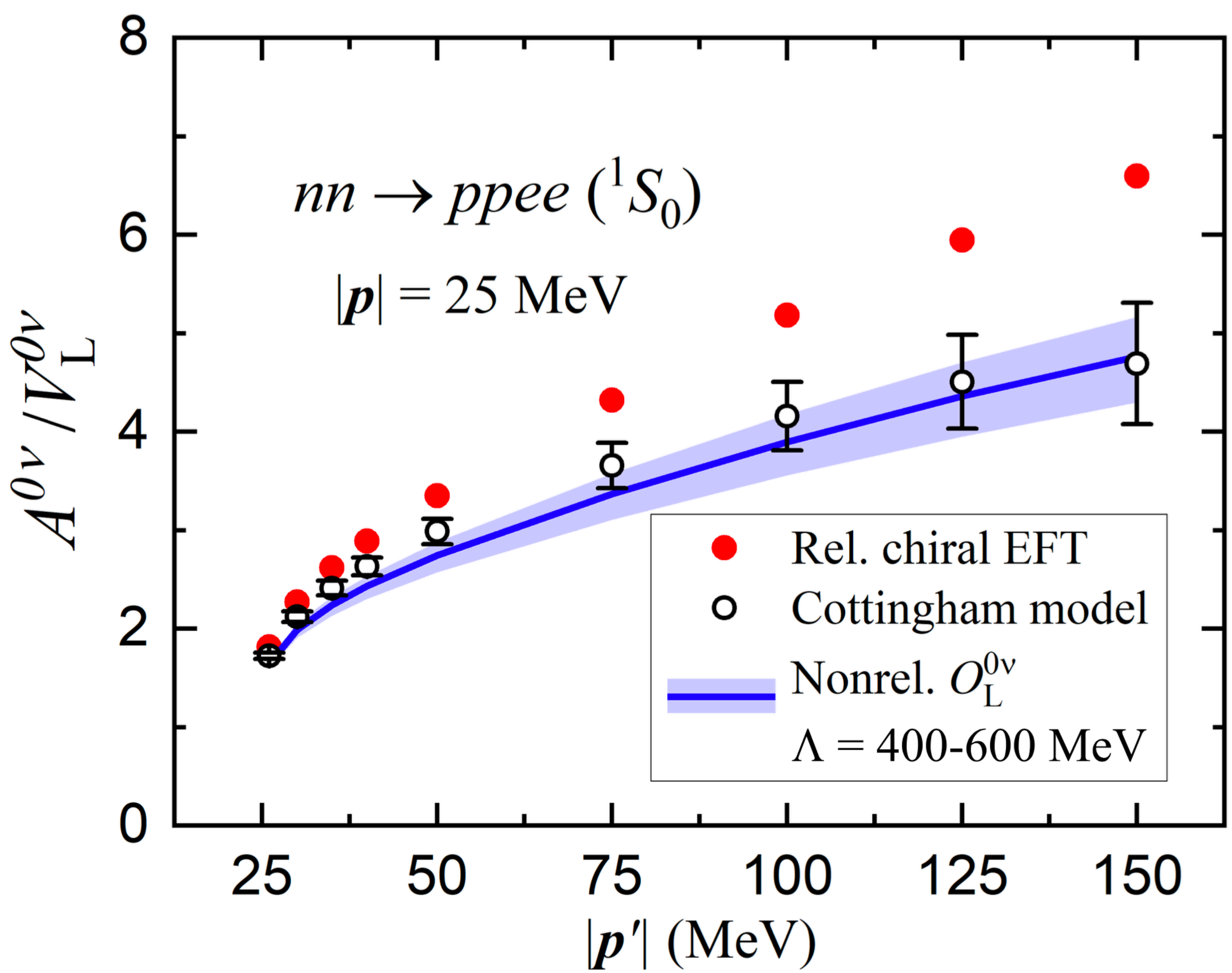




Wang, PWZ, Meng, Science Bulletin 69, 2017-2020 (2024)

The short-range uncertainty in NME

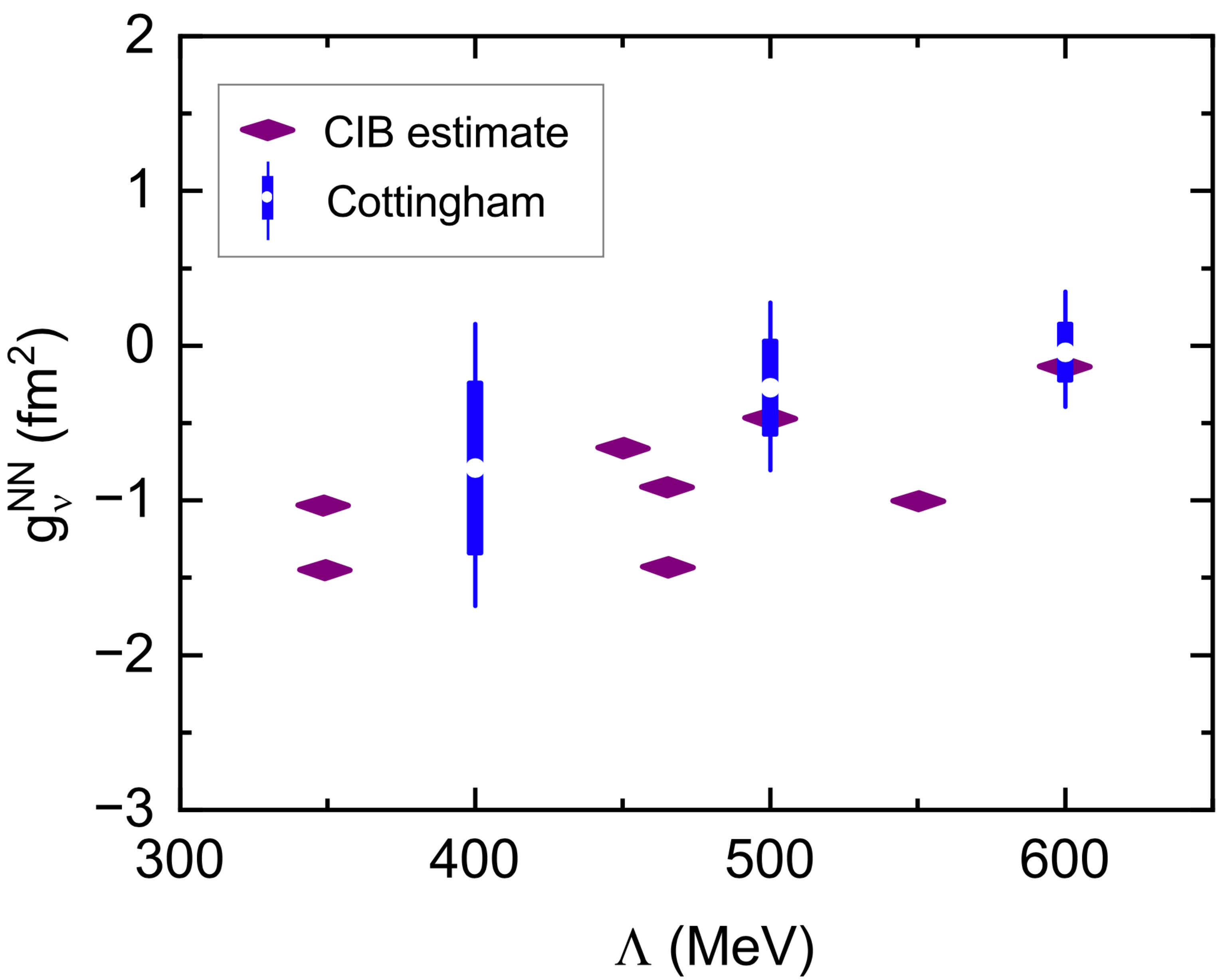
Wang, Yang, PWZ, submitted



The nonrelativistic results depend on cutoff, introducing a short-range term with unknown coupling g_ν^{NN}

Determine the g_ν^{NN}

- Thick bars: systematic error of the Cottingham model
- Thin bars: errors after considering the dependence on final momentum |p'|



