Cosmic Track Reconstruction
of BESIII MDC

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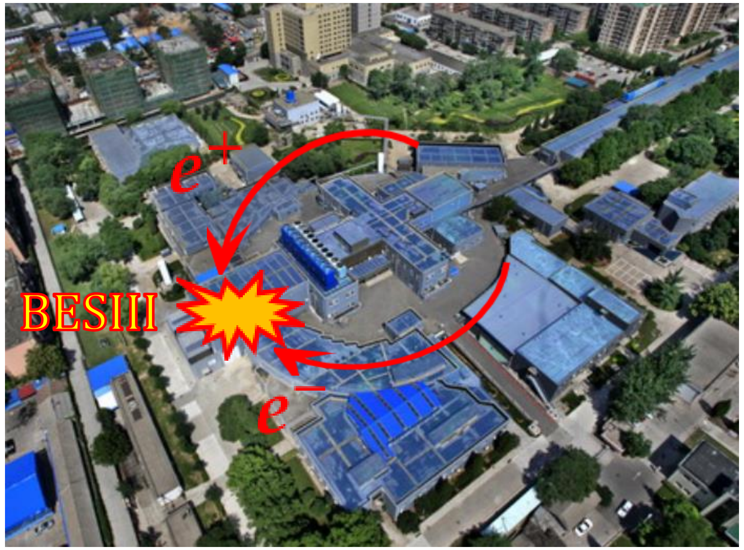
Outline

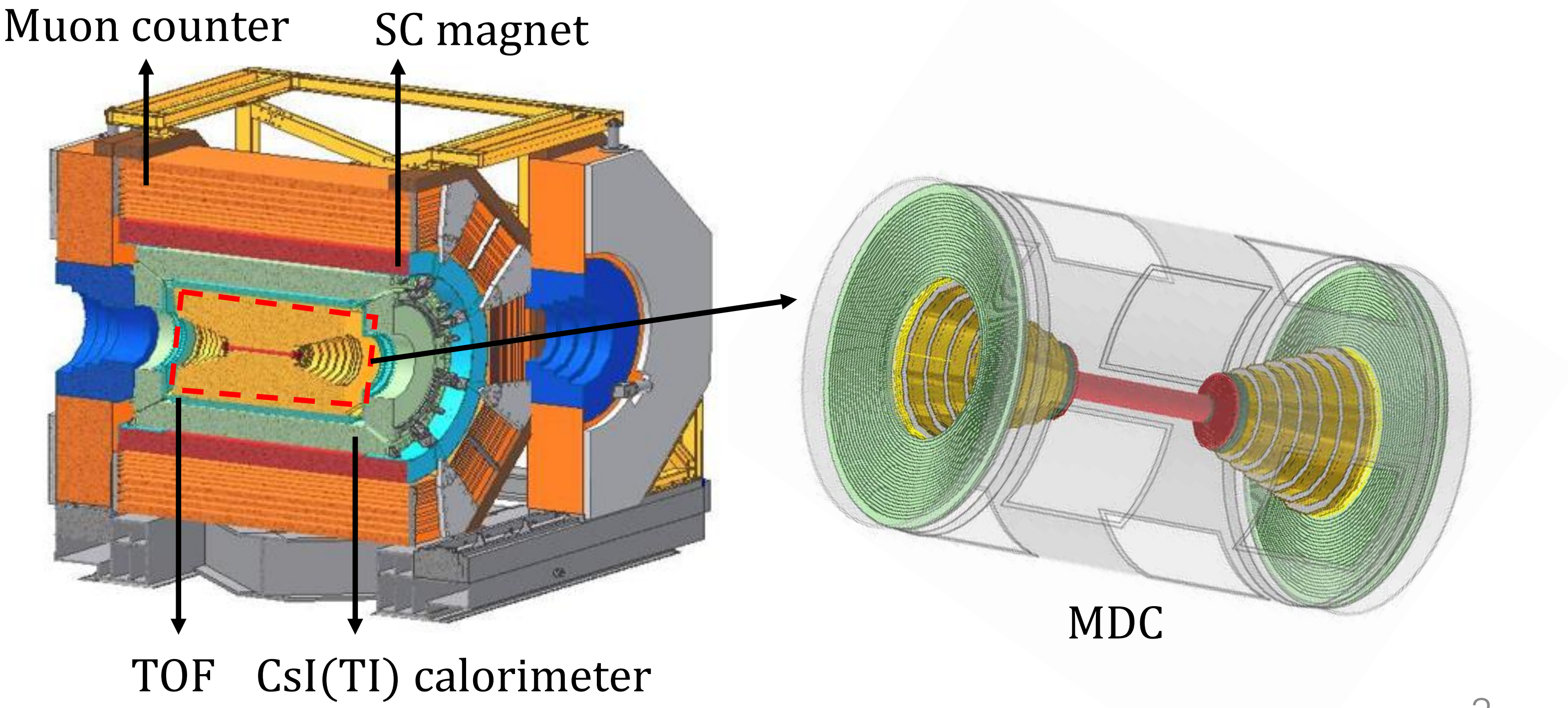
- Introduction
- Principle of Legendre Transform & Hough Transform
- 2D Track finder Using Legendre Transform in magnetic field
- Track Reconstruction in magnetic field
- 2D Track finder Using Hough Transform without magnetic field
- Track Reconstruction without magnetic field
- Algorithm Applications & Performances
- Summary

MDC: the Central Tracker on BESIII

BESIII: Beijing Spectrometer III experiment

- located at the BEPCII (Beijing Electron–Positron Collider II) in Beijing, China.
- focus on precision measurements in the τ-charm energy region
  (hadron spectroscopy, charmonium states, light hadrons, new physics etc.)
- Luminosity : 1×10^{33} $\mathrm{cm^{-2}s^{-1}}$ @ 3.773 GeV





MDC: the Central Tracker on BESIII

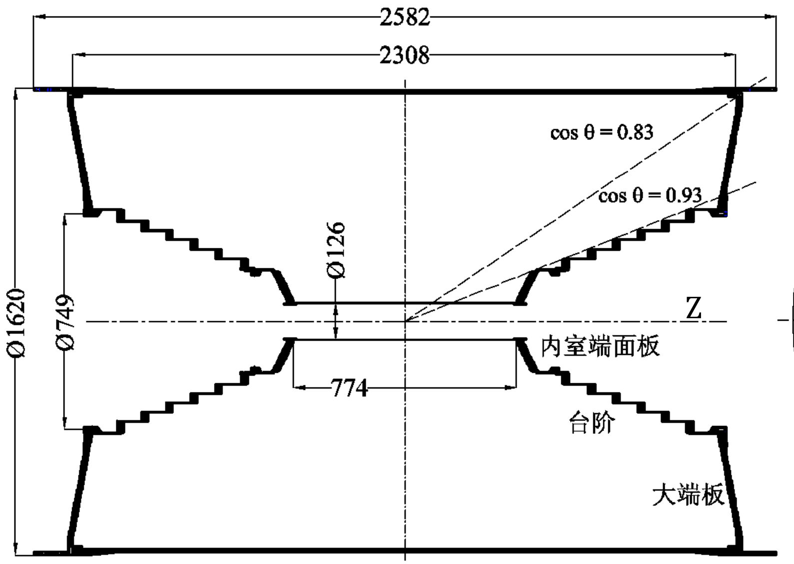
MDC: Main Drift Chamber

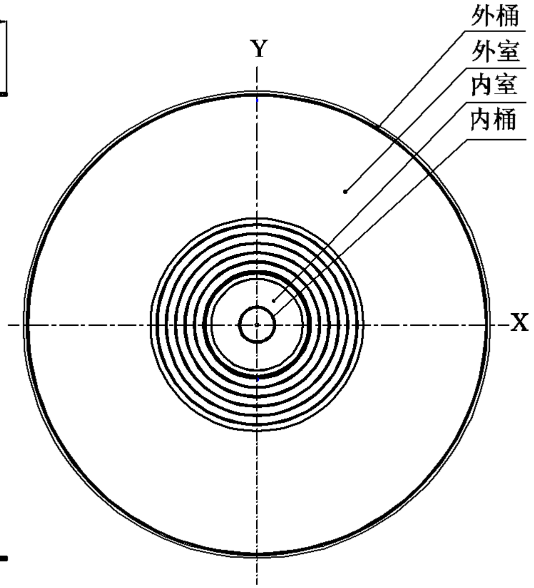
- The central tracking detector of the BESIII experiment
- Spatial resolution ~130 μm

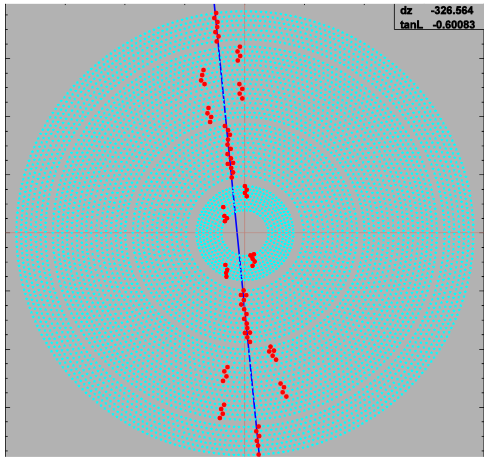
  Momentum resolution ~0.5% @ 1 GeV/c

  dE/dx information for particle identification
- **Inner chamber** : layer 1~8 (stereo wires)

  Outer chamber : layer 9~20 (axial wires), layer 21~36 (stereo wires), layer 37~43 (axial wires)







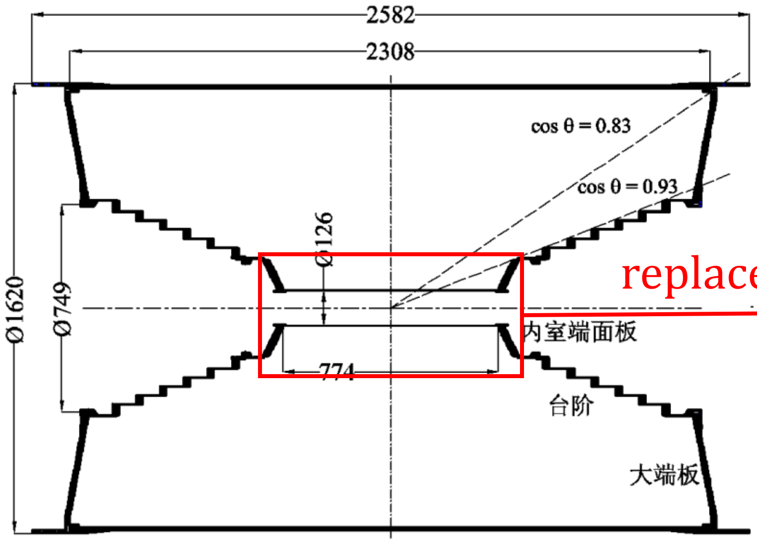
Inner Tracker Replacement in BESIII Upgrade

Aging of the inner chamber of MDC (iMDC)

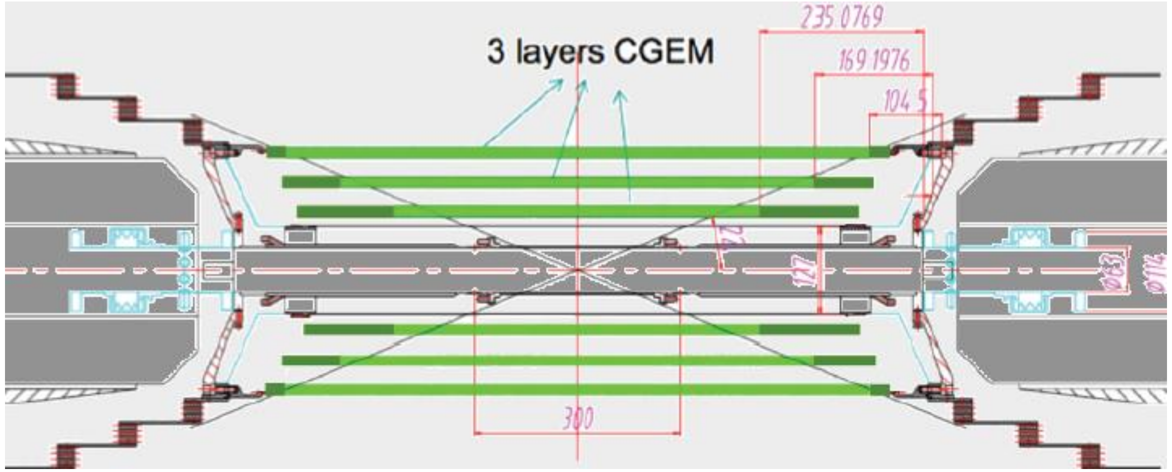
- due to the high luminosities ($1 \times 10^{33}\ cm^{-2}s^{-1}$ @ 3.773 GeV) & long-term operation(2009~2024)
- will affect the detector performances significantly

