Quantum & quantum-inspired algorithms for reconstruction

2nd workshop on Tracking in Particle Physics Experiments, July 21-23, 2025

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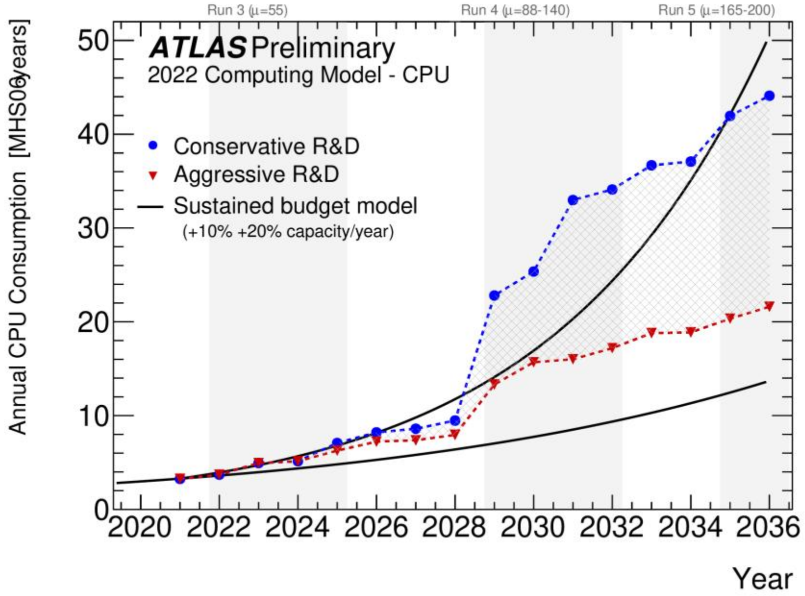
Picture credits: Qianhui Shi, Yan Liang

Reconstruction at LHC & HL-LHC



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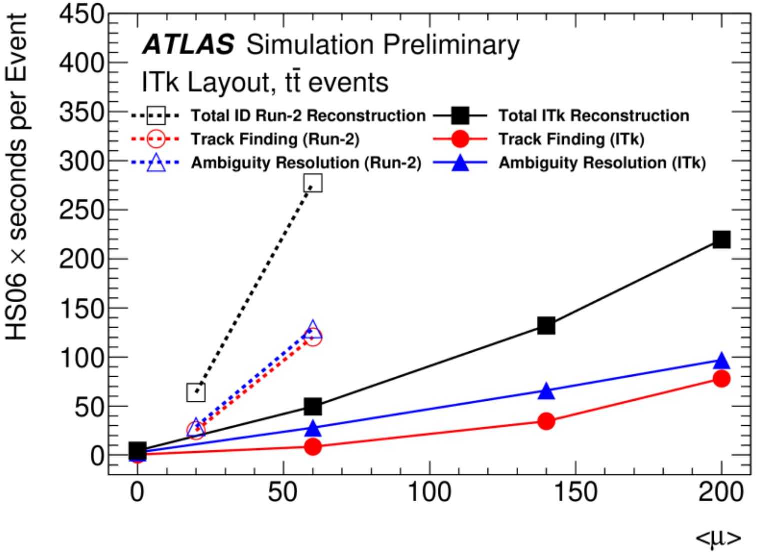
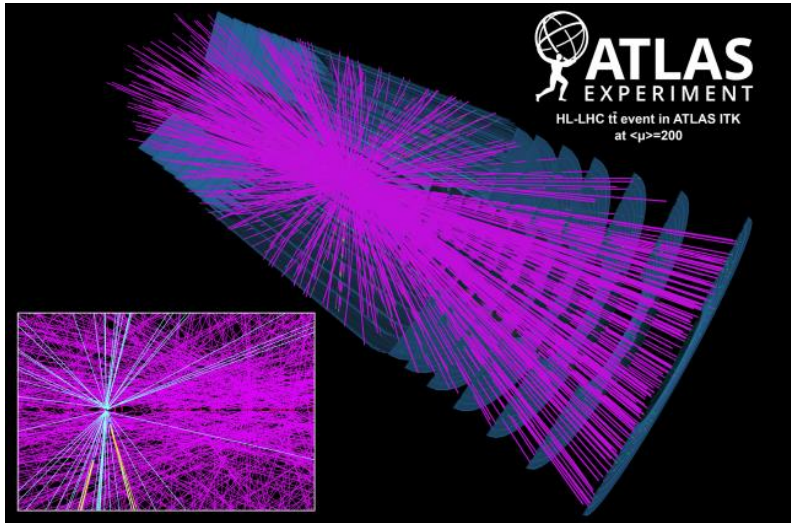




- At the HL-LHC, **CPU time increases exponentially with more pileup**, leading to increase in annual computing cost by a factor of 10-20.
- **Tracking & jet clustering are CPU-consuming tasks.**
- GPU & ML-based approaches are actively investigated, but **quantum algorithms may also bring in innovations.**

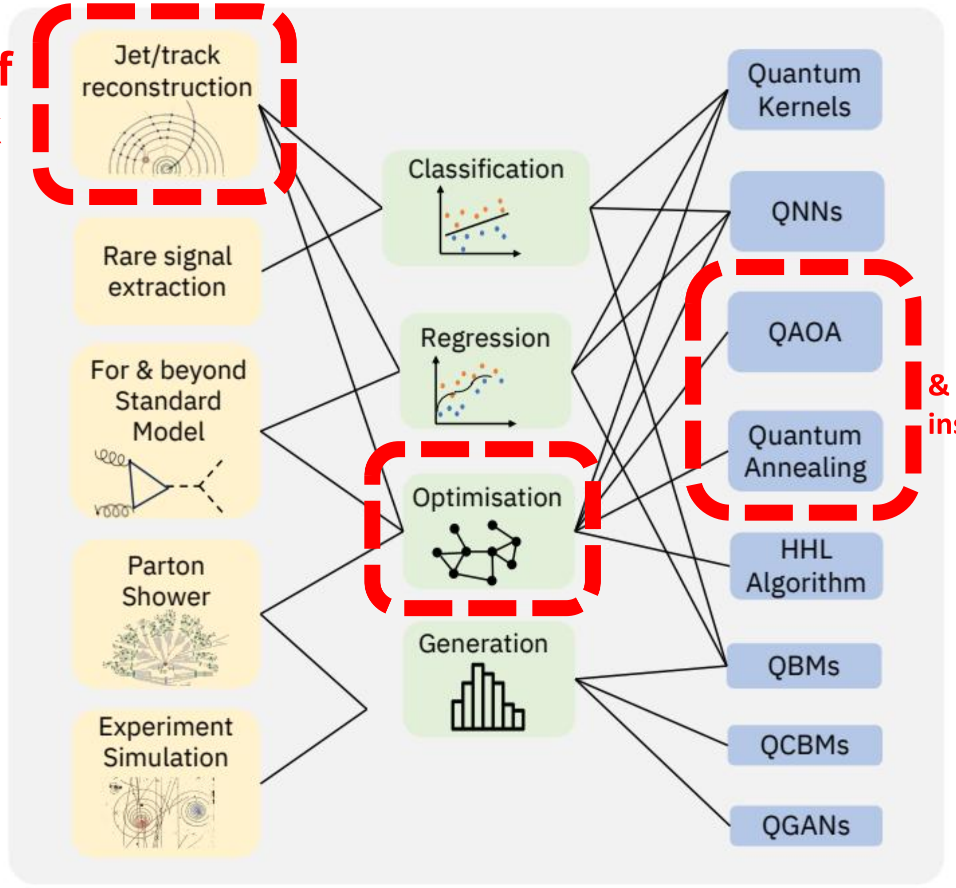
	Run 1	Run 2	HL-LHC
μ	21	40	150-200
Tracks	~280	~600	**~7-10k**

