Good LO Hunting

—Chiral EFT for Nuclear Forces

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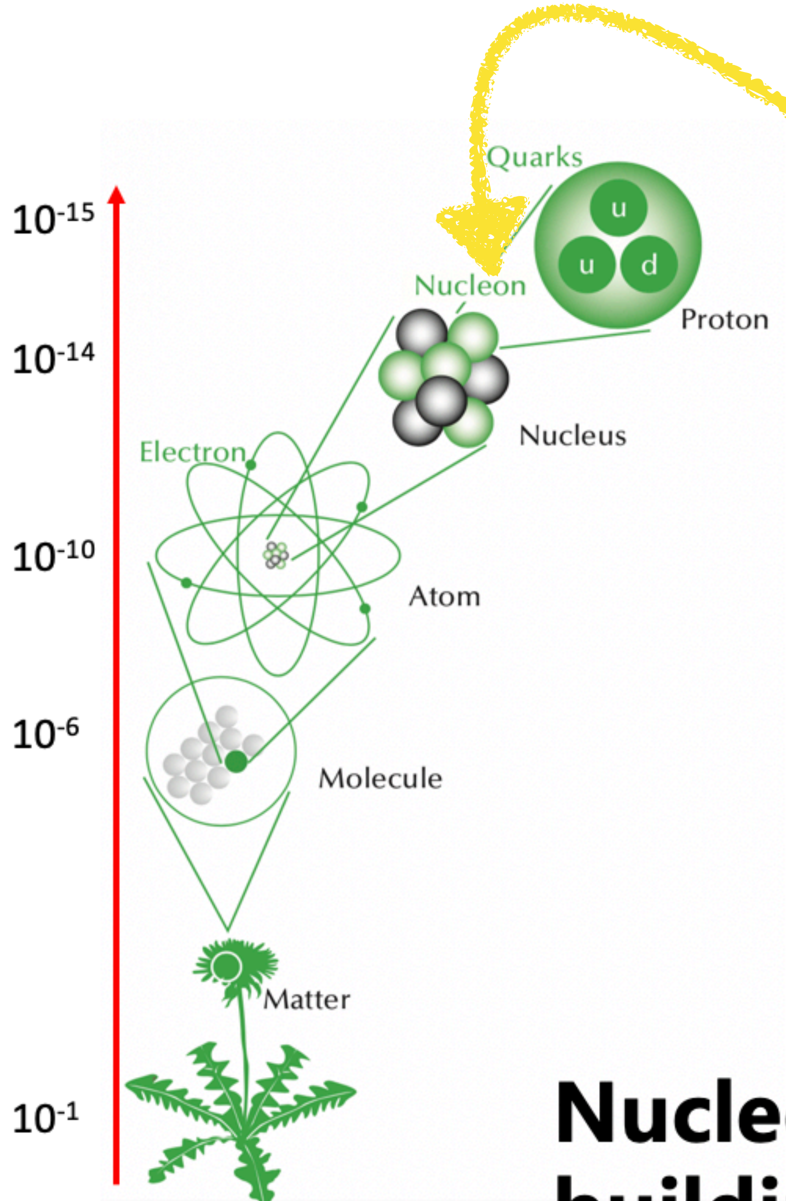
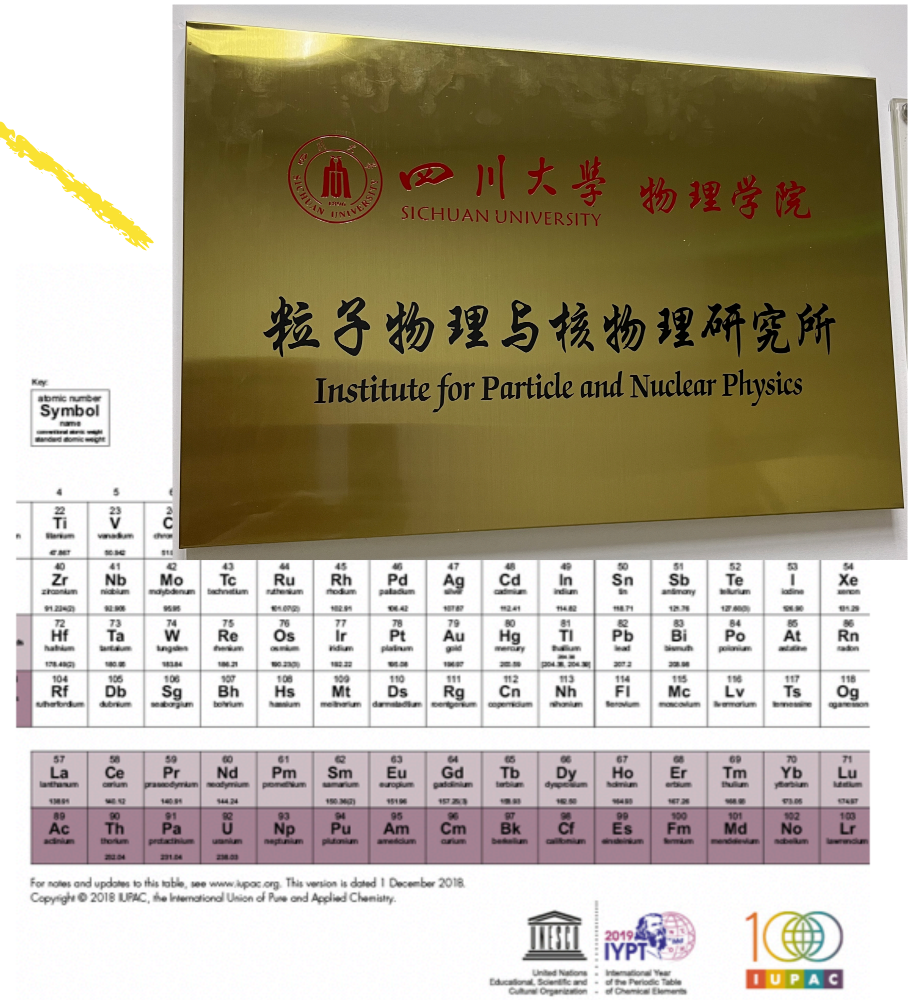
Outline

- From chiral Lagrangian to nuclear forces
- Simplifying EFT nuclear forces:
  - Towards perturbative-pion interaction: Why and How?

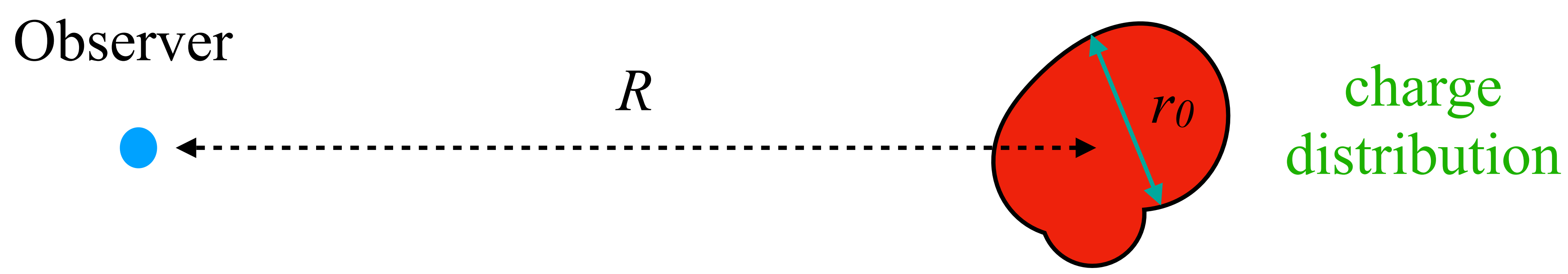
粒子物理与核物理结合，大有可为





Nucleons are the essential building blocks of Matter!

Multipole expansion
A classical example of EFT

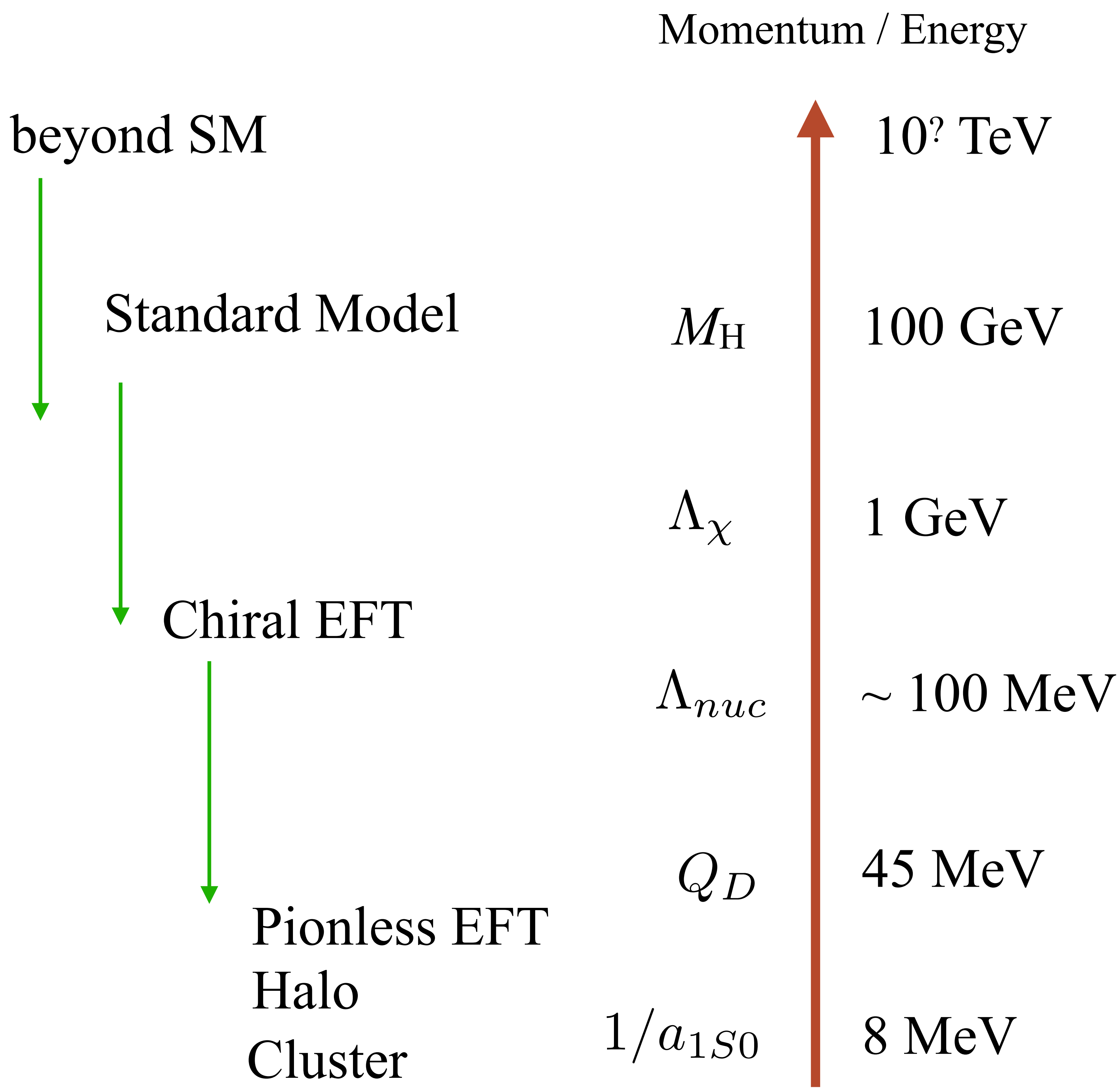


- Separation of scales: $R >> r_0$
- Controlled approximation, able to estimate uncertainty