Replace the iMDC with 3 layers CGEM (Cylindrical Gas Electron Multiplier)

- 2024-09-14, iMDC pulled out
- 2024-10-05, CGEM installed
- 2025-02~03, MDC cosmic-ray test with & without magnetic field
- 2025-03~06, MDC+CGEM combining cosmic-ray test with & without magnetic field



replaced by

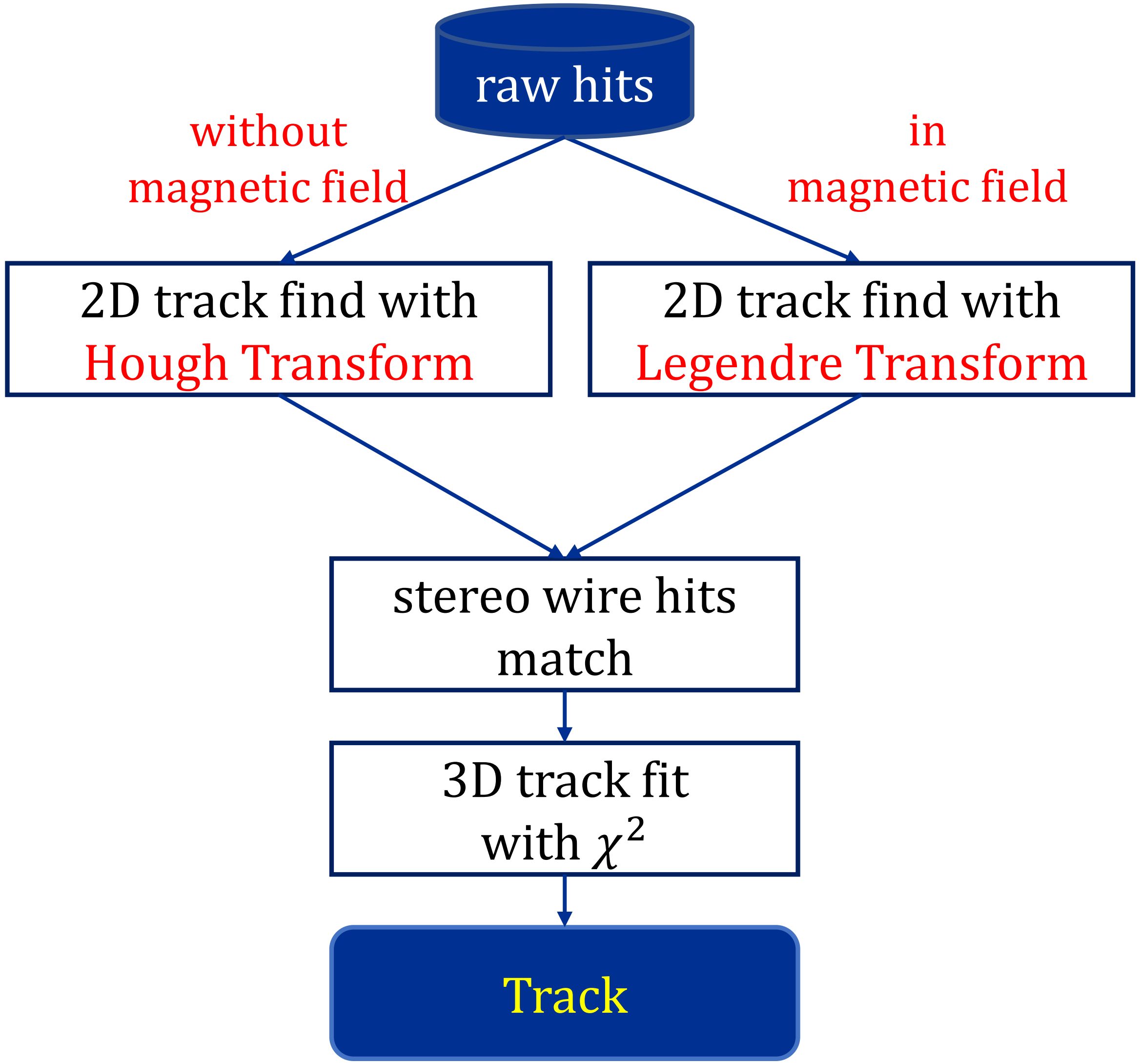


Reconstruction Algorithm for Cosmic-Ray Test

To evaluate the MDC performance and optimize the operation parameters, the cosmic reconstruction algorithm is developed.

The algorithm

using Hough/Legendre Transform to find track and using Least Squares Method to fit track .



Legendre Transform

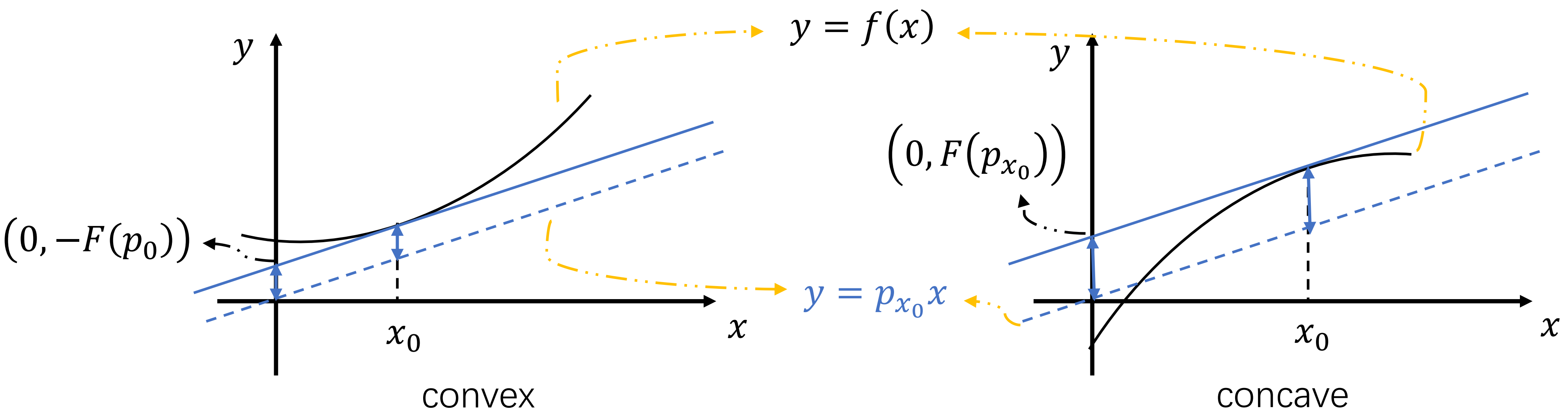
Convex function : $f(x): R \to R,\ \frac{d^2 f}{dx^2} > 0 \quad \to \quad F(p) = \sup_x [px - f(x)]$

Concave function: $f(x): R \to R,\ \frac{d^2 f}{dx^2} < 0 \quad \to \quad F(p) = \sup_x [f(x) - px]$

$(x_0, f(x_0)) \quad \to \quad (p_0, F(p_0))$ in Legendre space

p_0 : the slope of the tangent line at $\left(x_0, f(x_0)\right)$

$\pm F(p_0)$: the intercept of the tangent line at $\left(x_0, f(x_0)\right)$



Cite from: Alexopoulos, T. *Implementation of the Legendre transform for track segment reconstruction in drift tube chambers*. NIMA, (2008).

Legendre Transform

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$(x_0, f(x_0)) \quad \to \quad (p_0, F(p_0))$in Legendre space

p_0 : the slope of the tangent line at $(x_0, f(x_0))$

$\pm F(p_0)$: the intercept of the tangent line at $(x_0, f(x_0))$

a curve $f(x)$ $\to$ a curve $F(p)$ in Legendre space

$$\boxed{\begin{cases} p = \frac{df}{dx} \\ F(p) = px - f(x) \text{ for convex function} \\ F(p) = f(x) - px \text{ for concave function} \end{cases}}$$

a point $(x_0, f(x_0))$

$f(x_0) = kx_0 + b$

$$\begin{cases} p = \frac{df}{dx} = k \\ F(p) = f(x) - px = \mathrm{b} \end{cases}$$

Hough Transform

a point $(x_0, f(x_0))$ $\to$ a line $F(p) = f(x_0) - x_0 p$ in Legendre (Hough) space

2D Track Finder with Legendre Transform

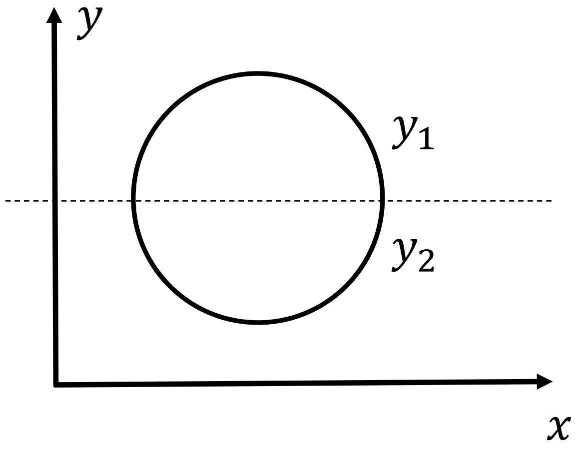
a circle: $(x_0, y_0), R$

$$\begin{cases} y_1 = y_0 + \sqrt{R^2-(x-x_0)^2} \\ y_2 = y_0 - \sqrt{R^2-(x-x_0)^2} \end{cases} \Rightarrow (\mathcal{L}) \begin{cases} r_1 = x_0 \cos\theta + y_0 sin\theta + R, \text{ concave part} \\ r_2 = x_0 \cos\theta + y_0 sin\theta - R, \text{ convex part} \end{cases} \rightarrow \begin{cases} r_1 = r_0 \cos(\theta - \phi) + R \\ r_2 = r_0 \cos(\theta - \phi) - R \end{cases}$$

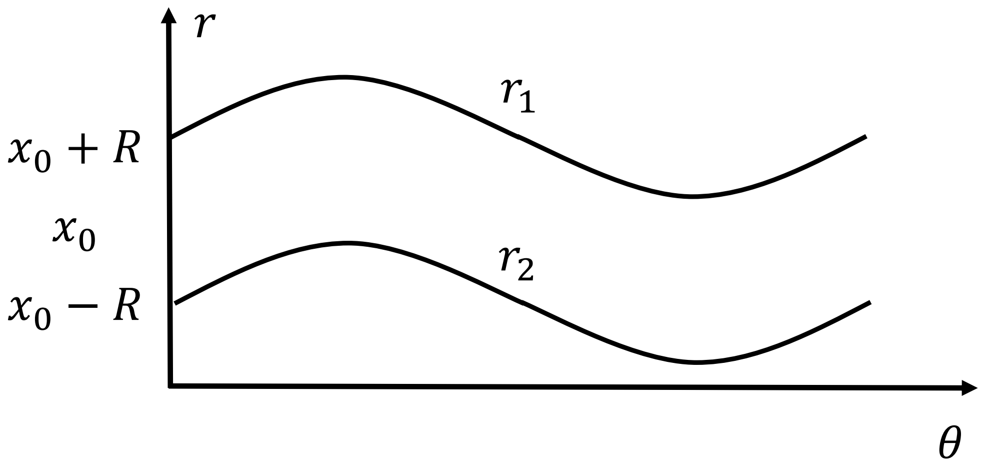
$r_0 = \sqrt{x_0^2 + y_0^2}$ $\qquad$ $\phi = \arctan(\frac{y_0}{x_0})$

$$\Downarrow$$

$$\boxed{\begin{cases} r_1 + r_2 = 2r_0 \cos(\theta - \phi) \\ r_1 - r_2 = 2R \end{cases}}$$



