



Institute of Theoretical Physics
Chinese Academy of Sciences

有效场论观点下的中微子 $0\nu\beta\beta$

EFT Perspective on Neutrino $0\nu\beta\beta$

于江浩 (Jiang-Hao Yu)

中国科学院理论物理研究所 (Institute of Theoretical Physics, CAS)

第一届中微子、原子核物理与新物理研讨会

2025年7月24日@兰州

Outline

- **EFT description for neutrino $0\nu\beta\beta$**

- **Quark level EFTs: SMEFT and LEFT**

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2404.15047]

- **Nucleon level EFTs: ChPT and ChEFT**

[Xuan-He Li, Hao Sun, Feng-Jie Tang, **J.H.Yu**, 2404.14152]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2501.14018]

- **UV completion of neutrino masses and $0\nu\beta\beta$**

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2501.09787]

[Gang Li, Chuan-Qiang Song, **J.H.Yu**, 2507.02538]

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in preparation]

[Xu-Xiang Li, Zhe Ren, **J.H.Yu**, 2307.10380]

- **Summary**

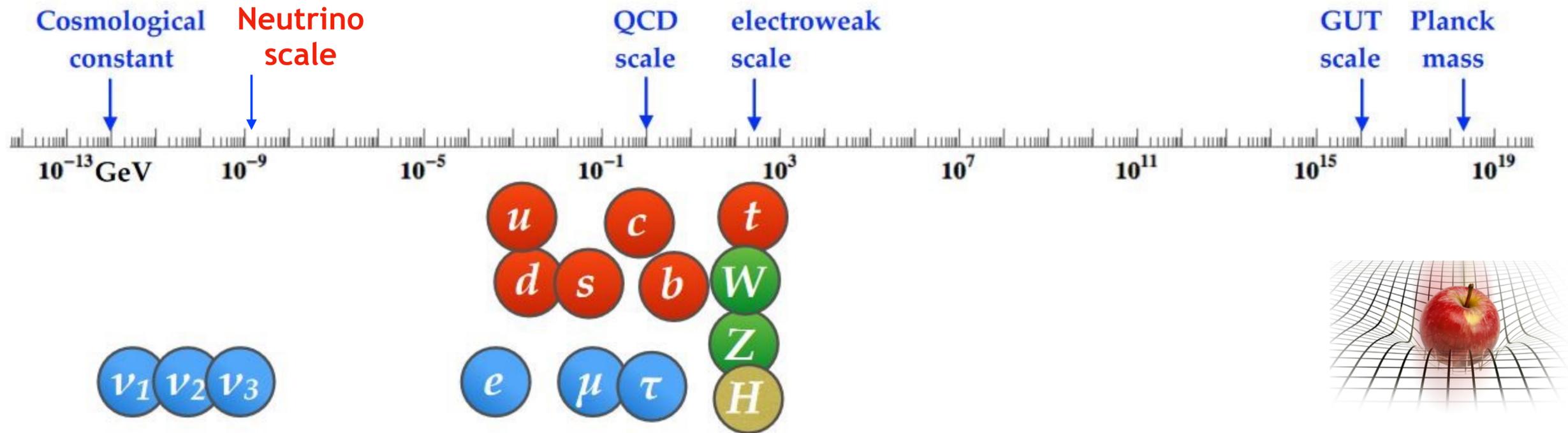
[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, in preparation]

[Gang Li, **J.H.Yu**, Xiang Zhao, 2311.1

EFT description for neutrino $0\nu\beta\beta$

Origin of Neutrino Mass

The existence of neutrino masses is the first evidence of new physics beyond Standard Model



Why neutrino masses so tiny?

Why Higgs mass so light?

Elucidate the Mysteries of Neutrinos
Reveal the Secrets of the Higgs Boson



Decipher the Quantum Realm

Dirac vs Majorana Neutrino

Simplest way to give neutrino masses is introducing the right-handed neutrino

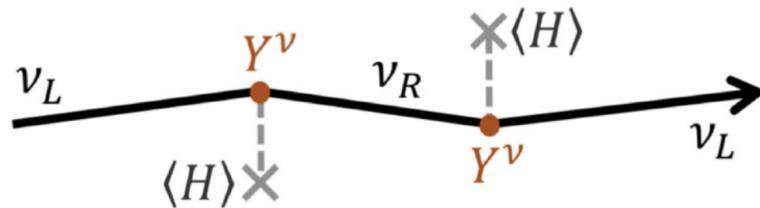
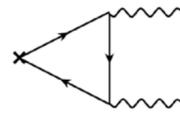
$$\mathcal{L}_m = \underbrace{m_D \bar{\psi}_R \psi_L}_{\text{Dirac term}} + \underbrace{\frac{1}{2} m_L \bar{\psi}_L^c \psi_L + \frac{1}{2} m_R \bar{\psi}_R \psi_R^c}_{\text{Majorana terms}} + \text{h.c.}$$

$$\mathbf{M} = \begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}$$

How to forbid the Majorana term?

Lepton number conservation



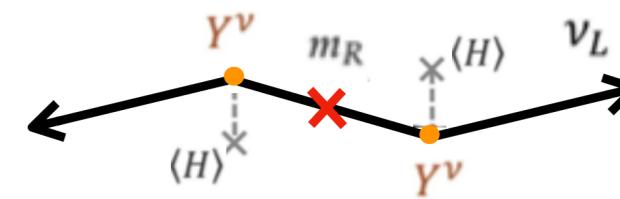
$$m_\nu = Y^\nu v_{EW}$$

Tiny Yukawa coupling

$$\frac{m_\nu}{v_{EW}} \leq 10^{-12}$$

The Majorana term should be there

Lepton number violation



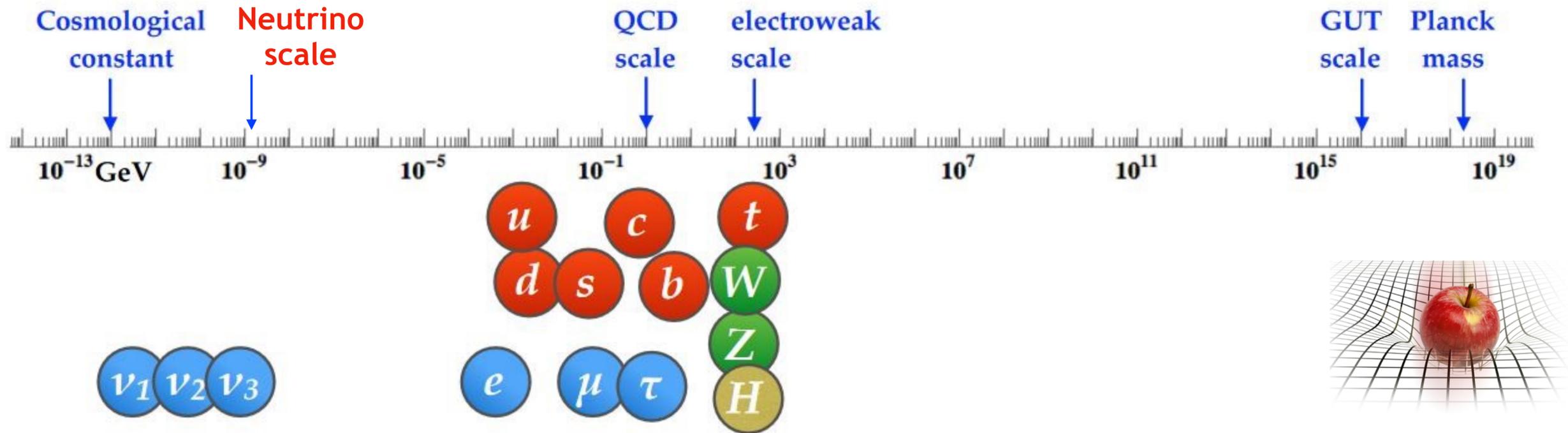
$$m_\nu = \frac{(Y^\nu v_{EW})^2}{m_R}$$

Yukawa coupling not small, but m_R heavy

[See Zhi-Zhong Xing's talk]

Majorana Neutrino: Origin of Matter

Majorana neutrino also explains matter-antimatter asymmetry



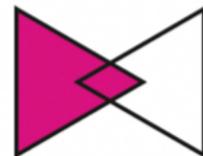
Why neutrino masses so tiny?

Leptogenesis

Why Higgs mass so light?

Electroweak baryogenesis

Elucidate the Mysteries of Neutrinos
Reveal the Secrets of the Higgs Boson

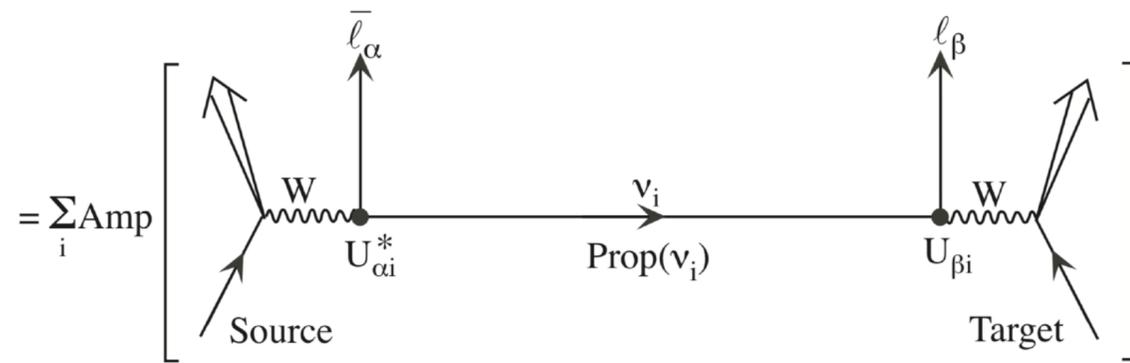
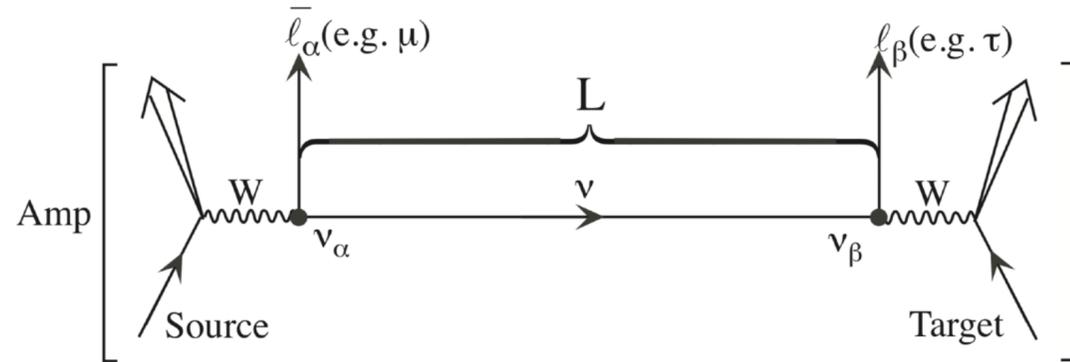


Decipher the Quantum Realm

[See Ramsey-Musolf's talk]

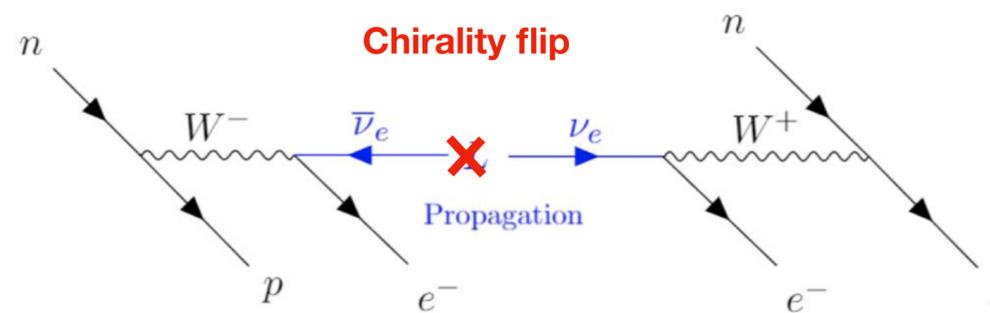
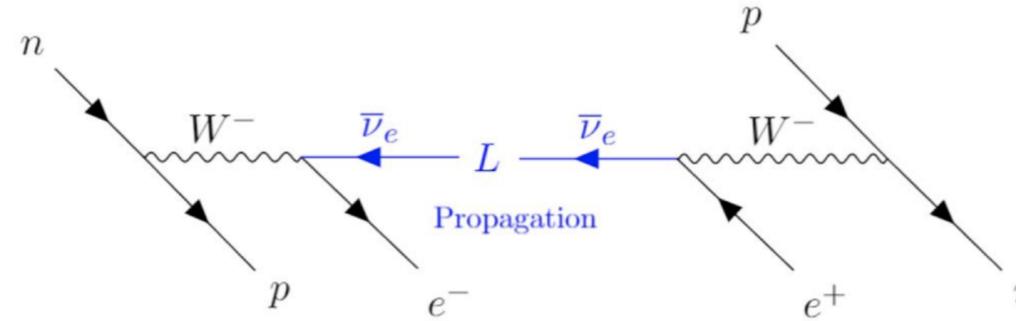
Nature of Majorana Neutrino

How to distinguish Dirac vs Majorana neutrino?



$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_{i=1}^3 U_{\alpha i}^* e^{-i\frac{m_i^2}{2E}L} U_{\beta i}$$

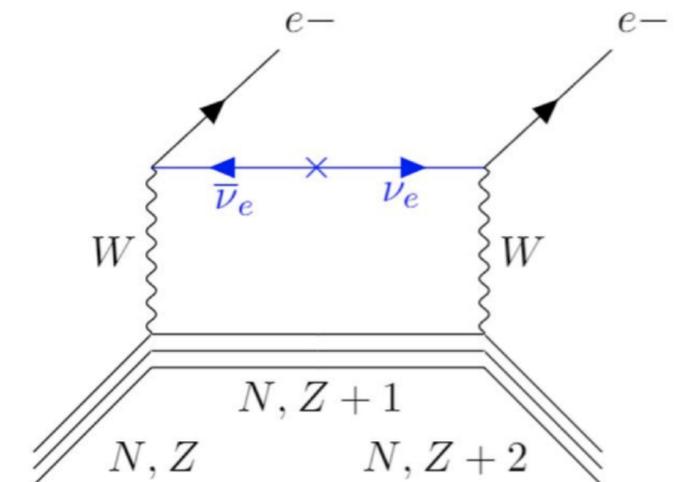
Neutrino Oscillation experiments



$$\mathcal{M} = \mathcal{A} \sum_i (U_{ei})^2 m_{\nu_i} = \mathcal{A} m_{\beta\beta}$$

$$m_{\beta\beta} = c_{12}^2 c_{13}^2 m_1 e^{2i\lambda_a} + s_{12}^2 c_{13}^2 e^{2i\lambda_b} \sqrt{m_1^2 + \Delta m_{12}^2} + s_{13}^2 \sqrt{m_1^2 \pm |\Delta m_{23}^2|}$$

suppressed
by a factor m_ν^2/E^2
0νββ exp.

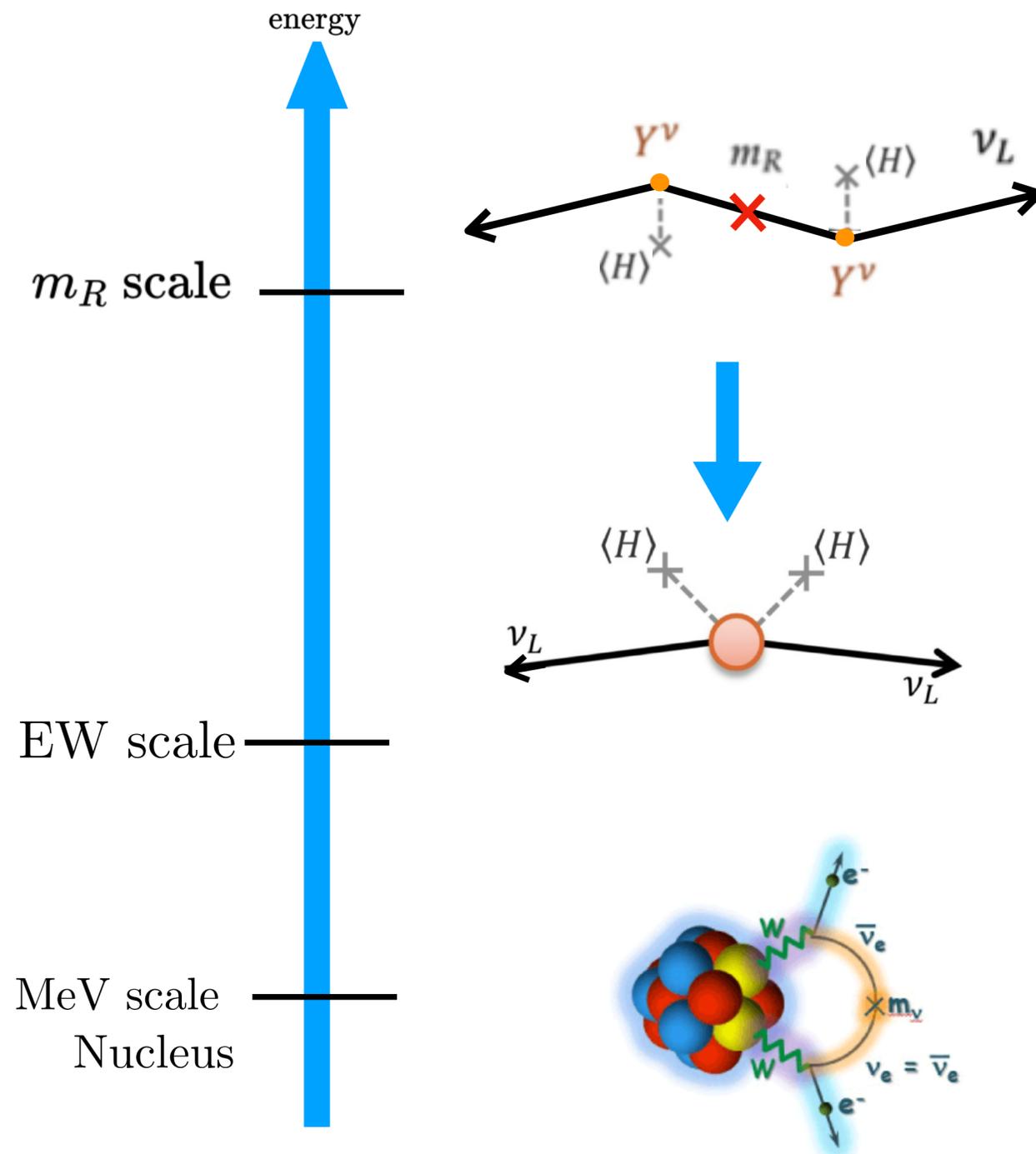


Neutrinoless Double Beta Decay

Low energy probe of high energy new physics



[Weinberg, 1979]



Weinberg operator

$$\mathcal{L}_{\text{Weinberg}} = \frac{c_{ij}}{\Lambda} (L_i H)(L_j H) + \text{h.c.} \dots$$

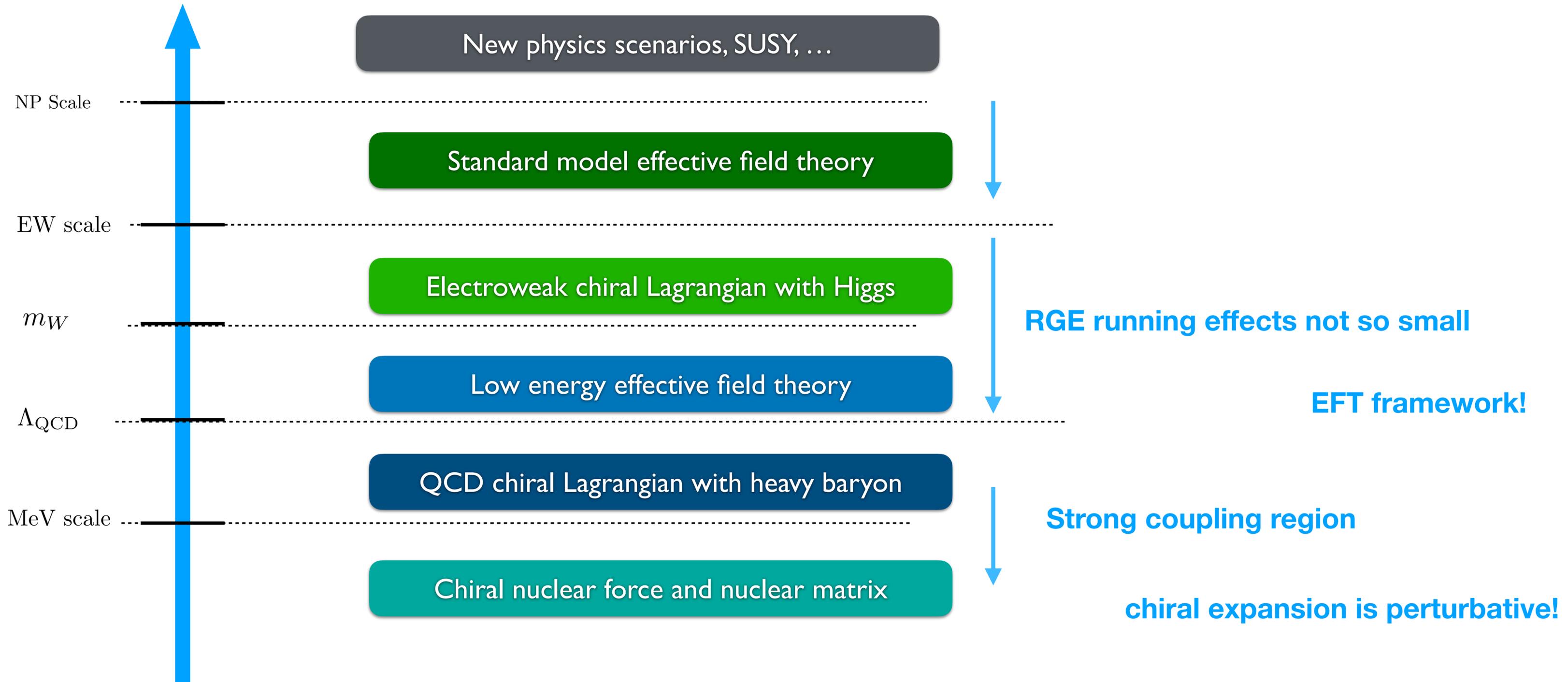
$$\rightarrow c_{ij} \frac{v^2}{\Lambda} \nu_i \nu_j + \text{h.c.}$$

... the effective field theory point view had predicted the neutrino masses

[Weinberg, 2021]

Tower of effective field theories

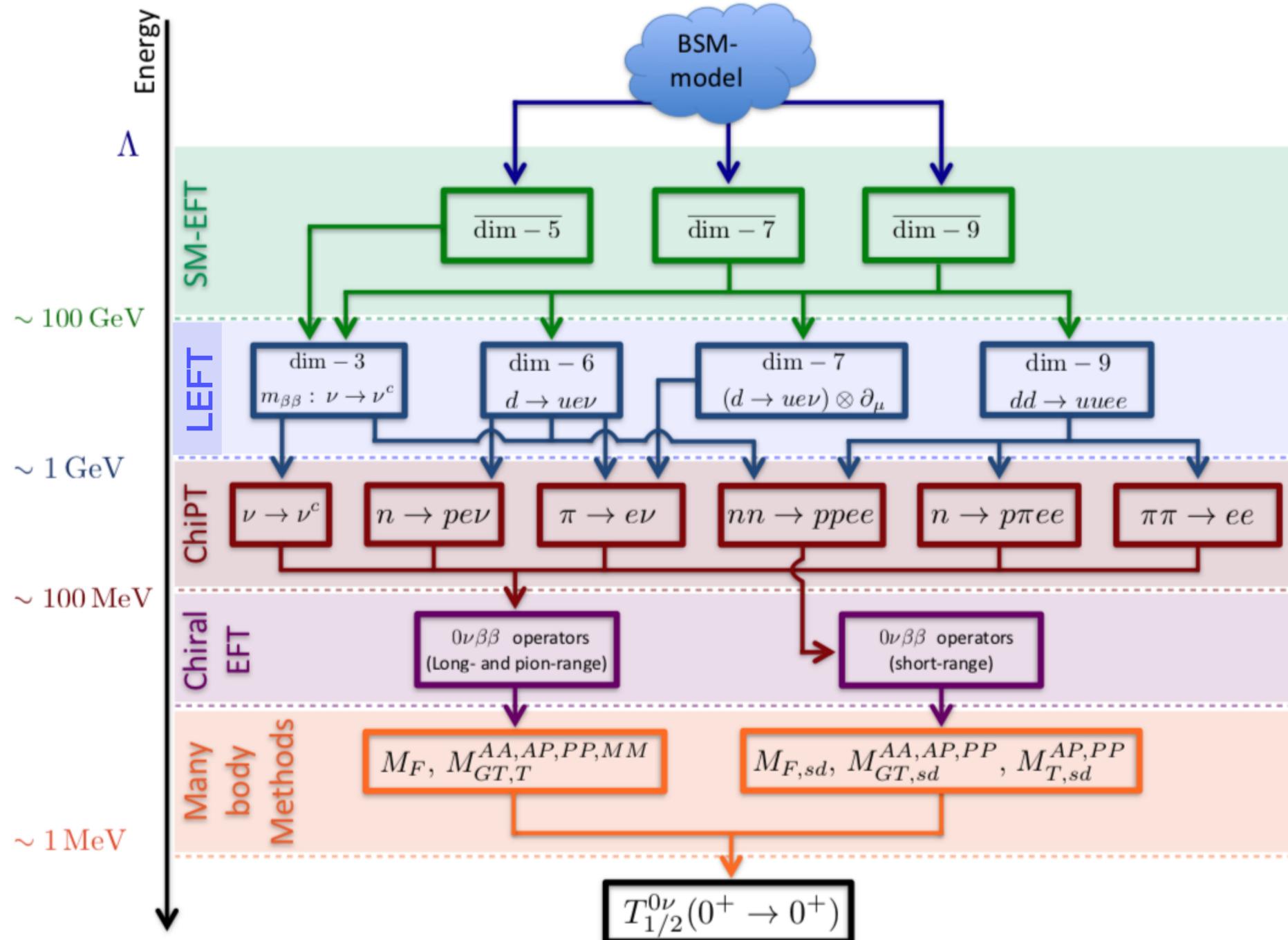
Why EFT framework? it is natural to consider different EFTs to avoid large Logs



Complete EFT Framework for $0\nu\beta\beta$

Operator bases for different EFTs are necessarily needed to provide most general description

[Cirigliano, Dekens, de Vries, Graesser, Mereghetti, 2018]

