From ideal to real – where is my detector?

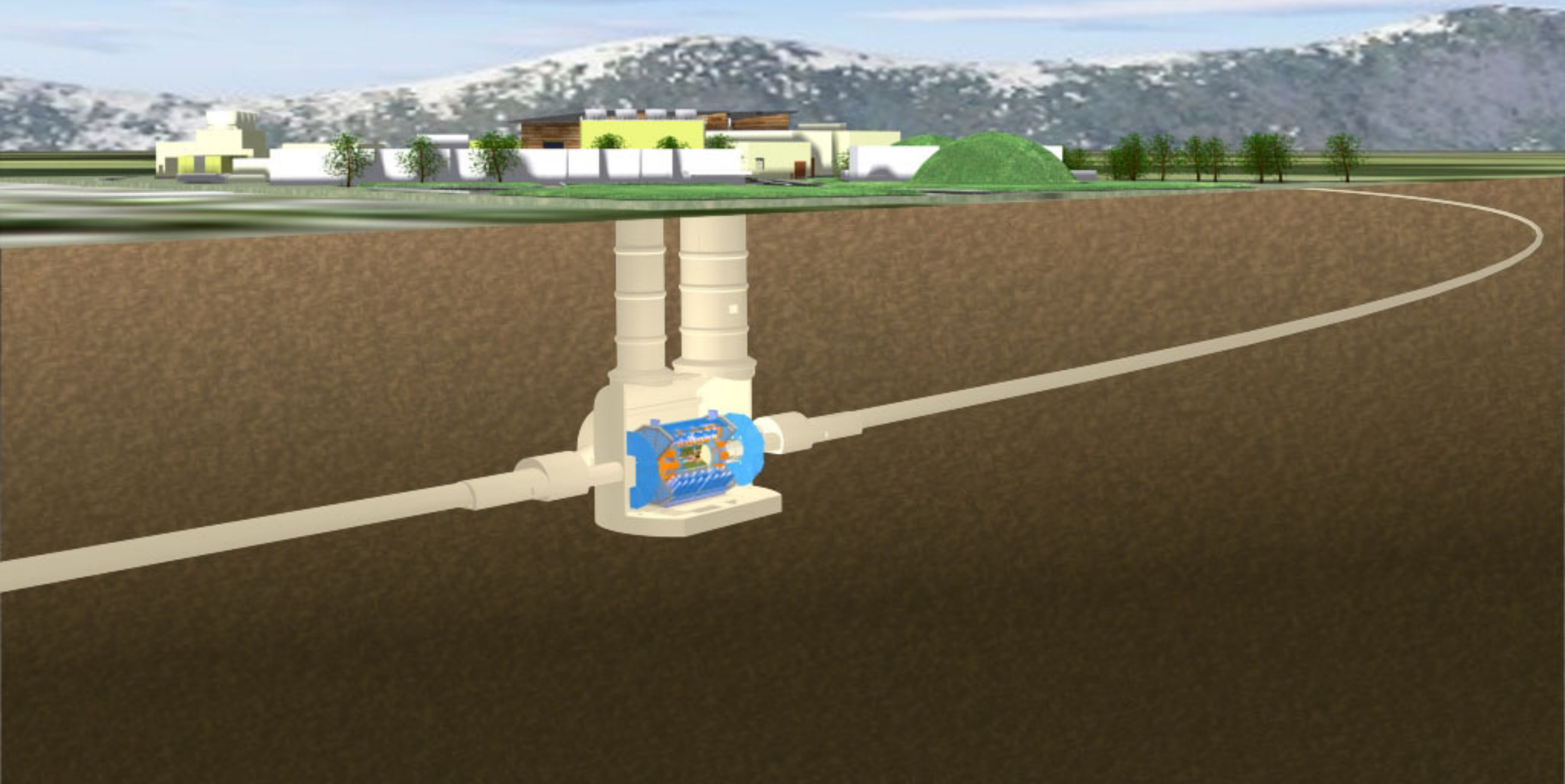
(on the ATLAS example)

Pawel Brückman de Renstrom (IFJ PAN, Krakow)

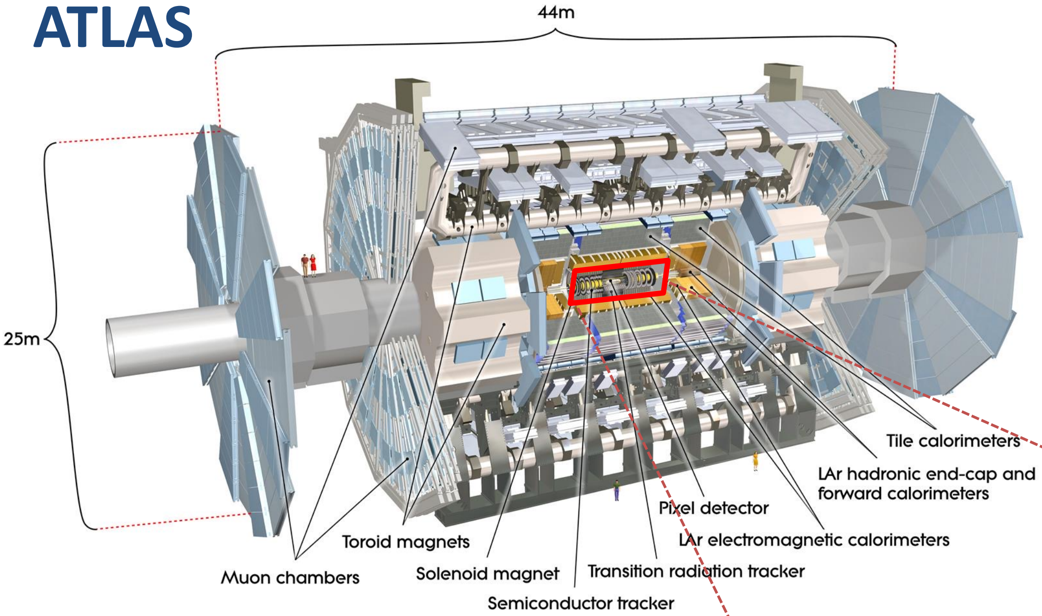
Second Workshop on Trajectory Reconstruction in Particle Physics Experiments
Huizhou, 21-23.07.2025

On the example of ATLAS…

On the example of ATLAS…

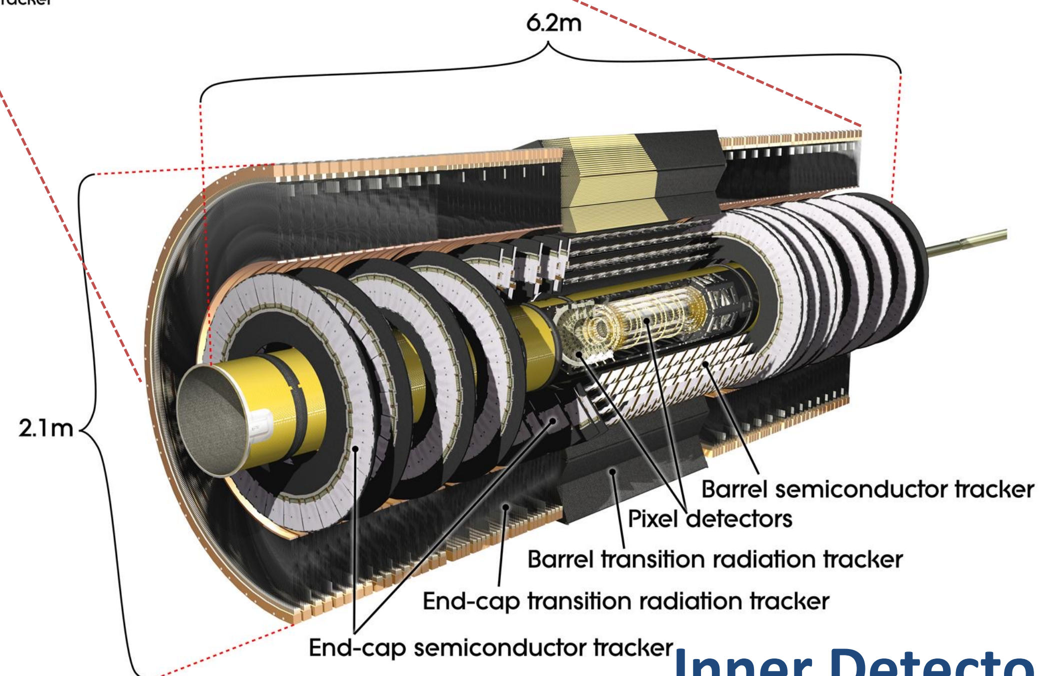


ATLAS



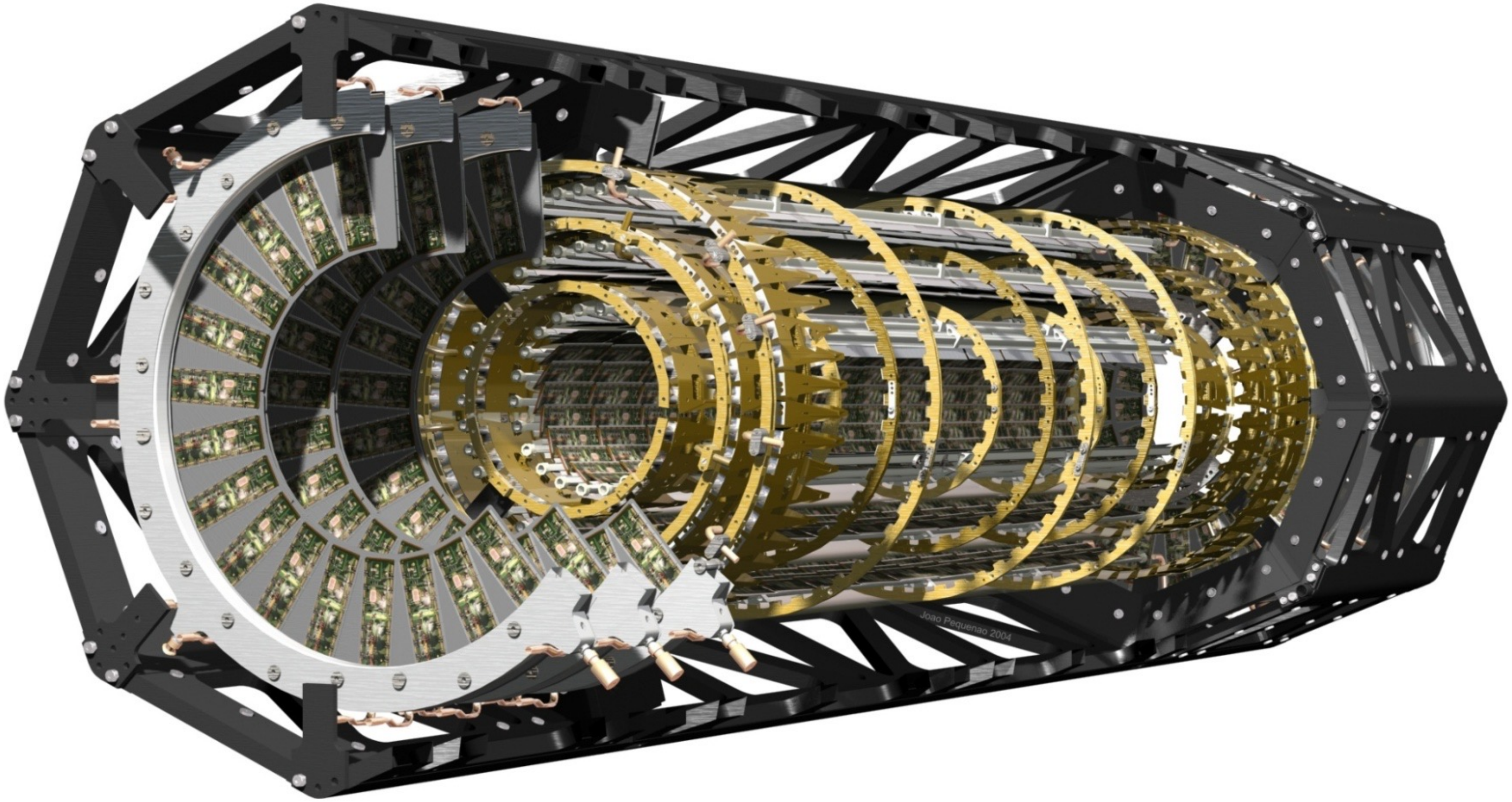
Placed in the very heart of the ATLAS spectrometer…

…the Inner Detector is of rather macroscopic size!