CIB estimates from

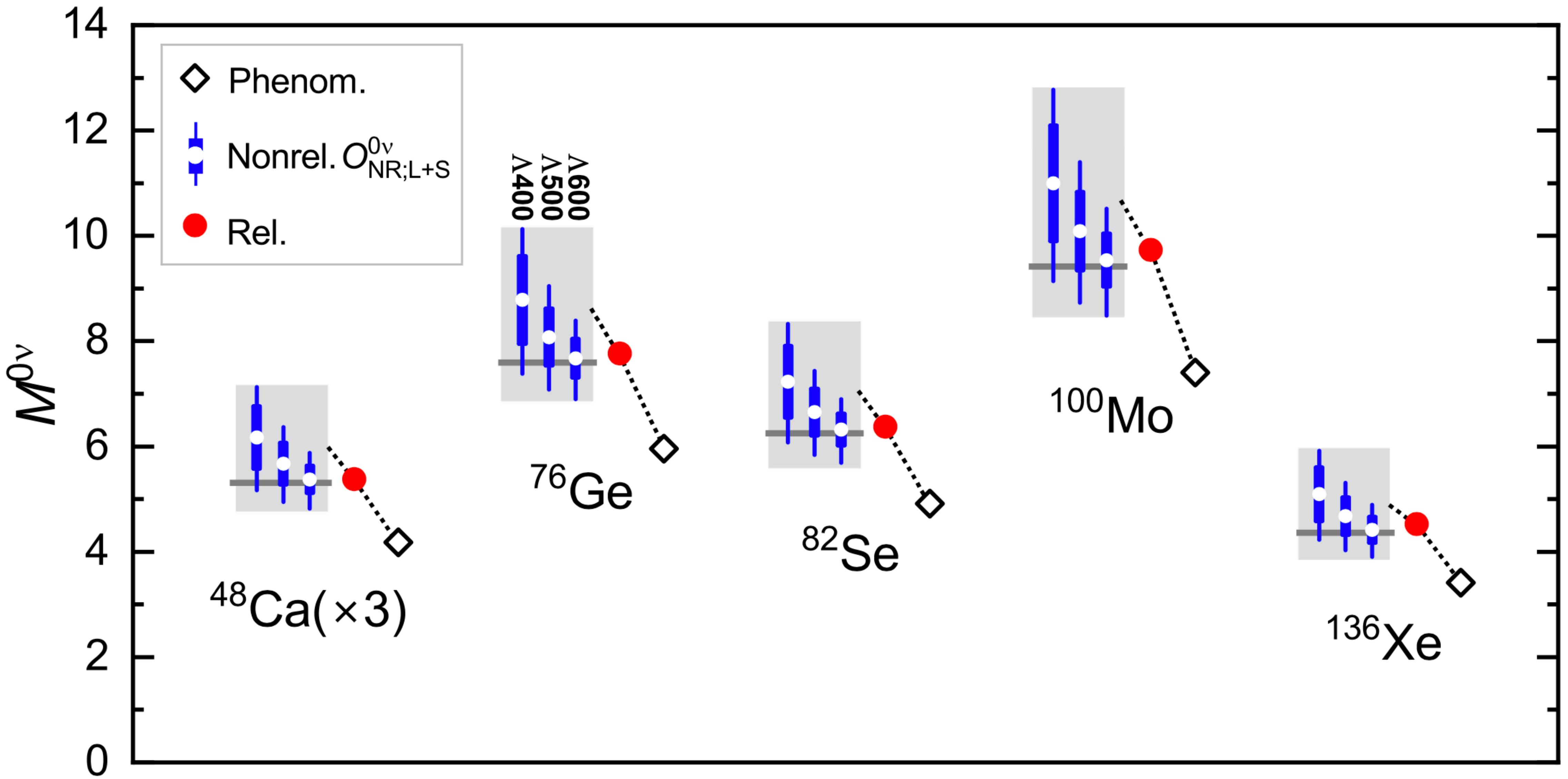
Jokiniemi, Soriano, Menéndez, PLB 823, 136720 (2021)

Cirigliano, et al., PRC 100, 055504 (2019)

Wang, Yang, PWZ, submitted

NMEs based on relativistic decay operator

ReCD theory ⇒ wavefunctions; **Relativistic EFT ⇒ decay operator**



Wang, Yang, PWZ, submitted

The relativistic NMEs are free from the short-range uncertainty

The relativistic NMEs lie within the uncertainty range of nonrelativistic NMEs

Summary

A **model-free** prediction of the $nn \to ppee$ decay amplitude is obtained by the **relativistic framework based on chiral EFT**…

- No contact term is needed for renormalization at LO.
- Consistent with the previous estimation within 10%~40%.
- Better performance for the charge-dependent *NN* scattering.
- First NLO prediction for $nn \to ppee$ decay amplitude (the most accurate!)

$$|\mathcal{A}_\nu^{\text{NLO}}|(p_i = 25\ \text{MeV}, p_f = 30\ \text{MeV}) = 0.0209(7)\ \text{MeV}^{-2}$$

- NMEs for candidate nuclei free from the short-range uncertainty

Benchmark with lattice QCD …

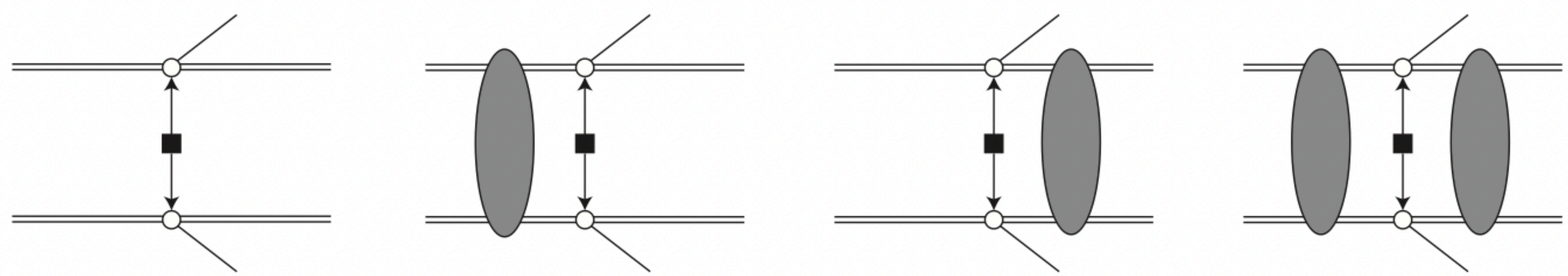

Thank you.

Analysis of UV divergence

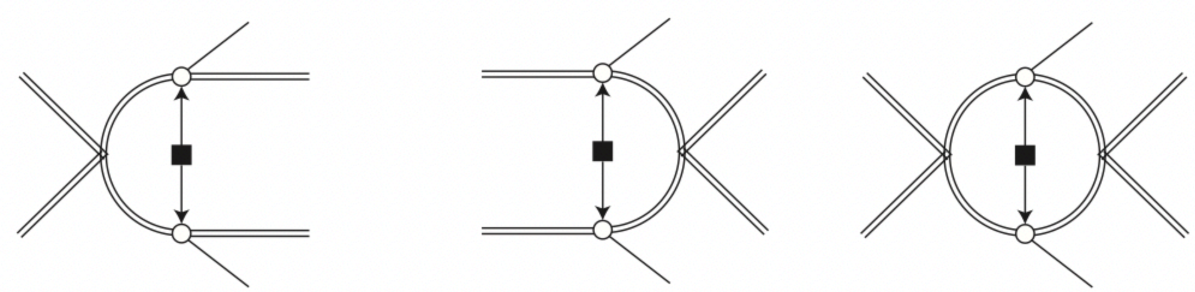
- Relativistic scattering equation has a milder ultraviolet (UV) behavior

	Relativistic		Nonrelativistic
Propagator:	$\dfrac{M^2}{\boldsymbol{k}^2+M^2}\dfrac{1}{E-2\sqrt{\boldsymbol{k}^2+M^2}+\mathrm{i}0^+}$	$\rightarrow$	$\dfrac{1}{E_{\mathrm{kin}}-\boldsymbol{k}^2/M+\mathrm{i}0^+}$
UV behavior ($k\sim\Lambda$):	$O(\Lambda^{-3})$		$O(\Lambda^{-2})$

- The degree of divergence of $nn\rightarrow ppee$ decay amplitude：



UV structure: –

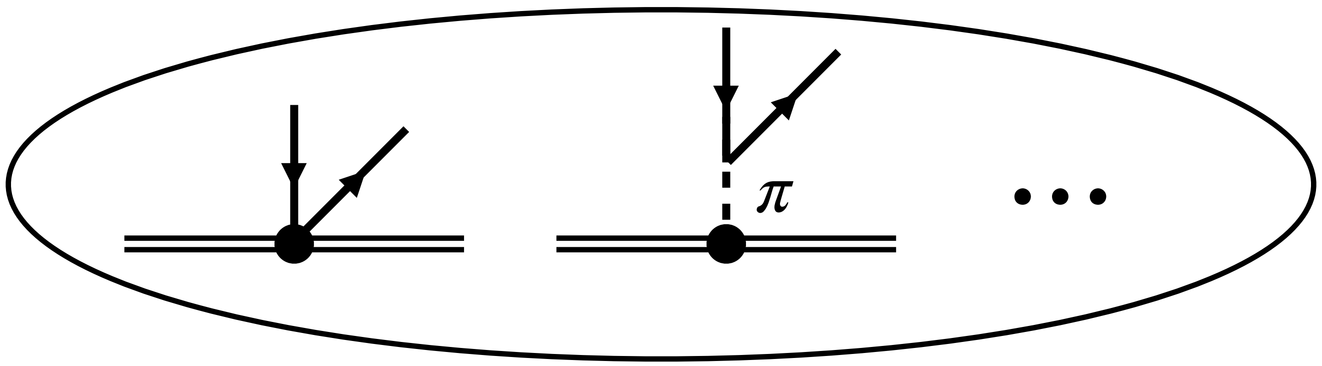


					Renormalizable?
Relativistic:	–	$O(\Lambda^{-2})$	$O(\Lambda^{-2})$	$O(\Lambda^{-2})$	Yes
Nonrelativistic:	–	$O(\Lambda^{-1})$	$O(\Lambda^{-1})$	$O(\log\Lambda)$	No

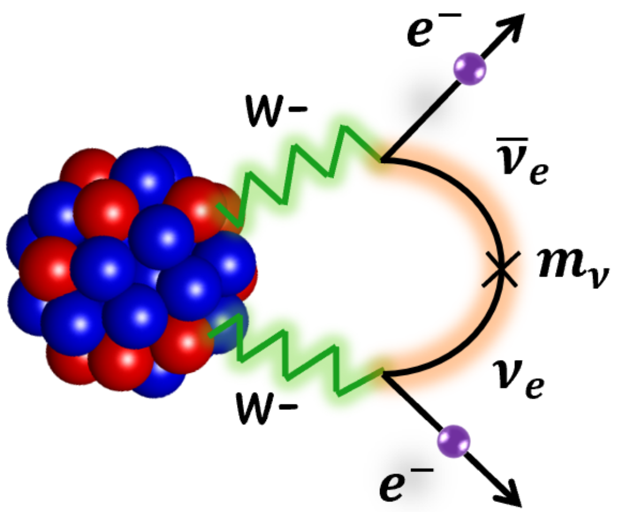
Future

Relativistic calculations of $0\nu\beta\beta$ nuclear matrix elements with LO and NLO chiral decay operators

