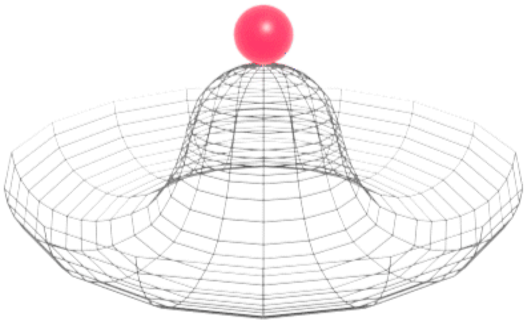
Neutrino Theory —— A Personal Overview 2025

Zhi-zhong Xing
【IHEP Beijing】



Majorana
nature

⟷

Yukawa
interactions

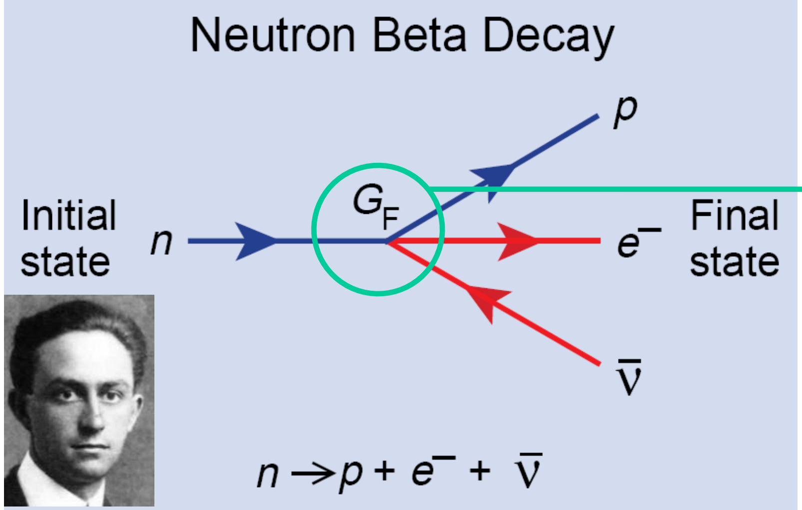
Neutrino, Nuclear Physics and New Physics Symposium, Lanzhou, 21—25.8.2025

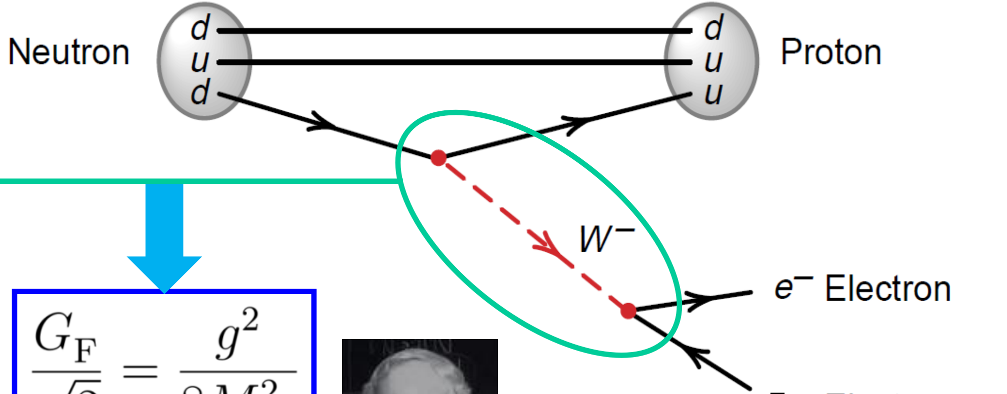
OUTLINE

- Historical roles of lepton flavors
- Origin of small neutrino masses
- Possible lepton favor symmetry
- Charged lepton flavors can help

A role of the "1 G" leptons

♦ **Fermi's EFT for beta decays with "1 G" leptons and quarks (1933/1934):**





$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

gauge seesaw

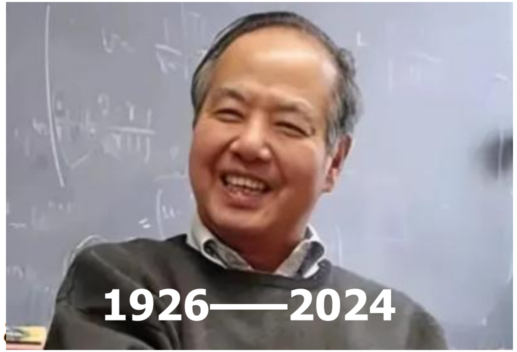
Fermi coupling constant **Weak interaction coupling constant**

$$G_F \simeq 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$g \simeq 0.65$ **vs** $M_W \simeq 80.4\ \mathrm{GeV}$

A good lesson: a small effective quantity at low energies is very likely to originate from some new and heavy degrees of freedom in a more fundamental theory at much higher energy scales. History repeats itself, as we will see again and again.

Fermi's intuition is a mystery



In 2001 Fermi's PhD student T.D. Lee made the remarks on Fermi's EFT for beta decays 〖Int. J. Mod. Phys. A 16 (2001) 3633——3658, Review article〗:

$$G_F(\overline{\psi}_p \gamma_\mu \psi_n)(\overline{\psi}_e \gamma^\mu \gamma_5 \psi_\nu) \leftarrow$$ **Fermi (1933/1934)**

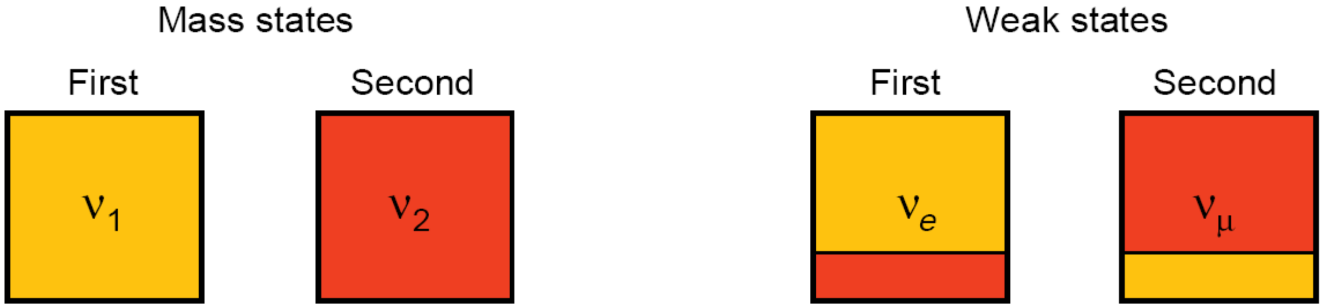
V A

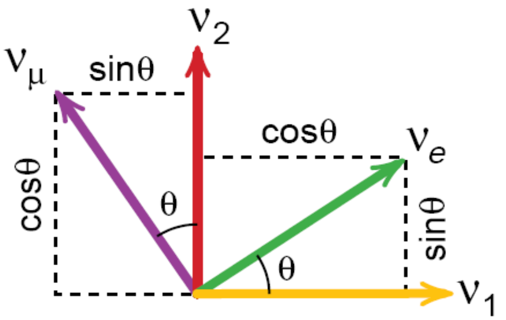
Fermi told me that his interaction was modelled after the electromagnetic forces between charged particles, and his coupling G was inspired by Newton's constant. His paper was, however, rejected by *Nature* for being unrealistic. It was published later in Italy, and then in *Zeitschrift für Physik.*[13] Fermi wrote his γ matrices explicitly in terms of their matrix elements. His lepton current differs from his hadron current by a γ_5 factor; of course the presence of this γ_5 factor has no physical significance. Nevertheless, it is curious why Fermi should choose this particular expression, which resembles the V–A interaction, but with parity conservation. Unfortunately, by 1956, when I noticed this, it was too late to ask Fermi.

