Vertex reconstruction at STCF

Mingyu Yu   Xingtao Huang   Teng Li
Shandong University

Workshop of Track Reconstruction in Particle Physics ExPerments，Huizhou，7/23/2025

目 录 *Contents*

01 Motivation

part one

Motivation

➢**Proposal of the Super Tau-Charm Facility (STCF)**

- **BEPCII/BESIII is expected to run for another decade**

  Major advances in XYZ states, light-flavor exotics, and charm physics have brought Chinese particle physics to the forefront internationally.

- **Further exploration of hadron structure and new physics beyond the SM**

STCF

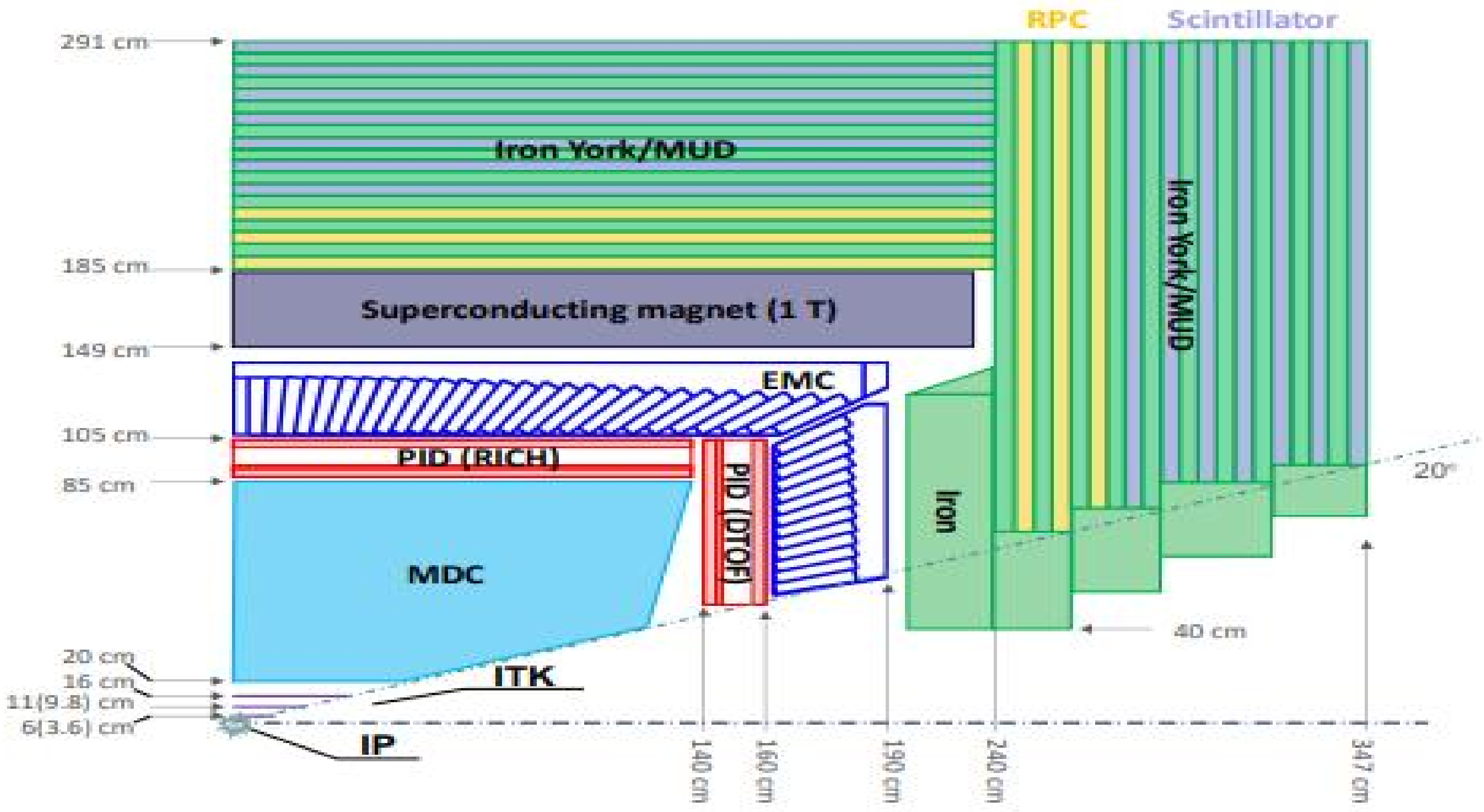
peak luminosity: $0.5 \times 10^{35} cm^{-2} s^{-1}$

center-of-mass energy: $2 \sim 7 Gev$

BesIII

peak luminosity: $1 \times 10^{33} cm^{-2} s^{-1}$

center-of-mass energy: $2 \sim 4.6 Gev$



SuperTau-Charm Facility, STCF

➢**STCF Data Characteristics**

- High-statistics + High-background + High-precision

Motivation

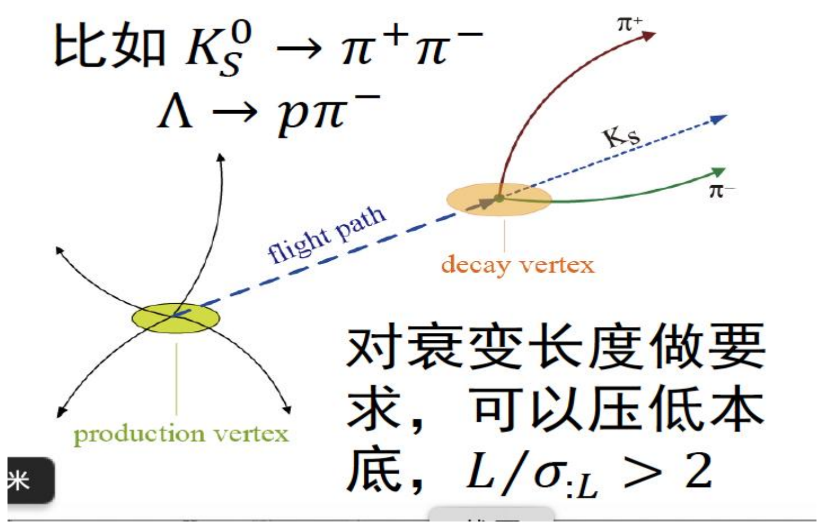
➤ **The vertex fitting tool at STCF is adapted from BESIII**

Event vertex reconstruction is a key component of STCF data analysis and a prerequisite for physics studies.

- **VertexFit:**

  Vertex constraint requires all final-state tracks come from a common point in 3D space

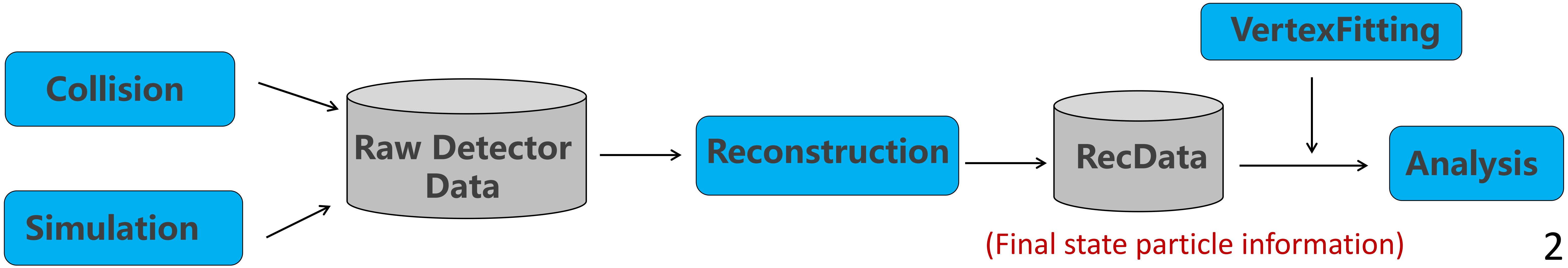
  ①Optimize track parameters
  ②Precise reconstruction of intermediate states
  ③Suppresses fake tracks in tracking



- **SecondVertexFit:**

  Precision Lifetime Measurement of Intermediate Particles

  ①optimizes intermediate particle parameters
  ②suppresses backgrounds



SHANDONG UNIVERSITY

Motivation

➢ Limitations of VertexFit

- Each decay vertex is fitted separately
- Neutral particles are not fully utilized — vertex fitting is not applicable to neutrals