Weak charged current



Nuclear-structure calculations

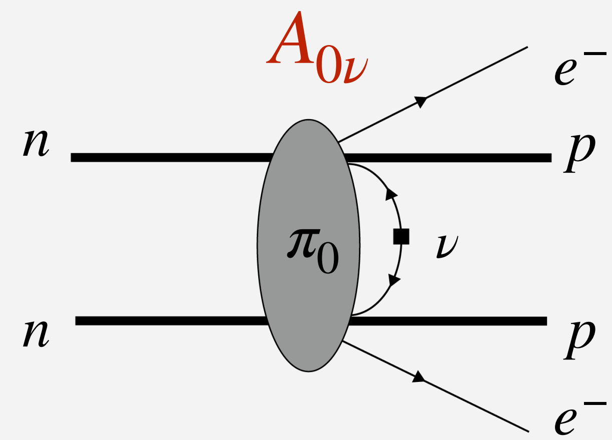


$$\langle p'|J^\mu_{\mathrm{LO}}(x)|p\rangle = \mathrm{e}^{\mathrm{i}qx}\bar{u}(p')\left(g_V\gamma^\mu - g_A\gamma^\mu\gamma_5 + g_A\frac{2m_N q^\mu}{m_\pi^2+\boldsymbol{q}^2}\gamma_5\right)u(p)$$

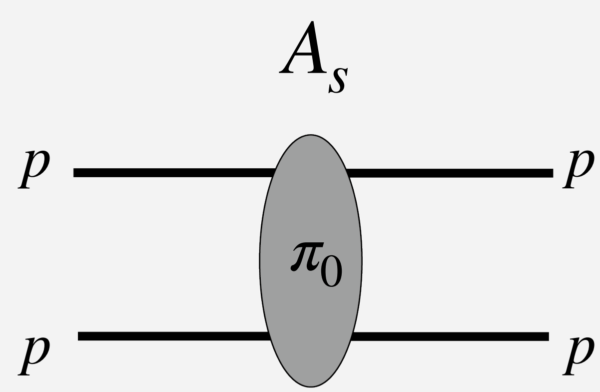
No need for contact term!

Neutrino and photon exchange

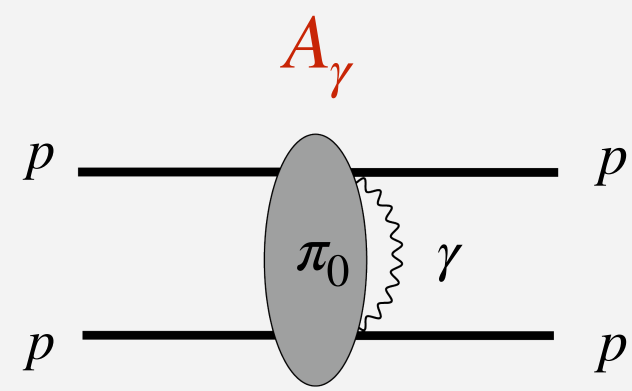
$nn \to ppee$



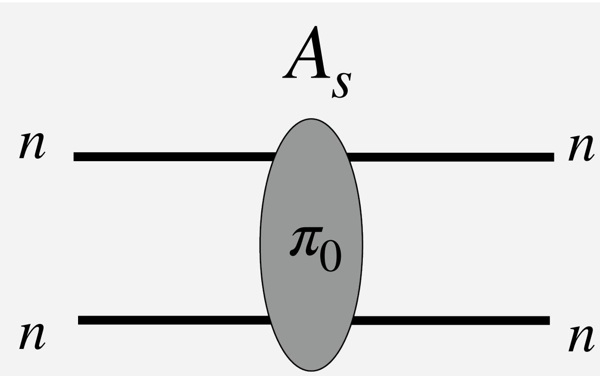
$pp \to pp$



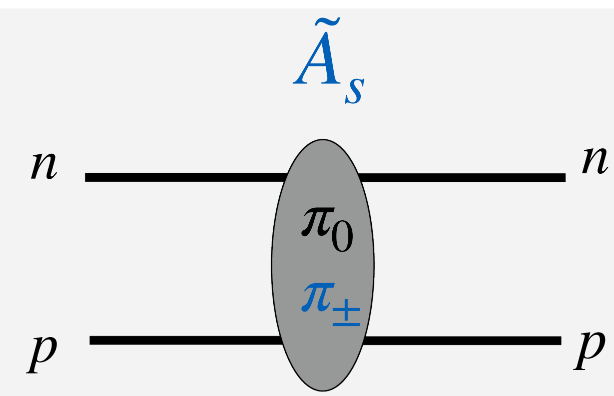
$+$



$nn \to nn$



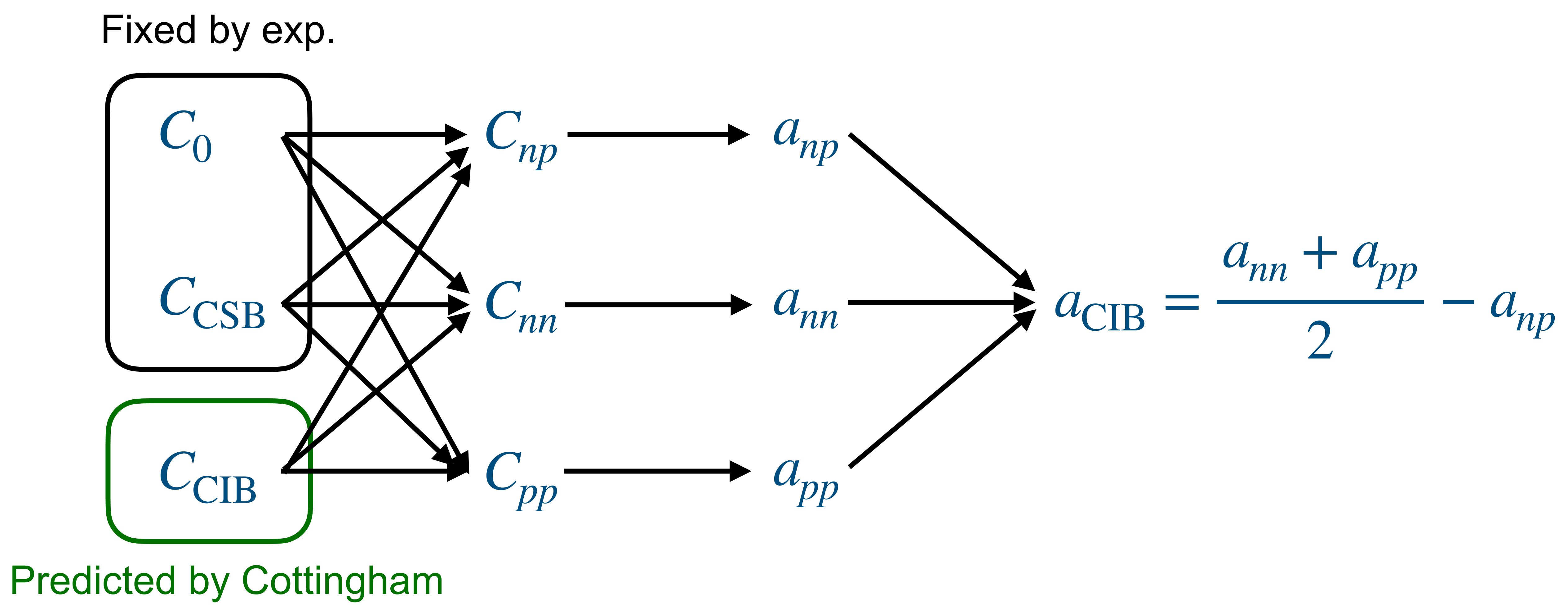
$np \to np$



CSB: $A_{nn} - A_{pp} \to -A_\gamma$

CIB: $\frac{1}{2}(A_{nn} + A_{pp}) - A_{np} \to \frac{1}{2}A_\gamma + A_s - \tilde{A}_s$

Scattering length



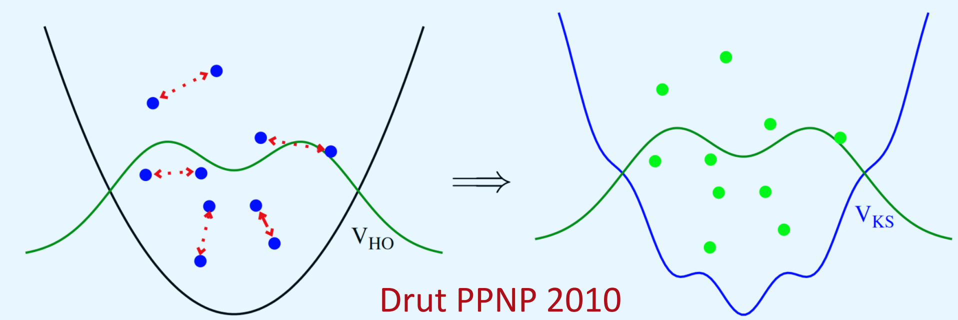
To predict $a_{\rm CIB}$, all the three contact terms in nn, np, pp must be known. The Cottingham model provides the value of one contact term and the other two are fixed by data. Here, the goal is to test the prediction of the Cottingham model for contact terms. In practice, all the three contact terms can be determined by the data.