Parity violation (1956/1957) → V—A theory (1958) → Electroweak theory (1967)

A role of the "2 G" leptons

♦ In **1962** all the four "**2 G**" lepton members went home, making it possible to consider **lepton flavor mixing** (**Z. Maki**, **M. Nakagawa**, **S. Sakata**):





$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

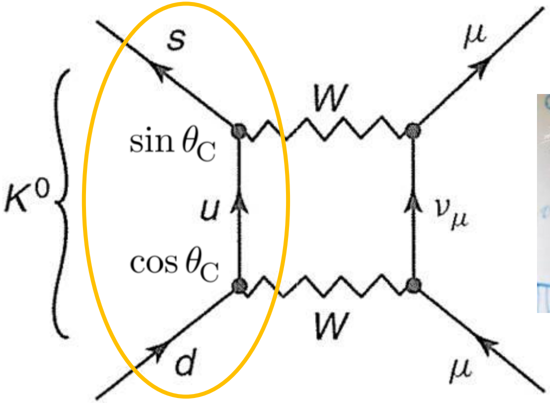
In **1967**, **B. Pontecorvo** formulated the neutrino oscillation probability

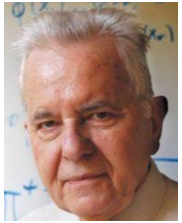
♦ In **1964** the **lepton-quark symmetry** motivated **J. Bjorken** and **S. Glashow** to propose a new quark "**charm**" with respect to ν_μ .

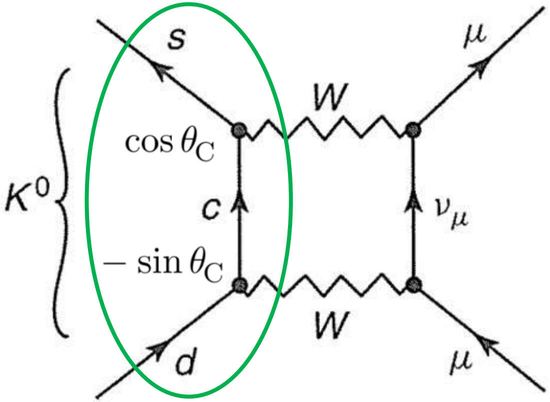
The successful GIM mechanism

♦ In 1970, S. Glashow, J. Iliopolous and L. Maiani found that the SU(4) quark model could successfully suppress the FCNC effects of the SU(3) quark model, improved by incorporating the Cabibbo flavor mixing —— the GIM mechanism.





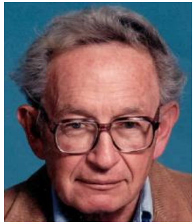




Hidden new and heavy degrees of freedom:

→ $m_c \sim 2$ GeV

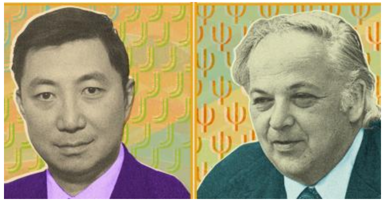
More = New dynamics?



P. Anderson

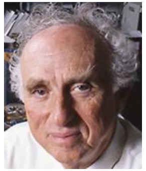
More is different (1972)

♦ In November 1974, the charm quantum number was independently discovered by S. Ting and B. Richter. A brand new "GeV" era began, calling for much higher energy machines to produce new heavy particles.



A role of the "3 G" leptons

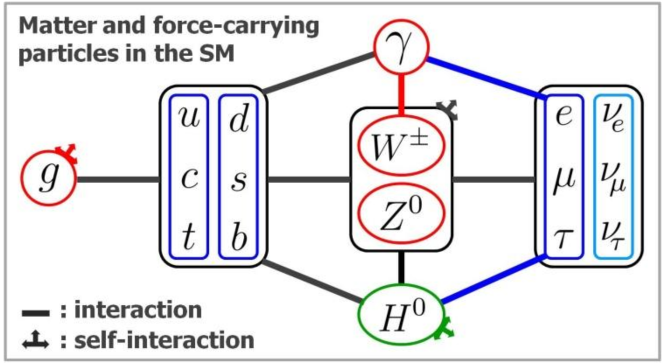
♦ In 1975 the third and heaviest charged lepton —— τ lepton was discovered by M. Perl, opening the "3 G" era of leptons and quarks.



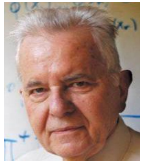
♦ Fermilab: let's do the rest on behalf of Fermi.

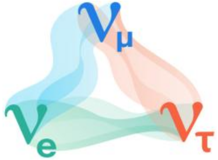
- ♦ 1977: the bottom quark
- ♦ 1995: the top quark
- ♦ 2001: the tau neutrino

Then the "3 G" picture of fermions is complete.



♦ The probabilities of 3-flavor neutrino oscillations with CP/T violation w/o matter effects were first formulated by N. Cabibbo in 1978 and by V. Barger, K. Whisnant, R. Phillips in 1980.





♦ A global analysis of various neutrino oscillation data in the standard 3-flavor scheme was first made by G. Fogli, E. Lisi and D. Montanino in 1994 —— *proof of concept* to show its potential (predictive) power!

Weinberg's 3rd law

♦ Going beyond the SM in the flavor sector may naturally mean going beyond the "3 G" paradigm of fundamental fermions, especially the "3 G" neutrinos, as motivated by the understanding of *neutrino mass generation* or by explaining some *puzzling anomalies*.

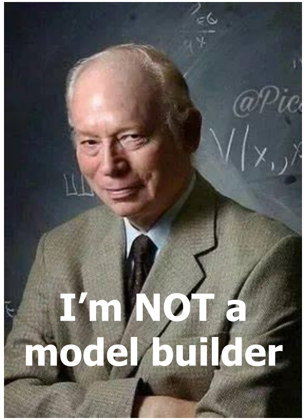
sterile species

- ♦ 3 + 1: light (eV, keV), LSND, warm DM….
- ♦ 3 + 2: heavy (the minimal seesaw)
- ♦ 3 + 3: heavy (the canonical seesaw)
- ♦ 3 + 6: the double or inverse seesaw
- ♦ 3 + n: arbitrary number and mass scales

♦ S. Weinberg's third Law of Progress in Theoretical Physics (1983):

> You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry.

"more" maybe stupid



♦ A good lesson: the history of particle physics tells us that a *real* new degree of freedom must be able to help solve at least one fundamental problem and make the theory more natural, consistent and powerful.