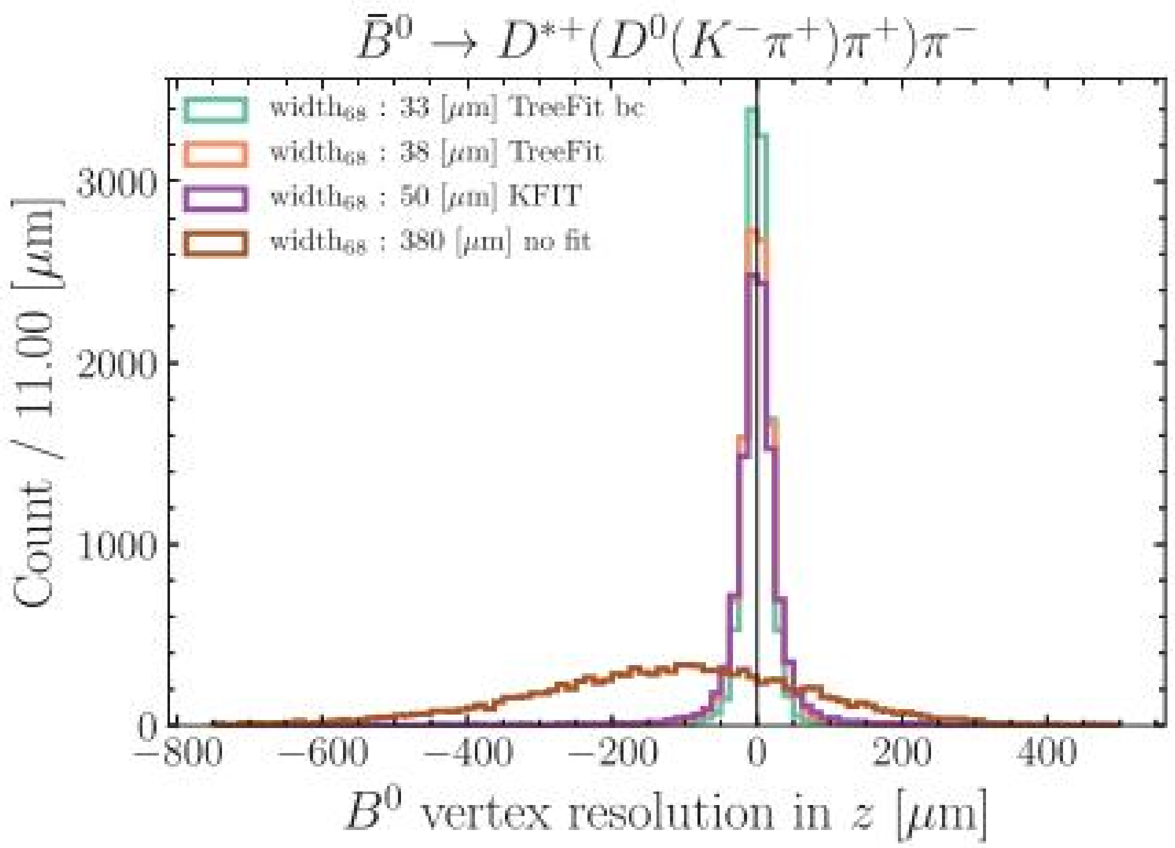
(Long-Lived Particles)$\Sigma^+ \to P\gamma$ —Decay Vertex: Not Available→ Affects the four-momentum of neutral particles in kinematic fitting

（assumed to originate from the interaction point）

➢ Global Vertex Fitting in Belle II

Developed based on the Kalman filter

- KFit：Fits each decay vertex separately
- TreeFit:Performs global vertex fitting across the decay chain



Global vertex fitting is planned for STCF! ！ ！

02 Fitting Method

part two

Lagrange Multiplier Method

Motivation

Fitting Method

performance

Summary

- **Vertex Fit & Global Vertex Fit — Core Objective**

  To find the optimal set of physical parameters

  - **Least Squares Method**

  Estimate parameters by minimizing the sum of squared differences between measured values and theoretical predictions.

Cannot handle constrained cases

$$\chi^2 = (X - X_0)^T V^{-1} (X - X_0) = min$$

（X −measured values）

（X_0 − *theoretical predictions*)

- **Lagrange Multiplier Method (Constrained Least Squares)**

  Use Lagrange multipliers (λ) to convert a constrained least squares problem (n variables, m constraints) into an unconstrained one with (n + m) variables.

$$\chi^2 = (X - X_0)^T V_0^{-1} (X - X_0) + 2\lambda^T h(X) = min$$

（V^{-1}: Covariance Matrix of Parameters)

（$h(X)$ − constraint equations)

Lagrange Multiplier Method

Motivation

Fitting Method

performance

Summary

➢ **State vector solution**

$$\chi^2 = (X - X_0)^T V_0^{-1} (X - X_0) + 2\lambda^T h(X) = min$$

- **Minimizing the Chi-Square Function：**

$$\frac{d\chi^2}{dX} = 0\ , \qquad \frac{d\chi^2}{d\lambda} = 0\ , \qquad X_0\ \text{（initial parameters）} -> X\text{(optimal solution)}$$

- **Nonlinear constraints complicate the solution; Taylor expansion is applied to linearize them**

$$h(X) = h(X_{\alpha-1}) + H(X - X_{\alpha-1})\ , \qquad H = -\frac{\partial h(X)}{\partial X}$$

$$\Big\downarrow \text{代入}$$

Iteration

$$\frac{d\chi^2}{dX} = (X_\alpha - X_{\alpha-1})^T V_{\alpha-1}^{-1} (X_\alpha - X_{\alpha-1}) + 2\lambda^T h(X_\alpha) = 0$$

- **Solution of the state vector**

$$X_\alpha = X_{\alpha-1} - V_0 H^T \lambda\ , \qquad \lambda = V_D h(X_\alpha)$$

$$V_D = (H V_0 H^T)^{-1}$$

Motivation

Fitting Method

performance

Summary

Lagrange Multiplier Method

➢Constraint equations for vertex fitting

Vertex constraint: All final-state charged tracks from the decay are constrained to pass through a common vertex in three-dimensional space.

$$\begin{cases} \text{r–φ Plane Constraint:} \quad p_{x_i}\Delta y_i - p_{y_i}\Delta x_i - \frac{a_i}{2}(\Delta x_i^2 + \Delta y_i^2) = 0 \\ \text{3D Surface Constraint:} \quad \Delta z_i - \frac{p_{z_i}}{a_i} sin^{-1}[a_i(p_{x_i}\Delta x_i + p_{y_i}\Delta y_i)/p_{T_i}^2] = 0 \end{cases}$$

(i:The i-th charged track)

（$\Delta x_i = x_x - x_i \rightarrow$ Decay Vertex - Particle Position ）

（Shared among all charged tracks）

Inserting into the χ² expression, a simultaneous fit is applied to all tracks.

Reconstruct the decay vertex and momentum of the intermediate particle

➢Constraint equations for second vertex fitting

- for neutral particle：

$$\begin{cases} x_p - x_d + \frac{p_x}{m} c\tau = 0; \\ y_p - y_d + \frac{p_y}{m} c\tau = 0; \\ z_p - z_d + \frac{p_z}{m} c\tau = 0; \end{cases}$$

- for charged particle:

$$\begin{cases} x_p - x_d + \frac{p_x}{a} sin(ac\tau/m) + \frac{p_y}{a}(1 - cos(ac\tau/m))=0; \\ y_p - y_d + \frac{p_y}{a} sin(ac\tau/m) + \frac{p_x}{a}(1 - cos(ac\tau/m))=0; \\ z_p - z_d + \frac{p_z}{m} c\tau = 0; \end{cases}$$

Extracting cτ, (related to the decay length)

（Subscriptp:production vertex； Subscriptd:decay vertex）

Kalman filter

➢ Belle II Global Vertex Fitting — based on the Kalman filter

Recursive estimation of particle states under uncertainties and constraints, with optimal parameters and error matrix output.。

$$\chi^2 = (X_\alpha - X_{\alpha-1})^T C_{\alpha-1}^{-1}(X_\alpha - X_{\alpha-1}) + (m - h(X_\alpha))^T V_m^{-1}(m - h(X_\alpha)) = min$$

（$C_{\alpha-1}^{-1}$: Covariance Matrix of Parameters) （V^{-1}: Measurement covariance matrix) （$m - h(X_\alpha)$: Residual)

Matrix inversion is limited to the constraint dimension， simplify calculation

➢ State vector solution

$$\frac{d\chi^2}{dX} = 0$$

Linear approximation： $h(X) = h(X_{\alpha-1}) + H(X - X_{\alpha-1})$

Iteration

$$\downarrow$$

$$(X_\alpha - X_{\alpha-1})^T C_{\alpha-1}^{-1}(X_\alpha - X_{\alpha-1}) + (m - h(X_\alpha))^T V_m^{-1}(m - h(X_\alpha)) = 0$$

- **Solution of the state vector**

$$X_\alpha = X_{\alpha-1} - K_\alpha r_\alpha$$

（$K_\alpha = C_{\alpha-1} H R_\alpha^{-1}$ – Kalman gain matrix)
（$r_\alpha = m - h(X_\alpha)$ – Residual)
（$R_\alpha = V + H_\alpha^{\alpha-1} C_{\alpha-1} (H_\alpha^{\alpha-1})^T$ – residual covariance matrix)

Motivation

Fitting Method

performance

Summary

Kalman filter

➢ Global Vertex Fitting based on Kalman Filter

- **Global state vector:**

  includes all particles' information

  - Final-state particle: (p_x, p_y, p_z)
  - Intermediate particle
    - Resonance: (p_x, p_y, p_z, E)
    - Long-lived: $(\boxed{x, y, z, \theta,} p_x, p_y, p_z, E)$

Improve fitting performance degraded by poorly measured particles

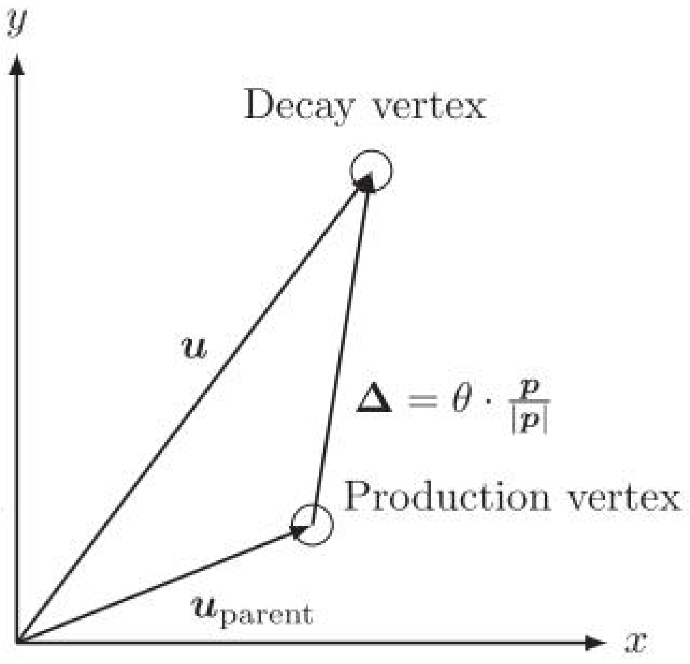
- **Decay Vertex:**

  Global vertex fitting uses a geometric method to compute the initial decay vertex:

  - If two or more charged tracks exist, the two with highest momentum are selected, and their point of closest approach in 3D space is calculated as the initial decay vertex.
  - This vertex is further refined during fitting with additional constraints.