Nuclear many-body approaches

Nuclear Density Functional Theory (DFT)

The exact ground-state energy is a **universal functional** of local densities

⇒ *full model space, limited correlations*

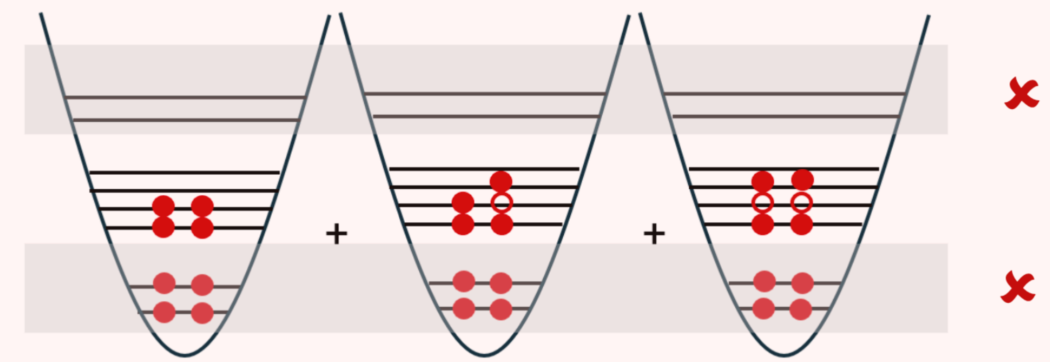


$$E[\rho] \Rightarrow \hat{h} = \frac{\partial E[\rho]}{\partial \rho}, \hat{h}\phi_i = \varepsilon_i \phi_i \Rightarrow \rho = \sum_i |\phi_i|^2$$

Nuclear Shell Model (SM)

The full nuclear Hamiltonian in the complete model space is replaced by **an effective Hamiltonian in a limited model space**

⇒ *limited model space, sufficient correlations*

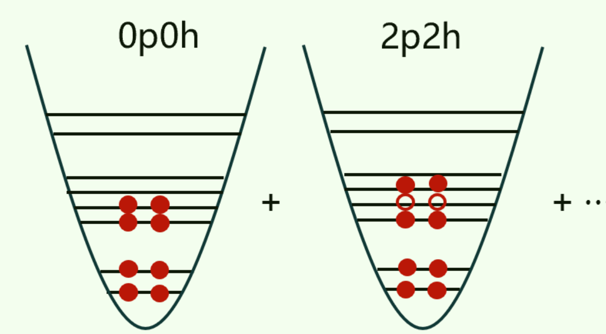


$$\hat{H}|\Psi\rangle = E|\Psi\rangle \;\;\Rightarrow\;\; \hat{H}_{\text{eff}}|\tilde{\Psi}\rangle = E_{\text{eff}}|\tilde{\Psi}\rangle$$

Random Phase Approximation (RPA)

A particle-hole theory with **ground-state correlations** (based on DFT or Bonn potential)

⇒ *larger model space and less correlations compared to the Shell Model*

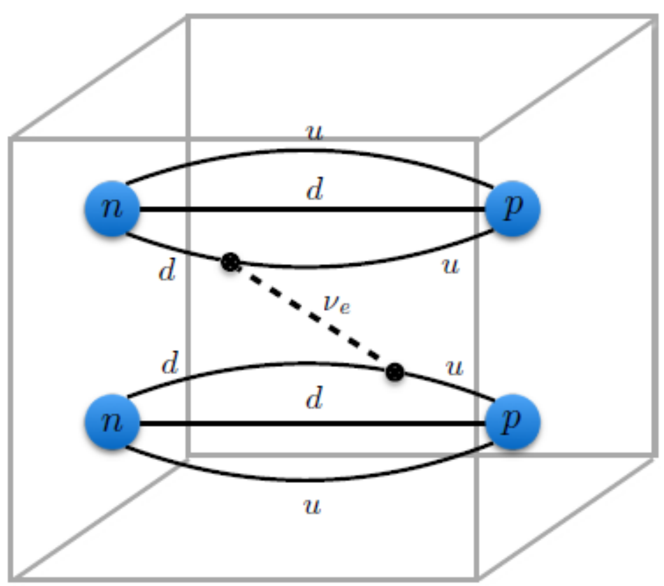


$$|\text{RPA}\rangle = N_0 \exp(\hat{Z})|\text{HF}\rangle$$

$$\hat{Z} = \frac{1}{2} \sum_{minj} Z_{minj} \hat{a}^\dagger_m \hat{a}_i \hat{a}^\dagger_n \hat{a}_j$$

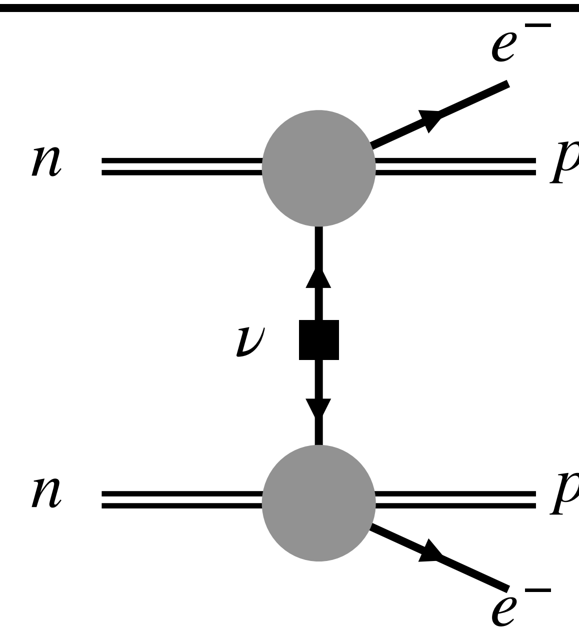
Benchmark with Lattice QCD

Benchmark with lattice QCD calculations of $0\nu\beta\beta$ decay will provide a stringent test on present framework