OUTLINE

- Historical roles of lepton flavors
- Origin of small neutrino masses
- Possible lepton favor symmetry
- Charged lepton flavors can help

Going beyond SM: ν's = Majorana

♦ **Fundamentals** of the electroweak SM structure → **reasons** for zero ν-masses:

♦ Quantum mechanics + Lorentz invariance
♦ Local $\mathrm{SU(2)_L \times U(1)_Y}$ gauge symmetries
♦ The Higgs mechanism
♦ Renormalizability (no d ≥ 5 operators)

Plus *economical* particle content:
- No right-handed neutrino fields
- Only one Higgs doublet

$$m_1 = m_2 = m_3 = 0$$

Go beyond the d=4 operators

The "unique" d=5 operator

SMEFT

$$\mathcal{O}_{\mathrm{W}} = \frac{\overline{\ell_{\mathrm{L}}}\widetilde{H}\widetilde{H}^{T}\ell^{c}_{\mathrm{L}}}{\Lambda}$$

S. Weinberg 1979

E. Majorana 1937

SSB

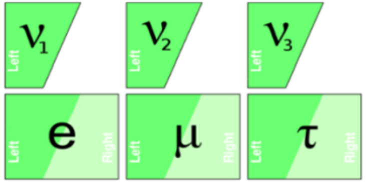
LNV

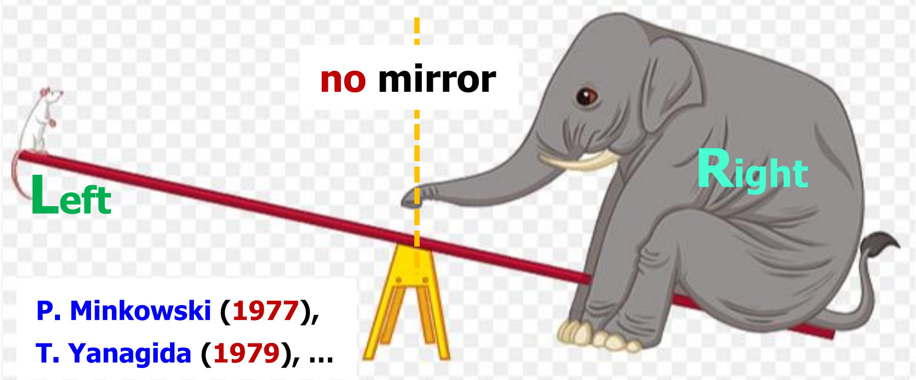
Talk by Shun

Supported by ν-oscillations ← ν-masses   ν's = the Majorana fermions

A simplest way to realize $\mathcal{O}_{\rm W}$

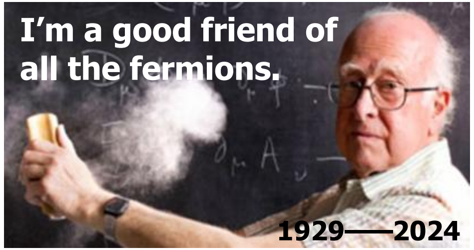
- **Right-handed neutrino fields are added, not mirror counterparts of left-handed ones.**





N_1 N_2 N_3

- **Yukawa interactions —— the Higgs fields play a crucial role, as they do in generating masses for the charged fermions in the SM**

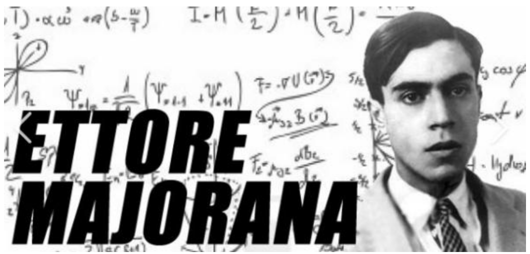


- **The Majorana nature of massive neutrinos:**

  N **and** N^c **may have** ***self-interactions*****, respecting all the fundamental symmetries of the SM.**

$$\frac{1}{2}\overline{(N_{\rm R})^c}M_{\rm R}N_{\rm R}$$

Gell-Mann's totalitarian principle (1956)
Everything not forbidden is compulsory!



- **If you try to forbid this term for Prof. Dirac, you'll have to invoke uneasy new physics**

Seesaw works before and after SSB

- The **seesaw** mechanism formally works above the **SM** electroweak scale **before SSB**.

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}} Y_l H l_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$= \overline{l_{\rm L}} Y_l l_{\rm R} \phi^0 + \frac{1}{2}\overline{\begin{bmatrix}\nu_{\rm L} & (N_{\rm R})^c\end{bmatrix}} \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + \overline{\nu_{\rm L}} Y_l l_{\rm R} \phi^+ - \overline{l_{\rm L}} Y_\nu N_{\rm R} \phi^- + {\rm h.c.}$$

- The **basis transformation** for the origin of three active **Majorana** neutrino masses:

$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N\end{pmatrix}$$

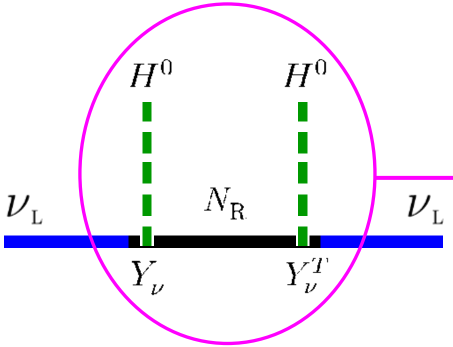
working masses:

$$\begin{cases} D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\} & \text{light} \\ D_N \equiv {\rm Diag}\{M_1, M_2, M_3\} & \text{heavy}\end{cases}$$

SSB

$$\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}$$

6 × 6 mass matrix



Integrating out the heavy degrees of freedom:

$$-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}} M_\nu \nu_{\rm L}^c + {\rm h.c.} \qquad M_\nu \simeq -Y_\nu \frac{\langle H\rangle^2}{M_{\rm R}} Y_\nu^T$$

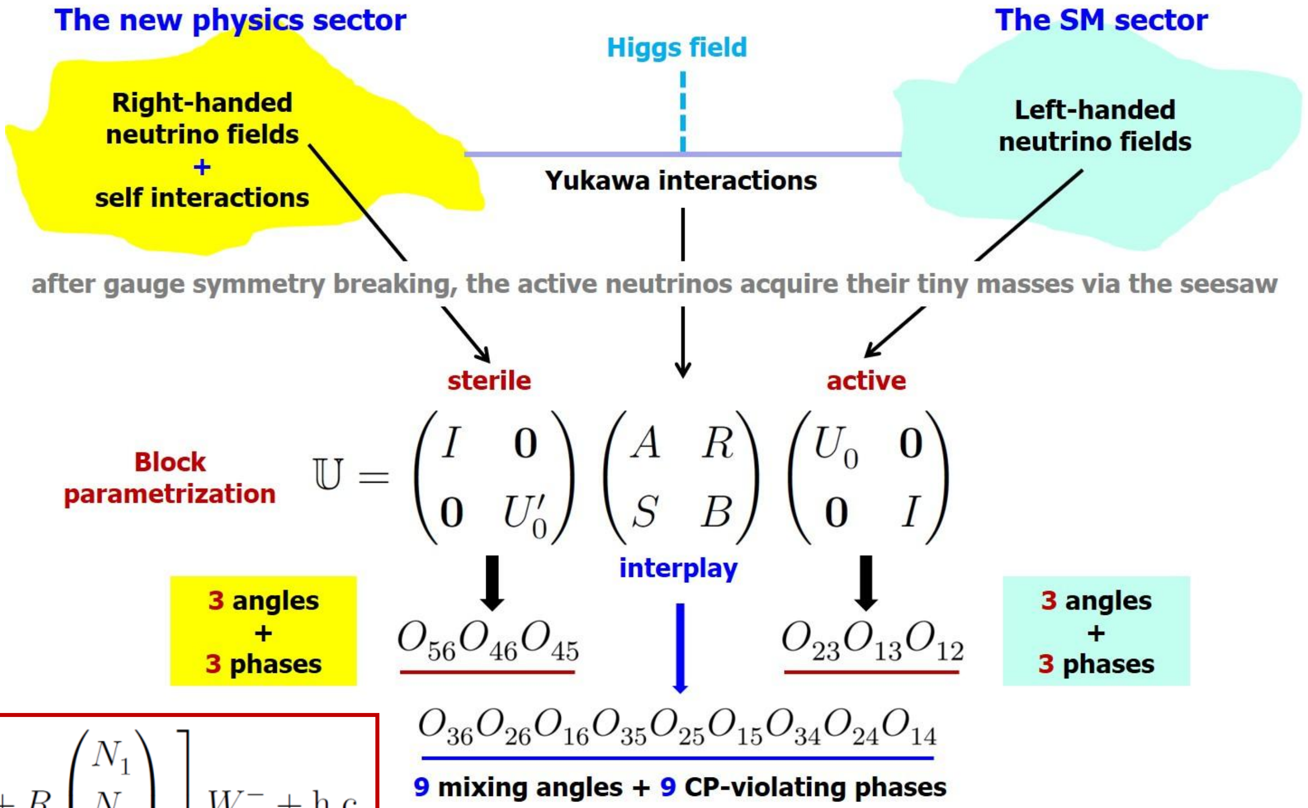
Consistent with the dim-5 Weinberg operator!