- **decay lengthθ**

  The decay length is extracted using a vector triangle based on the decay geometry.



03 Performance

part three

VertexFit - SecondVerexFit

Motivation

Fitting Method

rformance

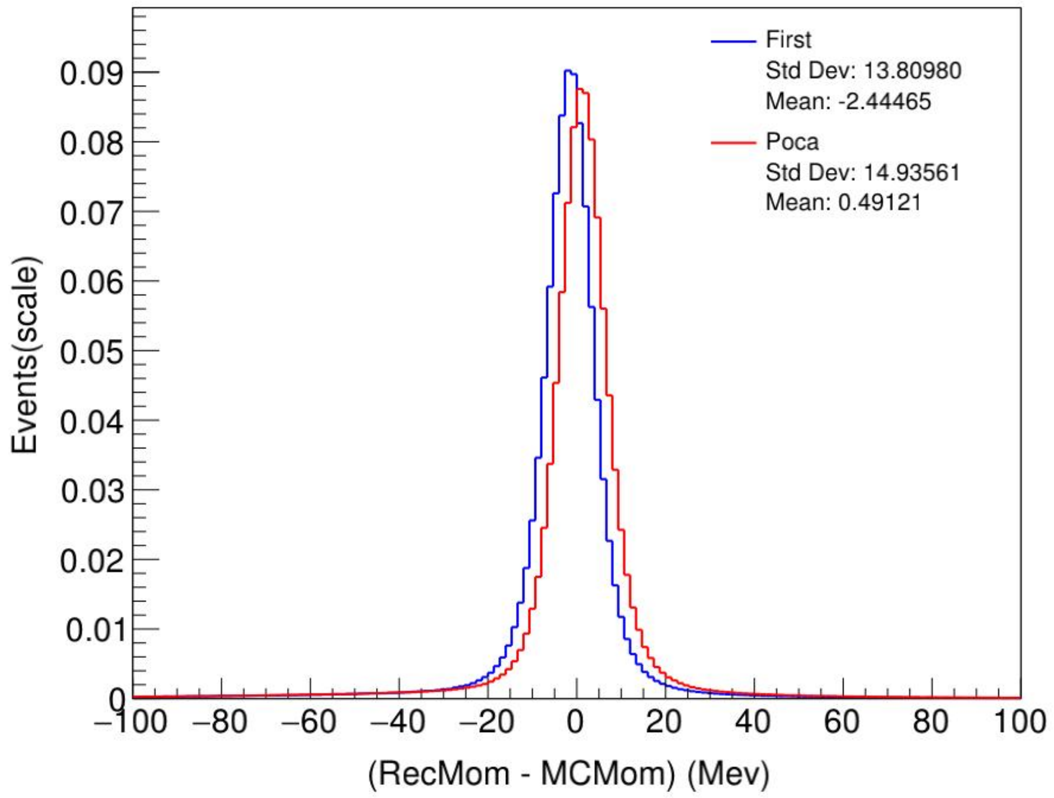
Summary

➢ **Evaluate Vertex Fitting Performance using charged track information from different spatial locations**

$$J/\psi \rightarrow \Lambda\bar{\Lambda} \rightarrow (p\pi^-)(\bar{p}\pi^+)$$

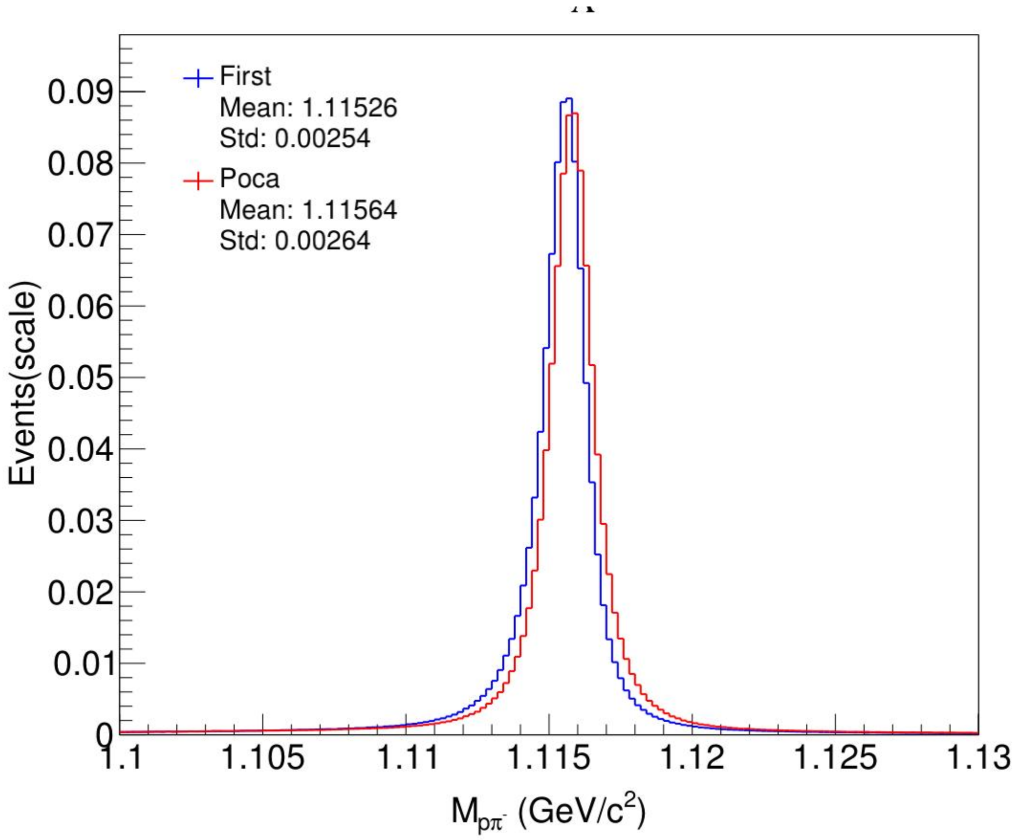
- **First**： Reconstructed position of the first hit of the charged track; material effects before this point are not considered.
- Poca： The point where the charged track is extrapolated to be closest to a reference point in the xy plane, with material effects during propagation taken into account.

➢ Residual of Reconstructed Momentum of Λ



Poca:
Taking more material effects
First:
Taking less material effects

➢ Invariant Mass Spectrum of Λ



VertexFit - SecondVerexFit

➢ **Evaluate Vertex Fitting Performance using charged track information from different spatial locations**