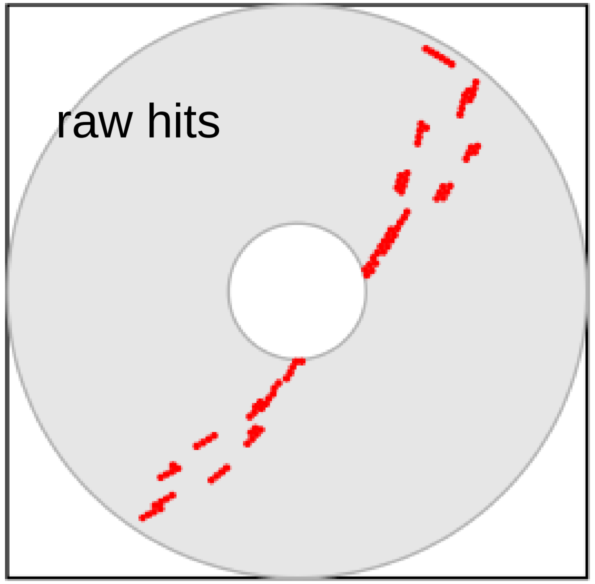
$\Rightarrow$

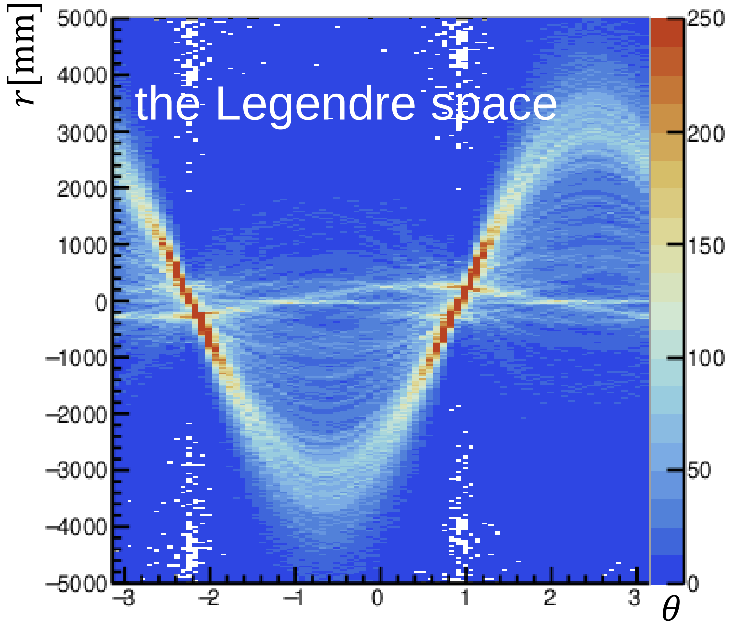


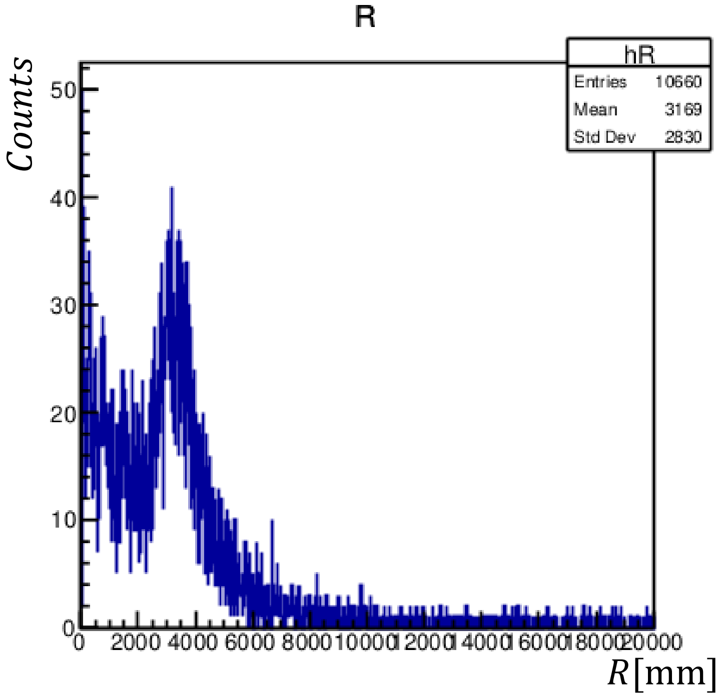
one circle $\rightarrow$ one sine curve $+$ one value

whose parameter corresponds to (x_0, y_0) $\qquad$ R

2D Track Finder with Legendre Transform







a series of circles → a series of sine curves + a series of values of R

each triplet of the axial wire hits defines a circle

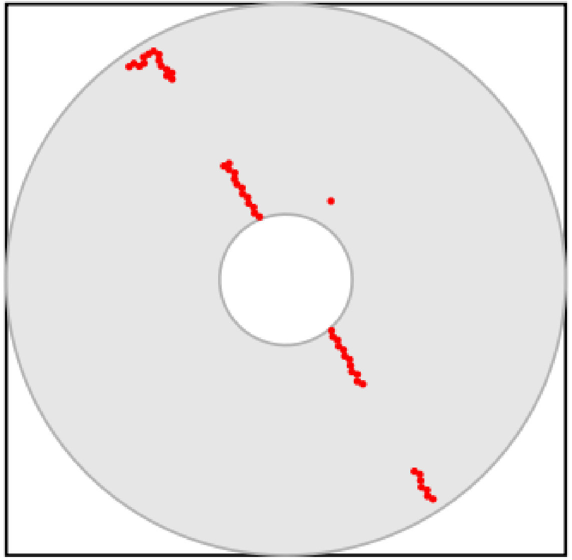
the brightest band corresponds to (x_0, y_0) of 2D trajectory

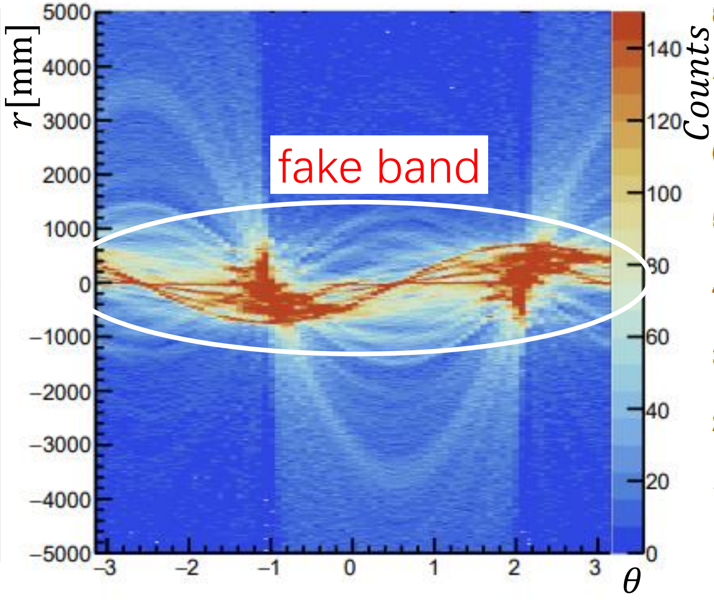
the peak corresponds to R of 2D trajectory

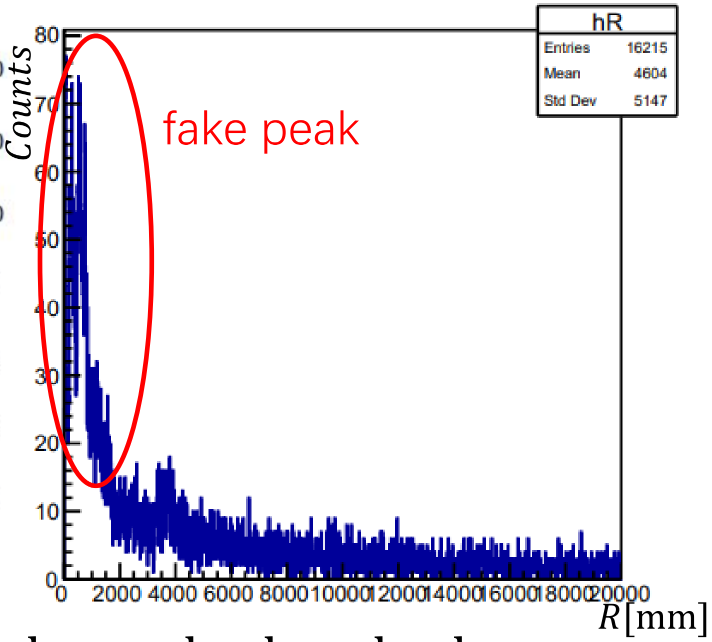
to determine (x_0, y_0)

to determine R

Optimizations of the 2D Track Finder







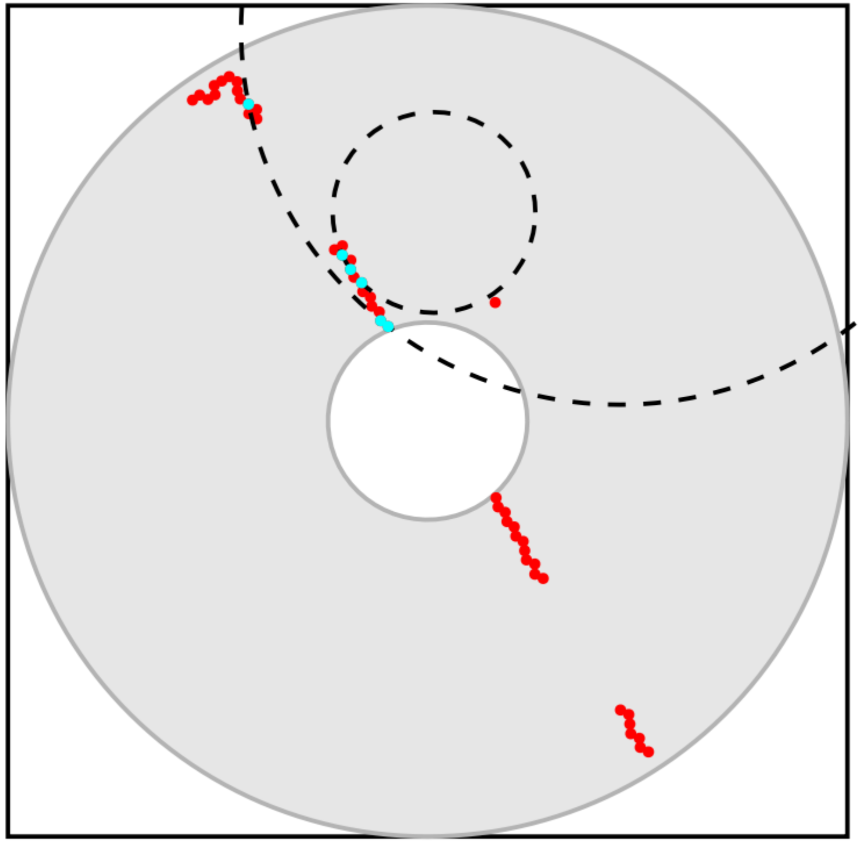
- The Cosmic-ray events are selected with barrel EMC shower back-to-back
- A rough analysis shows that the momentum of cosmic-ray tracks is more than 1 GeV/c

  $\rightarrow$ R > 3 m.
- The radius of MDC is around 0.8 m

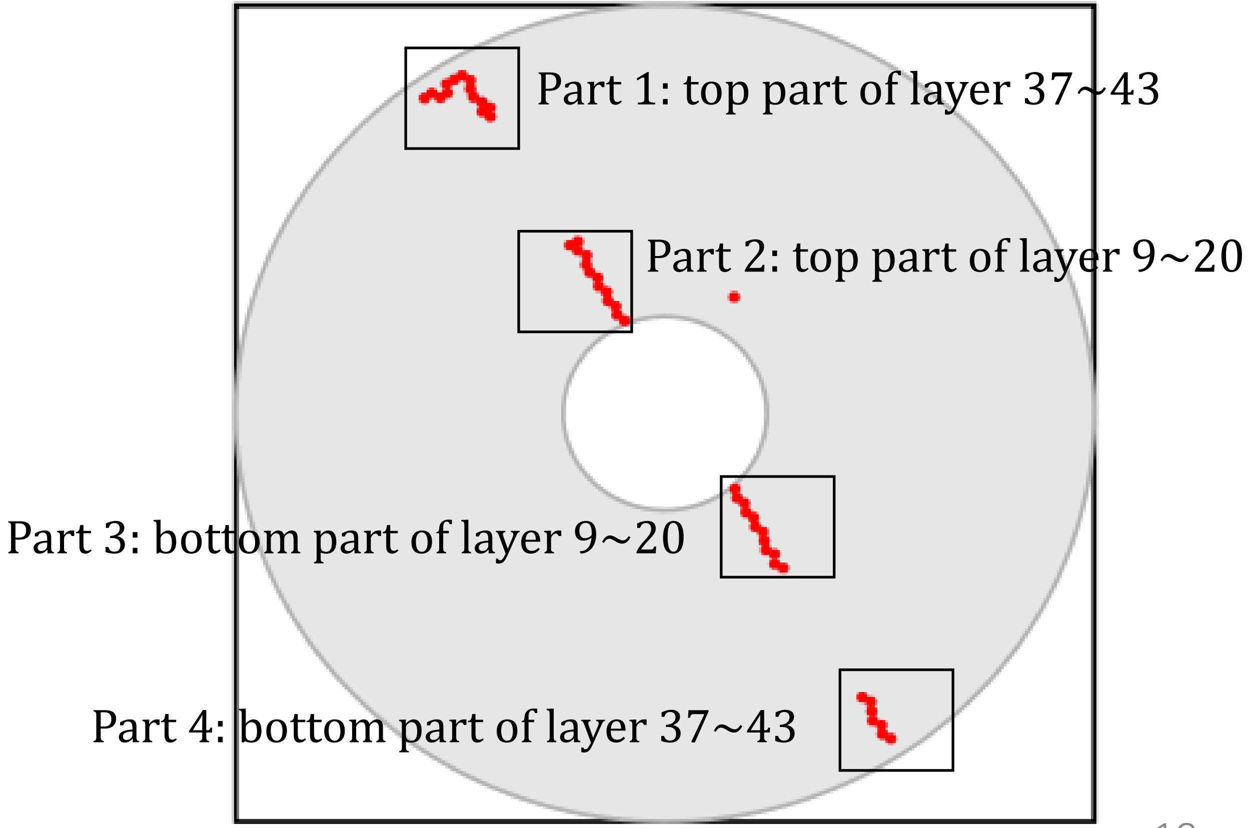
  $\rightarrow r_0 >> 0.8$ m.
- Fake band and peak in Legendre space comes from:
  - triplets with closely spaced hits
  - triplets with noise hits