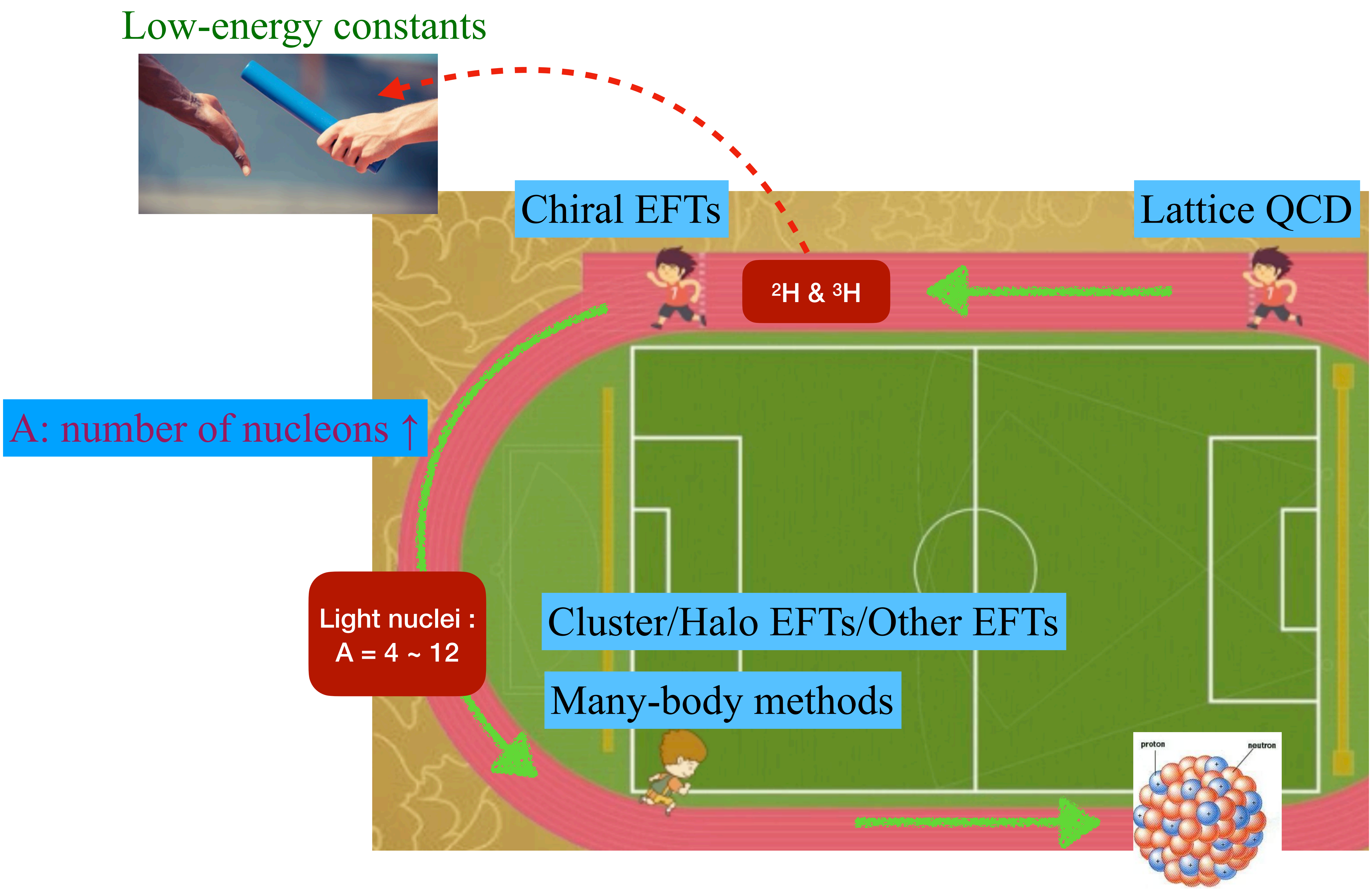
$$V = \frac{q}{R} + \frac{d_i R_i}{R^3} + \frac{Q_{ij} R_i R_j}{R^5} + \cdots$$

- Naturalness $|d_i| \sim q r_0$ $|Q_{ij}| \sim q r_0^2$ $\Rightarrow$ power counting based on naive dim. analysis (NDA)
- What if it is a rod?
  Slow convergence of a regular PC $\Rightarrow$ possible fine-tuning $\Rightarrow$ change PC

Hierarchy of EFTs



Relay: from quarks & gluons to Uranium



QCD → Chiral Lagrangian

External sources: v, a, s, p_s

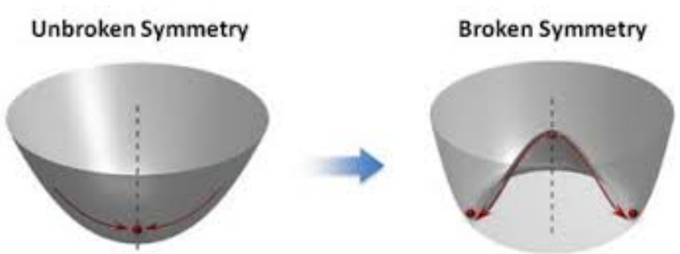
$$\mathscr{L}_{QCD} = \bar{q}_R i\gamma_\mu D^\mu q_R + \bar{q}_L i\gamma_\mu D^\mu q_L - \frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}(s - i\gamma_5 p_s)q + \ldots$$

$$r_\mu = v_\mu + a_\mu = -eQA_\mu\,,$$

$$l_\mu = v_\mu - a_\mu = -eQA_\mu - \frac{g}{\sqrt{2}}(W^\dagger_\mu T_+ + h.c)\,,$$

$$s = \mathcal{M} \equiv \mathrm{diag}(m_q, m_q)\,,$$

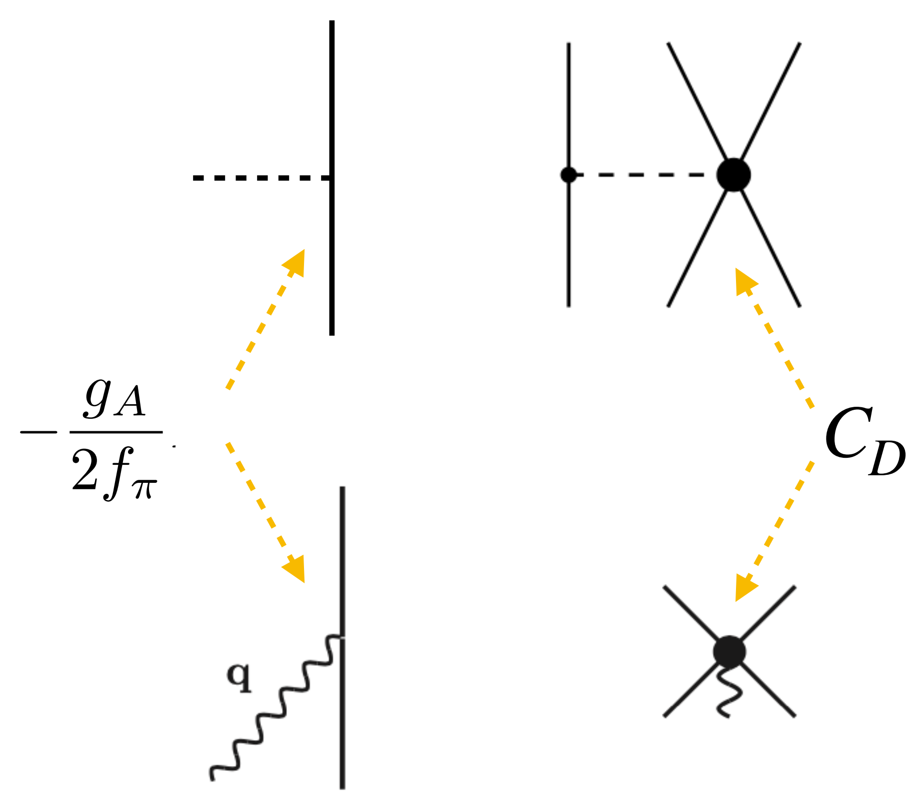
$$p_s = 0\,。$$



nonlinear realization

CCWZ; Weinberg; ...

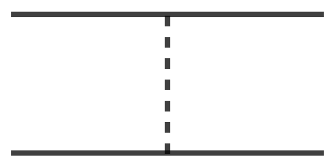
Constraints by chiral symmetry



Wavy lines: axial currents

Potential diagrams

- One-pion exchange



Tensor F

$$V_{1\pi}(\vec{r}\,) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 [T(r)S_{12} + Y(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2]$$

$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

- Tensor force dominant

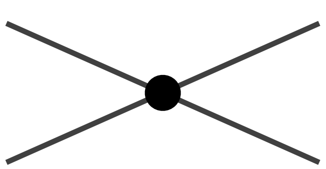
$$T(r) = \frac{e^{-m_\pi r}}{m_\pi r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right]$$

$$r = m_\pi^{-1}: \quad \frac{\langle {}^3S_1|\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \hat{S}_{12} T(1)|{}^3D_1\rangle}{\langle {}^1S_0|\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 Y(1)|{}^1S_0\rangle} = 14\sqrt{2},$$

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r}$$

- Many contact terms parametrizing NN short-range forces, e.g.,

$$V_{1S0} = c_0^{1S0} + c_2^{1S0}(p^2 + p'^2) + \cdots$$

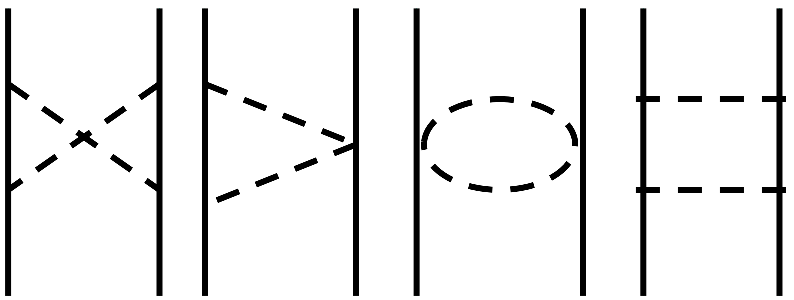


$$V_{3P0} = c_0^{3P0} pp' + \cdots$$

- For Q << mpi —> pionless EFT w/ only contact interactions

Power counting for loops (HBChPT)

- Nucleon propagator — 1/Q or m_N/Q^2 $\dfrac{i}{p_0 - \frac{p^2}{2m_N}}$
- Pion propagator — $1/Q^2$ $\dfrac{i}{k_0^2 - \vec{k}^2 - m_\pi^2}$ $Q \sim m_\pi$
- Loop integral — $Q^4/(16\pi^2)$ $\displaystyle\int \frac{d^4 l}{(2\pi)^4}$