Lattice simulation of $0\nu\beta\beta$

Currently only possible at heavy quark (neutrino) masses

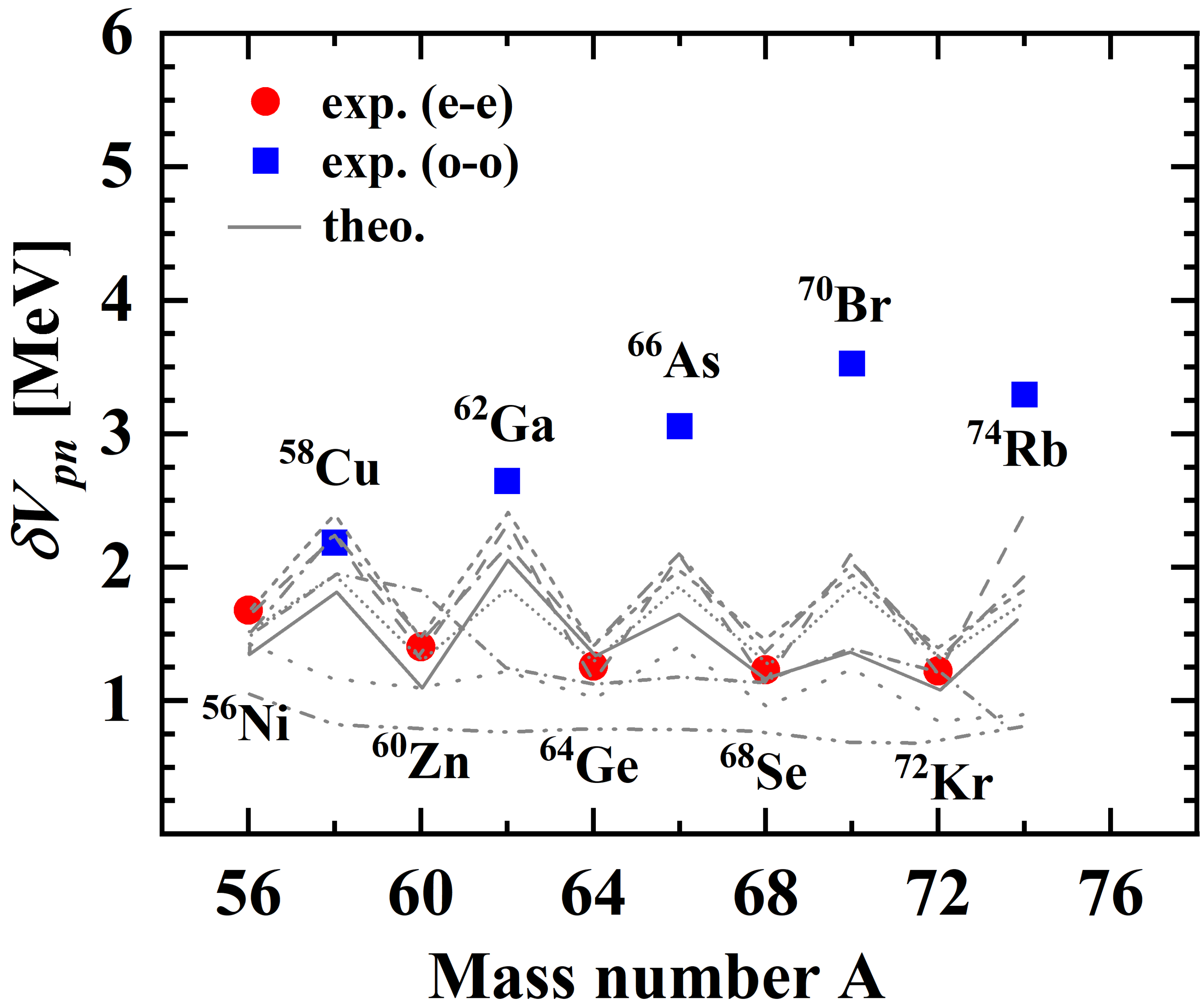


Rel.$\not\pi$EFT (χEFT) prediction

Fixed by $a_{np}(,\ g_A)$ at lattice-QCD conditions

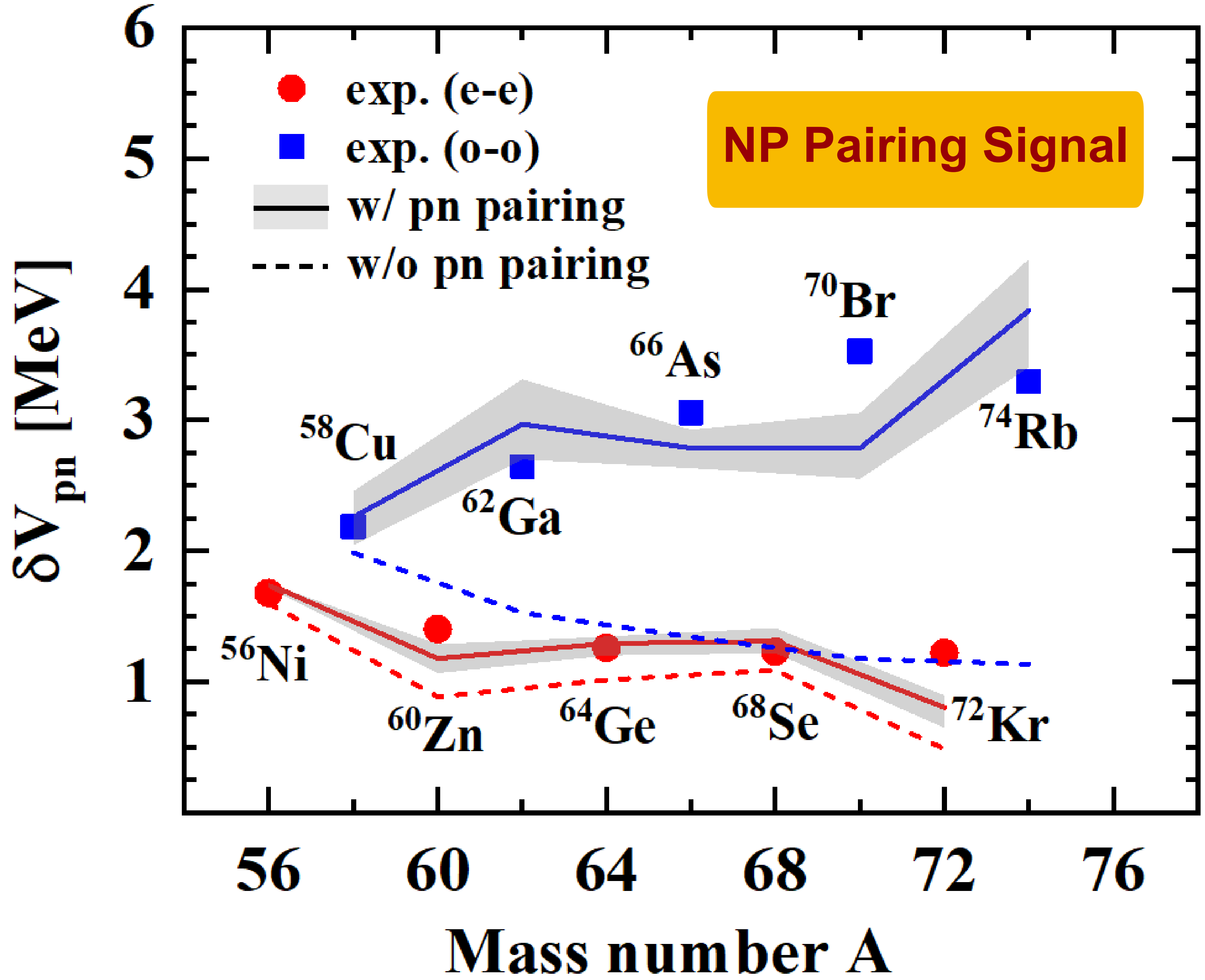
New mass measurement of upper fp-shell nuclei

This bifurcation of double binding energy eifferences δVpn cannot be reproduced by the available mass models.



Exp: M. Wang et al., PRL 130, 192501 (2023)

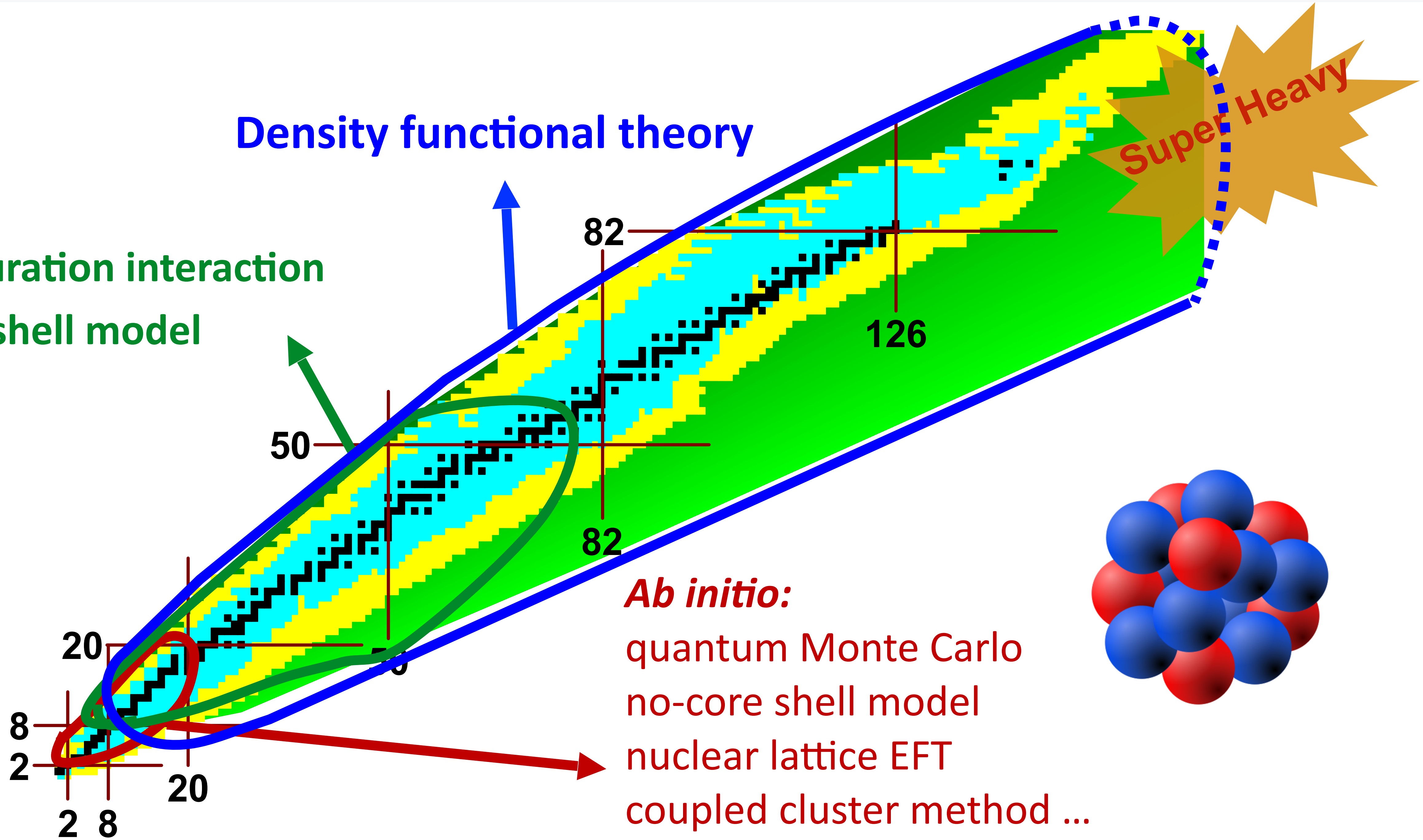
New mass measurement of upper fp-shell nuclei



Exp: M. Wang et al., PRL 130, 192501 (2023)

Wang, Wang, Xu, **PWZ**, Meng, PRL 132, 232501 (2024)

State-of-the-art theories for nuclei



It would be interesting to investigate the intersections between different theories for a unified and comprehensive description of nuclei.

Density functional theory

The many-body problem is mapped onto a one-body problem

Hohenberg-Kohn Theorem

The exact ground-state energy of a quantum mechanical many-body system is a universal functional of the local density.

$$E[\rho] = T[\rho] + U[\rho] + \int V(\mathbf{r})\rho(\mathbf{r})\, \mathrm{d}^3\mathbf{r}$$