Quantum Applications in HEP

QC4HEP Whitepaper

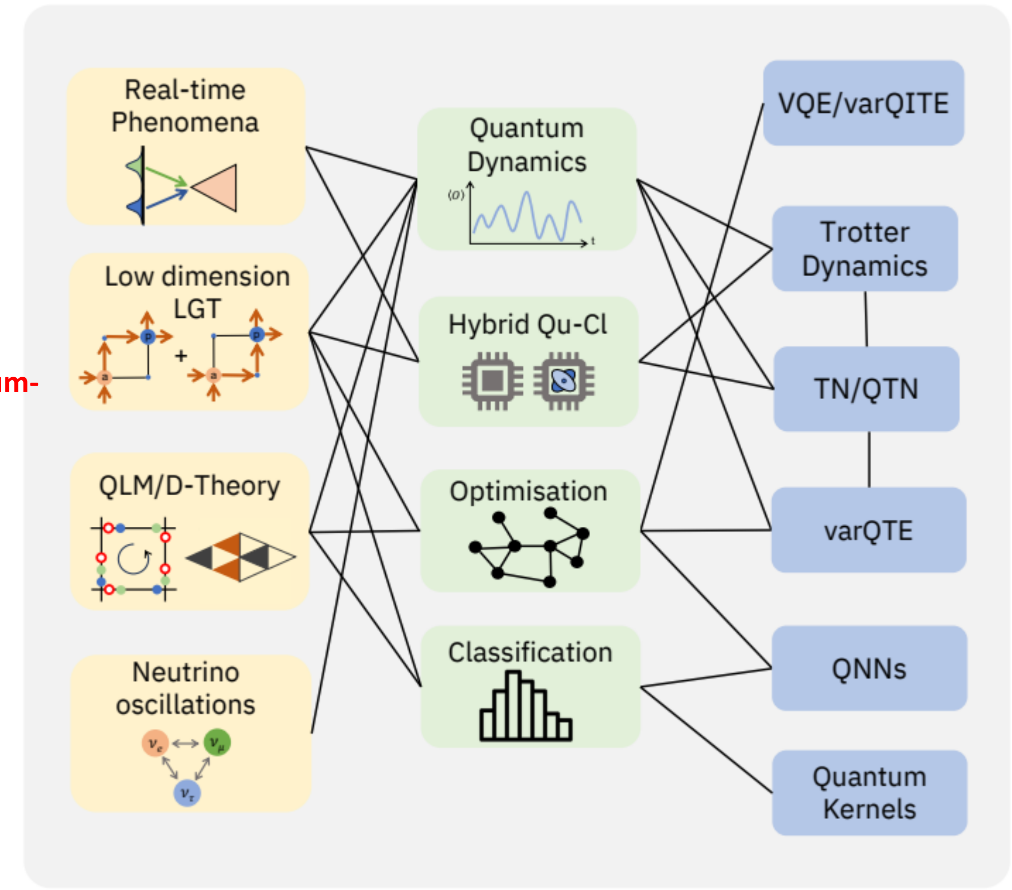
Scope of this talk

Experiment



& Quantum-inspired

Theory

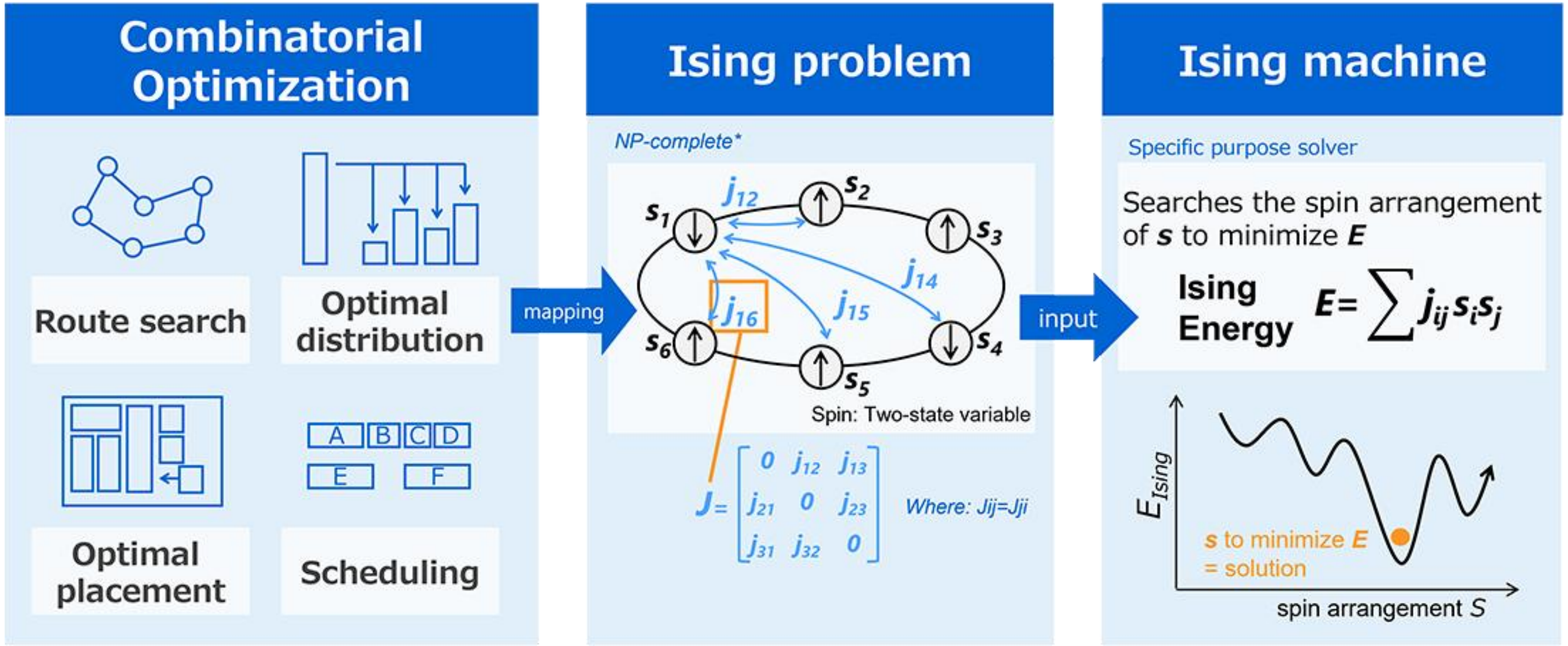


About quantum simulation, see also: C.W. Bauer et al., PRX Quantum 4 (2023) 2, 027001

Iterative or Global Reconstruction?

- Both track & jet reconstruction algorithms can be categorized into iterative & global approaches.
- **Iterative:**
  - Tracking: combined Kalman filter, etc.
  - Jet clustering: all(?) traditional jet algorithms (anti-k_t, Cambridge-Aachen, ee-k_t, etc.)
- **Global:**
  - Tracking: Hough transform, GNN, etc.
  - Jet clustering: ML-based algorithms (though not used in actual experiments so far)
- **In quantum algorithms, global reconstruction can be formulated as combinatorial optimization problems.**

Combinatorial Optimization Problems



- Practically complicated problems can often be formulated as combinatorial optimization problems/Ising problems → **Ground state of Ising Hamiltonian corresponds to the answer.**

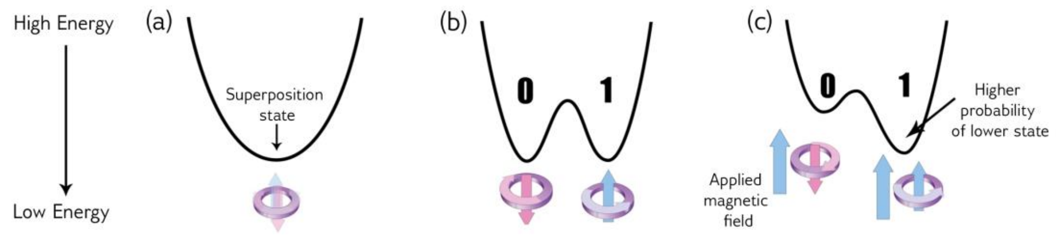
Non-deterministic Polynomial time

- **They are generally NP-complete problem**: no efficient algorithm exists to find the solution.
  → But quantum approaches can provide quasi-optimal answers!

Quantum Hardware Solvers

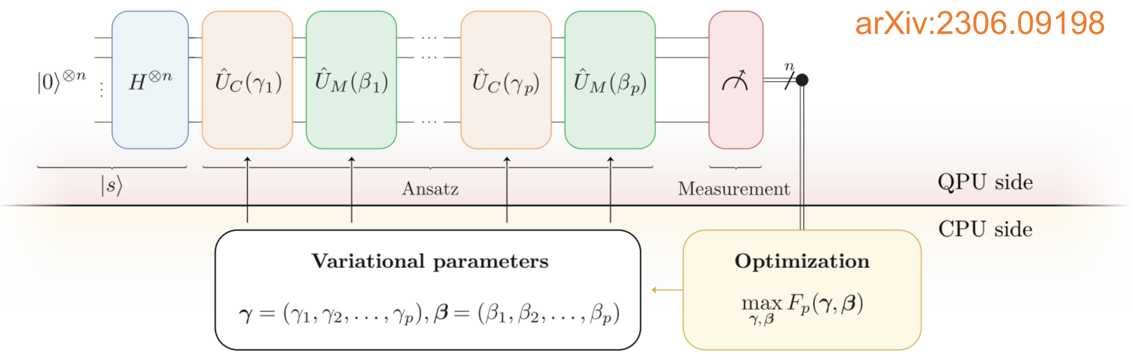
Quantum annealing

©D-wave



- Quantum annealer looks for the global minimum based on the quantum adiabatic theorem and also uses quantum tunneling.
- Higher number of qubits available than quantum gates (4000+ qubits for D-Wave Advantage2)
- Connectivity among qubits is currently limited (6-ways for 2000Q & 20-ways for D-Wave Advantage2).

Quantum Gates + Classical Hybrid

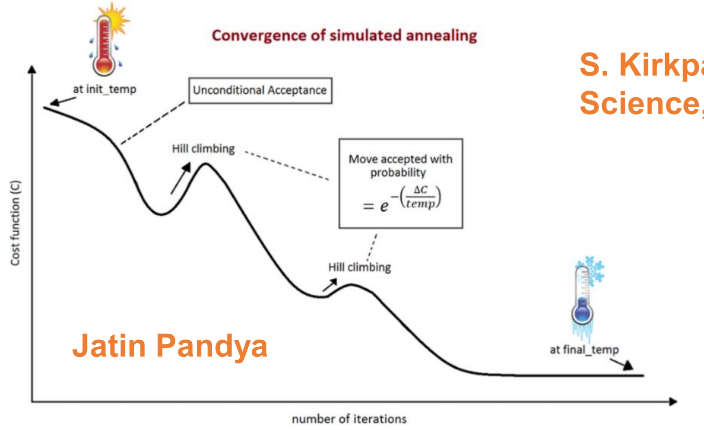


- Quantum gate machines can also solve Ising problems with variational circuits:
  - e.g. **Variational Quantum Eigensolver (VQE), Quantum Approximate Optimization Algorithm (QAOA)**, **imaginary Hamiltonian variational ansatz (iHVA), Imaginary Time Evolution-Mimicking Circuit (ITEMC; next talk by Yahui Chai)** etc.
- Search for the ground state by scanning the variational parameters **with classical optimizers**.

Quantum-Inspired Solvers

"Quantum-inspired" algorithms search for ground state through the classical time evolution of differential equations.

Simulated annealing



S. Kirkpatrick, et al., Science, 220 (1983) 671

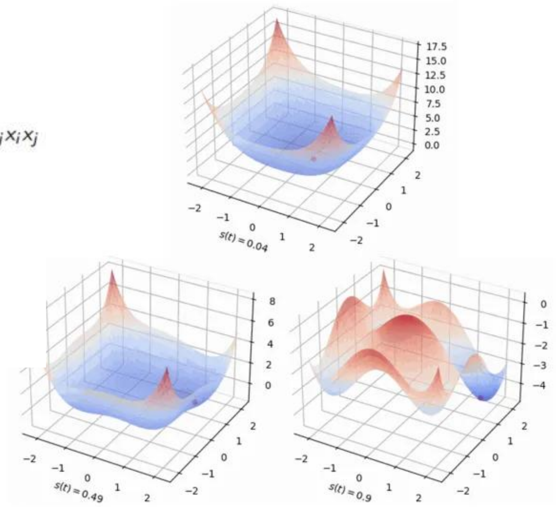
- **Simulated annealing (SA) uses random moves in the solution space.**
- In each random displacement, if lower energy $\Delta E < 0$ is obtained, it is automatically accepted.
- If $\Delta E > 0$, it is accepted probabilistically according to the Boltzmann factor: $P(\Delta E) = \exp(-\Delta E / kT)$.

Simulated bifurcation

$$H_{\mathrm{SB}}(\boldsymbol{x}, \boldsymbol{y}, t) = \sum_{i=1}^{N} \frac{\Delta}{2} y_i^2 + \sum_{i=1}^{N} \left[\frac{K}{4} x_i^4 + \frac{\Delta - p(t)}{2} x_i^2\right] - \frac{\xi_0}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} J_{ij} x_i x_j$$

$$\dot{x}_i = \frac{\partial H_{\mathrm{SB}}}{\partial y_i} = \Delta y_i$$

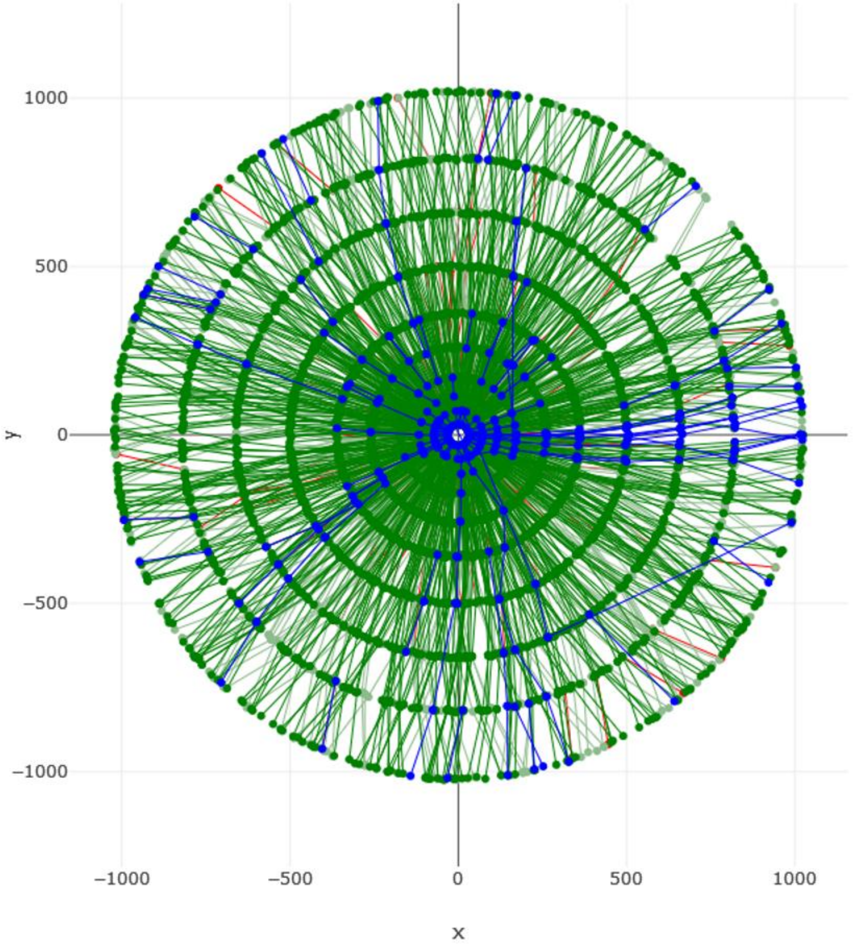
$$\dot{y}_i = -\frac{\partial H_{\mathrm{SB}}}{\partial x_i} = -\left[K x_i^2 - p(t) + \Delta\right] x_i + \xi_0 \sum_{j=1}^{N} J_{i,j} x_j$$