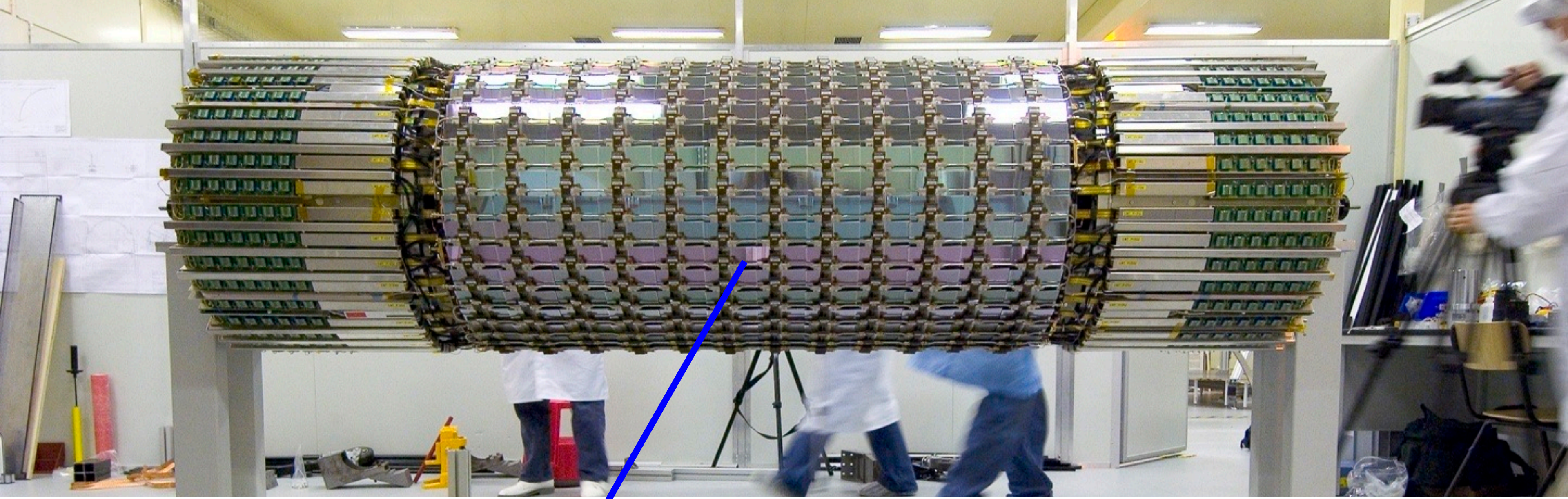


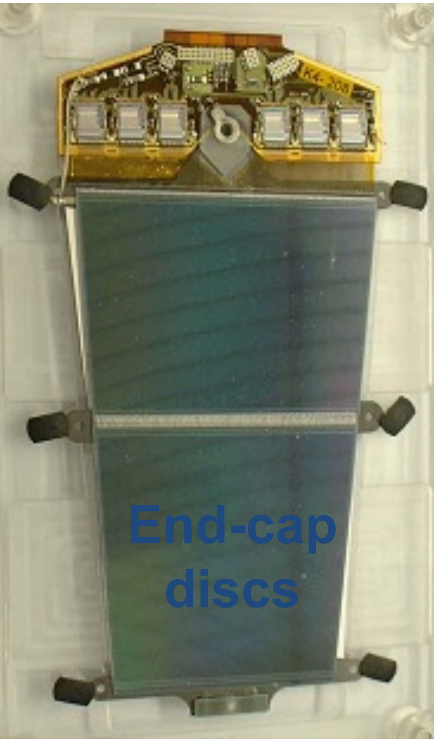
Inner Detector

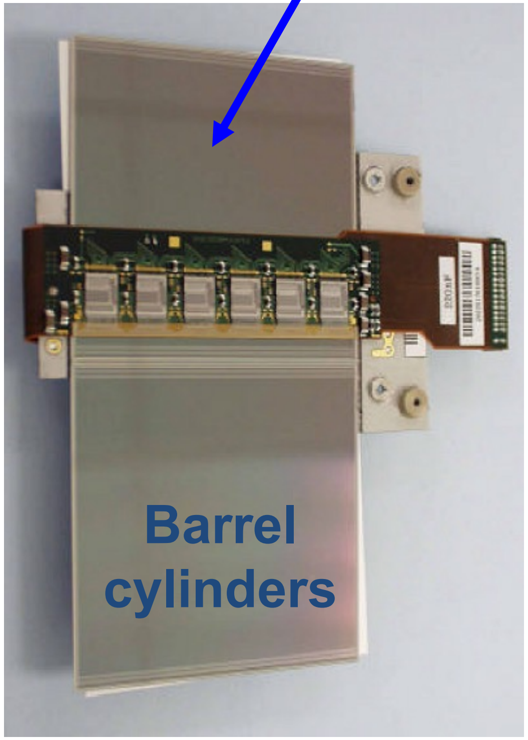
ATLAS Pixel detector



- ❖ 3+1 concentric cylinders
- ❖ 2x3 discs in the forward region



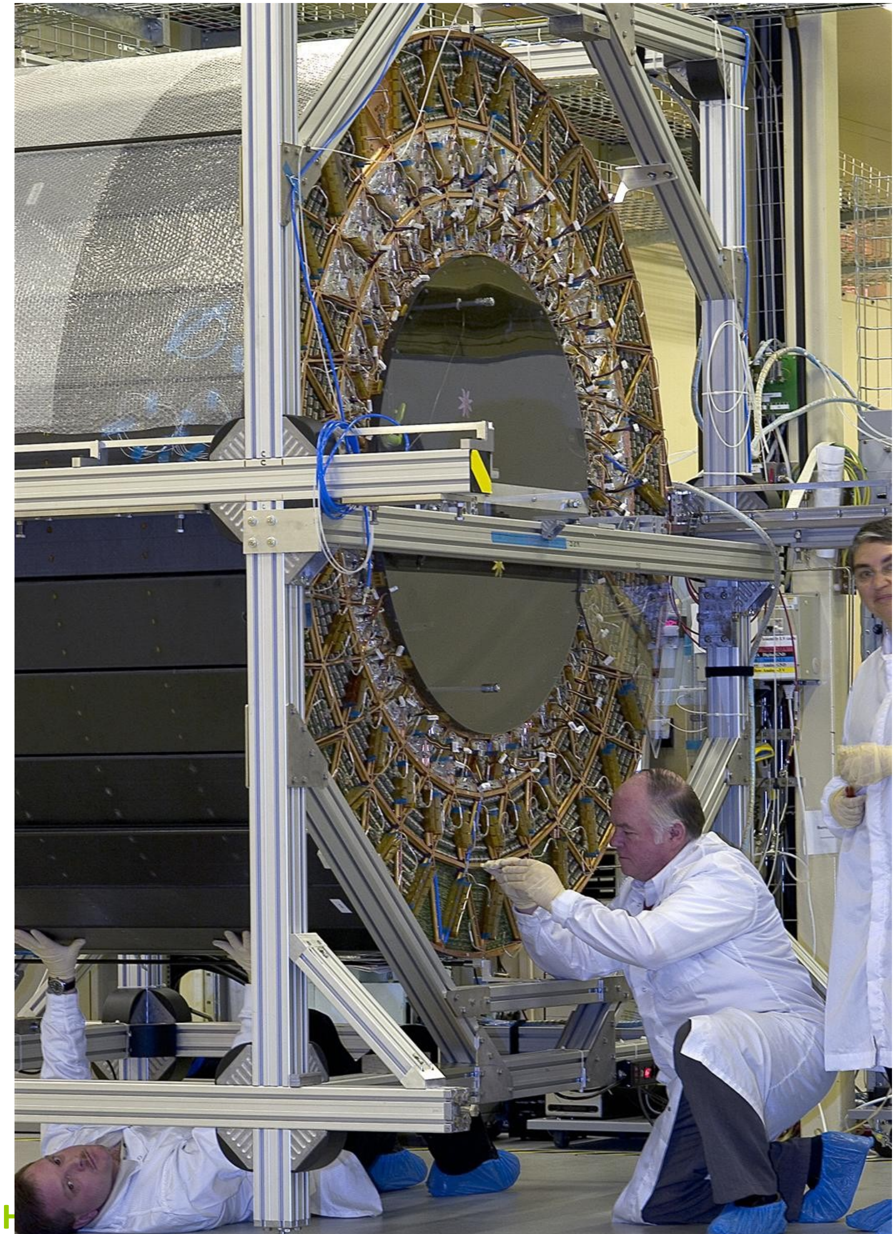




ATLAS Semiconductor Tracker (SCT)

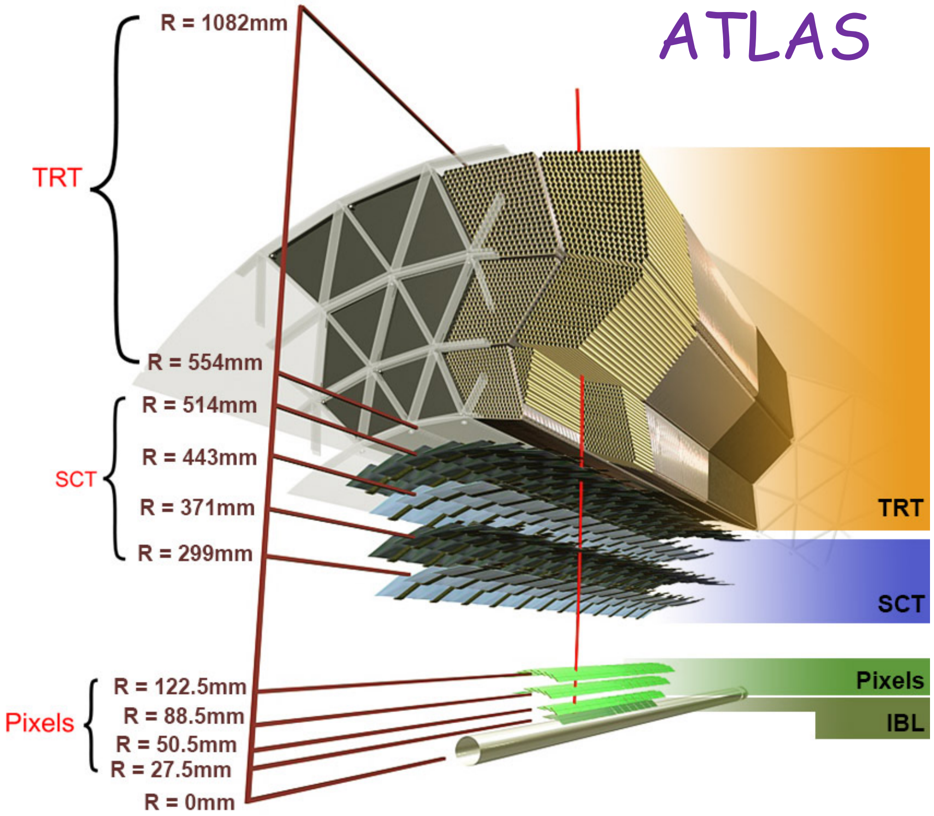
- 4 concentric cylinders
- 2x9 discs in the forward region

ATLAS Transition Radiation Tracker (TRT)



Inner Detector at a closer look

High resolution
Silicon & gas detectors

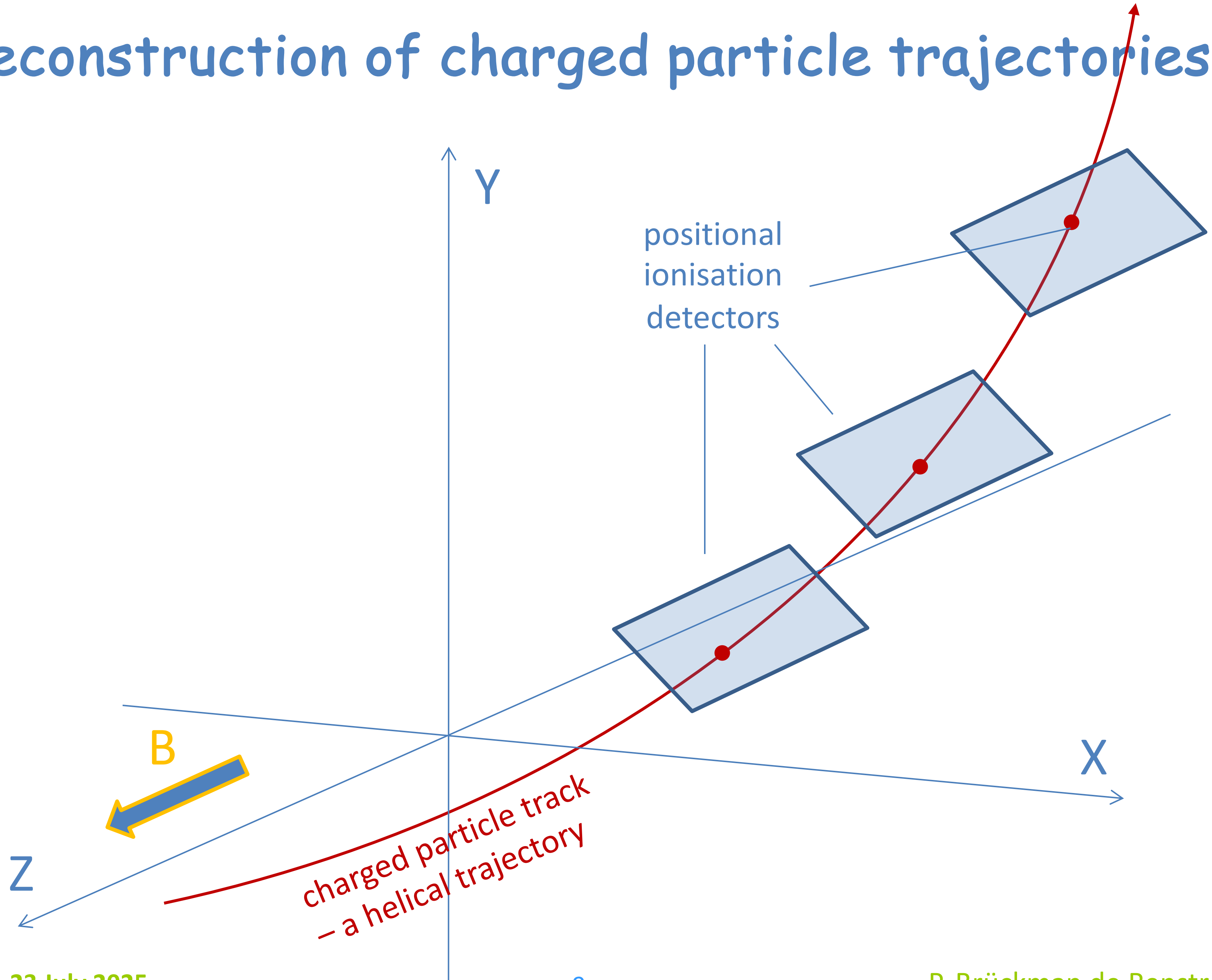


TRT (gas proportional):
- 350,048 straws (701,696 par’s)
- Size: 4 mm × 71/39 cm
- Resolution: 130 μm

SCT (Si ministrips):
- 4088 modules (24,528 par’s)
- Strip dimensions: 80 μm × 12 cm
- Resolution: 17 μm × 580 μm

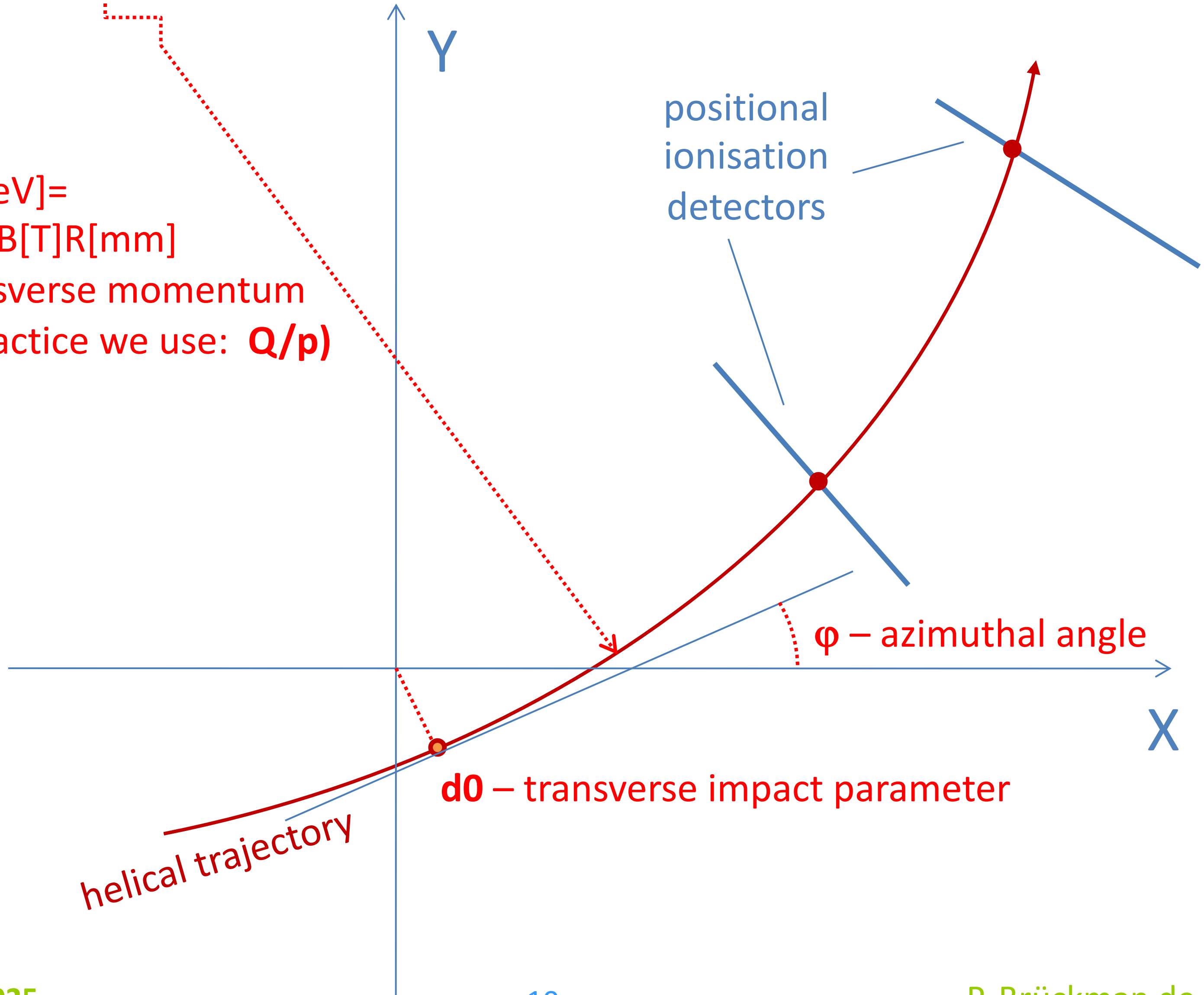
Pixels+IBL (Si pads):
- 1744+224 modules (11,808 par’s)
- Pixel size: 50 μm × 400(250) μm
- Resolution : 10 μm × 115(72) μm

Reconstruction of charged particle trajectories

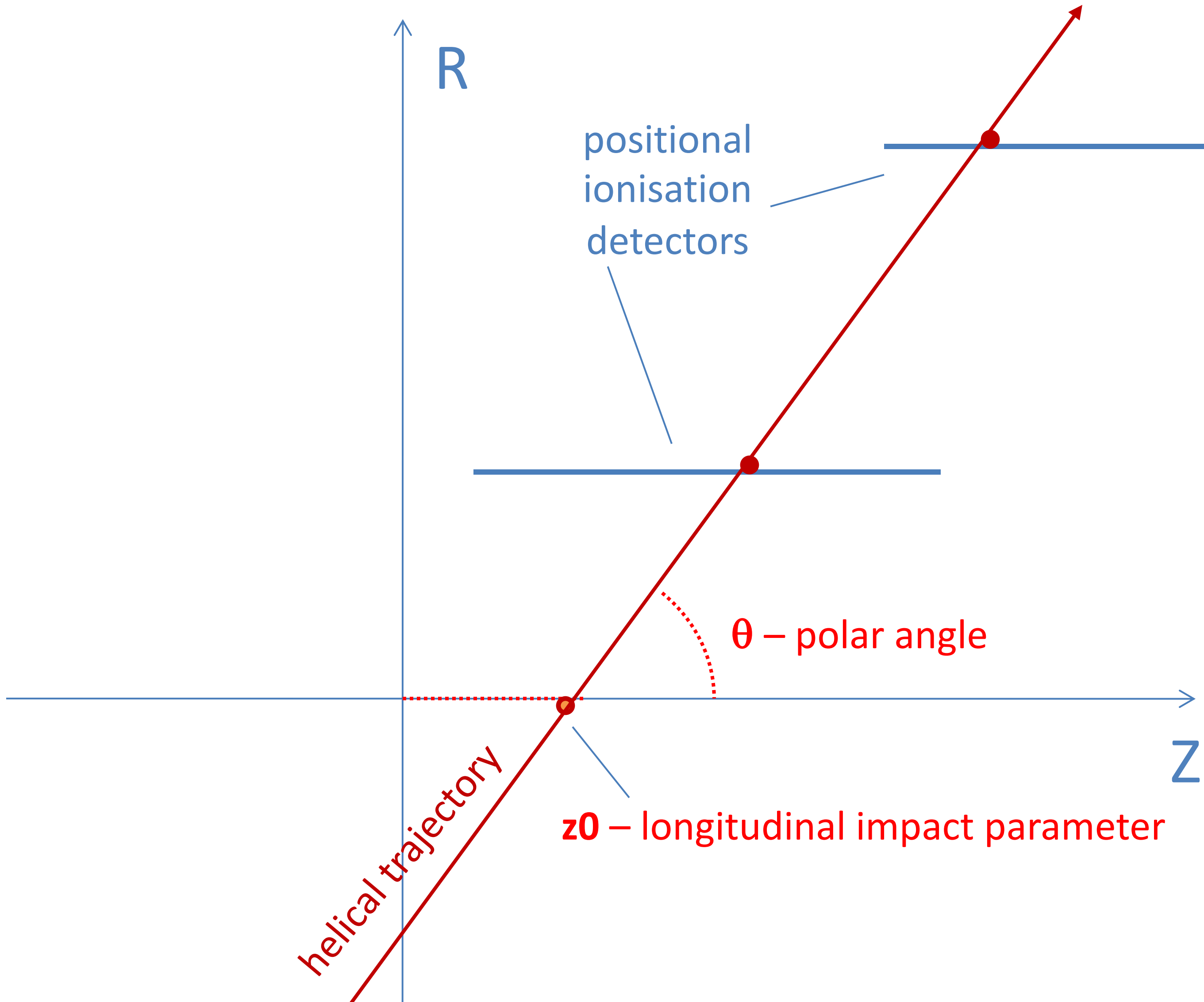


Track reconstruction – kinematical parameters

p_T [MeV]=
=0.3qB[T]R[mm]
- transverse momentum
(In practice we use: **Q/p**)



Track reconstruction - kinematical parameters



How does it work?