Kohn-Sham DFT

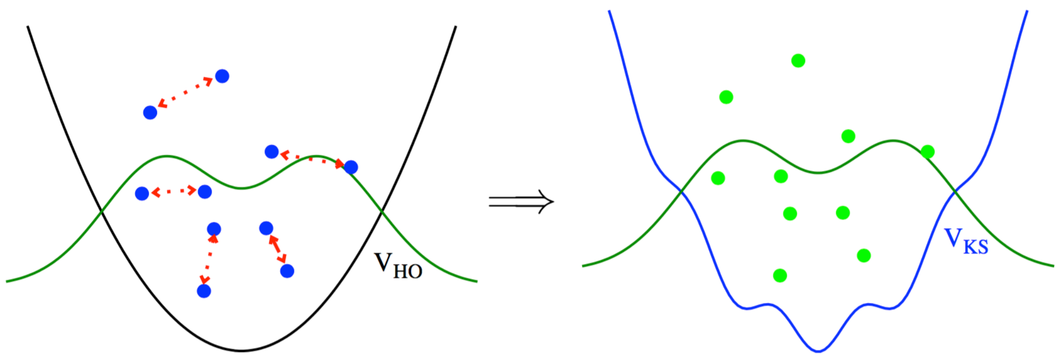


Figure from Drut PPNP 2010

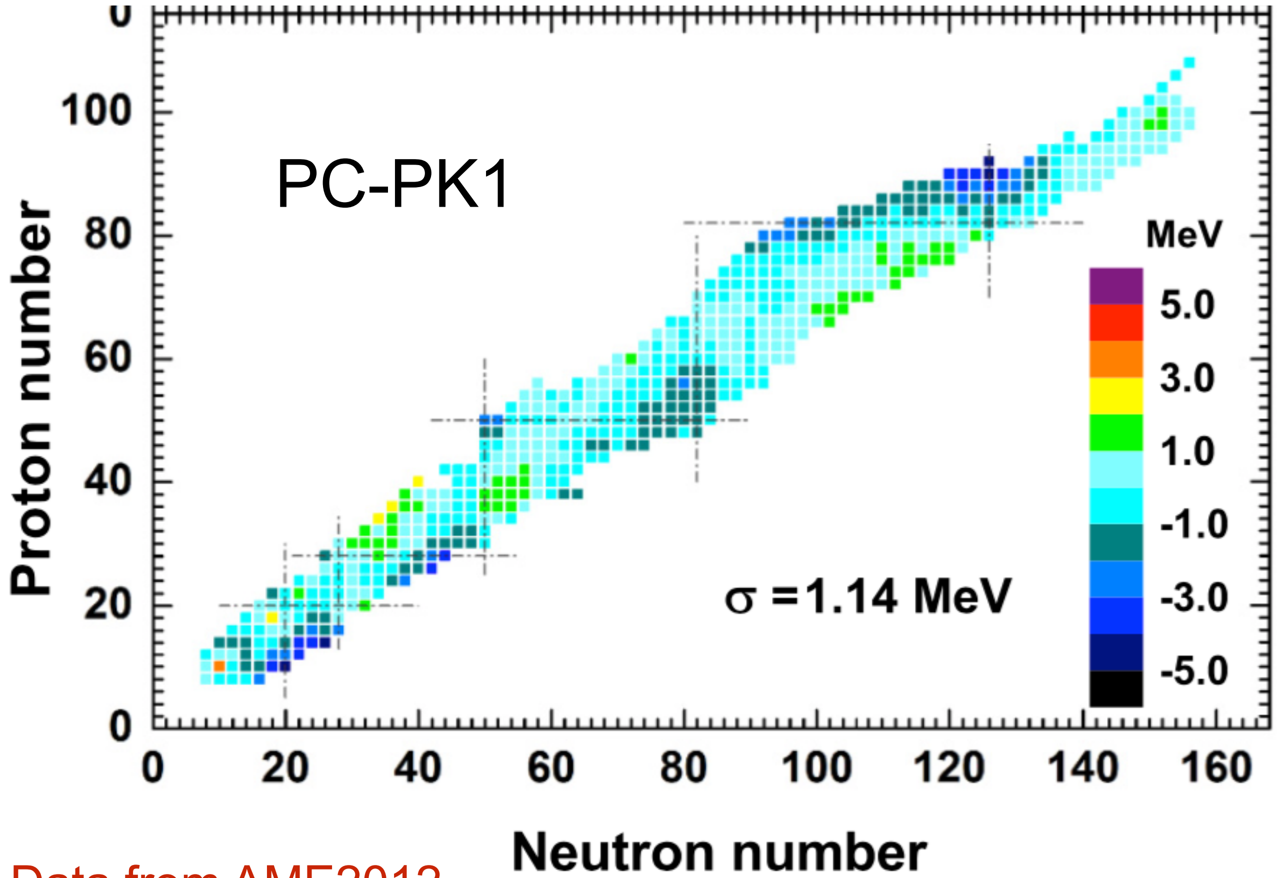
$$T[\rho] \doteq \sum_{i=1}^{N} \left\langle \varphi_i \left| -\frac{\hbar^2}{2m}\nabla^2 \right| \varphi_i \right\rangle$$

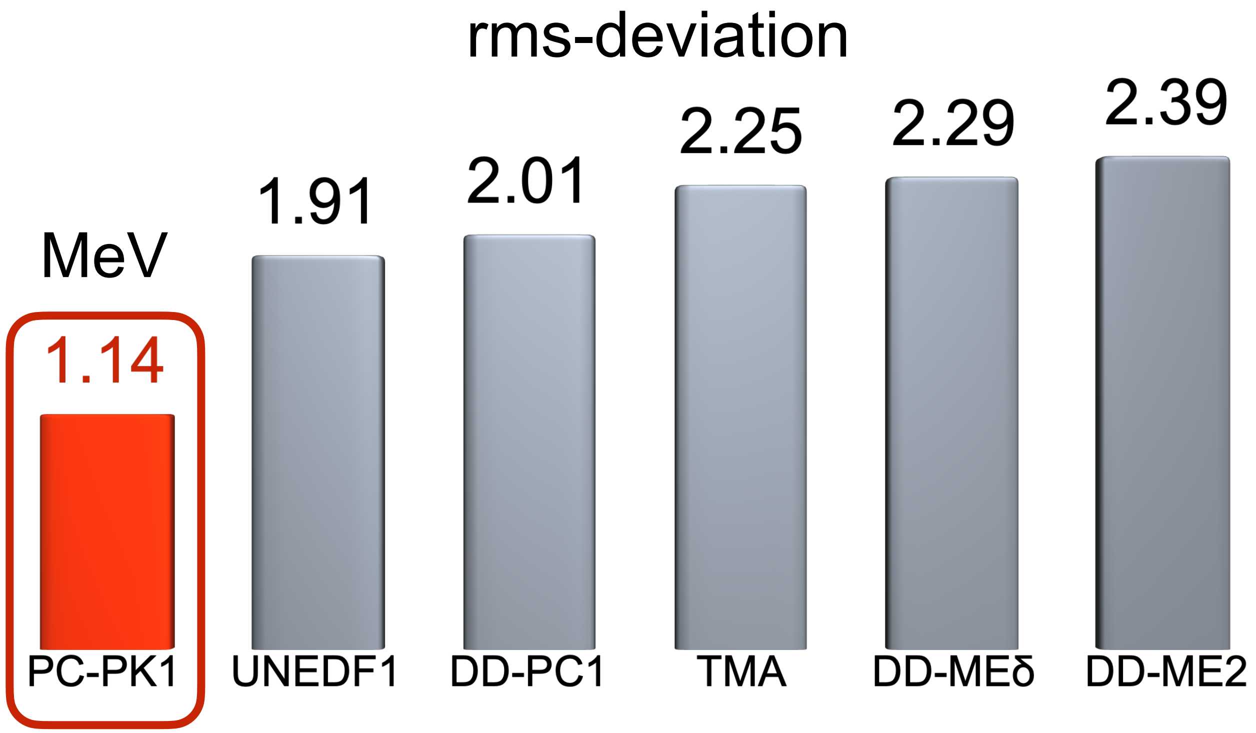
$$E[\rho] \Rightarrow \boxed{\hat{h} = \frac{\delta E}{\delta \rho}} \Rightarrow \boxed{\hat{h}\varphi_i = \varepsilon_i \varphi_i} \Rightarrow \boxed{\rho = \sum_{i=1}^{A} |\varphi_i|^2}$$

The practical usefulness of the Kohn-Sham theory depends entirely on whether an **Accurate Energy Density Functional** can be found!

Covariant density functional: PC-PK1

Mass Differences: M_{cal} - M_{exp}





Data from AME2012

PWZ, Li, Yao, Meng, PRC 82, 054319 (2010)

Lu, Li, Li, Yao, Meng, PRC 91, 027304 (2015)

http://nuclearmap.jcnp.org

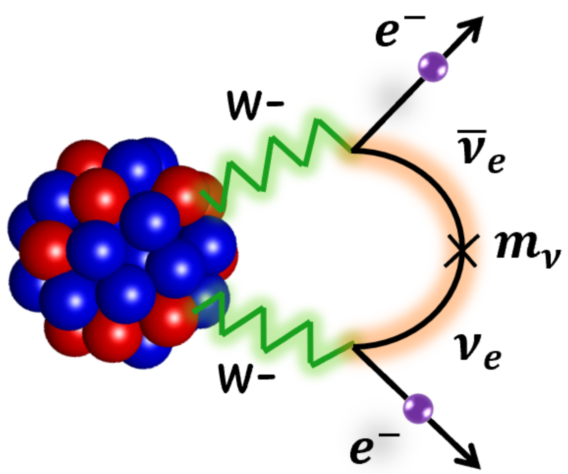
Yang, Wang, PWZ, Li, PRC 104, 054312 (2021)

Yang, PWZ, Li, PRC 107, 024308 (2023)

Among the best density-functional description for nuclear masses!

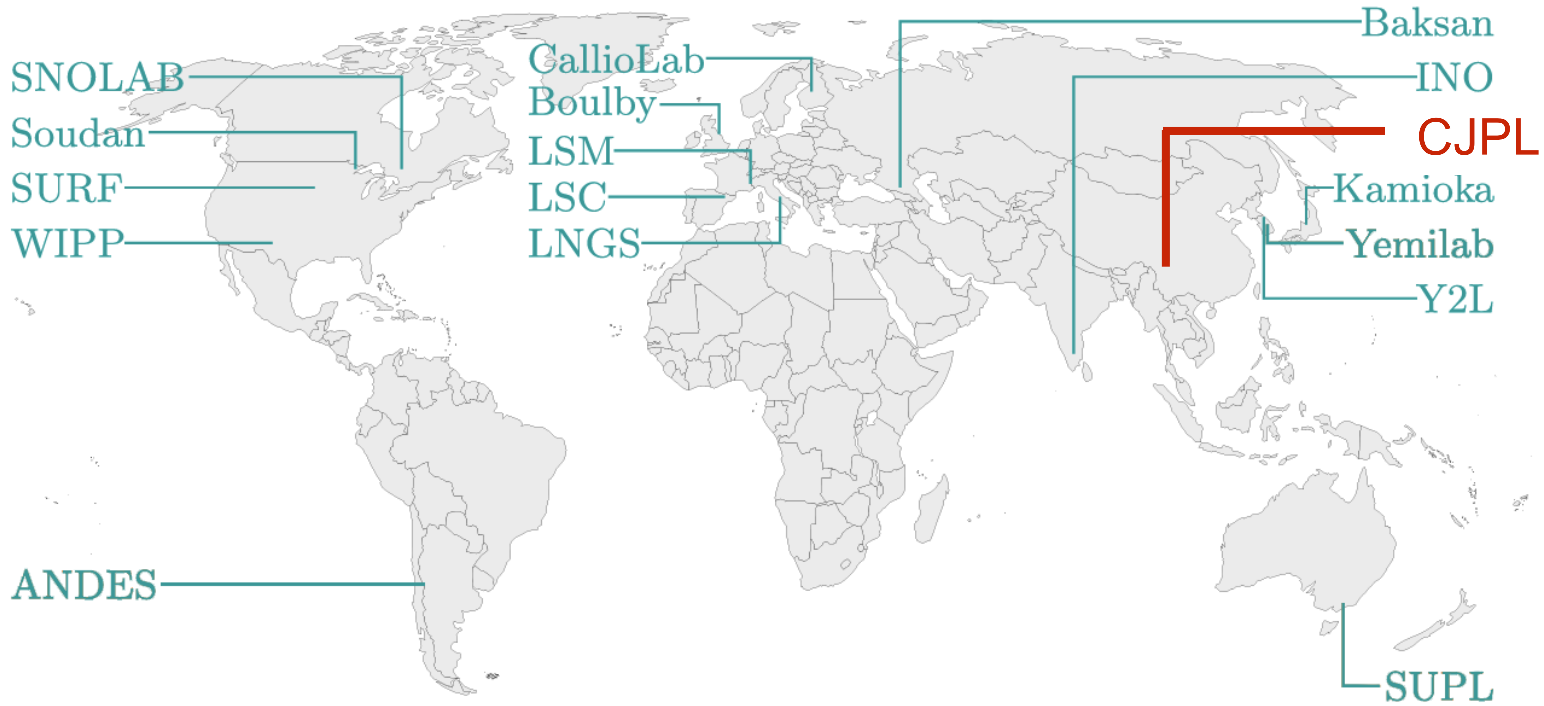
Neutrinoless double-beta decay

- Neutrinoless $\beta\beta$ decay ($0\nu\beta\beta$): $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$
  - ✓ Lepton number violation
  - ✓ Majorana nature of neutrinos
  - ✓ Neutrino mass scale and hierarchy
  - ✓ matter-antimatter asymmetry