- **If you can untie Weinberg's knot, you will find *new* and *heavier* degrees of freedom**

A full parameterization of seesaw

A ***block parametrization*** **of active-sterile flavor mixing in the seesaw framework:**

- ♦ **reflects salient features of the seesaw dynamics**
- ♦ **offers *generic* + *explicit* expressions of observables using the Euler-like angles and phases (ZZX, 2012)**



The weak charged-current interactions of leptons:

U **=** ***AU***$_0$**: the PMNS matrix;**
R **: an analogue for heavy.**

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \;\; \mu \;\; \tau)_{\rm L}} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

seesaw + unitarity:

$$\begin{cases} U D_\nu U^T + R D_N R^T = \mathbf{0} \\ U U^\dagger + R R^\dagger = I \end{cases}$$

oscillations ⟵ light　　heavy ⟶ leptogenesis (Talk by Michael)

The Euler-like parametrization

♦ **The 1st full Euler-like parametrization of $U = AU_0$ and R is useful for calculating flavor structures.**

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

derivable from the parameters of A and R

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} - \hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} & c_{24}c_{25}c_{26} & 0 \\ \begin{matrix} -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{matrix} & \begin{matrix} -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} \\ -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} \end{matrix} & c_{34}c_{35}c_{36} \end{pmatrix}$$

ZZX
0709.2220/1110.0083

$$R = \begin{pmatrix} \hat{s}^*_{14}c_{15}c_{16} & \hat{s}^*_{15}c_{16} & \hat{s}^*_{16} \\ -\hat{s}^*_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} + c_{14}\hat{s}^*_{24}c_{25}c_{26} & -\hat{s}^*_{15}\hat{s}_{16}\hat{s}^*_{26} + c_{15}\hat{s}^*_{25}c_{26} & c_{16}\hat{s}^*_{26} \\ \begin{matrix} -\hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} - c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} + c_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{matrix} & \begin{matrix} -\hat{s}^*_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +c_{15}c_{25}\hat{s}^*_{35}c_{36} \end{matrix} & c_{16}c_{26}\hat{s}^*_{36} \end{pmatrix}$$

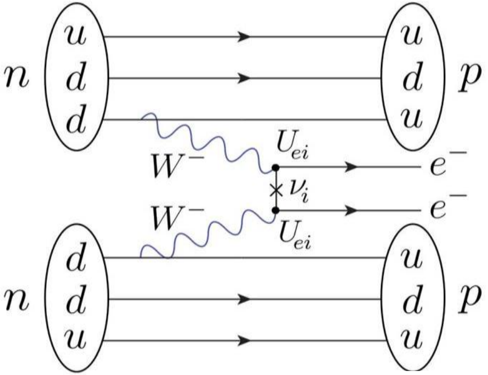
The latest stringent *bounds* on possible PMNS nonunitarity.
M. Blennow et al. 2023

$$\begin{cases} \theta_{1j} < 2.92^\circ \\ \theta_{2j} < 0.27^\circ \\ \theta_{3j} < 2.56^\circ \\ [j = 4, 5, 6] \end{cases}$$

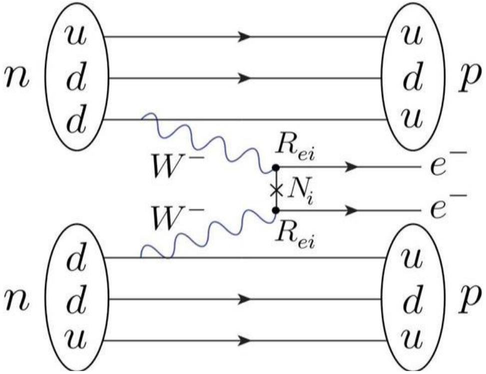
ZZX, J.y. Zhu 2412.17698

To see Prof. Majorana in 0ν2β decays

♦ The **seesaw**-induced **Majorana** nature of massive neutrinos assure the **0ν2β** decays to occur, a unique **LNV** place to meet Prof. **Majorana**.



light neutrinos



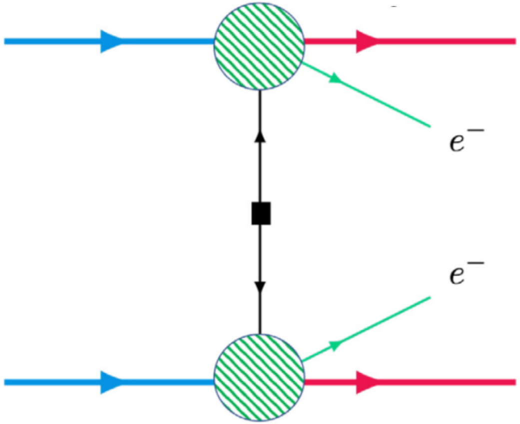
heavy neutrinos

Seesaw + Unitarity:

$$\sum_{i=1}^{3} m_i U_{ei}^2 + \sum_{j=1}^{3} M_j R_{ej}^2 = 0$$

$$\sum_{i=1}^{3} |U_{ei}|^2 + \sum_{j=1}^{3} |R_{ej}|^2 = 1$$

Interplay between propagators + NMEs



Talks by **Chun-Lin**, et al.

♦ **Stupid question**: which channel is more fundamental?

♦ **Correct answer**: they are equally fundamental, thanks to the **Yukawa** interactions (i.e., $R = 0 \implies m_i = 0$).

♦ In most cases, the contribution from heavy **Majorana** neutrinos to the **0ν2β decays** are negligibly small in the seesaw mechanism (**ZZX**, **2009**; **W. Rodejohann**, **2010**).

Pros and cons of the seesaw

♦ **Pros A**: neutrinos have the **right** to be **right** (***handed***) to keep a **left-right symmetry** —— the most natural and economical extension of the SM: **high gain** + **low costs**.

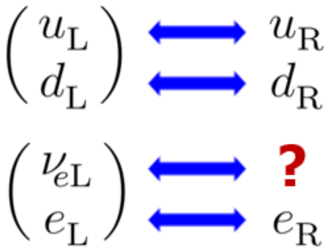
♦ **Pros B**: The **Majorana** mass term as **new dof** is highly nontrivial and has a profound effect on the SM, making the **seesaw** framework consistent with **Weinberg**'s EFT.

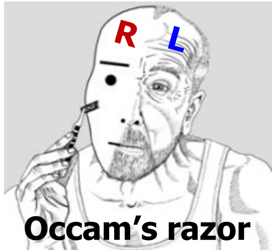
♦ **Pros C**: A big bonus is **baryogenesis** via **leptogenesis**, making it possible to **kill two birds with one stone**.