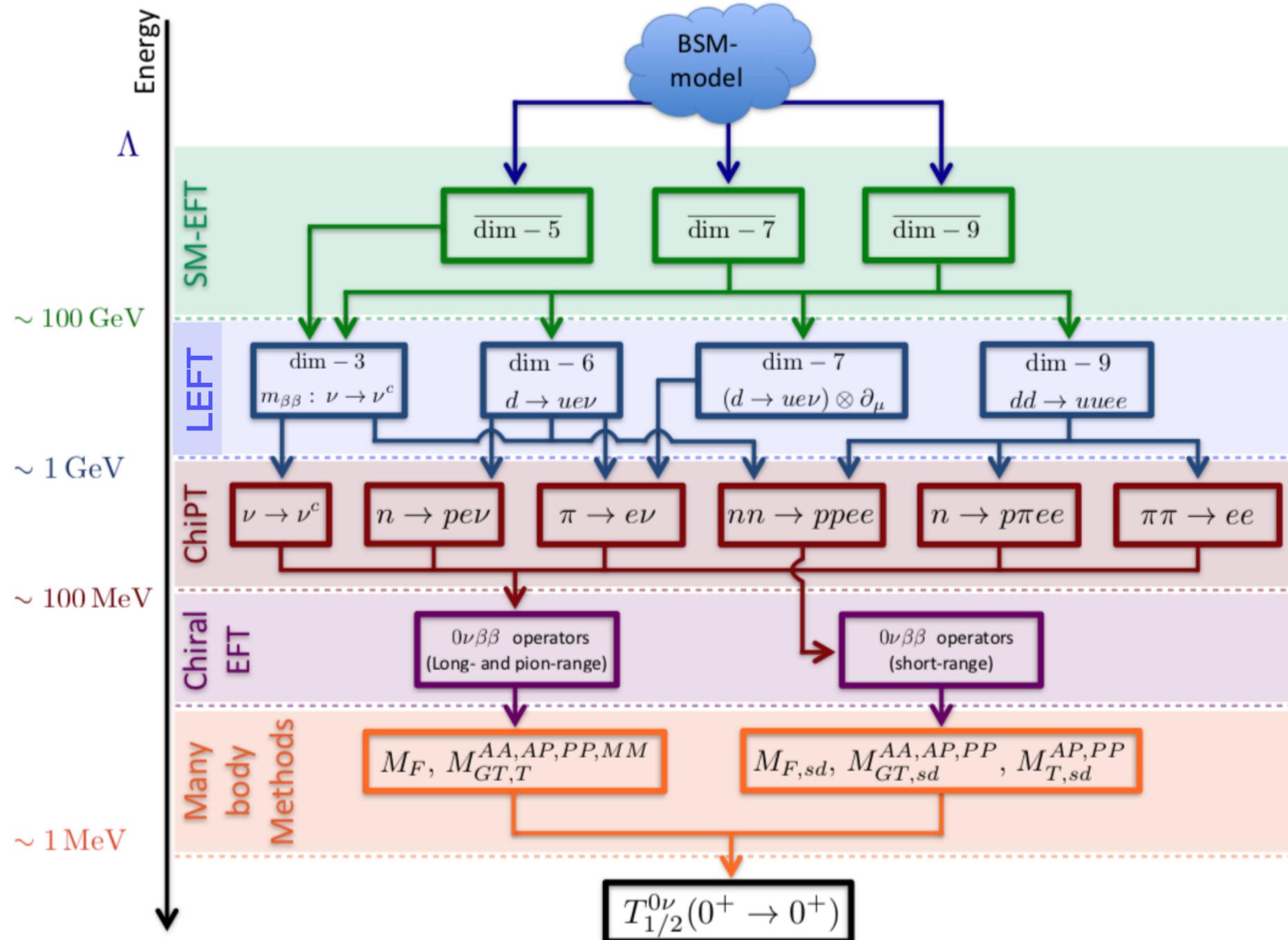


[See Shun Zhou's talk]

Complete EFT Framework for $0\nu\beta\beta$

Operator bases for different EFTs are necessarily needed to provide most general description

[Cirigliano, Dekens, de Vries, Graesser, Mereghetti, 2018]



UV models new particles

[See Shun Zhou and Jingyu Zhu's talk]

Quark level operators

[See Gang Li's talk]

Hadronic level operators

[See Bingwei Long and Tai-Xing Liu's talk]

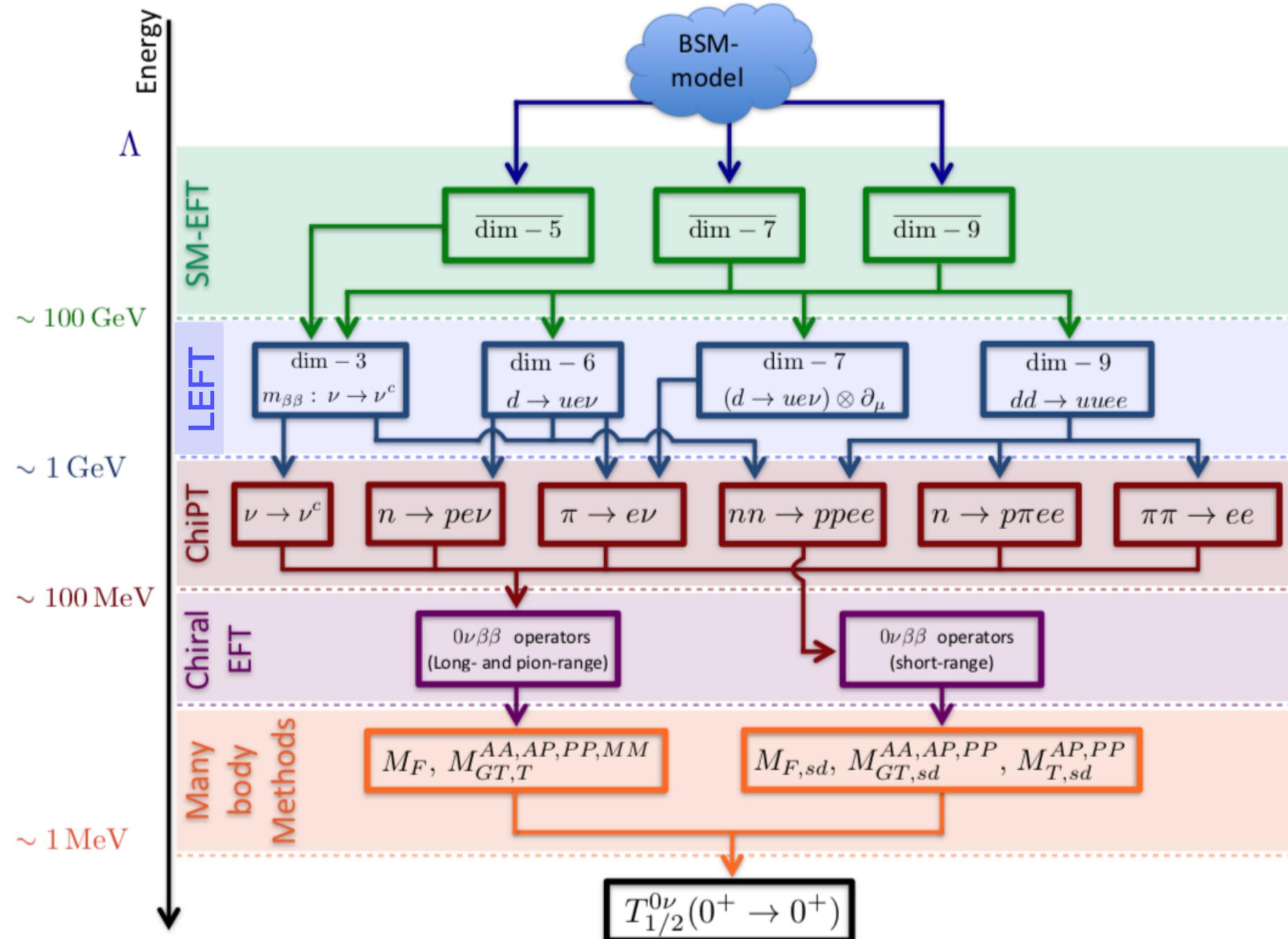
Nuclear level operators

[See Pengwei Zhao and Chun-Lin Bai's talk]

Complete EFT Framework for $0\nu\beta\beta$

Operator bases for different EFTs are necessarily needed to provide most general description

[Cirigliano, Dekens, de Vries, Graesser, Mereghetti, 2018]



UV models new particles

Repeat for each model

Quark level operators

Operator correlation

Hadronic level operators

External source not enough

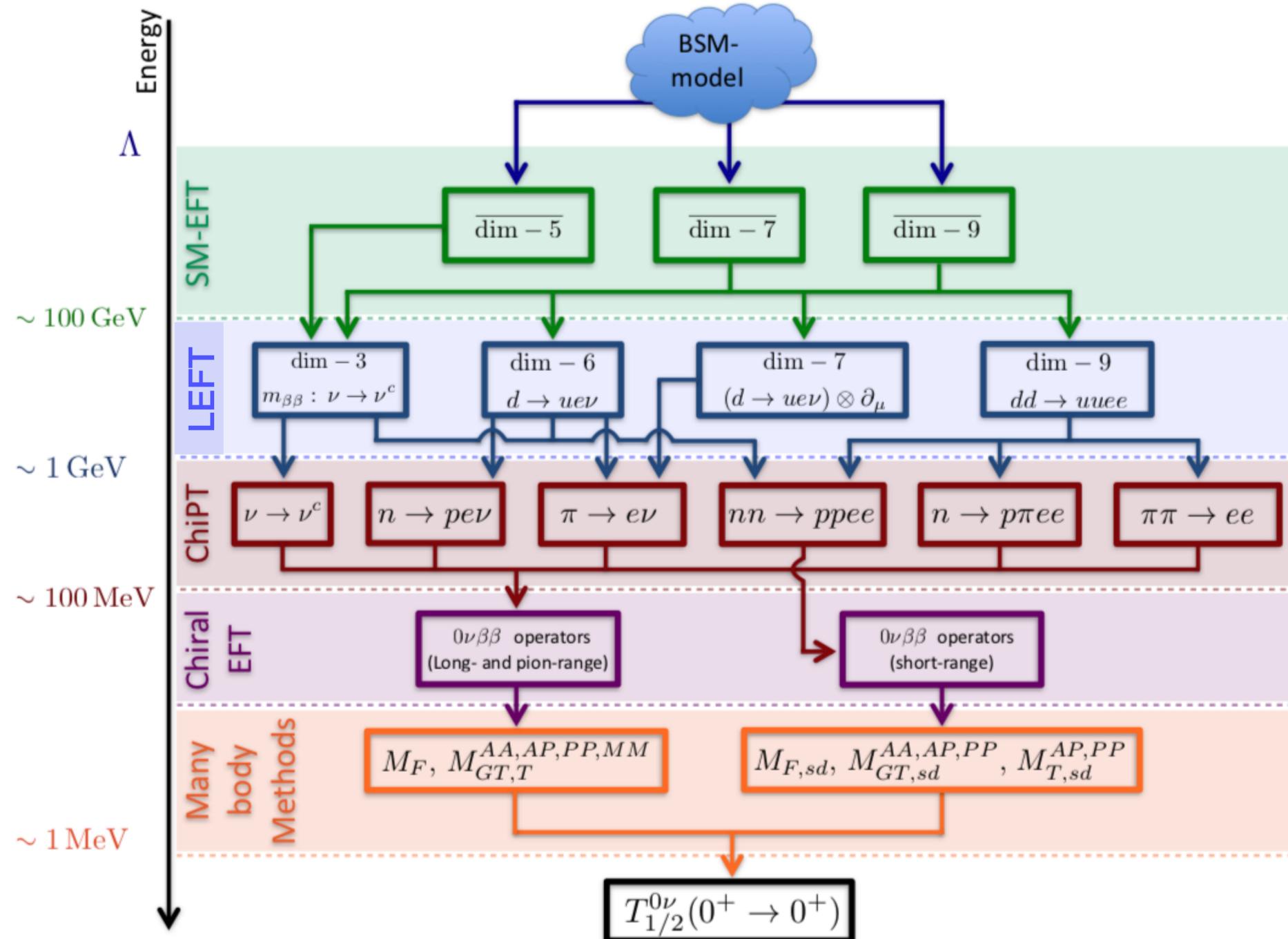
Nuclear level operators

Nucleon matrix element incomplete

Complete EFT Framework for $0\nu\beta\beta$

Operator bases for different EFTs are necessarily needed to provide most general description

[Cirigliano, Dekens, de Vries, Graesser, Mereghetti, 2018]



UV models new particles

Quark level operators

Hadronic level operators

Nuclear level operators

Setup a general framework

Need complete operators at each level

Quark level EFTs: SMEFT and LEFT

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2201.04639]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2012.09188]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2007.07899]

[Hao-Lin Li, Jing Shu, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2005.00008]

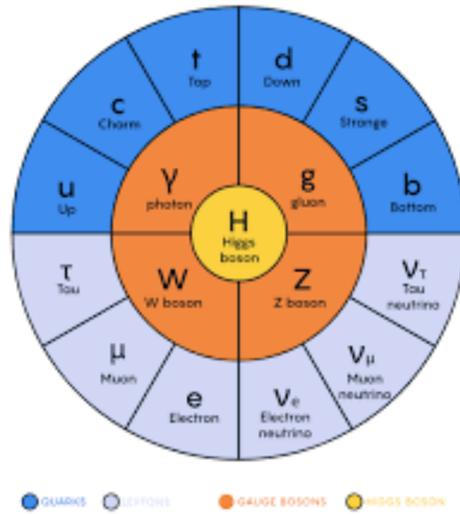
Standard Model Effective Field Theory

Standard model is viewed as the leading renormalizable terms of most general effective field theory

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\mathcal{D} \leq 4} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \frac{\mathcal{L}_7}{\Lambda^3} + \frac{\mathcal{L}_8}{\Lambda^4} + \frac{\mathcal{L}_9}{\Lambda^5} + \dots \quad [\text{Weinberg, 1980}]$$

Standard Model Weinberg Operator

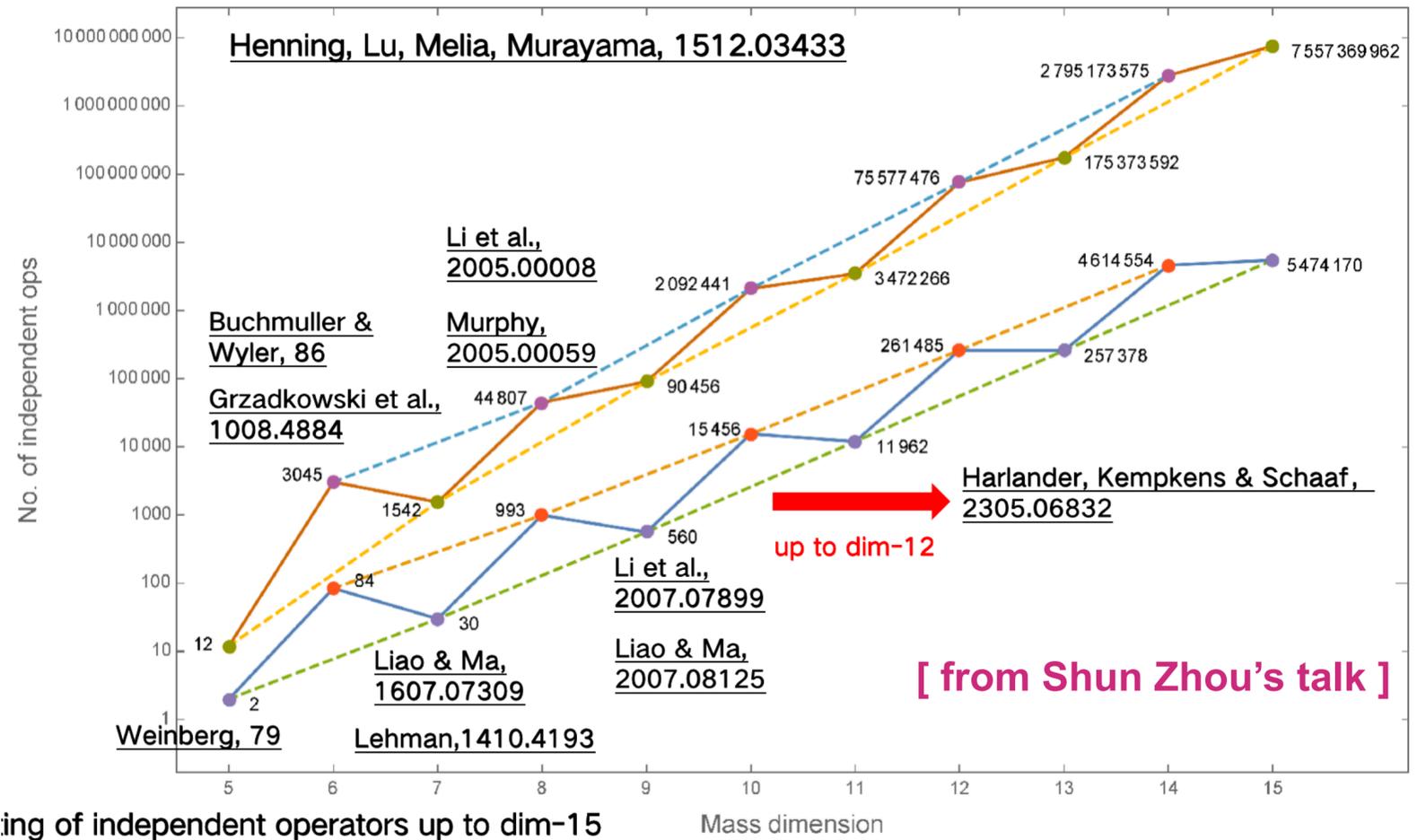
$$\frac{c_{ij}}{\Lambda} (L_i H)(L_j H) + \text{h.c.}$$



17 elementary particles

$$\begin{aligned} \mathcal{L}_{SM} = & \underbrace{\frac{1}{4}W_{\mu\nu} \cdot W^{\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}G_{\mu\nu}^{\alpha} G^{\mu\nu}_{\alpha}}_{\text{kinetic energies and self-interactions of the gauge bosons}} \\ & + \underbrace{\bar{L}\gamma^{\mu} \left(i\partial_{\mu} - \frac{1}{2}g\tau \cdot W_{\mu} - \frac{1}{2}g'YB_{\mu} \right) L + \bar{R}\gamma^{\mu} \left(i\partial_{\mu} - \frac{1}{2}g'YB_{\mu} \right) R}_{\text{kinetic energies and electroweak interactions of fermions}} \\ & + \frac{1}{2} \left[\left(i\partial_{\mu} - \frac{1}{2}g\tau \cdot W_{\mu} - \frac{1}{2}g'YB_{\mu} \right) \phi \right]^2 - V(\phi) \\ & + \underbrace{W^{\pm}, Z, \gamma \text{ and Higgs masses and couplings}}_{\text{}} \\ & + \underbrace{g^{\alpha} (\bar{q}\gamma^{\mu} T_{\alpha} q) G_{\mu}^{\alpha}}_{\text{interactions between quarks and gluons}} + \underbrace{(G_1 \bar{L}\phi R + G_2 \bar{L}\phi_c R + \text{h.c.})}_{\text{fermion masses and couplings to Higgs}} \end{aligned}$$

19 parameters, all measured but theta term



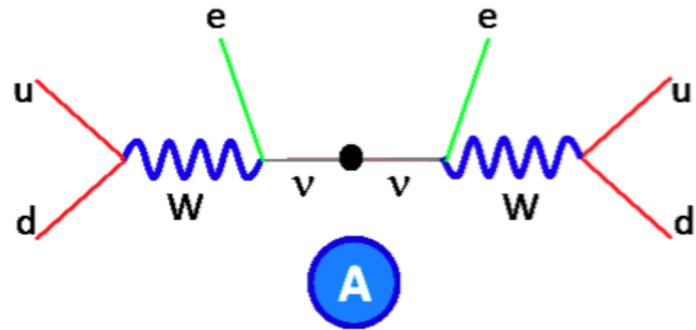
up to dim-12

[from Shun Zhou's talk]

Standard Model Effective Field Theory (SMEFT) provides systematic parameterization of all possible Lorentz-inv. new physics

Operators for 0vbb

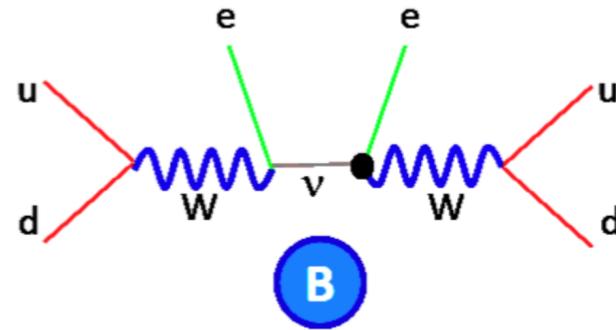
SMEFT dim-5,7,9 operators provides most general parametrization of new physics for 0vbb



Dim-5, 7

$$\mathcal{O}_5 \quad \epsilon^{ik} \epsilon^{jl} (\ell_i^T C \ell_j) H_k H_l$$

$$\mathcal{O}_{LH} \quad \epsilon^{ik} \epsilon^{jl} (\ell_i^T C \ell_j) H_k H_l (H^\dagger H)$$



Dim-7

$$\mathcal{O}_{LeHD} \quad \epsilon^{ij} \epsilon^{kl} (\ell_i^T C \gamma^\mu e) H_j H_k (i D_\mu H_l)$$

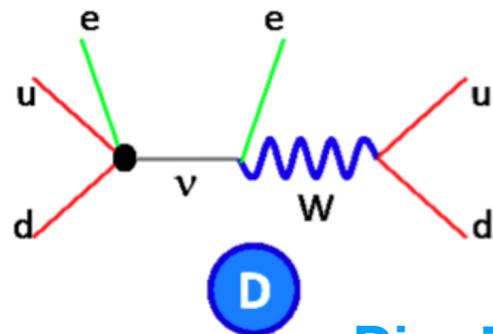
$$\mathcal{O}_{LHW} \quad -\epsilon^{ik} (\epsilon \tau^I)^{jl} (\ell_i^T C i \sigma^{\mu\nu} \ell_j) H_k H_l W_{\mu\nu}^I$$

Dim-7, 9

$$\mathcal{O}_{LHD1} \quad \epsilon^{ij} \epsilon^{kl} (\ell_i^T C D_\mu \ell_j) (H_k D^\nu H_l)$$

$$D^2 H^\dagger L^2$$

$$D^2 H^\dagger L^2 WL$$



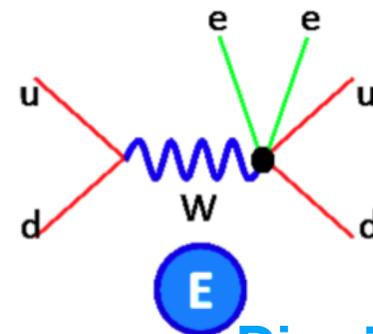
Dim-7

$$\mathcal{O}_{dLQLH1} \quad \epsilon^{ij} \epsilon^{kl} (\bar{d}^a \ell_i) (q_{aj}^T C \ell_k) H_l$$

$$\mathcal{O}_{dLQLH2} \quad \epsilon^{ik} \epsilon^{jl} (\bar{d}^a \ell_i) (q_{aj}^T C \ell_k) H_l$$

$$\mathcal{O}_{dLueH} \quad \epsilon^{ij} (\bar{d}^a \ell_i) (u_a^T C e) H_j$$

$$\mathcal{O}_{QuLLH} \quad \epsilon^{ij} (\bar{q}^{ak} u_a) (\ell_k^T C \ell_i) H_j$$

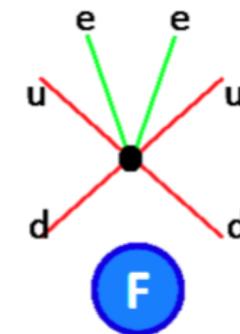


Dim-7, 9

$$\mathcal{O}_{duLLD} \quad \epsilon^{ij} (\bar{d}^a \gamma^\mu u_a) (\ell_i^T C i D_\mu \ell_j)$$

$$D \text{ dc}^\dagger L^\dagger \text{ uc}$$

$$\text{dc}^\dagger \text{ ec H}^\dagger L^\dagger \text{ uc WL}$$



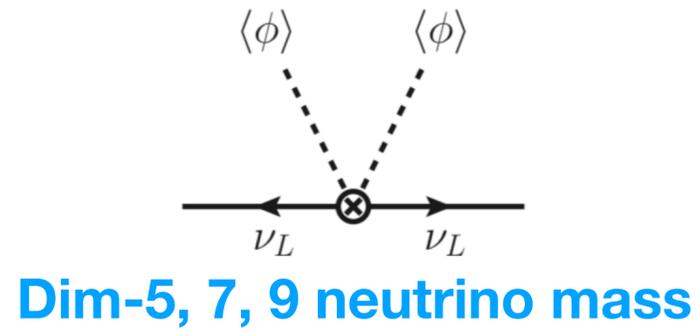
Dim-9

$$\text{dc}^2 L^2 Q^2, \text{dc}^2 \text{dc}^\dagger L^2 \text{uc}^\dagger, \text{dc} L^2 \text{uc uc}^\dagger, \text{dc}^2 \text{ec}^\dagger L Q \text{uc}^\dagger,$$

$$\text{dc}^\dagger \text{ec}^2 \text{uc}^2, \text{dc} L^2 Q Q^\dagger \text{uc}^\dagger, \text{dc}^\dagger \text{ec} L^\dagger Q \text{uc}^2, L^\dagger L^2 Q^2 \text{uc}^2$$

Operator Correlation for ν mass/ $0\nu\beta\beta$

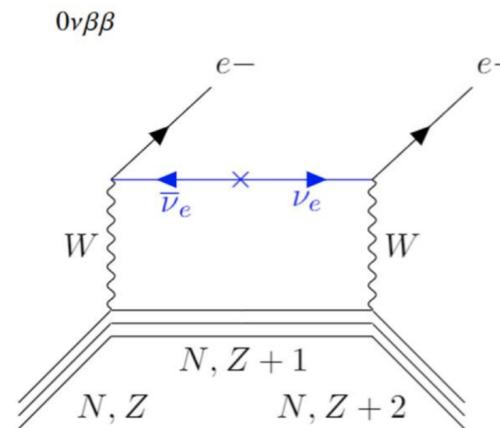
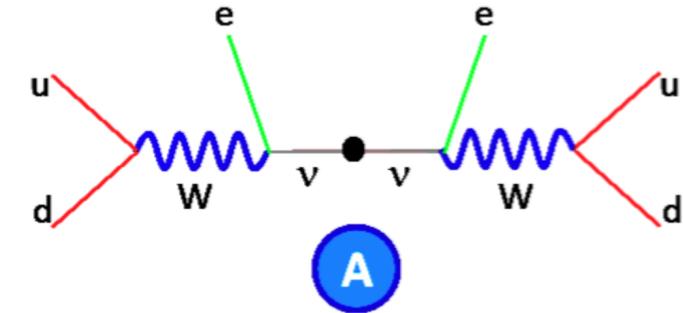
SMEFT dim-5,7,9 operators provides most general parametrization of new physics for neutrino masses and $0\nu\beta\beta$



$$\mathcal{O}_5 \quad \epsilon^{ik} \epsilon^{jl} (l_i^T C l_j) H_k H_l$$

$$\mathcal{O}_{LH} \quad \epsilon^{ik} \epsilon^{jl} (l_i^T C l_j) H_k H_l (H^\dagger H)$$

Correlated between ν mass and $0\nu\beta\beta$



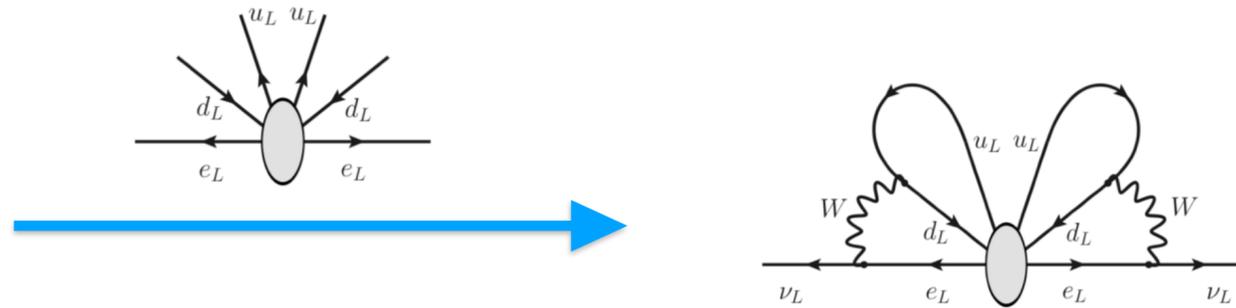
$0\nu\beta\beta$ does not need to be related to the total neutrino masses

Lepton number violation operator



$$dc^2 L^2 Q^2, dc^2 dc^\dagger L^2 uc^\dagger, dc L^2 uc uc^\dagger, dc^2 ec^\dagger L Q uc^\dagger,$$