Optimizations of the 2D Track Finder

Cause 1: When two points in a triplet are close to each other, the circle tends to have a smaller radius, since the wire position, rather than the drift circle, is taken as the hit position)

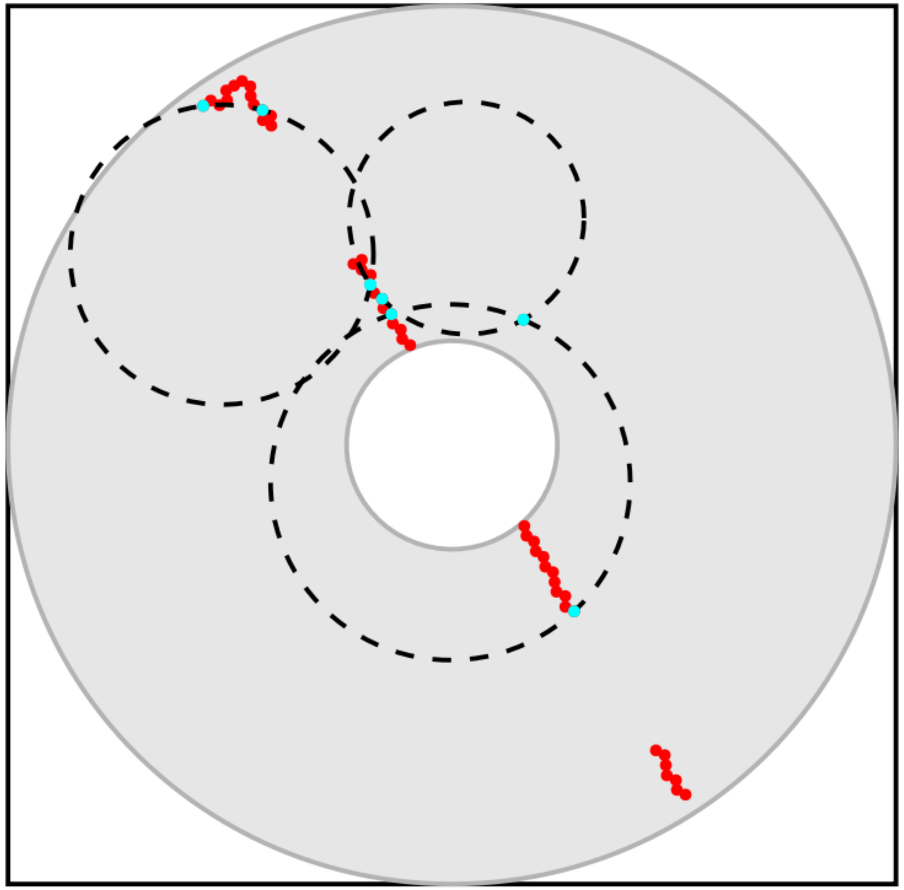


Optimization:
chose points from different parts as a triplet to use



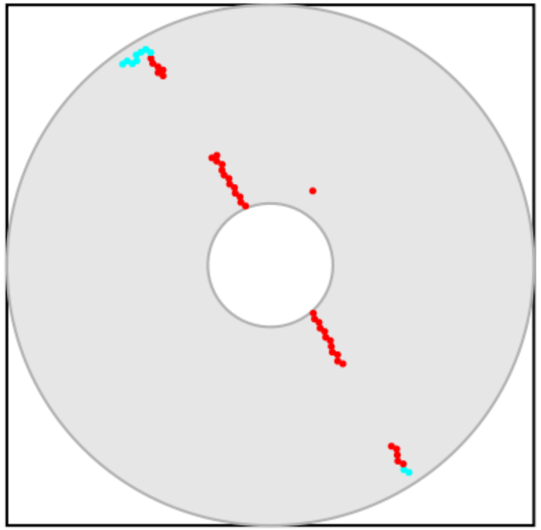
Optimizations of the 2D Track Finder

Cause 2: interfering hits from noise or secondary particles

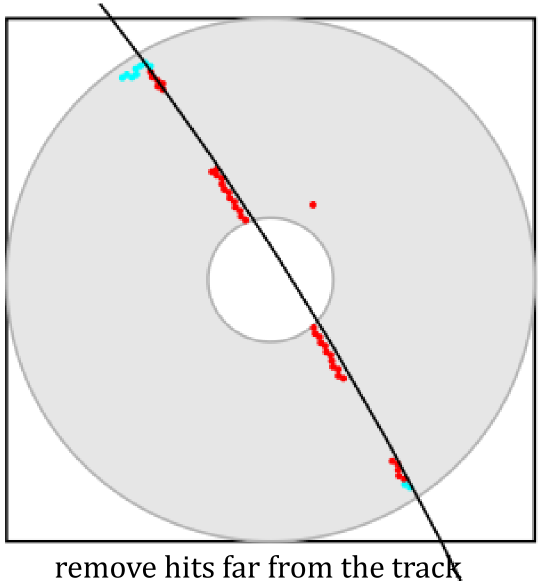


Optimization:

1. exclude hits from layers with nhits>3 (secondary particles with low momentum)
2. exclude circles defined by triplets with unreasonable (x_0, y_0) or R value
3. remove hits far from the track and fit the circle with the remaining ones