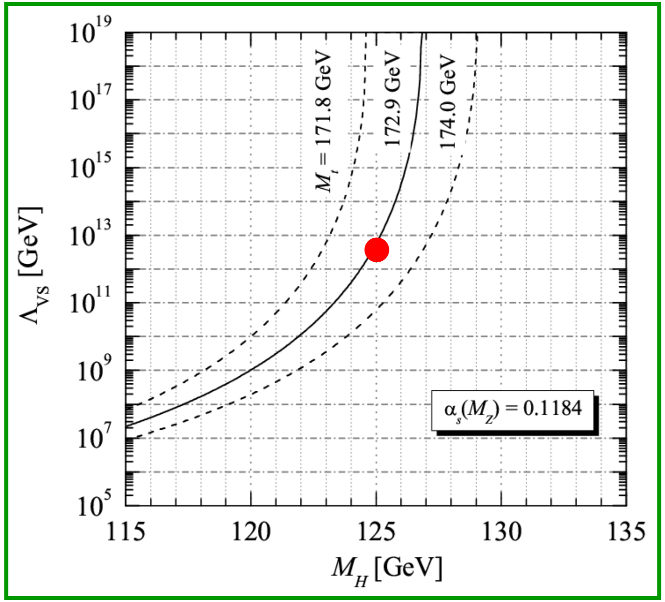
♦ **Cons A**: **Naturalness** of the **seesaw** demands its scale far above the **Fermi** scale, making its **testability** dim.

♦ **Cons B**: **Seesaw**-induced **fine-tuning** issue associated with the **Higgs** mass (**F. Vissani 1998**, **Casas et al 2004**, **Abada et al 2007**).

The scale of the SM vacuum stability seems consistent with the seesaw + leptogenesis scale —— *suggestive?* (J. Elias-Miro et al 2012, ZZX, H. Zhang, S. Zhou 2012)







The SM vacuum stability for a light Higgs

Recent TH progress for seesaw

♦ Complete one-loop matching of the seesaw onto the SMEFT (D. Zhang, S. Zhou 2021; Y. Du, X.X. Li, J.H. Yu 2022)

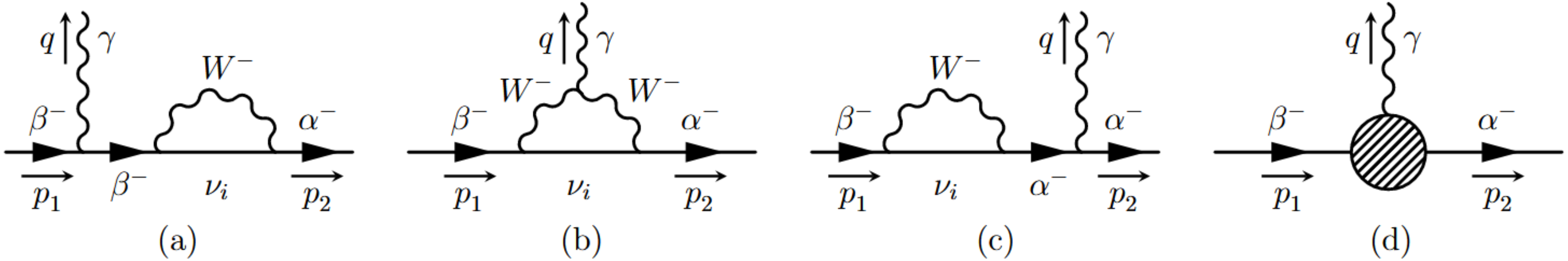


Diagram (d) is generated by the *dim-6* operator at the *one-loop* level and is crucial for the seesaw EFT to correctly calculate the *cLFV decays*, consistent with the full seesaw.

♦ Complete one-loop RGEs in the seesaw EFT framework including the effects of PMNS non-unitarity (Y. Wang, D. Zhang, S. Zhou 2023)

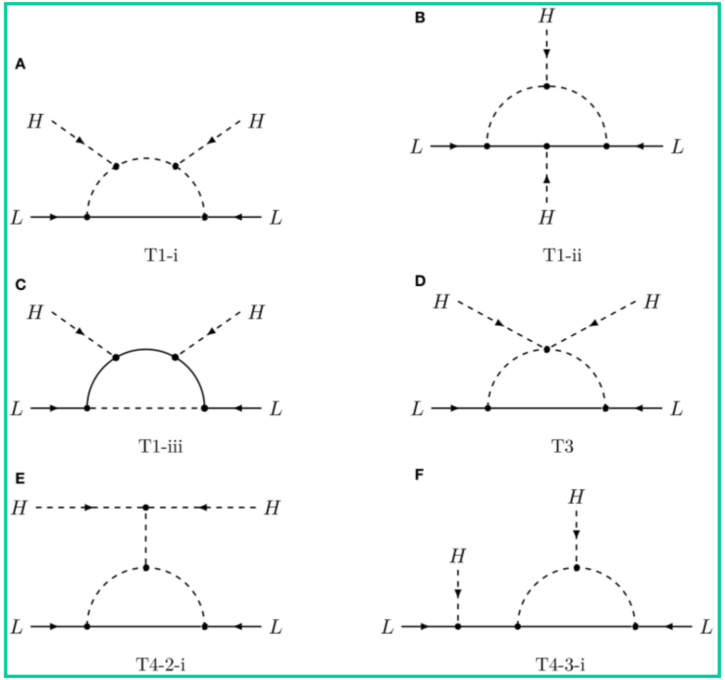
♦ First calculations of the flavor parameters of light Majorana neutrinos in terms of the original seesaw parameters without any special assumptions:

- A generic analytical connection between CPV in neutrino oscillations and CPV in heavy neutrino decays (ZZX, 2023, PLB);
- Generic analytical expressions of light neutrino masses, flavor mixing angles, and the Dirac CPV phase associated with neutrino oscillations (ZZX, Jing-yu Zhu, 2024, NPB in press).

Is there a better alternative? Not really!

- ♦ Radiative origin of charged-lepton and neutrino masses (S. Weinberg 1972, 2020; A. Zee 1980 …)

- ♦ A review by Y. Cai, J.H. Garcia, M.A. Schmidt, A. Vicente, R.R. Volkas in Front. in Phys. 5 (2017) 63 "from the trees to the forest: a review of radiative neutrino mass models"



Talk by Xiao-Gang

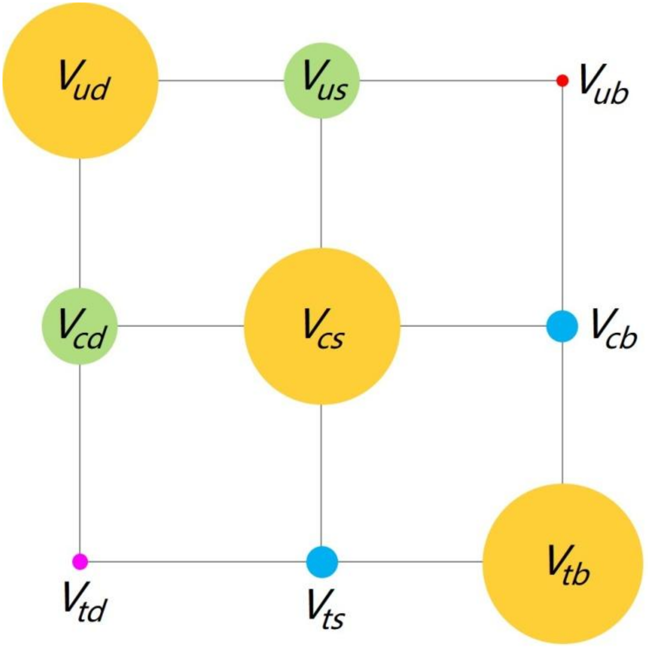
Feynman diagram topologies for one-loop radiative neutrino mass generation with the d5 Weinberg operator, where a dashed line can be scalars or gauge bosons if allowed.