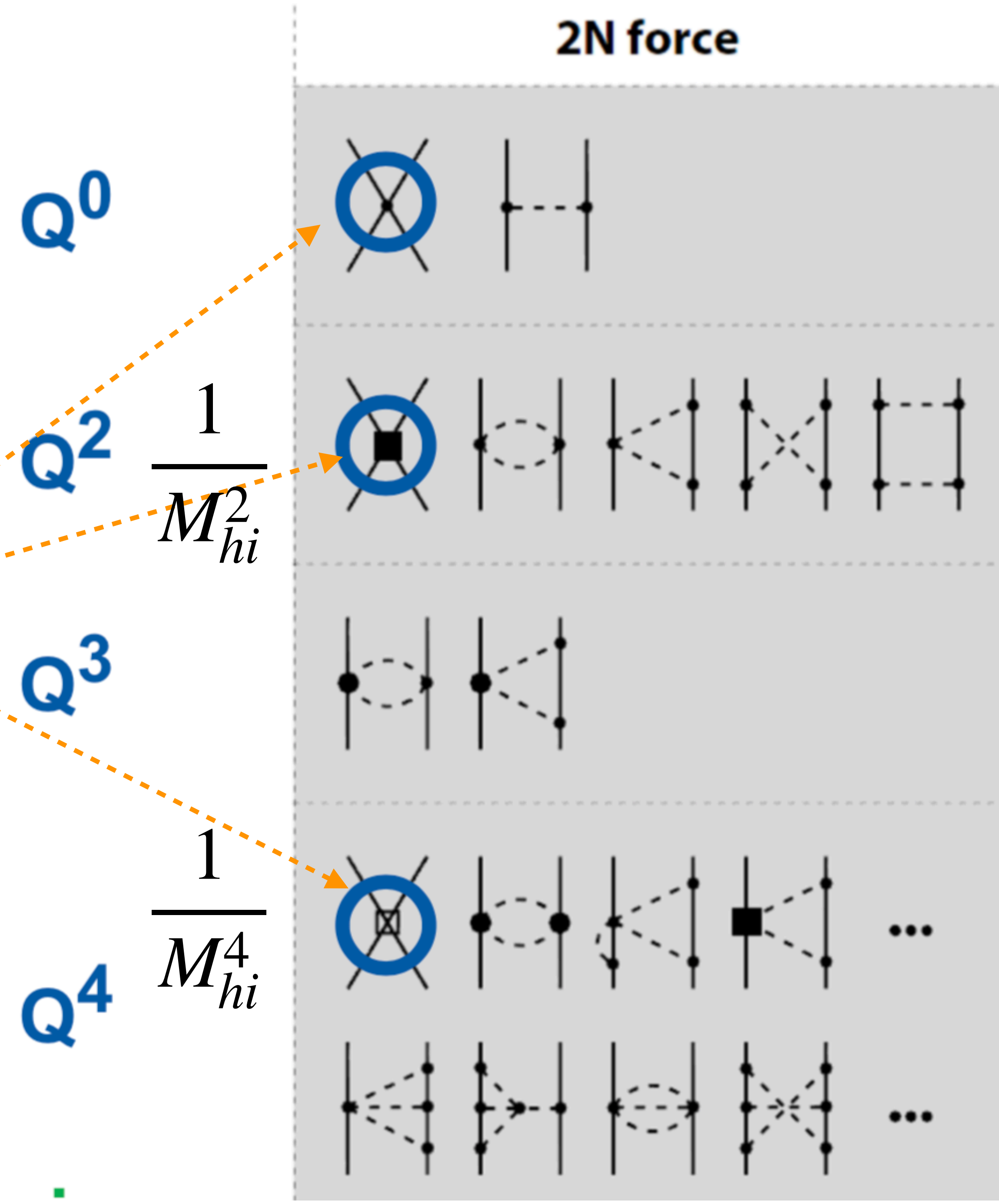
- Vertex with ν derivatives — Q^ν $-\frac{g_A}{2f_\pi} N^\dagger \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a N$
- A pion loop brings a suppression factor of $\left(\dfrac{Q}{4\pi f_\pi}\right)^2$

NN potentials from ChEFT

- Contact forces doing heavy lifting

24 contact terms
in NDA up to Q^4

NDA = naive dim. analysis



van Kolck et al. ’92

Entem & Machleidt ’03

Epelbaum et al. ’99

Pastore, Piarulli, et al. ’09

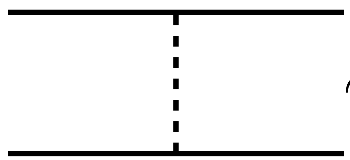
Ekstrom, et al. ’13

…

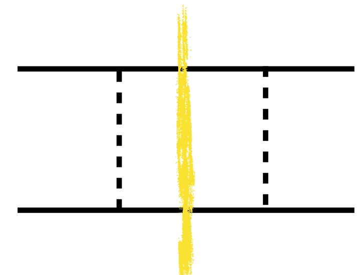
To promote, or not to promote ?

Strength of OPE

Typical size of external momenta: $Q \sim m_\pi$



$$\sim \frac{1}{f_\pi^2}\frac{Q^2}{m_\pi^2+Q^2} \sim \frac{1}{f_\pi^2}$$



$$\sim \frac{1}{f_\pi^2}\frac{m_N}{4\pi f_\pi}\frac{Q}{a_l f_\pi}$$

NN reducible

- Focus on loop momenta ~ external momenta Q
- Pion line or photon line ~ $1/Q^2$, nucleon line in irreducible diagrams ~ $1/Q$
- Nucleon line in reducible diagrams ~ m_N/Q^2
  ⇒ Explain why we solve the Schrodinger eqn
  ⇒ Explain why nuclei bound
- Strength of OPE ~ $a_l f_\pi$ (numerical factor $a_l \sim 1$ for small l, $a_l \gg 1$ for large l by centrifugal suppression)

Power counting: short-range physics

$$\sim \frac{1}{f_\pi^2}\frac{m_N}{4\pi f_\pi}\frac{Q}{a_l f_\pi}$$

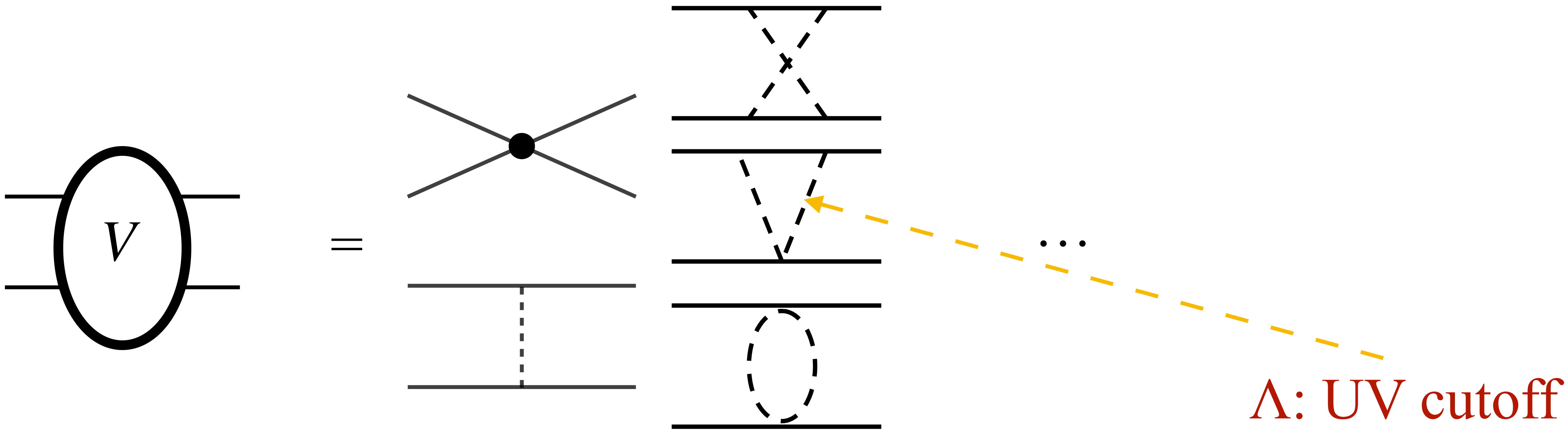
- Strength of OPE $a_l f_\pi$ may have impact on contacts through renormalization

- Coexistence of $a_l f_\pi$ and M_{hi} makes NDA no longer reliable

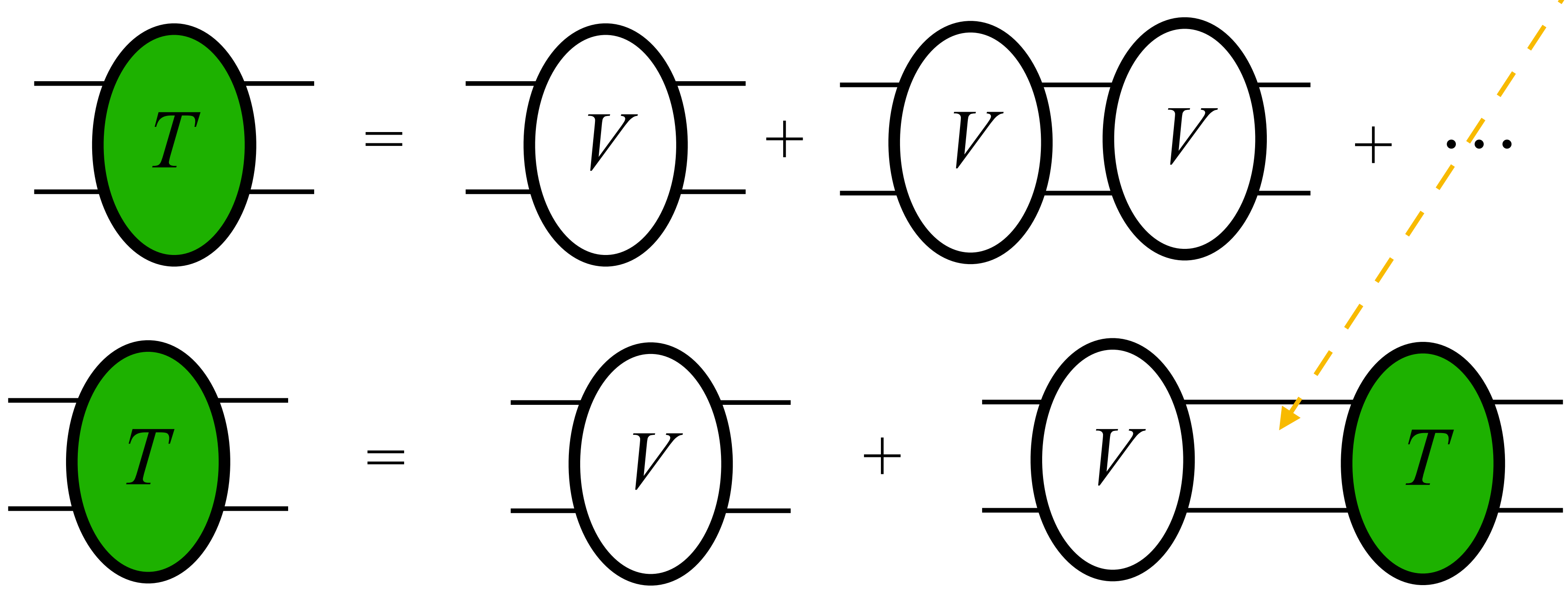
- Operators gaining large anomalous dimension through nuclear dynamics $\rightarrow$ "irrelevant" operators become relevant