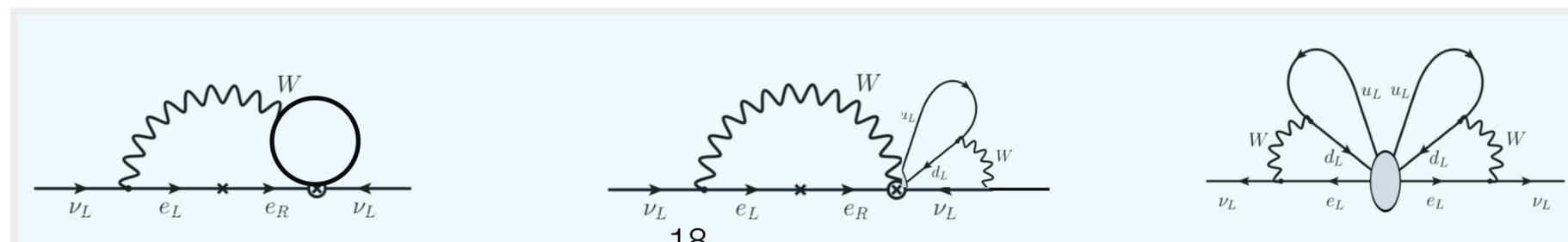
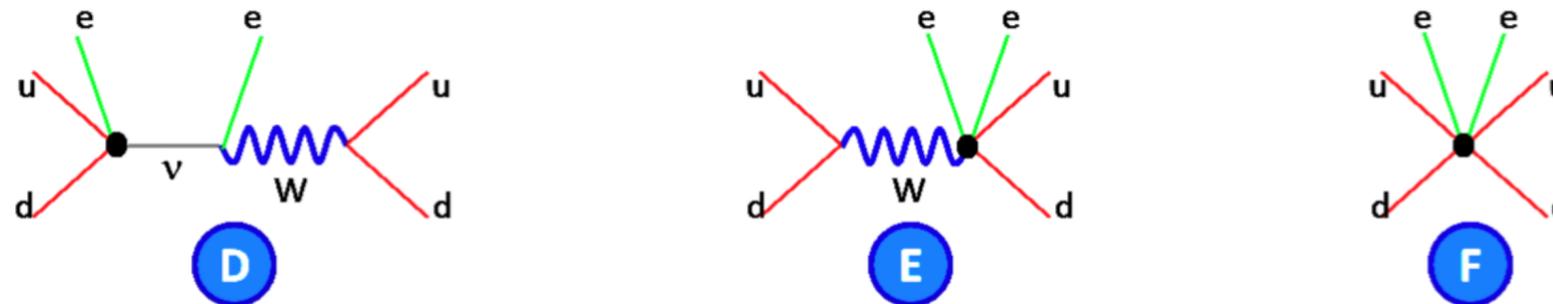
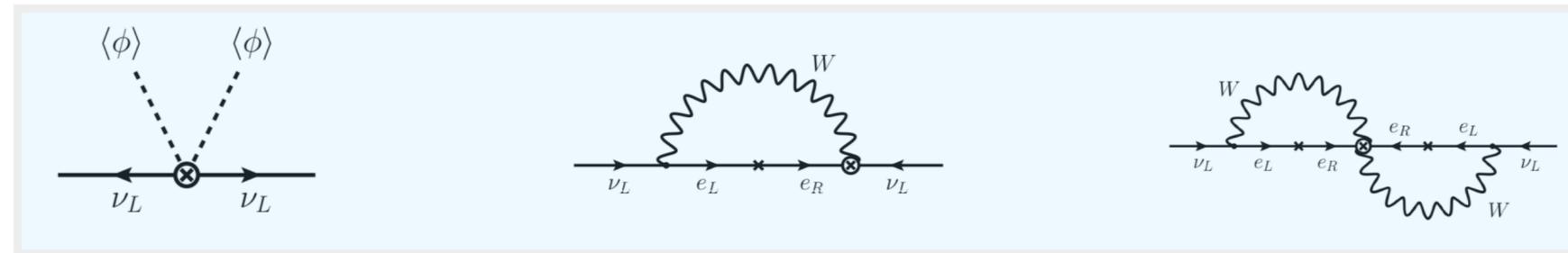
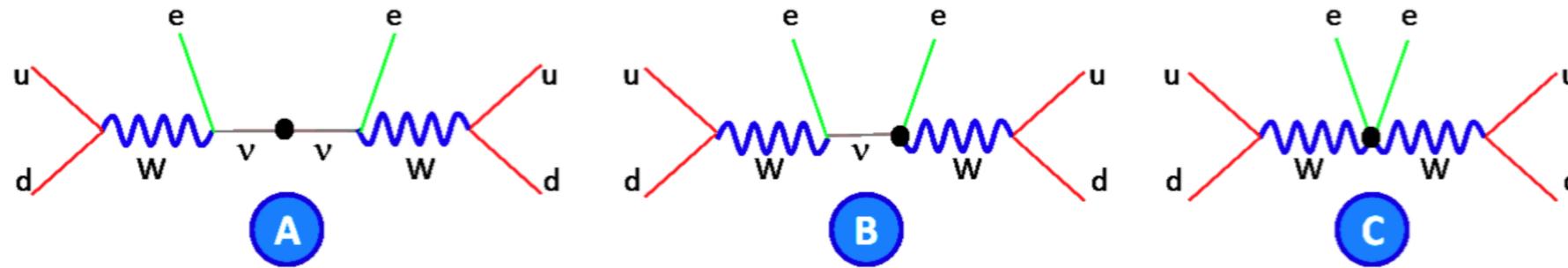
$$dc^\dagger ec^2 uc^2, dc L^2 Q Q^\dagger uc^\dagger, dc^\dagger ec L^\dagger Q uc^2, L^\dagger L^2 Q^2 uc^2$$



Very tiny loop-level neutrino mass

0vbb and Neutrino Masses

Schechter-Valle Theorem: Whatever processes cause 0vbb, its observation would imply existence of Majorana mass term



SMEFT Dimension-6 Operators

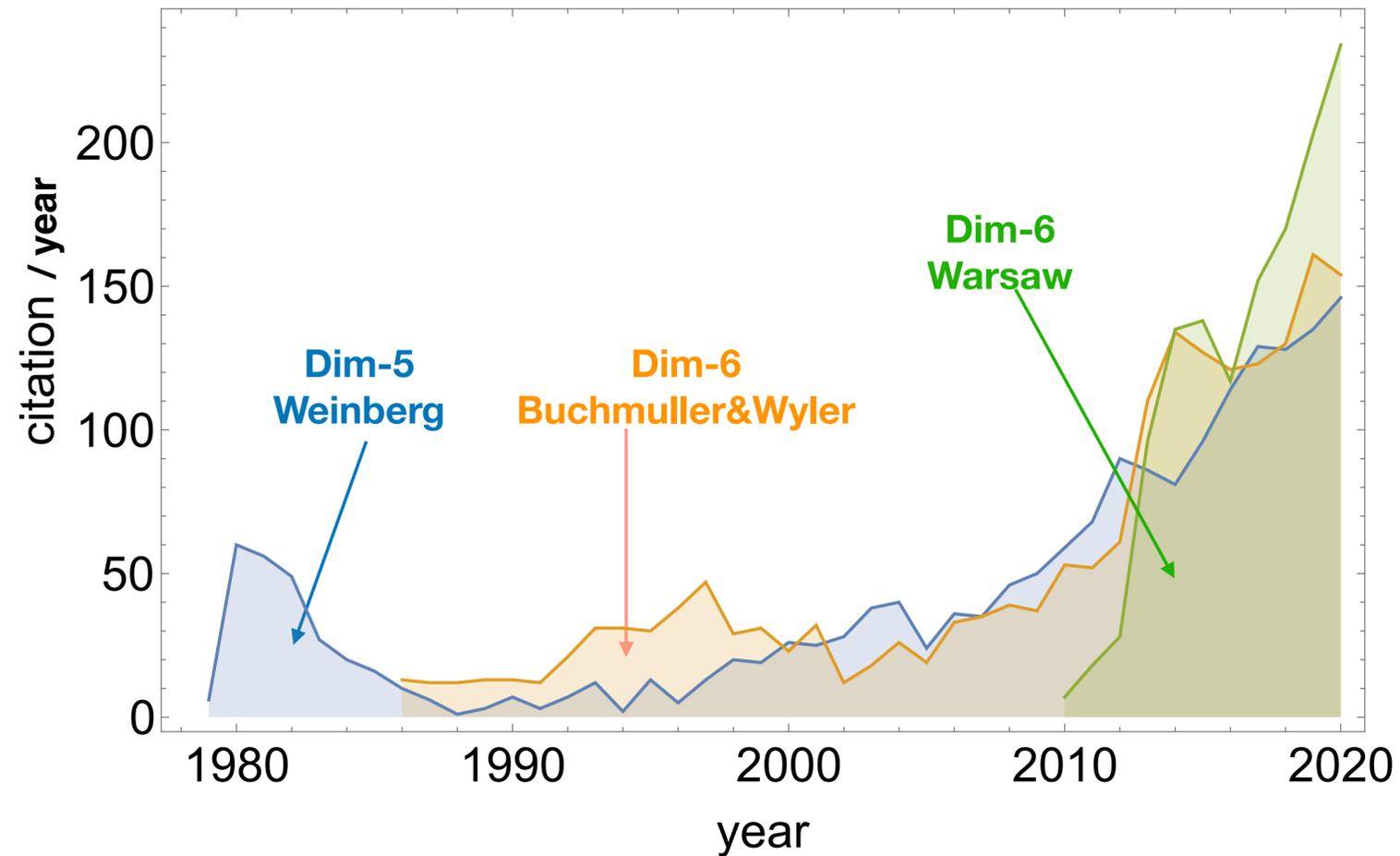
Dimension-6 operators parametrize new physics effects at low energy, electroweak precision, and Higgs boson physics, etc

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\mathcal{D} \leq 4} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \frac{\mathcal{L}_7}{\Lambda^3} + \frac{\mathcal{L}_8}{\Lambda^4} + \frac{\mathcal{L}_9}{\Lambda^5} + \dots$$

[Buchmuller and Wyler, 1986]

One important task of LHC run-3 : dim-6 operator Wilson coefficients

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]



Why take 25 years to write down complete and independent dimension-6 operators?

SMEFT Dimension-6 Operators

[Buchmuller and Wyler, 1986]

$$\begin{aligned}
 O_\varphi &= \frac{1}{3}(\varphi^\dagger \varphi)^3, & O_G &= f_{ABC} G_\mu^{A\nu} G_\nu^{B\lambda} G_\lambda^{C\mu}, \\
 O_{\partial\varphi} &= \frac{1}{2}\partial_\mu(\varphi^\dagger \varphi) \partial^\mu(\varphi^\dagger \varphi), & O_{\tilde{G}} &= f_{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\lambda} G_\lambda^{C\mu}, \\
 O_{e\varphi} &= (\varphi^\dagger \varphi)(\bar{\ell}e\varphi), & O_W &= \varepsilon_{IJK} W_\mu^{I\nu} W_\nu^{J\lambda} W_\lambda^{K\mu}, \\
 O_{u\varphi} &= (\varphi^\dagger \varphi)(\bar{q}u\tilde{\varphi}), & O_{\tilde{W}} &= \varepsilon_{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\lambda} W_\lambda^{K\mu}, \\
 O_{d\varphi} &= (\varphi^\dagger \varphi)(\bar{q}d\varphi), & O_{\ell B} &= i\bar{\ell}\gamma_\mu D_\nu \ell B^{\mu\nu}, \\
 & & O_{qB} &= i\bar{q}\gamma_\mu D_\nu q B^{\mu\nu}, \\
 O_{\varphi G} &= \frac{1}{2}(\varphi^\dagger \varphi) G_{\mu\nu}^A G^{A\mu\nu}, & O_{\varphi \tilde{G}} &= (\varphi^\dagger \varphi) \tilde{G}_{\mu\nu}^A G^{A\mu\nu}, \\
 O_{\varphi W} &= \frac{1}{2}(\varphi^\dagger \varphi) W_{\mu\nu}^I W^{I\mu\nu}, & O_{\varphi \tilde{W}} &= (\varphi^\dagger \varphi) \tilde{W}_{\mu\nu}^I W^{I\mu\nu}, \\
 O_{\varphi B} &= \frac{1}{2}(\varphi^\dagger \varphi) B_{\mu\nu} B^{\mu\nu}, & O_{\varphi \tilde{B}} &= (\varphi^\dagger \varphi) \tilde{B}_{\mu\nu} B^{\mu\nu}, \\
 O_{WB} &= (\varphi^\dagger \tau^I \varphi) W_{\mu\nu}^I B^{\mu\nu}, & O_{\tilde{W}B} &= (\varphi^\dagger \tau^I \varphi) \tilde{W}_{\mu\nu}^I B^{\mu\nu}, \\
 O_\varphi^{(1)} &= (\varphi^\dagger \varphi)(D_\mu \varphi^\dagger D^\mu \varphi), & O_\varphi^{(3)} &= (\varphi^\dagger D^\mu \varphi)(D_\mu \varphi^\dagger \varphi), \\
 O_{\ell W} &= i\bar{\ell}\tau^I \gamma_\mu D_\nu \ell W^{I\mu\nu}, & O_{\varphi\ell}^{(1)} &= i(\varphi^\dagger D_\mu \varphi)(\bar{\ell}\gamma^\mu \ell), \\
 O_{eB} &= i\bar{e}\gamma_\mu D_\nu e B^{\mu\nu}, & O_{\varphi\ell}^{(3)} &= i(\varphi^\dagger D_\mu \tau^I \varphi)(\bar{\ell}\gamma^\mu \tau^I \ell), \\
 O_{qG} &= i\bar{q}\lambda^A \gamma_\mu D_\nu q G^{A\mu\nu}, & O_{\varphi e} &= i(\varphi^\dagger D_\mu \varphi)(\bar{e}\gamma^\mu e), \\
 O_{qW} &= i\bar{q}\tau^I \gamma_\mu D_\nu q W^{I\mu\nu}, & O_{\varphi q}^{(1)} &= i(\varphi^\dagger D_\mu \varphi)(\bar{q}\gamma^\mu q), \\
 O_{uG} &= i\bar{u}\lambda^A \gamma_\mu D_\nu u G^{A\mu\nu}, & O_{\varphi q}^{(3)} &= i(\varphi^\dagger D_\mu \tau^I \varphi)(\bar{q}\gamma^\mu \tau^I q), \\
 O_{uB} &= i\bar{u}\gamma_\mu D_\nu u B^{\mu\nu}, & O_{\varphi u} &= i(\varphi^\dagger D_\mu \varphi)(\bar{u}\gamma^\mu u), \\
 O_{dG} &= i\bar{d}\lambda^A \gamma_\mu D_\nu d G^{A\mu\nu}, & O_{\varphi d} &= i(\varphi^\dagger D_\mu \varphi)(\bar{d}\gamma^\mu d), \\
 O_{dB} &= i\bar{d}\gamma_\mu D_\nu d B^{\mu\nu}, & & \\
 O_{D_e} &= (\bar{\ell}D_\mu e)D^\mu \varphi, & O_{\tilde{D}_e} &= (D_\mu \bar{\ell}e)D^\mu \varphi, \\
 O_{D_u} &= (\bar{q}D_\mu u)D^\mu \tilde{\varphi}, & O_{\tilde{D}_u} &= (D_\mu \bar{q}u)D^\mu \tilde{\varphi}, \\
 O_{D_d} &= (\bar{q}D_\mu d)D^\mu \varphi, & O_{\tilde{D}_d} &= (D_\mu \bar{q}d)D^\mu \varphi, \\
 O_{eW} &= (\bar{\ell}\sigma^{\mu\nu} \tau^I e)\varphi W_{\mu\nu}^I, & O_{eB} &= (\bar{\ell}\sigma^{\mu\nu} e)\varphi B_{\mu\nu}, \\
 O_{uG} &= (\bar{q}\sigma^{\mu\nu} \lambda^A u)\tilde{\varphi} G_{\mu\nu}^A, & O_{uq}^{(1,1)} &= \frac{1}{2}(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q), \\
 O_{uW} &= (\bar{q}\sigma^{\mu\nu} \tau^I u)\tilde{\varphi} W_{\mu\nu}^I, & O_{uq}^{(1,3)} &= \frac{1}{2}(\bar{q}\gamma_\mu \tau^I q)(\bar{q}\gamma^\mu \tau^I q), \\
 O_{dG} &= (\bar{q}\sigma^{\mu\nu} \lambda^A d)\varphi G_{\mu\nu}^A, & O_{dq}^{(1)} &= (\bar{\ell}\gamma_\mu \ell)(\bar{q}\gamma^\mu q), \\
 O_{dW} &= (\bar{q}\sigma^{\mu\nu} \tau^I d)\varphi W_{\mu\nu}^I, & O_{dW} &= (\bar{q}\sigma^{\mu\nu} d)\varphi B_{\mu\nu}, \\
 O_{ee} &= \frac{1}{2}(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e), & O_{\ell e} &= (\bar{\ell}e)(\bar{e}\ell), \\
 O_{uu}^{(1)} &= \frac{1}{2}(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u), & O_{\ell u} &= (\bar{\ell}u)(\bar{u}\ell), \\
 O_{dd}^{(1)} &= \frac{1}{2}(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d), & O_{\ell d} &= (\bar{\ell}d)(\bar{d}\ell), \\
 O_{eu} &= (\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u), & O_{qe} &= (\bar{q}e)(\bar{e}q), \\
 O_{ed} &= (\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d), & O_{qu}^{(1)} &= (\bar{q}u)(\bar{u}q), \\
 O_{ud}^{(1)} &= (\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d), & O_{qd}^{(1)} &= (\bar{q}d)(\bar{d}q), \\
 O_{ud}^{(8)} &= (\bar{u}\gamma_\mu \lambda^A u)(\bar{d}\gamma^\mu \lambda^A d), & O_{qde} &= (\bar{\ell}e)(\bar{d}q).
 \end{aligned}$$

Equation of motion (field redefinition)

$$\begin{aligned}
 (D^\mu D_\mu \varphi)^j &= m^2 \varphi^j - \lambda (\varphi^\dagger \varphi) \varphi^j - \bar{e} \Gamma_e^{lj} + \varepsilon_{jkl} \bar{q}^k \Gamma_u u - \bar{d} \Gamma_d^{lj} \\
 i\cancel{D}l &= \Gamma_e e \varphi, & i\cancel{D}e &= \Gamma_e^\dagger \varphi^\dagger l, & i\cancel{D}q &= \Gamma_u u \tilde{\varphi} + \Gamma_d d \varphi, & i\cancel{D}u &= \Gamma_u^\dagger \tilde{\varphi}^\dagger q, \\
 (D^\rho W_{\rho\mu})^I &= \frac{g}{2} (\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi + \bar{l} \gamma_\mu \tau^I l + \bar{q} \gamma_\mu \tau^I q),
 \end{aligned}$$

Covariant derivative commutator

$$[D_\rho, D_\alpha] \sim X_{\rho\alpha}$$

80

Bianchi identity $D_{[\rho} X_{\mu\nu]} = 0$

Integration by part (total derivatives)

$$(D^n \varphi)^\dagger (D^m \varphi) = -(D^{n+1} \varphi)^\dagger (D^{m-1} \varphi) + \partial \left[(D^n \varphi)^\dagger (D^{m-1} \varphi) \right]$$

Fierz identity

$$\begin{aligned}
 T_{\alpha\beta}^A T_{\kappa\lambda}^A &= \frac{1}{2} \delta_{\alpha\lambda} \delta_{\kappa\beta} - \frac{1}{6} \delta_{\alpha\beta} \delta_{\kappa\lambda} \\
 \tau_{jk}^I \tau_{mn}^I &= 2\delta_{jn} \delta_{mk} - \delta_{jk} \delta_{mn}
 \end{aligned}$$

$$80 - 1 - 16 - 5 + 1 = 59$$

$$\begin{aligned}
 O_{qq}^{(1)} &= (\bar{q}u)(\bar{q}d), \\
 O_{qq}^{(8)} &= (\bar{q}\lambda^A u)(\bar{q}\lambda^A d), \\
 O_{\ell q} &= (\bar{\ell}e)(\bar{q}u), \\
 O_{qu}^{(8)} &= (\bar{q}\lambda^A u)(\bar{u}\lambda^A q), \\
 O_{qd}^{(8)} &= (\bar{q}\lambda^A d)(\bar{d}\lambda^A q),
 \end{aligned}$$

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger \varphi)\Box(\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$
$Q_{\tilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
$Q_{\varphi \tilde{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi \tilde{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \tilde{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
$Q_{\varphi \tilde{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating			
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s^k q_t^l)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jkl} [(d_p^\alpha)^T C u_r^\beta] [(q_s^\gamma)^T C l_t^k]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jkl} (\bar{q}_s^k d_t^l)$	Q_{quq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jkl} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(u_s^\gamma)^T C e_t]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jkl} (\bar{q}_s^k T^A d_t^l)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jkl} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^\gamma)^T C l_t^l]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jkl} (\bar{q}_s^k u_t^l)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^\gamma)^T C e_t]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jkl} (\bar{q}_s^k \sigma^{\mu\nu} u_t^l)$				

Operator with Spinor Indices

Operator has more symmetries than what we expected

Field as irreducible rep of Lorentz group

Field transforming under Little group of Poincare

SO(3,1)

SL(2,C) $SU(2)_l \times SU(2)_r$

Spinor-helicity

Building blocks in spinor-helicity form

ϕ

$\phi \in (0,0)$

$$D^{r_i} \phi_i \Leftrightarrow \lambda_i^{r_i} \tilde{\lambda}^{i,r_i},$$

ψ

$\psi_\alpha \in (1/2,0)$
 $\psi_\alpha^\dagger \in (0,1/2)$

λ_α

$$D^{r_i-1/2} \psi_i^{(\dagger)} \Leftrightarrow \lambda_i^{r_i \pm 1/2} \tilde{\lambda}^{i,r_i \mp 1/2},$$

$F_{\mu\nu}$

$F_{L\alpha\beta} = \frac{i}{2} F_{\mu\nu} \sigma_{\alpha\beta}^{\mu\nu} \in (1,0)$
 $F_{R\dot{\alpha}\dot{\beta}} = -\frac{i}{2} F_{\mu\nu} \bar{\sigma}_{\dot{\alpha}\dot{\beta}}^{\mu\nu} \in (0,1)$

$\lambda_\alpha \lambda_\beta$

$$D^{r_i-1} F_{L/R i} \Leftrightarrow \lambda_i^{r_i \pm 1} \tilde{\lambda}^{i,r_i \mp 1},$$

$R_{\mu\nu\rho\sigma}$

$C_{\alpha\beta\gamma\delta} = C_{\mu\nu\rho\sigma} \sigma_{\alpha\beta}^{\mu\nu} \sigma_{\gamma\delta}^{\rho\sigma} \in (2,0)$

$\lambda_\alpha \lambda_\beta \lambda_\gamma \lambda_\delta$

$$D^{r_i-2} C_{L/R i} \Leftrightarrow \lambda_i^{r_i \pm 2} \tilde{\lambda}^{i,r_i \mp 2},$$

D_μ

$D_{\alpha\dot{\alpha}} = D_\mu \sigma_{\alpha\dot{\alpha}}^\mu \in (1/2,1/2)$

$\lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$

On-shell operator

$$\mathcal{O}_N^{(d)} = (\epsilon^{\alpha_i \alpha_j})^{\otimes n} (\tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_j})^{\otimes \tilde{n}} \prod_{i=1}^N (D^{r_i - |h_i|} \Psi_{i, a_i})_{\alpha_i}^{\dot{\alpha}_i} \lambda_i^{r_i - h_i} \tilde{\lambda}_{\dot{\alpha}_i}^{i, r_i + h_i}$$

$$\mathcal{M} \rightarrow e^{i h_i \varphi} \mathcal{M}$$

$$\lambda_i \rightarrow e^{-i\varphi/2} \lambda_i, \quad \tilde{\lambda}^i \rightarrow e^{i\varphi/2} \tilde{\lambda}^i.$$

$$(\epsilon^{\alpha_i \alpha_j})^{\otimes n} (\tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_j})^{\otimes \tilde{n}} \prod_{i=1}^N \lambda_i^{r_i - h_i} \tilde{\lambda}_{\dot{\alpha}_i}^{i, r_i + h_i}$$

$$\langle ij \rangle = \epsilon^{\beta\alpha} \lambda_{i\alpha} \lambda_{j\beta}$$

$$[ij] = \tilde{\lambda}_{i\dot{\alpha}} \tilde{\epsilon}^{\dot{\alpha}\beta} \tilde{\lambda}_{j\dot{\beta}}$$

On-shell Brackets

$$\langle 12 \rangle^2 \langle 34 \rangle [34]$$

$$F_{L1}^{\alpha\beta} F_{L2\alpha\beta} (D\phi_3)^\gamma_{\dot{\alpha}} (D\phi_4)_{\dot{\gamma}}^{\dot{\alpha}}$$

EOM and CDC

$$D^2 \phi_i \Leftrightarrow p_i^2 = 0$$

$$[D_\mu, D_\nu] \Leftrightarrow [p_\mu, p_\nu] = 0$$

Operator as On-shell Amplitude

[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]

[Li, Ren, Xiao, **Yu**, Zheng, 2012.09188]

[Li, Ren, Xiao, **Yu**, Zheng, 2007.07899]

[Li, Ren, Shu, Xiao, **Yu**, Zheng, 2005.00008]

Operator

$$W_{\mu\lambda} (e_{cp} \sigma^{\nu\lambda} D^\mu L_r) D_\nu H^\dagger$$



Spinor Tensor



Symmetrize indices

$$D_{[\alpha\dot{\alpha}} D_{\beta]\dot{\beta}} = D_\mu D_\nu \sigma_{[\alpha\dot{\alpha}}^\mu \sigma_{\beta]\dot{\beta}}^\nu = -D^2 \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} + \frac{i}{2} [D_\mu, D_\nu] \epsilon_{\alpha\beta} (\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}},$$

$$D_{[\alpha\dot{\alpha}} \psi_{\beta]} = D_\mu \sigma_{[\alpha\dot{\alpha}}^\mu \psi_{\beta]} = -\epsilon_{\alpha\beta} (D_\mu \sigma^\mu \psi)_{\dot{\alpha}},$$

$$D_{[\alpha\dot{\alpha}} F_{L\beta]\gamma} = \frac{i}{2} D_\mu F_{L\nu\rho} \sigma_{[\alpha\dot{\alpha}}^\mu \sigma_{\beta]\gamma}^{\nu\rho} = i D^\mu F_{L\mu\nu} \epsilon_{\alpha\beta} \sigma_{\gamma\dot{\alpha}}^\nu,$$

$$(D\psi)_{\alpha\beta\dot{\alpha}} = -\frac{1}{2} \epsilon_{\alpha\beta} (D\psi)_{\dot{\alpha}} + \frac{1}{2} (D\psi)_{(\alpha\beta)\dot{\alpha}}.$$

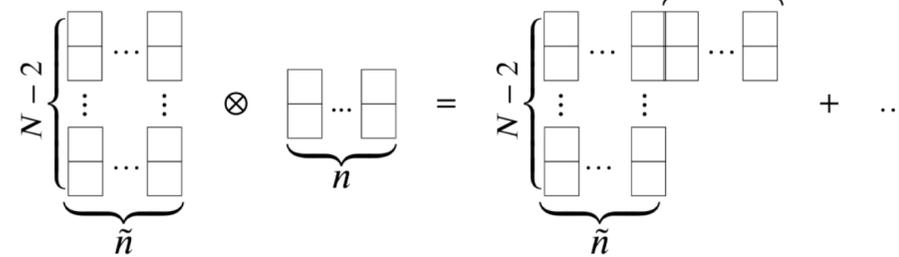
$$\epsilon_{\alpha_1\alpha_3} \epsilon_{\alpha_1\alpha_3} \epsilon_{\alpha_2\alpha_4} \epsilon^{\dot{\alpha}_3\dot{\alpha}_4} F_1^{\alpha_1^2} \psi_2^{\alpha_2} (D\psi_3)_{\dot{\alpha}_3}^{\alpha_3} (D\phi_4)_{\dot{\alpha}_4}^{\alpha_4}$$

$$\lambda_i \rightarrow \sum_j U_i^j \lambda_j, \quad \tilde{\lambda}^i \rightarrow \sum_k U^{\dagger i}_k \tilde{\lambda}^k,$$