H. Goto et al., Sci. Adv. 2019; 5: eaav2372
H. Goto et al., Sci. Adv. 2021; 7: eabe7953

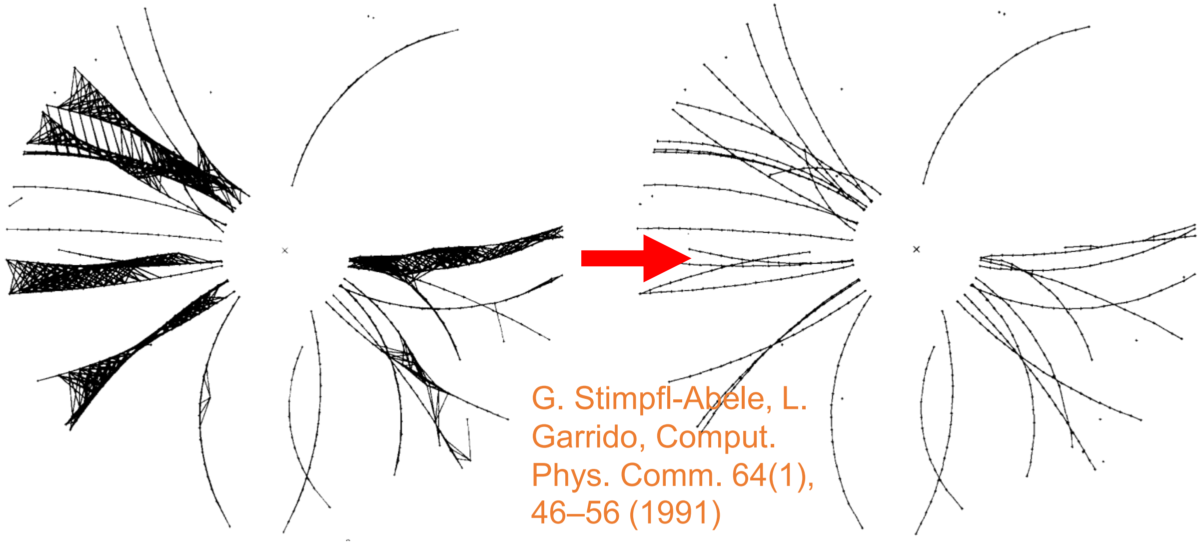
- **Simulated bifurcation (SB) emulates quantum adiabatic evolution of Kerr-nonlinear parametric oscillators, exhibiting bifurcation phenomena.**
- **Several variants exist** depending on the continuous treatment of the spins (x_i): e.g. **aSB, bSB, dSB**

Track Reconstruction

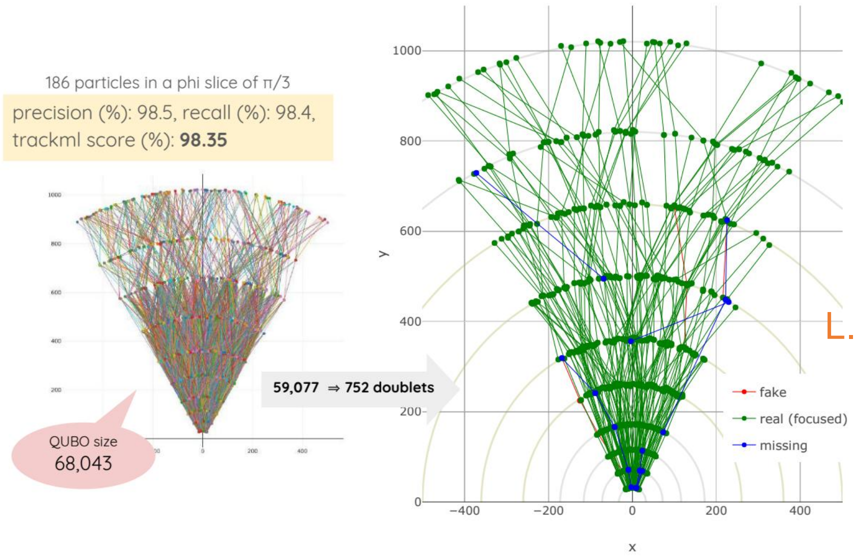


Tracking as Optimization Problem

- **Tracking as an optimization problem: a global approach to reconstruct tracks in one-go.** (↔iterative approach: Combined Kalman Filter)
- **Stimple-Abele & Garrido (1990):** generate all potential doublets with some cuts applied & pursue a binary classification task (i.e. solve an Ising/QUBO problem) to determine which ones should be kept.
- **Modern quantum computing versions:** quantum annealers w/ doublets (A. Zlokapa et al.) & triplet-based (F. Bapst et al.) approaches; quantum gate machines (L. Funcke et al., etc.)



G. Stimpfl-Abele, L. Garrido, Comput. Phys. Comm. 64(1), 46–56 (1991)



L. Linder

Track Finding as Ising/QUBO Problem

- Doublets/triplets are connected to reconstruct tracks & it can be regarded as a **quadratic unconstrained binary optimization (QUBO)/Ising** problem.
- **In our study, we adopted a triplet-based QUBO formulation**.

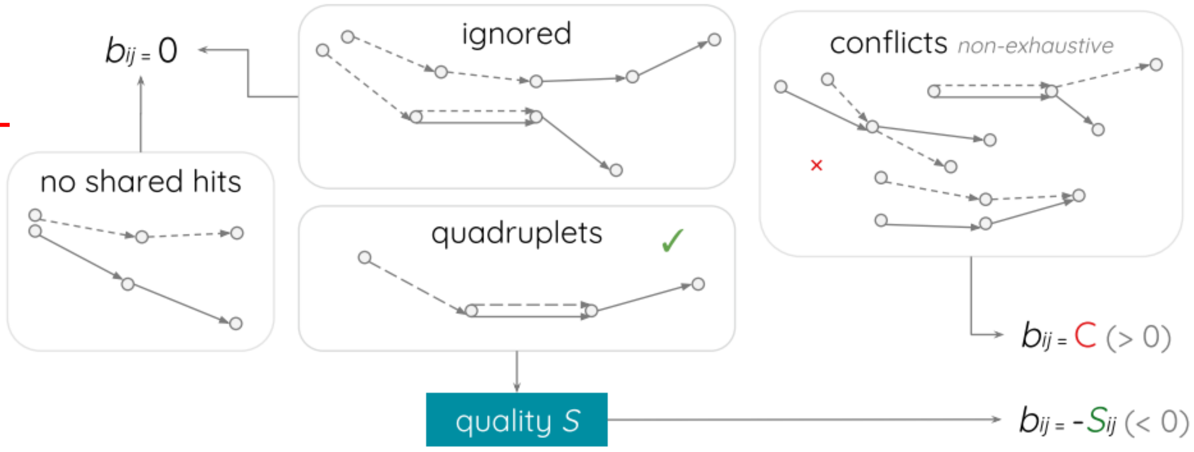
$$O(a,b,T) = \underbrace{\sum_{i=1}^{N} a_i T_i}_{\text{Quality of triplets}} + \underbrace{\sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j}_{\text{Compatibility b/w triplet pairs}}$$

$$a_i = \alpha \left(1 - e^{\frac{|d_0|}{\gamma}}\right) + \beta \left(1 - e^{\frac{|z_0|}{\lambda}}\right),$$

b_{ij} = 0 (if no shared hit), 1 (if conflict)
= $-S_{ij}$ (if two hits are shared)

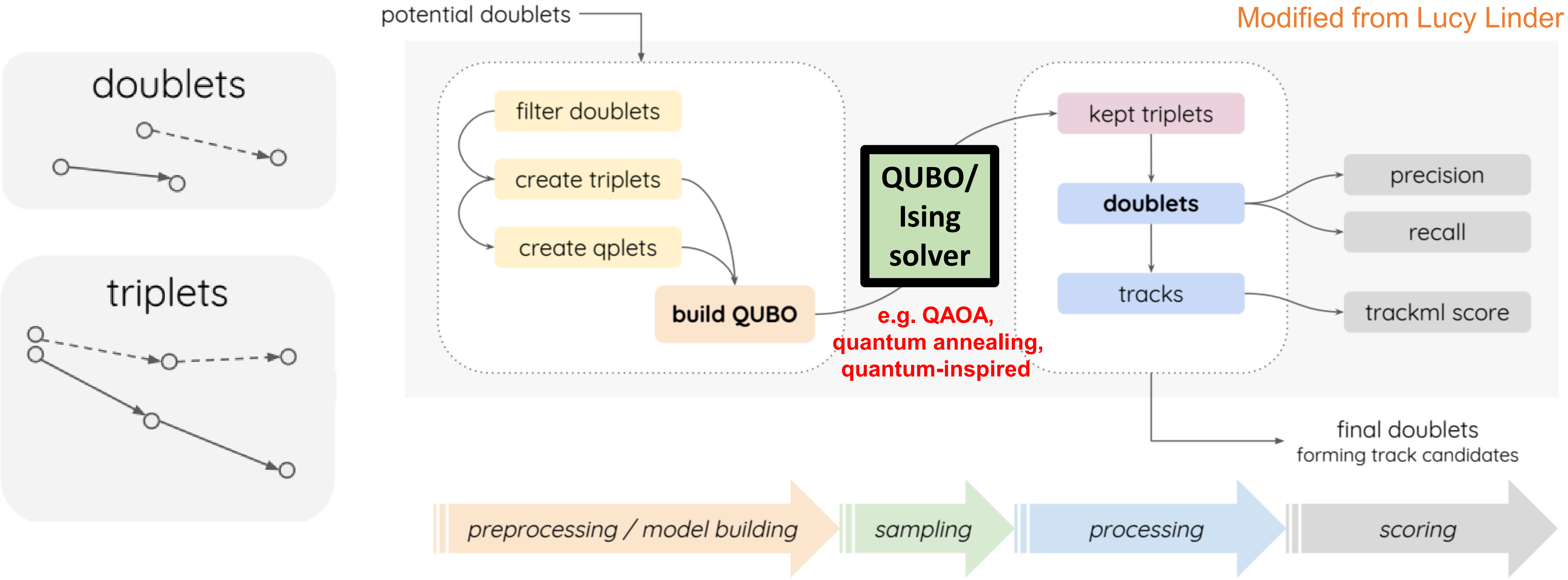
$$S_{ij} = \frac{1 - \frac{1}{2}(|\delta(q/p_{Ti}, q/p_{Tj})| + max(\delta\theta_i, \delta\theta_j))}{(1 + H_i + H_j)^2},$$

F. Bapst et al. Comp. Soft. Big Sci. 4 (2019) 1.



Minimizing QUBO is equivalent to searching for the ground state of the Hamiltonian.