UV cutoff

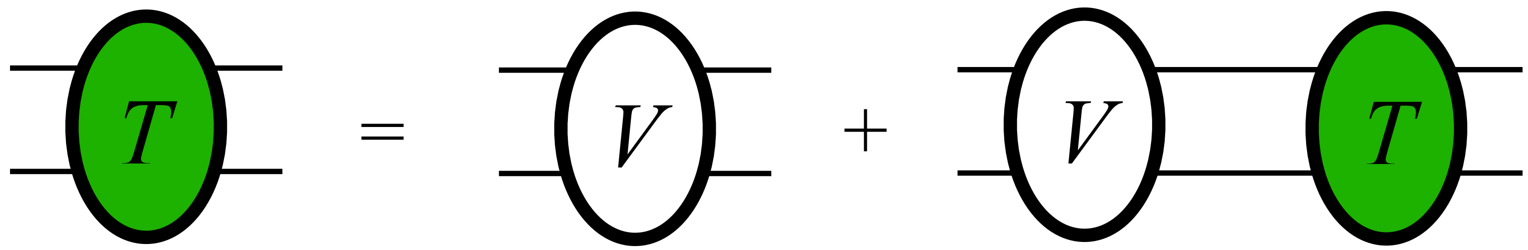
- "*NN* Potential": two-nucleon irreducible diagrams



- Lippmann-Schwinger eqn (equivalent to Schrodinger eqn)



Lippmann-Schwinger equation



$$T(\vec{p}\,', \vec{p}) = V(\vec{p}\,', \vec{p}) + \int \frac{d^3 l}{(2\pi)^3} V(\vec{p}\,', \vec{l}) \frac{T(\vec{l}, \vec{p})}{E - \vec{l}^2/m_N + i\epsilon}$$

- Nucleons only propagate "forward", so l_0 can be integrated out
  Otherwise, antifermion-fermion pairs make this a many-body problem

- p << m_N, relativistic corrections added as perturbations

- A = 3, 4 … nonrelativistic few-body diagrams similarly resummed

- Regularized by UV cutoffs, numerically solved

Renormalization group and power counting

- Power counting of subleading pot. = how (ir)relevant they are in Wilson's RG

$$\frac{\Lambda}{T^{(\nu)}(Q,\Lambda)}\frac{dT^{(\nu)}(Q,\Lambda)}{d\Lambda} = \mathcal{O}\left(\frac{Q^{\nu+1}}{M_{hi}^{\nu}\Lambda}\right)$$

- Program1: Explicitly solving RG equation → PC of contacts

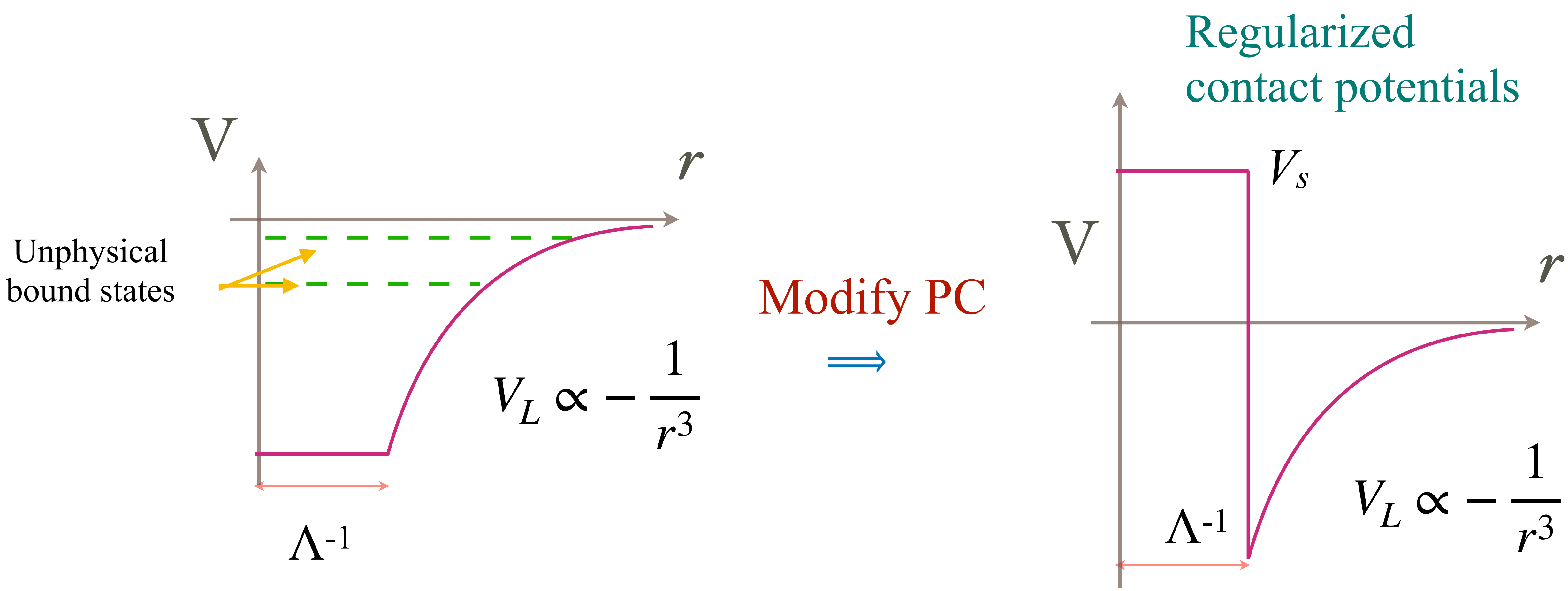
Birse et al. ’99

Pavon Valderrama & Phillips ‘15

- Program 2: Speculating a PC and testing it against cutoff indep
  → one possible solution of RGE, thus an acceptable PC

Renormalizing singular attraction

Beane et al '01
Pavon Valderrama & Ruiz Arriola '05, '07
Nogga et al '05
BwL & van Kolck '07

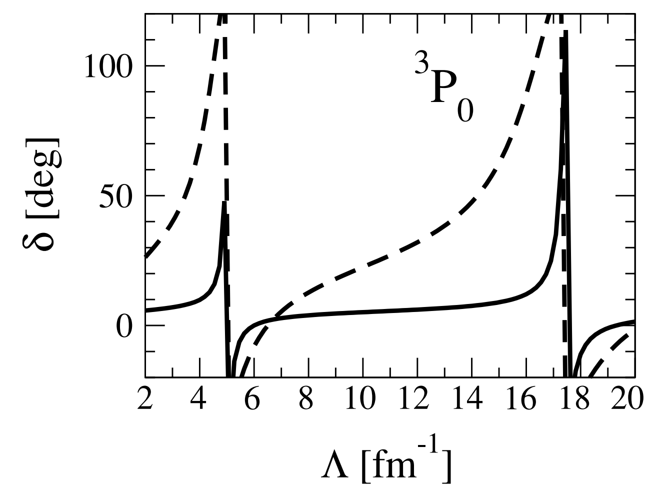


Renormalizing singular attraction

Nogga, Timmerman & van Kolck (2005)

- Manifestation via cutoff dependence

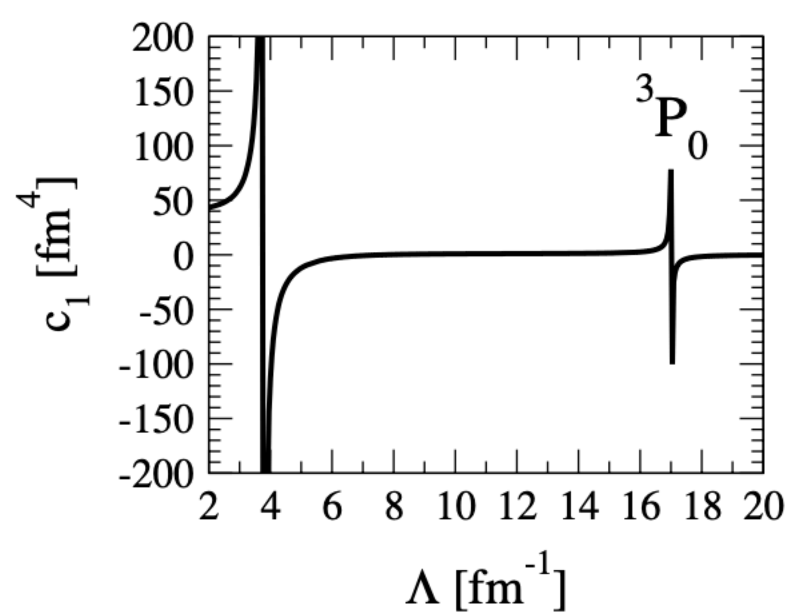
Phase shifts vs. Λ :

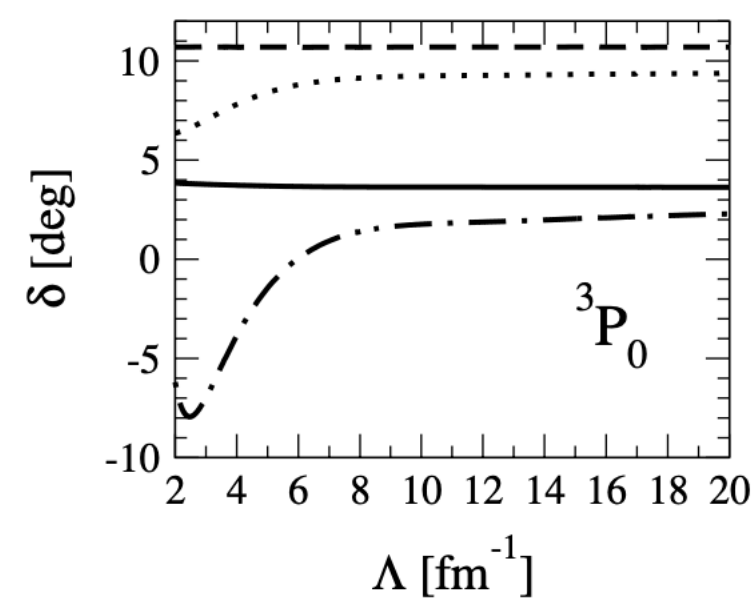


Solid: Tlab = 10 MeV, dashed: 50 MeV

- Solution: change power counting

$$C_{3P0}\vec{p}\cdot\vec{p}' \sim \frac{Q^2}{m_{hi}^2} \xrightarrow{\text{promote}} C_{3P0}\vec{p}\cdot\vec{p}' \sim \frac{Q^2}{m_{lo}^2}$$