SL(2,C) x SU(N)

$$(\epsilon^{\alpha_i\alpha_j})^{\otimes n} (\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}})^{\otimes \tilde{n}} \prod_{i=1}^N \lambda_i^{r_i-h_i} \tilde{\lambda}^{i,r_i+h_i}$$



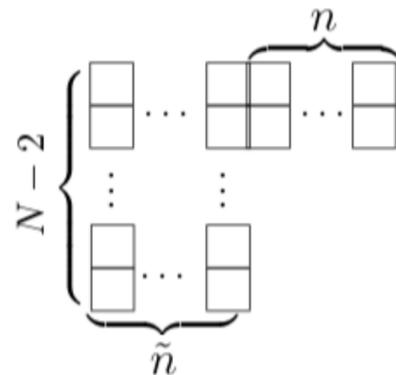
$$\begin{array}{|c|} \hline i \\ \hline j \\ \hline \end{array} \Leftrightarrow \langle ij \rangle$$

Momentum conservation

$$\delta^{(4)} \left(\sum_{i=1}^N \lambda_i \tilde{\lambda}_i \right)$$

$$\sum_i \lambda_i \tilde{\lambda}^i \rightarrow \sum_i \sum_j \sum_k U_i^j U^{\dagger i}_k \lambda_j \tilde{\lambda}^k = \sum_j \lambda_j \tilde{\lambda}^j$$

SSYT



$$\{ \underbrace{1, \dots, 1}_{\#1}, \underbrace{2, \dots, 2}_{\#2}, \dots, \underbrace{N, \dots, N}_{\#N} \}$$

$$\#i = \tilde{n} - 2h_i$$

On-shell Amplitude

1	1	1	2
2	3	3	4

$\langle 13 \rangle \langle 13 \rangle \langle 24 \rangle [34]$

$$F_{L1}^{\alpha\beta} \psi_2^\gamma (D\psi_3)_{\alpha\beta\dot{\alpha}} (D\phi_4)_{\gamma\dot{\alpha}}$$

1	1	1	3
2	2	3	4

$\langle 12 \rangle \langle 13 \rangle \langle 34 \rangle [34]$

$$F_{L1}^{\alpha\beta} \psi_{2\alpha} (D\psi_3)_{\beta\gamma\dot{\alpha}} (D\phi_4)_{\gamma\dot{\alpha}}$$

On-shell Amplitude correspondence

Operator as Spinor Young Tensor

Dim-8 operators: 993 (44807) operators for 1 (3) generations

Traditional method

$$BWHH^\dagger D^2$$

[Hays, Martin, Sanz, Setford, 2018]

$$\begin{aligned}
 & (D^2 H^\dagger) H B_{L\mu\nu} W_L^{\mu\nu}, (D^\mu D_\nu H^\dagger) H B_{L\mu\rho} W_L^{\nu\rho}, (D_\nu D^\mu H^\dagger) H B_{L\mu\rho} W_L^{\nu\rho}, (D_\mu H^\dagger) (D^\mu H) B_{L\nu\rho} W_L^{\nu\rho}, \\
 & (D_\mu H^\dagger) (D^\nu H) B_{L\nu\rho} W_L^{\mu\rho}, (D^\nu H^\dagger) (D_\mu H) B_{L\nu\rho} W_L^{\mu\rho}, (D_\mu H^\dagger) H (D^\mu B_{L\nu\rho}) W_L^{\nu\rho}, (D_\mu H^\dagger) H (D^\nu B_{L\nu\rho}) W_L^{\mu\rho}, \\
 & (D^\nu H^\dagger) H (D_\mu B_{L\nu\rho}) W_L^{\mu\rho}, (D_\mu H^\dagger) H B_{L\nu\rho} (D^\mu W_L^{\nu\rho}), (D^\nu H^\dagger) H B_{L\nu\rho} (D_\mu W_L^{\mu\rho}), \\
 & H^\dagger (D^2 H) B_{L\mu\nu} W_L^{\mu\nu}, H^\dagger (D^\mu D_\nu H) B_{L\mu\rho} W_L^{\nu\rho}, H^\dagger (D^\nu D^\mu H) B_{L\mu\rho} W_L^{\nu\rho}, H^\dagger (D^\mu H) (D_\mu B_{L\nu\rho}) W_L^{\nu\rho}, \\
 & H^\dagger (D^\nu H) (D_\mu B_{L\nu\rho}) W_L^{\mu\rho}, H^\dagger (D_\mu H) (D^\nu B_{L\nu\rho}) W_L^{\mu\rho}, H^\dagger (D^\mu H) B_{L\nu\rho} (D_\mu W_L^{\nu\rho}), H^\dagger (D^\nu H) B_{L\nu\rho} (D_\mu W_L^{\mu\rho}), \\
 & H^\dagger (D_\mu H) B_{L\nu\rho} (D^\nu W_L^{\mu\rho}), H^\dagger H (D^2 B_{L\mu\nu}) W_L^{\mu\nu}, H^\dagger H (D^\mu D_\nu B_{L\mu\rho}) W_L^{\nu\rho}, H^\dagger H (D_\nu D^\mu B_{L\mu\rho}) W_L^{\nu\rho}, \\
 & H^\dagger H (D^\mu B_{L\nu\rho}) (D_\mu W_L^{\nu\rho}), H^\dagger H (D^\nu B_{L\nu\rho}) (D_\mu W_L^{\mu\rho}), H^\dagger H (D_\mu B_{L\nu\rho}) (D^\nu W_L^{\mu\rho}), H^\dagger H B_{L\mu\nu} (D^2 W_L^{\mu\nu}), \\
 & H^\dagger H B_{L\mu\rho} (D^\mu D_\nu W_L^{\nu\rho}), H^\dagger H B_{L\mu\rho} (D_\nu D^\mu W_L^{\nu\rho}). \tag{14}
 \end{aligned}$$

EOM

$$\begin{aligned}
 & (DH^\dagger)_{\alpha\dot{\alpha}} (DH)_{\beta\dot{\beta}} B_{L\{\gamma\delta\}} W_{L\{\xi\eta\}} \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\alpha\beta} \epsilon^{\gamma\xi} \epsilon^{\delta\eta} \\
 & (DH^\dagger)_{\alpha\dot{\alpha}} (DH)_{\beta\dot{\beta}} B_{L\{\gamma\delta\}} W_{L\{\xi\eta\}} \frac{1}{2} \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\delta\xi} (\epsilon^{\alpha\gamma} \epsilon^{\beta\eta} + \epsilon^{\beta\gamma} \epsilon^{\alpha\eta}) \\
 & (DH^\dagger)_{\alpha\dot{\alpha}} H (DBL)_{\{\beta\gamma\delta\},\dot{\beta}} W_{L\{\xi\eta\}} \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\alpha\beta} \epsilon^{\gamma\xi} \epsilon^{\delta\eta} \\
 & (DH^\dagger)_{\alpha\dot{\alpha}} H B_{L\{\xi\eta\}} (DWL)_{\{\beta\gamma\delta\},\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\alpha\beta} \epsilon^{\gamma\xi} \epsilon^{\delta\eta} \\
 & H^\dagger (DH)_{\alpha\dot{\alpha}} (DBL)_{\{\beta\gamma\delta\},\dot{\beta}} W_{L\{\xi\eta\}} \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\alpha\beta} \epsilon^{\gamma\xi} \epsilon^{\delta\eta} \\
 & H^\dagger (DH)_{\alpha\dot{\alpha}} B_{L\{\xi\eta\}} (DWL)_{\{\beta\gamma\delta\},\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\alpha\beta} \epsilon^{\gamma\xi} \epsilon^{\delta\eta} \\
 & H^\dagger H (DBL)_{\{\alpha\beta\gamma\},\dot{\alpha}} (DWL)_{\{\xi\eta\delta\},\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\alpha\xi} \epsilon^{\beta\eta} \epsilon^{\gamma\delta}
 \end{aligned}$$

IBP

$$\begin{aligned}
 & B_L^{\alpha\beta} W_{L\alpha\beta} (DH^\dagger)^\gamma{}_{\dot{\alpha}} (DH)_\gamma{}^{\dot{\alpha}} \\
 & B_L^{\alpha\beta} W_{L\alpha}{}^\gamma (DH^\dagger)_{\beta\dot{\alpha}} (DH)_\gamma{}^{\dot{\alpha}}
 \end{aligned}$$

$\omega \backslash \bar{\omega}$	0	2	4	6	8
0					
2					
4					
6					
8					

$$BWHH^\dagger D^2$$

#1 = 3, #2 = 3, #3 = 1, #4 = 1

1	1	1	3
2	2	2	4

1	1	1	2
2	2	3	4

2

$\langle 13 \rangle \langle 13 \rangle \langle 24 \rangle [34]$

$$B_L^{\alpha\beta} W_{L\alpha\beta} (DH^\dagger)^\gamma{}_{\dot{\alpha}} (DH)_\gamma{}^{\dot{\alpha}}$$

$\langle 12 \rangle \langle 13 \rangle \langle 34 \rangle [34]$

$$B_L^{\alpha\beta} W_{L\alpha}{}^\gamma (DH^\dagger)_{\beta\dot{\alpha}} (DH)_\gamma{}^{\dot{\alpha}}$$

EFT Operator Bases

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\mathcal{D} \leq 4} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \frac{\mathcal{L}_7}{\Lambda^3} + \frac{\mathcal{L}_8}{\Lambda^4} + \frac{\mathcal{L}_9}{\Lambda^5} + \dots$$

Standard Model Effective Field Theory

SU(3) x SU(2) x U(1) gauge symmetry

Dim-5

[Weinberg, 1979]

Dim-6

[Buchmuller, Wyler, 1986]

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

Dim-7

[Lehman, 2014]

[Henning, Lu, Melia, Murayama, 2015]

[Liao, Ma, 2016]

Dim-8

[Li, Ren, Shu, Xiao, **Yu**, Zheng, 2020]

[Murphy, 2020]

Dim-9

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

[Liao, Ma, 2020]

Low Energy Effective Field Theory

SU(3) x U(1) gauge symmetry

Dim-5

[Dirac, 1932]

Dim-6

[Fermi, 1934]

[Lee, Yang, 1956]

[Jenkins, Manohar, Stoffer, 2017]

Dim-7

[Liao, Ma, Wang, 2020]

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

Dim-8

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

[Murphy, 2020]

Dim-9

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

ABC4EFT Package

Amplitude Basis Construction for Effective Field Theory

[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]

- Home
- Repo
- Downloads
- Contact

Welcome to the HEPForge Project: ABC4EFT

This is the website for the Mathematica package: Amplitude Basis Construction for Effective Field Theory

Package

This package has the following features:

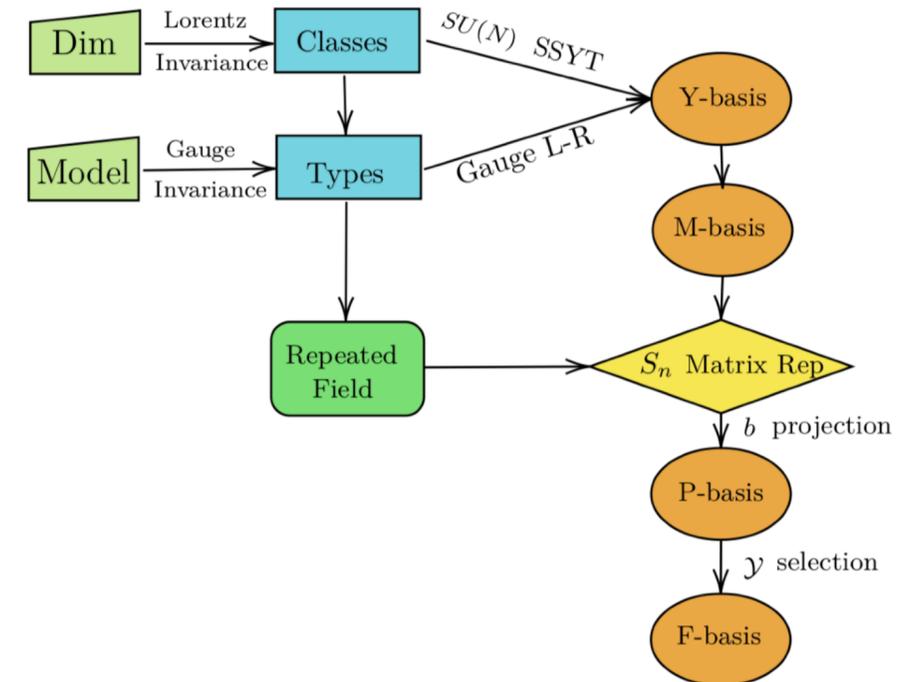
- It provides a general procedure to construct the independent and complete operator bases for generic Lorentz invariant effective field theory, given any kind of gauge symmetry and field content, up to any mass dimension.
- Various operator bases have been systematically constructed to emphasize different aspects: operator independence (y-basis), flavor relation (p-basis) and conserved quantum number (j-basis).
- It provides a systematic way to convert any operator into our on-shell amplitude basis and the basis conversion can be easily done.

Authors

The collaboration group at Institute of Theoretical Physics, CAS Beijing (ITP-CAS)

- Hao-Lin Li (previously postdoc at ITP-CAS, now postdoc at UC Louvain)
- Zhe Ren (4th-year graduate student at ITP-CAS)
- Ming-Lei Xiao (previously postdoc at ITP-CAS, now postdoc at Northwestern and Argonne)
- **Jiang-Hao Yu** (professor at ITP-CAS)
- Yu-Hui Zheng (5th-year graduate student at ITP-CAS)

<https://abc4eft.hepforge.org/>



Fully Automatic

Dark matter EFT
Sterile neutrino EFT
Gravity EFT
Axion EFT

...

Hadronic level EFTs: ChPT and ChEFT

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2404.15047]

[Xuan-He Li, Hao Sun, Feng-Jie Tang, **J.H.Yu**, 2404.14152]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2501.14018]

[Gang Li, Chuan-Qiang Song, **J.H.Yu**, 2507.02538]

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2501.09787]

EFTs in Broken Phase

Standard Model Effective Field Theory

Matching
Running

Low Energy Effective Field Theory

approximate custodial symmetry
 $SU(2) \times SU(2)$

$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} \rightarrow g_L \Sigma g_R^\dagger$$

$$\langle \Sigma \rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \neq 0$$

Electroweak Chiral Lagrangian

$$\Phi \equiv \frac{1}{\sqrt{2}} \vec{\sigma} \cdot \vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}} \varphi^0 \end{pmatrix}$$

SM fields and Goldstone

SM Fermion masses from Higgs VEV

approximate chiral symmetry
 $SU(3) \times SU(3)$

$$\mathbf{q}_L \rightarrow g_L \mathbf{q}_L, \quad \mathbf{q}_R \rightarrow g_R \mathbf{q}_R,$$

$$\langle 0 | (\bar{\mathbf{q}}_L \mathbf{q}_R + \bar{\mathbf{q}}_R \mathbf{q}_L) | 0 \rangle \neq 0$$

QCD Chiral Lagrangian

$$\Phi \equiv \frac{\vec{\lambda}}{\sqrt{2}} \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}$$

meson and baryon

Baryon masses around 1 GeV from Trace anomaly

CCWZ Chiral Lagrangian

Define the nonlinear Goldstone matrix

$$\Omega(\Pi) \equiv \exp \left[\frac{i}{2f} \Pi(x) \right] \rightarrow \Omega(\Pi^{(\mathfrak{g})}) = \mathfrak{g} \Omega(\Pi) \mathfrak{h}^{-1}(\Pi; \mathfrak{g})$$

[Callan, Coleman, Wess, Zumino, 1969]

CCWZ Coset

$$-i\Omega^\dagger \partial_\mu \Omega = d_\mu^{\hat{a}} T^{\hat{a}} + E_\mu^a T^a \equiv d_\mu + E_\mu$$

$$d_\mu \rightarrow \mathfrak{h} d_\mu \mathfrak{h}^{-1}, \quad E_\mu \rightarrow \mathfrak{h} E_\mu \mathfrak{h}^{-1} - i \mathfrak{h} \partial_\mu \mathfrak{h}^{-1}$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i A_\mu$$

$$A_\mu = A_\mu^{\hat{a}} T^{\hat{a}} + A_\mu^a T^a$$

$$[T_{\hat{a}}, T_{\hat{b}}] \propto T_c$$

Symmetric Coset

$$\Omega \rightarrow \mathfrak{g} \Omega \mathfrak{h}^{-1}, \quad \Omega \rightarrow \mathfrak{h} \Omega \mathfrak{g}_R^{-1}$$

$$U \equiv \Omega^2 \rightarrow \mathfrak{g} U \mathfrak{g}_R^{-1}$$

$$D_\mu U \equiv \partial_\mu U + i A_\mu U - i U A_\mu^{(R)}$$

$$A_\mu^{(R)} \equiv A_\mu^a T^a - A_\mu^{\hat{a}} T^{\hat{a}}$$

Building block

$$d_\mu(\Pi), \quad E_{\mu\nu}(\Pi) \quad \nabla_\mu \equiv \partial_\mu + i E_\mu$$

$$f_{\mu\nu} = \Omega^\dagger F_{\mu\nu} \Omega = f_{\mu\nu}^{-\hat{a}} T^{\hat{a}} + f_{\mu\nu}^{+a} T^a$$

$$E_{\mu\nu} = -i[u_\mu, u_\nu] + f_{\mu\nu}^+$$

$$d_\mu = u_\mu$$

Building block

$$u_\mu = i\Omega(D_\mu U)^\dagger \Omega \quad D_\mu$$

$$f_{\mu\nu}^\pm = \frac{1}{2} (f_{\mu\nu} \pm f_{\mu\nu}^{(R)}) = \Omega^\dagger F_{\mu\nu} \Omega \pm F_{\mu\nu}^{(R)}$$

$$\Omega(\Pi) \equiv \begin{bmatrix} u(\Pi) & 0 \\ 0 & u^\dagger(\Pi) \end{bmatrix} \quad u \rightarrow \sqrt{\mathfrak{g}_L U \mathfrak{g}_R^\dagger} = \mathfrak{g}_L u \mathfrak{h}^{-1} = \mathfrak{h}^{-1} u \mathfrak{g}_R$$

$$U(\Pi) \equiv u^2(\Pi) \rightarrow \mathfrak{g}_L U(\Pi) \mathfrak{g}_R^\dagger$$

QCD Chiral Lag

$$u_\mu \rightarrow \mathfrak{h} u_\mu \mathfrak{h}^{-1}$$

$$u_\mu = i \left[u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i l_\mu) u^\dagger \right]$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u,$$

$$f_{\mu\nu}^\pm = u f_{\mu\nu}^L u^\dagger \pm u^\dagger f_{\mu\nu}^R u,$$

EW Chiral Lag

$$\mathbf{V}_\mu \rightarrow \mathfrak{g}_L \mathbf{V}_\mu \mathfrak{g}_L^{-1}$$

$$\mathbf{V}_\mu(x) = i \mathbf{U}(x) D_\mu \mathbf{U}(x)^\dagger$$

$$\mathbf{T} = \mathbf{U} \mathbf{T}_R \mathbf{U}^\dagger \rightarrow \mathfrak{g}_L \mathbf{T} \mathfrak{g}_L^\dagger \quad \hat{W}_{\mu\nu} \rightarrow \mathfrak{g}_L \hat{W}_{\mu\nu} \mathfrak{g}_L^\dagger$$

$$\mathbf{Y} = \mathbf{U} \mathbf{Y}_R \mathbf{U}^\dagger \rightarrow \mathfrak{g}_L \mathbf{Y} \mathfrak{g}_L^\dagger \quad \hat{B}_{\mu\nu} \rightarrow \mathfrak{g}_R \hat{B}_{\mu\nu} \mathfrak{g}_R^\dagger.$$

Adler Zero Condition for Goldstone

Chiral symmetry

$$\alpha \rightarrow \beta \quad \xleftrightarrow{\text{At low energy}} \quad \alpha + n_1\pi \rightarrow \beta + n_2\pi$$

Goldberger-Trieman, Callan-Trieman, Adler-Weisberger, etc

Adler Zero condition

$$T(\alpha + \phi(p), \beta) = -\frac{p_\mu}{F} R^\mu(p) \xrightarrow{p \rightarrow 0} 0$$

Amplitude (soft limit of external leg s)

$$\mathcal{A}(1, \dots, N, s) \xrightarrow{p_s \rightarrow 0} \begin{cases} (S^{(0)}(s) + S^{(\text{sub})}(s)) \mathcal{A}(1, \dots, N) \\ \mathcal{O}(p_s^\sigma) & \text{for Goldstone Boson} \end{cases}$$

[Adler, 1965]

$\{-1/2, -1/2, 1, 0, 0\}$

$$\begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 4 \\ \hline 2 & 2 & 2 & 5 \\ \hline 4 & 5 & & \\ \hline \end{array}, \quad \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 5 & 5 \\ \hline 4 & 4 & & \\ \hline \end{array}, \quad \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 4 & 4 \\ \hline 5 & 5 & & \\ \hline \end{array}, \quad \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 4 & 5 \\ \hline 4 & 5 & & \\ \hline \end{array},$$

[Sun, Xiao, Yu, 2210.14939]

[Sun, Xiao, Yu, 2206.07722]

[Low, Shu, Xiao, Zheng, 2022]

Expand the soft-limit amplitude into the SSYT basis

Put constraints on the SSYT basis

$$\mathcal{B}_i^{(N)}(p_\pi \rightarrow 0) = \sum_{l=1}^{d_N} \mathcal{K}_{il} \mathcal{B}_l^{(N)}$$

$$\begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 4 \\ \hline 2 & 2 & 2 & 5 \\ \hline 4 & 5 & & \\ \hline \end{array}, \quad \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 4 & 5 \\ \hline 4 & 5 & & \\ \hline \end{array},$$

custodial/chiral symmetry breaking: spurion

Higher Order Chiral Lagrangian

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{p^2} + \mathcal{L}_{p^3} + \mathcal{L}_{p^4} + \mathcal{L}_{p^5} + \mathcal{L}_{p^6} + \dots$$

ChPT and Chiral EFT

LO Lagrangian

[Weinberg, 1979]

Pure mesonic

[Gasser, Leutwyler, 1984, 1985]

[Fearing, Scherer 1994]

[Bijmans, Colangelo, Ecker, 1999]

[Jiang, Ge, Wang, 2014]

[Bijmans, Hermansson, Wang, 2018]

nucleon-meson

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2404.15047]

[Krause, 1990]

[Ecker, 1994]

[Fettes, Meisner, Mojzis, Steininger, 2000]

[Oller, Verbeni, Prades, 2006]

[Frink, Meisner, 2006]

[Jiang, Chen, Liu, 2017]

[Xuan-He Li, Hao Sun, Feng-Jie Tang, **J.H.Yu**, 2404.14152]

[Weinberg 1990]

[van Kolck, Ordonez, 1992]

[Petschauer, Kaiser, 2013]

[Petschauer, Haidenbauer, Kaiser, Meisner, Weise, 2020]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2501.14018]

EW Chiral Lagrangian = HEFT

LO Lagrangian

[Weinberg, 1979]

NLO bosonic

[Appelquist, Bernard, 1980]

[Longhitano, 1980, 1981]

[Feruglio, 1993]

NLO 2-fermion

[Buchalla, Cata, Krause, 2014]

NLO 4-fermion

[Buchalla, Cata, Krause, 2014]

[Pich, Rosell, Santos, Sanz-Cillero, 2015, 2018]

[Sun, Xiao, **Yu**, 2206.07722]

$$\begin{aligned} \mathcal{O}_{33}^{U\psi^4} &= (\bar{q}_L \gamma_\mu \tau^I \mathbf{T} q_L) (\bar{q}_R \gamma^\mu \mathbf{U}^\dagger \tau^I \mathbf{U} q_R) \mathcal{F}_{33}^{U\psi^4}(h), \\ \mathcal{O}_{34}^{U\psi^4} &= (\bar{q}_L \gamma_\mu \lambda^A \tau^I \mathbf{T} q_L) (\bar{q}_R \gamma^\mu \lambda^A \mathbf{U}^\dagger \tau^I \mathbf{U} q_R) \mathcal{F}_{34}^{U\psi^4}(h), \\ \mathcal{O}_{39}^{U\psi^4} &= (\bar{q}_L \gamma_\mu \tau^I \mathbf{T} q_L) (\bar{q}_R \gamma^\mu \tau^I \mathbf{U}^\dagger \mathbf{T} \mathbf{U} q_R) \mathcal{F}_{39}^{U\psi^4}(h), \\ \mathcal{O}_{107}^{U\psi^4} &= (\bar{q}_L \gamma_\mu \tau^I \mathbf{T} q_L) (\bar{q}_R \gamma^\mu \tau^I q_R) \mathcal{F}_{107}^{U\psi^4}(h), \\ \mathcal{O}_{113}^{U\psi^4} &= (\bar{q}_L \gamma_\mu \tau^I \mathbf{T} q_L) (\bar{q}_R \gamma^\mu \tau^I q_R) \mathcal{F}_{113}^{U\psi^4}(h), \\ \mathcal{O}_{119}^{U\psi^4} &= (\bar{q}_L \gamma_\mu \mathbf{U}^\dagger \tau^I \mathbf{T} q_L) (\bar{q}_R \gamma^\mu \tau^I q_R) \mathcal{F}_{119}^{U\psi^4}(h), \\ \mathcal{O}_{125}^{U\psi^4} &= (\bar{q}_L \gamma_\mu \tau^I \mathbf{T} q_L) (\bar{q}_R \gamma^\mu \mathbf{U}^\dagger \tau^I \mathbf{U} q_R) \mathcal{F}_{125}^{U\psi^4}(h), \\ \mathcal{O}_{140}^{U\psi^4} &= \mathcal{Y} \left[\square \right]^{abc, km} (\mathbf{T}_L^T)_{pm} C(\mathbf{T} q_L)_{an} (q_L^T_{ak} C q_{Lcl}) \mathcal{F}_{140}^{U\psi^4}(h), \\ \mathcal{O}_{160}^{U\psi^4} &= \mathcal{Y} \left[\square \right]^{abc, km} e^{ln} (\mathbf{T}_R^T)_{pm} C(\mathbf{T} q_R)_{an} (q_R^T_{ak} C q_{Rcl}) \mathcal{F}_{160}^{U\psi^4}(h). \end{aligned}$$

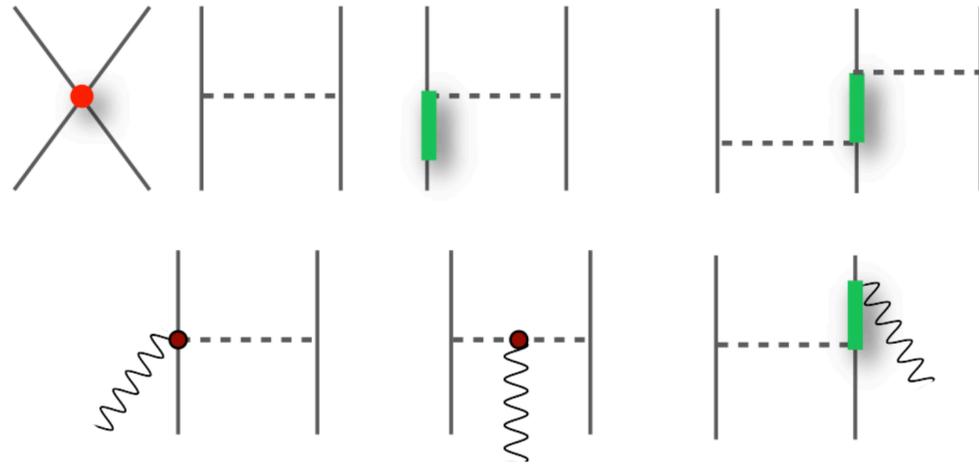
6 term missing

NNLO Basis

[Sun, Xiao, **Yu**, 2210.14939]