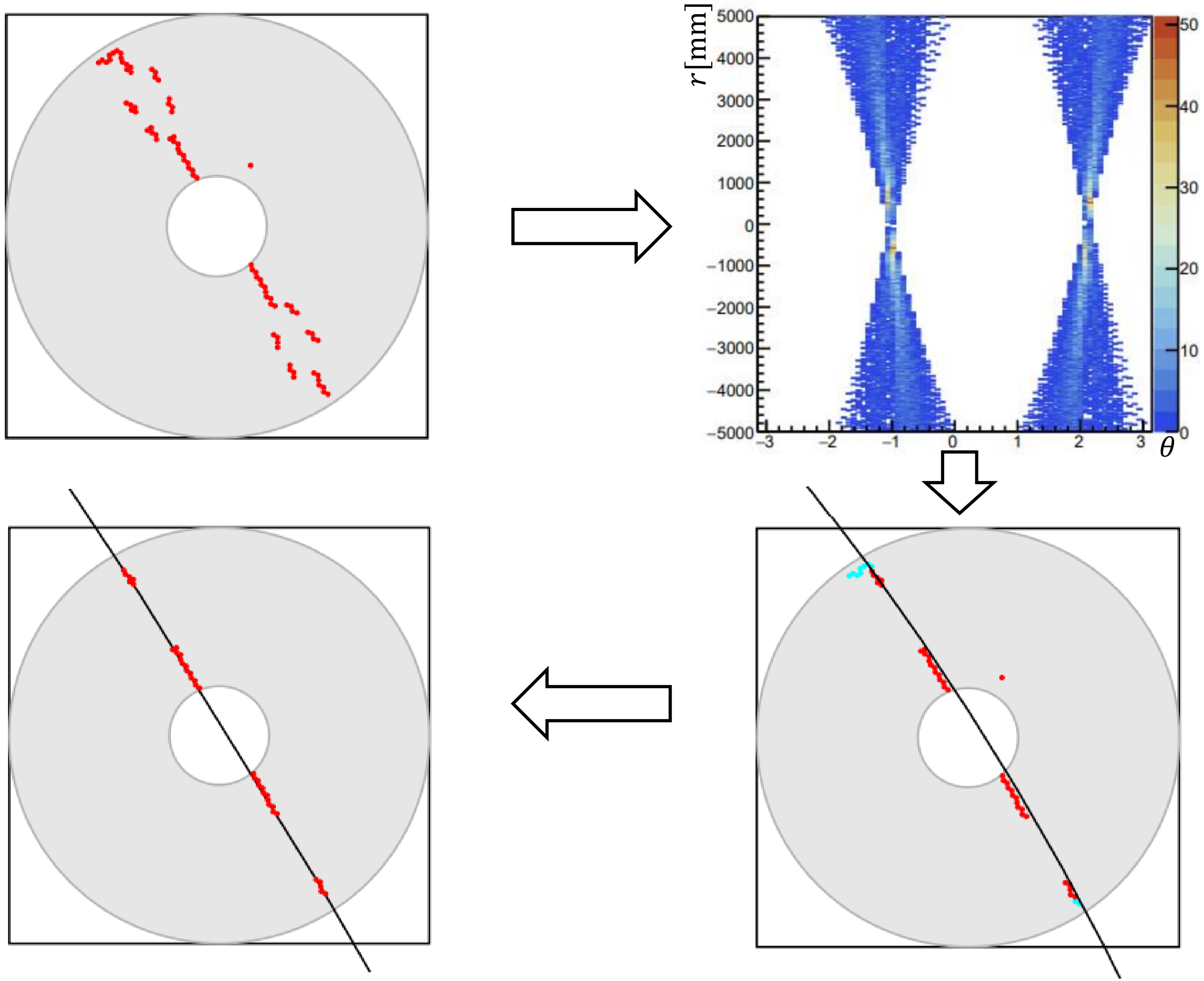


exclude hits from layers with nhits>3



remove hits far from the track

2D Track Found with Legendre Transform



Track Reconstruction in Magnetic Filed

raw hits

↓

2D track find with Legendre Transform

↓

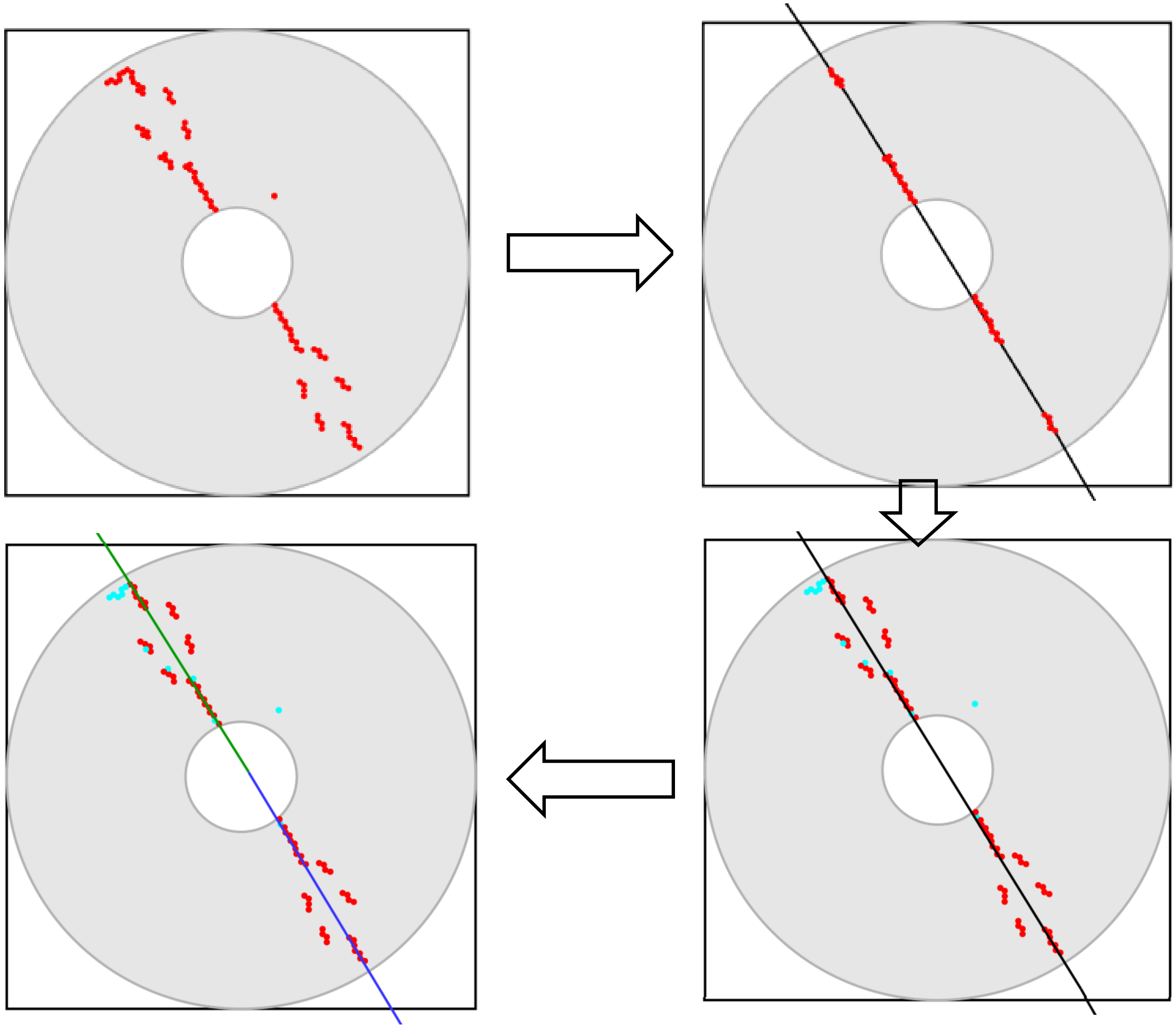
stereo wire hits match

↓

3D track fit with χ^2

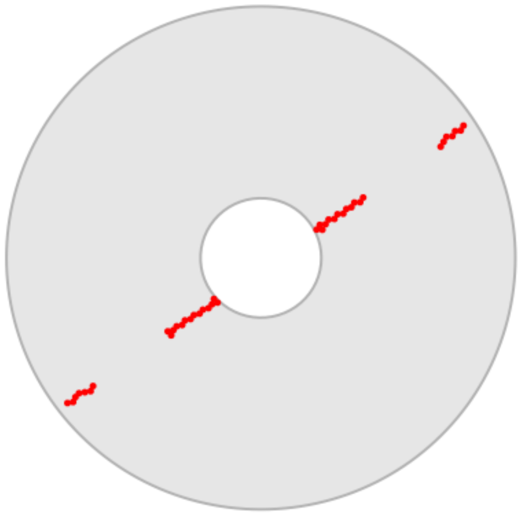
↓

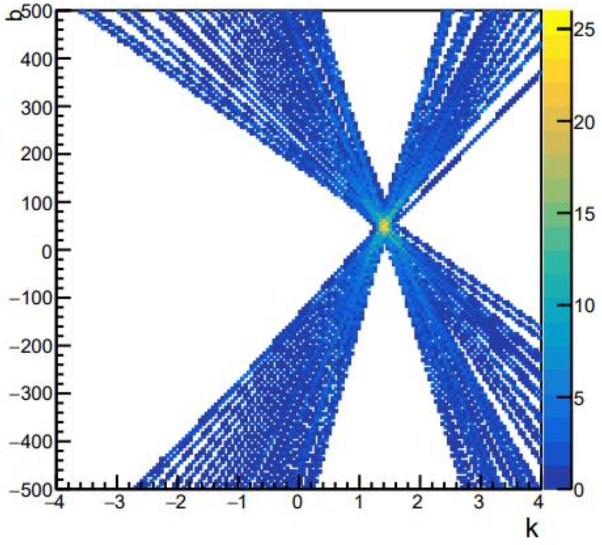
2 tracks

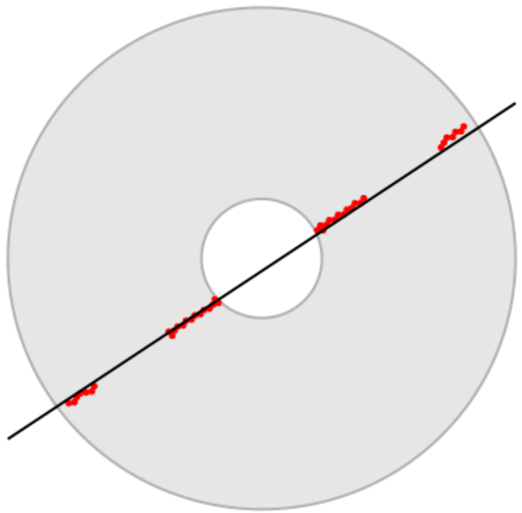


2D Track Finder with Hough Transform

one hit → one line on Hough plane







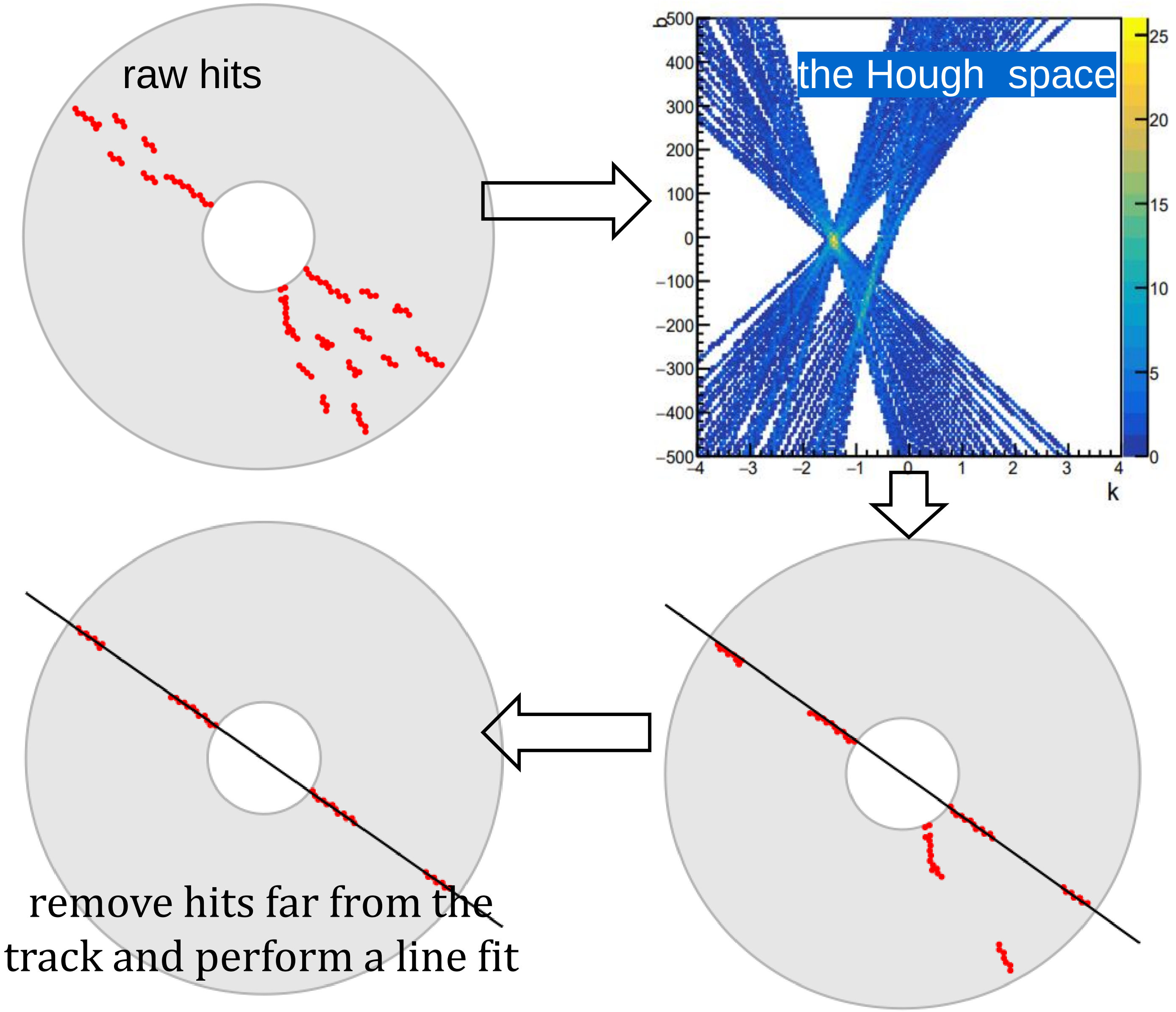
a series of hits → a series of lines on Hough plane