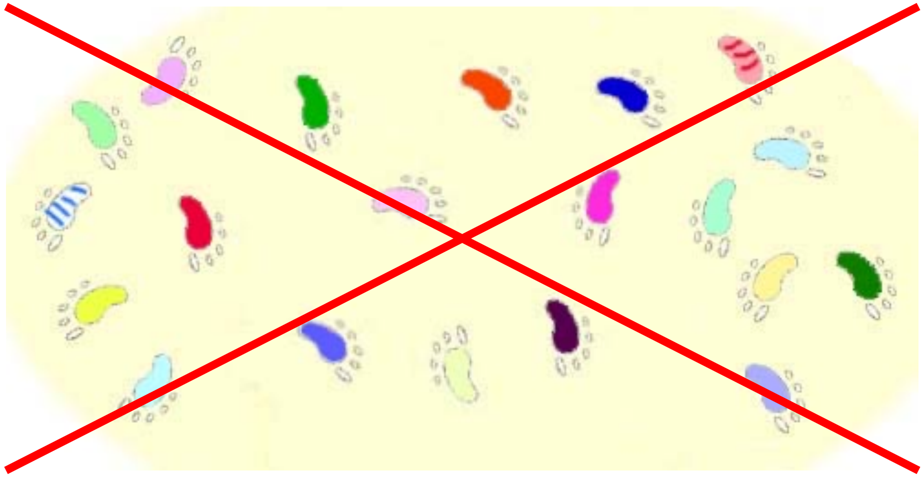
❖Cannot see trajectories, only scattered „footprints".

❖Have to „reconstruct" them on this basis.

How does it work?

❖Cannot see trajectories, only scattered „footprints".
❖Have to „reconstruct" them on this basis.

❖Each „footprint" is „photographed" separately.
❖To reconstruct trajectories one has to know how to arrange the pictures:

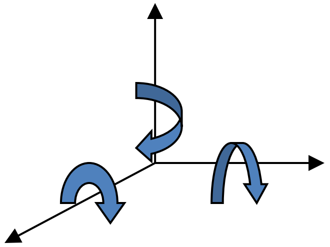


❖Only one hypothesis is correct!

When did the issue of geometry reconstruction (alignment) became relevant?

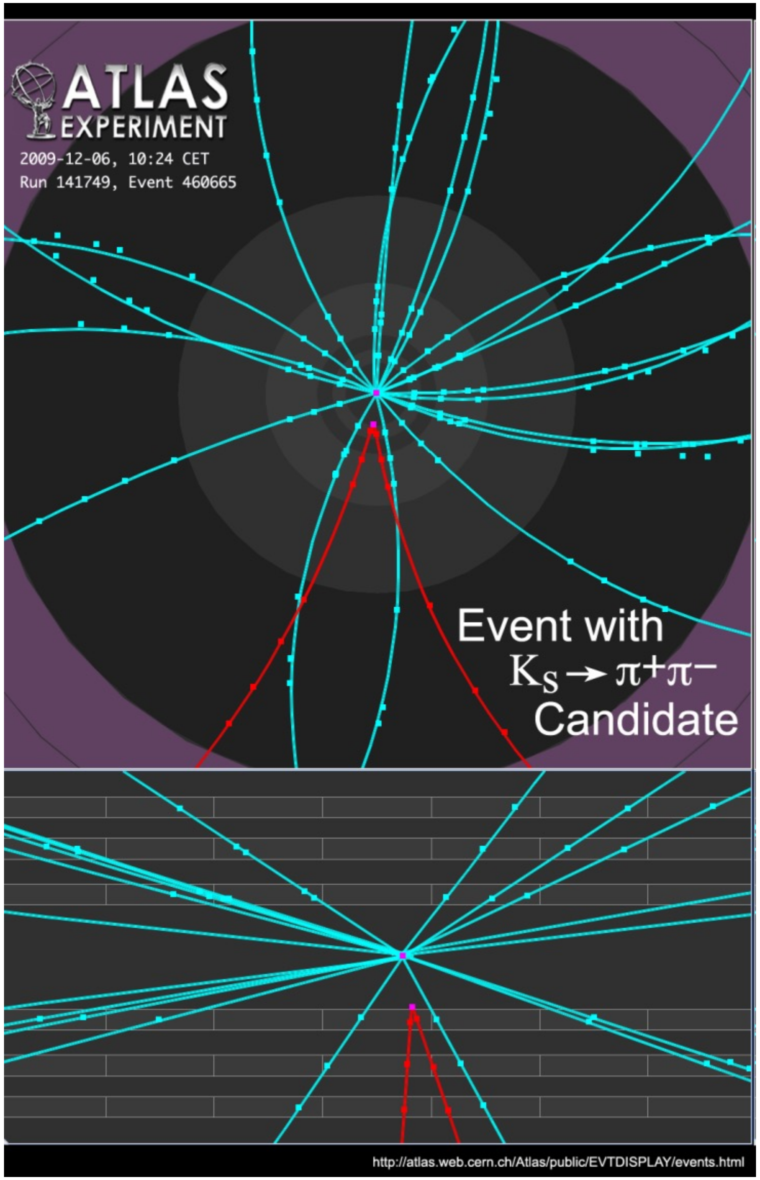
1. Detector consists of more than one position-sensitive element,
2. Intrinsic resolution of sensors is better than the placing accuracy or their positions survey.

Ad1: The ATLAS Inner Detector consists of ~360,000 sensing devices.
Each has 6 Degrees of Freedom (DoF).



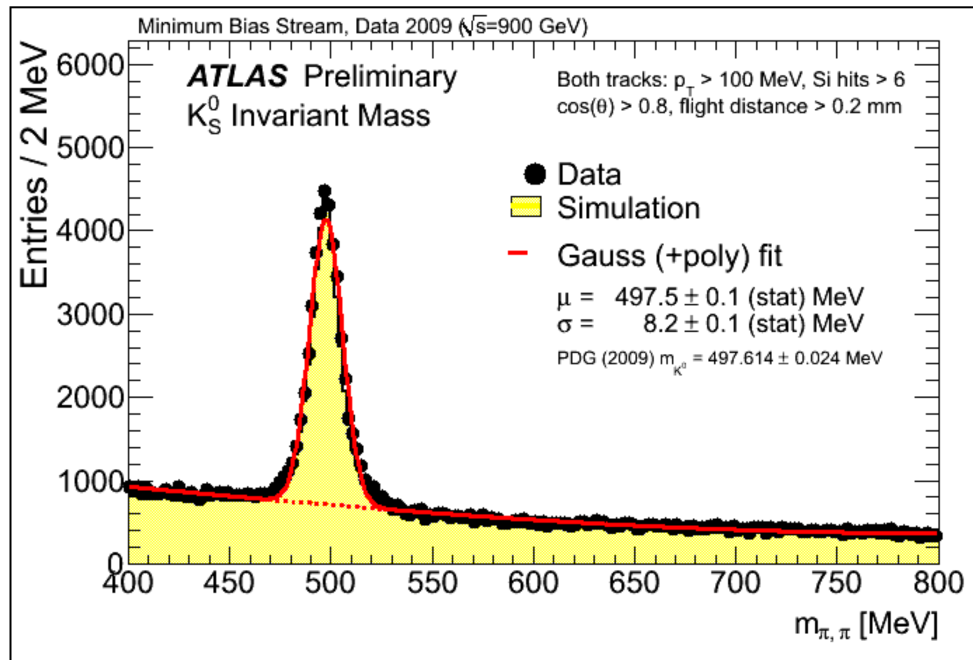
Ad2: In ATLAS ID the survey precision is from one to two orders of magnitude worse than the intrinsic resolution.

Why do we care?



Example of on of the first measurements in ATLAS:

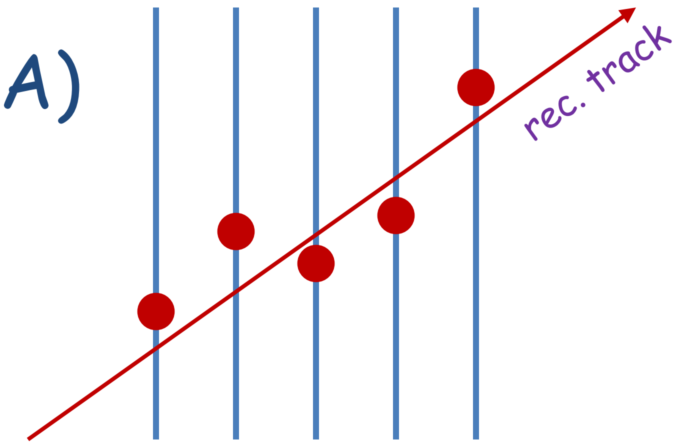
- invariant mass,
- K_s^0 decay vertex,
- primary event vertex.



Inferring the actual geometry -Alignment

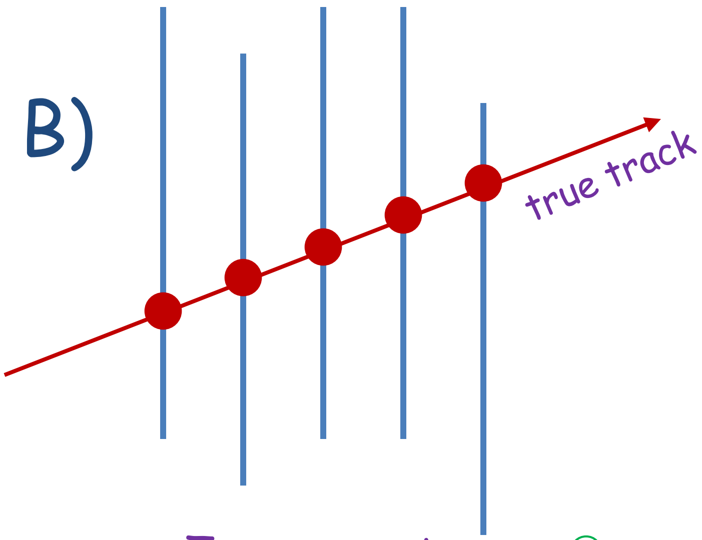
- Kinematical parameters are reconstructed from the recorded spacepoints:

A)



Assumed geometry ☹
reduced resolution (fit quality),
biased parameter estimator

B)

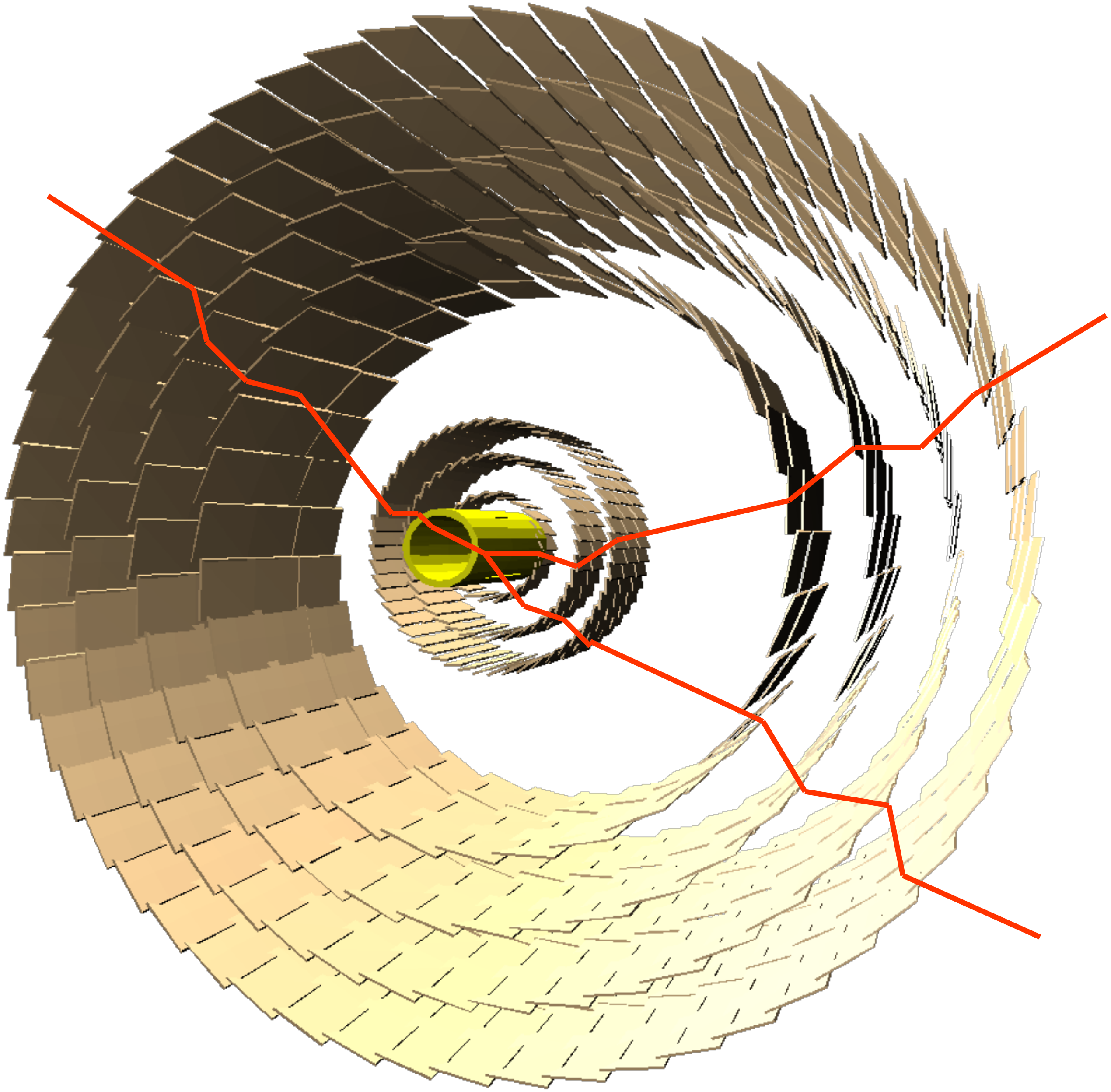


True geometry ☺
optimal resolution,
unbiased parameters

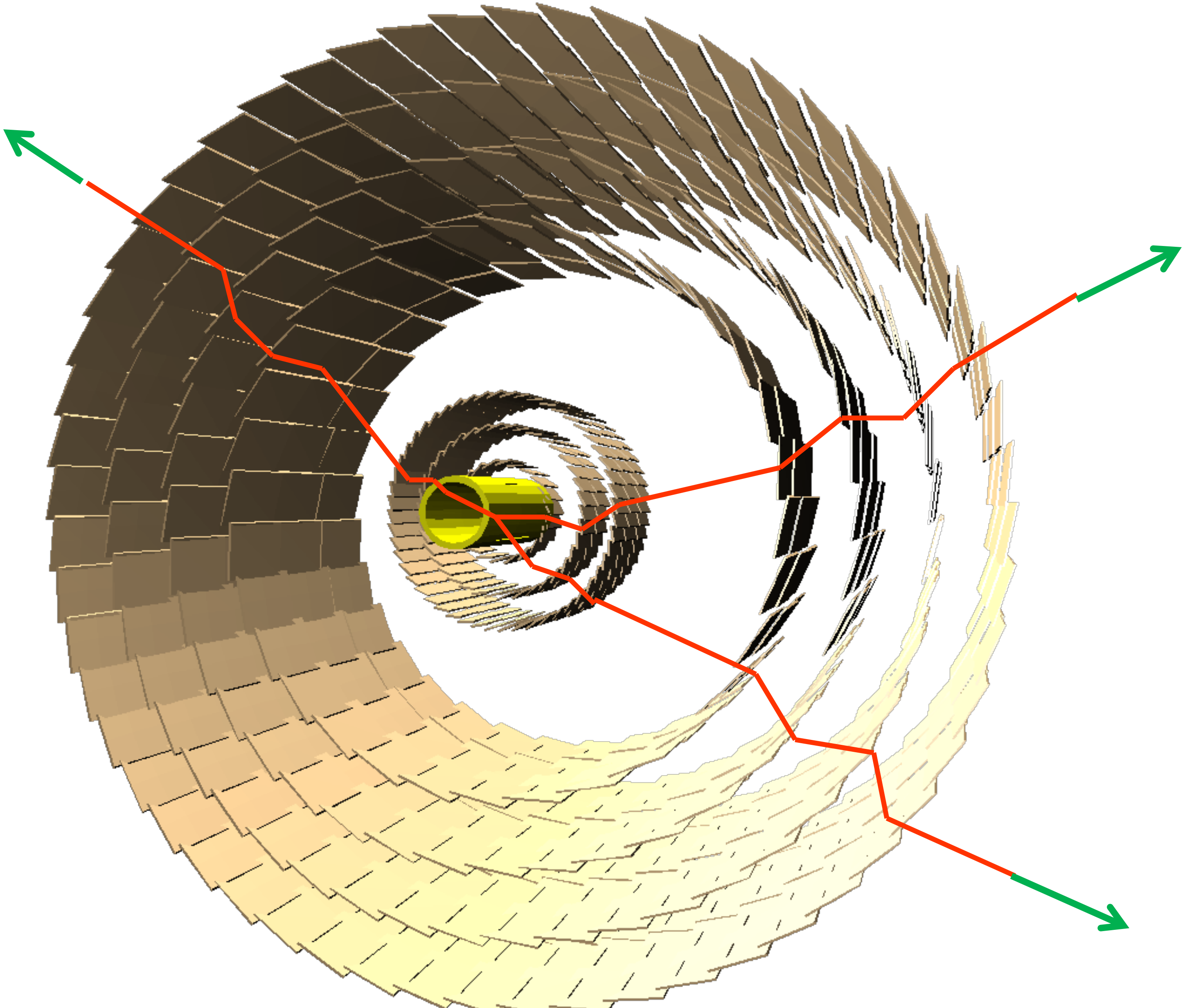
- Alignment should bring us from A to B!