OUTLINE

- Historical roles of lepton flavors
- Origin of small neutrino masses
- **Possible lepton favor symmetry**
- Charged lepton flavors can help

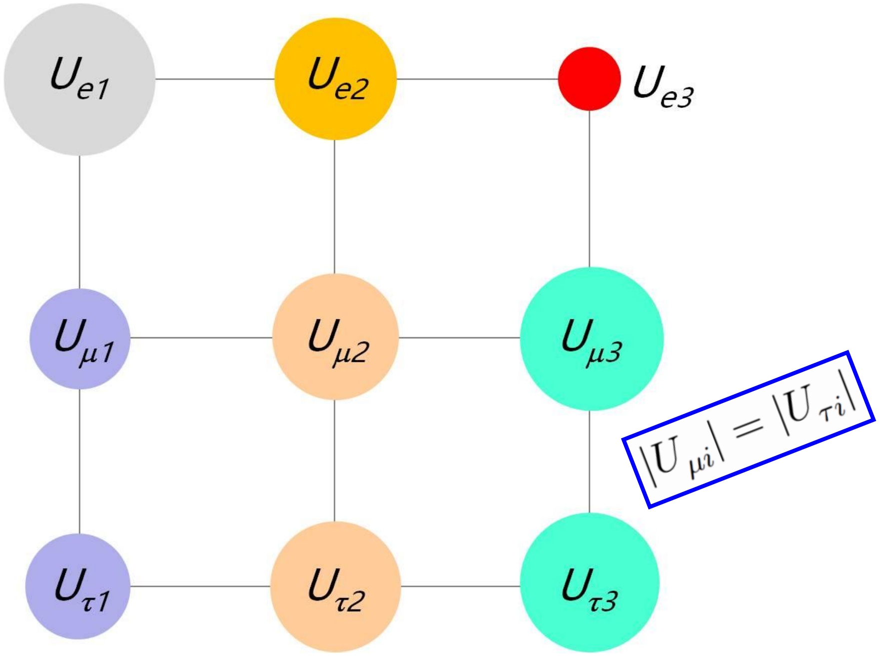
The patterns of flavor mixing

♦ The data tell us that quarks and leptons have rather different flavor mixing patterns:



The CKM quark flavor mixing

$$V = I + \mathcal{O}(\lesssim \theta_{\rm C})$$



The PMNS lepton flavor mixing

$$U = U_0 + \mathcal{O}(\lesssim \theta_{13})?$$

♦ Quarks: approximate up-down parallelism

♦ Leptons: approximate μ-τ interchange symmetry

C

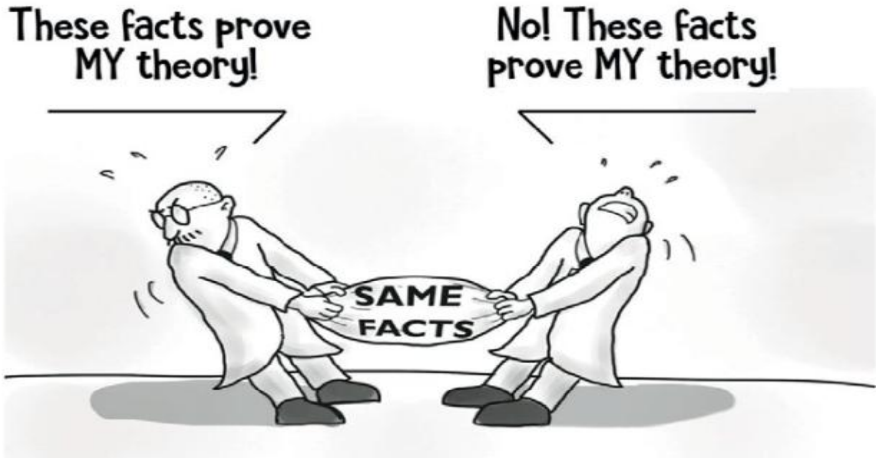
Which symmetry is closer to the truth?

♦ So far a lot of flavor symmetries have been taken into account for model building [recent reviews: ZZX 2020 (Phys. Rept.); F. Feruglio, A. Romanino 2020 (Rev. Mod. Phys.); G.J. Ding, S. King 2024 (Rept. Prog. Phys.)]

S_3 , S_4 , A_4 , A_5 , D_4 , D_7 , T_7 , T′, Δ(27), Δ(48), …
$U(1)_F$, $SU(2)_F$, modular, translational, ….

♦ What is the guiding principle? The bottom line is that the models should be compatible with data

But data is King?



νfit
www.nu-fit.org
(T. Schwetz et al)

♦ Almost all the flavor symmetries cannot explain tiny ν-masses. Many of them invoke the seesaw.

Modular invariance is the best seller

♦ The modular invariant model building (G. Altarelli, F. Feruglio 2006; F. Feruglio 2017)

♦ Orbifold compactification: 10D string theory → 4D SM + 3 copies of 2D torus.

♦ A single complex modulus τ is enough to parameterize the shape of torus. The modular invariant super-potential gives rise to the modular form of the Yukawa coupling matrices which depend on τ.

♦ The "*seesaw mechanism*" is invoked.

Comment A: physical meaning of the complex modular parameter τ is unclear?

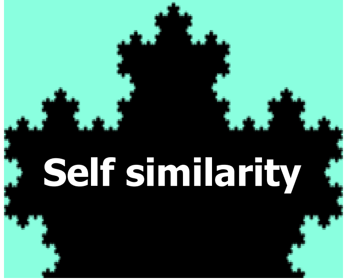
Comment B: flavor textures are not transparent due to a *nonlinear* realization of modular symmetry, and hence a careful numerical fitting has to be done?

Comment C: no good reason for a strong mass hierarchy of charged fermions?

♦ In contrast, the conventional (*discrete*) flavor symmetries can linearly predict flavor mixing with CG coefficients, and thus more transparent in physics. *None is simple!*

Symmetry breaking is more subtle

♦ **Symmetry** or **form invariance** of a theory means that ***behind it*** there is **something unobservable**. But **symmetry breaking** is highly nontrivial as it usually makes things observable.



♦ A natural source of **symmetry breaking** is from ***quantum corrections*** from a super-high energy scale down to the **Fermi** scale.

♦ Other ways of **symmetry breaking** is often of high costs and low gain.