Avignone, Elliott, Engel, Rev. Mod. Phys. 80, 481 (2008)

- $0\nu\beta\beta$ search in worldwide experimental facilities



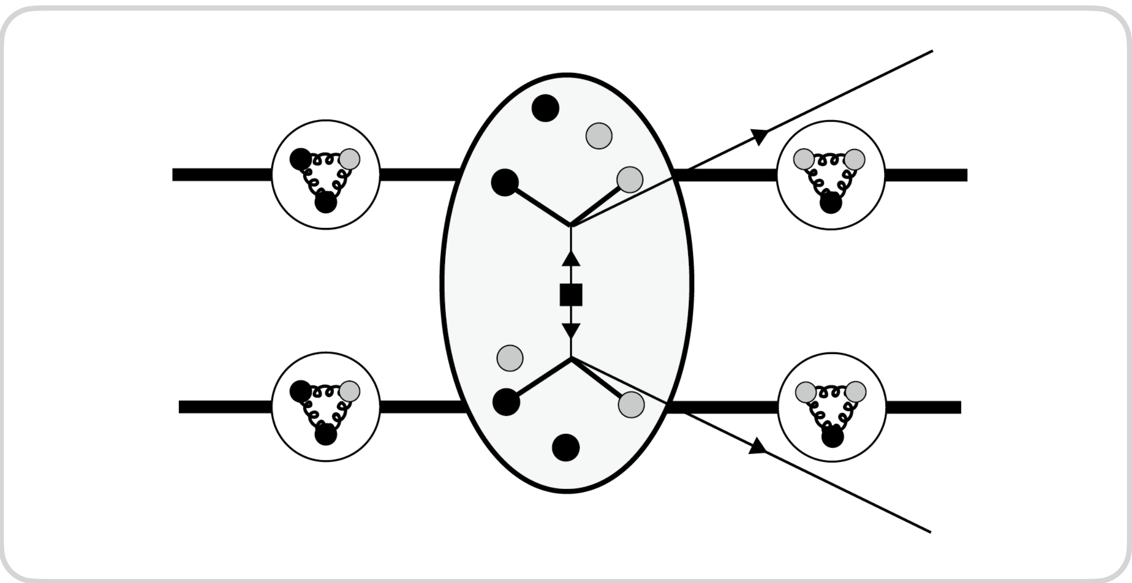
Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)

$0\nu\beta\beta$ decay occurs only if neutrinos are Majorana particles!

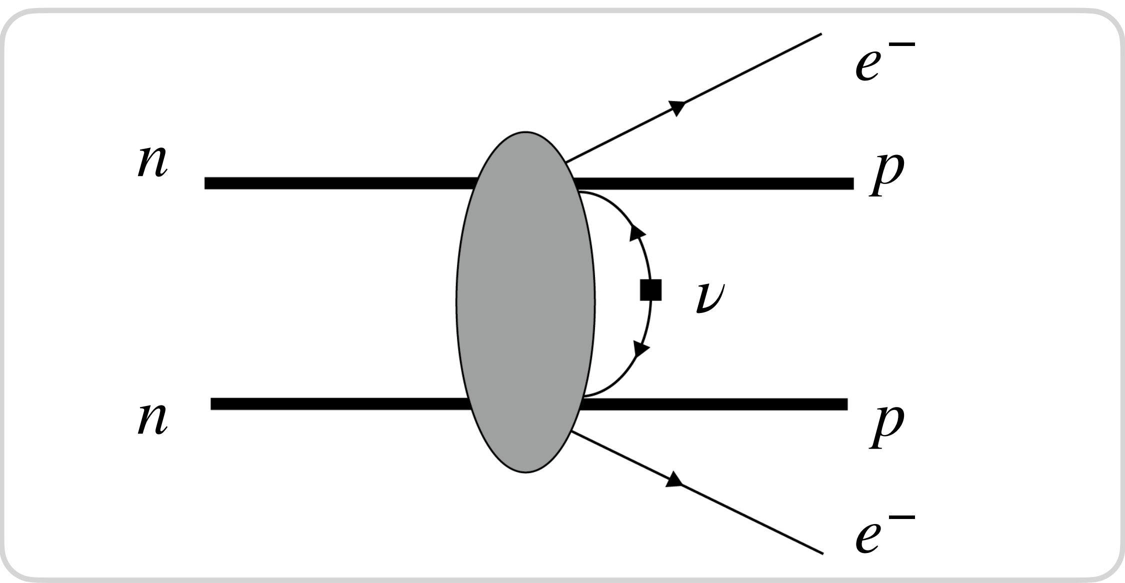
$0\nu\beta\beta$ decay operator

- Nucleons are the relevant degrees of freedom in nuclei.
- The hadronic decay operator needs to be derived from the quark-gluon level.

Quark-gluon level



Hadronic level



- Derivation of $0\nu\beta\beta$ decay operator

Standard mechanism

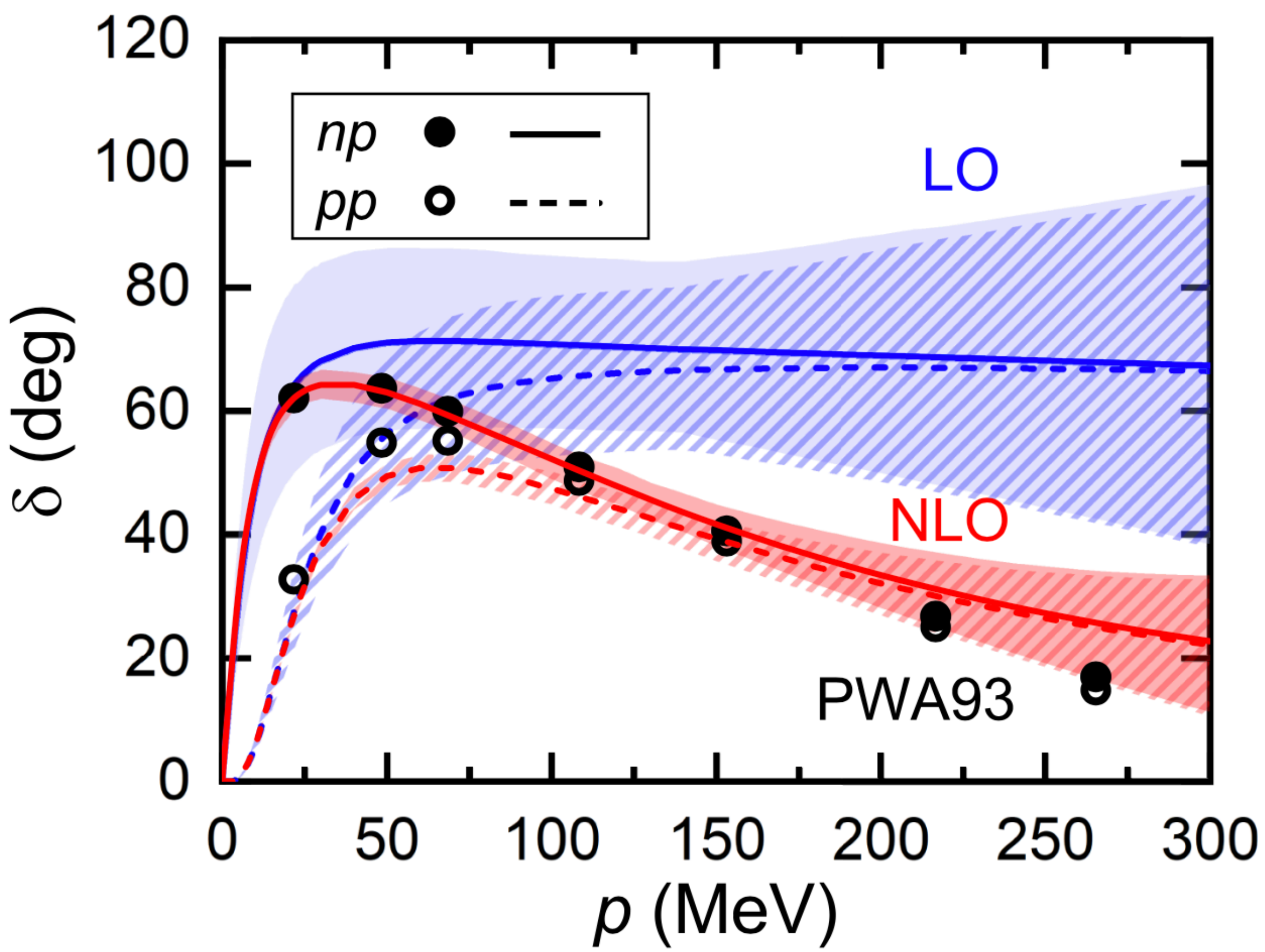
Avignone, Elliott, Engel,
Rev. Mod. Phys. 80, 481 (2008)
Cirigliano, Dekens, Mereghetti, Walker-Loud,
PRC 97, 065501 (2018)
…