The principle - „reversed engineering"

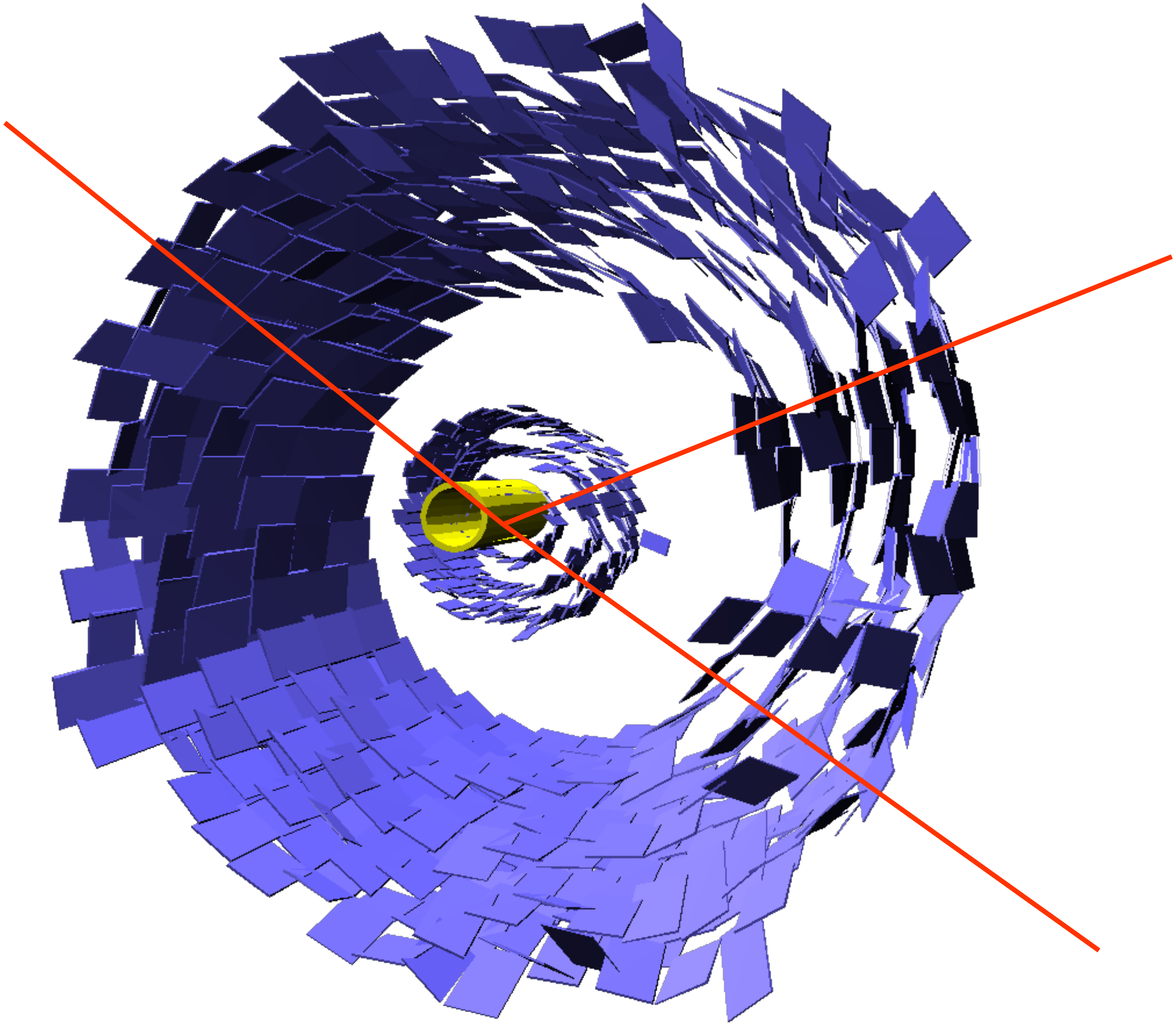
Use particle trajectories as our rulers!



The principle - „reversed engineering"
Use particle trajectories as our rulers!



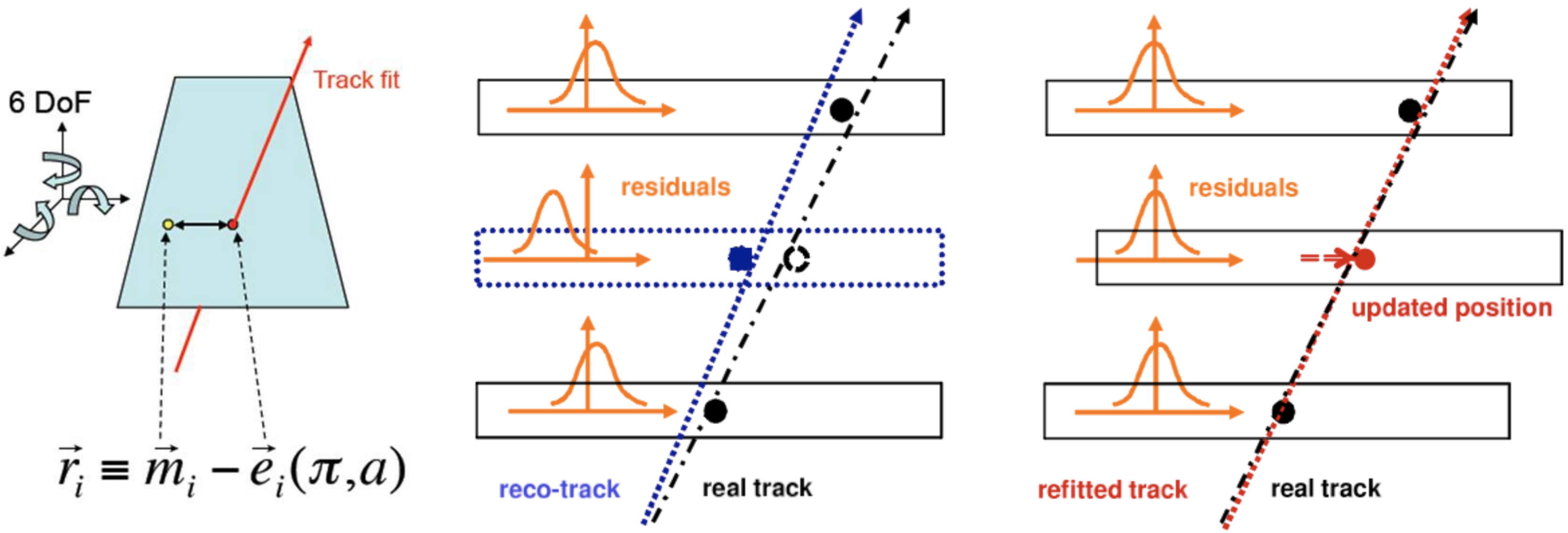
The principle - „reversed engineering"
Use particle trajectories as our rulers!



How can we do that?

Basically two approaches on the market

1. **Local:** optimise sensor positions to previously reconstructed tracks (inherently iterative)



2. **Global:** simultaneous optimisation (fit) of all track parameters AND geometry DoF's (~instantaneous + constraints naturally integrated)

Global χ^2 approach

$$\chi^2_{\text{Global}} = \sum_i \chi^2_{\text{Track}\,i}$$

$$\boxed{\frac{\mathrm{d}\chi^2}{\mathrm{d}\boldsymbol{\alpha}} = 0, \qquad \boldsymbol{\rho} \equiv \boldsymbol{\rho}(\boldsymbol{\pi}(\boldsymbol{\alpha}), \boldsymbol{\alpha})}$$

$$\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\alpha}} = \frac{\partial}{\partial\boldsymbol{\alpha}} + \frac{\mathrm{d}\boldsymbol{\pi}}{\mathrm{d}\boldsymbol{\alpha}}\frac{\partial}{\partial\boldsymbol{\pi}} = \frac{\partial}{\partial\boldsymbol{\alpha}} - A^T V^{-1} H C \frac{\partial}{\partial\boldsymbol{\pi}}$$

$$A \equiv \frac{\partial\boldsymbol{\rho}}{\partial\boldsymbol{\alpha}}, \quad H \equiv \frac{\partial\boldsymbol{\rho}}{\partial\boldsymbol{\pi}}, \quad \boxed{C} = \frac{1}{2}\left(\frac{\mathrm{d}^2\chi^2}{\mathrm{d}\boldsymbol{\pi}^2}\right)^{-1} = \underbrace{\left(H^T V^{-1} H\right)^{-1}}_{Cov(\boldsymbol{\pi})}$$

extractable for KF tracks! (see Xiaocong talk)

$$\boxed{R = V - HCH^\top}$$

full covariance of the residuals of the track fit

$$Y \equiv \left(\frac{\mathrm{d}\chi^2_{\text{Global}}}{\mathrm{d}\boldsymbol{\alpha}}\right)^\top = 2\sum_{\text{tracks}} A^\top V^{-1}(V - \underbrace{HCH^\top)V^{-1}\boldsymbol{\rho}}_{=0} = \boxed{2\sum_{\text{tracks}} A^\top V^{-1}\boldsymbol{\rho}}$$

"locality ansatz"
big simplification!!!

$$\mathcal{M} \equiv \frac{\mathrm{d}^2\chi^2_{\text{Global}}}{\mathrm{d}\boldsymbol{\alpha}^2} = 2\sum_{\text{tracks}} A^\top V^{-1}\underbrace{(V - HCH^\top)}_{R}V^{-1}A\,.$$

Global χ^2 approach

$$\boldsymbol{X} \equiv \Delta\boldsymbol{\alpha} = -\left(\left.\frac{\mathrm{d}^2\chi^2_{\mathrm{Global}}}{\mathrm{d}\boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}_0} \right)^{-1} \left.\left(\frac{\mathrm{d}\chi^2_{\mathrm{Global}}}{\mathrm{d}\boldsymbol{\alpha}}\right)^{\mathsf{T}}\right|_{\boldsymbol{\alpha}_0} \equiv -\mathcal{M}^{-1}\boldsymbol{Y}$$

vector of size N

$$\begin{pmatrix} \delta a \end{pmatrix} = \begin{bmatrix} \mathbf{M} \end{bmatrix}^{-1} \begin{pmatrix} \mathbf{Y} \end{pmatrix}$$

NxN
symmetric matrix

vector of size N

N = numer of DoF's
(align pars.)

ITERATIONS

Fetch the current ID geometry

ACCUMULATION

Full tracking event reconstruction:

- using current ID geometry
- pattern recognition
- track fit
- may include constraints on track parameters (pseudo-measurements)
- Get: r, $\frac{\partial r}{\partial \pi}$, V

Accumulate alignment for the event:

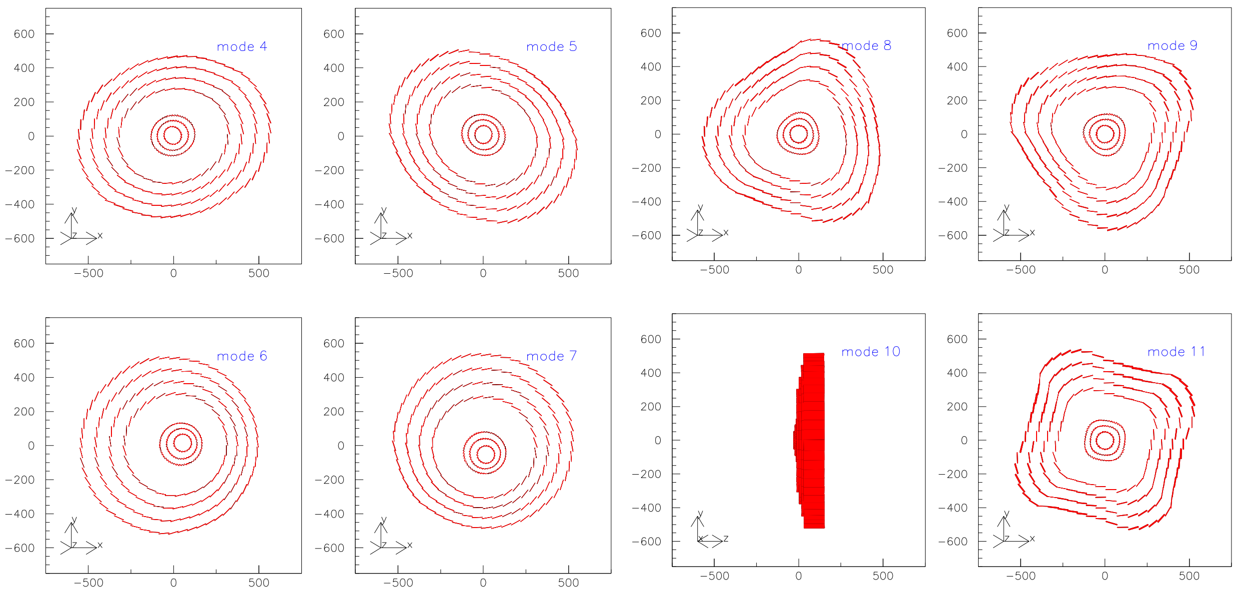
- full covariance of the measurements
- calculate derivatives w.r.t alignment parameters $\frac{\partial r}{\partial a}$
- accumulate alignment fit info for the tracks
- Increment: $\frac{d\chi^2}{da}$, $\frac{d^2\chi^2}{da^2}$

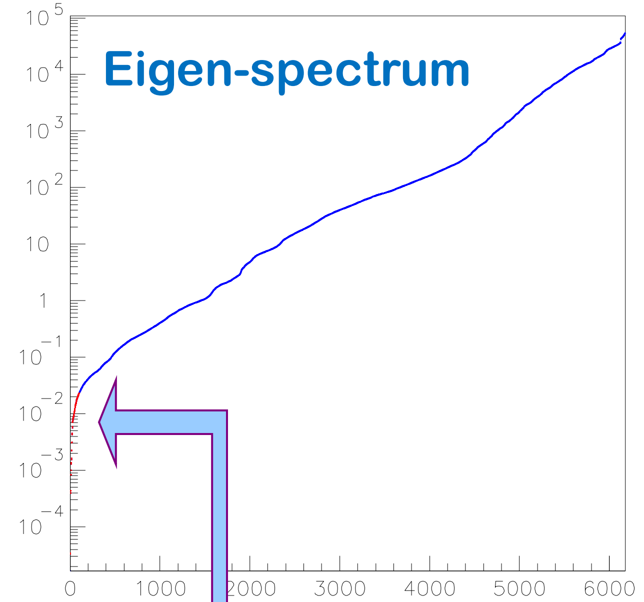
SOLVING

Solve for alignment corrections: δa

Create updated ID geometry

„Weak modes" – underbelly of alignment





- Weak modes correspond to the lowest eigenvalues in the spectrum (**including degenerate 6 DoF's!**).
- They contribute the most to the uncertainty of the solution.
- Have next to no impact on the fit quality – the χ^2.
- Most importantly, they are source of biases on the reconstructed track parameters (**systematics!**).

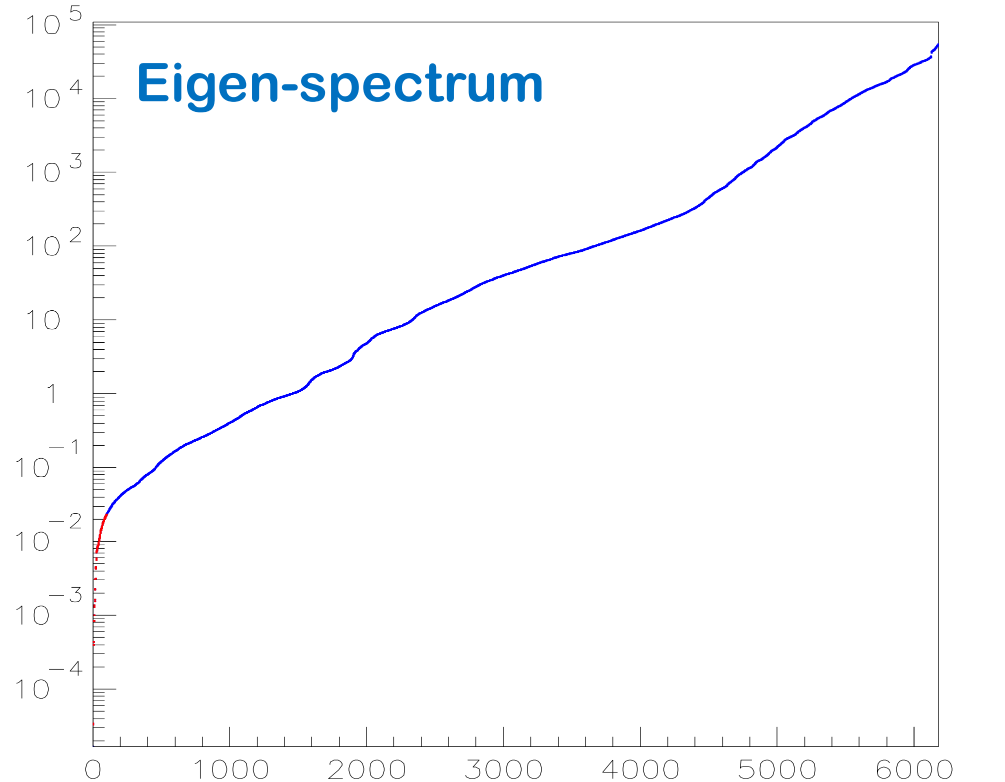
Singular (and weak) modes need to be removed from the solution.
What can we do for very large systems? - soft-mode-cut

$$\mathbf{M}X = Y, \quad \mathbf{U}\mathbf{M}\mathbf{U}^{\mathrm{T}}\mathbf{U}X = \mathbf{U}Y \Rightarrow \mathbf{D}X_D = Y_D$$

$$X = \sum_i E_i \frac{1}{\lambda_i} Y_D$$

$$\boxed{\mathbf{M} \to \mathbf{M} + \kappa 1, \quad \mathbf{D} \to \mathbf{D} + \kappa 1, \quad \lambda_i \to \lambda_i + \kappa}$$

$$\begin{bmatrix} \lambda_1 & & & & \\ & \lambda_2 & & & \\ & & \lambda_3 & & \\ & & & \ddots & \\ & & & & \lambda_n \end{bmatrix}$$