Ab initio nuclear structure

Effective Hamiltonians and consistent currents

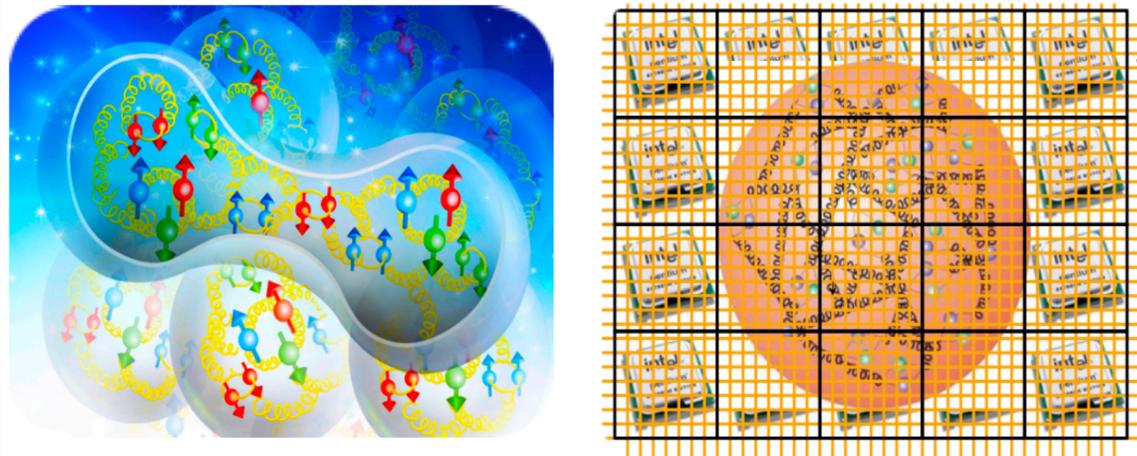


Accurate nuclear many-body methods

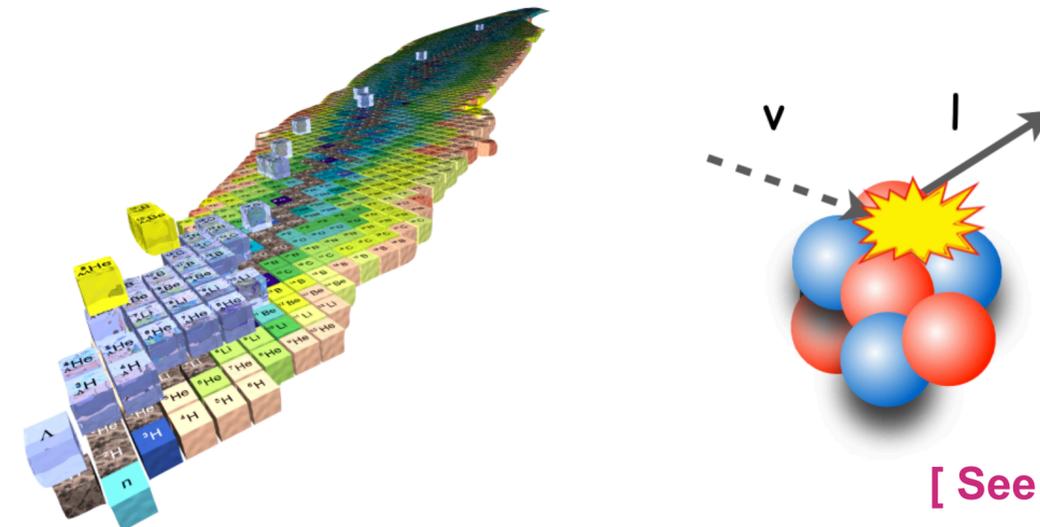


$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$
$$J_{mn} = \langle\Psi_m|J|\Psi_n\rangle$$

Quantum Chromodynamics

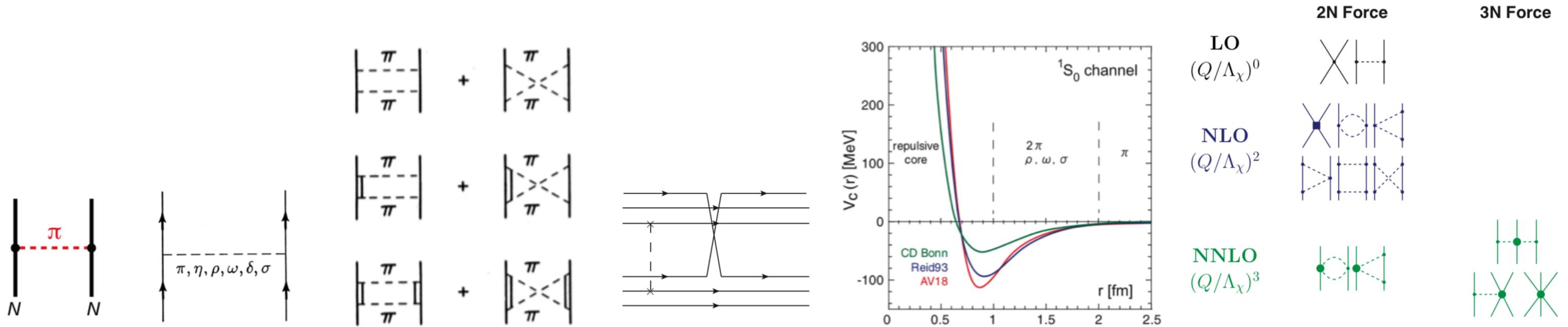


Nuclei and electroweak interactions



[See Pengwei Zhao's talk]

Nuclear force



Yukawa Pion Theory
1935

One-Boson Exchange Model
1936 - 1960

Two-Pion Exchange
1950 - 1980

N-N from quark/chiral-bag model
1970 - 1980

High precision potential
1990 -

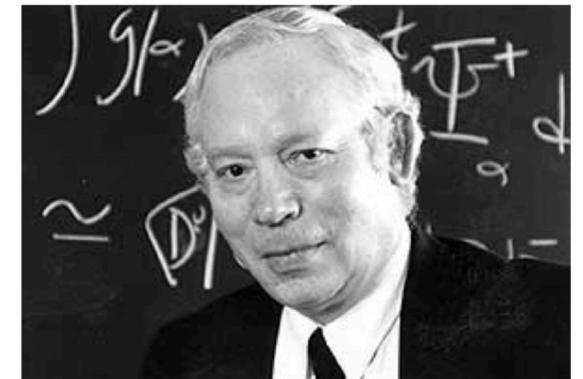
Weinberg nuclear chiral EFT
1990 -



Proca, Kemmer, Moller, Rosenfeld and Schwinger, Pauli, ...

Taketani, Nakamura Sasaki, Bruckner, Watson, ...

AV18, CD Bonn, Nijm, Reid93 ...

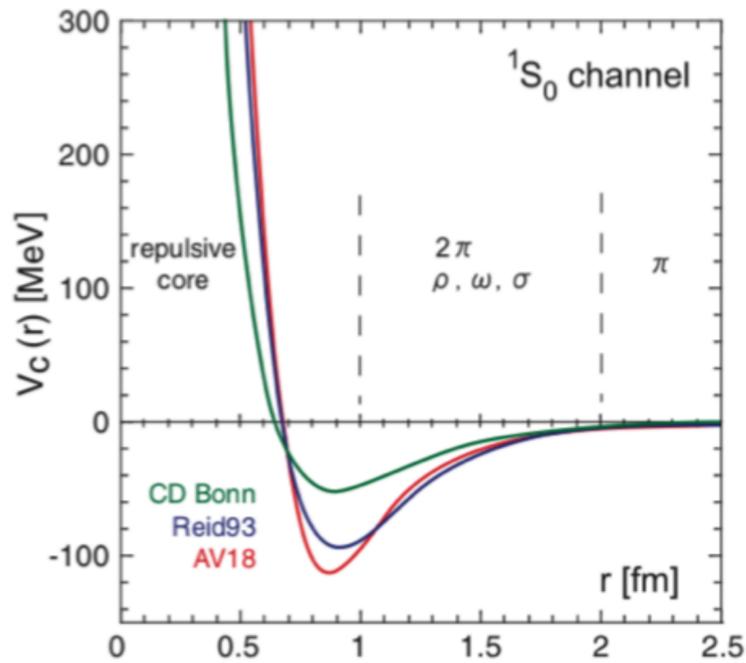
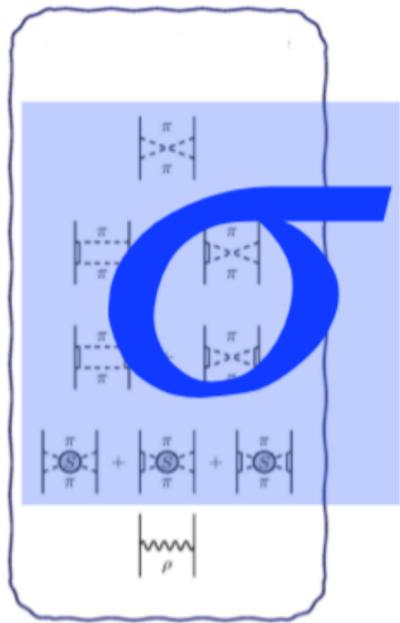


Chiral nuclear force

Meson Exchange Model

$$\mathcal{L}_\sigma = \bar{N}_L i \not{D} N_L + \bar{N}_R i \not{D} N_R - g \bar{N}_R \Sigma N_L - g \bar{N}_L \Sigma^\dagger N_R$$

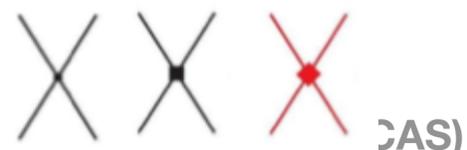
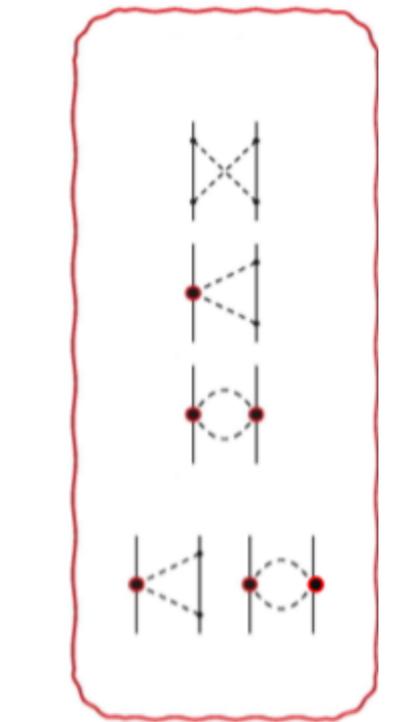
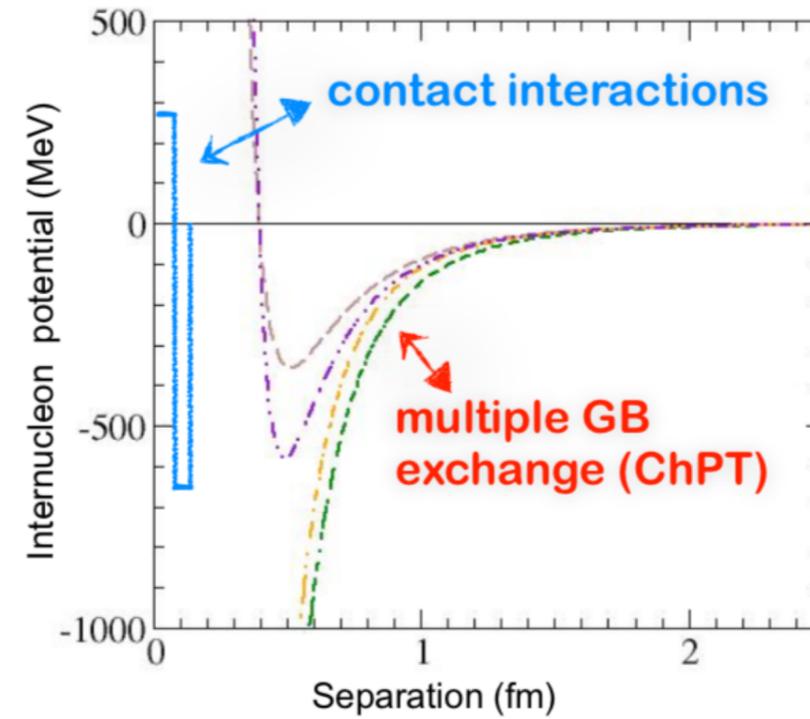
$$V(\mathbf{r}) = \left\{ C_c + C_\sigma \sigma_1 \cdot \sigma_2 + C_T \left(1 + \frac{3}{m_\alpha r} + \frac{3}{(m_\alpha r)^2} \right) S_{12}(\hat{r}) + C_{SL} \left(\frac{1}{m_\alpha r} + \frac{1}{(m_\alpha r)^2} \right) \mathbf{L} \cdot \mathbf{S} \right\} \frac{e^{-m_\alpha r}}{m_\alpha r} (1, \tau_1 \cdot \tau_2)$$



Repulsive central

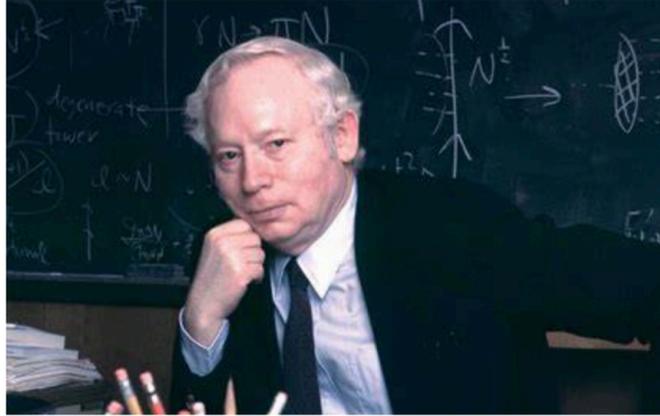
Chiral EFT

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\psi} (i \not{D} - m_N + \frac{1}{2} g_A \gamma_\mu \gamma_5 u^\mu) \psi$$



Weinberg's nuclear force

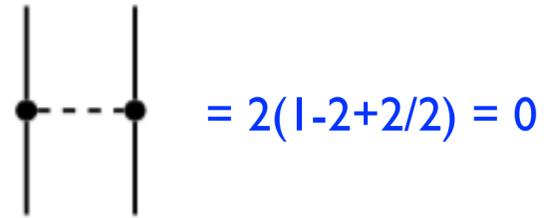
Hard-core nucleon-nucleon interaction



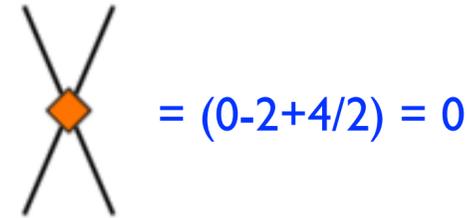
[Weinberg 1933 - 2021]

Weinberg power counting

$$D = 2 - A + 2L + \sum_d V_d \left(d - 2 + \frac{f}{2} \right)$$



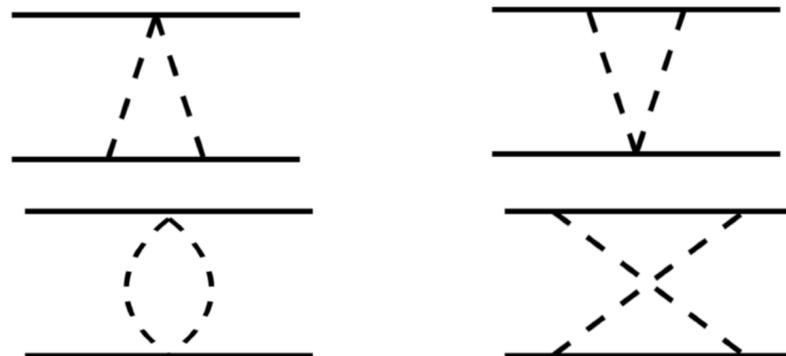
$$V_{1\pi} = -\left(\frac{g_A}{2F_\pi}\right)^2 \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \sim \mathcal{O}(1)$$



It had taken me a decade to realize that four divided by two is two. This sort of interaction is just the kind of hard-core nucleon-nucleon interaction that nuclear physicists had always known would be needed to understand nuclear forces. But now we had a rationale for it.

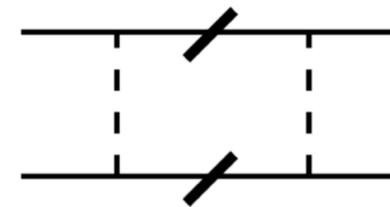
Weinberg 2021

2PI Diagrams



Nuclear potential from Irre. 2PI only

2PR Diagrams

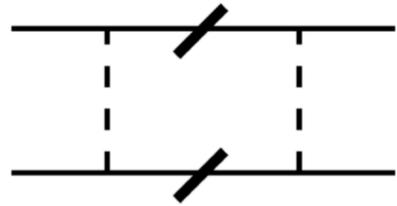


Breakdown in perturbation theory = nuclear bound states

Chiral effective field theory

Chiral EFT with Pion

[Weinberg, 1990]



$$\sim \left(\frac{g_A}{F_\pi} \right)^2 \frac{Q}{\Lambda_{NN}}$$

$$I \sim \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^0 - \frac{\vec{p}^2 - \vec{q}^2}{2M} - i\epsilon} \frac{1}{-q^0 + \frac{\vec{p}^2 - \vec{q}^2}{2M} + i\epsilon} \frac{1}{(q+p)^2 + i\epsilon} \frac{1}{(q-p)^2 + i\epsilon}$$

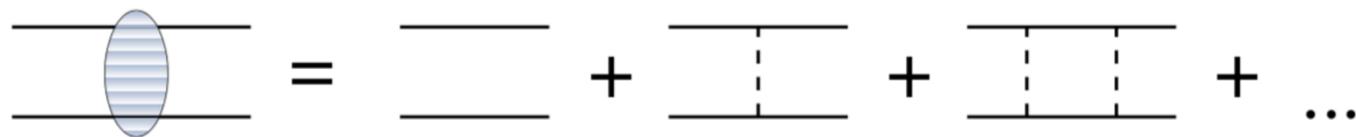
Pinch singularity

Infrared enhancement!

1. calculate nuclear potential from irreducible diagrams

pinch diagrams subtracted

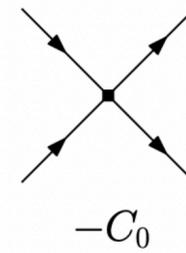
2. Truncated nuclear potential is iterated to all order



Solve Schrodinger equation

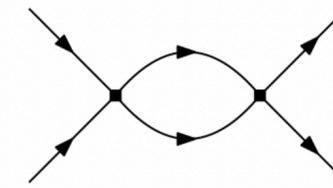
Pionless EFT

Kaplan, Savage, Wise 1998



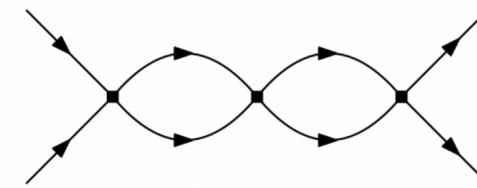
$$-C_0$$

+



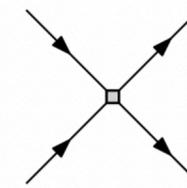
$$\frac{Q^5}{4\pi M} \times C_0 \left(\frac{M}{Q^2} \right) \left(\frac{M}{Q^2} \right) C_0 \sim C_0^2 \frac{MQ}{4\pi}$$

Pinch singularity



$$C_0^3 \left(\frac{Mp}{4\pi} \right)^2$$

+



$$-C_2 p^2$$

$$\sigma_{tot} \rightarrow 4\pi a^2 \text{ as } p \rightarrow 0. \quad a_0 \approx -23.7 \text{ fm} \quad 1/m_\pi \approx 1.4 \text{ fm}$$

a. Natural scattering length

$$\mathcal{A} = -\frac{4\pi a}{M} \left[1 - iap + \left(\frac{1}{2} ar_0 - a^2 \right) p^2 + O(p^3/\Lambda^3) \right] \quad C_0 \sim 4\pi a/M$$

Irrelevant

b. Unnatural scattering length

$$\mathcal{A} = -\frac{4\pi}{M} \frac{1}{(1/a + ip)} \left[1 + \frac{r_0/2}{(1/a + ip)} p^2 + \frac{(r_0/2)^2}{(1/a + ip)^2} p^4 + \dots \right]$$

$$C_0 \sim 4\pi/MQ$$

Relevant

Power counting schemes

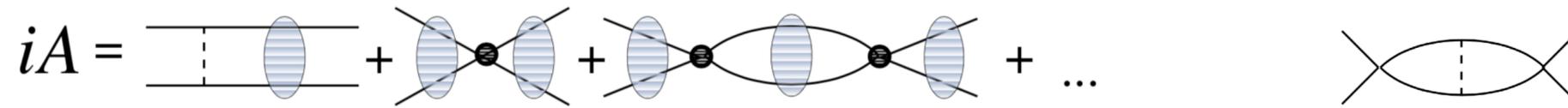
[See Bingwei Long's talk]

Complicated due to non-perturbative natures and renormalization problems

Weinberg Scheme

$$V_{\text{Weinberg}}^{\text{LO}} \sim \mathcal{O}(1), \quad V_{\text{Weinberg}}^{\text{NLO}} \sim \mathcal{O}(p^2)$$

[i.e. scaling of C_{2n} according to NDA ($\sim \mathcal{O}(1)$)]



Renormalization problem!

KSW Scheme

$$V_{\text{KSW}}^{\text{LO}} \sim \mathcal{O}(p^{-1}), \quad V_{\text{KSW}}^{\text{NLO}} \sim \mathcal{O}(1)$$

[i.e. scaling of C_{2n} as $C_{2n} \sim \mathcal{O}(p^{-1-n})$]

Pion are perturbative



Converge problem!

Modified Weinberg

[Nogga, Timmermans, van Kolck, 2005]

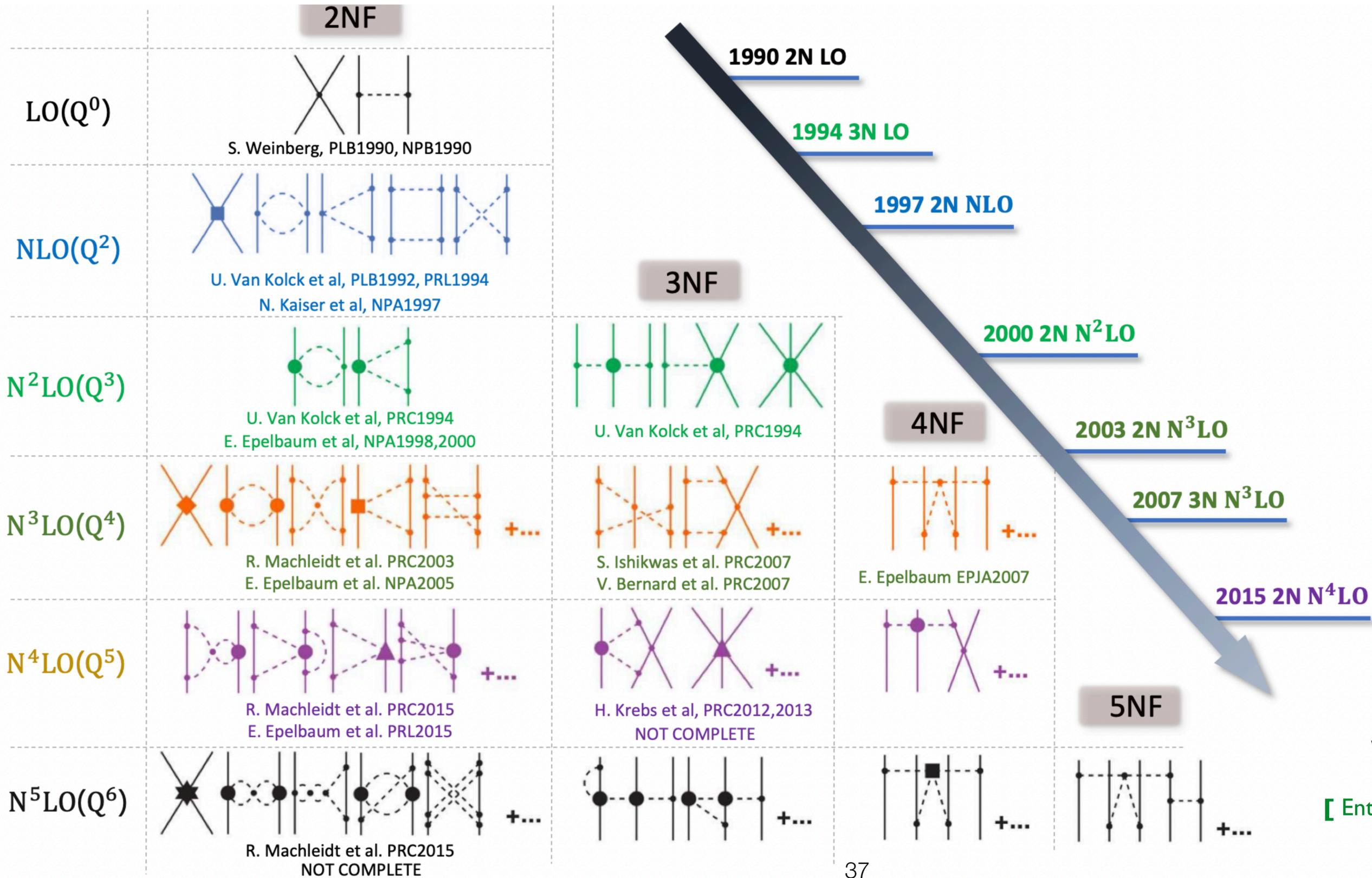
[Epelbaum, Gegelia, 2012]

[S. Wu, B. W. Long, 2019]

$$V_{\text{LO}}^{\text{WPC}}(\mathbf{p}, \mathbf{p}') = \frac{g_A^2}{4f_\pi^2} \tau_1 \cdot \tau_2 \frac{(\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q})}{m_\pi^2 + \mathbf{q}^2} + \tilde{C}_{1S_0} + \tilde{C}_{3S_1} \quad \longrightarrow \quad V_{\text{LO}}^{\text{MWPC}}(\mathbf{p}, \mathbf{p}') = V_{\text{LO}}^{\text{WPC}}(\mathbf{p}, \mathbf{p}') + (\tilde{C}_{3P_0} + \tilde{C}_{3P_2})pp'$$

Solve both but why?