Track Finding Workflow w/ QUBO

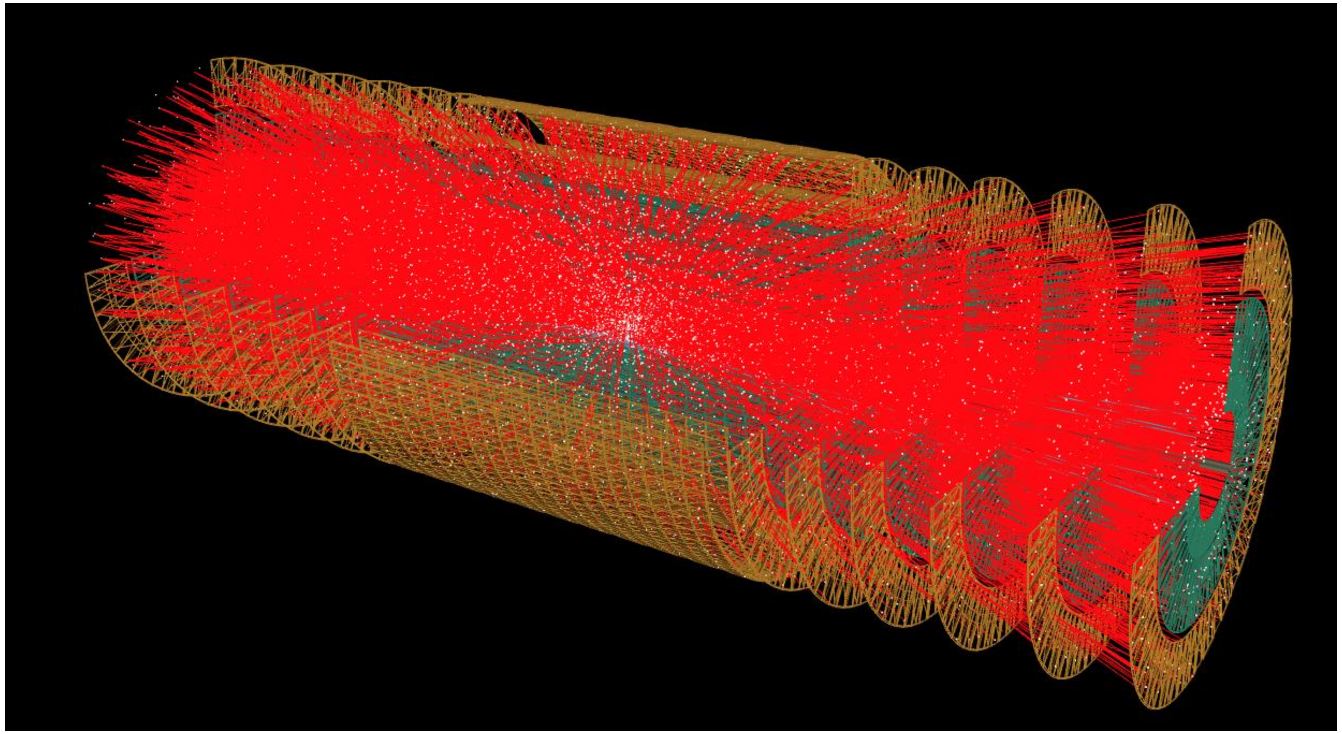


- We build QUBO on an event-by-event basis from the silicon detector hits.
- Predicted ground state will define which triplets should be kept (binary=1). Connecting the adopted triplets will give us the tracks.

Dataset (TrackML)

- TrackML is an open-source dataset prepared for TrackML Challenges (two competitions hosted by CERN & Kaggle).
- It is **designed w/ HL-LHC conditions (200 pileup) & run w/ fast simulation (e.g. noise, inefficiency, parametrized material effects, etc.)**
- Only tracks w/ p_T>1 GeV in the barrel are considered.
- QUBO is computed event by event using hepqpr-qallse framework.

Amrouche, S., et al., arXiv:1904.06778 (2019);
Amrouche, S., et al., Comput. Softw. Big Sci. 7(1), 1 (2023)

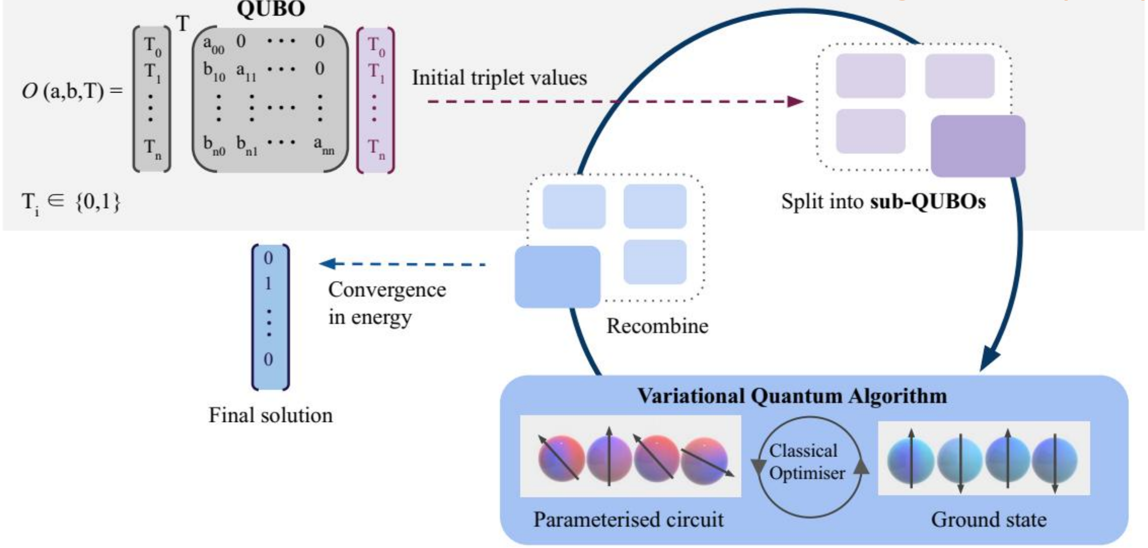


Thanks to Andreas Salzburger for suggestions and discussions!

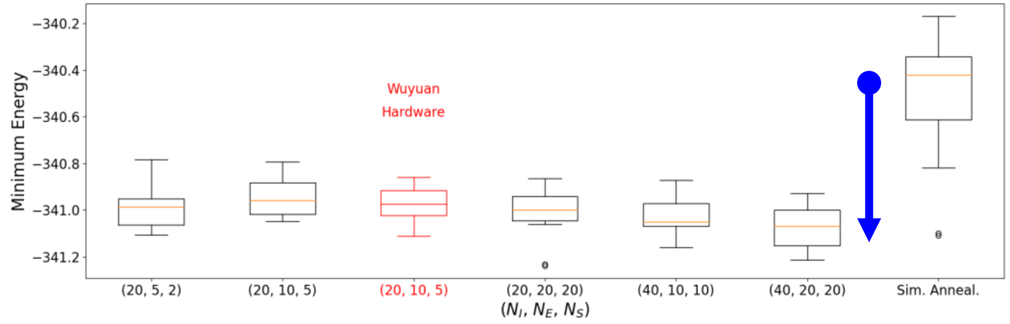
Quantum Hardware Approaches

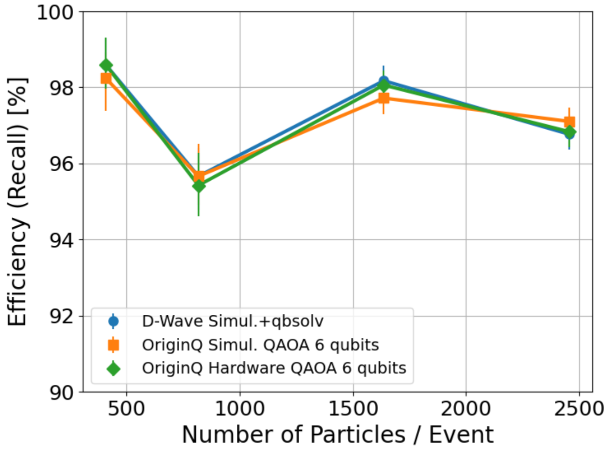
- # of triplet candidates determines # of qubits required → **HL-LHC conditions (O(0.1M) qubits) do not fit into the current scale of quantum annealing & gate computers**
- **QUBO is split into sub-QUBOs.** There is no visible degradation in Ising solving precision, but **the computation speed degrades by a few orders of magnitude.**

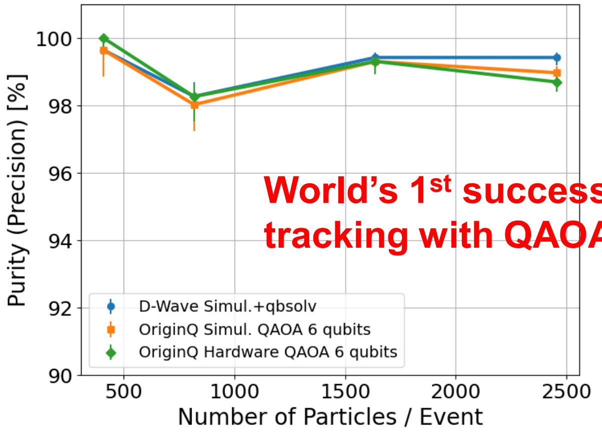
A.Crippa et al., Comput Softw Big Sci 7, 14 (2023)



1st usage of theoretically robust sub-QUBO algorithm in HEP

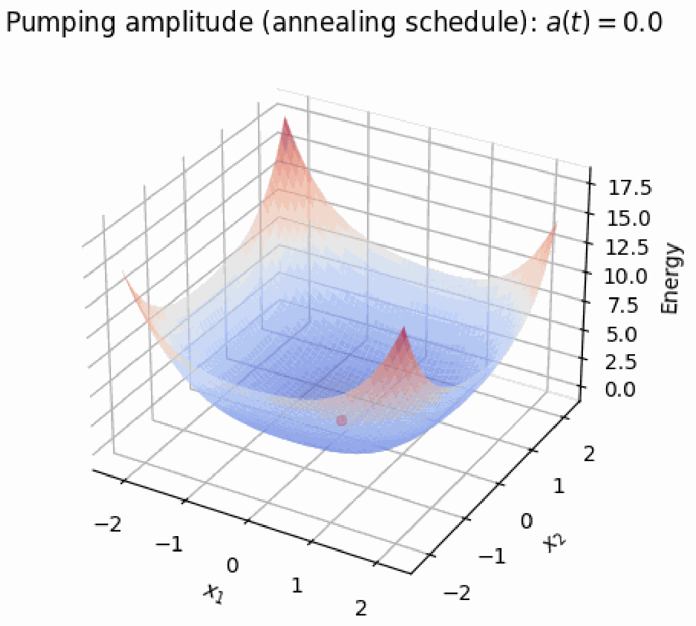


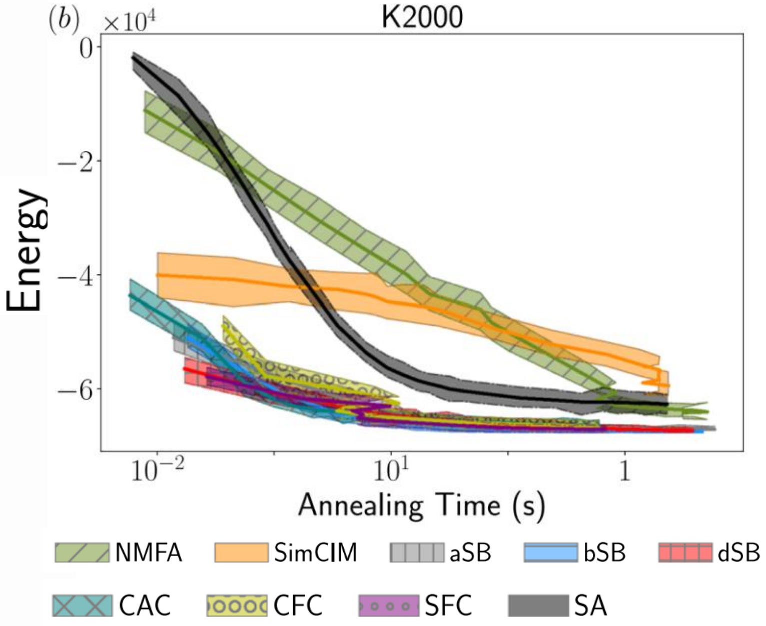




World's 1st successful tracking with QAOA!