the intersection point of lines corresponds
to the line parameter of 2D track

2D Track Finder with Hough Transform

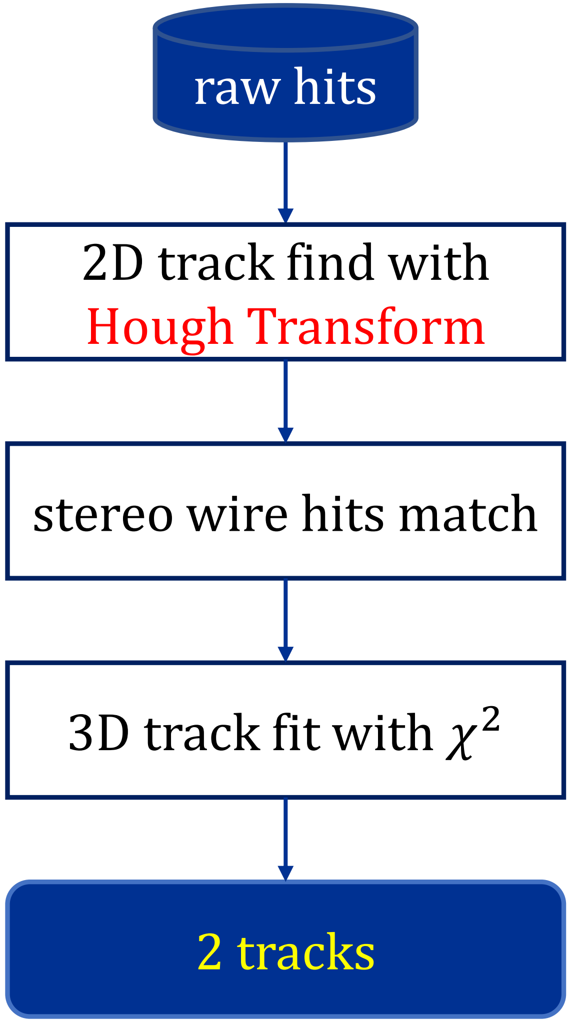
A hit-to-track distance cut

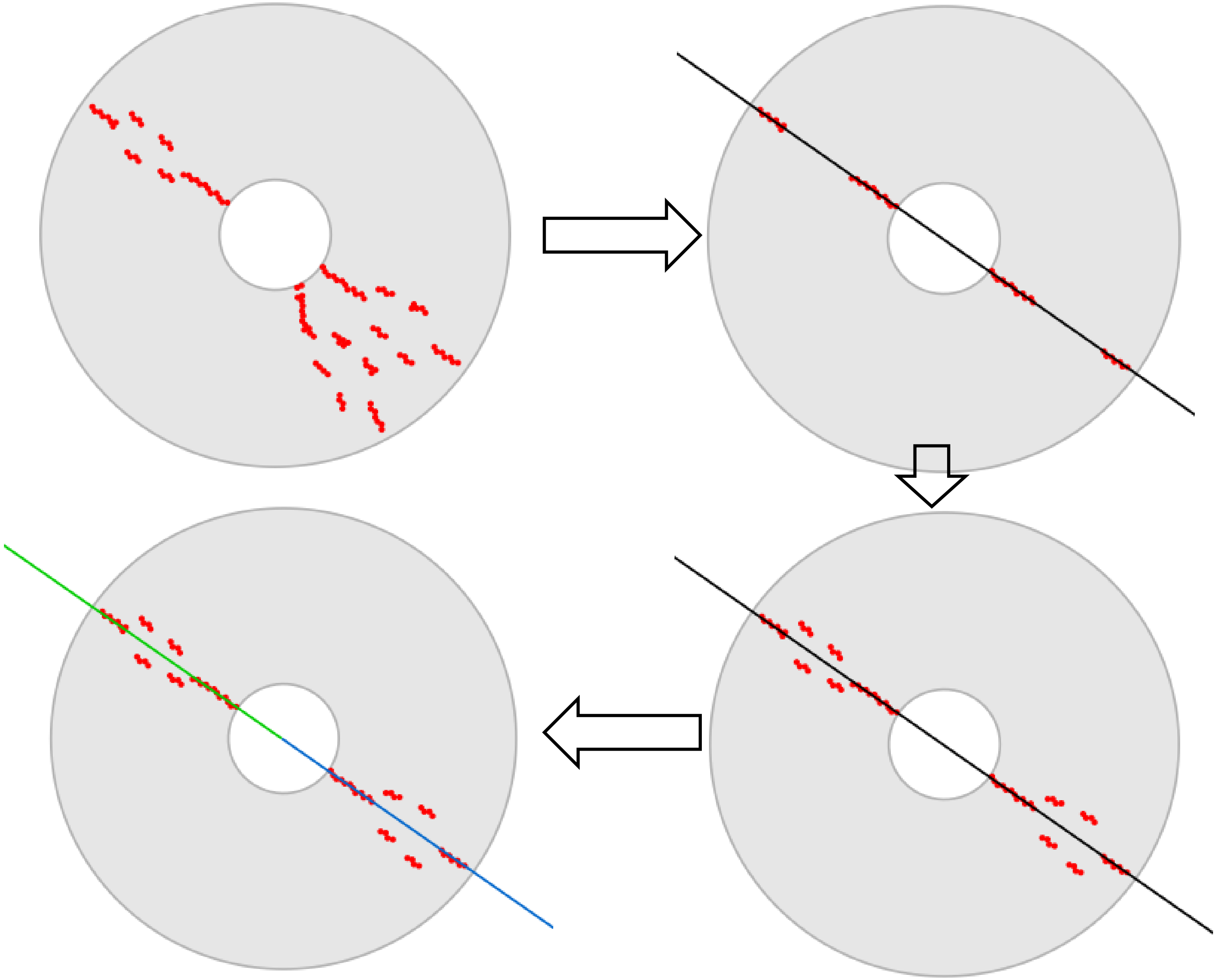
to exclude interfering hits



remove hits far from the track and perform a line fit

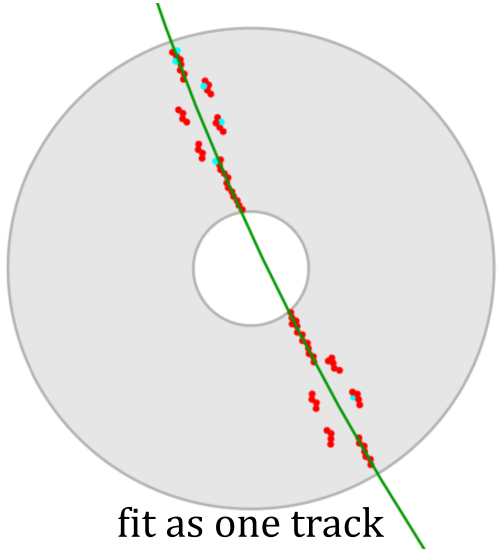
Track Reconstruction without Magnetic Filed



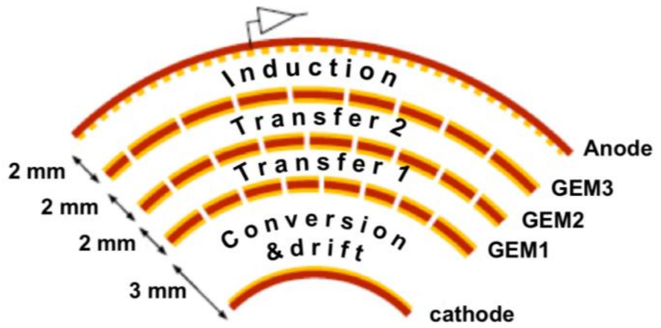


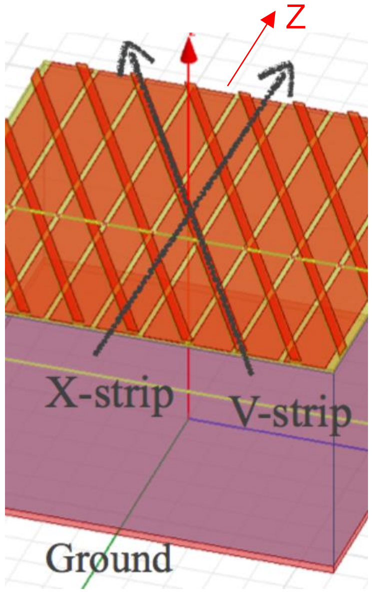
Algorithm extension to integrate CGEM-IT

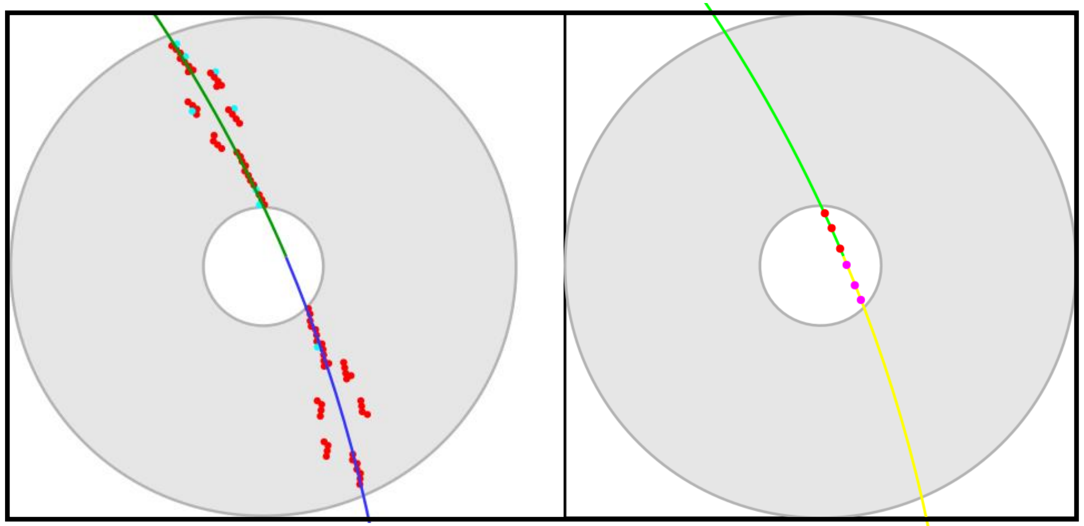
- For alignment use, Extend the functionality to support fitting as one track
- For CGEM data analysis, Extend the functionality to associate CGEM clusters and perform joint track fitting.
  - CGEM: 2D Readout using X and V strips
  - CGEM: clusters (formed by combining adjacent fired strips on both X and V views) serve as "hits" in track reconstruction.



fit as one track







MDC track reconstruction

CGEM association & combining fit

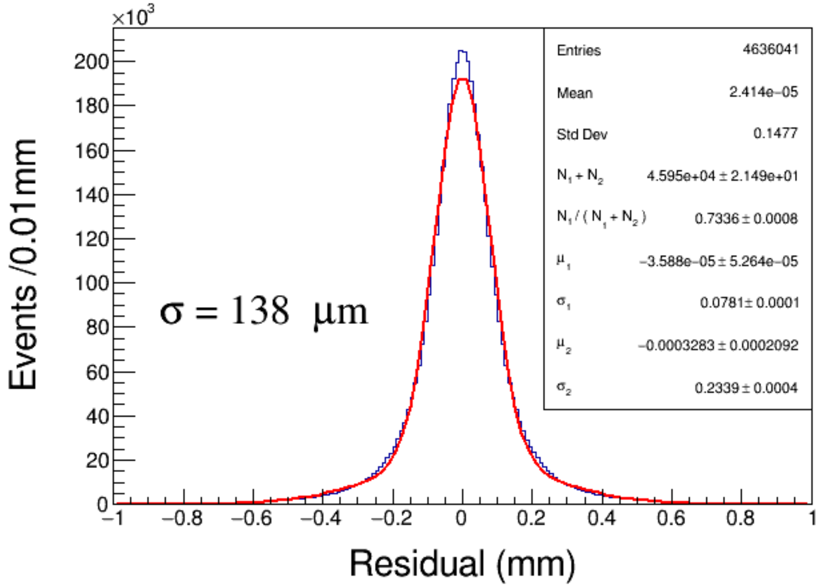
Algorithm Application & Performance

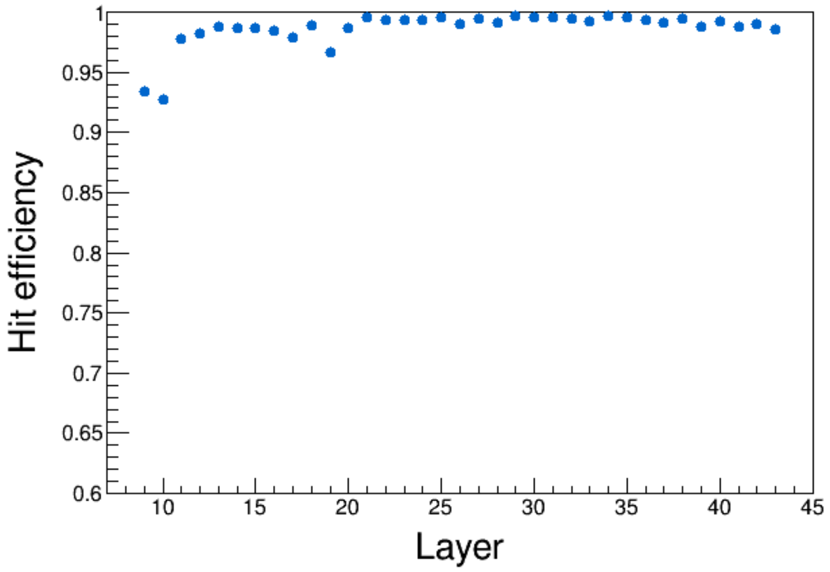
- Efficiency>90%(based on real data)[not the key performance concern]

MDC reconstruction	in mag	without mag
All cosmic ray events selected	9782	8648
events with successful track reconstruction	8944	8087
Ratio(success/all)	91.4%	93.5%

MDC + CGEM joint reconstruction	in mag	without mag
cosmic ray events with CGEM clusters	8356	7836
events with successful track reconstruction	8159	7616
Ratio(success/all)	97.6%	97.2%

- Works well in MDC performance evaluation





- Works well in CGEM alignment [details in Jiajv Wang's report]

Summary

- To analyze cosmic-ray data in the BESIII upgrade, a new track reconstruction algorithm has been developed, for scenarios :

  without the MDC inner chamber

  with newly installed CGEM-IT
- The algorithm
  - performs track finding using:

    the Hough Transform (for cases without a magnetic field)

    the Legendre Transform (for cases with a magnetic field)
  - fit tracks with the Least Squares Method
- The algorithm works well in both MDC and CGEM data analysis.

THANKS!

BACK UP

Legendre Transform

a circle: (x_0, y_0), R

(canonical form)

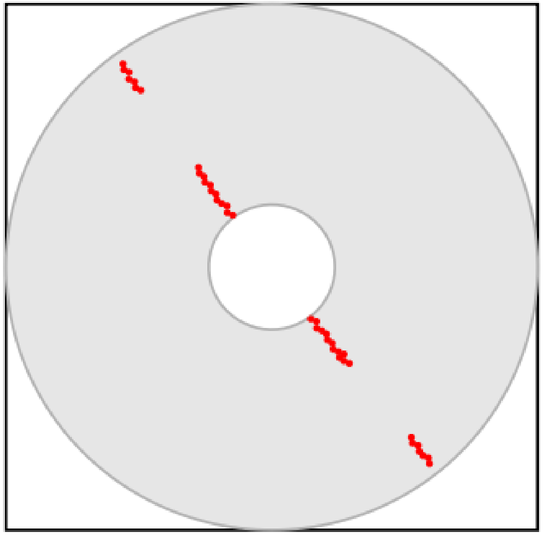
$$p = -cot\theta$$
$$F(p) = r/sin\theta$$

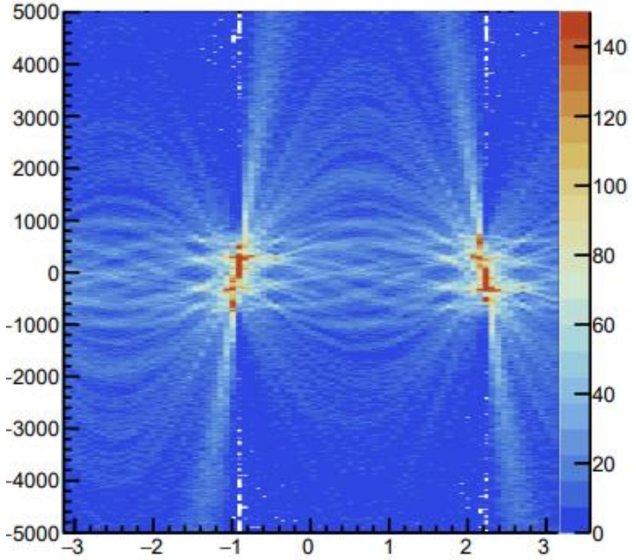
$$y = \mathrm{y}_0 + \sqrt{\mathrm{R}^2 - (x - \mathrm{x}_0)^2} \quad \rightarrow (\mathcal{L}) \begin{cases} p = -\dfrac{\mathrm{x} - \mathrm{x}_0}{\sqrt{\mathrm{R}^2 - (\mathrm{x} - \mathrm{x}_0)^2}} \\ F(p) = \mathrm{y}_0 - \mathrm{x}_0 p + \mathrm{R}\sqrt{p^2 + 1} \end{cases} \rightarrow \quad \frac{r}{\sin\theta} = \mathrm{y}_0 + \mathrm{x}_0 \frac{\cos\theta}{\sin\theta} + \frac{\mathrm{R}}{\sin\theta}$$