$$J/\psi \to \Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$$

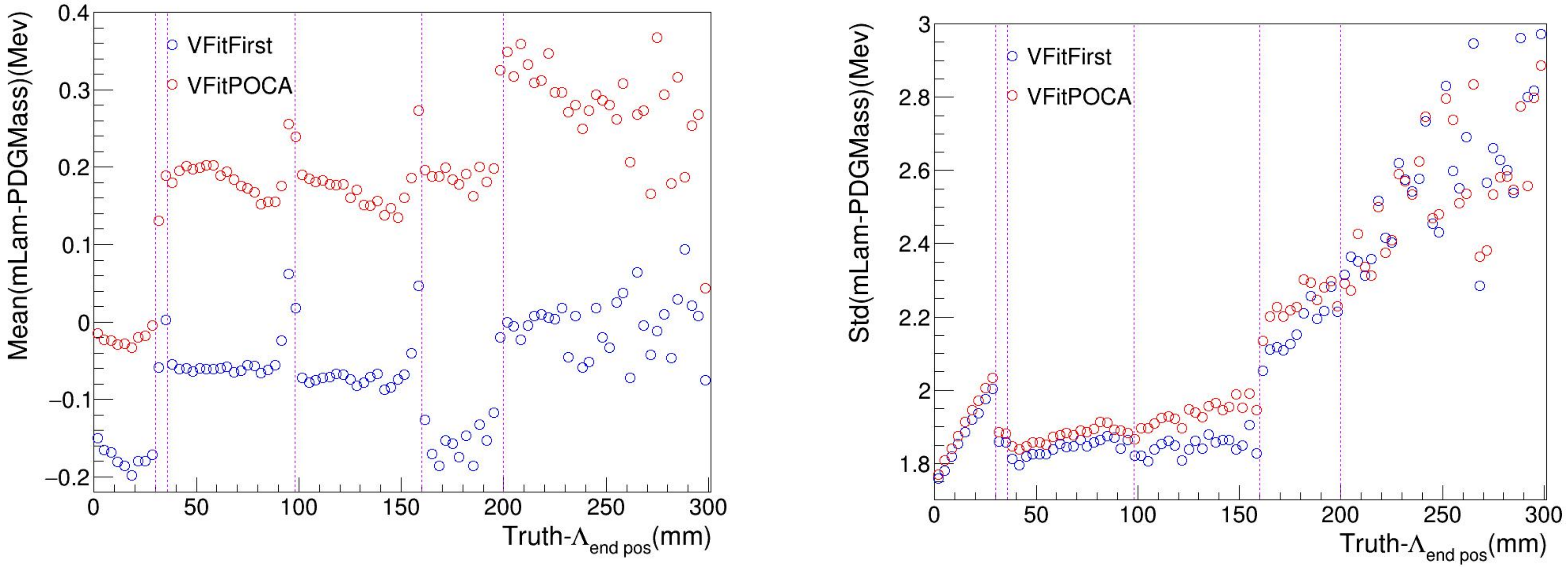


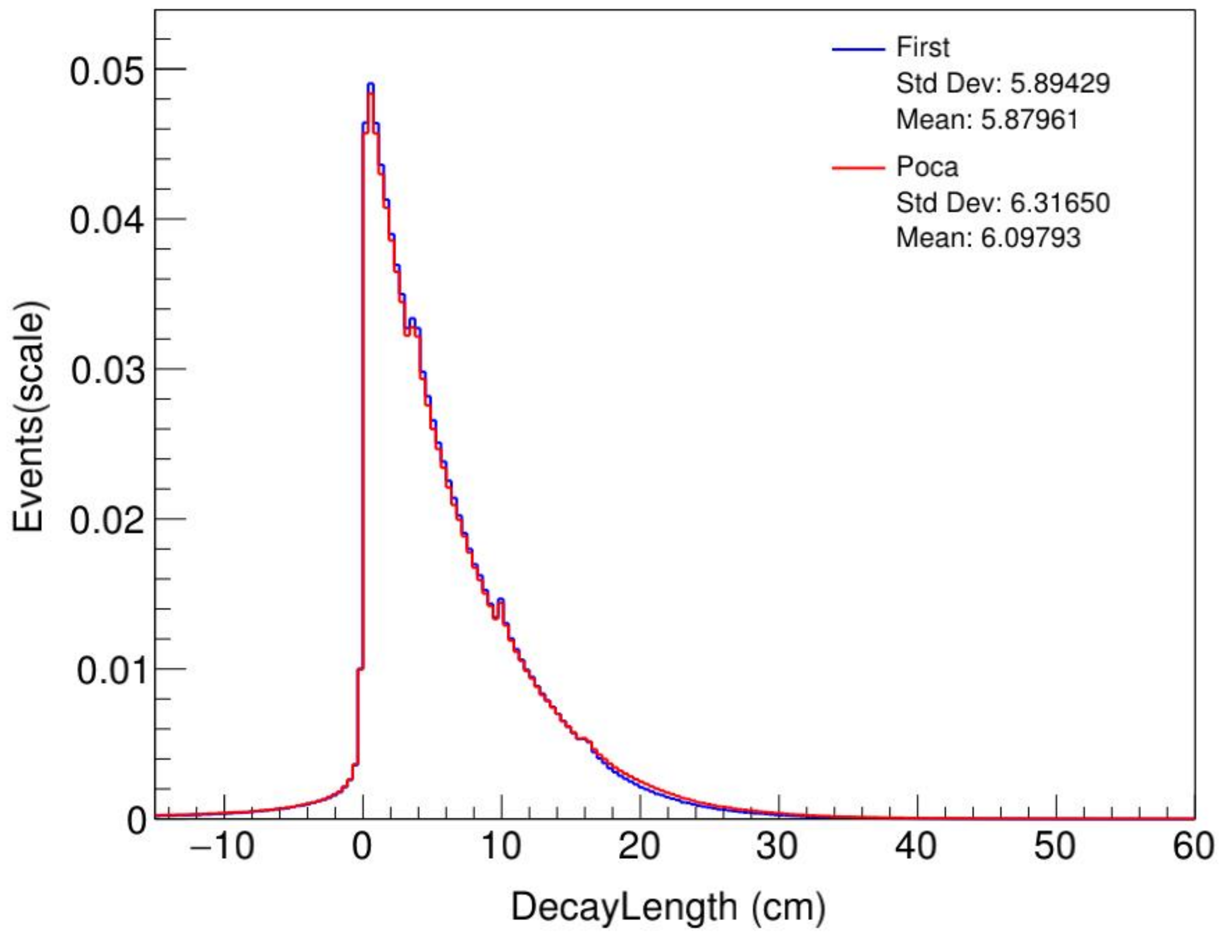
Figure. Invariant Mass Residuala vs Λ Decay Vertex in xy Plane

VertexFit - SecondVerexFit

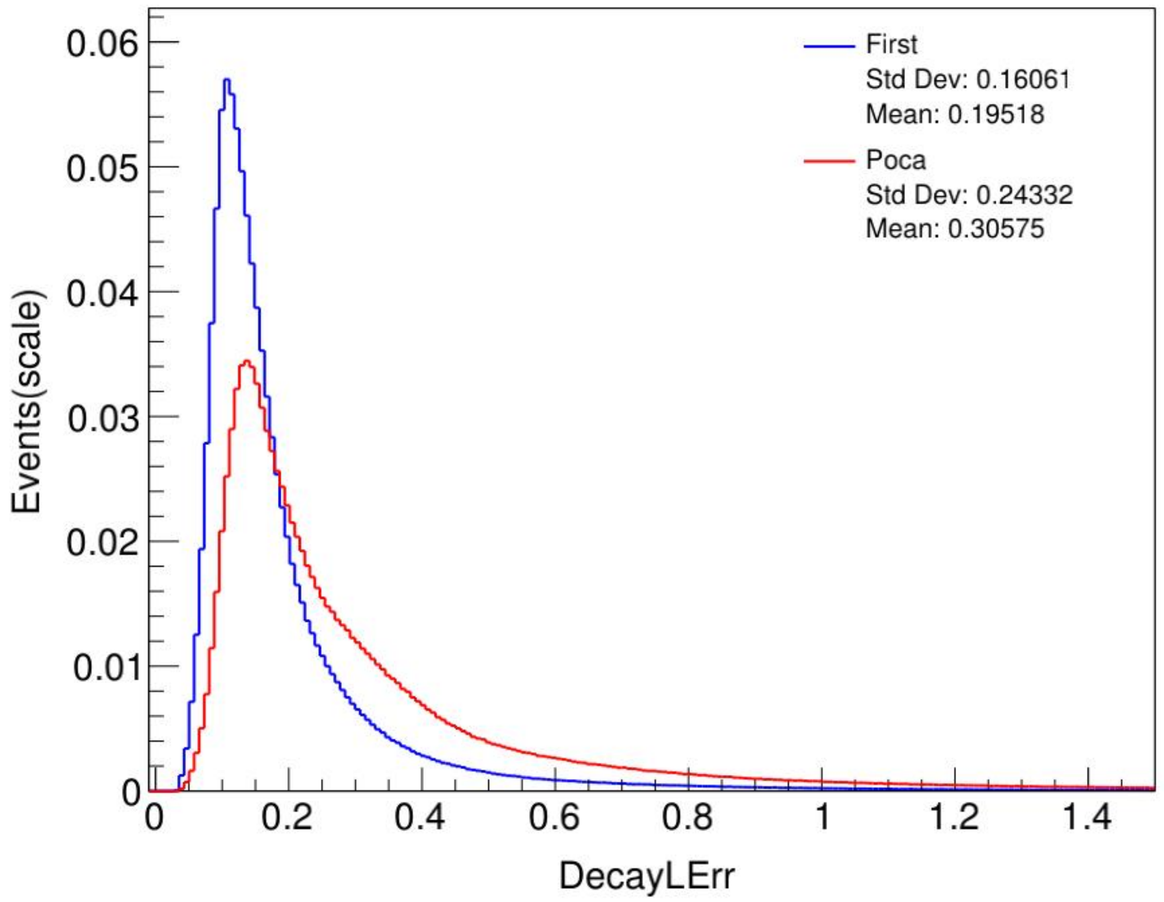
- **Evaluate Vertex Fitting Performance using charged track information from different spatial locations**

$$J/\psi \to \Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$$

- DecayLength



- Err of DecayLength



Motivation

Fitting Method

rformance

Summary

VertexFit - SecondVerexFit

➢ **Evaluate Vertex Fitting Performance using charged track information from different spatial locations**

$$J/\psi \rightarrow \Lambda\bar{\Lambda} \rightarrow (p\pi^-)(\bar{p}\pi^+)$$

Material effects primarily affect the reconstructed momentum of the particle!