Singular (and weak) modes need to be removed from the solution.
What can we do for very large systems – soft-mode-cut

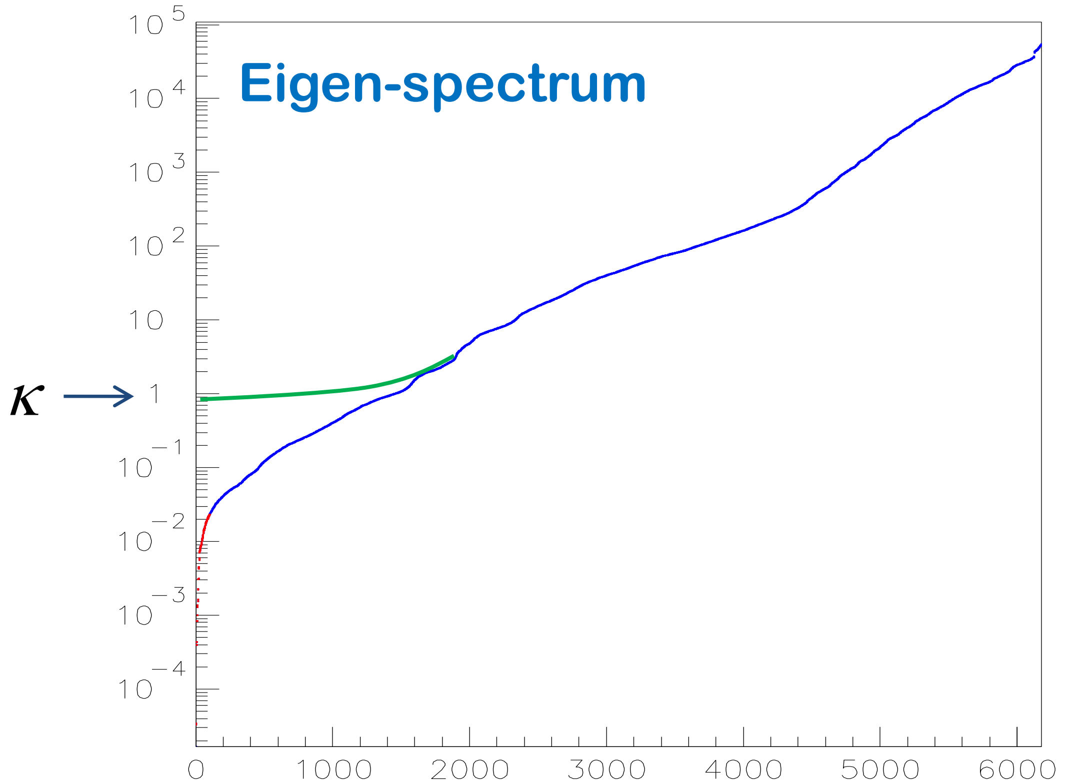
(…and use fast solvers)

$$\mathbf{M}X = Y, \quad \mathbf{U}\mathbf{M}\mathbf{U}^{\mathrm{T}}\mathbf{U}X = \mathbf{U}Y \Rightarrow \mathbf{D}X_D = Y_D$$

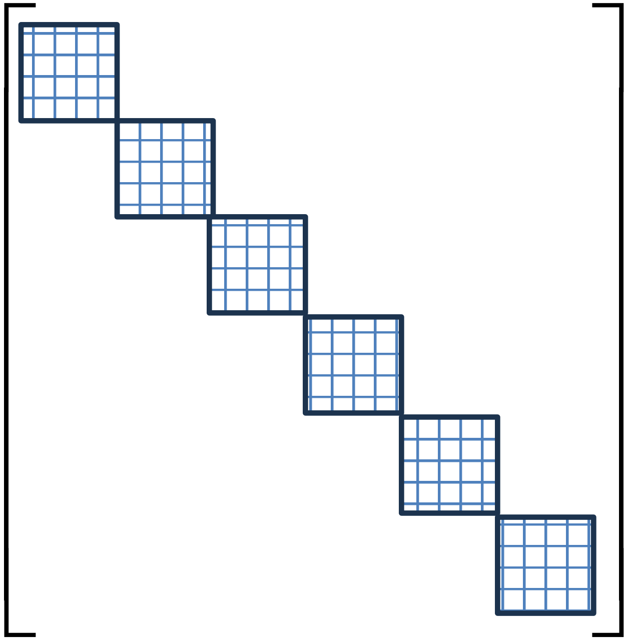
$$X = \sum_i E_i \frac{1}{\lambda_i} Y_D$$

$$\boxed{\mathbf{M} \to \mathbf{M} + \kappa 1, \quad \mathbf{D} \to \mathbf{D} + \kappa 1, \quad \lambda_i \to \lambda_i + \kappa}$$

$$\begin{bmatrix} \lambda_1 + \kappa & & & & \\ & \lambda_2 + \kappa & & & \\ & & \lambda_3 + \kappa & & \\ & & & \ddots & \\ & & & & \lambda_n + \kappa \end{bmatrix}$$



The **Local solution**: solve for small blocks on the diagonal (e.g. 6x6 for individual modules).
The **Local accumulation**: accumulate assuming $\boxed{\dfrac{\mathrm{d}}{\mathrm{d}\alpha} \Rightarrow \dfrac{\partial}{\partial \alpha}}$



The solution is numerically simple quick and stable.

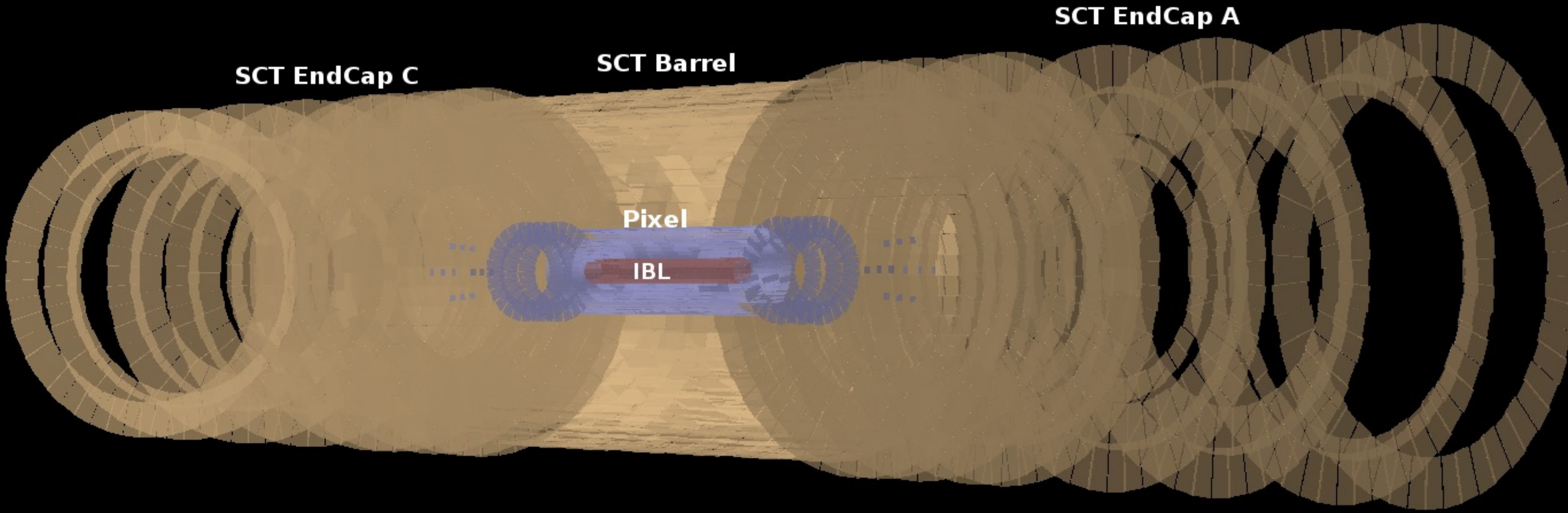
The nice properties of the Global approach is lost, however.

The method needs to resort to multiple iterations.

The alignment „**Levels**"

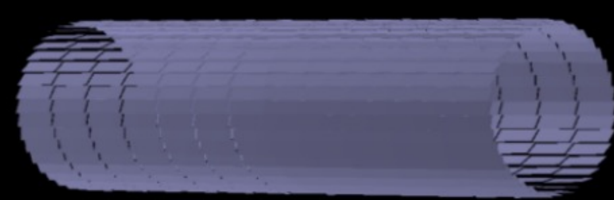
Jacobian
(mode itself)

$$M_{ij} = \frac{dr^T}{da}\Omega^{-1}\frac{dr}{da} \qquad Y_i = \frac{dr^T}{da}\Omega^{-1} r \quad \Longrightarrow \quad \frac{dr}{dA_l} = \frac{dr}{da_k}\frac{da_k}{dA_l}$$



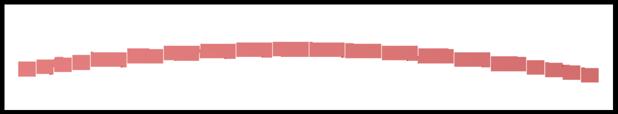
Level 1 (11): 4 (5) alignable structures
SCT ECC, Barrel, ECA, Pixel, (IBL)

Level 2: 32 alignable structures
2x9 SCT discs, 4 SCT Barrel layers,
2x3 Pixel discs, 3 Pixel layers,
IBL layer



Level 3: 6112 modules
4088 SCT modules
1744 Pixel modules
280 IBL modules

Level 16:IBL stave bowing



Strategy adopted by ATLAS:

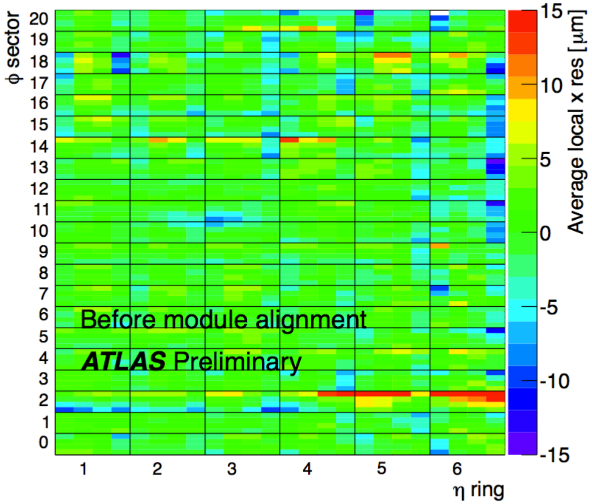
- Sequential procedure of alignment at different levels of granularity – starting from big structures down to individual modules.

- Heavily rely on the Beam Spot constraint.

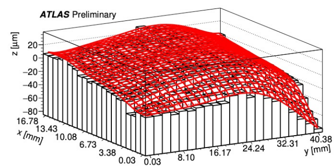
- Global χ^2 employed to systems not larger than Pixel + SCT ~35,000 DoF's (diagonalisation L1/L2, or „fast solvers" L3)

- TRT straw-level alignment (currently 2 DoF's/straw => ~700,000 parameters) utilises the Local method.

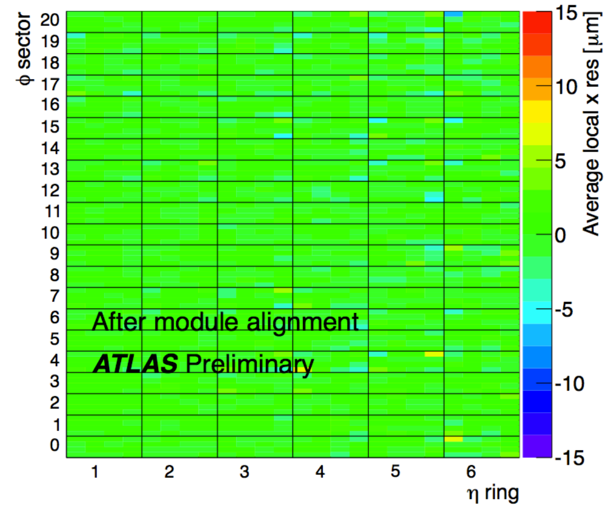
Low-level corrections from 2010 $\sqrt{s}$=7 TeV data

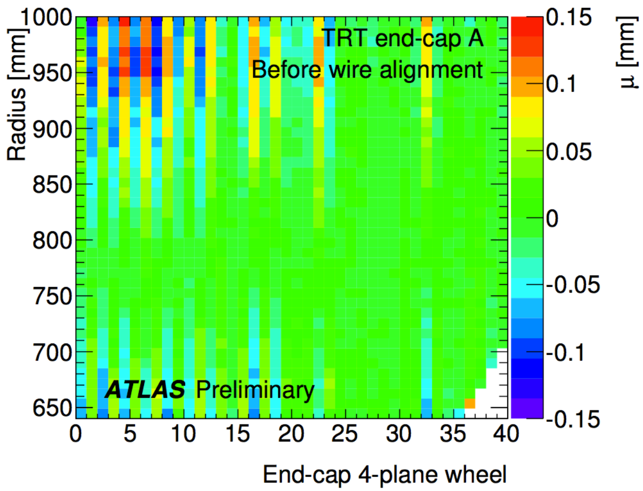


Correcting for out-of-the-plane Pixel sensor deformations as of the survey

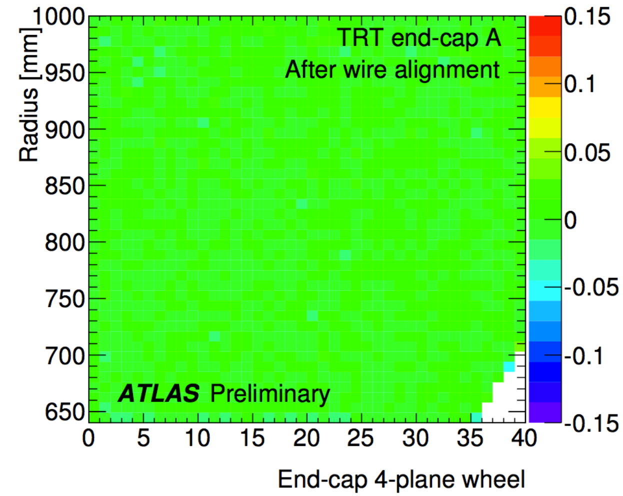


(IBL measurement)

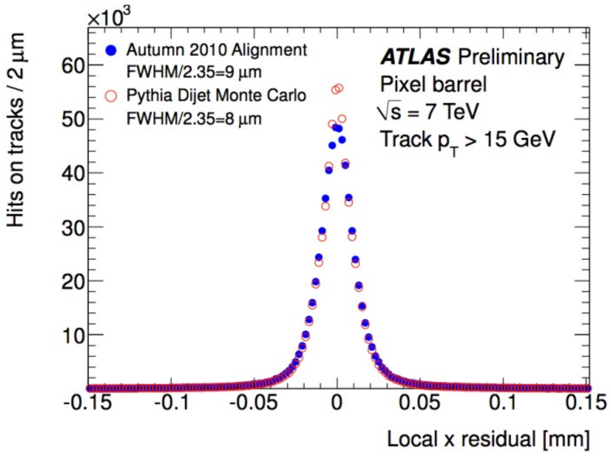


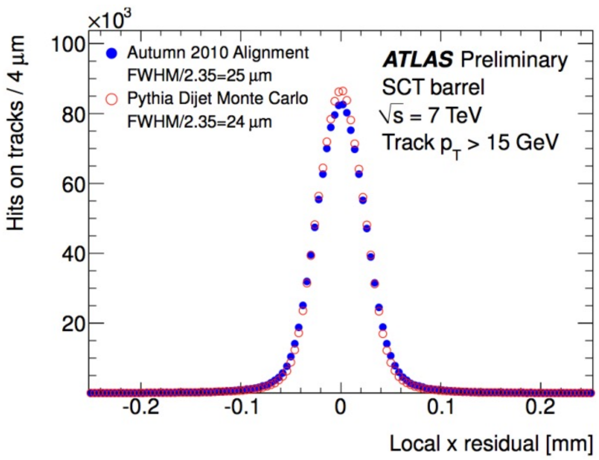


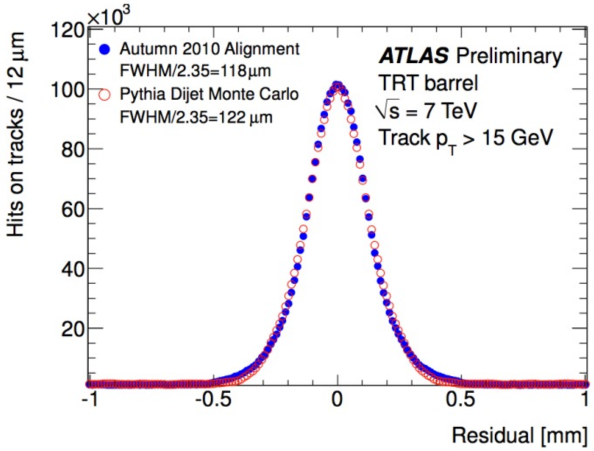
TRT straw (L3) alignment



Alignment of 2010 data ($\sqrt{s}$=7 TeV)







As a result of the Global χ^2 procedure we got a seemingly perfect detector…

…well not necesarily. Do we really know where our sensors are?

Additional constraints on track parameters are needed!