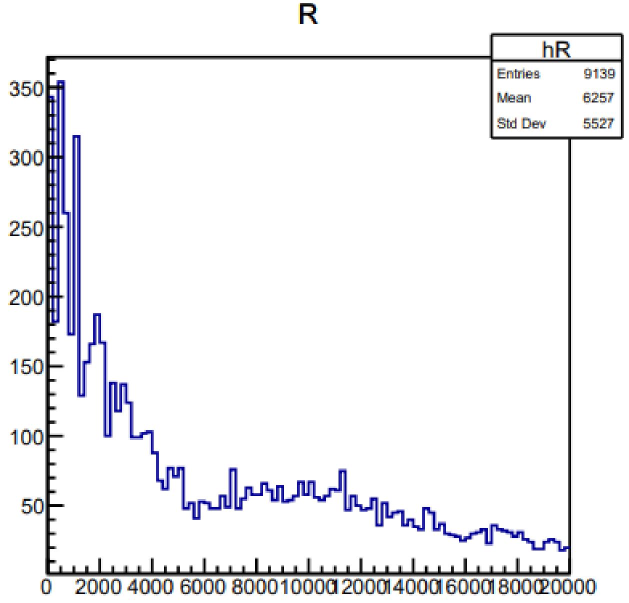
$$p = -cot\theta$$
$$-F(p) = r/sin\theta$$

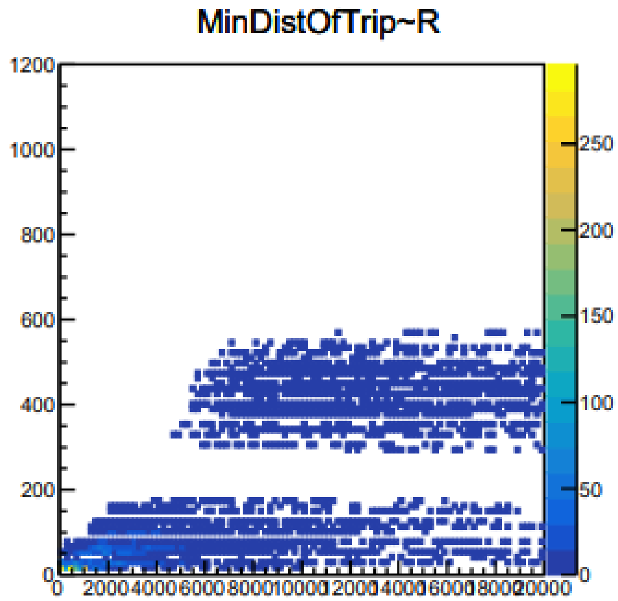
$$y = \mathrm{y}_0 - \sqrt{\mathrm{R}^2 - (x - \mathrm{x}_0)^2} \quad \rightarrow (\mathcal{L}) \begin{cases} p = \dfrac{\mathrm{x} - \mathrm{x}_0}{\sqrt{\mathrm{R}^2 - (\mathrm{x} - \mathrm{x}_0)^2}} \\ F(p) = -\mathrm{y}_0 + \mathrm{x}_0 p + \mathrm{R}\sqrt{p^2 + 1} \end{cases} \rightarrow \quad -\frac{r}{\sin\theta} = -\mathrm{y}_0 - \mathrm{x}_0 \frac{\cos\theta}{\sin\theta} + \frac{\mathrm{R}}{\sin\theta}$$