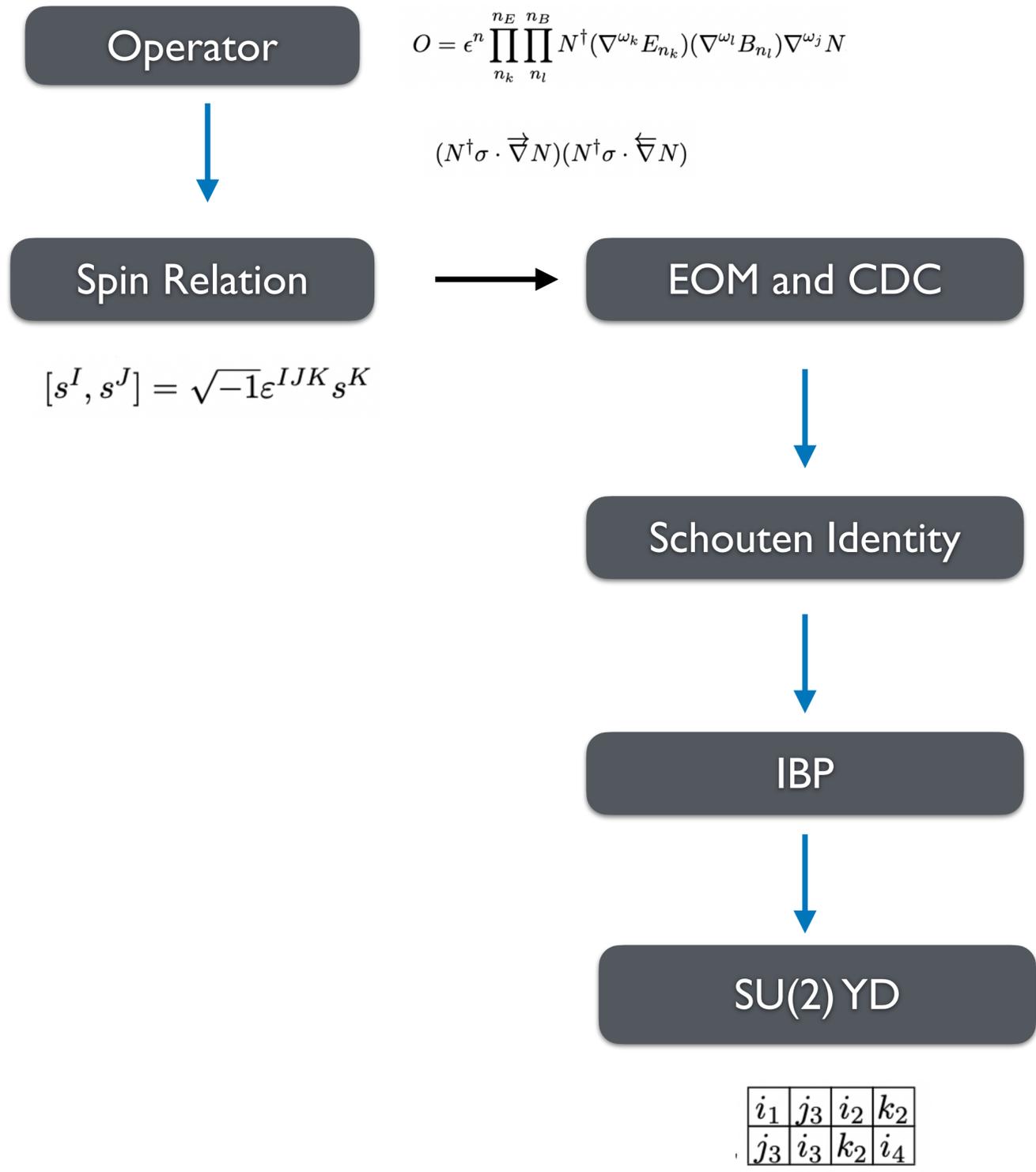
High precision nuclear force



Weinberg scheme
[Entem, Machleidt, Nosyk, 2020]

Non-relativistic operators

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in preparation]



$$\Psi(x) = e^{-imv \cdot x} \left[\underbrace{e^{imv \cdot x} P_v^+ \Psi(x)}_{\equiv \mathcal{N}_v(x)} + \underbrace{e^{imv \cdot x} P_v^- \Psi(x)}_{\equiv \mathcal{H}_v(x)} \right]$$

Non-linearized Lorentz symmetry

$SO(3,1)/SO(3)$

$$v^\mu = L(\vec{\eta}(v)) v_0^\mu \quad L(\vec{\eta}) = e^{i\vec{\eta} \cdot \vec{K}}$$

Missing Goldstone

$$\left. \begin{array}{l} |v_0, 0, \sigma\rangle \\ |v_0, \vec{k}\rangle \end{array} \right\} \longrightarrow |v_0, \vec{k}, \sigma\rangle$$

On-shell Amplitude

$$\epsilon^{i_1 j_3} \epsilon^{j_3 i_3} \epsilon^{i_2 k_2} \epsilon^{k_2 i_4} \sim \langle i_1 j_3 \rangle \langle j_3 i_3 \rangle \langle i_2 k_2 \rangle \langle k_2 i_4 \rangle$$

NN and 3N operators

Nucleon-nucleon sector

3 nucleon sector

LO

[Weinberg 1990] [Weinberg 1991]

[van Kolck, Ordonez, 1992]

[Petschauer, Haidenbauer, Kaiser, Meisner, Weise, 2016]

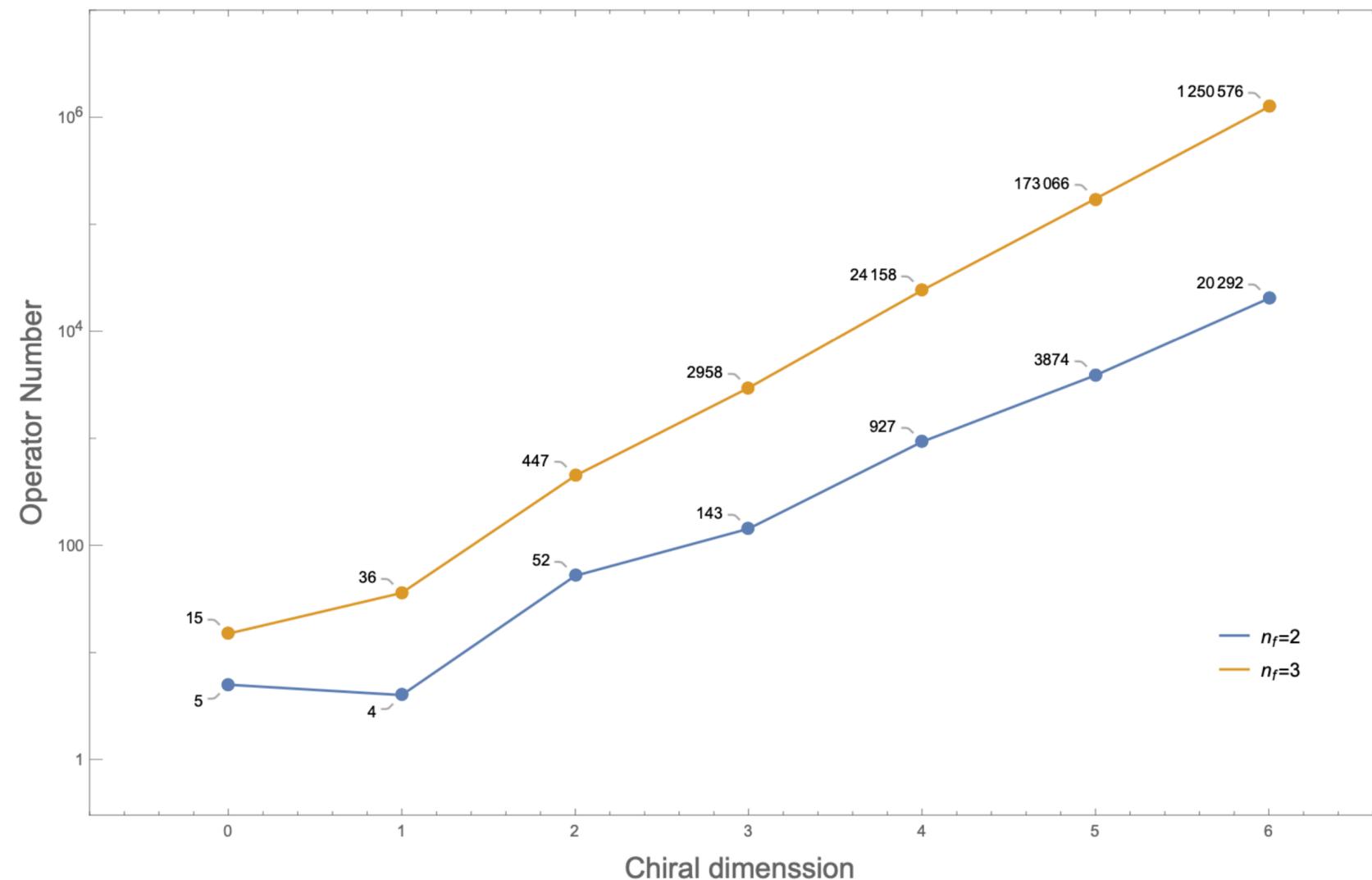
NLO

[Girlanda, Pastore, Schiavilla, Viviani, 2010]

[Nasoni, Filandri, Girlanda, 2023]

[Petschauer, Kaiser, 2013] [Xiao, Geng, Ren, 2019]

NNLO



In order to obtain the most general contact Lagrangian in flavor $SU(3)$, we follow the same procedure as used for the four-baryon contact terms in Ref. [47]. Generalizing these construction rules straightforwardly to six-baryon contact terms, we end up with a (largely) overcomplete set of terms for the leading covariant Lagrangian:

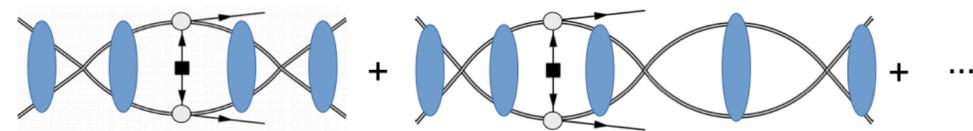
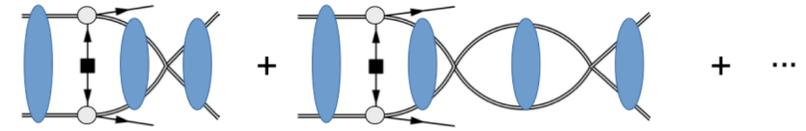
[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2501.14018]

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in preparation]

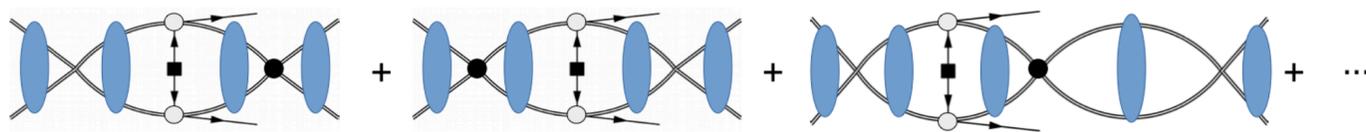
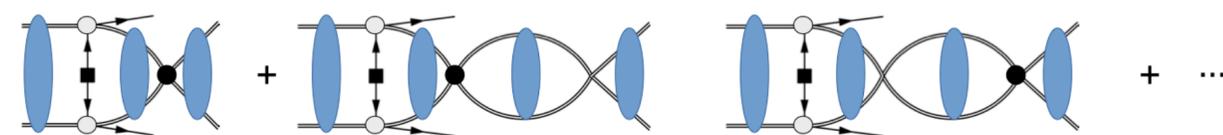
Nuclear many body effects for $0\nu\beta\beta$

Long-range and short-range weak currents

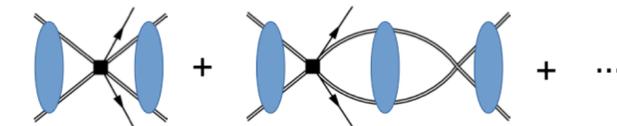
LO



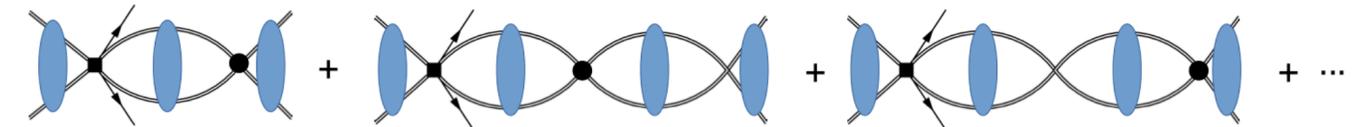
NLO



LO



NLO

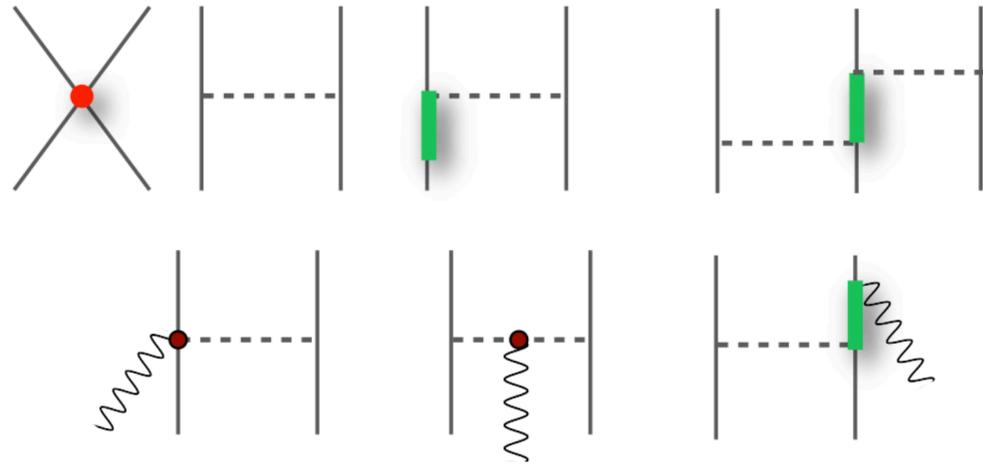


[Cirigliano, Dekens, de Vries, Graesser, Mereghetti, 2019]

[See Pengwei Zhao's talk]

Ab initio nuclear structure

Effective Hamiltonians and consistent currents

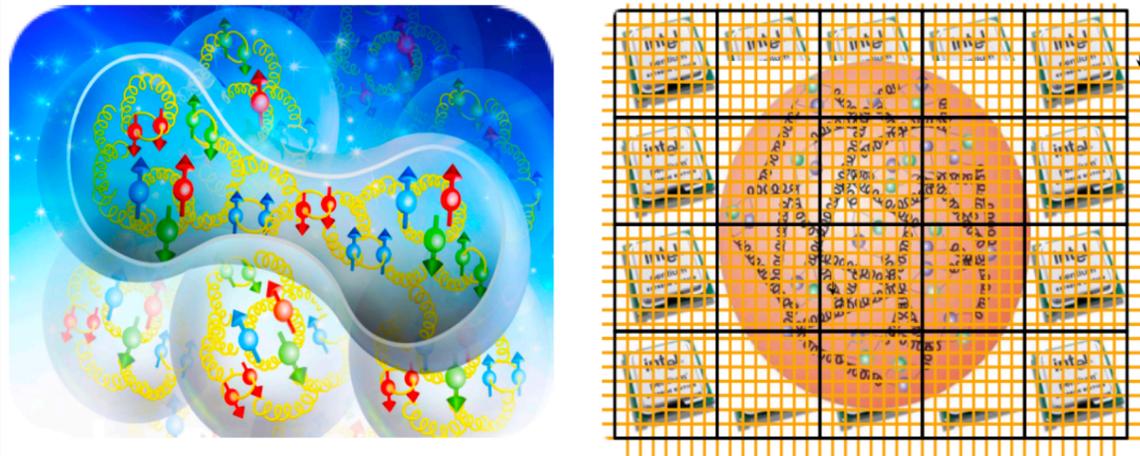


Accurate nuclear many-body methods

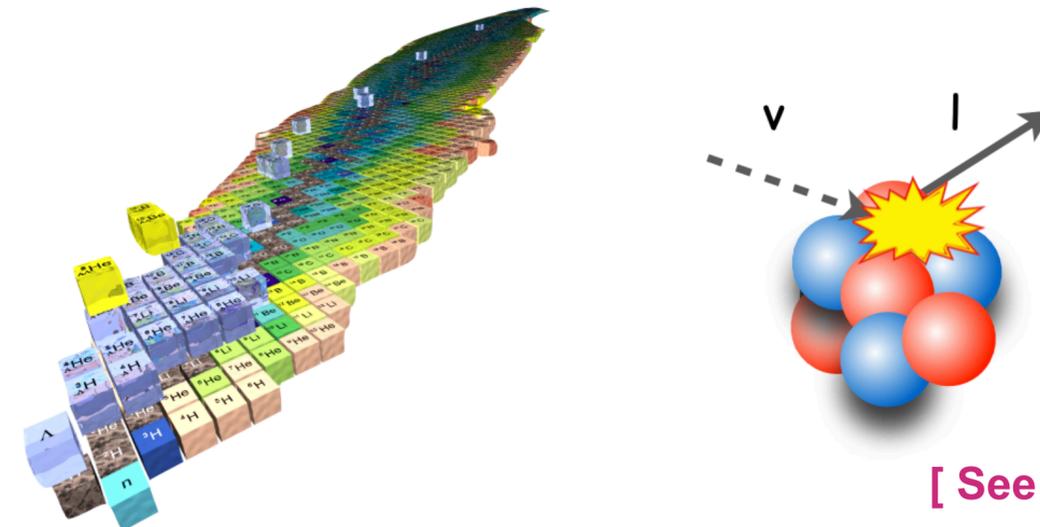


$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$
$$J_{mn} = \langle\Psi_m|J|\Psi_n\rangle$$

Quantum Chromodynamics



Nuclei and electroweak interactions

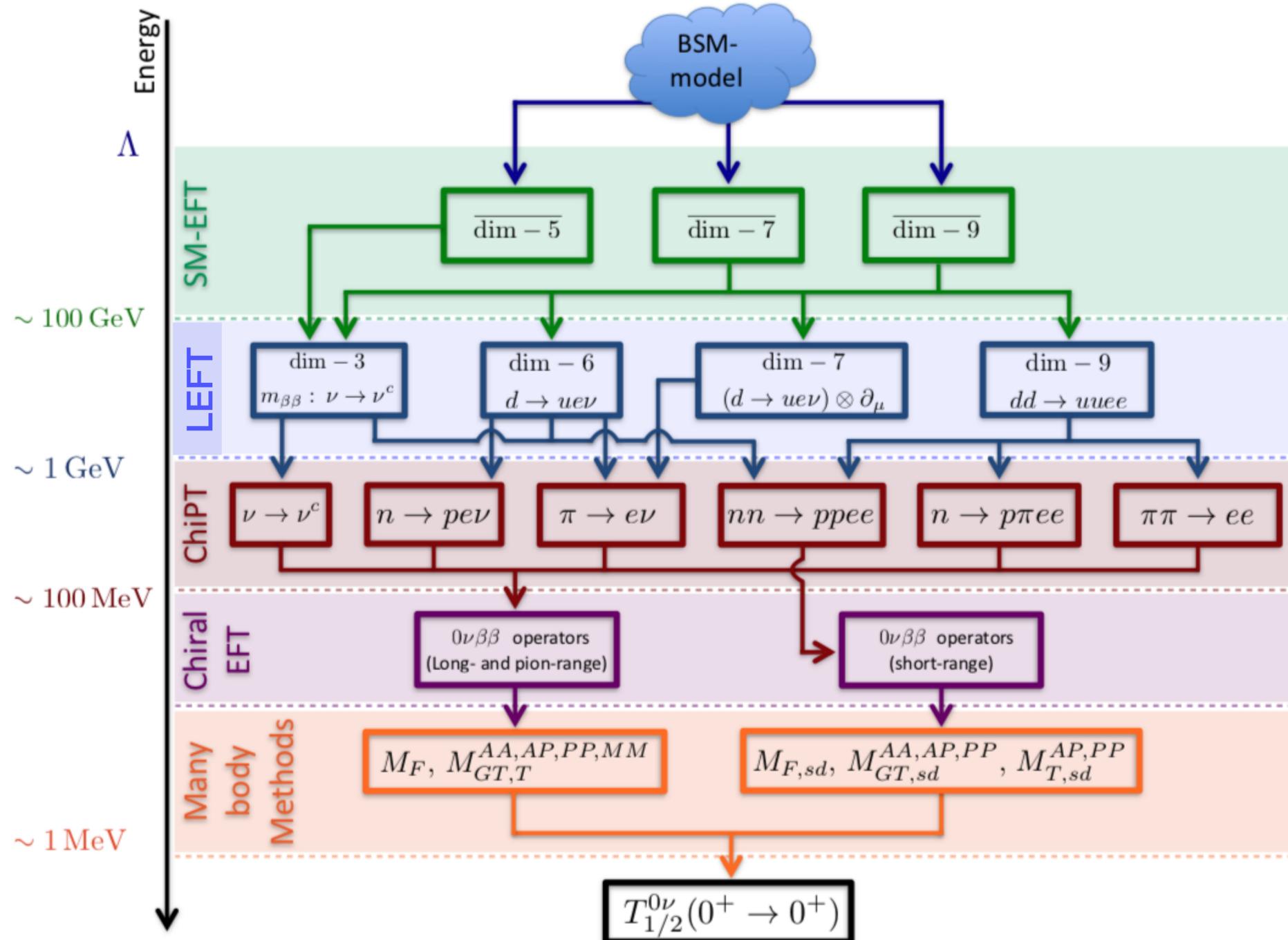


[See Pengwei Zhao's talk]

Complete EFT Framework for $0\nu\beta\beta$

Operator bases for different EFTs are necessarily needed to provide most general description

[Cirigliano, Dekens, de Vries, Graesser, Mereghetti, 2018]



UV models new particles

Quark level operators

Hadronic level operators

Nuclear level operators

Setup a general framework

How to find complete UV?

UV Completion for nv mass and $0\nu\beta\beta$

[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, 2204.03660]

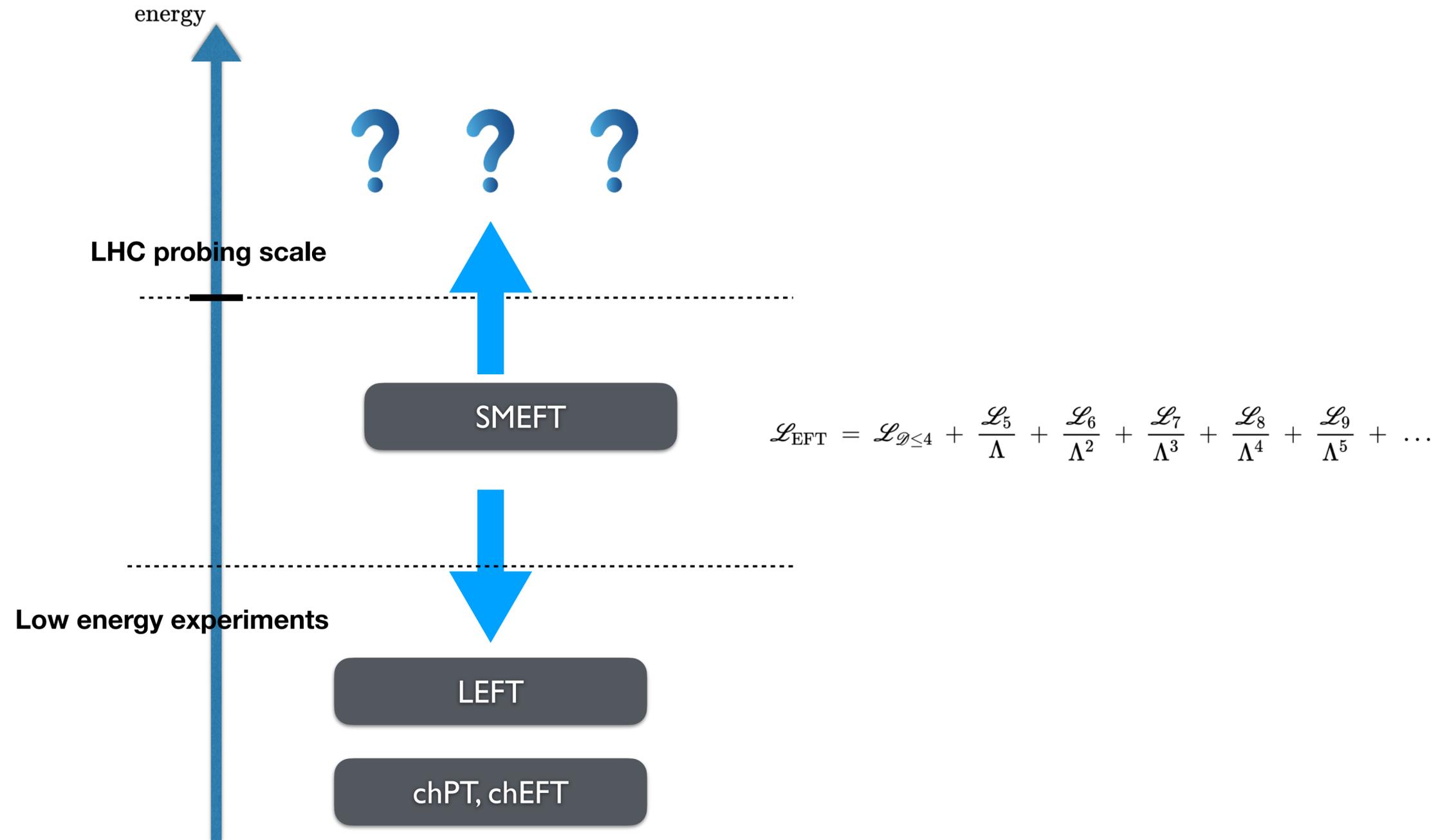
[Xu-Xiang Li, Zhe Ren, **J.H.Yu**, 2307.10380]

[Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **J.H.Yu**, in preparation]

[Gang Li, **J.H.Yu**, Xiang Zhao, 2311.15422]

SMEFT Inverse Problem

Given effective operators, how to find complete UV ?