Most relevant distortions

- Charge-antisymmetric momentum bias (sagitta):

$$q/p_{\mathrm{T}} \longrightarrow q/p_{\mathrm{T}} + \delta_{\mathrm{sagitta}} \quad \text{or} \quad p_{\mathrm{T}} \longrightarrow p_{\mathrm{T}}(1 + q p_{\mathrm{T}}\, \delta_{\mathrm{sagitta}})^{-1}.$$

$$p \longrightarrow p(1 + q p_{\mathrm{T}}\, \delta_{\mathrm{sagitta}})^{-1}.$$

- Charge-symmetric momentum bias (radial):

$$r \longrightarrow (1 + \epsilon_{\mathrm{radial}}(\phi, \eta))r.$$

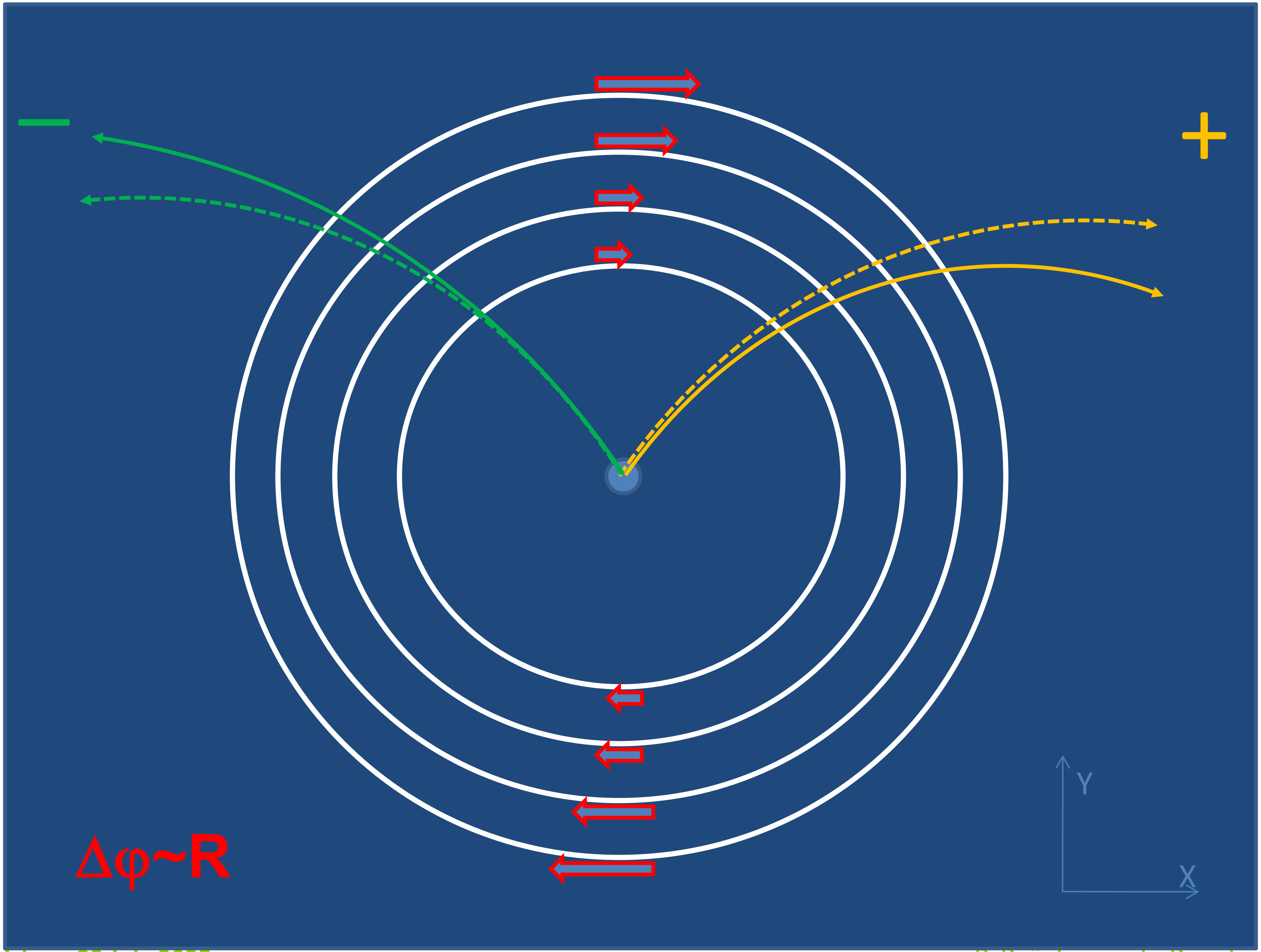
$$p_{\mathrm{T}} \longrightarrow p_{\mathrm{T}}(1 + 2\epsilon_{\mathrm{radial}}) \qquad \text{for small } \epsilon_{\mathrm{radial}}.$$

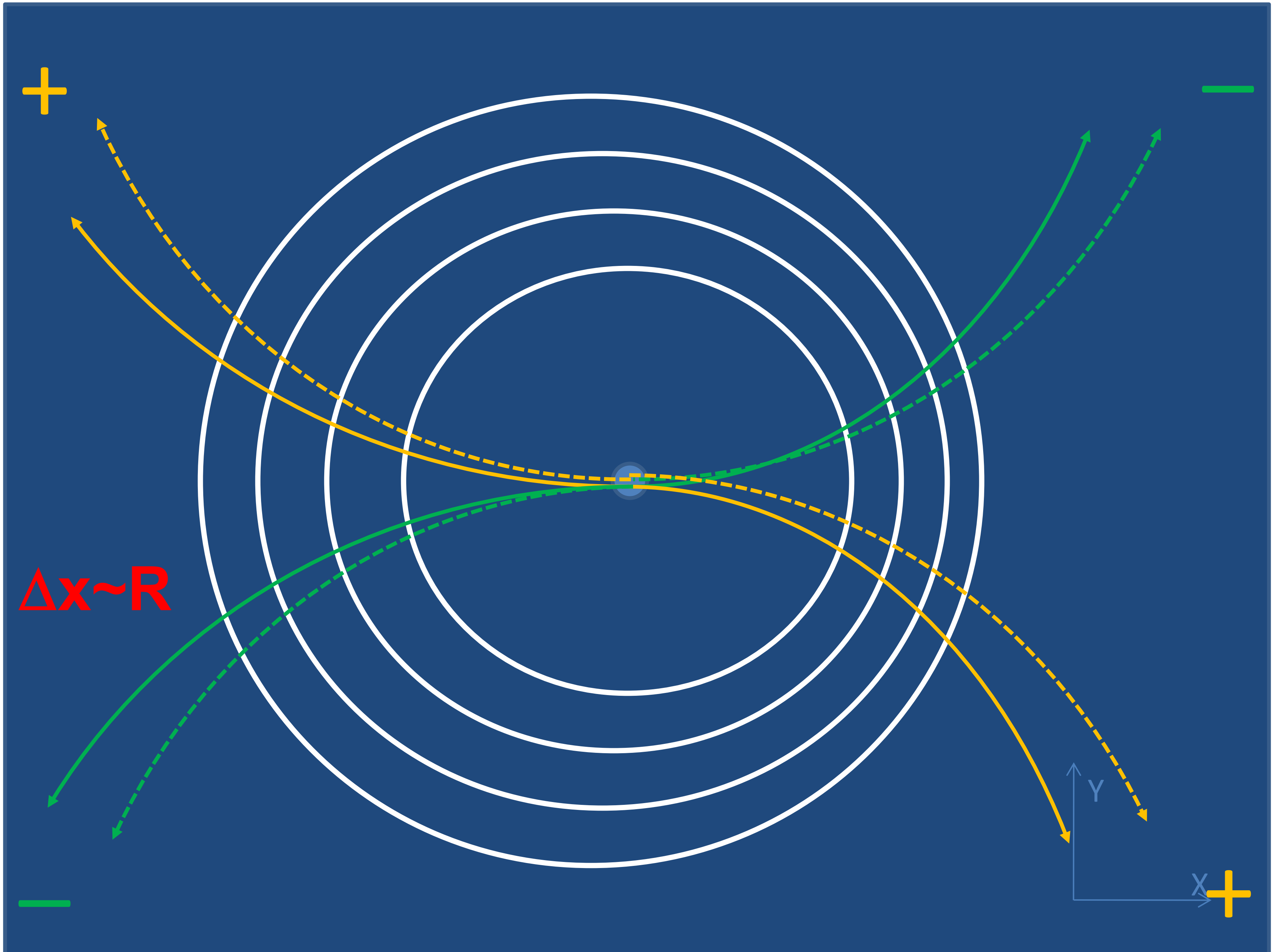
$$p_{\mathrm{z}} \longrightarrow p_{\mathrm{z}}(1 + \epsilon_{\mathrm{radial}}).$$

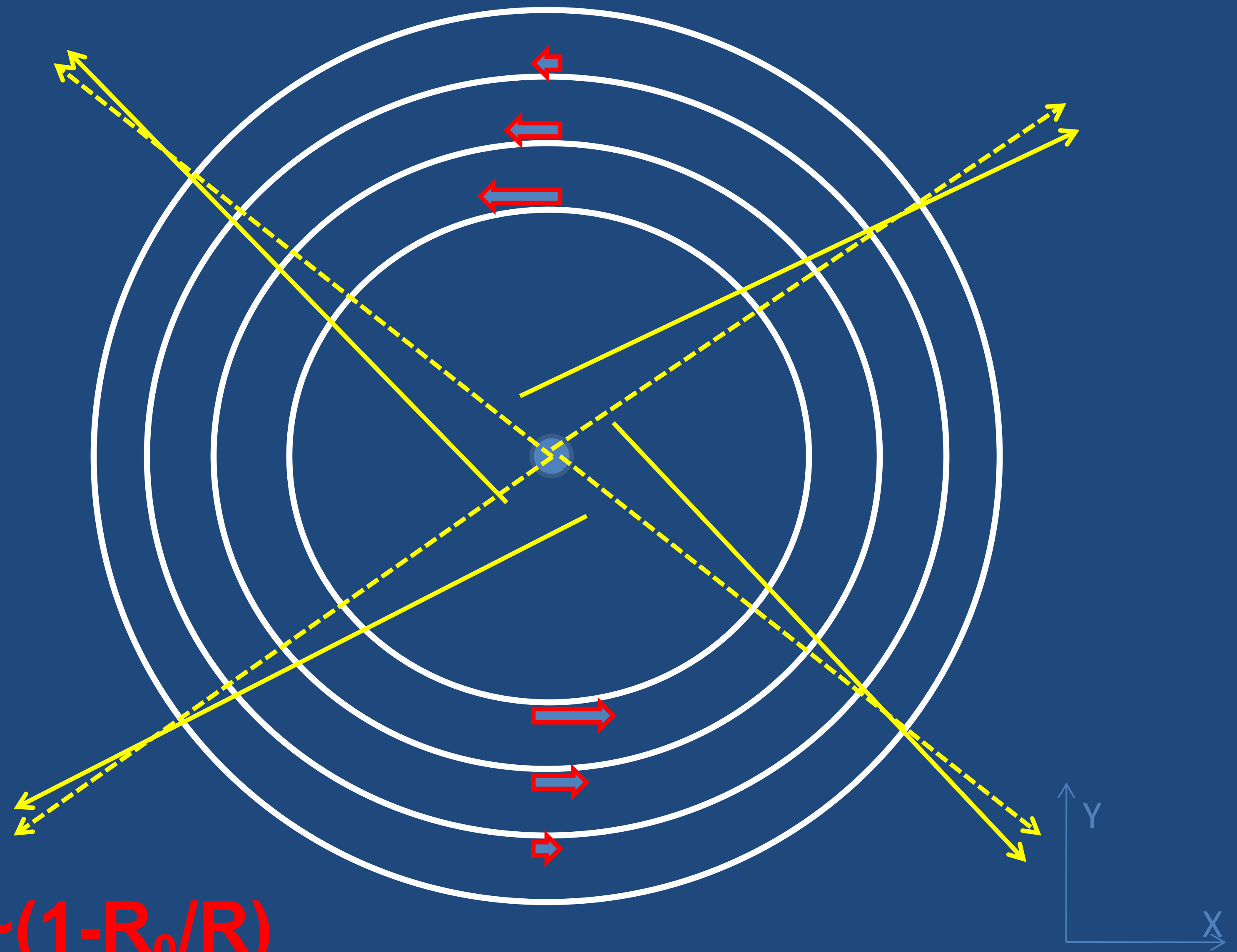
- Bias on the Impact Parameter (d0 or z0)

$$t \longrightarrow t + \delta d_0,$$

$$\Delta\phi[\mathrm{rad}] = \frac{\delta d_0}{r},$$

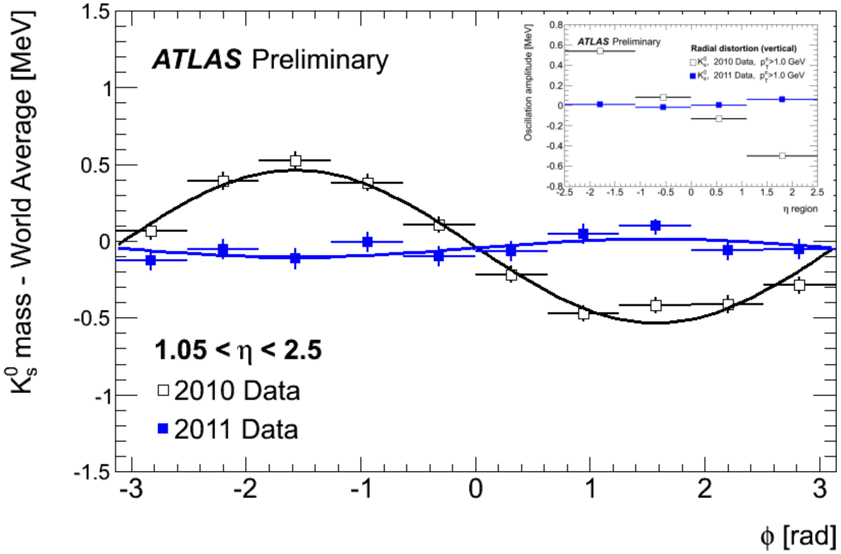

−
+
Δφ~R
Y
X

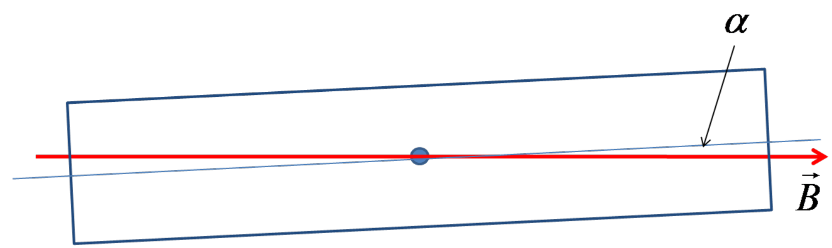

+
−
Δx~R
−
Y
X
+



$\Delta\varphi \sim (1 - R_0/R)$

Use physics signals to understand and constrain the tracker geometry?

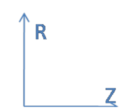




$$R = R_0 + \sin\varphi * \alpha * z, \qquad z = R_0 * \cot\theta$$

$$R = R_0(1 + \sin\varphi * \alpha * \cot\theta)$$

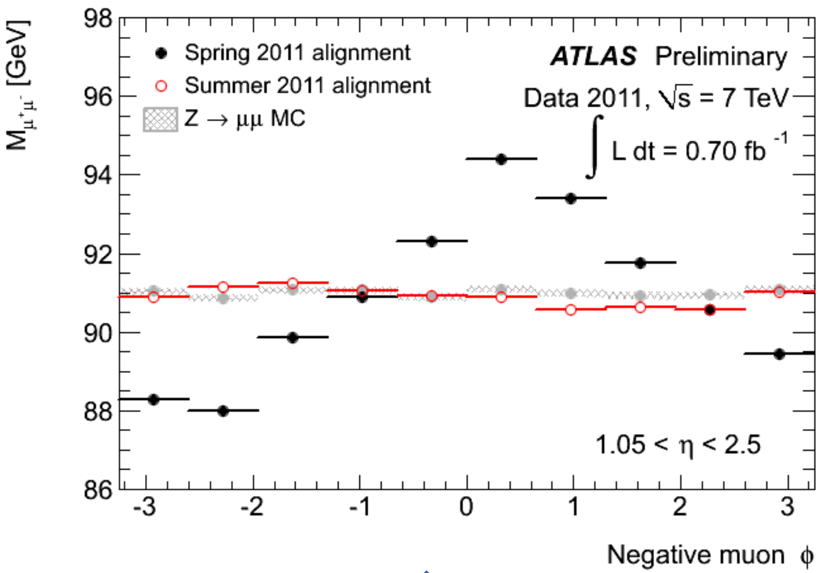
Which mimicks exactly the „radial" deformation proportional to cot(θ).

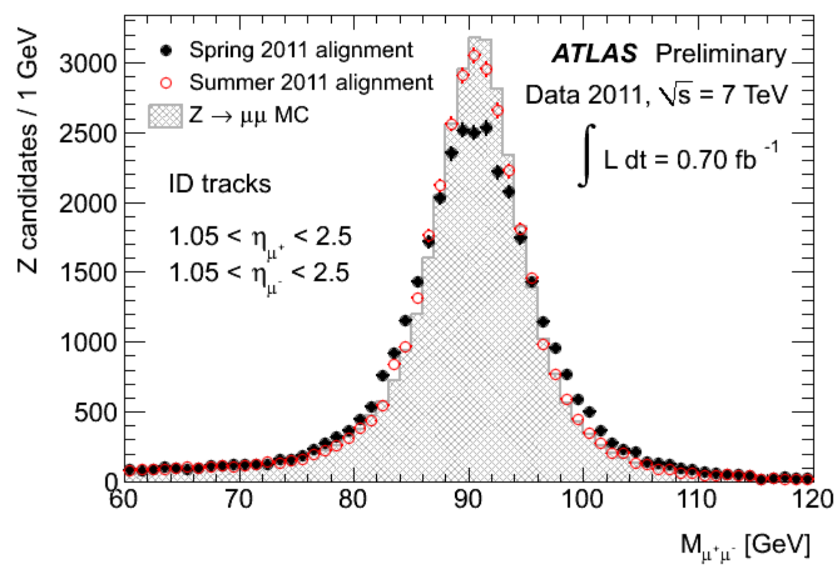


Rotate the B-field by +0.55 mrad around X

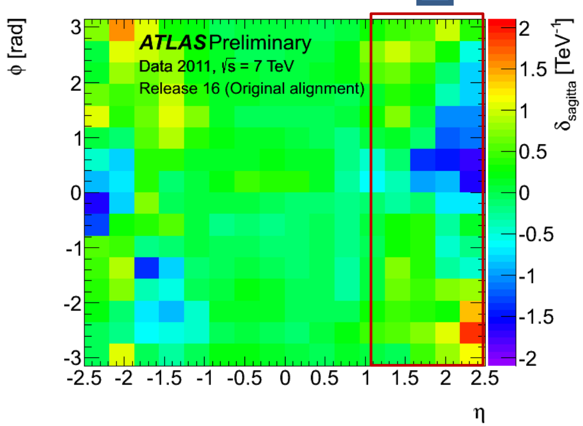
This way we measured and corrected the relative alignment of the tracker and the B-field to better than 0.1 mrad!