2D Track Finder with Legendre Transform

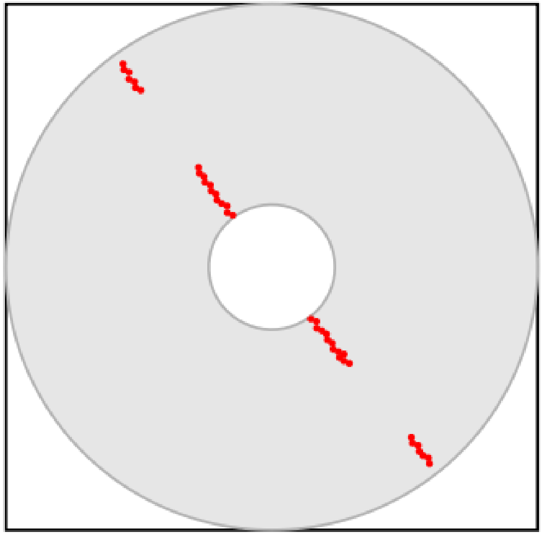


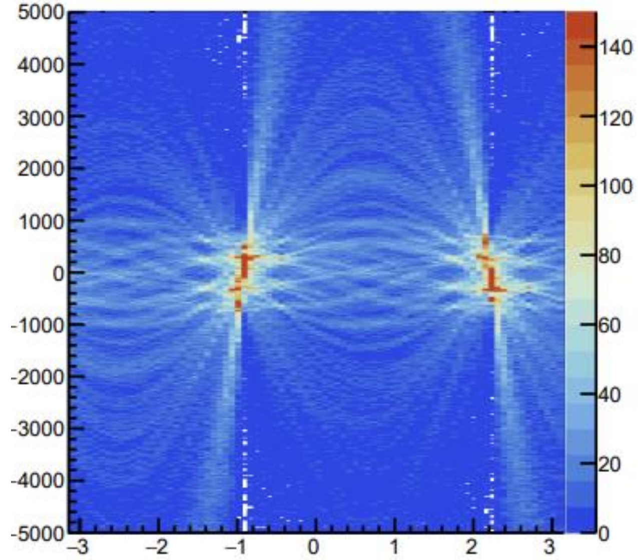


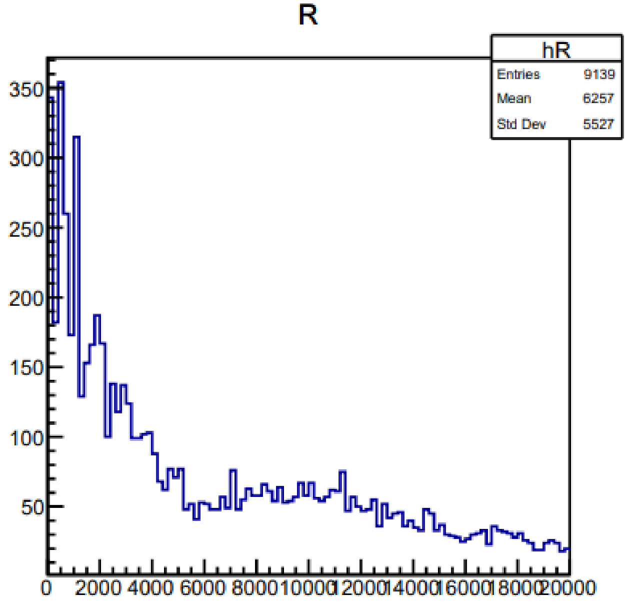




2D Track Finder with Legendre Transform

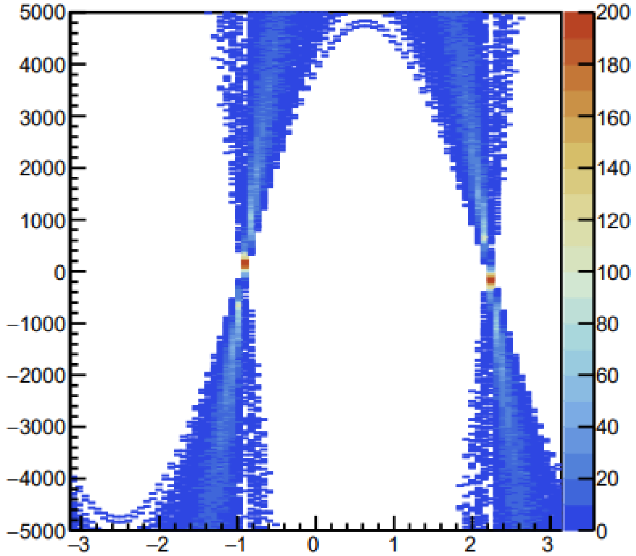


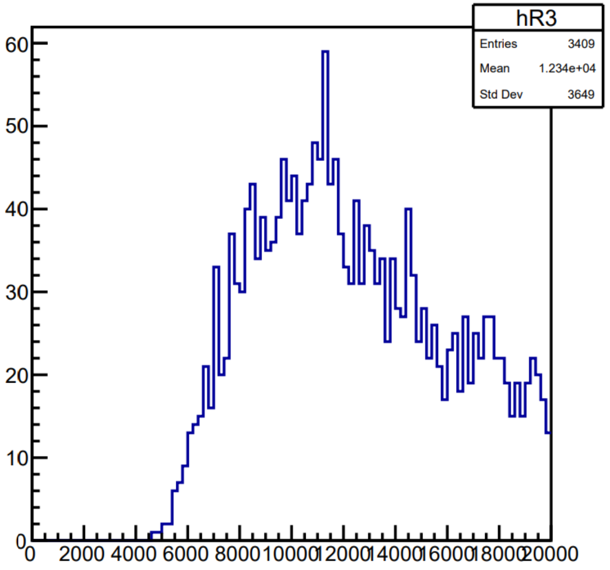




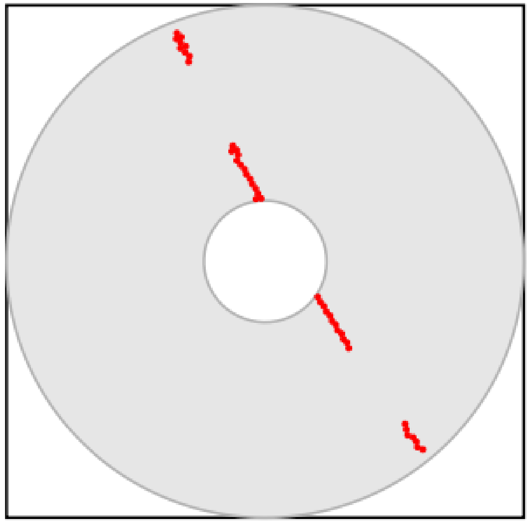
exclude triplets

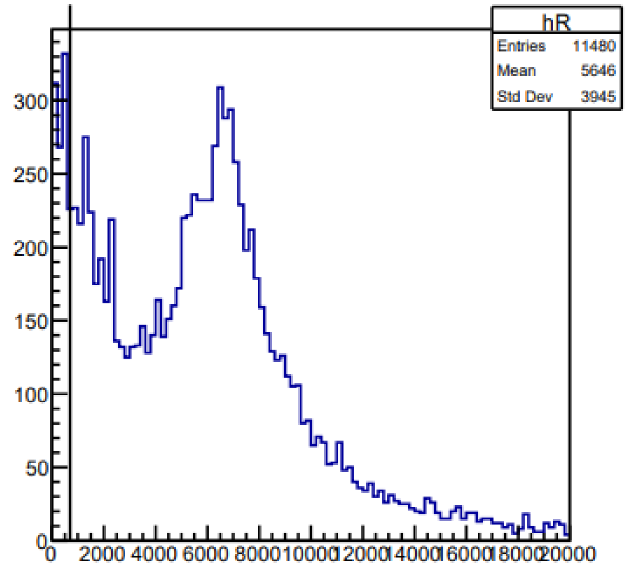
with minD<230

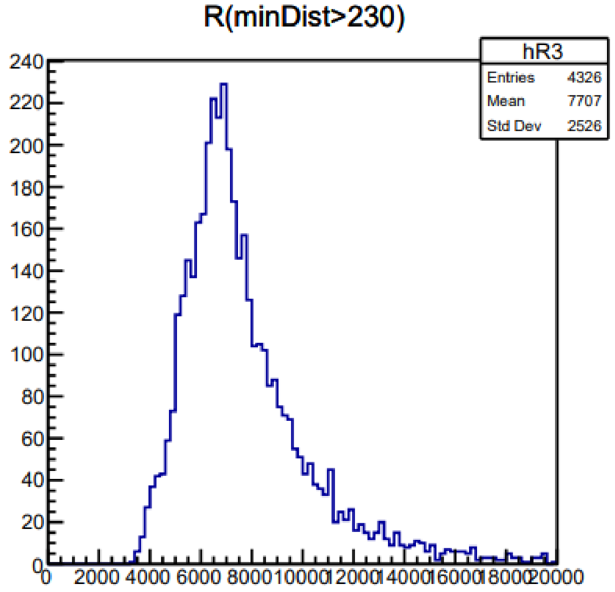


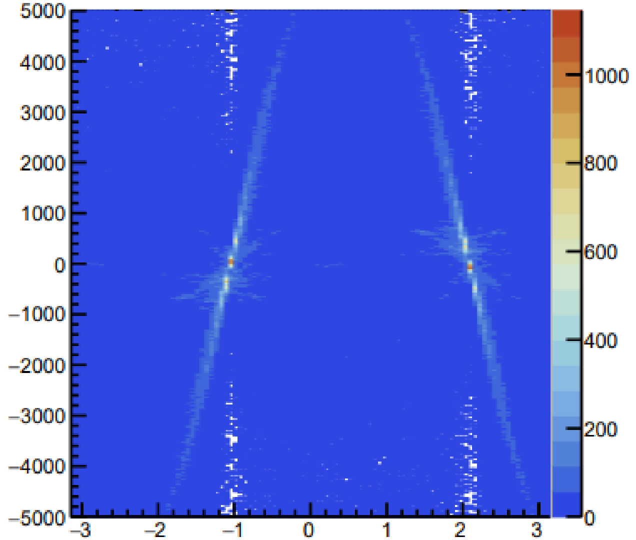


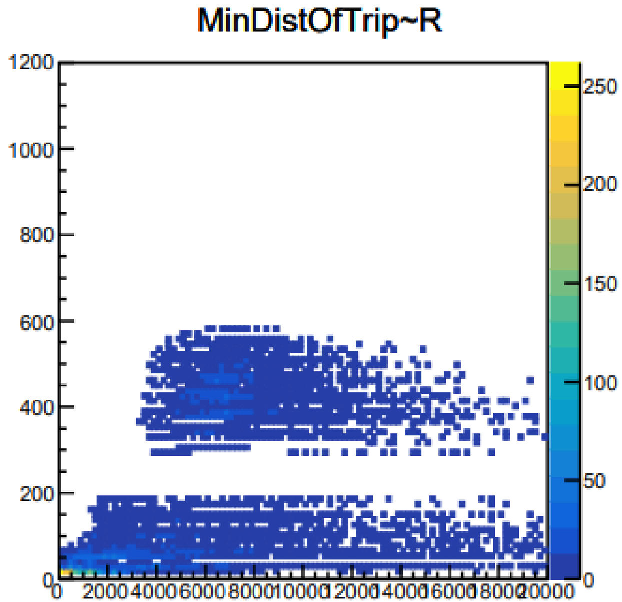
2D Track Finding with Legendre Transform

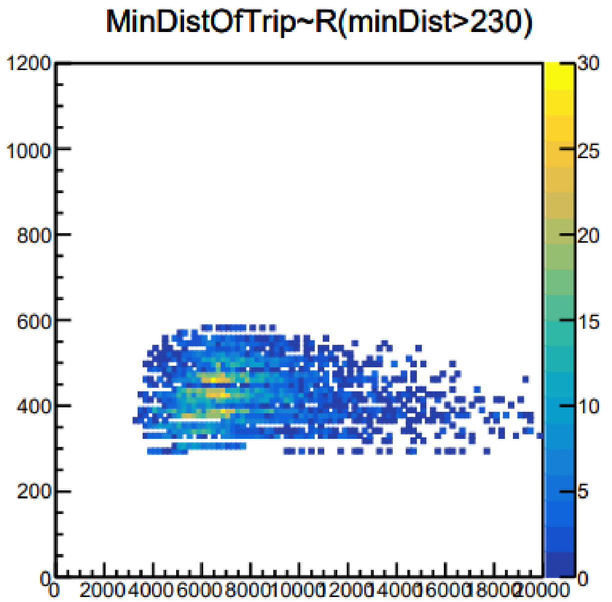




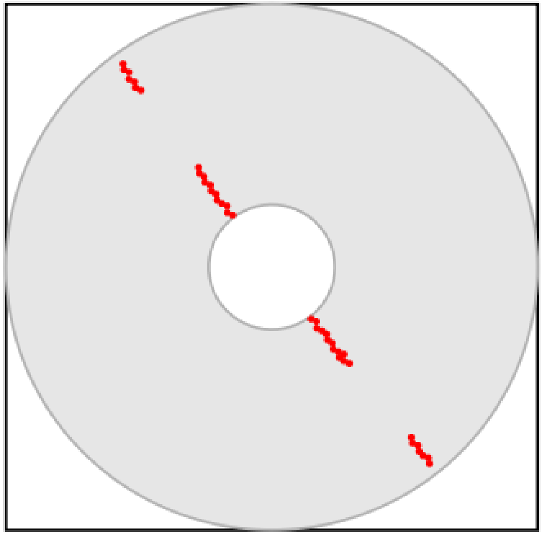


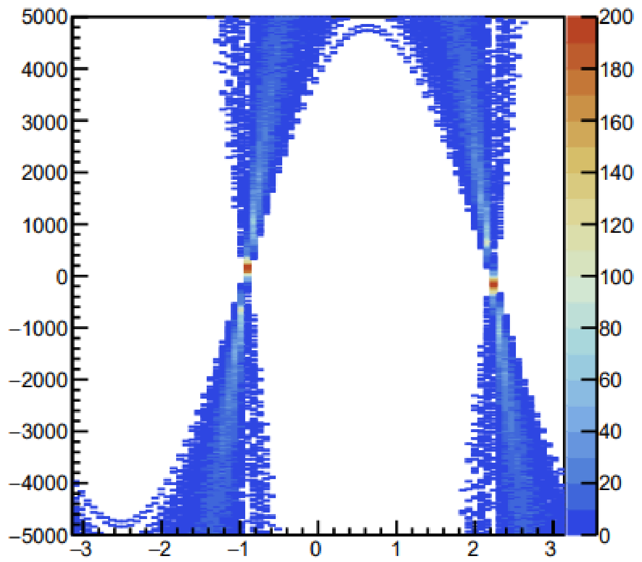


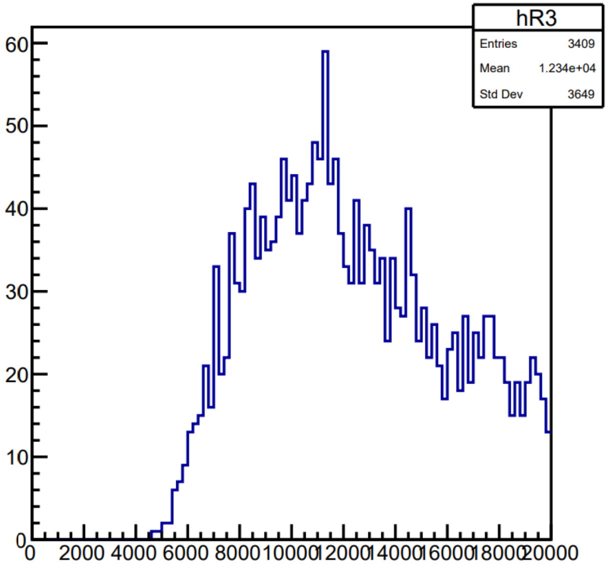




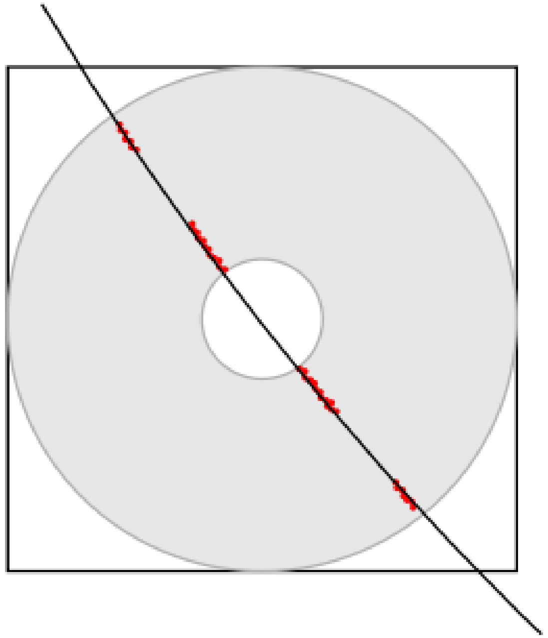
2D Track Finder with Legendre Transform



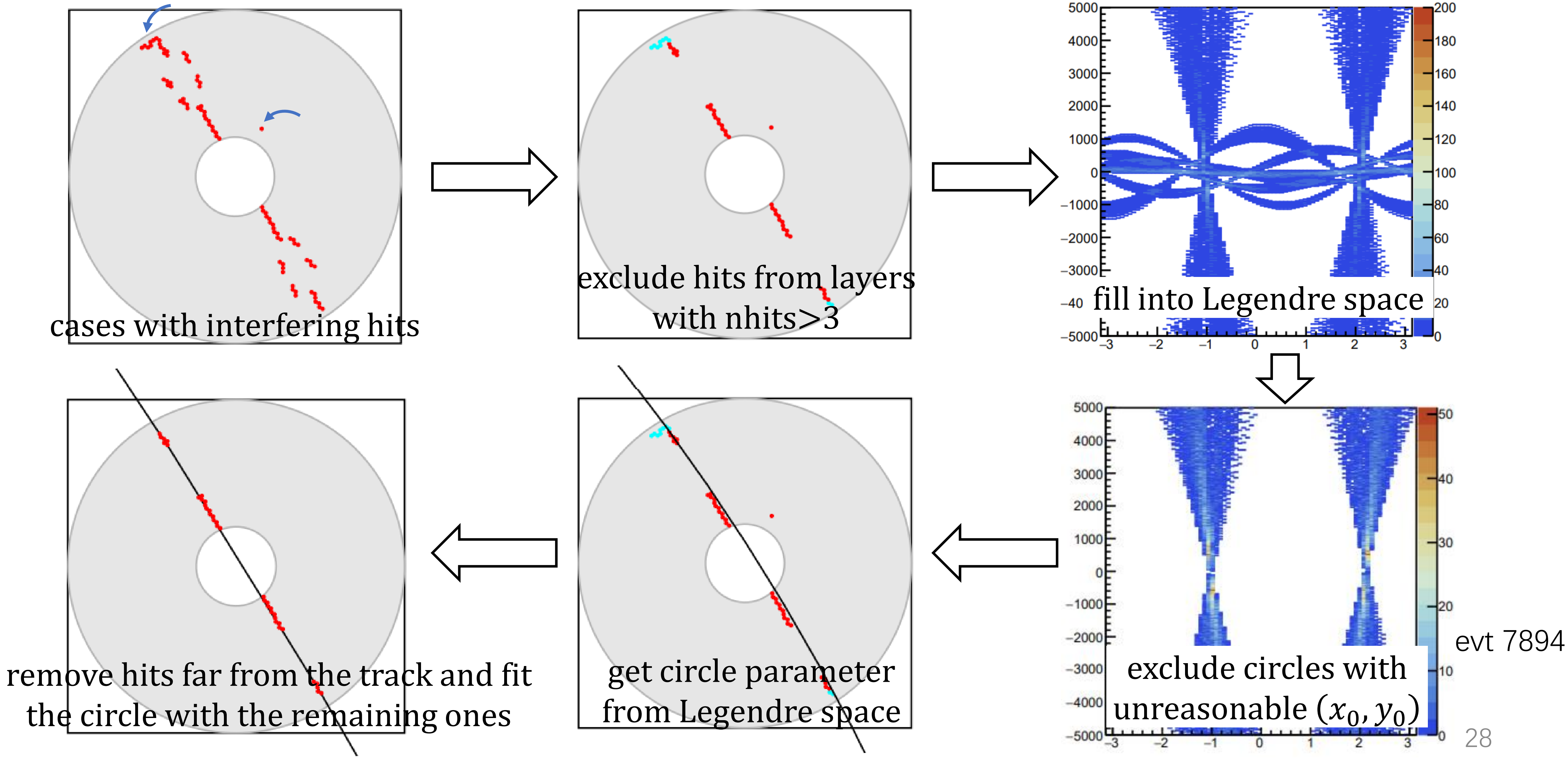




1. Determine (x_0, y_0) with the brightest band in Legendre Space
2. Iterative peak finding in R by minimizing the hit-to-track distance



2D Track Finder with Legendre Transform



Track Reconstruction with Magnetic Filed