UV of Weinberg Operator

[See Xiao-Gang He's talk]

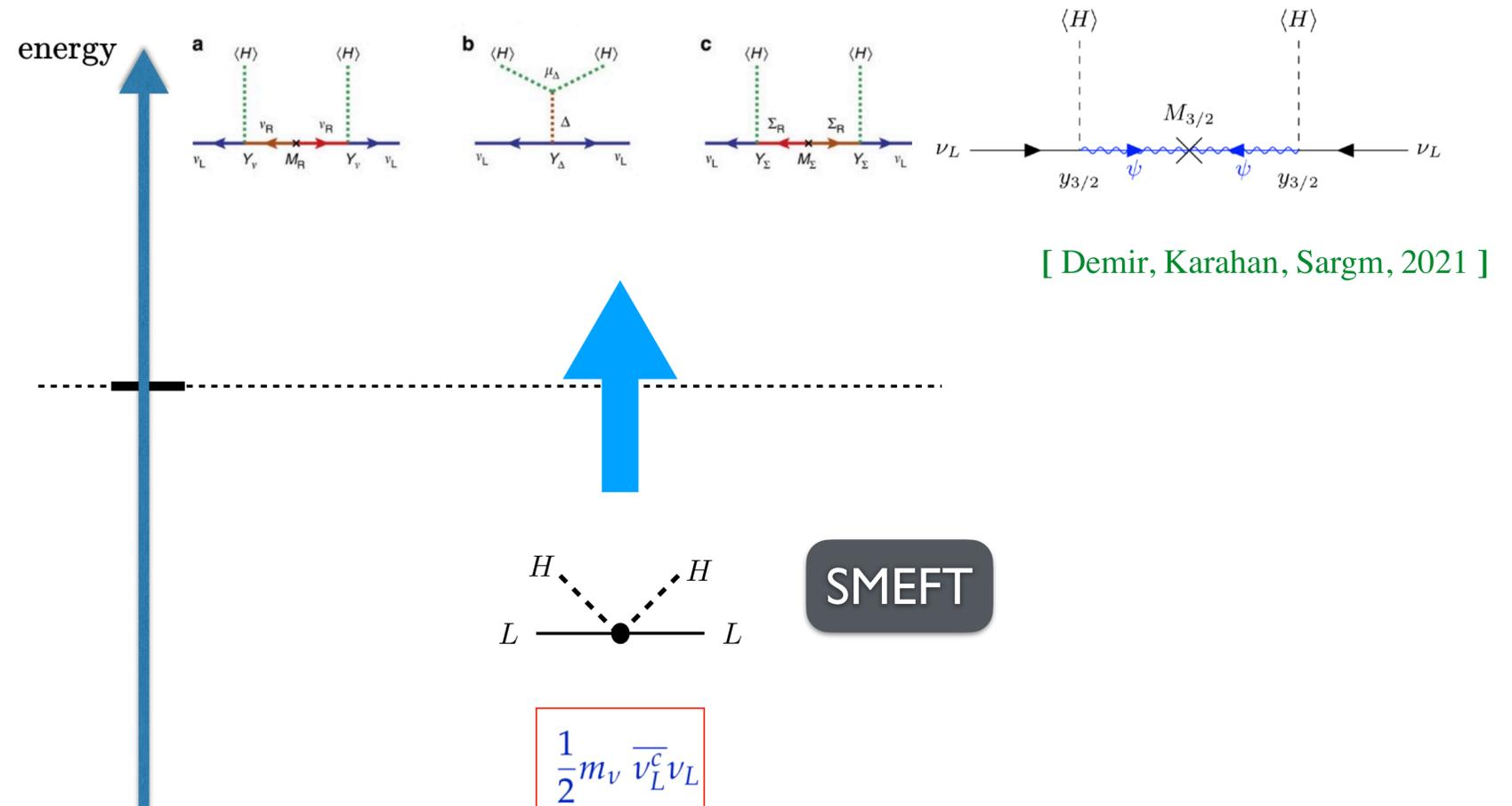
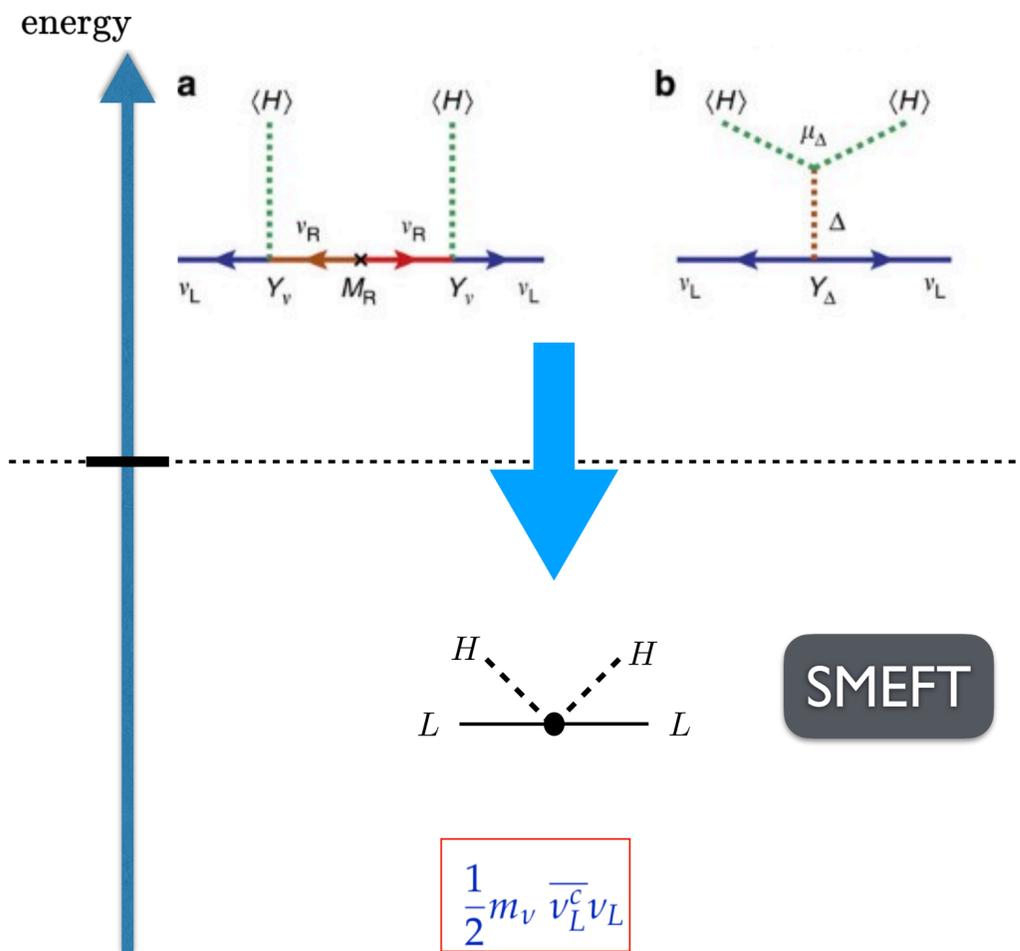
Top-down Approach

Bottom-up Approach

[Yanagida 1979, Gell-Mann, Ramond, Slansky 1979, Mahapatra, Senjanovic, 1980]

[Schechter, Valle, 1980, Cheng, Li 1980, Magg and Wetterich 1980]

[Foot, Lew, He, Joshi 1989]



Consider Angular momentum conservation

Why only 3 Tree-level Seesaw?

Angular momentum conservation for **space-time Poincare symmetry**

[Li, Ni, Xiao, Yu, 2204.03660]

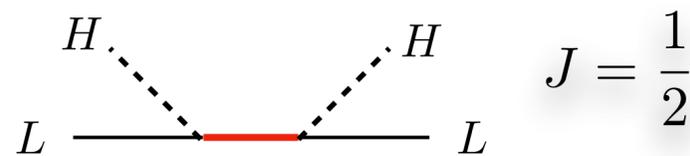
Pauli-Lubanski Casimir $\mathbf{W}^2 \langle p_L, h_L; p_H, h_H | P, J, J_z \rangle = -P^2 J(J+1) \langle p_L, h_L; p_H, h_H | P, J, J_z \rangle$



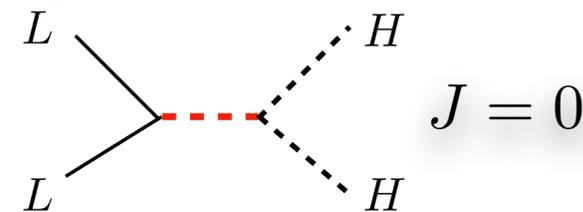
Generalized partial wave analysis for Poincare/Gauge Casimir

$$\mathbf{W}^2 \mathcal{B}^J = -sJ(J+1) \mathcal{B}^J$$

$$W_{\{1,3\}}^2 \mathcal{B}^y = -\frac{3}{4} s_{13} \langle 12 \rangle \quad LH \rightarrow LH \text{ channel}$$



$$LL \rightarrow HH \text{ channel} \quad W_{\{1,2\}}^2 \mathcal{B}^y = 0$$



Type-I and III: **SU(2) singlet and triplet**

Type-II: **SU(2) triplet**, or singlet (excluded by repeated field)

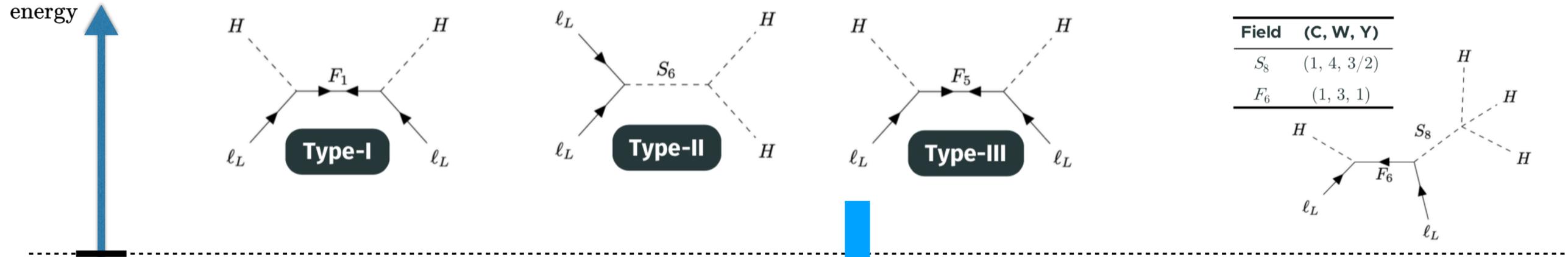
j-basis	Model
$\mathcal{O}_{HL \rightarrow HL}^{(1/2,1)} = \mathcal{O}^S + \mathcal{O}^A$	type I
$\mathcal{O}_{HL \rightarrow HL}^{(1/2,3)} = \mathcal{O}^S - 3\mathcal{O}^A$	type III

$$\mathcal{O}^S = (HL)(HL), \quad \mathcal{O}^A = (HH)(LL)$$

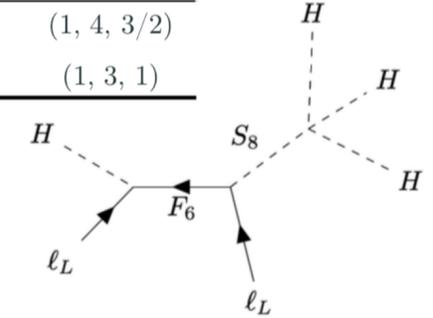
j-basis	Model
$\mathcal{O}_{HH \rightarrow LL}^{(0,1)} = \mathcal{O}^A$	N/A
$\mathcal{O}_{HH \rightarrow LL}^{(0,3)} = \mathcal{O}^S$	type II

Top-Down from Bottom-up

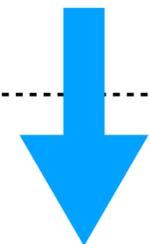
[Xu-Xiang Li, Zhe Ren, **J.H.Yu**, 2307.10380]



Field	(C, W, Y)
S_8	(1, 4, 3/2)
F_6	(1, 3, 1)



Operators	F_1	S_6	F_5	$S_8 \& F_6$	$0\nu\beta\beta$	other
\mathcal{O}_5	✓	✓	✓	✗	✓	
\mathcal{O}_H	✗	✓	✗	✓		
$\mathcal{O}_{H\Box}$	✗	✓	✗	✗		
\mathcal{O}_{HD}	✗	✓	✗	✗		
\mathcal{O}_{eH}	✗	✓	✓	✗		
\mathcal{O}_{uH}	✗	✓	✗	✗		
\mathcal{O}_{dH}	✗	✓	✗	✗		
$\mathcal{O}_{Hl}^{(1)}$	✓	✗	✓	✓		
$\mathcal{O}_{Hl}^{(3)}$	✓	✗	✓	✓		
\mathcal{O}_{ll}	✗	✓	✗	✗		
\mathcal{O}_{LH}	✓	✓	✓	✓	✓	τ
\mathcal{O}_{LeHD}	✓	✗	✓	✗	✓	τ
\mathcal{O}_{LHD1}	✗	✓	✓	✗	✓	τ, K
\mathcal{O}_{LHD2}	✓	✓	✓	✗	✗	τ
\mathcal{O}_{LHW}	✓	✗	✓	✗	✓	τ, K, Dip
\mathcal{O}_{LHB}	✗	✗	✗	✗	✗	Dip
\mathcal{O}_{eLLLH}	✓	✗	✓	✗	✗	τ
\mathcal{O}_{dLQLH1}	✓	✗	✓	✗	✓	τ
\mathcal{O}_{dLQLH2}	✓	✗	✓	✗	✓	τ
\mathcal{O}_{QuLLH}	✓	✗	✓	✗	✓	τ



Operators	Seesaw models			$Y_{\nu, \Sigma} \stackrel{?}{=} 0$
	Type-I	Type-II	Type-III	
One-loop matching				
\mathcal{O}_W	✗	✓	$\sqrt{-N_\Sigma}$	No
\mathcal{O}_{HW}	✗	✓	✗	Yes
\mathcal{O}_{HB}	✗	✓	✗	Yes
\mathcal{O}_{HWB}	✗	✓	✗	Yes
\mathcal{O}_{eW}	✓	$\sqrt{(-\frac{3}{5})}$	$\sqrt{(\frac{9}{5})}$	No
\mathcal{O}_{eB}	✓	$\sqrt{(6)}$	$\sqrt{(3)}$	No
$\mathcal{O}_{Hl}^{(1)}$	✗	✓	✗	Yes
$\mathcal{O}_{Hl}^{(3)}$	✗	✓	✓	Yes
$\mathcal{O}_{Hq}^{(1)}$	✗	✓	✗	Yes
$\mathcal{O}_{Hq}^{(3)}$	✗	✓	✓	Yes
$\mathcal{O}_{ll}^{(1)}$	✗	✓	✓	Yes
$\mathcal{O}_{qq}^{(1)}$	✗	✓	✗	No
$\mathcal{O}_{qq}^{(3)}$	✗	✓	$\sqrt{(4N_\Sigma)}$	No
$\mathcal{O}_{lq}^{(1)}$	✗	✓	✗	Yes
$\mathcal{O}_{lq}^{(3)}$	✗	✓	$\sqrt{(4N_\Sigma)}$	Yes
\mathcal{O}_{ee}	✗	✓	✗	No
\mathcal{O}_{uu}	✗	✓	✗	No
\mathcal{O}_{dd}	✗	✓	✗	No
\mathcal{O}_{eu}	✗	✓	✗	No
\mathcal{O}_{ed}	✗	✓	✗	No
$\mathcal{O}_{ud}^{(1)}$	✗	✓	✗	No
\mathcal{O}_{le}	✗	✓	✗	Yes
\mathcal{O}_{lu}	✗	✓	✗	Yes
\mathcal{O}_{ld}	✗	✓	✗	Yes
\mathcal{O}_{qe}	✗	✓	✗	No
$\mathcal{O}_{qu}^{(1)}$	✗	✓	✗	Yes
$\mathcal{O}_{qu}^{(8)}$	✗	✓	✗	Yes
$\mathcal{O}_{qd}^{(1)}$	✗	✓	✗	Yes
$\mathcal{O}_{qd}^{(8)}$	✗	✓	✗	Yes
\mathcal{O}_{ledq}	✗	✓	✗	Yes
$\mathcal{O}_{quqd}^{(1)}$	✗	✓	✗	Yes
$\mathcal{O}_{lequ}^{(1)}$	✗	✓	✗	Yes

One-loop CDE

[Du, Li, **Yu**, 2201.04646]

[Zhang, Zhou, 2107.12133]

[Li, Zhang, Zhou, 2201.05082]

Complete Dim-7 UV Resonances

[Li, Ni, Xiao, Yu, 2204.03660]

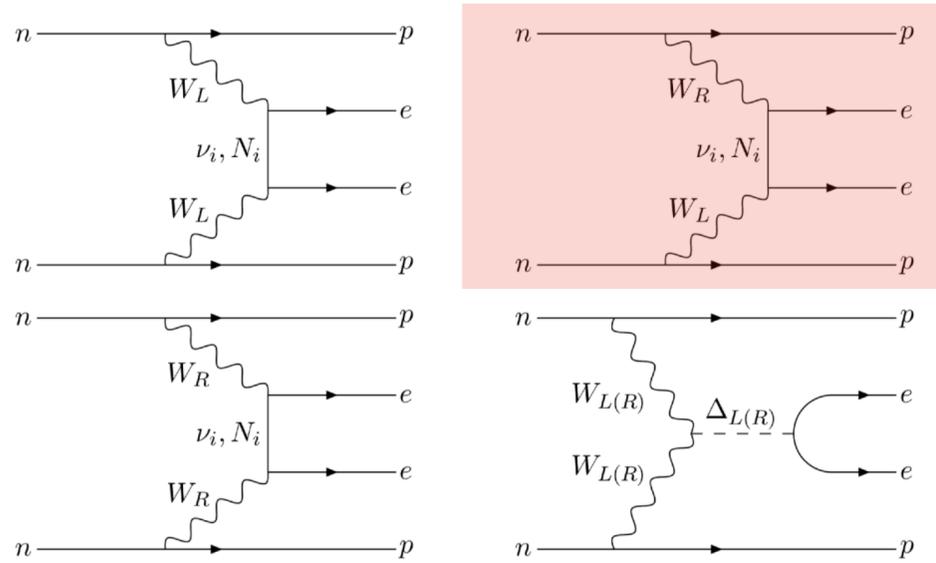
Scalar	
(SU(3) _c , SU(2) ₂ , U(1) _y)	
S1 (1, 1, 0)	$H^3 H^\dagger L^2[(S6), (F5), (F1), (S4, S6), (S4, F5), (S4, F1), (F3, F5), (F1, F3), (S6, F3)]$
S2 (1, 1, 1)	$e_C HL^3[(S4), (F4), (F1)] \quad d_C HL^2 Q[(S4), (F10), (F9)]$ $HL^2 Q^\dagger u_C^\dagger[(S4), (F8), (F12)] \quad De_C H^\dagger L^3[(F1), (F3), (V3)]$
S4 (1, 2, 1/2)	$e_C HL^3[(S6), (S2), (F5), (F1)] \quad d_C HL^2 Q[(S6), (S2), (F5), (F1)]$ $HL^2 Q^\dagger u_C^\dagger[(S6), (S2), (F5), (F1)] \quad H^3 H^\dagger L^2[(S6), (F5), (F1), (S5, S6), (S1, S6), (S6, F5), (S6, F1), (S5, F5), (S5, F1), (S1, F5), (S1, F1)]$
S5 (1, 3, 0)	$H^3 H^\dagger L^2[(S6), (F1, F5), (S6, S7), (S4, S6), (S7, F5), (S4, F5), (S4, F1), (F5, F7), (F3, F5), (F1, F3), (S6, F7), (S6, F3)]$
S6 (1, 3, 1)	$D^2 H^2 L^2 \quad e_C HL^3[(S4), (F4), (F5)] \quad d_C HL^2 Q[(S4), (F10), (F14)]$ $HL^2 Q^\dagger u_C^\dagger[(S4), (F13), (F12)]$ $De_C H^\dagger L^3[(F5), (F3), (V3)] \quad H^2 L^2 W_L[(F7)]$ $H^3 H^\dagger L^2[(S4), (S8), (S7), (S5), (S1), (F5, F6), (F1, F6), (S5, S7), (S4, S5), (S1, S4), (S7, F5), (S4, F5), (S4, F1), (F5, F7), (F3, F5), (F1, F3), (S8, F6), (F6, F7), (F3, F6), (S5, F7), (S5, F3), (S1, F3)]$ $H^2 L^2 W_L \quad B_L H^2 L^2 \quad e_C HL^3$ $HL^2 Q^\dagger u_C^\dagger \quad d_C HL^2 Q \quad De_C^\dagger H^3 L$
S7 (1, 4, 1/2)	$H^3 H^\dagger L^2[(S6), (F5), (S5, S6), (S6, F5), (S5, F5)]$
S8 (1, 4, 3/2)	$H^3 H^\dagger L^2[(S6), (F6), (S6, F6)]$
S10 (3, 1, -1/3)	$d_C^2 HLu_C[(S12), (F10), (F1)] \quad d_C HL^2 Q[(S12), (F10), (F1)]$ $d_C e_C^\dagger HLu_C^\dagger[(S12), (F10), (F1)] \quad d_C HLQ^{\dagger 2}[(S12), (F10), (F1)]$
S11 (3, 1, 2/3)	$d_C^3 H^\dagger L[(S12), (F11), (F2)] \quad d_C^2 HLu_C[(F11), (S13), (F1)] \quad d_C^2 e_C^\dagger HQ^\dagger[(S13), (F3), (F8)]$
S12 (3, 2, 1/6)	$d_C^3 H^\dagger L[(S11), (F11)] \quad d_C^2 HLu_C[(F11), (S10), (F10)]$ $d_C HL^2 Q[(S10), (S14), (F5), (F1), (F14), (F9)] \quad d_C e_C^\dagger HLu_C^\dagger[(S10), (F3), (F12)]$
S13 (3, 2, 7/6)	$d_C^2 HLu_C[(S11), (F10)] \quad d_C^2 e_C^\dagger HQ^\dagger[(S11), (F10)]$
S14 (3, 3, -1/3)	$d_C HL^2 Q[(S12), (F10), (F5)] \quad d_C HLQ^{\dagger 2}[(S12), (F10), (F5)]$

Fermion	
(SU(3) _c , SU(2) ₂ , U(1) _y)	
F1 (1, 1, 0)	$D^2 H^2 L^2 \quad e_C HL^3[(S4), (S2)] \quad d_C HL^2 Q[(S4), (S10), (S12)]$ $HL^2 Q^\dagger u_C^\dagger[(S4), (V5), (V8)] \quad De_C H^\dagger L^3[(F3), (V2)]$ $d_C^2 HLu_C[(S11), (S10)] \quad d_C e_C^\dagger HLu_C^\dagger[(S10), (V5)] \quad d_C HLQ^{\dagger 2}[(S10), (V8)]$ $H^2 L^2 W_L[(F5)]$ $H^3 H^\dagger L^2[(S4), (S5, F5), (S1), (S6, F6), (F3, F5), (F3), (F3, F6), (S4, S6), (S6, F3), (S4, S5), (S1, S4), (S5, F3), (S1, F3)]$ $H^2 L^2 W_L \quad B_L H^2 L^2 \quad e_C HL^3$ $HL^2 Q^\dagger u_C^\dagger \quad d_C HL^2 Q \quad De_C^\dagger H^3 L$
F2 (1, 1, 1)	$d_C^3 H^\dagger L[(S11)]$
F3 (1, 2, 1/2)	$De_C H^\dagger L^3[(F5), (F1), (S6), (V2)] \quad d_C e_C^\dagger HLu_C^\dagger[(S12), (V8)]$ $d_C^2 e_C^\dagger HQ^\dagger[(V8), (S11)] \quad H^3 H^\dagger L^2[(F5), (F1, F5), (F1), (F5, F6), (F1, F6), (S6, F5), (S6, F1), (S5, F5), (S5, F1), (S1, F5), (S1, F1), (S6, F6), (S5, S6), (S1, S6)]$
F4 (1, 2, 3/2)	$e_C HL^3[(S6), (S2)]$
F5 (1, 3, 0)	$e_C HL^3[(S4), (S6)] \quad d_C HL^2 Q[(S4), (S12), (S14)] \quad HL^2 Q^\dagger u_C^\dagger[(S4), (V9), (V8)]$ $D^2 H^2 L^2 \quad De_C H^\dagger L^3[(S6), (F3), (V5)] \quad d_C HLQ^{\dagger 2}[(S14), (V8)]$ $H^2 L^2 W_L[(F7), (F1)] \quad H^3 H^\dagger L^2[(S4), (S7), (S5, F1), (S1), (S6, F6), (F7), (F3), (F1, F3), (F6, F7), (F3, F6), (S6, S7), (S4, S6), (S6, F7), (S6, F3), (S5, S7), (S4, S5), (S1, S4), (S5, F7), (S5, F3), (S1, F3)]$ $H^2 L^2 W_L \quad B_L H^2 L^2 \quad e_C HL^3$ $HL^2 Q^\dagger u_C^\dagger \quad d_C HL^2 Q \quad De_C^\dagger H^3 L$
F6 (1, 3, 1)	$H^3 H^\dagger L^2[(S8), (S6, F5), (S6, F1), (F5, F7), (F3, F5), (F1, F3), (S6, S8), (S6, F7), (S6, F3)]$
F7 (1, 4, 1/2)	$H^2 L^2 W_L[(F5), (S6)] \quad H^3 H^\dagger L^2[(F5), (S6, F5), (F5, F6), (S6, F6), (S5, F5), (S5, S6)]$
F8 (3, 1, -1/3)	$HL^2 Q^\dagger u_C^\dagger[(S2), (V8)] \quad d_C HLQ^{\dagger 2}[(V8), (S12), (V5)] \quad d_C^2 e_C^\dagger HQ^\dagger[(V5), (S11)]$
F9 (3, 1, 2/3)	$d_C HL^2 Q[(S12), (S2)]$
F10 (3, 2, -5/6)	$d_C^2 HLu_C[(S12), (S10), (S13)] \quad d_C HL^2 Q[(S10), (S6), (S2), (S14)]$ $d_C e_C^\dagger HLu_C^\dagger[(S10), (V3), (V8)] \quad d_C HLQ^{\dagger 2}[(S10), (S14), (V9), (V5)]$
F11 (3, 2, 1/6)	$d_C^3 H^\dagger L[(S11), (S12)] \quad d_C^2 HLu_C[(S11), (S12)]$
F12 (3, 2, 7/6)	$HL^2 Q^\dagger u_C^\dagger[(S6), (S2), (V9), (V5)] \quad d_C e_C^\dagger HLu_C^\dagger[(V5), (S12), (V3)]$
F13 (3, 3, -1/3)	$HL^2 Q^\dagger u_C^\dagger[(S6), (V8)] \quad d_C HLQ^{\dagger 2}[(V8), (S12), (V9)]$
F14 (3, 3, 2/3)	$d_C HL^2 Q[(S12), (S6)]$

Complete Dim-7 UV for $0\nu\beta\beta$

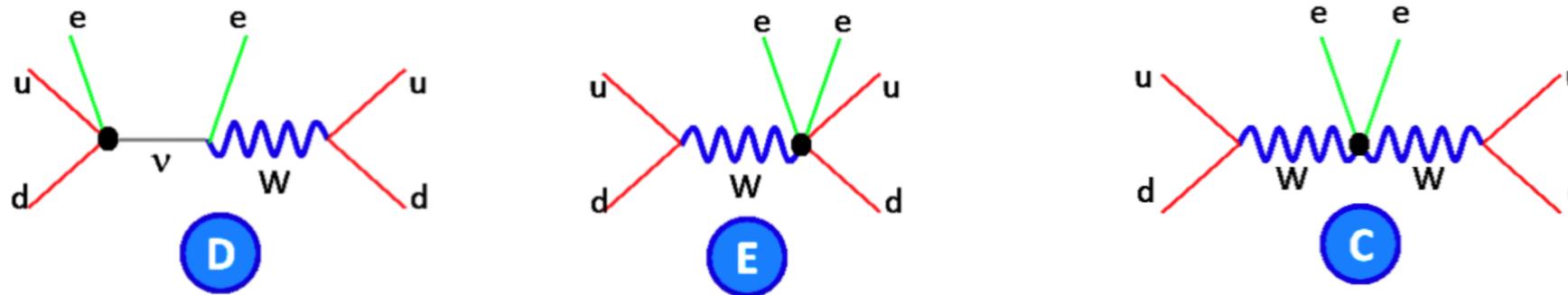
[Xu-Xiang Li, Zhe Ren, **J.H.Yu**, 2307.10380]

energy



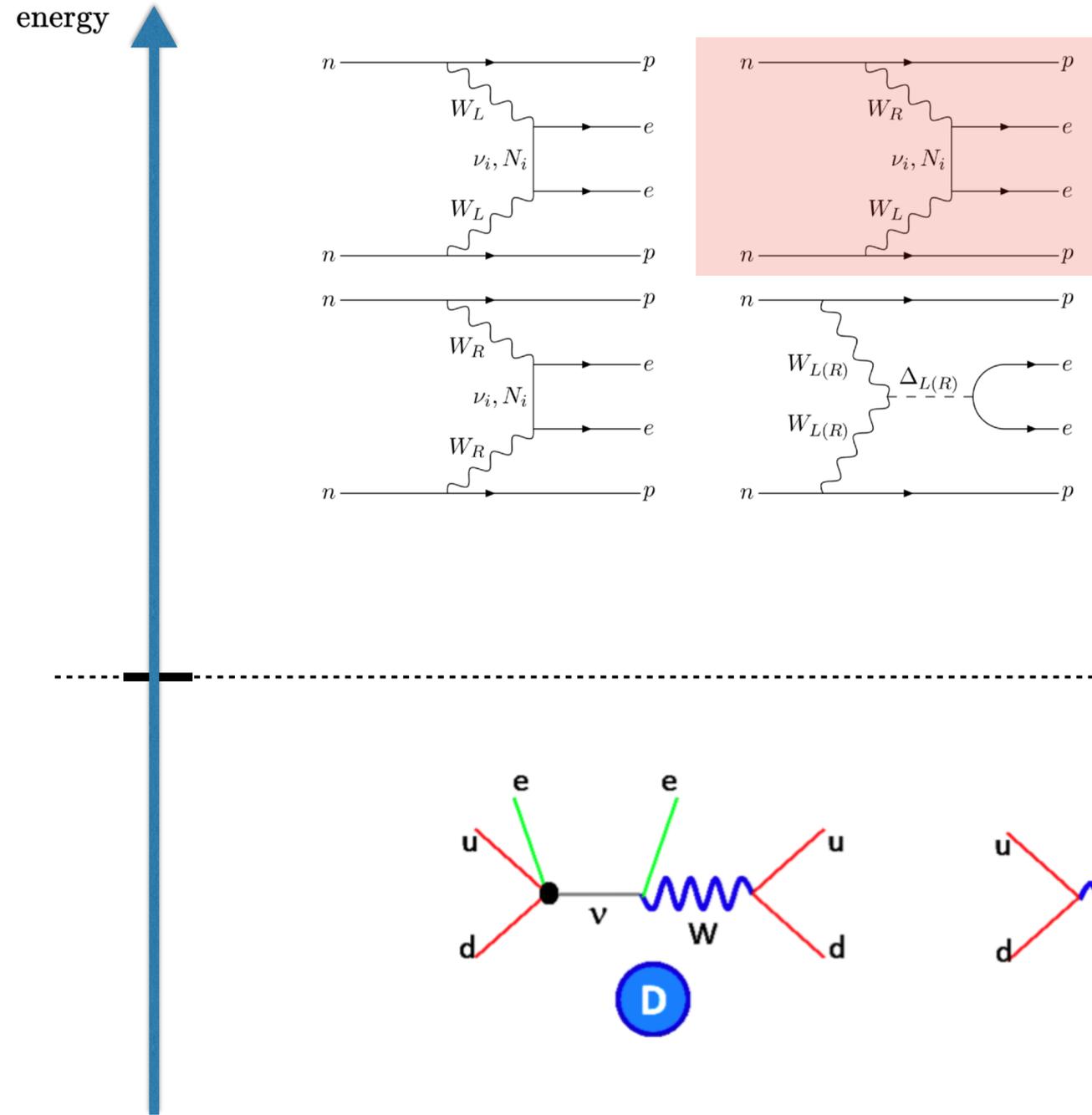
B preserving		B preserving		B violating	
$(S, 1, 1, 1)$	$(S, 1, 2, 1/2)$	$(S, 3, 2, 1/6)$	$(F, 3, 2, 7/6)$	$(S, 3, 1, -1/3)$	$(S, 3, 2, 1/6)$
$(S, 3, 2, 1/6)$	$(S, 3, 3, -1/3)$	$(S, 3, 2, 1/6)$	$(F, 3, 3, 2/3)$	$(S, 3, 1, -1/3)$	$(F, 3, 2, -5/6)$
$(S, 1, 1, 1)$	$(F, 3, 1, -1/3)$	$(S, 3, 3, -1/3)$	$(F, 3, 2, -5/6)$	$(V, 3, 2, 1/6)$	$(F, 1, 2, 1/2)$
$(S, 1, 1, 1)$	$(F, 3, 1, 2/3)$	$(V, 1, 1, 1)$	$(F, 1, 2, 1/2)$	$(V, 3, 2, 1/6)$	$(F, 3, 1, -1/3)$
$(S, 1, 1, 1)$	$(F, 3, 2, -5/6)$	$(V, 1, 2, 3/2)$	$(F, 3, 2, -5/6)$	$(V, 3, 2, 1/6)$	$(F, 3, 2, -5/6)$
$(S, 1, 1, 1)$	$(F, 3, 2, 7/6)$	$(V, 1, 2, 3/2)$	$(F, 3, 2, 7/6)$	$(V, 3, 2, 1/6)$	$(F, 3, 3, -1/3)$
$(S, 1, 2, 1/2)$	$(F, 1, 3, 0)$	$(V, 3, 1, 2/3)$	$(F, 3, 2, 7/6)$	$(V, 3, 1, 2/3)$	$(V, 3, 2, 1/6)$
$(S, 3, 2, 1/6)$	$(F, 1, 2, 1/2)$	$(V, 3, 3, 3/2)$	$(F, 3, 2, 7/6)$	$(V, 3, 2, 1/6)$	$(V, 3, 3, 2/3)$
$(S, 3, 2, 1/6)$	$(F, 3, 1, 2/3)$	$(V, 1, 1, 1)$	$(V, 1, 2, 3/2)$		