The differences in the reconstructed helix parameters d_ρ and d_z are relatively small

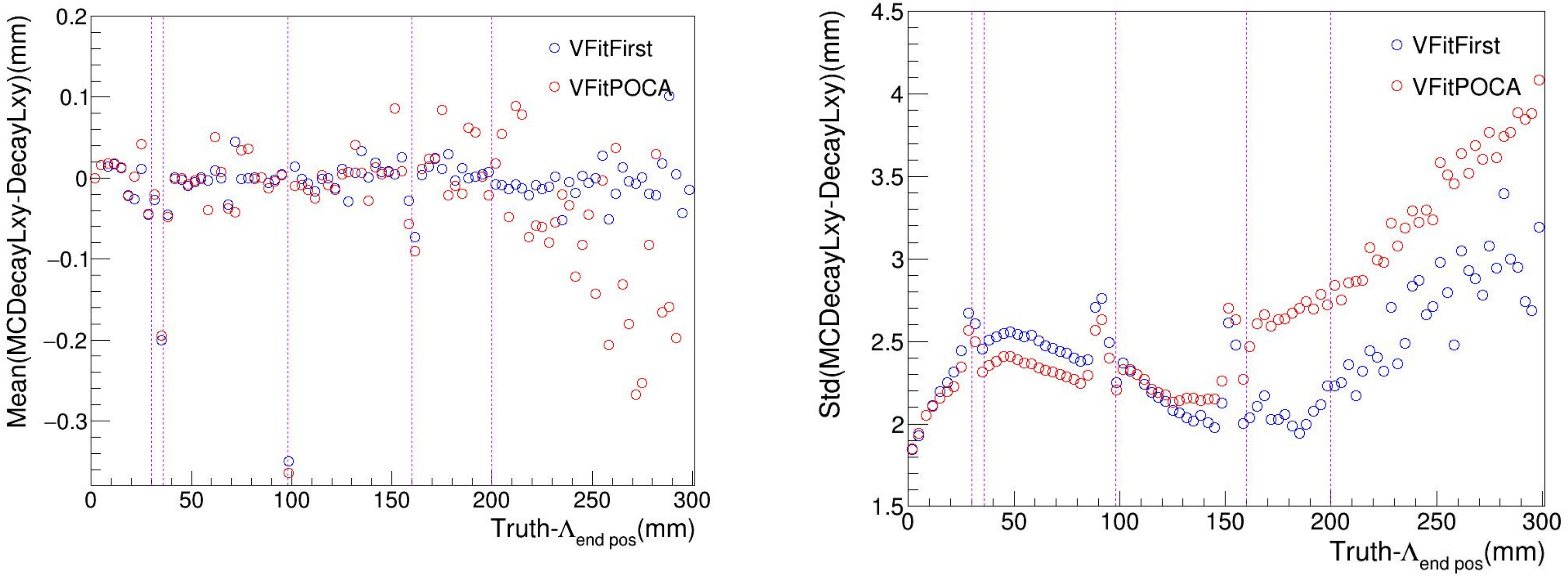


Figure. DecayLength Residuala vs Λ Decay Vertex in xy Plane

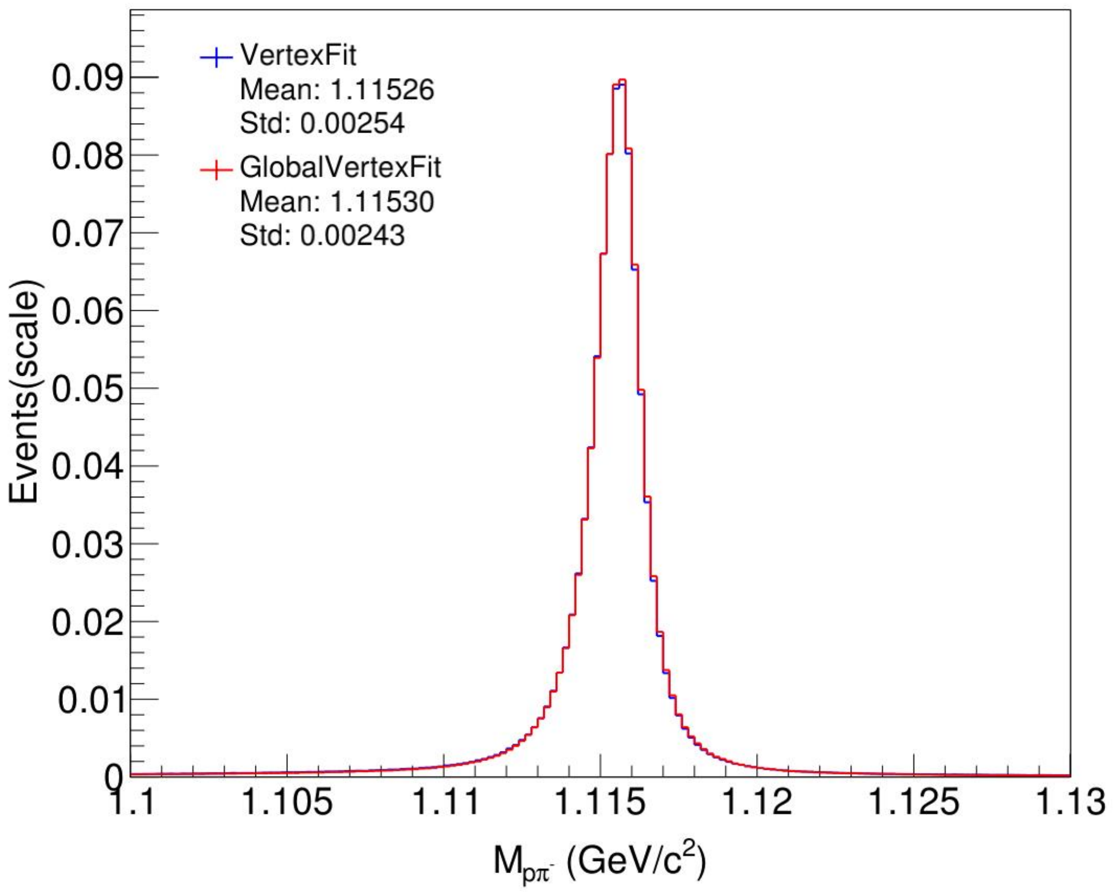
GlobalVertexFit

➢ **Evaluate Global Vertex Fitting performance based on different physics processes**