Renormalizing singular attraction

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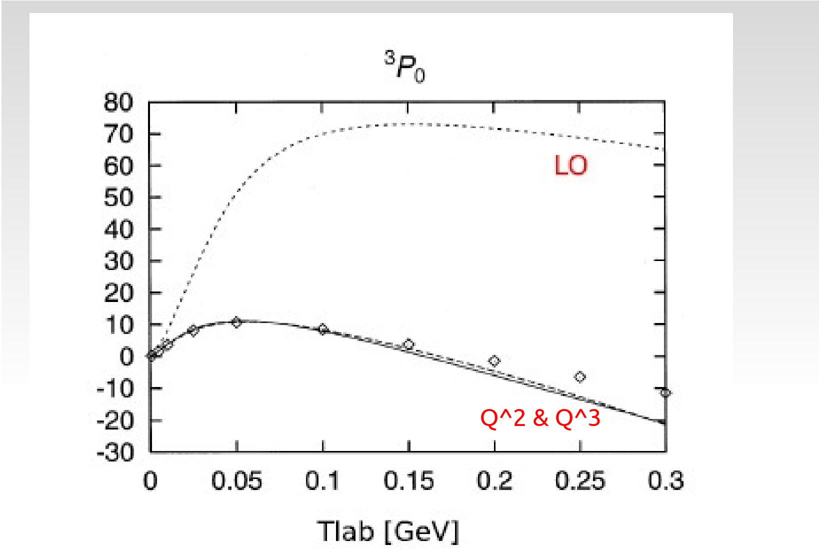
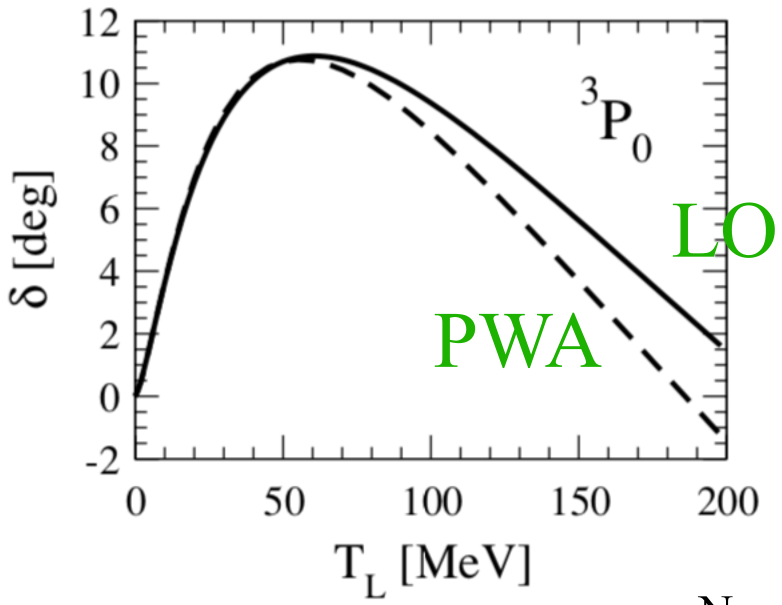


Epelgaum et at, NPA 671, 295



Nogga et al ('05)

Can we simplify these nuclear interactions?

A motivation: Wigner SU(4) symmetry

- Approximate SU(4) invariance of nuclear forces

$$\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix} \quad \text{SU(4) transformation}$$

JANUARY 15, 1937 PHYSICAL REVIEW VOLUME 51

On the Consequences of the Symmetry of the Nuclear Hamiltonian on the Spectroscopy of Nuclei

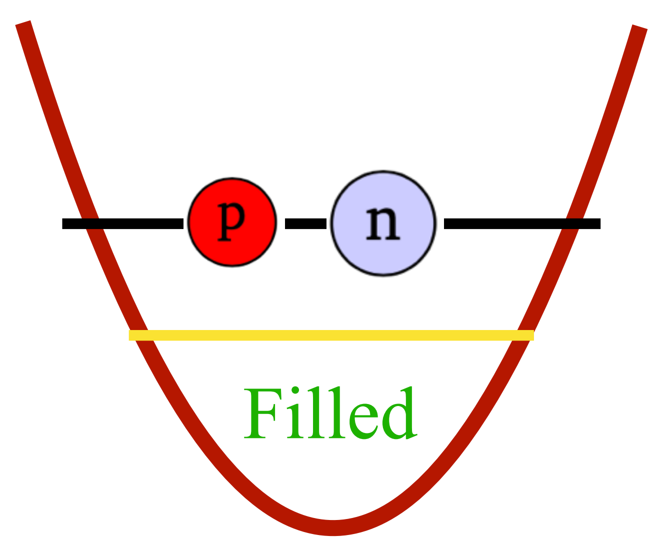
E. WIGNER*
Princeton University, Princeton, New Jersey
(Received October 23, 1936)

- Interactions between NN pairs : (S = 0, T = 1) ≈ (S = 1, T = 0)

2-body: $a_{1S0} \simeq -20$ fm ; $a_{3S1} \simeq 5$ fm

$$\frac{1}{-1/a + \frac{r}{2}k^2 + \cdots - ik}$$

Approximate degenerate states of nuclei:



^{4}He, ^{12}C, ^{16}O, etc. can be viewed as alpha clusters

Wigner symmetry via Chiral EFT?

- OPE tensor force breaks SU(4), badly

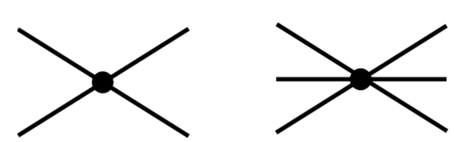
Tensor F

$$V_{1\pi}(\vec{r}\,) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2[T(r)S_{12} + Y(r)\vec{\sigma}_1\cdot\vec{\sigma}_2]$$

$$S_{12} = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

$$r = m_\pi^{-1}: \quad \frac{\langle {}^3S_1|\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\hat{S}_{12}T(1)|{}^3D_1\rangle}{\langle {}^1S_0|\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\vec{\sigma}_1\cdot\vec{\sigma}_2Y(1)|{}^1S_0\rangle} = 14\sqrt{2}$$

- But, SU(4) can implemented in pionless EFT by letting C_T to be perturbation



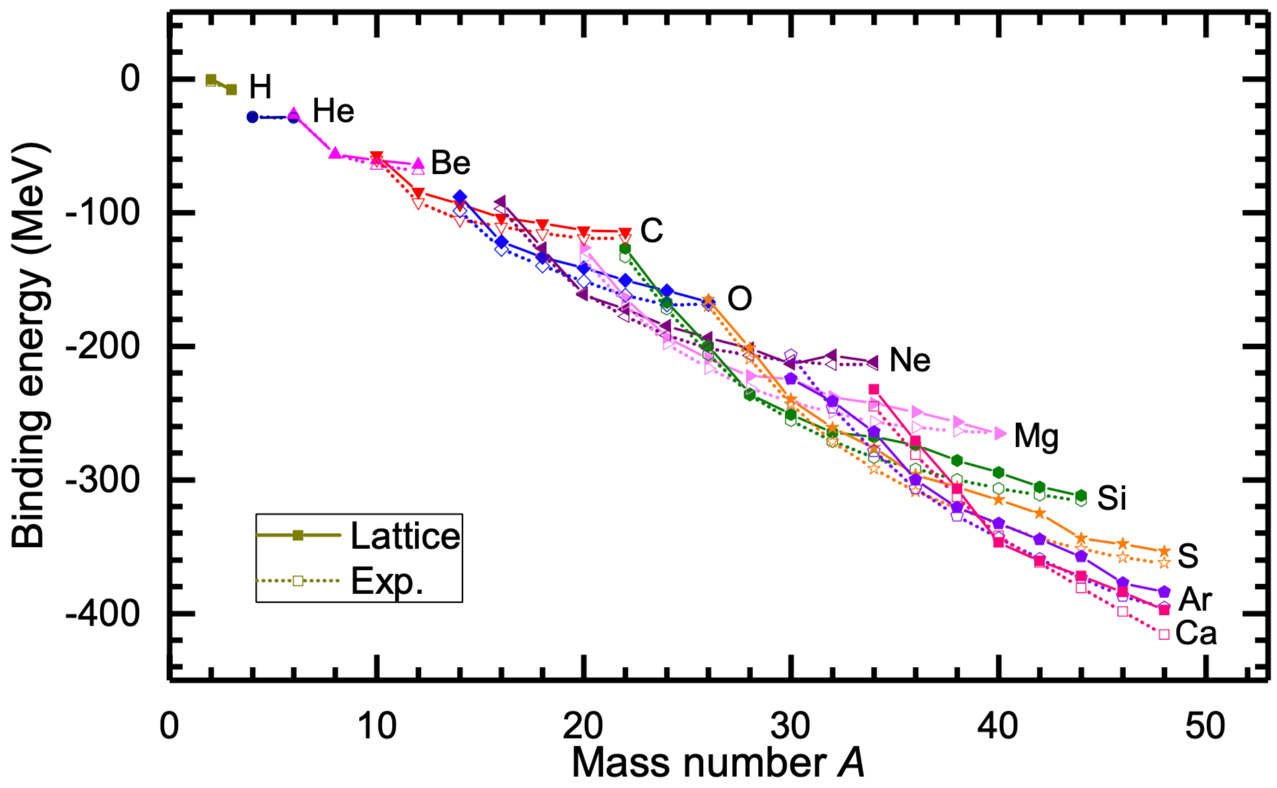
$$-\frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(N^\dagger\vec{\sigma}N)^2 \qquad -h_0(N^\dagger N)^3$$

Success of pionless-like interactions

- Microscopic explanation for recent pionless-like (short-ranged) structure calculations:

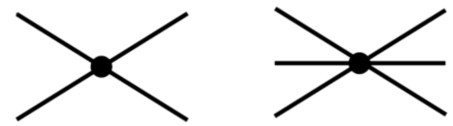
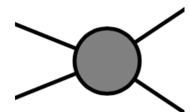
  Koenig et al. '16
  Lyu BN et al. '18
  Gattobigio et al. '19
  …

Lyu et al. '18, Lattice EFT calaboration

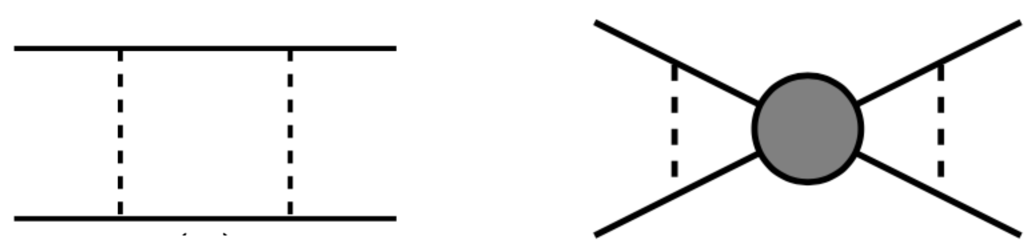


Perturbative pions?