18 B preserving UV + 8 B violating UV
($0\nu\beta\beta$ w/ no tree ν -mass)



Complete Dim-7 UV for 0vbb

[Xu-Xiang Li, Zhe Ren, **J.H.Yu**, 2307.10380]



B preserving		B preserving		B violating	
$(S, 1, 1, 1)$	$(S, 1, 2, 1/2)$	$(S, 3, 2, 1/6)$	$(F, 3, 2, 7/6)$	$(S, 3, 1, -1/3)$	$(S, 3, 2, 1/6)$
$(S, 3, 2, 1/6)$	$(S, 3, 3, -1/3)$	$(S, 3, 2, 1/6)$	$(F, 3, 3, 2/3)$	$(S, 3, 1, -1/3)$	$(F, 3, 2, -5/6)$
$(S, 1, 1, 1)$	$(F, 3, 1, -1/3)$	$(S, 3, 3, -1/3)$	$(F, 3, 2, -5/6)$	$(V, 3, 2, 1/6)$	$(F, 1, 2, 1/2)$
$(S, 1, 1, 1)$	$(F, 3, 1, 2/3)$	$(V, 1, 1, 1)$	$(F, 1, 2, 1/2)$	$(V, 3, 2, 1/6)$	$(F, 3, 1, -1/3)$
$(S, 1, 1, 1)$	$(F, 3, 2, -5/6)$	$(V, 1, 2, 3/2)$	$(F, 3, 2, -5/6)$	$(V, 3, 2, 1/6)$	$(F, 3, 2, -5/6)$
$(S, 1, 1, 1)$	$(F, 3, 2, 7/6)$	$(V, 1, 2, 3/2)$	$(F, 3, 2, 7/6)$	$(V, 3, 2, 1/6)$	$(F, 3, 3, -1/3)$
$(S, 1, 2, 1/2)$	$(F, 1, 3, 0)$	$(V, 3, 1, 2/3)$	$(F, 3, 2, 7/6)$	$(V, 3, 1, 2/3)$	$(V, 3, 2, 1/6)$
$(S, 3, 2, 1/6)$	$(F, 1, 2, 1/2)$	$(V, 3, 3, 3/2)$	$(F, 3, 2, 7/6)$	$(V, 3, 2, 1/6)$	$(V, 3, 3, 2/3)$
$(S, 3, 2, 1/6)$	$(F, 3, 1, 2/3)$	$(V, 1, 1, 1)$	$(V, 1, 2, 3/2)$		

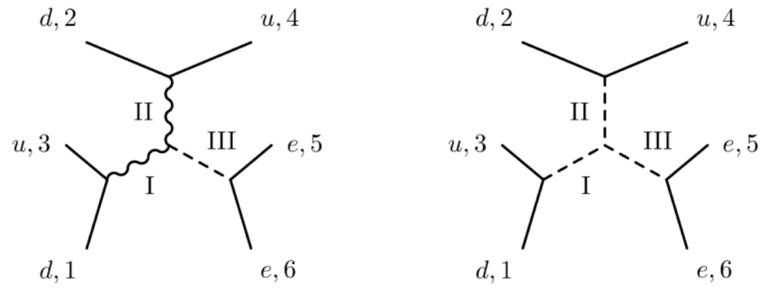
	\mathcal{O}_{LeHD}	\mathcal{O}_{eLLLH}	\mathcal{O}_{dLQLH1}	\mathcal{O}_{dLQLH2}	\mathcal{O}_{dLueH}	\mathcal{O}_{QuLLH}
S_2		$S_4/F_1/F_4$		$S_4/F_9/F_{10}$		$S_4/F_8/F_{12}$
S_4		$S_2/S_6/F_5$	$S_6/F_1/F_5$	$S_2/S_6/F_1/F_5$		$S_2/S_6/F_1/F_5$
S_6	F_3/F_5	$S_4/F_4/F_5$	$S_4/F_{10}/F_{14}$	$S_4/F_{10}/F_{14}$		$S_4/F_{12}/F_{13}$
S_{12}			$F_1/F_5/F_{14}$	$F_5/F_9/F_{14}$	F_3/F_{12}	
F_1	F_3/V_2	S_2	S_4/S_{12}	S_4	V_2/V_5	S_4/V_5
F_3	$S_6/F_1/F_5/V_2$				S_{12}/V_2	
F_4		S_2/S_6				
F_5	S_6/F_3	S_4/S_6	S_4/S_{12}	S_4/S_{12}		S_4/V_9
F_8						S_2
F_9				S_2/S_{12}		
F_{10}			S_6	S_2/S_6	V_3	
F_{12}					$S_{12}/V_3/V_5$	$S_2/S_6/V_5/V_9$
F_{13}						S_6
F_{14}			S_6/S_{12}	S_6/S_{12}		
V_2	$F_1/F_3/V_3$				$F_1/F_3/V_3$	
V_3	V_2				$F_{10}/F_{12}/V_2$	
V_5					F_1/F_{12}	F_1/F_{12}
V_9						F_5/F_{12}

Complete Dim-9 UV for 0vbb

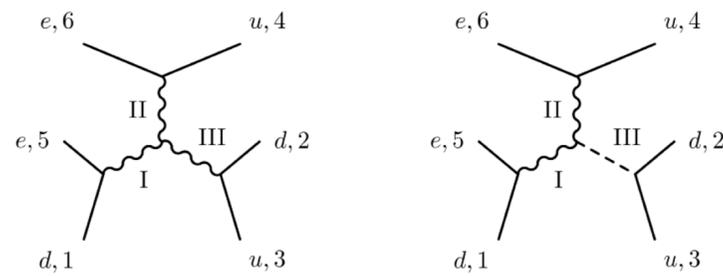
[Li, Ni, Xiao, Yu, in preparation]

[Li, Yu, Zhao, 2311.15422]

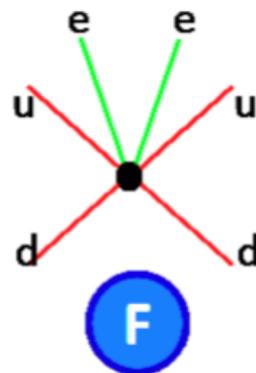
energy



(\mathbf{r}_i, J_i)	$(1, 1, 0)$	$(0, 0, 0)$
$(\mathbf{8}_2, \mathbf{8}_2, \mathbf{1}_3)$	$\frac{4}{3}\mathcal{O}_1 + 2\mathcal{O}_2$	$-\frac{2}{3}\mathcal{O}_2$
$(\mathbf{8}_2, \mathbf{8}_2, \mathbf{1}_1)$	$-2\mathcal{O}_2$	$\frac{4}{3}\mathcal{O}_1 + \frac{2}{3}\mathcal{O}_2$
$(\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_3)$	$-4\mathcal{O}_1 - 6\mathcal{O}_2$	$2\mathcal{O}_2$
$(\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_1)$	$6\mathcal{O}_2$	$-4\mathcal{O}_1 - 2\mathcal{O}_2$



(\mathbf{r}_i, J_i)	$(1, 1, 1)$	$(1, 1, 0)$
$(\mathbf{3}_3, \mathbf{8}_2, \mathbf{3}_2)$	$-\frac{8}{3}\mathcal{O}_1$	$\frac{4}{3}\mathcal{O}_1 + \frac{8}{3}\mathcal{O}_2$
$(\mathbf{3}_1, \mathbf{8}_2, \mathbf{3}_2)$	$-\frac{8}{3}\mathcal{O}_1$	0
$(\mathbf{3}_3, \mathbf{1}_2, \mathbf{3}_2)$	$-\frac{4}{3}\mathcal{O}_1$	$\frac{4}{3}\mathcal{O}_1 + \frac{8}{3}\mathcal{O}_2$
$(\mathbf{3}_1, \mathbf{1}_2, \mathbf{3}_2)$	$-\frac{4}{3}\mathcal{O}_1$	0



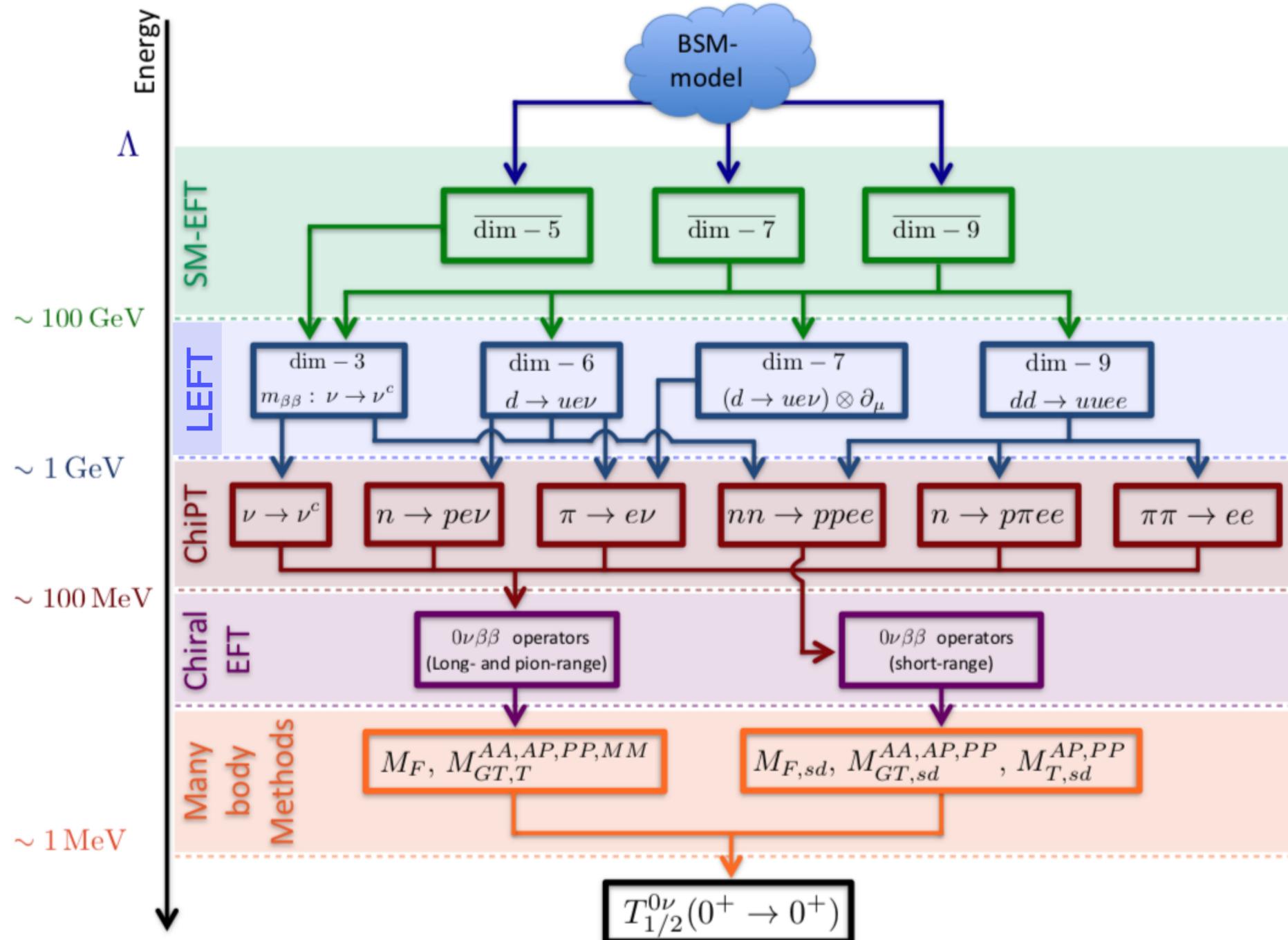
$$\mathcal{O}_1 = \frac{1}{4}(L^\dagger_u{}^j L^\dagger_v{}^i)(Q_{pai}Q_{rbj})(u_{cs}{}^b u_{ct}{}^a)$$

$$\mathcal{O}_2 = -\frac{1}{4}(L^\dagger_u{}^j L^\dagger_v{}^i)(Q_{pai}u_{cs}{}^b)(Q_{rbj}u_{ct}{}^a)$$

Summary

Summary

Operator bases for different EFTs are necessarily needed to provide most general description



UV models new particles
Repeat for each model

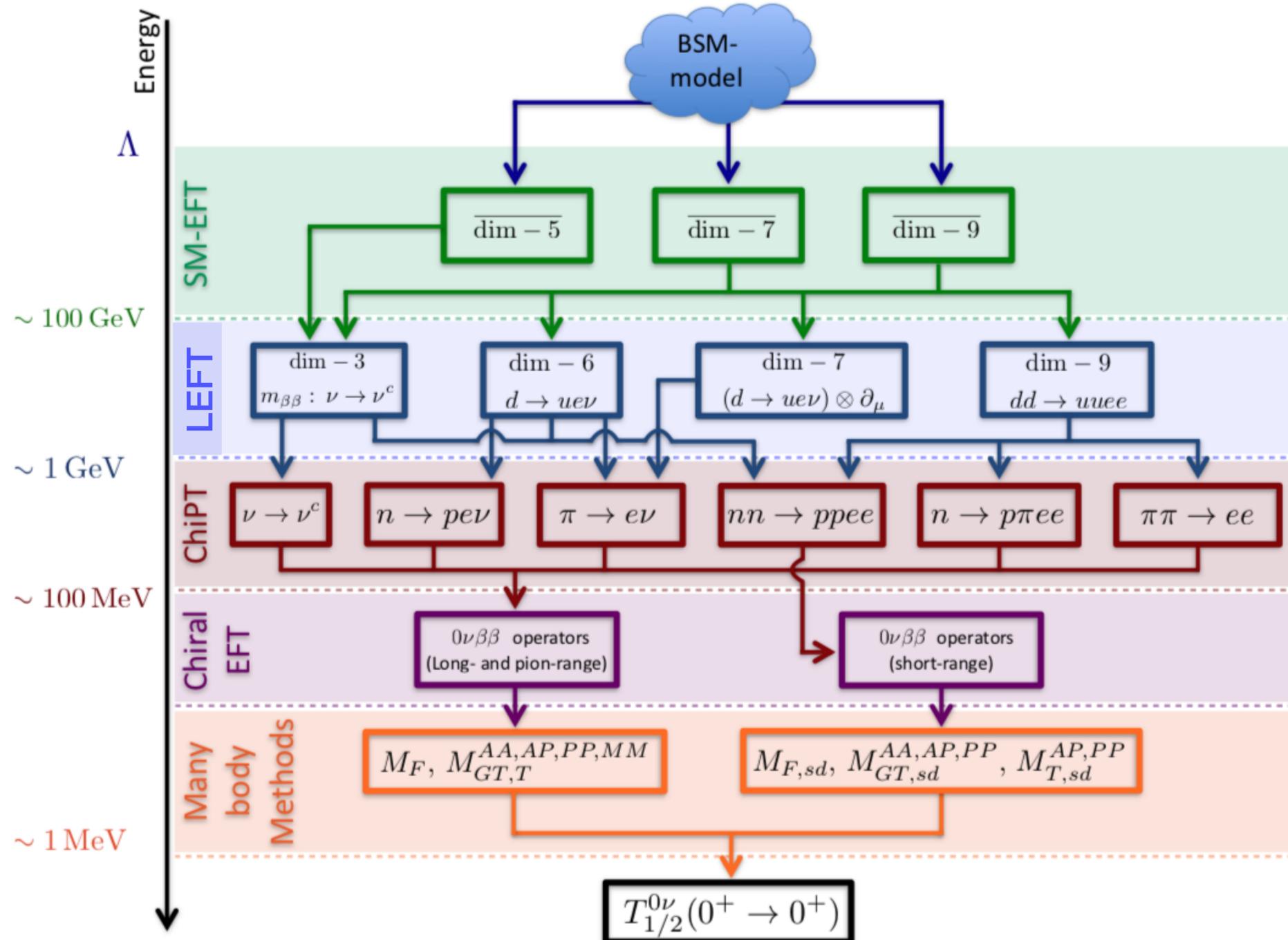
Quark level operators
Operator correlation

Hadronic level operators
How to do matching
External source not enough

Nuclear level operators
Nucleon matrix element incomplete

Summary

Operator bases for different EFTs are necessarily needed to provide most general description



UV models new particles

Quark level operators

Hadronic level operators

Nuclear level operators

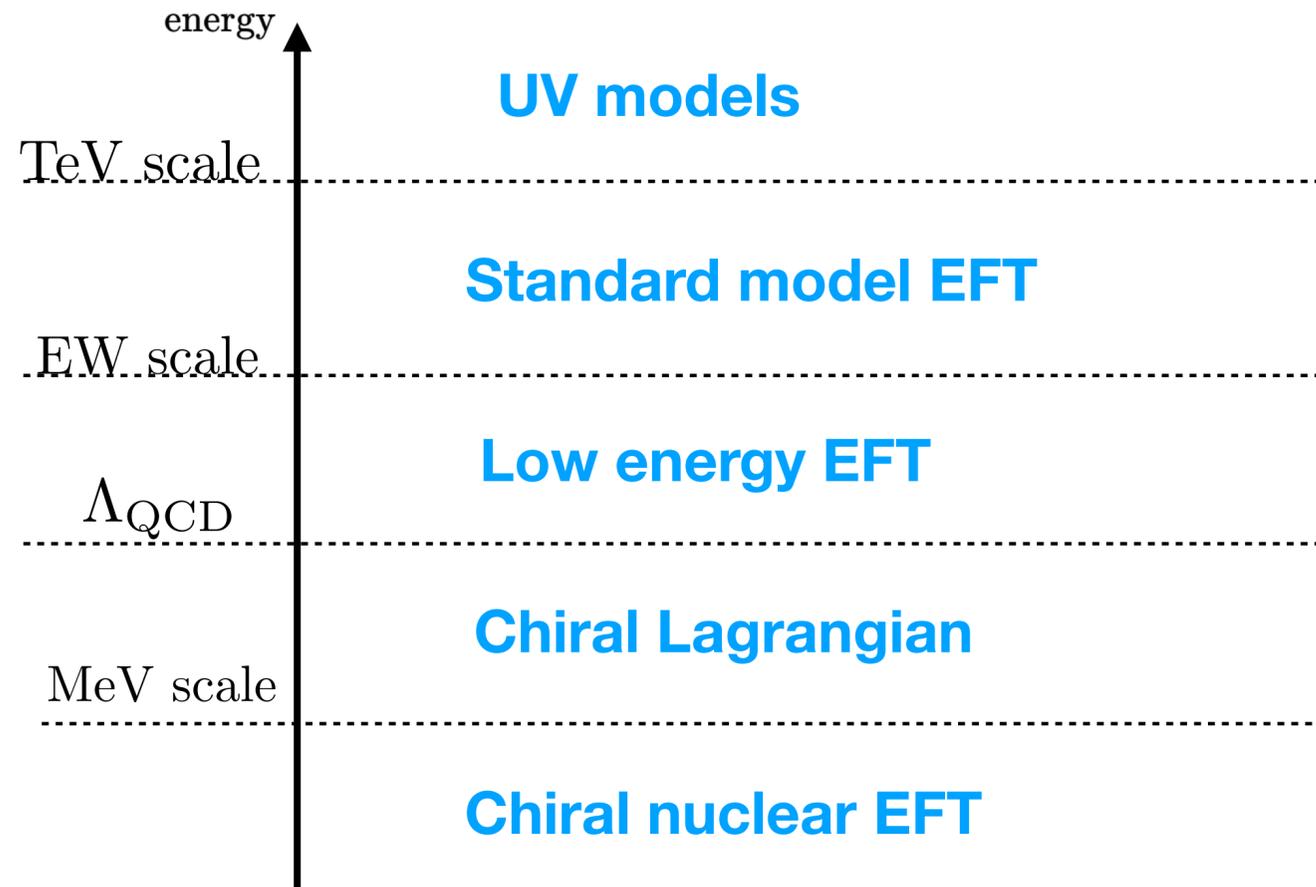
Setup a general framework

Need complete operators at each level

Summary

Neutrino experiments ($0\nu\beta\beta$, etc): low energy probe of high energy new physics

For physics involving in several energy scales, natural theoretical framework is EFT



EFT operators provide most general parametrization on new physics relevant to neutrino pheno

(Light sterile neutrino EFT not discussed in this talk)

The complete UV resonances can be explored using the Casimir projection

Thanks for your attention!

Matching: external source vs spurion

$$\mathcal{L} = \mathcal{L}_{\text{QCD}}^0 + \bar{q}\gamma^\mu [\nu_\mu + \gamma_5 a_\mu] q - \bar{q} [s - i\gamma_5 p] q$$

$$\nu^\mu \rightarrow \bar{e}\gamma^\mu e, \quad a^\mu \rightarrow \bar{e}\gamma^5\gamma^\mu e, \quad s \rightarrow \bar{e}e, \quad p \rightarrow \bar{e}\gamma^5 e$$

Λ_{QCD}

$$\langle u_\mu u^\mu \rangle \langle \Sigma_+ \rangle \quad \langle f_+^{\mu\nu} [u_\mu, u_\nu] \rangle \quad \langle u^\mu [D_\mu f_{+\nu\lambda}, f_-^{\nu\lambda}] \rangle \quad \langle D_\mu f_+^{\mu\nu} [u_\nu, \Sigma_-] \rangle$$

[Gang Li, Chuan-Qiang Song, **J.H.Yu**, 2507.02538]

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2501.09787]

Find same lepton structure, and same T spurion, and matching

$$\langle \bar{q} \overleftrightarrow{\partial}^\mu \mathbf{T} q \rangle (\bar{e}_L \gamma_\mu e_L),$$

$$\langle \bar{q} \overleftrightarrow{\partial}^\mu \mathbf{T} q \rangle (\bar{e}_R \gamma_\mu e_R),$$

$$\langle \bar{q} \gamma^5 \overleftrightarrow{\partial}^\mu \mathbf{T} q \rangle (\bar{e}_L \gamma_\mu e_L),$$

$$\langle \bar{q} \gamma^5 \overleftrightarrow{\partial}^\mu \mathbf{T} q \rangle (\bar{e}_R \gamma_\mu e_R).$$

$$\frac{\Lambda_{\text{EW}}^4}{16\pi^2} \left[\frac{\Lambda_\chi}{\Lambda_{\text{EW}}} \right]^{N_p + 2N_F + \frac{3}{2}N_\psi} \left[\frac{\partial}{\Lambda_\chi} \right]^{N_p} \left[\frac{F}{\Lambda_\chi^2} \right]^{N_F} \left[\frac{\psi}{\sqrt{\Lambda_\chi} f} \right]^{N_\psi}$$

Λ_{QCD}

$$\left[\frac{\Lambda_\chi}{\Lambda_{\text{EW}}} \right]^D \left(f^2 \Lambda_\chi^2 \left[\frac{\partial}{\Lambda_\chi} \right]^{N_p} \left[\frac{F}{\Lambda_\chi^2} \right]^{N_F} \left[\frac{\psi}{\sqrt{\Lambda_\chi} f} \right]^{N_\psi} \right)$$

$$\langle \Sigma_+ [u_\nu, D^\nu u_\mu] \rangle (\bar{e}_L \gamma^\mu e_L), \quad \langle [\Sigma_-, u_\mu] u^\nu u_\nu \rangle (\bar{e}_L \gamma^\mu e_L)$$

$$\langle \bar{B} \gamma^\mu \Sigma_+ B \rangle (\bar{e}_L \gamma_\mu e_L), \quad \langle \bar{B} \gamma^\mu B \Sigma_+ \rangle (\bar{e}_L \gamma_\mu e_L), \quad \langle \bar{B} \gamma^\mu B \rangle \langle \Sigma_+ \rangle (\bar{e}_L \gamma_\mu e_L).$$

Reformulate using SU(3) x SU(3)

$$\bar{q}_L \rightarrow \xi^\dagger, q_L \rightarrow \xi, \bar{q}_R \rightarrow \xi, q_R \rightarrow \xi^\dagger, \xi = \exp\left(\frac{i\Pi}{\sqrt{2}f}\right),$$

$$\bar{q}_L \rightarrow D^\mu \xi^\dagger, q_L \rightarrow D^\mu \xi, \bar{q}_R \rightarrow D^\mu \xi, q_R \rightarrow D^\mu \xi^\dagger,$$

Electroweak processes at nuclear scale

Many body nuclear effects in nuclear physics and particle physics community