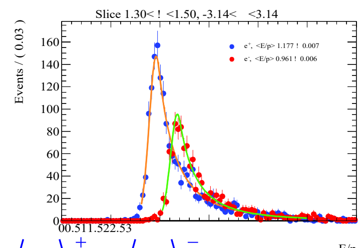
Decay of the Z boson to two muons



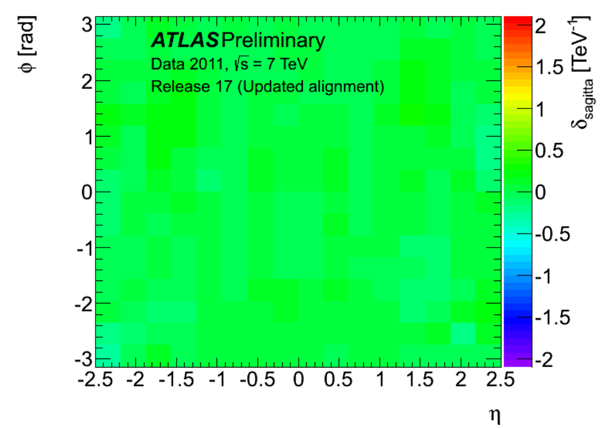


$$m^2_{\mu\mu} - m^2_Z \approx m^2_Z \left(p'^{+}_{\mathrm{T}}\, \delta_{\mathrm{sagitta}}(\eta^+,\phi^+) - p'^{-}_{\mathrm{T}}\, \delta_{\mathrm{sagitta}}(\eta^-,\phi^-)\right)$$





$$\left\langle \frac{E}{p} \right\rangle^{+} - \left\langle \frac{E}{p} \right\rangle^{-} = 2 < E_T > \delta$$



Alignment with constraints

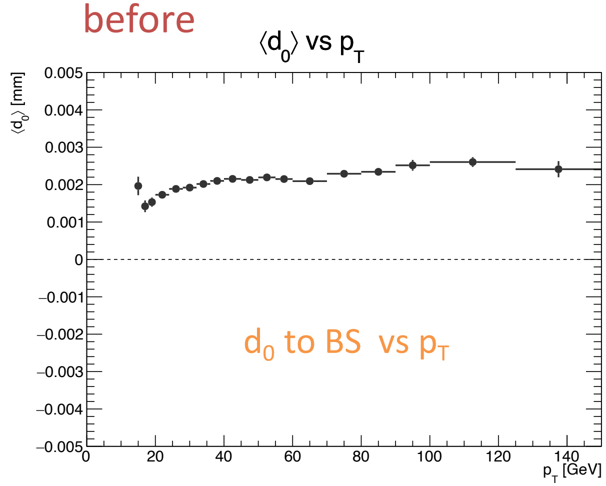
Alignment of 2022 ($\sqrt{s}$=13.6 TeV)

Standard procedure:

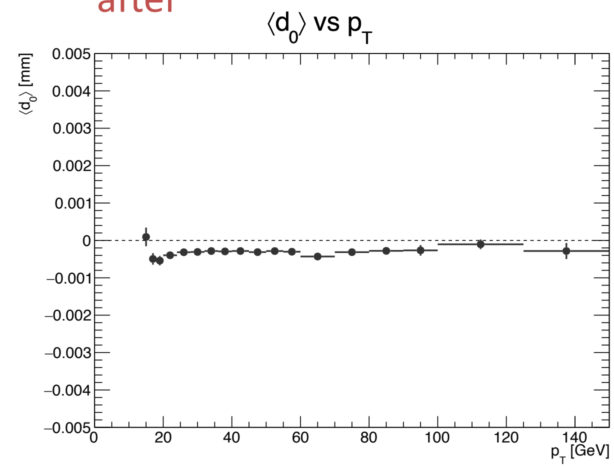
1. create (φ, η) maps of d0, z0, sagitta biases based on $Z \rightarrow \mu^+\mu^-$ decay events.
2. impose track parameter constraints (add pseudo-measurements) on subsequent alignment iteration.

d0 bias [mm]

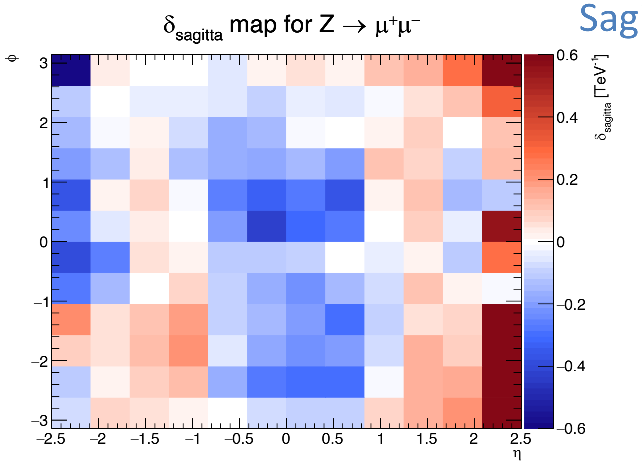
before

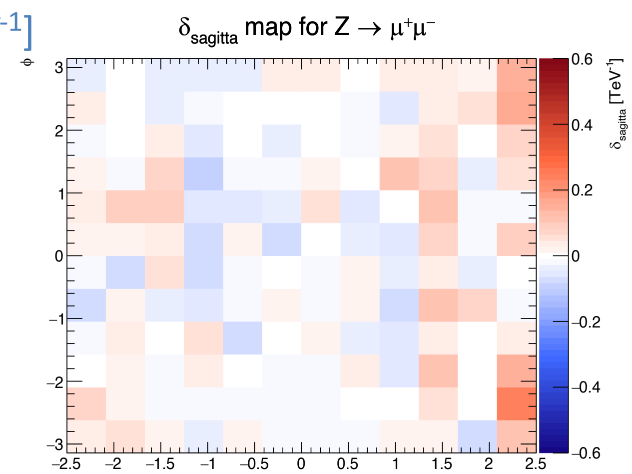


after



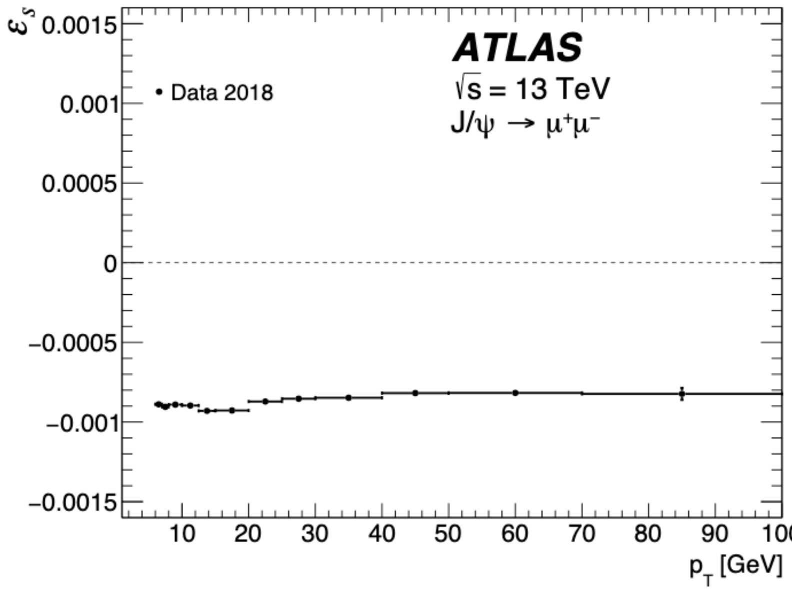
Sagitta dist. [TeV^{-1}]

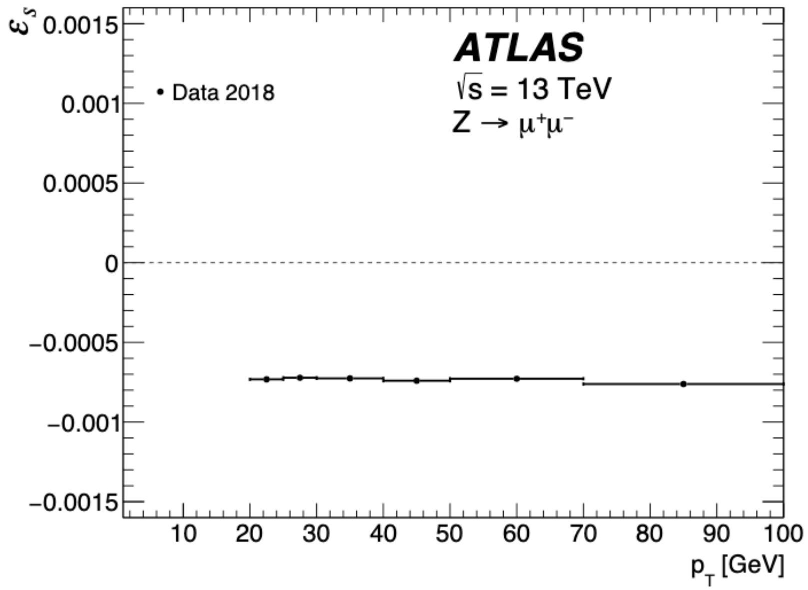




Alignment of 2018 ($\sqrt{s}$=13.0 TeV)

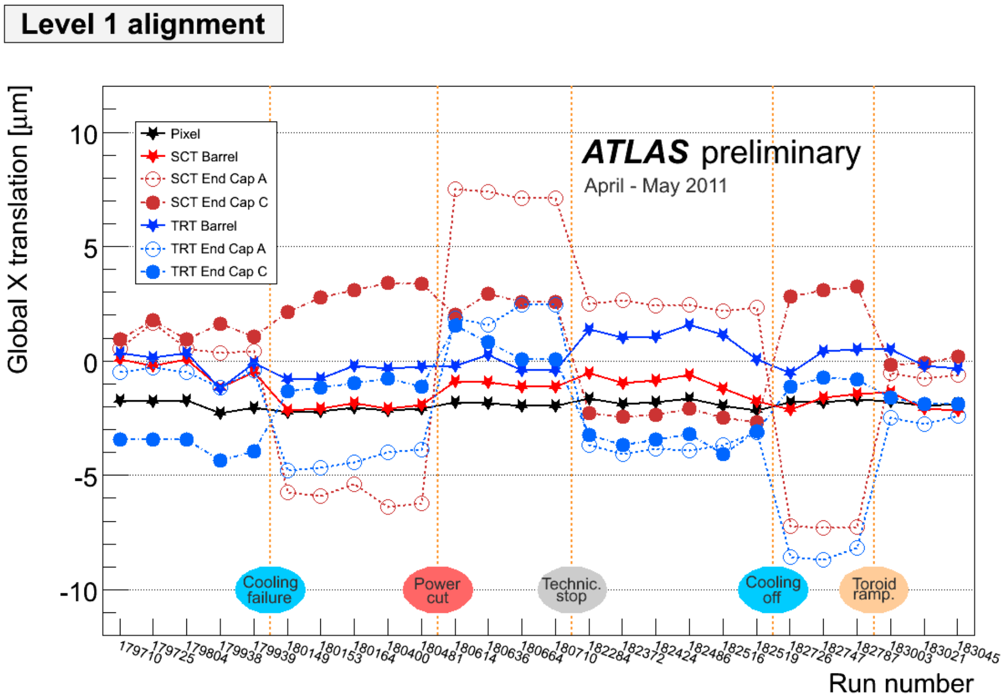
Study of the radial distortion (or B-field miscalibration)





The overall scale bias was found to be < 1×10^{-3}

Is our detector at rest?



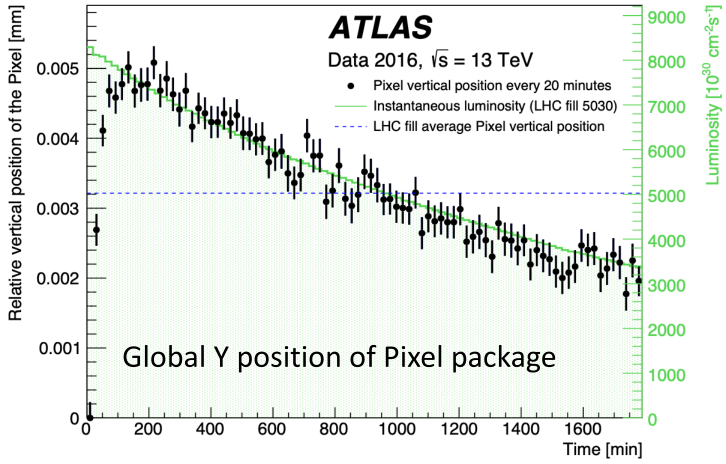
Unfortunately not!

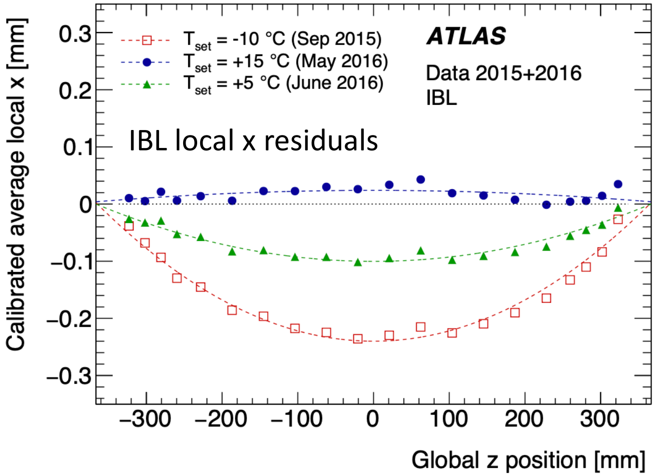
One needs to understand the extent of the motions and identify the affected DoF's - mostly L1.

- Cooling malfunction
- Magnet cycling
- Power cut
- etc.

Each „seismic event" requires re-evaluation of the geometry
A run-by-run L1 corrections run prior to the bulk reconstruction were introduced in Run 1.

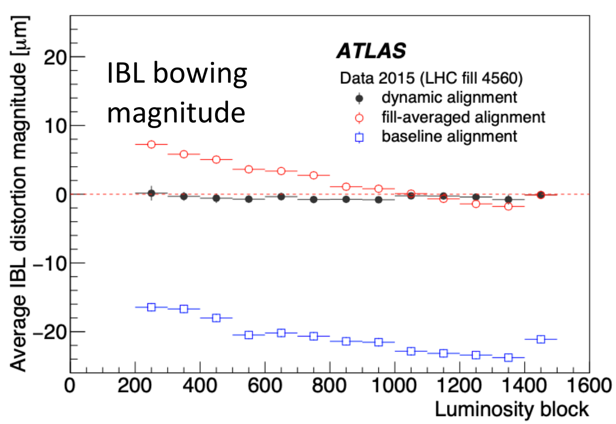
Fast movements and the Calibration Loop

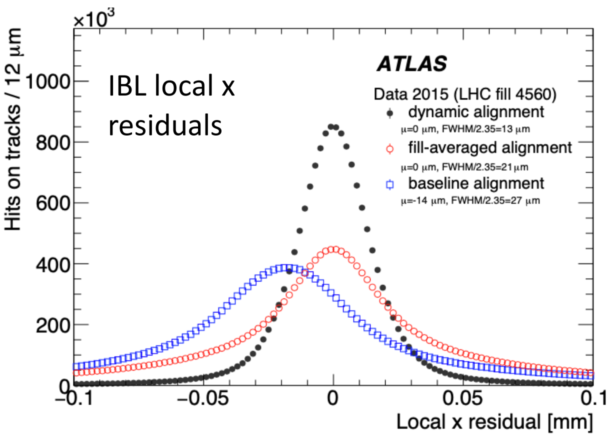




Dynamic L11/L16 per lumi-block interval (every 20/100 minutes) prior to the bulk reconstruction were introduced in Run 2.

- correct positions of all subsystems (SCT Barrel as reference) during a fill,
- correct the IBL stave bowing during a fill.





Conclusions

- Alignment is an indispensable element of modern experiments but potentially hazardous.
- In large systems Global χ^2 approach is preferred.
- Achieve good quality track fit is the easy part of the game (although involves solving linear systems with O(10-100)k parameters.
- Complete determination of the „true" geometry quasi impossible.
- Be pragmatic with systematics: try to measure relevant biases and eliminate them.

BONUS MATERIAL

Solving the alignment problem

In the most general case diagonalisation is the approach:

- Allows to control statistical significance of individual modes
- Full covariance matrix readily available
- Memory-demanding
- Time consuming ($\sim N^3$)
- Can be used for problems < O(10,000)

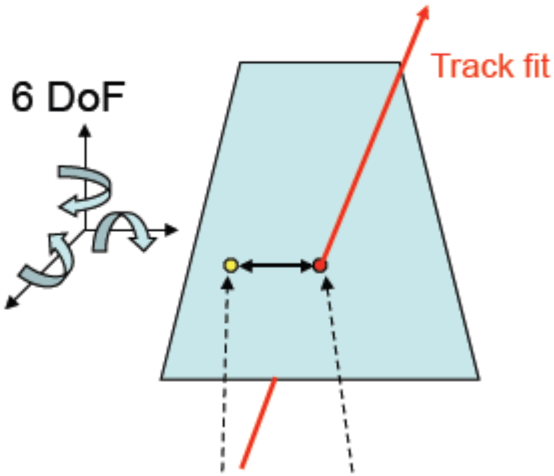
Sparse problems (usually the case) can be tackled using fast solvers (Gaussian elimination - MA27, Numerical norm minimization GMRES)):

- Much faster ($\leq N^2$) and less memory-demanding
- Require preconditioning to remove weak modes
- No direct error control – indirectly using soft-cuts

Local method does not present any numerical challenge (except for large number of iterations).