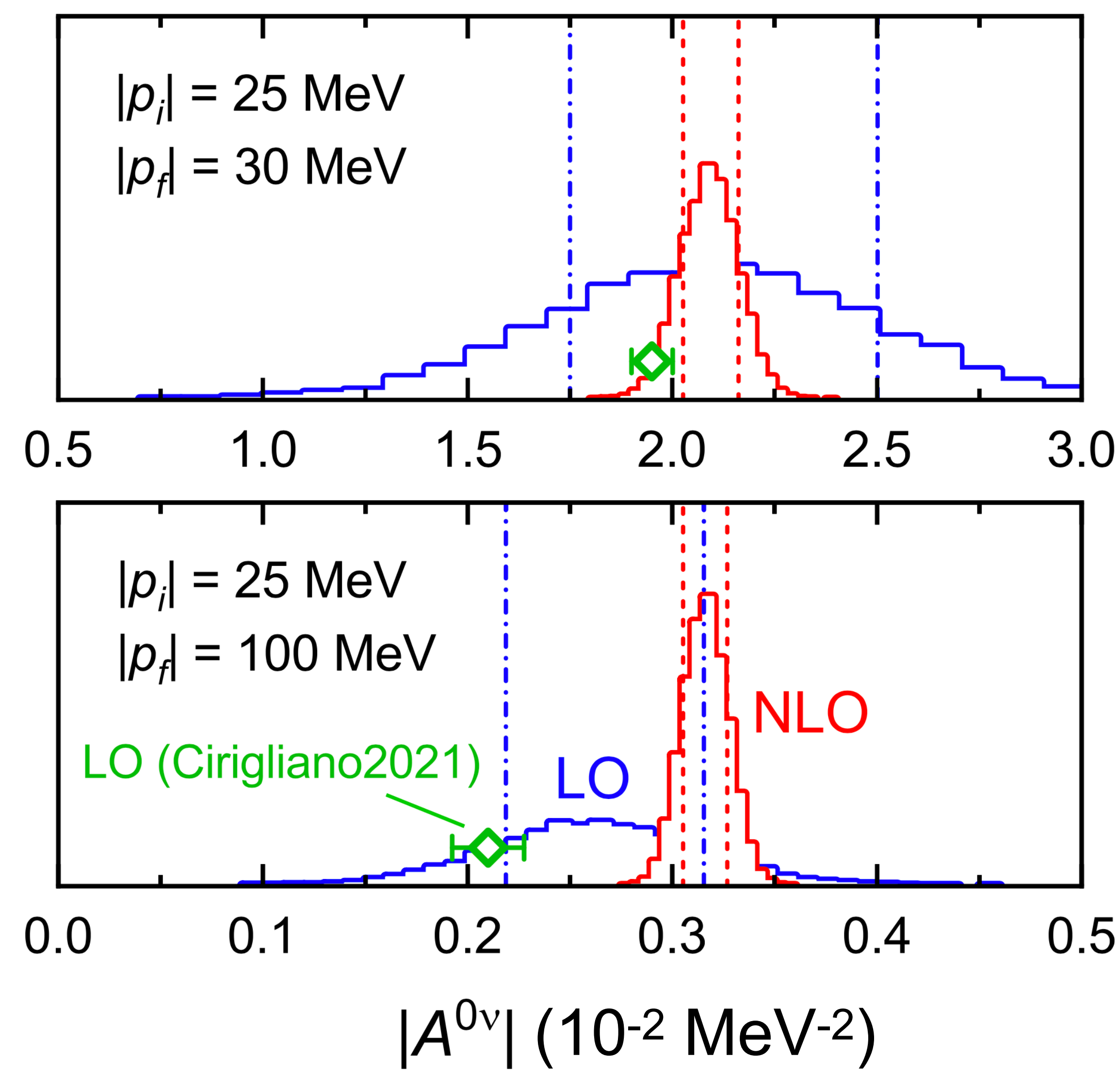
Non-standard mechanism

Cirigliano, Dekens, Mereghetti, Walker-Loud,
JHEP 12, 97 (2018)
Dekens, de Vries, Fuyuto, Mereghetii, Zhou,
JHEP 06, 97 (2020)
…

NLO prediction

The NLO (Q^1) effective-range corrections are included; significantly improves the description of phase shifts. The LECs are determined by a Bayesian approach.





Bayesian estimation of LECs in chiral EFT
Wesolowsk, Furnstahl, Melendez, and Phillips, JPG 46, 045102 (2019)

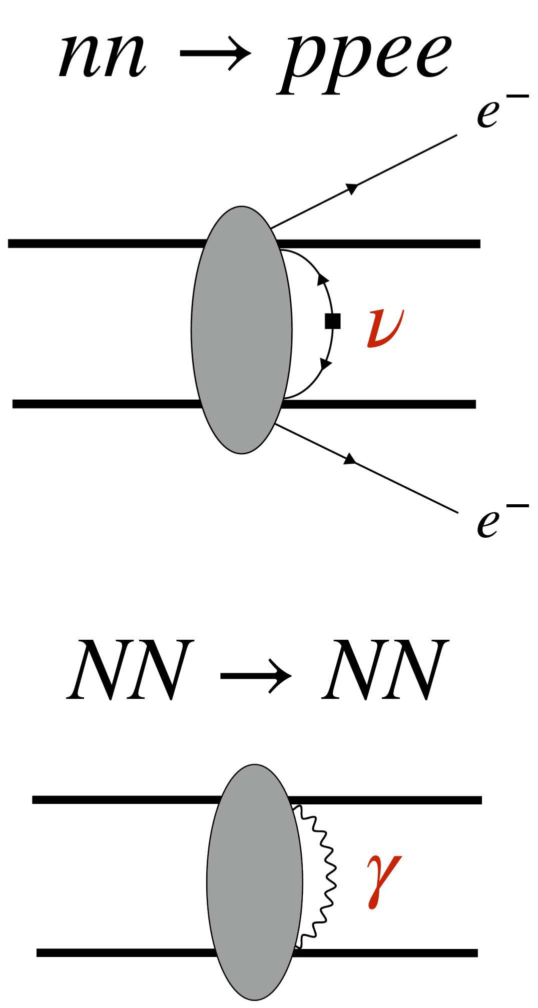
Yang and PWZ, PRL 134, 242502 (2025)

Validation

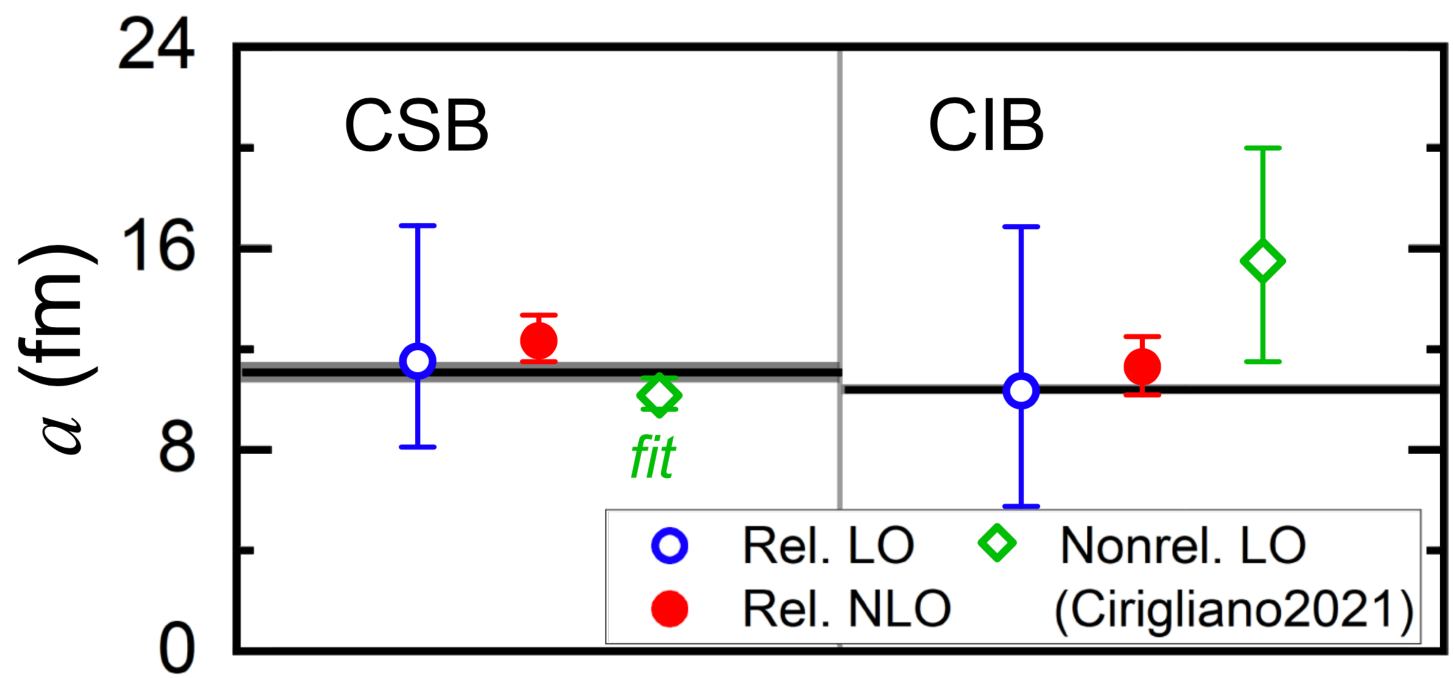
Replacing the neutrino exchange to γ exchange, the relativistic chiral EFT reproduces the CSB and CIB contributions to the *NN* scattering amplitude.

Prediction with NO free parameters.

Yang and PWZ, PRL 134, 242502 (2025)



Charge-Symmetry/
Independence Breaking



$$a_{\rm CSB} = a_{pp} - a_{nn} \qquad a_{\rm CIB} = \frac{a_{nn} + a_{pp}}{2} - a_{np}$$