Kaplan, Savage & Wise ’98
Fleming, Mehen & Stewart ’99
Beane, Kaplan & Vuorinen ’08

- Same LO as pionless: 
- LO of NN = bubble sums = 
- Subleading orders = distorted-wave expansion in OPE

NLO:  + ...

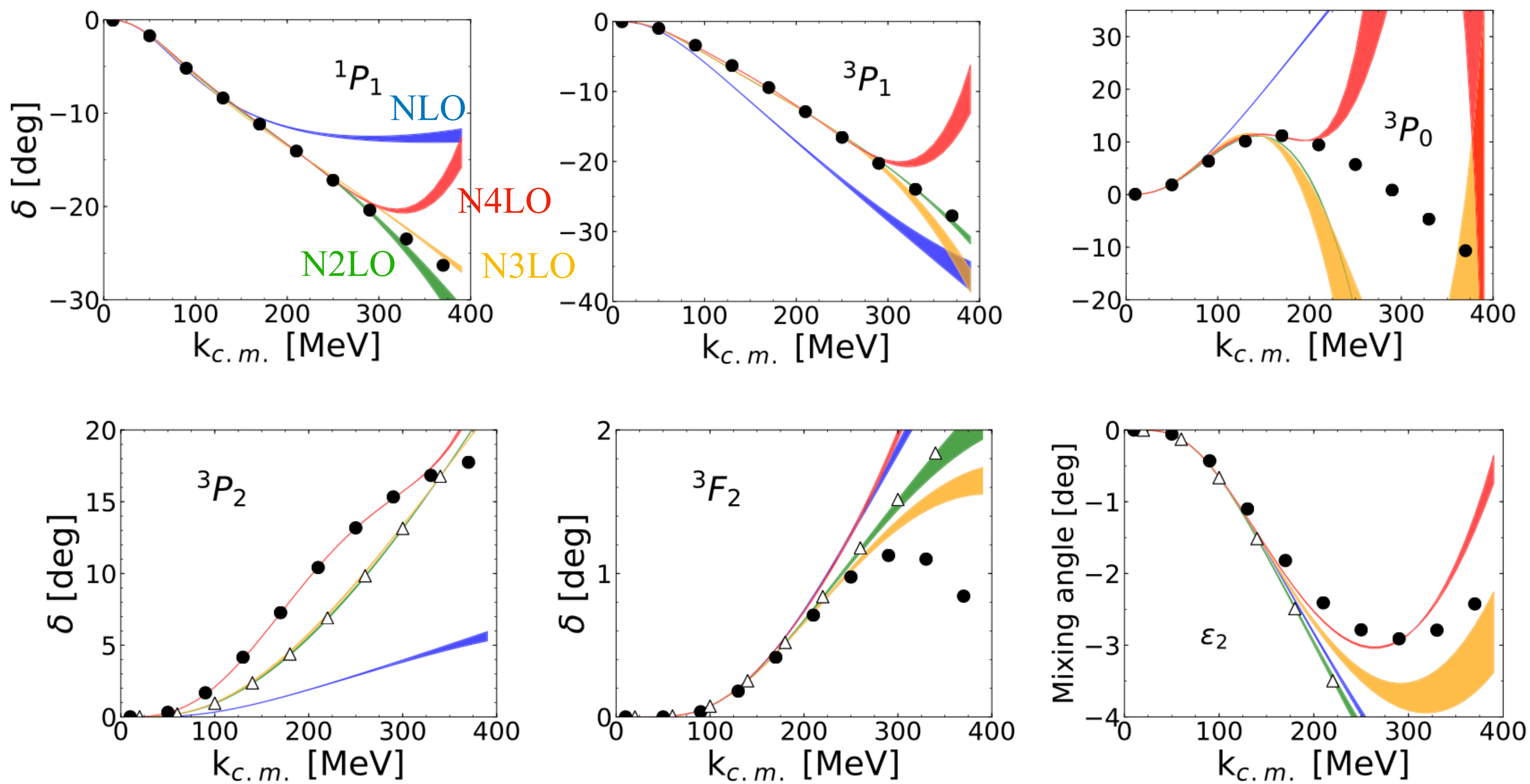
N2LO:  + ...

Where KSW works

- OK for $k < \Delta \simeq 290$ MeV for $l > 0$ except 3P0 — Wu & Long '18, Kaplan '19