$$J/\psi \to \Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$$

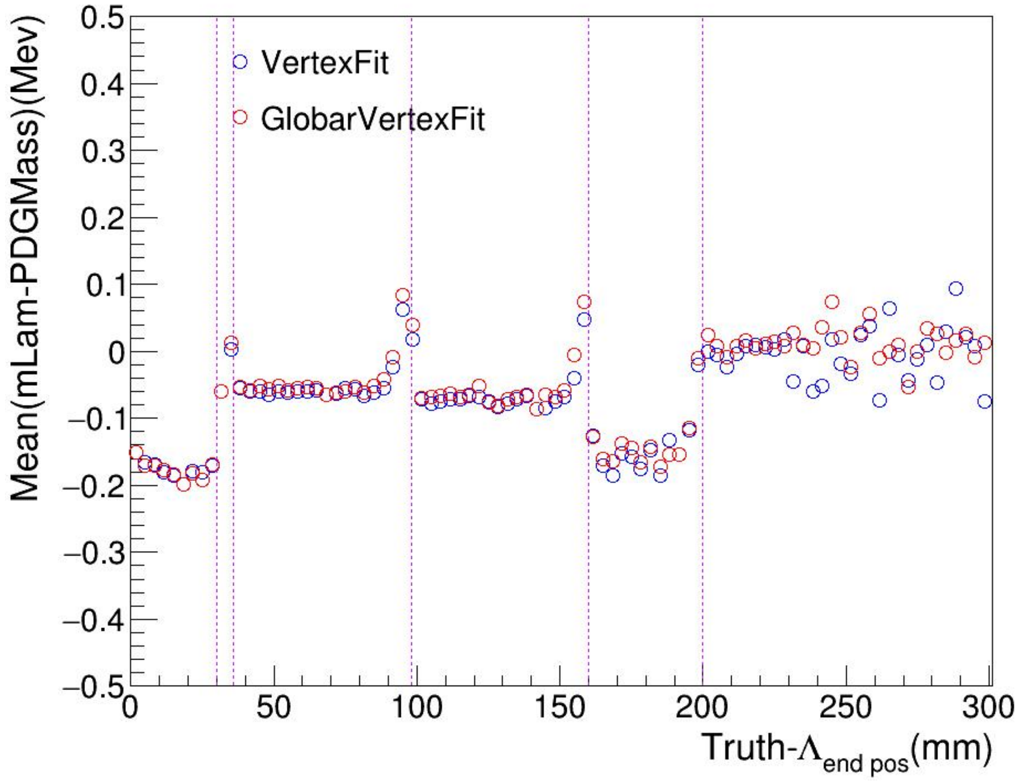
Using the First hit informaion

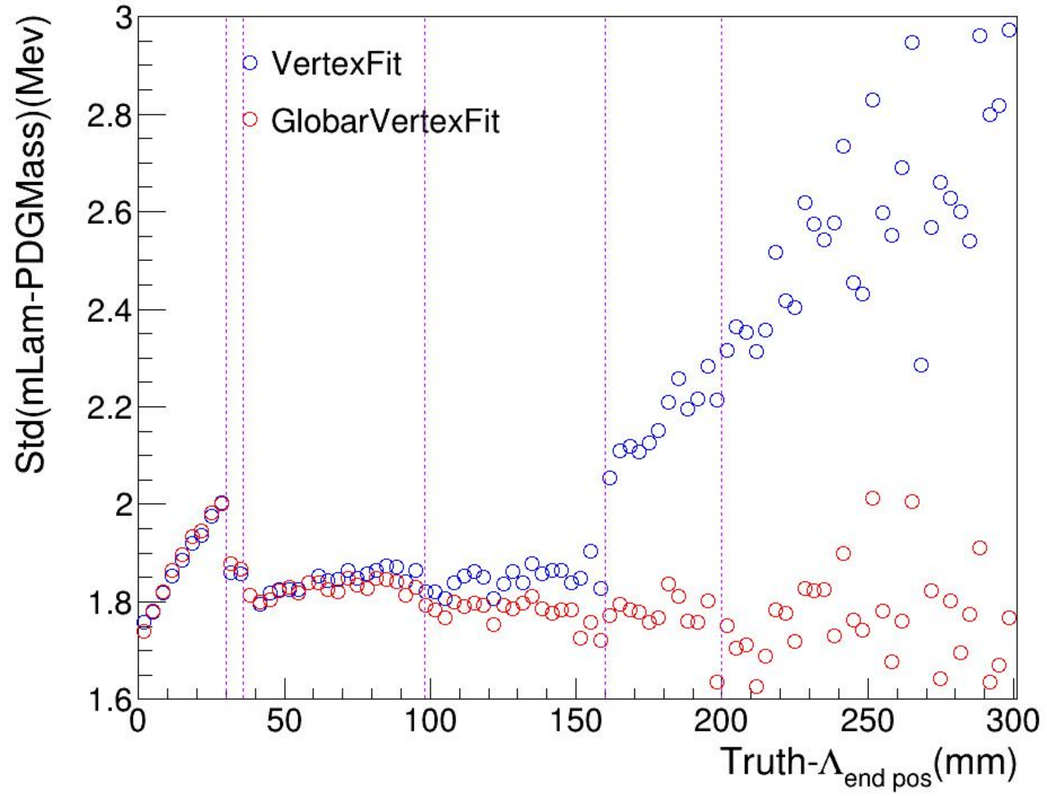
➢ Invariant Mass Spectrum of Λ



全局顶点拟合之后$\Lambda(\bar{\Lambda})$不变质量普分辨略高于VertexFit

➢ Invariant Mass Residuala vs Λ Decay Vertex in xy





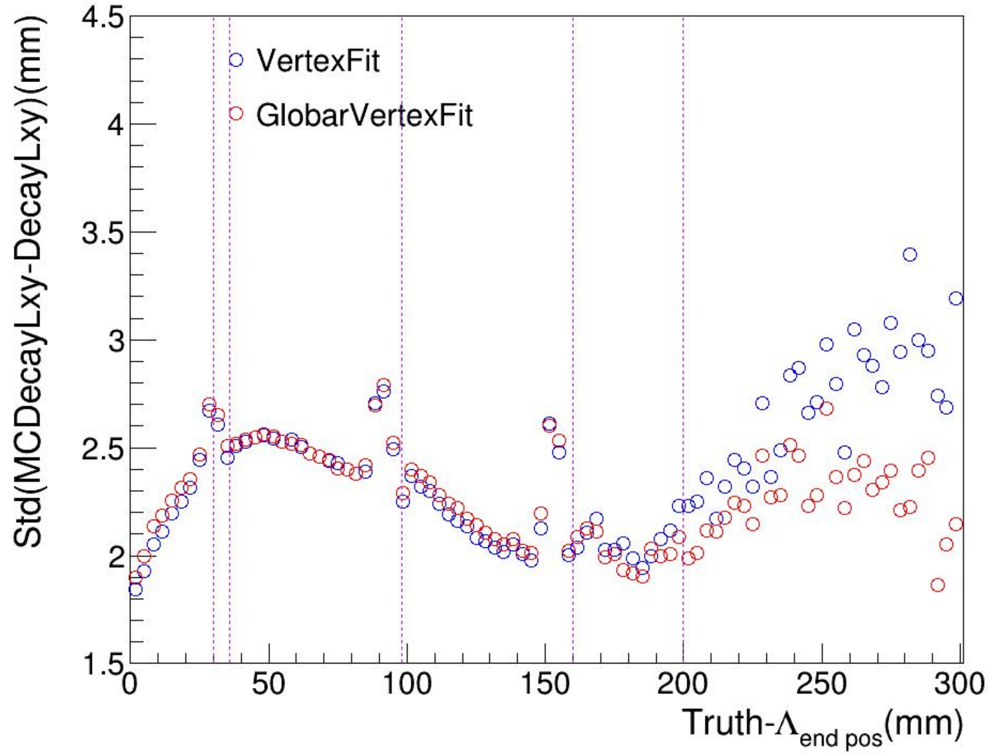
GlobalVertexFit

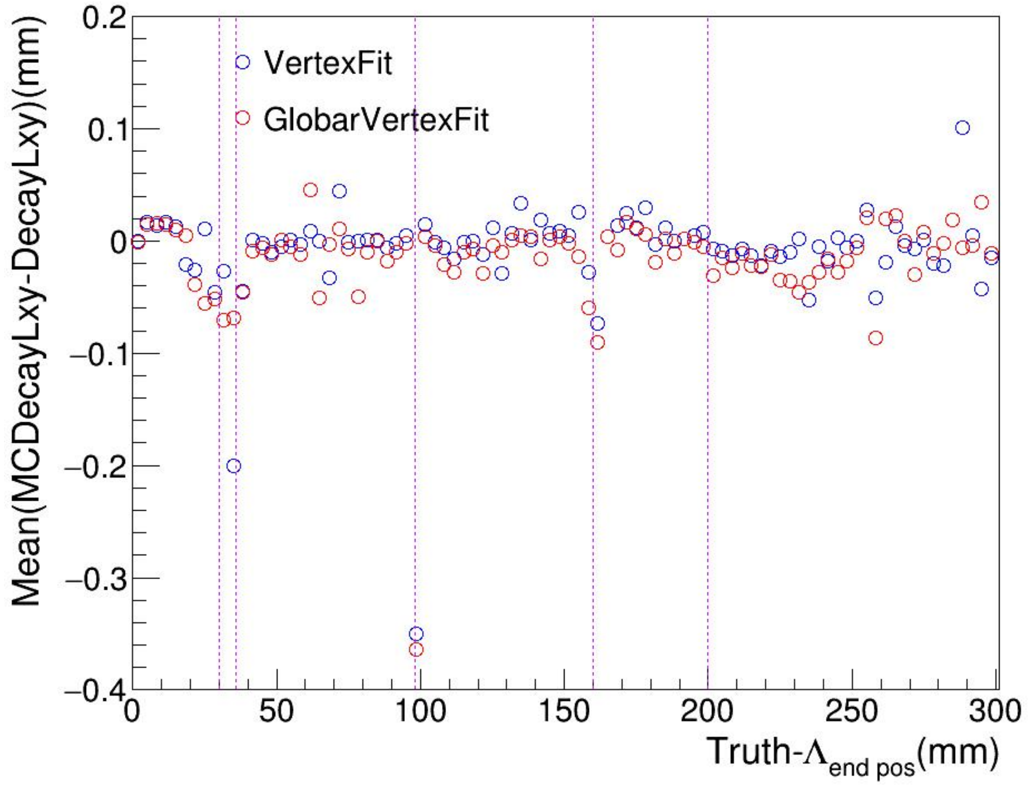
- **Evaluate Global Vertex Fitting performance based on different physics processes**