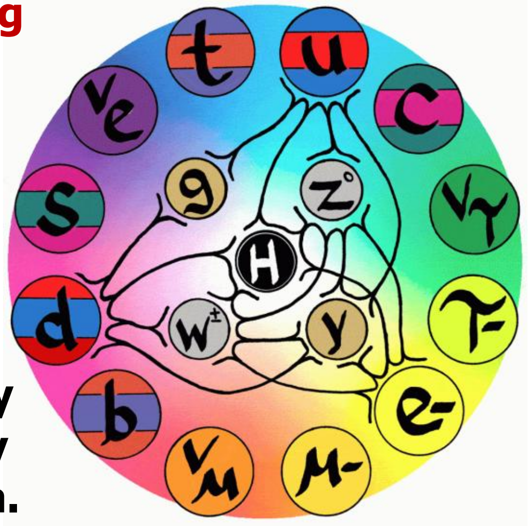
♦ Examples of **symmetry breaking** in the **SM** framework:

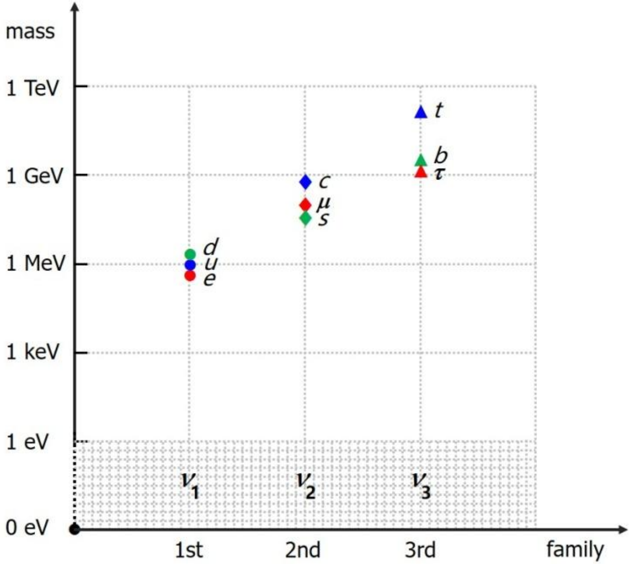
♦ **Parity:** weak **V–A** structure

♦ **Local gauge** $SU(2)_L \times U(1)_Y$: the **BEH** mechanism

♦ **CP violation:** the **KM** phase

♦ The flavor sector involves many free parameters, and we can only qualitatively understand the data.



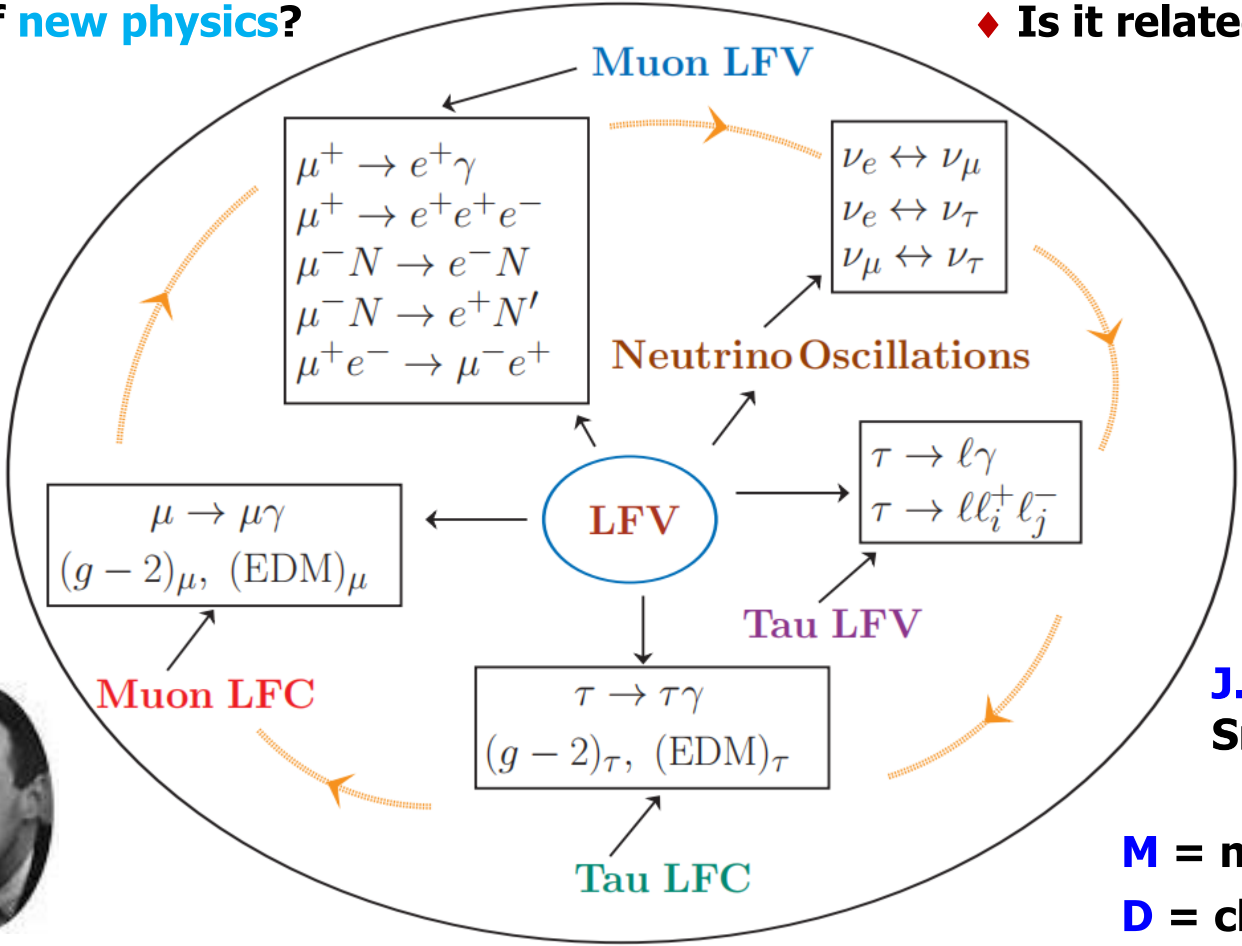


OUTLINE

- Historical roles of lepton flavors
- Origin of small neutrino masses
- Possible lepton favor symmetry
- Charged lepton flavors can help

Some typical LFV vs LFC processes

- What kind of new physics?
- Is it related to ν-masses?



J. Albrecht *et al*
Snowmass 2013

M = neutrinos
D = charged leptons

The 1st seesaw paper was on $\mu \to e + \gamma$!

Volume 67B, number 4 PHYSICS LETTERS 25 April 1977

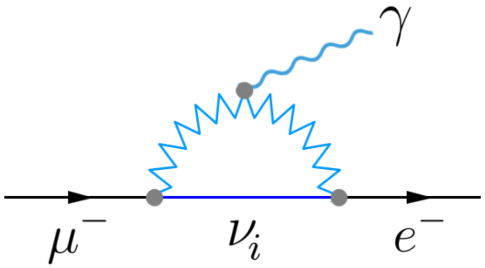
$\mu \to e\gamma$ AT A RATE OF ONE OUT OF 10^9 MUON DECAYS?

Peter MINKOWSKI

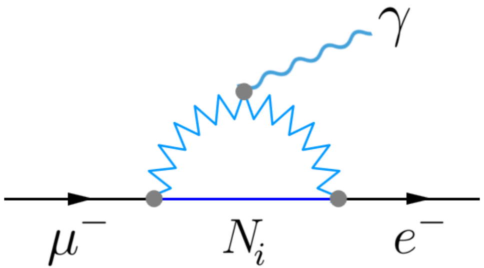
Institute for Theoretical Physics, University of Berne, Sidlerstrasse 5, 3012 Berne, Switzerland

Received 28 February 1977

It is proposed that lepton number conservation, purely left-handed charged weak currents and vanishing neutrino masses are a limiting case of a parity symmetric $SU2_L \times SU_R \times U2^V$ gauge theory. Right-handed neutrinos acquire a lepton number violating mass, leaving an $SU2_L \times U1$ subgroup unbroken. Consequences for the decay $\mu \to e\gamma$ are studied.