Quantum-Inspired Approach (SB)





Graph size	Algorithm	Hardware	Time(s)
	TTN	CPU 1 core	5.62
	Brute-force search[46]	GPU Titan V	$>10^{48}$
4 × 4 × 8	Exact belief propagation[13]	CPU 1 core	~0.96
	QA[13]	D-Wave	~0.05
	bSB	CPU 1 core	0.12
	bSB	GPU Tesla V100	<0.001
	TTN	CPU 1 core	32400
	TTN[44]	GPU Tesla V100	84
8 × 8 × 8	Brute-force search[46]	GPU Titan V	$>10^{190}$
	Exact belief propagation[13]	CPU 1 core	~2880
	dSB	CPU 1 core	17.64
	dSB	GPU Tesla V100	<0.68

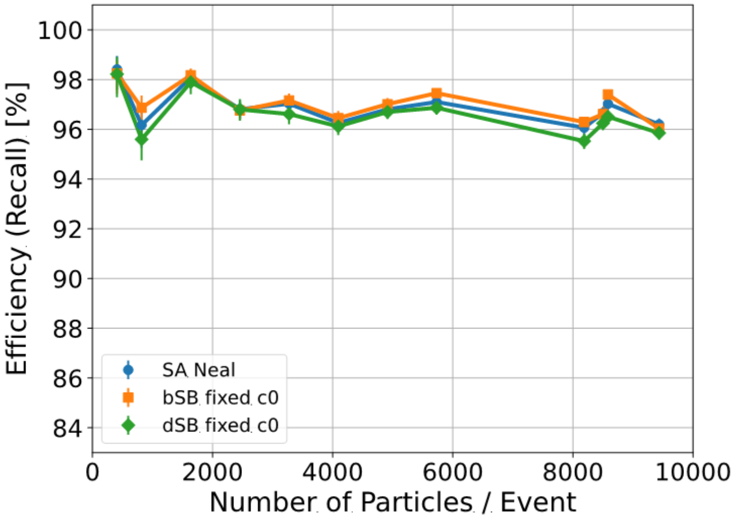
Q.G. Zeng et al., Comm. Phys. (2024) 7:249

- **SB is a powerful quantum-inspired algorithm & can directly handle up to ~1M-qubit-level problems.**
- **It can run in parallel** unlike simulated annealing. It also benefits from cutting-edge resources such as **GPUs and FPGAs**.
- **It is known to outperform other classical algorithms as well as quantum annealing (QA) for some existing problems: both in terms of minimum energy prediction & computing speed.**

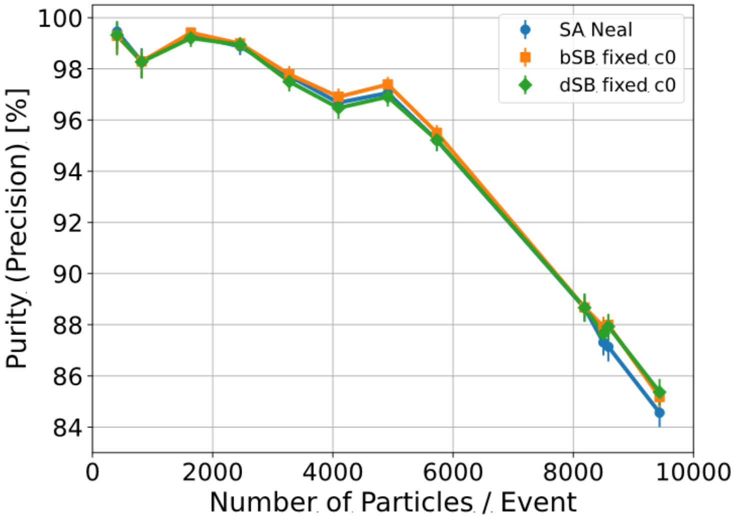
H. Okawa, Q.G. Zeng, X.Z. Tao, M.H. Yung, Springer Comput. Softw. Big Sci. 8, 16 (2024)

Quantum-Inspired Tracking

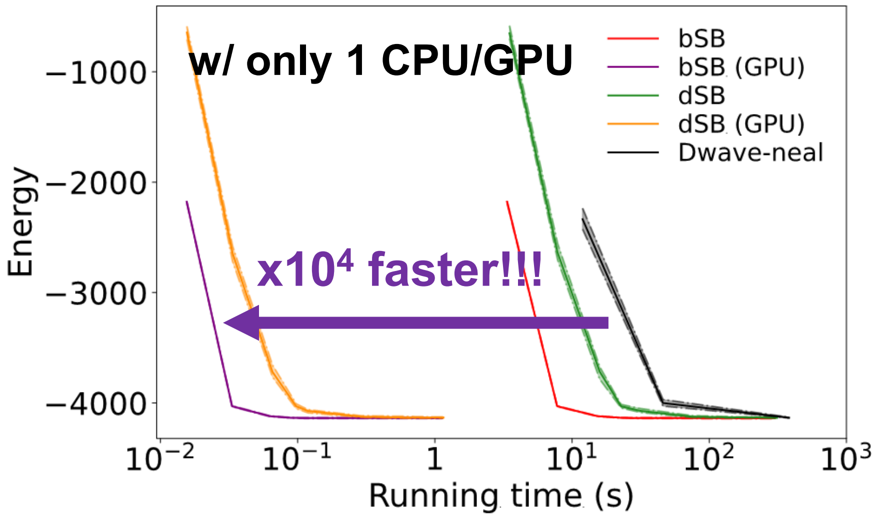
Efficiency



Purity

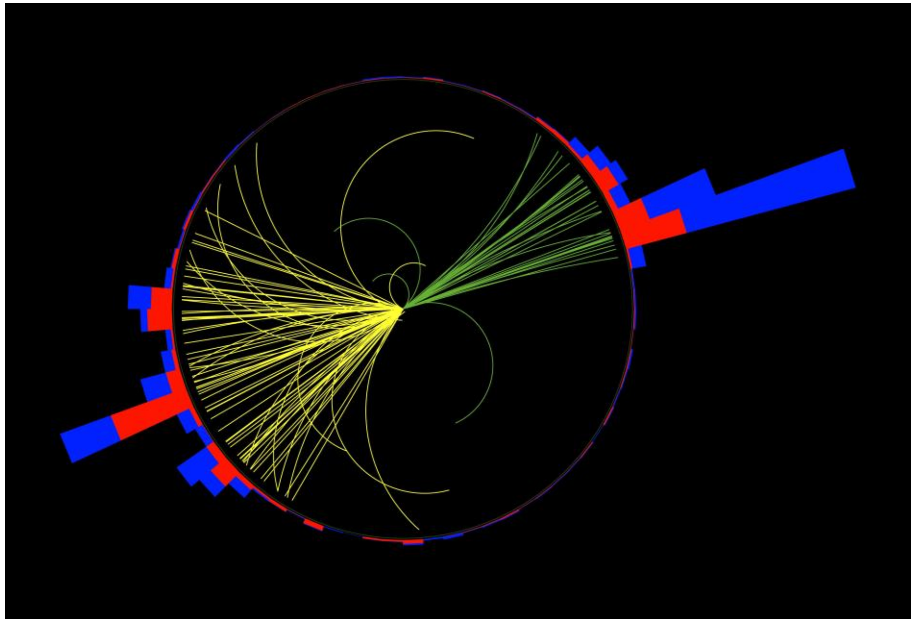


Ising Energy vs computing time



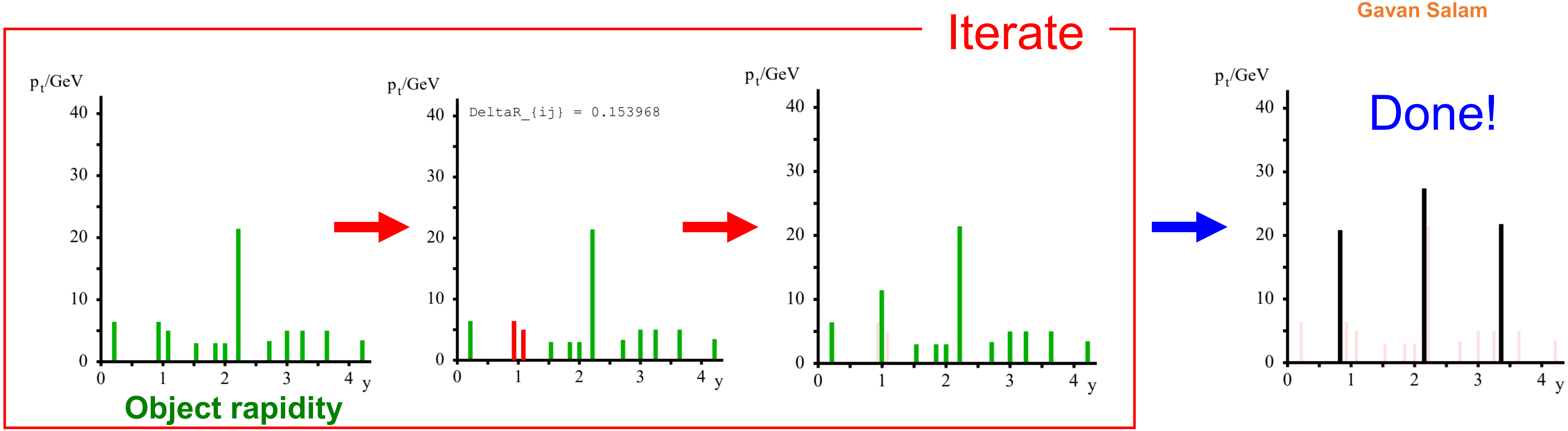
- **Quantum-annealing-inspired algorithm can DIRECTLY handle the HL-LHC dataset.**
- SB provides compatible or slightly better efficiency & purity than D-Wave Neal.
- bSB provides **4 orders of magnitude speed-up (23min → 0.14s) from D-Wave Neal** for HL-LHC data **(cf. D-Wave hardware w/ qbsolv is ~2 orders of magnitude slower than Neal).**
- **SB can effectively run w/ multiple processing, GPU & FPGA → Perfect match with HEP!!**

Jet Reconstruction



Traditional Approach

Modified figures from Gavan Salam



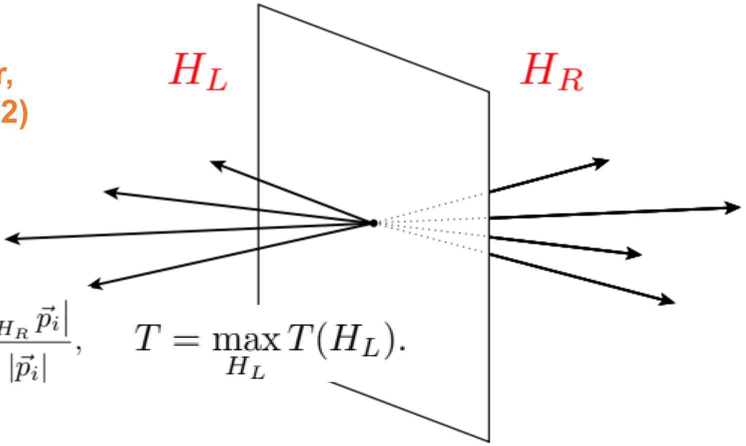
- Repeatedly recombine closest pair of objects (tracks, calorimeter clusters or particle flows etc.):
  - Terminate by a **user-defined distance R [inclusive clustering]**
  - Terminate by a **user-defined jet multiplicity [exclusive clustering]**
- **Users also define the distance**; i.e. how they call objects as "close" → $\Delta R_{ij}^2 = \Delta y_{ij}^2 + \Delta\phi_{ij}^2$

e.g. Distance adopted in Cambridge-Aachen jet algorithm

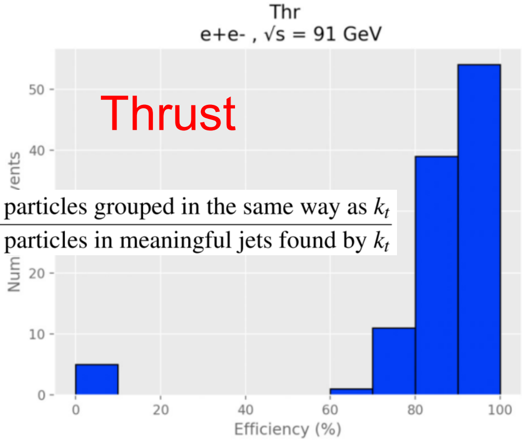
Jet Reconstruction as Ising Problem

Quantum Annealing (Thrust or Angle-based)