$$J/\psi \to \Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$$

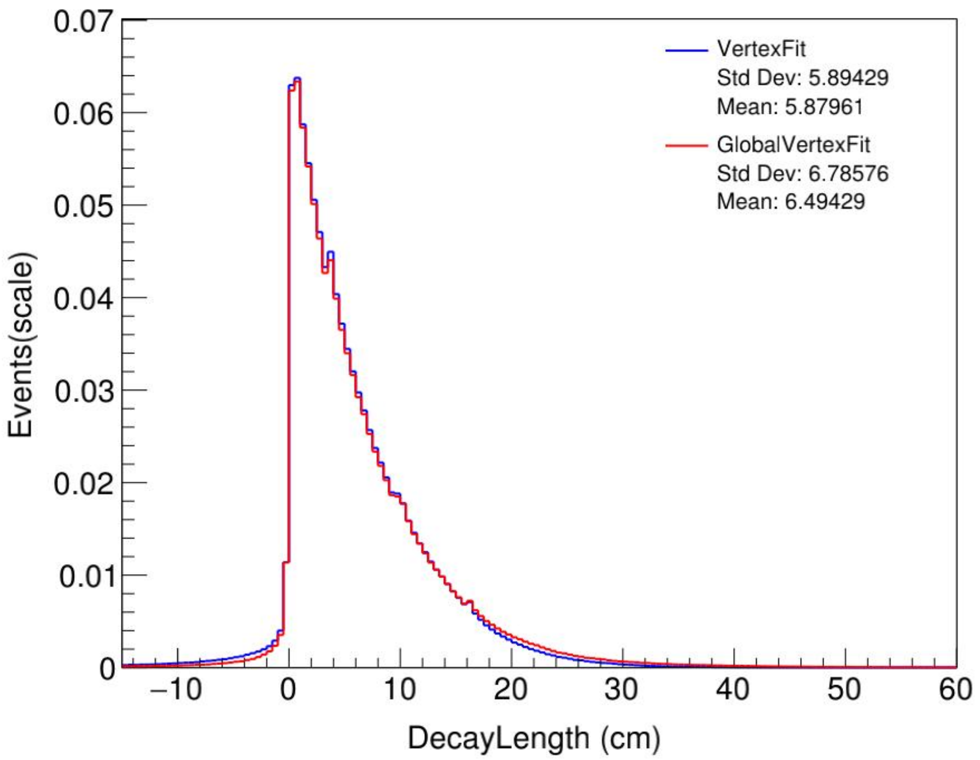
Using the First hit informaion

- DecayL Residuala vs Λ Decay Vertex in xy

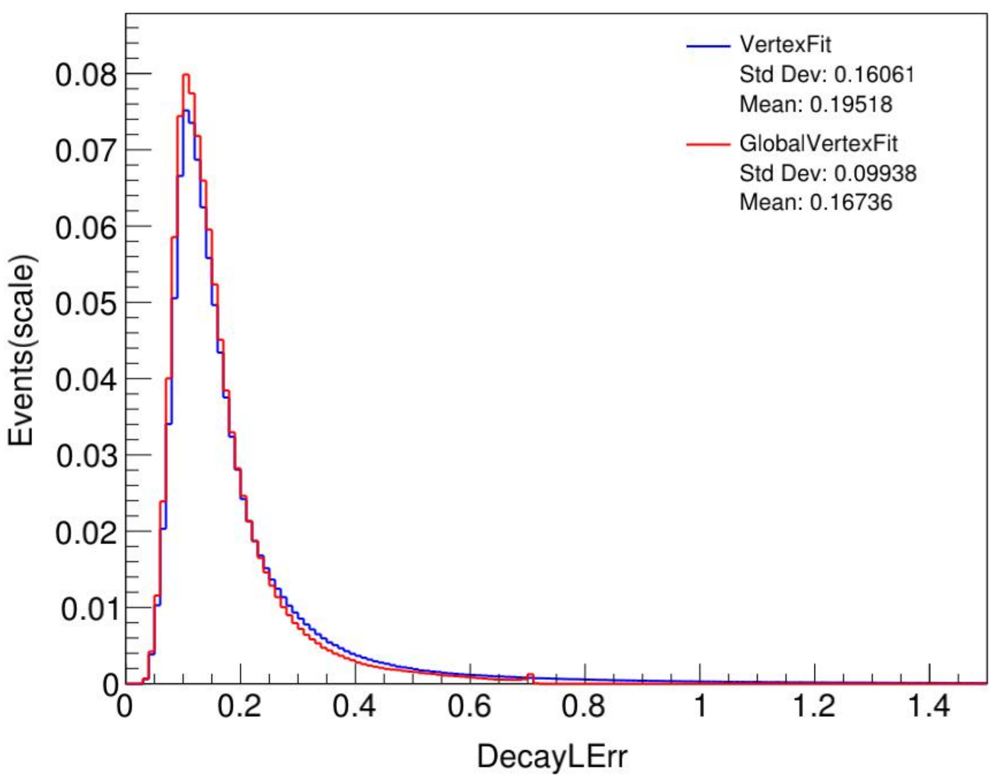




- DecayLength



- Err of DecayLength



SHANDONG UNIVERSITY

Motivation

Fitting Method

rformance

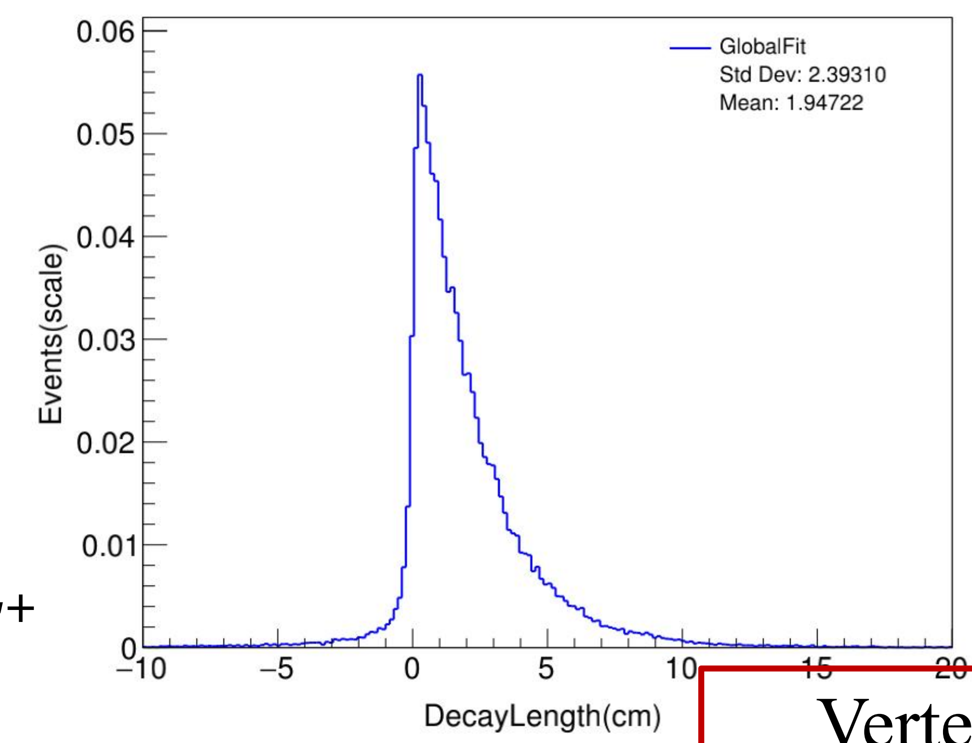
Summary

GlobalVertexFit

➢ **Evaluate Global Vertex Fitting performance based on different physics processes**