- TPEs included in N3LO & N4LO

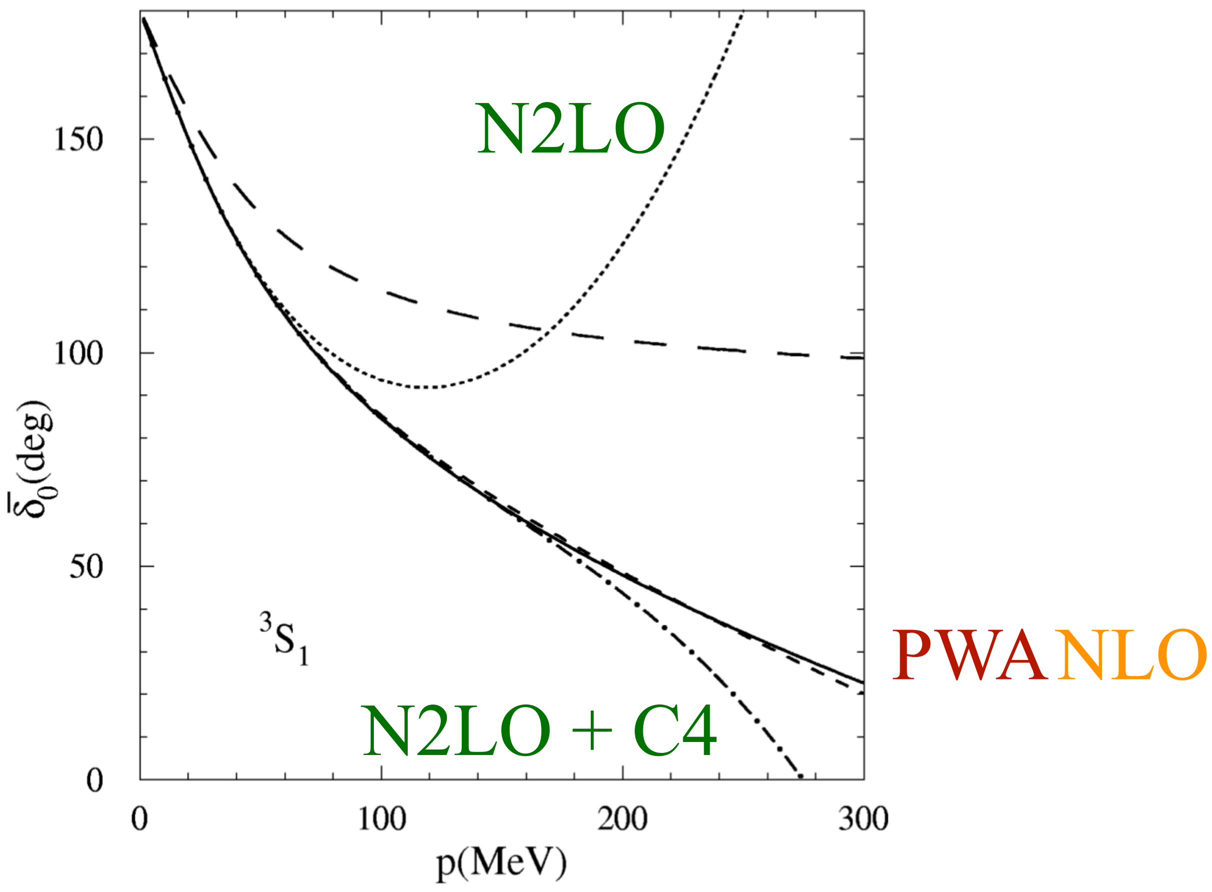
Wu & Long '18

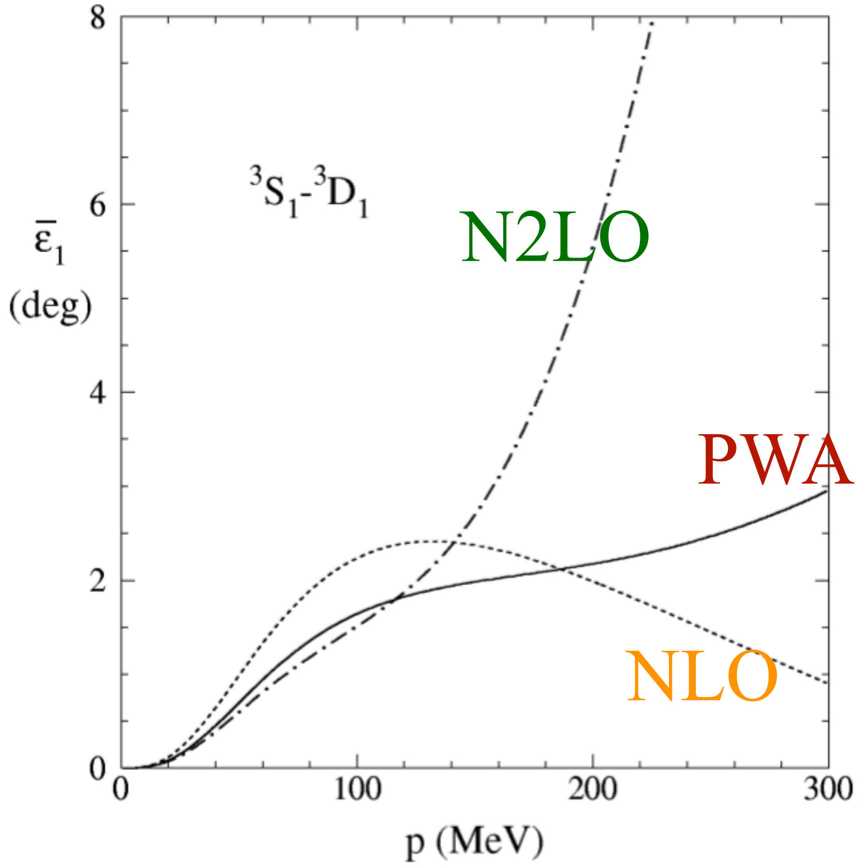


KSW

Fleming, Mehen & Stewart '99

- Convergence not better than pionless, esp. for higher waves





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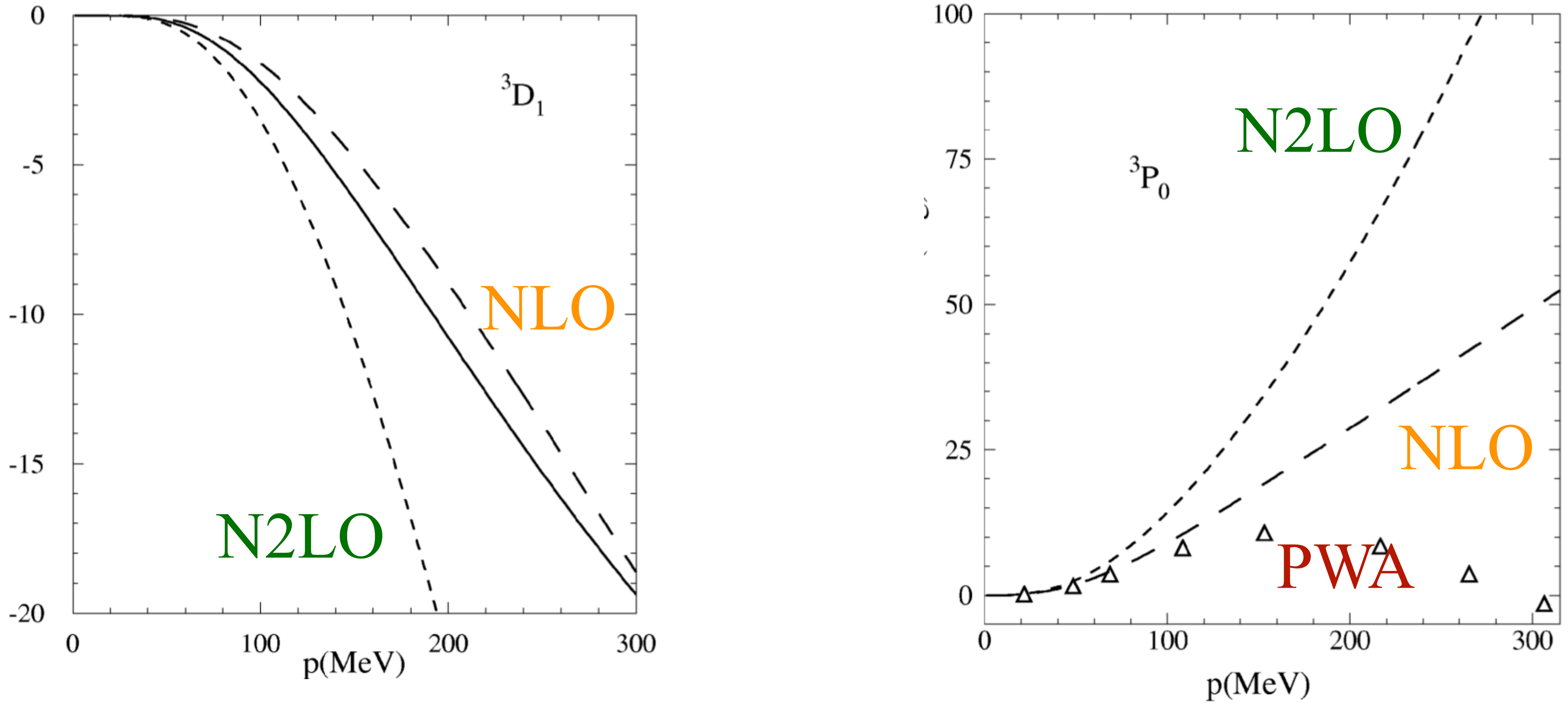
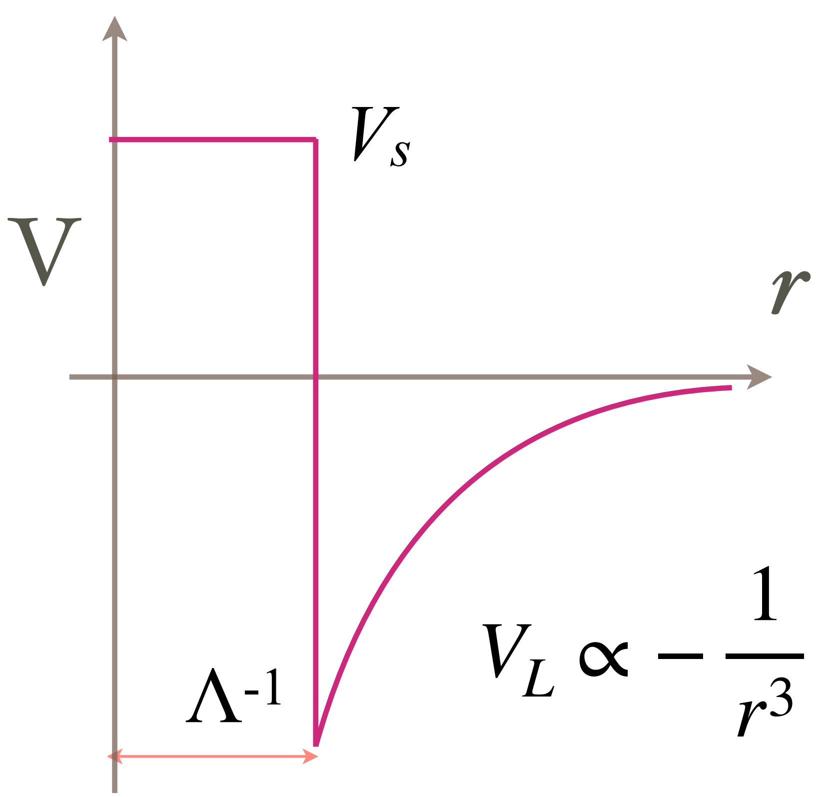


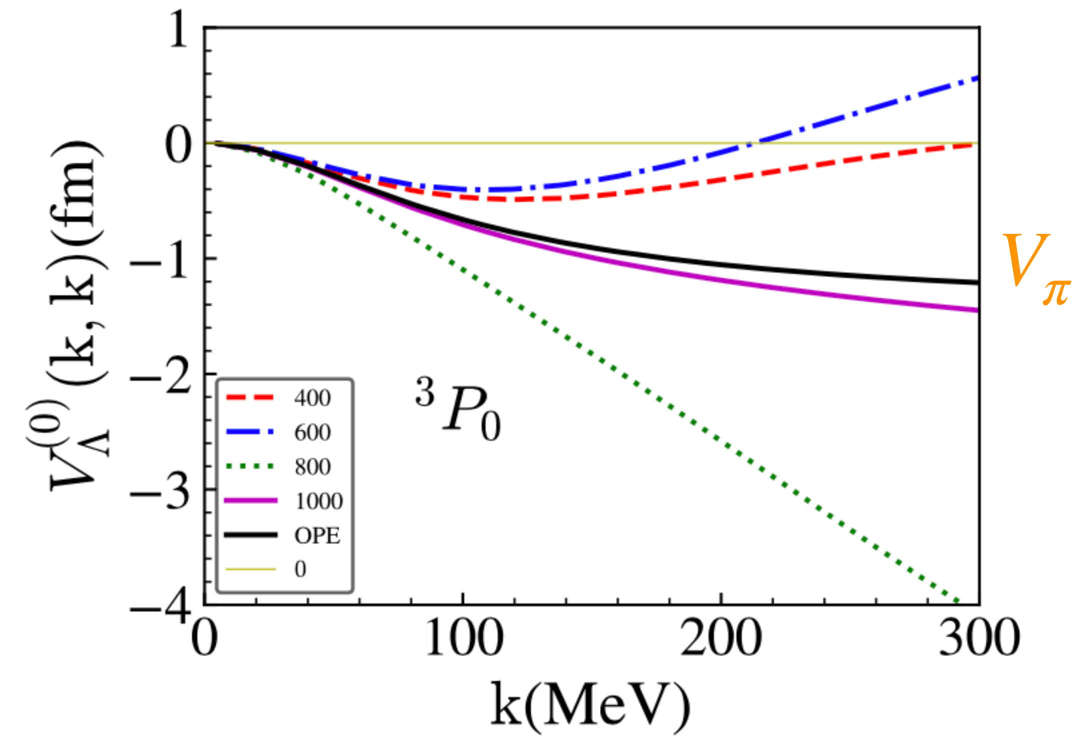
Pushing pert-pion interactions

- Re-organizing higher contact terms



Nonpert ren of OPE

$V_\pi + C_{3P0} p' p$



- From nonpert renormalization: V_s always repulsive (towards lower Λ)

- Using V_s to moderate the OPE tensor force

PPI in 3P0

Peng, Lyu & BwL '20

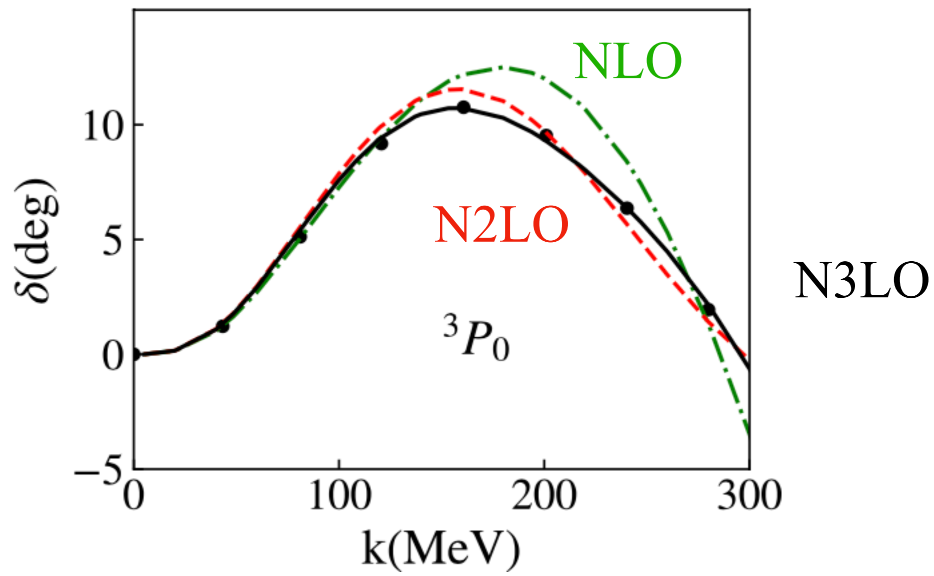
- Expansion in $V_\pi + C_{3P0}p'p$ (Born approximation)

$$V^{(1)}(^3P_0) = V_\pi + C_2^{^3P_0} p'p$$

- Higher-order contacts are identified when needed for renormalization at second, third-order Born approximation

	LO	NLO	N^2LO	N^3LO
π	OPE			TPE0
$^3P_0$		C_2	C_4	C_6

Updated fit $\Lambda = 800$ MeV

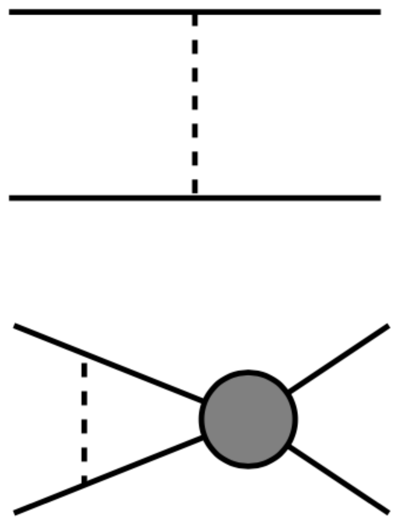


PPI: 3S1-3D1

Lyu, Zuo, Peng, Koenig & BwL

- Mixing angle vanishing at unitarity & chiral limits despite strong OPE tensor force

$$\epsilon_1 = \frac{m_N k}{4\pi} \frac{k}{(a_t^{-2} + k^2)^{\frac{1}{2}}} \times \left[\frac{1}{a_t k} \langle k, {}^3S_1 | V_\pi | k, {}^3D_1 \rangle - \frac{g_A^2}{\sqrt{2} f_\pi^2} \Pi(0, \frac{m_\pi}{k}) \right]$$