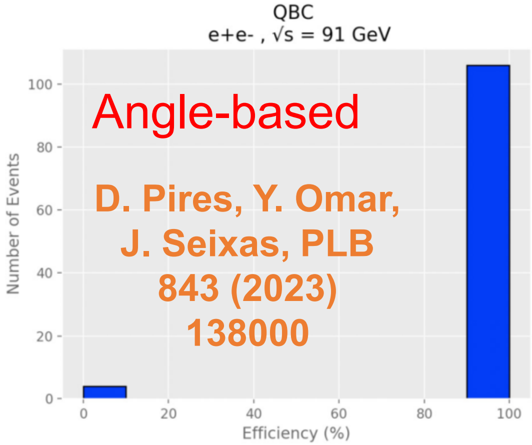
A. Delgado, J. Thaler, PRD 106, 094016 (2022)



$$T(H_L) = \frac{2\left|\sum_{i\in H_L} \vec{p}_i\right|}{\sum_{i=1}^{N} |\vec{p}_i|} = \frac{2\left|\sum_{i\in H_R} \vec{p}_i\right|}{\sum_{i=1}^{N} |\vec{p}_i|}, \qquad T = \max_{H_L} T(H_L).$$



$$\varepsilon = \frac{\text{\# of particles grouped in the same way as } k_t}{\text{\# of particles in meaningful jets found by } k_t}$$

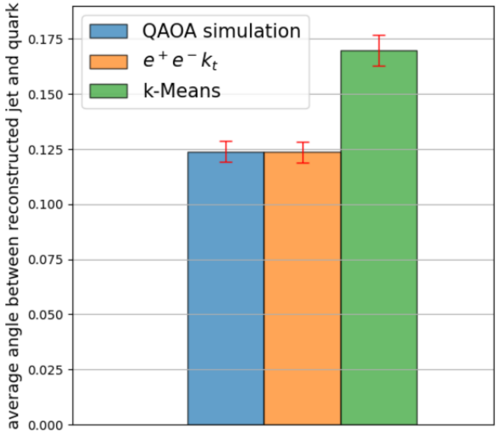


D. Pires, Y. Omar, J. Seixas, PLB 843 (2023) 138000

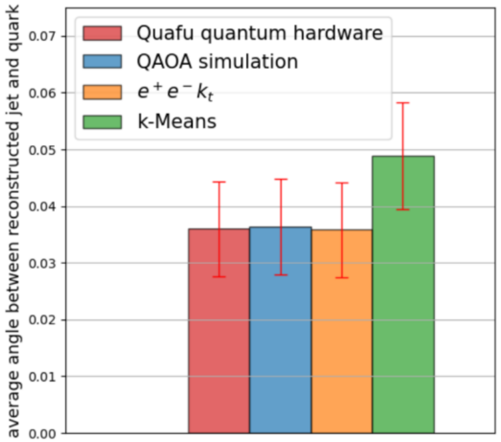
Quantum Gates (e.g. QAOA)

Y. Zhu et al., Science Bulletin 70 (2025) 460

30-particle data ($e^+e^- \rightarrow ZH \rightarrow \nu\nu ss$)



6-particle data ($e^+e^- \rightarrow ZH \rightarrow \nu\nu ss$)



- Jet reconstruction can also be considered as a QUBO problem. (There are also iterative quantum approaches; backup)
- **Quantum annealing:** Angle-based method has better performance than the Thrust-based, but **does not work for multijet (N_{jet}>2) events so far.**
- **QAOA:** Used small-size dataset & evaluated average angle.

Multijet QUBO Formulation (Our Study)

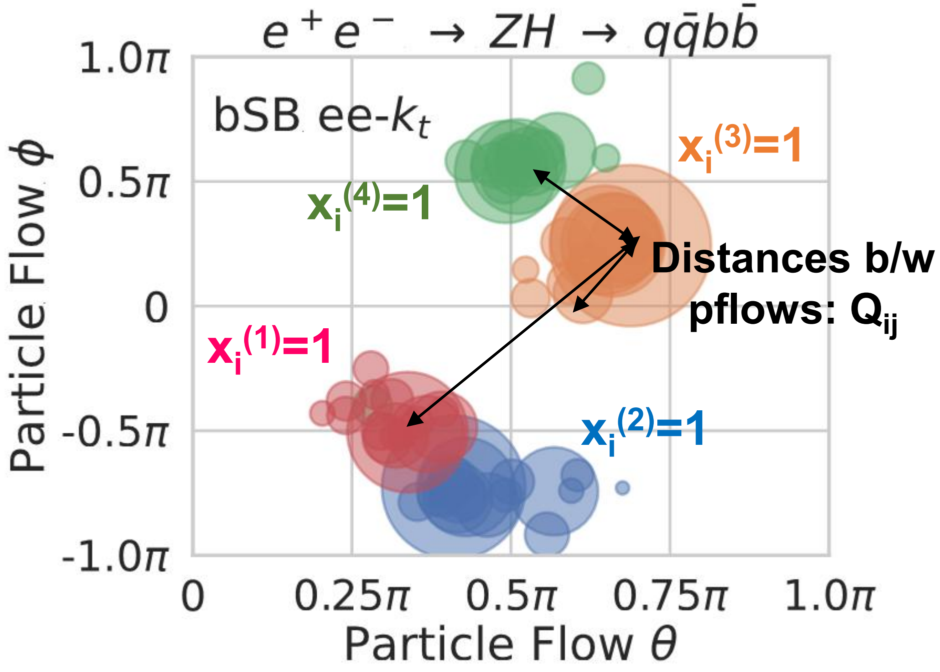
QUBO Formulation

$$O_{\text{QUBO}}^{\text{multijet}}(x_i) = \underbrace{\sum_{n=1}^{n_{\text{jet}}} \sum_{i,j=1}^{N_{\text{input}}} Q_{ij} x_i^{(n)} x_j^{(n)}}_{\text{Defines distances b/w particle flow candidates}} + \lambda \underbrace{\sum_{i=1}^{N_{\text{input}}} \left(1 - \sum_{n=1}^{n_{\text{jet}}} x_i^{(n)}\right)^2}_{\text{Avoids double/none-assignment of particle flow candidates}},$$

$$Q_{ij} = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij}). \quad \textbf{[ee-}k_t\textbf{ distance]}$$

$$Q_{ij} = -\frac{1}{2}\cos\theta_{ij} \quad \textbf{[angle-based]}$$

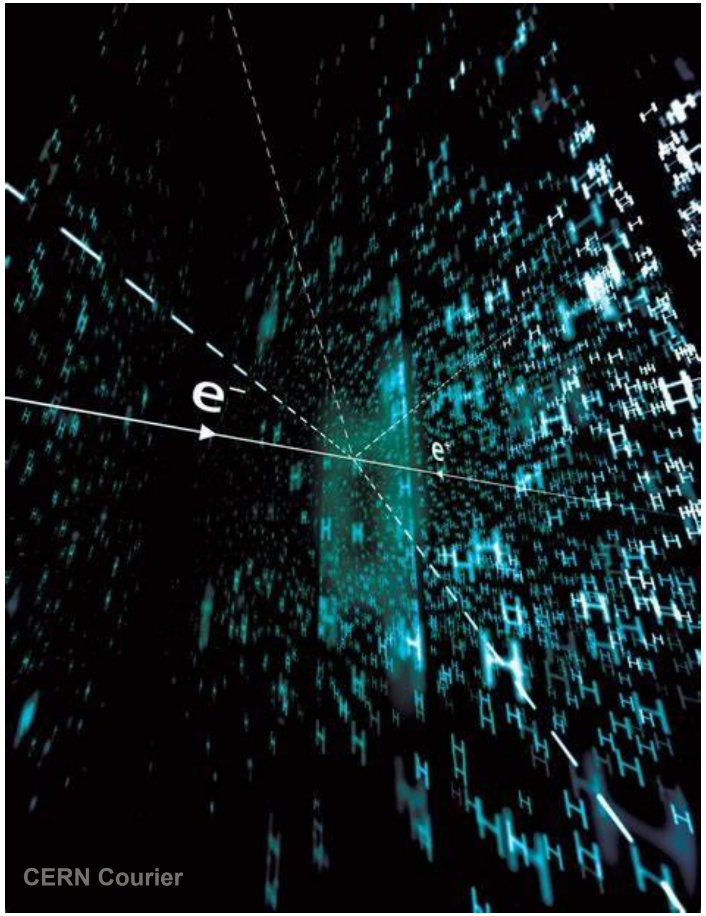
D. Pires, Y. Omar, J. Seixas, PLB 843 (2023) 138000



- **Exclusive jet finding (i.e. fixed number of jets) with the ee-k_t algorithm** is the baseline at CEPC & other e+e- future Higgs factories.
- **We adopt the same distance in the QUBO formulation. <u>QUBO is designed for general jet multiplicity beyond 2 jets.</u>** $x_i^{(n)}=1$ means i-th constituent belongs to n-th jet.
- Performance is also compared with the angle-based method from a previous study.

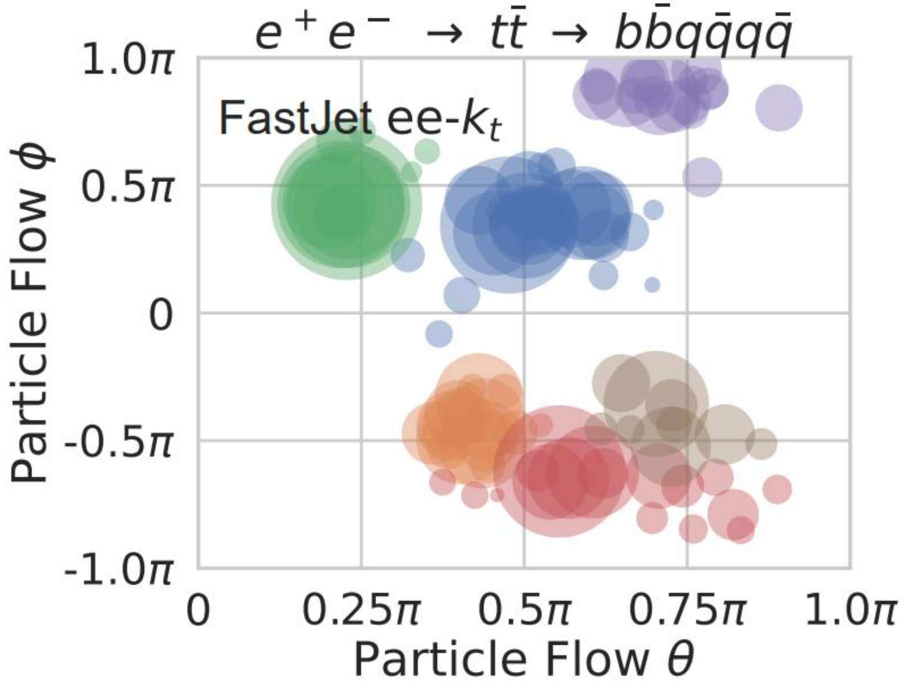
Dataset

- Three sets of e+e- collision events are generated to consider various jet multiplicity:
  - $Z \rightarrow q\bar{q}$ ($\sqrt{s}$=91 GeV, 2 jets),
  - $ZH \rightarrow q\bar{q}b\bar{b}$ ($\sqrt{s}$=240 GeV, 4 jets)
  - $t\bar{t} \rightarrow b\bar{b}q\bar{q}\bar{q}q$ ($\sqrt{s}$=360 GeV, 6 jets)
- **Delphes card with the CEPC 4th-detector concept** is used for the fast simulation.
  → Thanks to Gang Li, Shudong Wang and Xu Gao for feedback!
- Jets are reconstructed from **the particle flow candidates**.