Basic track fit (linearization)

$$\pi = (d, z, \varphi, \vartheta, Q/p_T), \qquad \vec{e} \equiv \vec{e}(\pi)$$



$$\vec{r}_i \equiv \vec{m}_i - \vec{e}_i(\pi, a)$$

$$\chi^2 = \mathbf{r}^T \mathbf{V}^{-1} \mathbf{r}, \qquad r_i \equiv (\vec{e} - \vec{m}) \bullet \hat{k}$$

$$\mathbf{r}(\pi) = \mathbf{r_0} + \frac{\partial \mathbf{r}}{\partial \pi}(\pi - \pi_0)$$

linear exp. aro. seed

$$\frac{d\chi^2}{d\pi} = 0$$

minimization condition

$$\boxed{\pi - \pi_0 = \left(\frac{\partial \mathbf{r}}{\partial \pi}^T \mathbf{V}^{-1} \frac{\partial \mathbf{r}}{\partial \pi} \right)^{-1} \frac{\partial \mathbf{r}}{\partial \pi}^T \mathbf{V}^{-1} \mathbf{r}_0}$$

Basic track fit

$$\pi = (d, z, \varphi, \vartheta, Q/p_T)$$

CAUTION:

Lots of simplifications in the above. In reality at least two more effects need to be accounted for:

A) Multiple Coulomb Scattering (track deflects at every intersected material)

B) Energy Loss (particle losses energy for ionisation – changes momentum)

$$\boxed{\pi - \pi_0 = \left(\frac{\partial \mathbf{r}}{\partial \pi}^T \mathbf{V}^{-1} \frac{\partial \mathbf{r}}{\partial \pi} \right)^{-1} \frac{\partial \mathbf{r}}{\partial \pi}^T \mathbf{V}^{-1} \mathbf{r}_0}$$

Global χ^2 approach

$$\mathbf{r}(\pi) = \mathbf{r}_0 + \frac{\partial \mathbf{r}}{\partial \pi}(\pi - \pi_0) + \frac{\partial \mathbf{r}}{\partial a}(a - a_0)$$

$$\boxed{\frac{d\chi^2}{d\pi} = \frac{d\chi^2}{da} = 0}$$

Simultaneous fit of all tracks & the geometry N+n*k parameters!

Practically unfeasible !!! ☹

Way out!:

Explicit solution for alignment only:

$$\mathbf{r}(\pi) = \mathbf{r}_0 + \underbrace{\overbrace{\left(\frac{\partial \mathbf{r}}{\partial \pi}\frac{d\pi}{da} + \frac{\partial \mathbf{r}}{\partial a}\right)}^{\frac{d\mathbf{r}}{da}}}(a - a_0)$$

$$\boxed{a - a_0 = \left(\sum_{\text{tracks}} \frac{d\mathbf{r}}{da}^T \mathbf{V}^{-1} \frac{d\mathbf{r}}{da}\right)^{-1} \sum_{\text{tracks}} \frac{d\mathbf{r}}{da}^T \mathbf{V}^{-1} \mathbf{r}_0}$$

„Locality ansatz" in the Global χ^2 approach

Having fitted the track, one satisfies:

$$\boxed{0 \equiv \left(\frac{\partial \mathbf{r}}{\partial \pi}^T \mathbf{V}^{-1} \frac{\partial \mathbf{r}}{\partial \pi} \right)^{-1} \frac{\partial \mathbf{r}}{\partial \pi}^T \mathbf{V}^{-1} \mathbf{r}_0}$$

$$\frac{d\chi^2}{d\pi} = \frac{d\chi^2}{da} 0$$

Most importantly, residuals not explicitly dependent on alignment parameters drop out. Only „actual" residuals survive:

$$\boxed{a - a_0 = \left(\sum_{\text{tracks}} \frac{d\mathbf{r}}{da}^T \mathbf{V}^{-1} \frac{d\mathbf{r}}{da} \right)^{-1} \sum_{\text{tracks}} \frac{\partial \mathbf{r}}{\partial a}^T \mathbf{V}^{-1} \mathbf{r}_0}$$

Idea of the Local χ^2 approach

$$\mathbf{r}(\pi) = \mathbf{r_0} + \cancel{\frac{\partial \mathbf{r}}{\partial \pi}(\pi - \pi_0)} + \frac{\partial \mathbf{r}}{\partial a}(a - a_0) \qquad \frac{d\chi^2}{da} 0$$

Fit of alignment parameters ignoring the correlations via tracks.
Numerically a lot easier. Problem breaks down to local (n=6) equations . Requires multiple iterations over the full reconstruction !

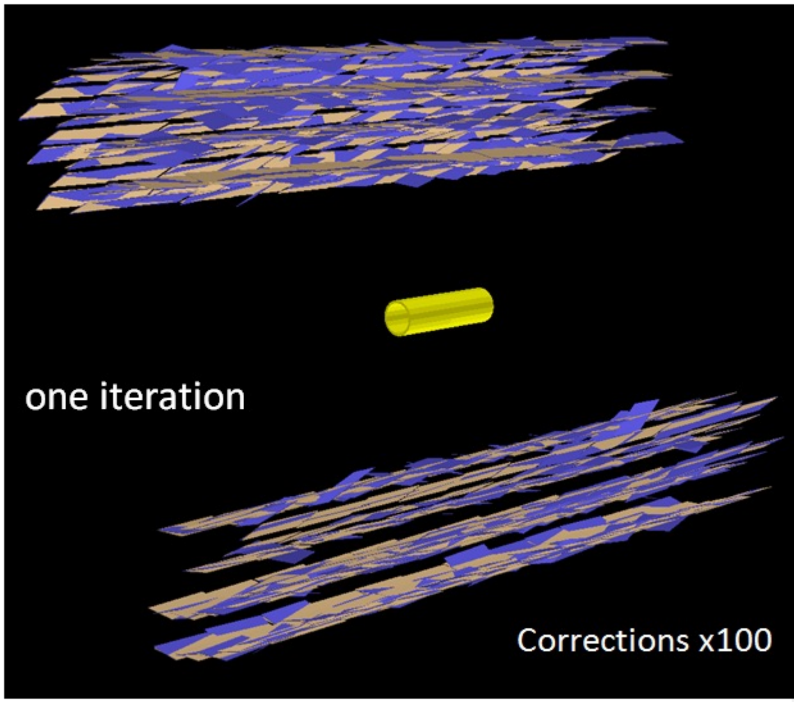
$$\mathbf{r}(\pi) = \mathbf{r_0} + \left(\cancel{\frac{\partial \mathbf{r}}{\partial \pi}\frac{d\pi}{da}} + \frac{\partial \mathbf{r}}{\partial a} \right)(a - a_0)$$

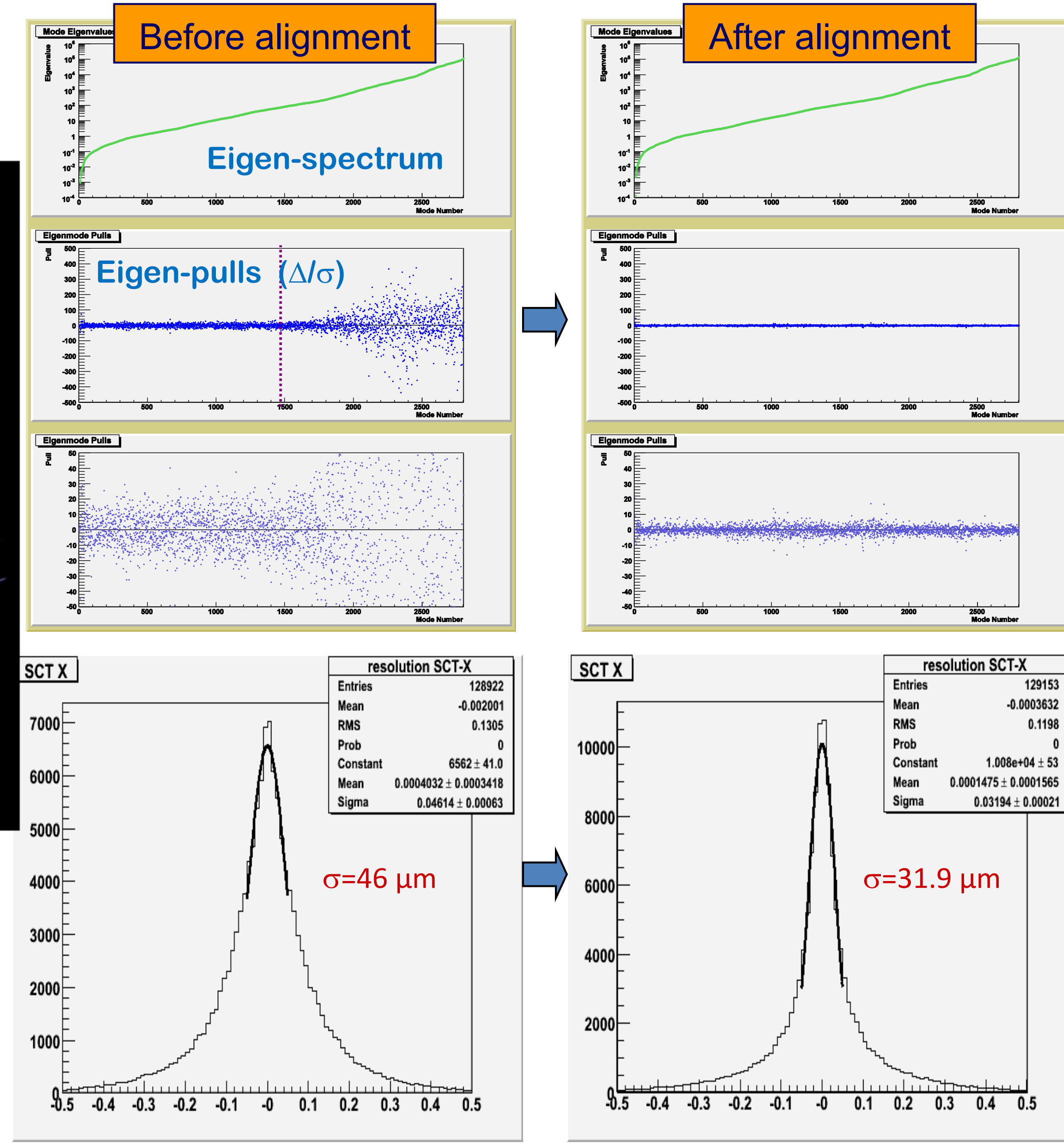
$$\boxed{a - a_0 = \left(\sum_{\text{tracks}} \frac{\partial \mathbf{r}}{\partial a}^T \mathbf{V}^{-1} \frac{\partial \mathbf{r}}{\partial a} \right)^{-1} \sum_{\text{tracks}} \frac{\partial \mathbf{r}}{\partial a}^T \mathbf{V}^{-1} \mathbf{r}_0}$$

Example: cosmic alignment with Global χ^2

2808 DoF's

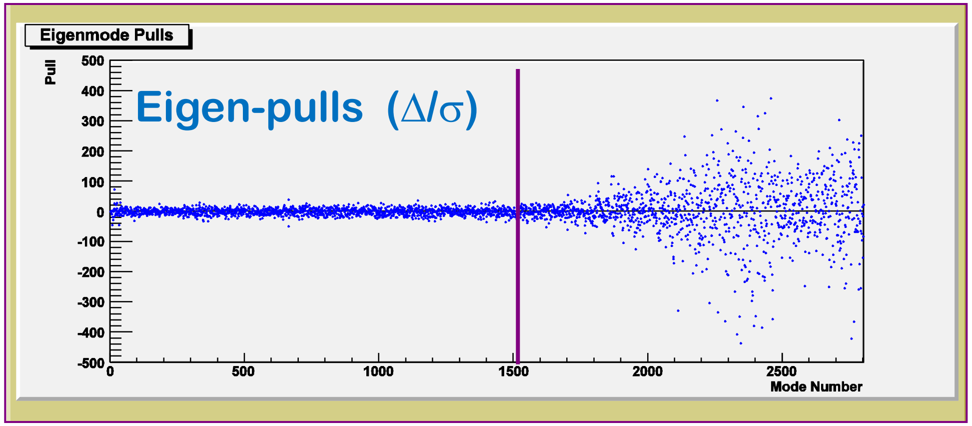
Corrections due to modes >1500



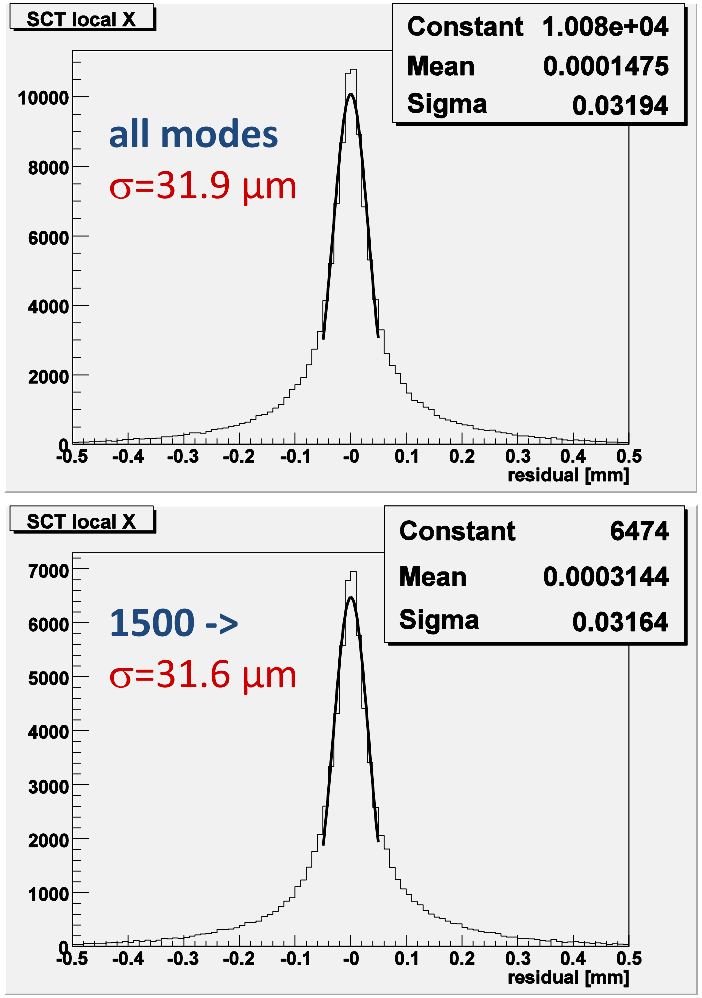


Example: cosmic alignment with Global χ^2

- **Possible trap: Do not try to exploit all apparent information:**



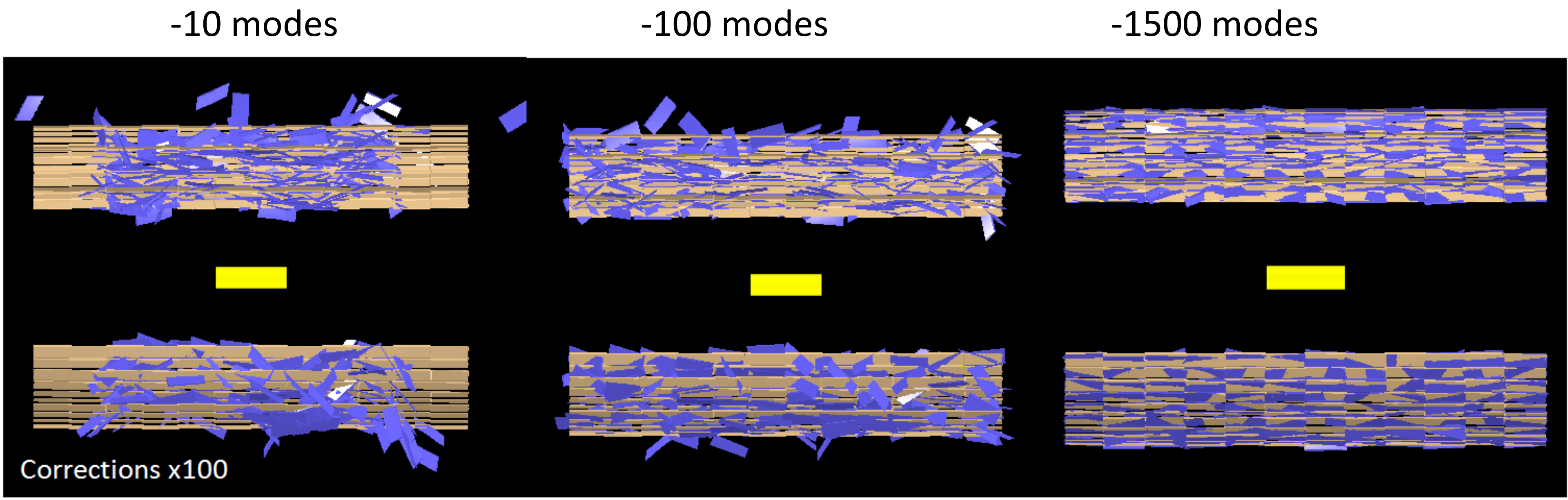
Residual distribution



- **Alignment quality the same for -1500!!!**

Example: cosmic alignment with Global χ^2

…but the resulting geometry is dramatically different!



I claim this one is physically justified!