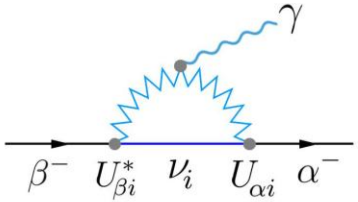
light neutrinos



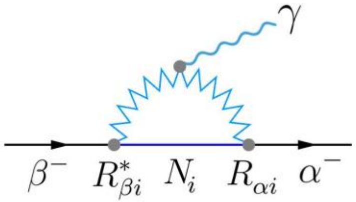
heavy neutrinos

A strongest constraint on PMNS nonunitarity

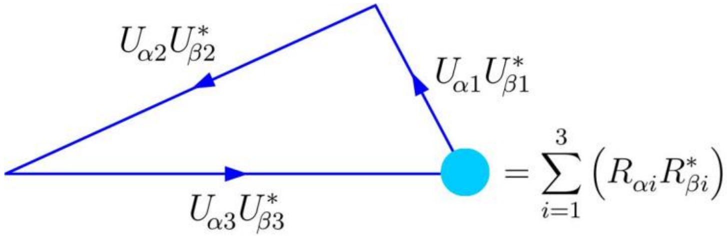
- It can help constrain unitarity of the 3×3 ***PMNS*** matrix through the ***cLFV*** processes.



light neutrinos



heavy neutrinos



In the full seesaw (ZZX, D. Zhang, 2009.09717) or its EFT with one-loop matching (D. Zhang, S. Zhou, 2107.12133):

$$\xi_{\alpha\beta} \equiv \frac{\Gamma(\beta^- \to \alpha^- + \gamma)}{\Gamma(\beta^- \to \alpha^- + \overline{\nu}_\alpha + \nu_\beta)} \simeq \frac{3\alpha_{\rm em}}{2\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\left(-\frac{5}{6} + \frac{1}{4}\cdot\frac{m_i^2}{M_W^2}\right) - \frac{1}{3}\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right|^2 \simeq \frac{3\alpha_{\rm em}}{8\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right|^2$$

which allows us to constrain the unitarity hexagon using current experimental data on three radiative cLFV decays:

$$\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right| = \left|\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right| \simeq \sqrt{\frac{8\pi\xi_{\alpha\beta}}{3\alpha_{\rm em}}} \simeq 33.88\sqrt{\xi_{\alpha\beta}}$$

$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\mu i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\mu i}\right| < 2.20 \times 10^{-5}$$

$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\tau i}\right| < 1.46 \times 10^{-2}$$

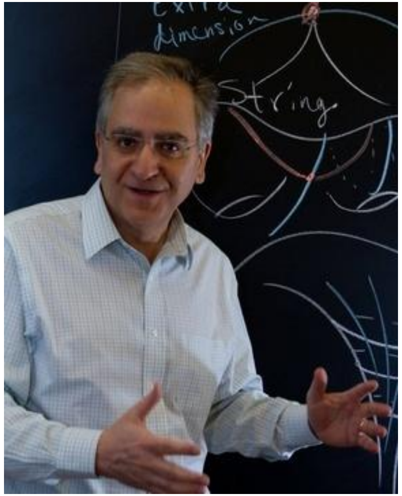
$$\left|\sum_{i=1}^{3} U_{\mu i}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{\mu i}R^*_{\tau i}\right| < 1.70 \times 10^{-2}$$

- **Imposing the mu-tau reflection symmetry:**

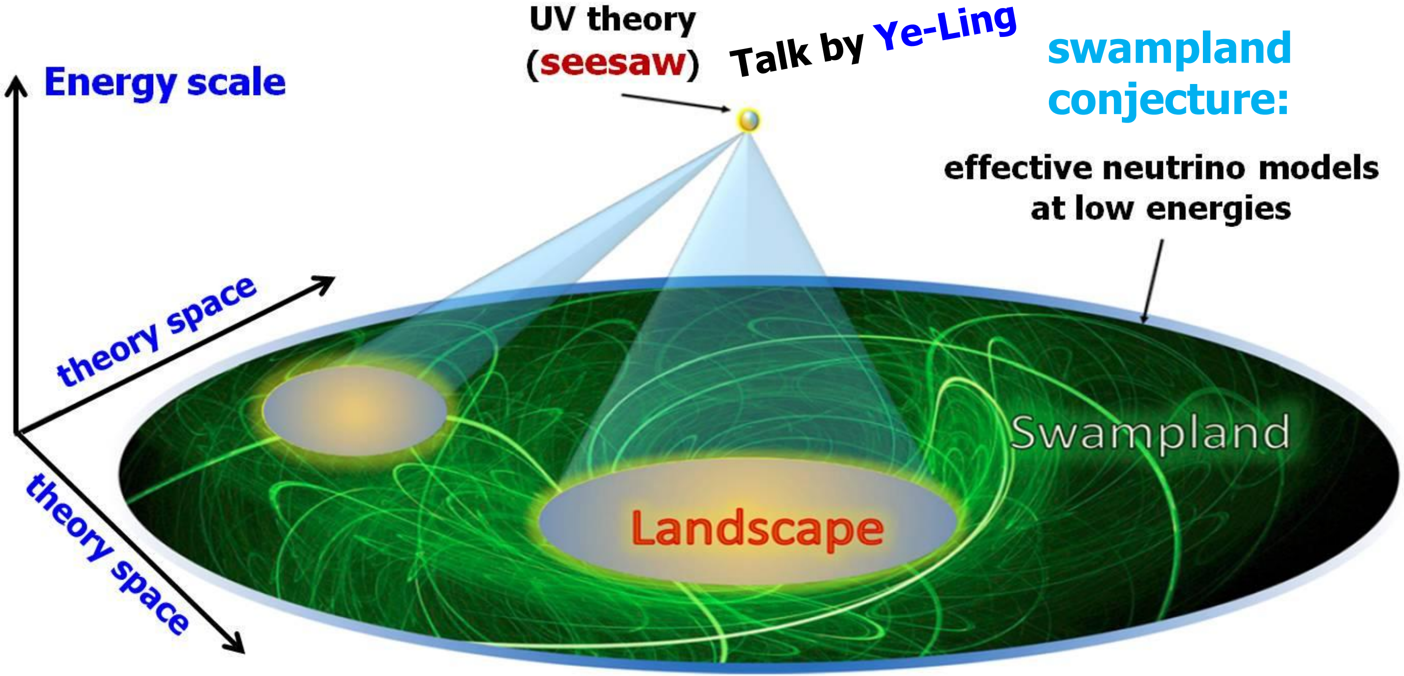
$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\tau i}\right| < 2.20 \times 10^{-5}$$

Conclusions

♦ Following the *naturalness* and *simplicity* principles to extend the SM, I foresee that the known neutrinos are **Majorana** fermions, and their very tiny masses originate from the **seesaw** mechanism. This picture is fully in agreement with the spirit of **Weinberg**'s EFT and thus should be located in **Vafa**'s **landscape** of particle physics.



Cumrun Vafa 2005



♦ In the **precision measurement** era, *model-independent* **TH** or **PH** studies are needed.

An example of this kind

Confronting the seesaw mechanism with neutrino oscillations: a general and explicit analytical bridge

e-Print: 2412.17698 [hep-ph]

SEESAWFIT

Zhi-zhong Xing[1,3,4] *, Jing-yu Zhu[2] †

With the help of a full Euler-like block parametrization of the flavor structure for the canonical seesaw mechanism, we present the *first* general and explicit analytical calculations of the two neutrino mass-squared differences, three flavor mixing angles and the effective Dirac CP-violating phase responsible for the primary behaviors of neutrino oscillations. Such model-independent results will pave the way for testing the seesaw mechanism at low energies.

MANY THANKS