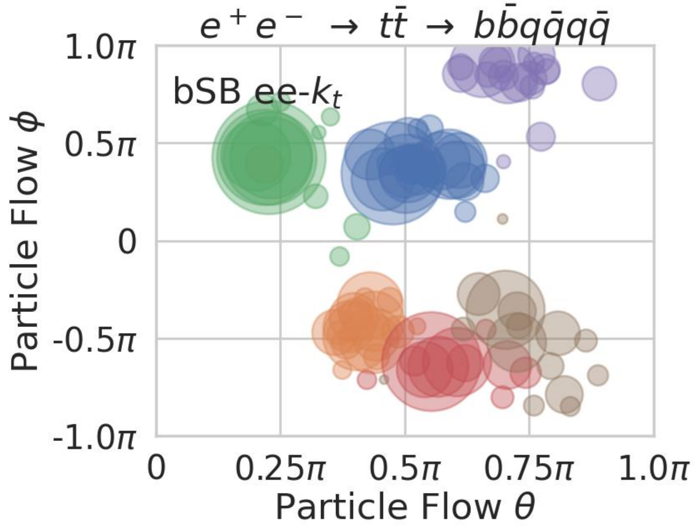
Event Displays ($t\bar{t}$)

Traditional (FastJet)

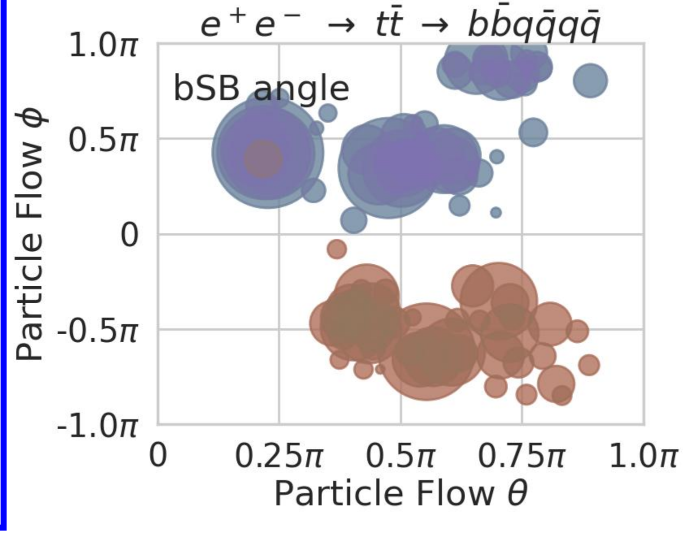


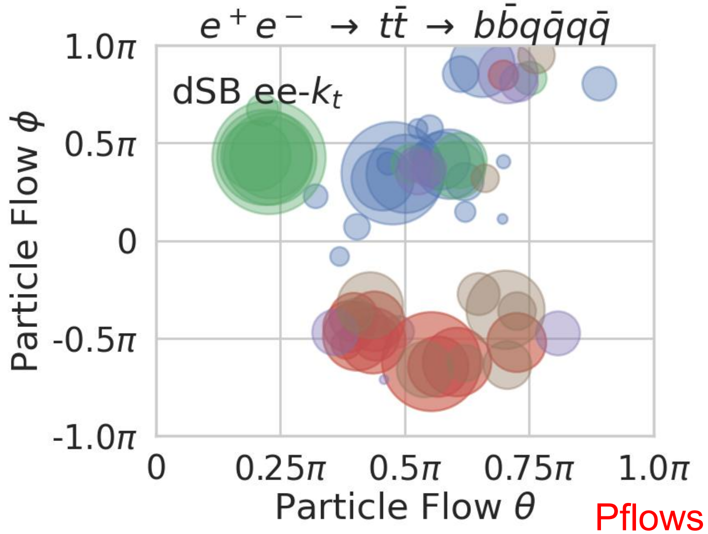
Quantum-Inspired

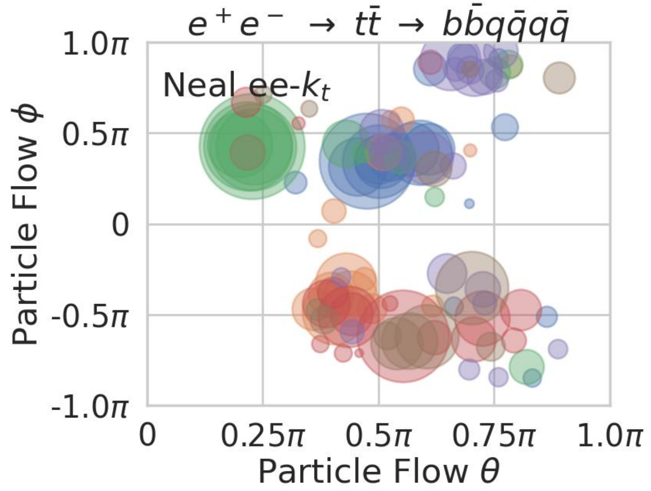
Success!



Missing some jets + double counting



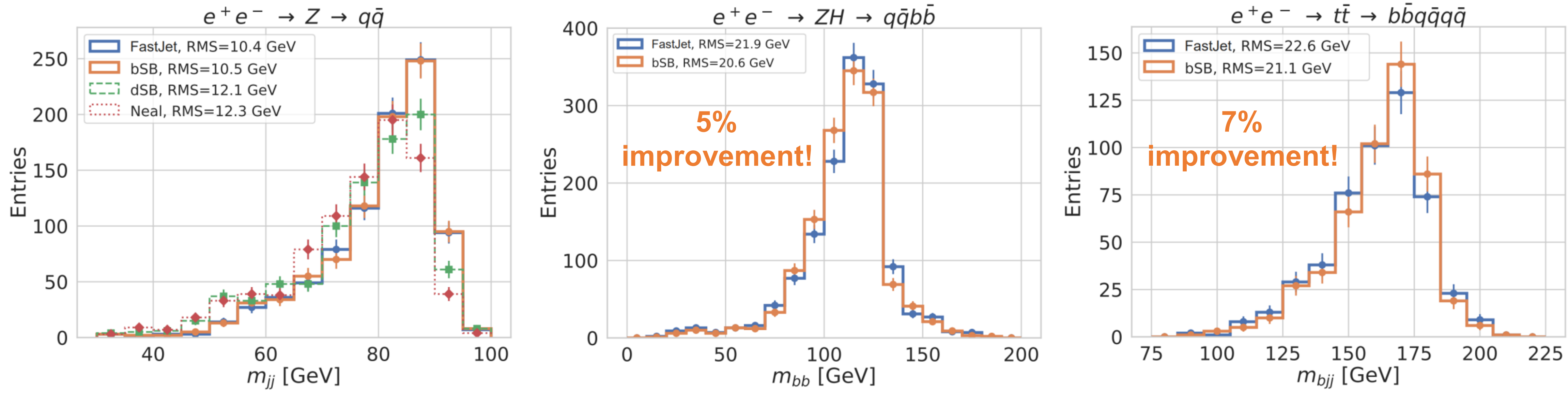




Pflows totally mixed up

- **Only bSB w/ ee-k_t QUBO model can reasonably reconstruct all jets.**
- **Angle QUBO model & other quantum-inspired algorithms miss some jets and/or PFlows are totally mixed up.**

Impact on Invariant Mass

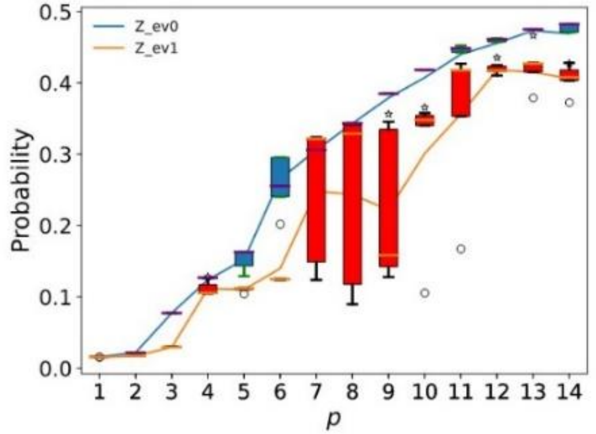


- **The clustering w/ bSB is slightly different from FastJet (see backup for quantitative comparison), <u>but it's OK</u>.** → Let's check the inv. mass resolutions.
- **<u>bSB improve mass resolution for multijet! (& comparable resolution for Z)</u>**
- Other quantum-inspired algorithms (dSB & Neal) already has ~20% degradation in Z mass resolution & unable to properly reconstruct jets in multijet events (thus not shown for ZH & $t\bar{t}$)

Comparison w/ Quantum Hardware

- Performance was compared with two simplified jet datasets (12 qubits) to quantum annealing and QAOA using simulator.
  - [QuantumAnnealing.jl package]() is used to evaluate D-Wave 2000Q performance (6-way connectivity).
- Even for such small datasets, **bSB exceeds the speed of quantum annealing by about two orders of magnitude & even more for QAOA** (w/ a caveat that QAOA should run faster on real quantum hardware).

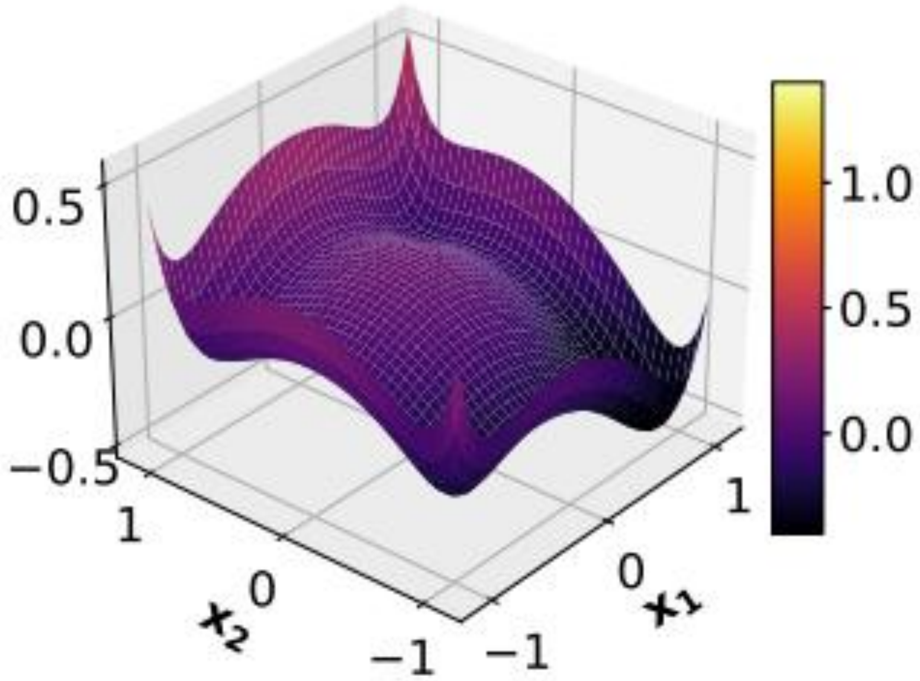
QAOA Performance



Time-to-solution for D-Wave 2000Q estimated by simulation, bSB, dSB, and QAOA on a quantum circuit simulator for two simplified $Z \to q\bar{q}$ events.