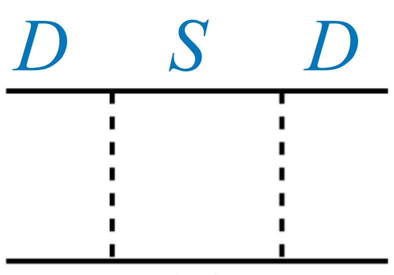


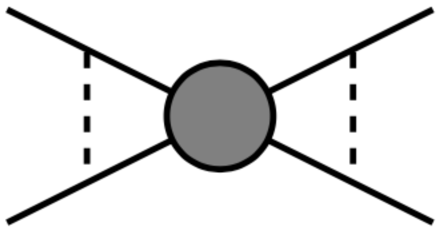
$$\Pi(kr, \frac{m_\pi}{k}) \equiv m_\pi^3 \int_r^\infty dr' r'^2 n_0(kr') T(m_\pi r') j_2(kr')$$

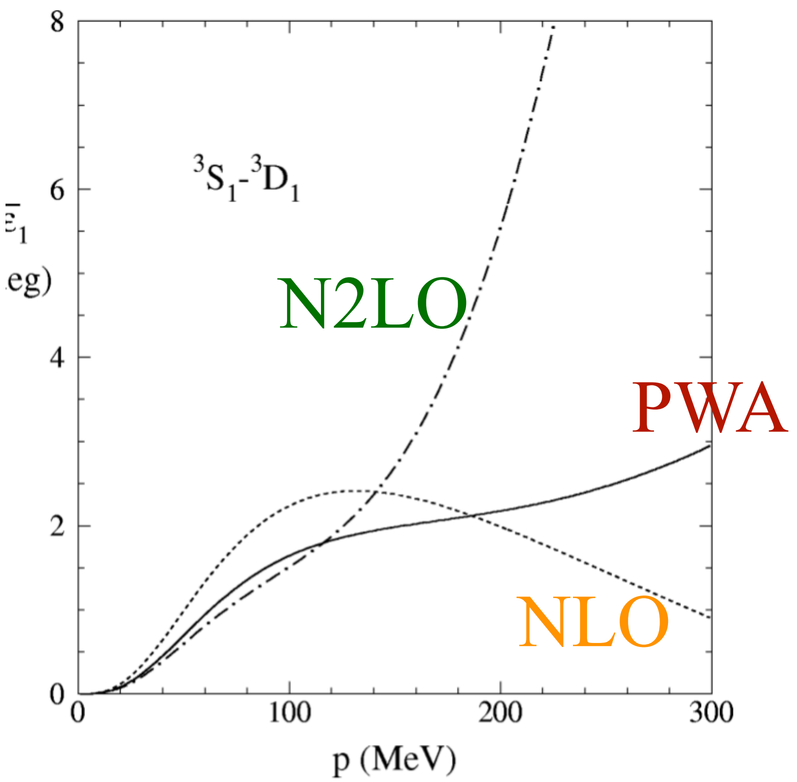
$$m_\pi \to 0: \quad \int_0^\infty dr r^2 n_0(kr) \frac{1}{r^3} j_2(kr) = 0$$

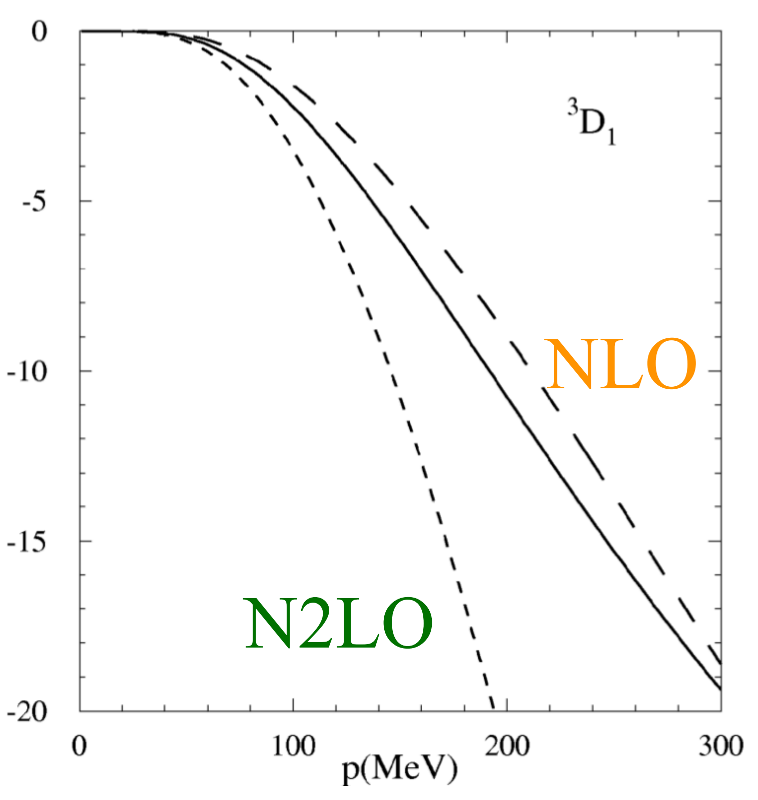
PPI: 3S1-3D1

- Not enough though
- Only works for on-shell kinematics
- N2LO 3D1, mixing angle are large





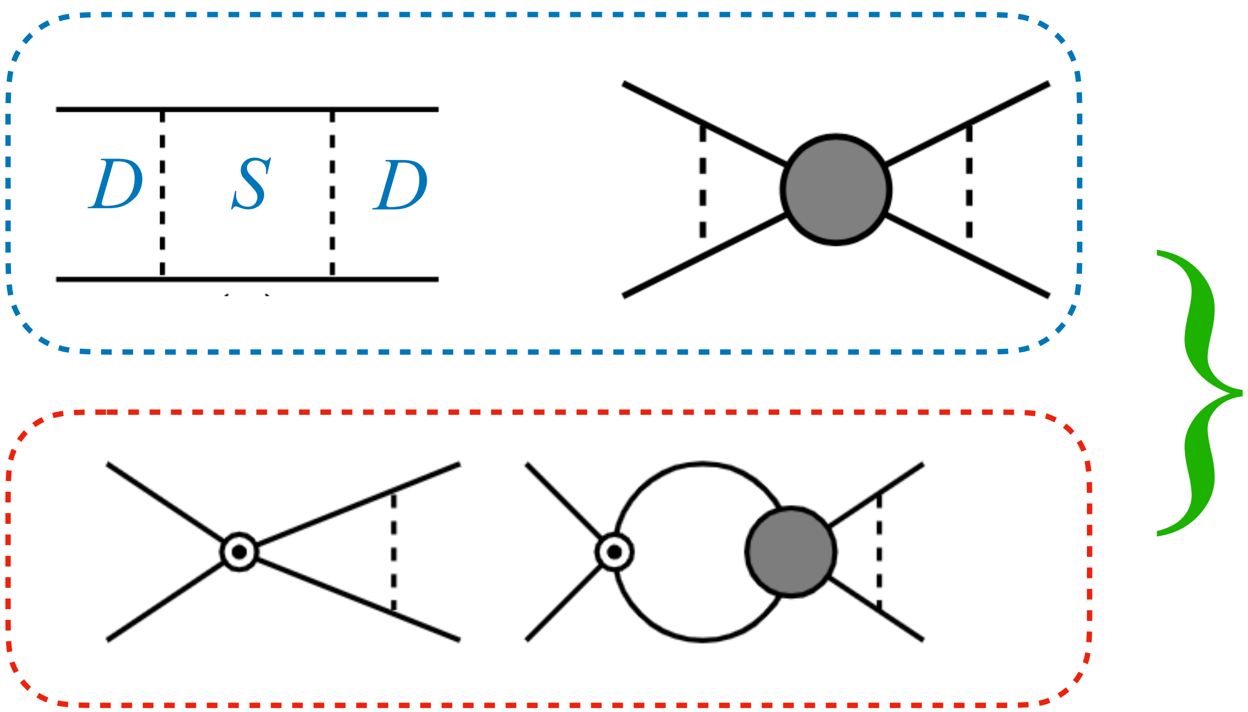




PPI: 3S1-3D1

- Expansion in (V_π + *SD* mixing contact)

$$V^{(1)}({}^3S_1 - {}^3D_1) = V_\pi + \begin{pmatrix} C_0^{(1)} + C_2^{{}^3S_1}(p'^2 + p^2) & -C_2^{SD} p^2 \\ -C_2^{SD} p'^2 & 0 \end{pmatrix}$$



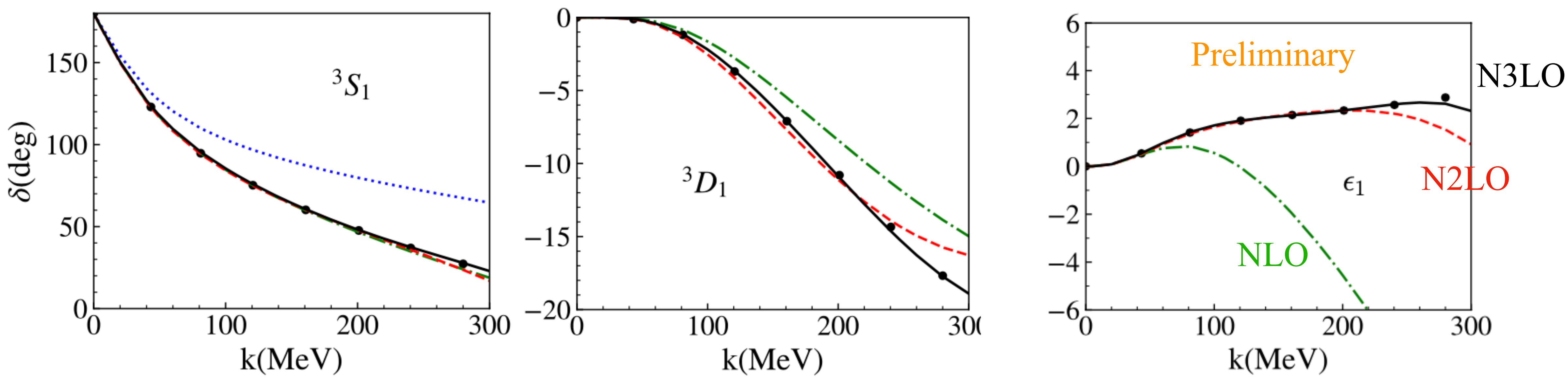
Cancellation

PPI: 3S1-3D1

- PC of high-order contacts by counting divergence

	LO	NLO	N^2LO	N^3LO
π		OPE		TPE0
$^3S_1$ $^1S_0$	C_0	C_2	C_4	C_6
SD		C_2	C_4	C_6
$^3D_1$				C_4

$\Lambda = 800$ MeV



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Summary

- A new perturbative-pion scheme with re-organized contacts
- Simplify LO nuclear interactions to emphasize accidental symmetries like SU(4)-Wigner, unitarity limits, etc.
- W/ larger validity range and 3NF at LO, implies 3NF important even for Q > m_π