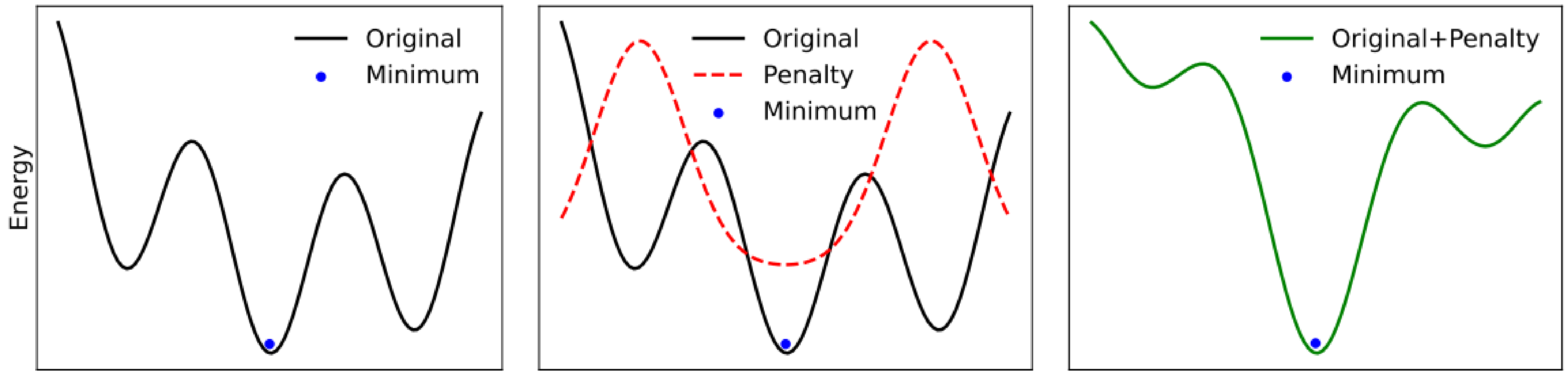
Event	D-Wave [s]	bSB [s]	dSB [s]	QAOA [s]
0	21.29	0.35	0.79	1.07×10^3
1	20.52	0.36	0.89	3.36×10^3

Further Improvement

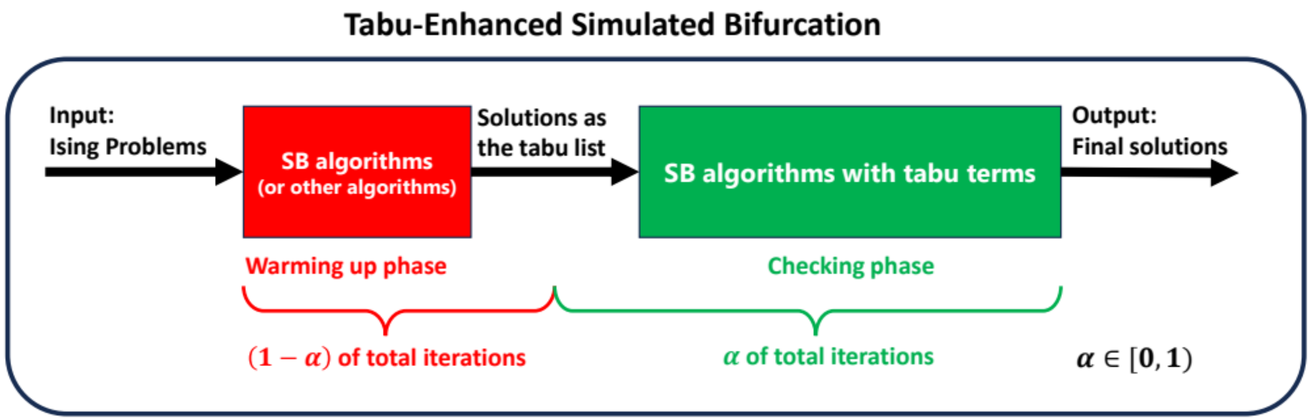


XZ Tao, QG Zeng, ZJ Huang, BW Zuo, J Zhuang, H Okawa, MH Yung, in preparation

Further Improvement in Quantum-Inspired



- We have developed a new variant of SB: Tabu-enhanced simulated bifurcation.
- Penalty is applied to local minima extracted from the warming-up phase.

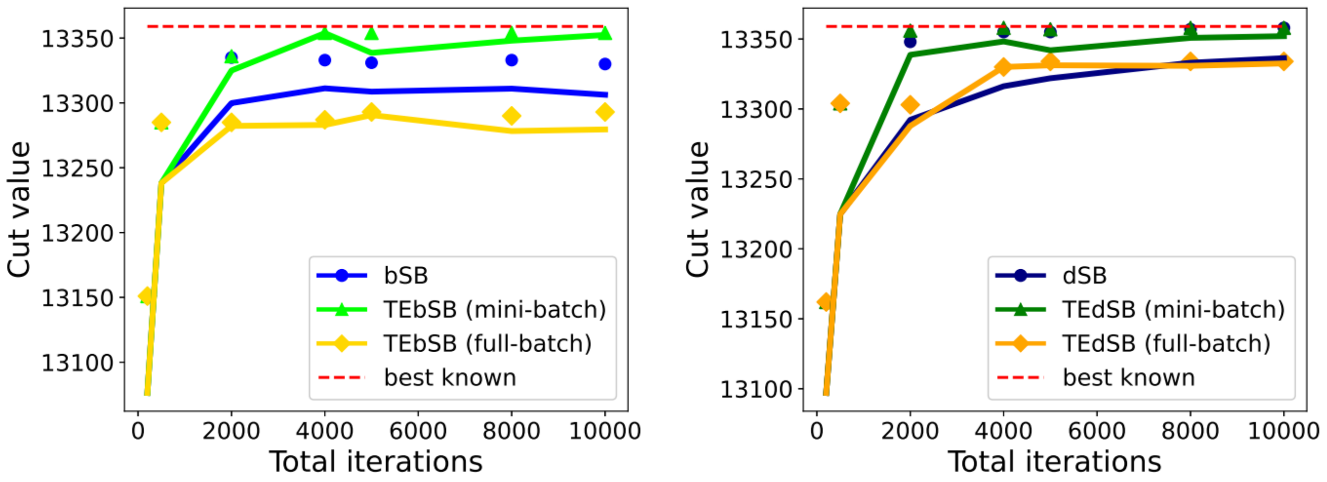


XZ Tao, QG Zeng, ZJ Huang, BW Zuo, J Zhuang, H Okawa, MH Yung, in preparation

Further Improvement in Quantum-Inspired

- Much improved values obtained for G-set Max-Cut benchmark dataset.
- **Visible improvement in minimum energy prediction & computing time** for TrackML datasets.

Max-cut values from G22 instance



Minimum energy predictions from TrackML datasets

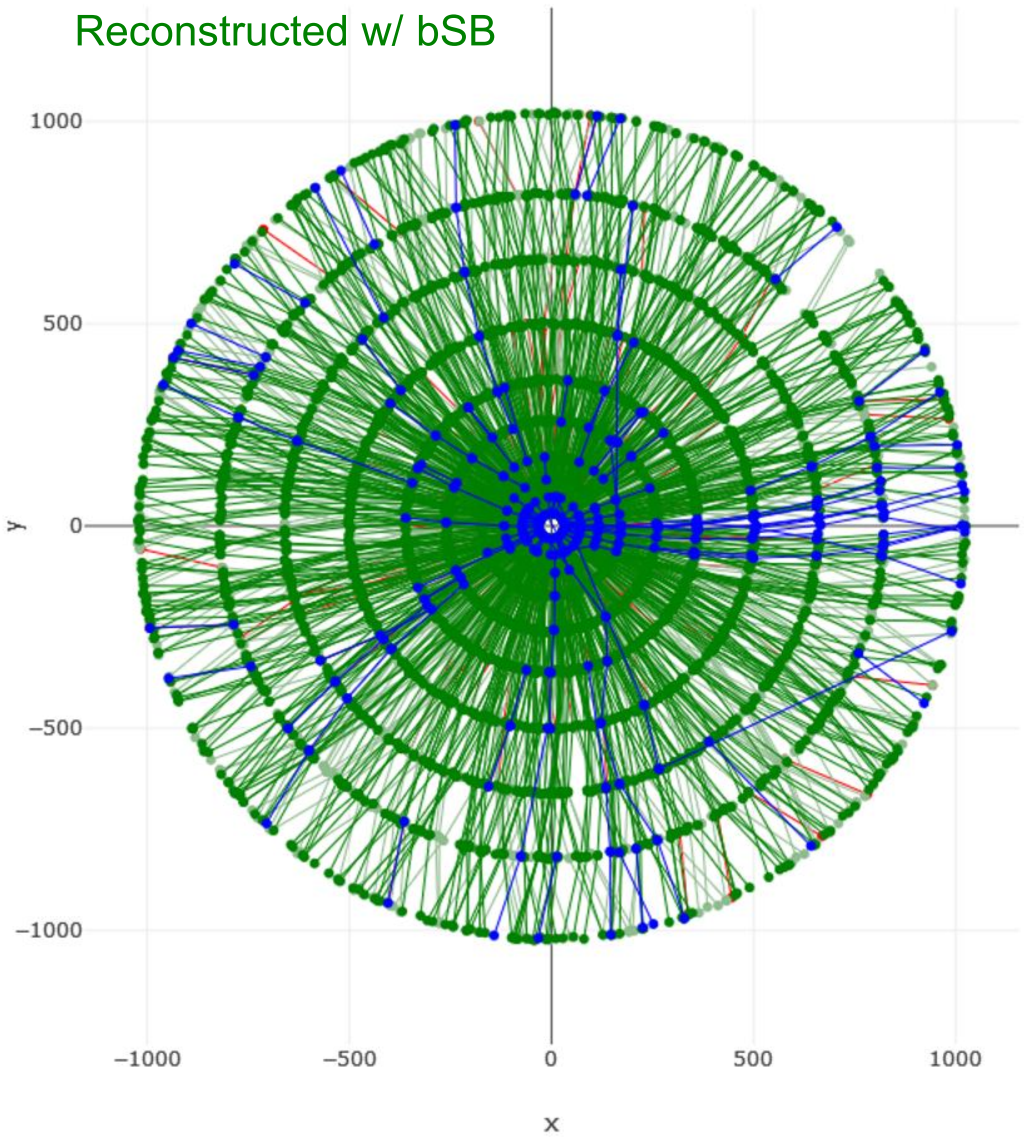
	bSB		TEbSB		dSB		TEdSB	
	Time (s)	Energy (a.u.)	Time (s)	Energy (a.u.)	Time (s)	Energy (a.u.)	Time (s)	Energy (a.u.)
ev1004 (N=109498)	8.67	-448998	7.25	**-449363**	9.02	-447488	7.43	-449349
ev1014 (N=78812)	5.06	-263353	4.27	**-263650**	5.24	-261860	4.33	-263641
ev1023 (N=80113)	5.33	-261244	4.42	-261345	5.48	-260928	4.80	**-261362**

References:

- H. Okawa, Spinger CCIS 2036 (2024) 272, arXiv:2310.10255
- H. Okawa, et al., Springer Comput. Softw. Big Sci. 8, 16 (2024)
- H. Okawa, et al., Phys. Lett. B 864 (2025) 139393
- XZ Tao et al., in preparation

Summary

- Reconstruction can be formulated as QUBO/Ising problems.
- **Quantum-inspired algorithms (bSB in particular) significantly outperforms QAOA & quantum annealing for QUBO tracking & jet clustering.**
- Tracking:
  - **This the world's first application of simulated bifurcation to high energy physics.**
  - **bSB can directly handle the HL-LHC datasets and provides four orders of magnitude speed-up with 1 GPU from D-Wave Neal** & can be considered for implementation **NOW**.
- Jet reconstruction:
  - **This is the world's first successful demonstration of multijet reconstruction w/ QUBO.**
  - At present, only bSB can predict reasonable Ising energy for multijet reconstruction QUBOs.
  - **bSB provides improved jet energy resolution for multijet events.**
- **We have also succeeded in improving the SB quantum-inspired algorithms further.** Applications to specific high-energy physics problems are under way.



Thank you for listening!

感谢聆听！