$$J/\psi \rightarrow \Sigma^{+}(Pr)\bar{\Sigma}(\pi_0(rr)\bar{P})$$

Using the First hit informaion

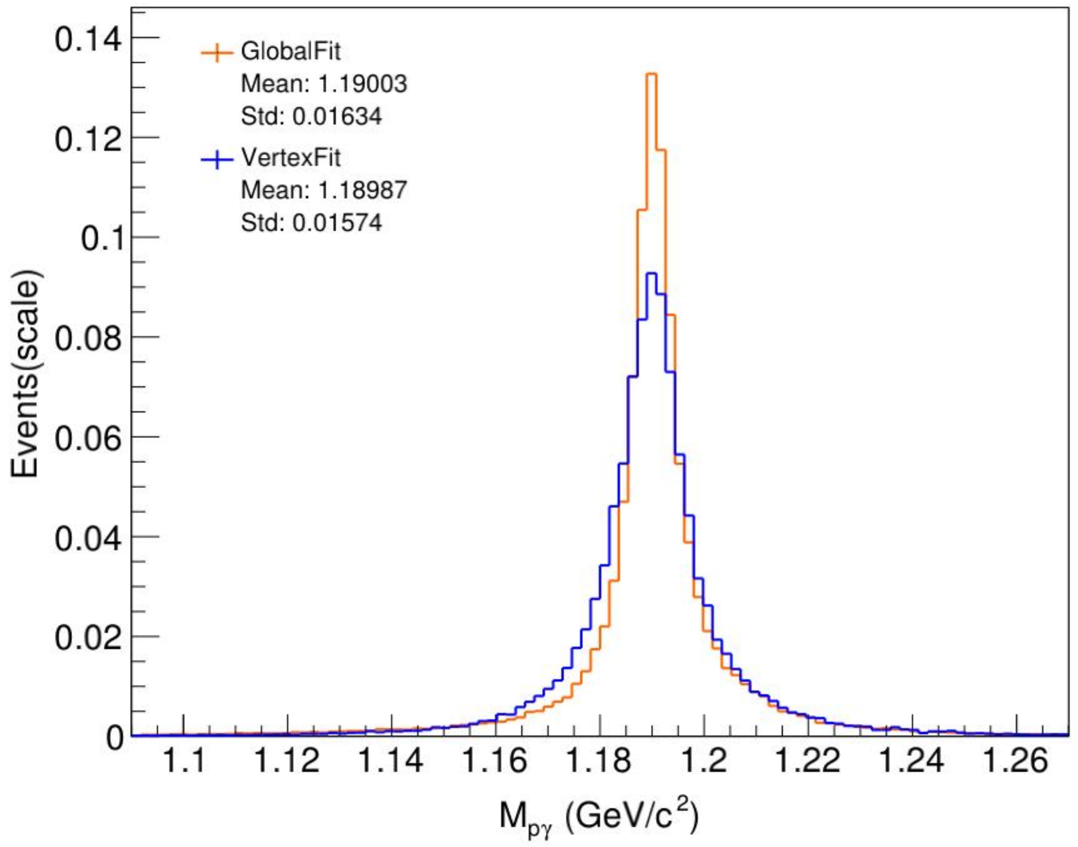


➢ Invariant Mass Spectrum of Σ^{+}

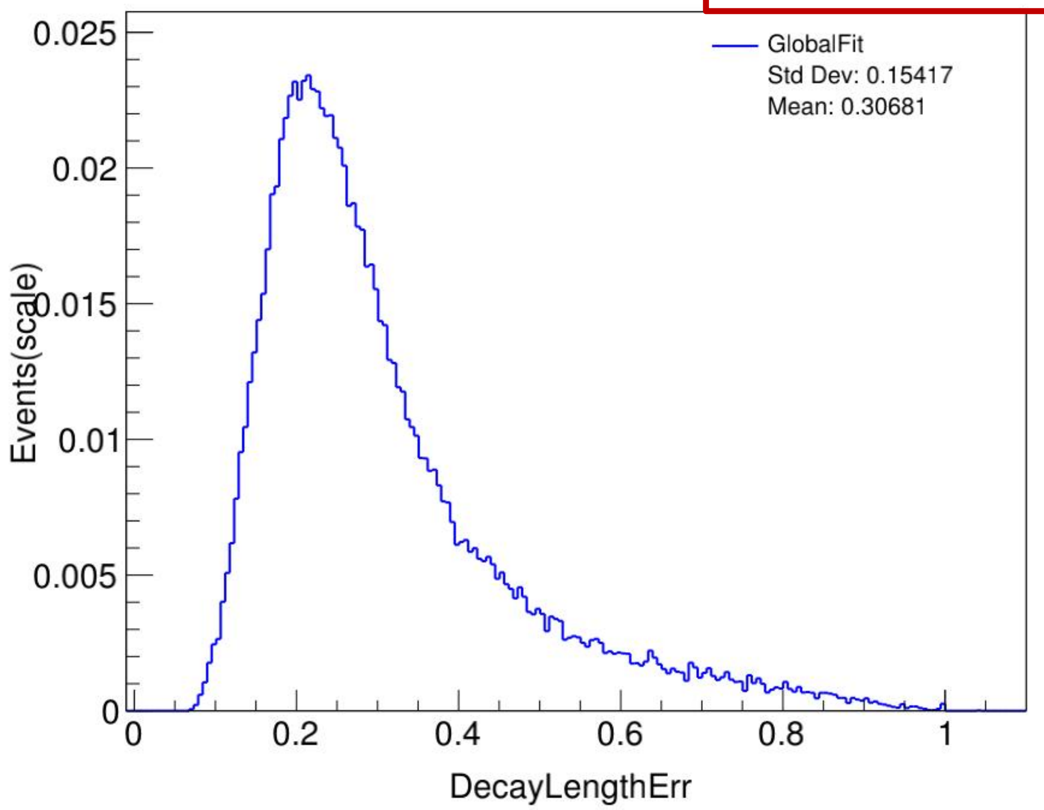
➢ DecayLength of Σ^{+}

Vertex fitting cannot achieve this

Sigma



➢ DecayLerr of Σ^{+}



Motivation

Fitting Method

rformance

Summary

GlobalVertexFit

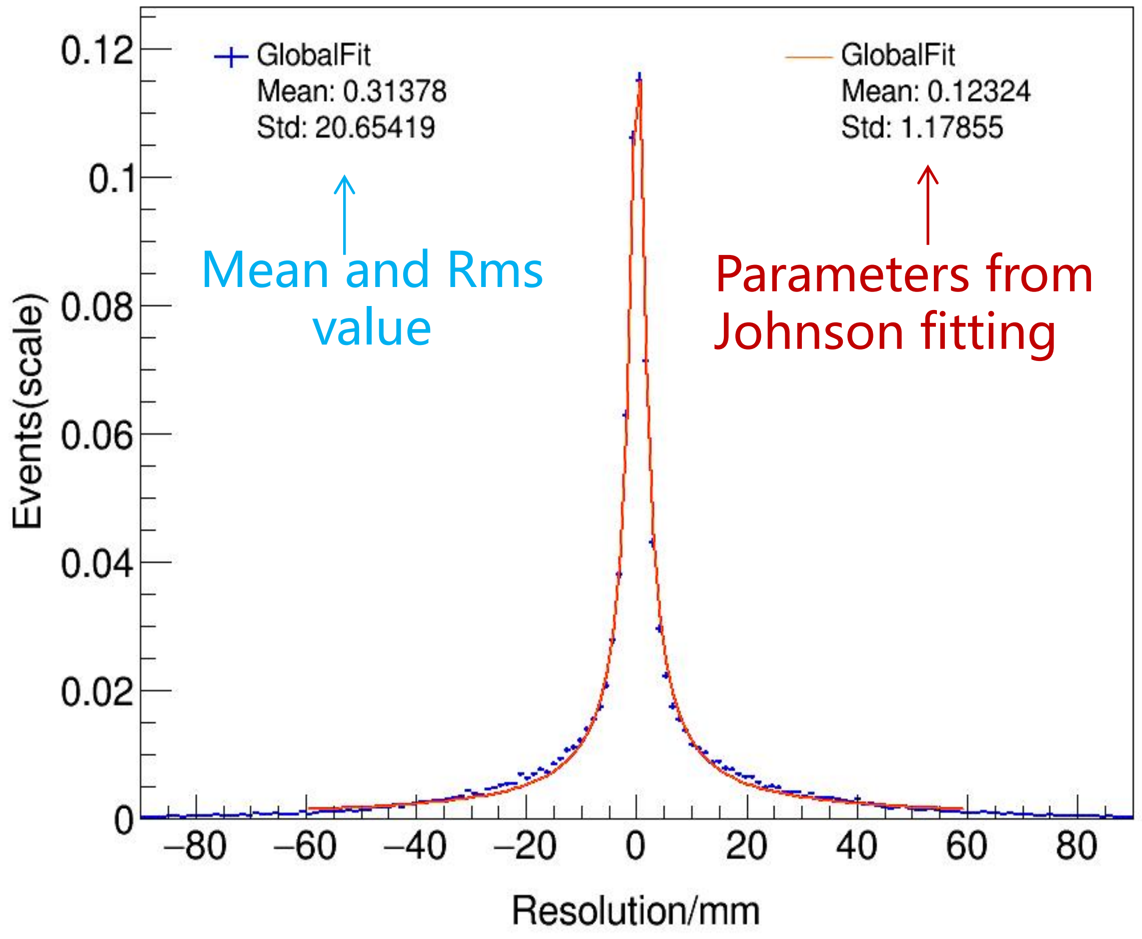
➢ **Evaluate Global Vertex Fitting performance based on different physics processes**

$$J/\psi \to \Sigma^{+}(Pr)\overline{\Sigma}(\pi_0(rr)\overline{P})$$

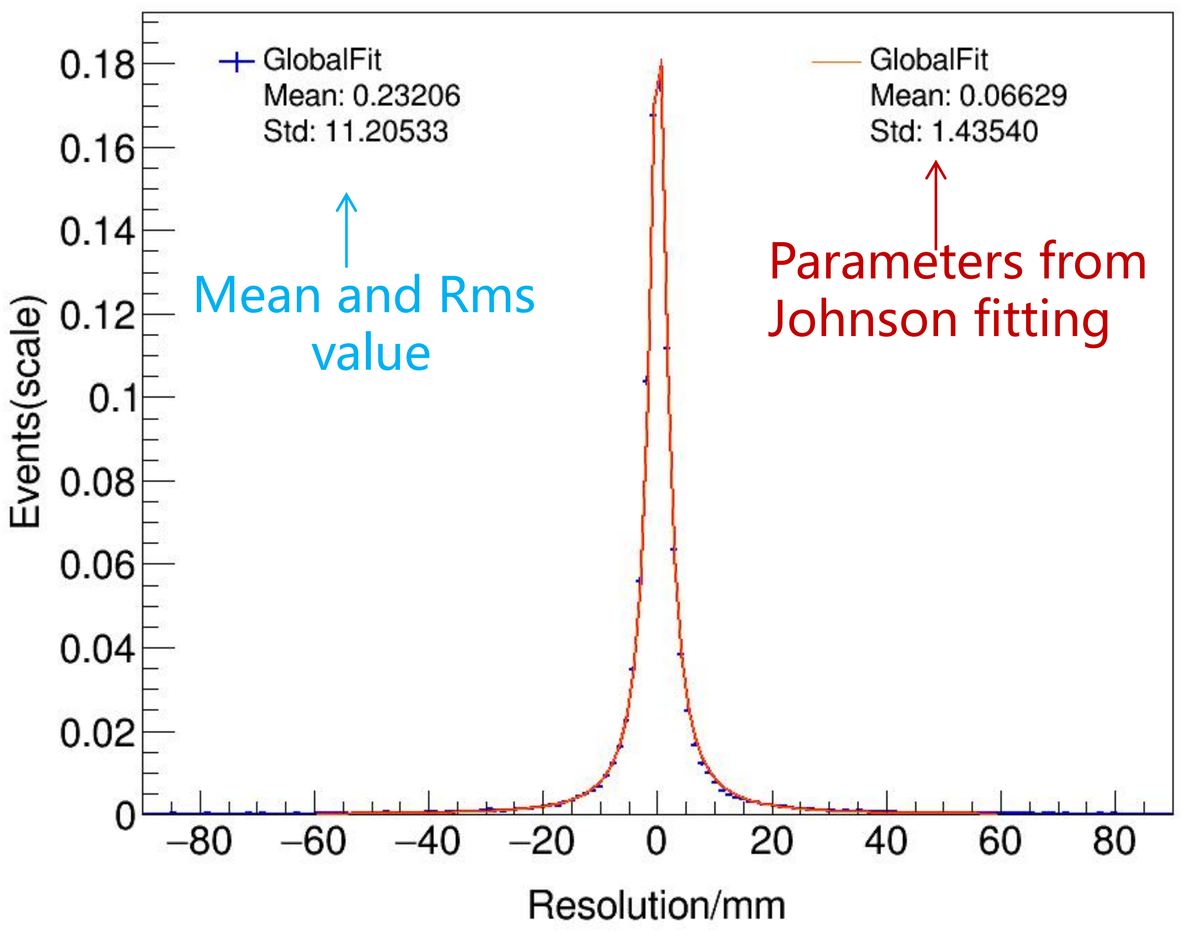
Using the First hit informaion

Global vertex fitting shows good performance on STCF and addresses the limitations of the VertexFit

➢Decay Length Resolution of Σ^{+}



➢Decay Length Resolution of$\overline{\Sigma}$



04

Summary

part four

Summary

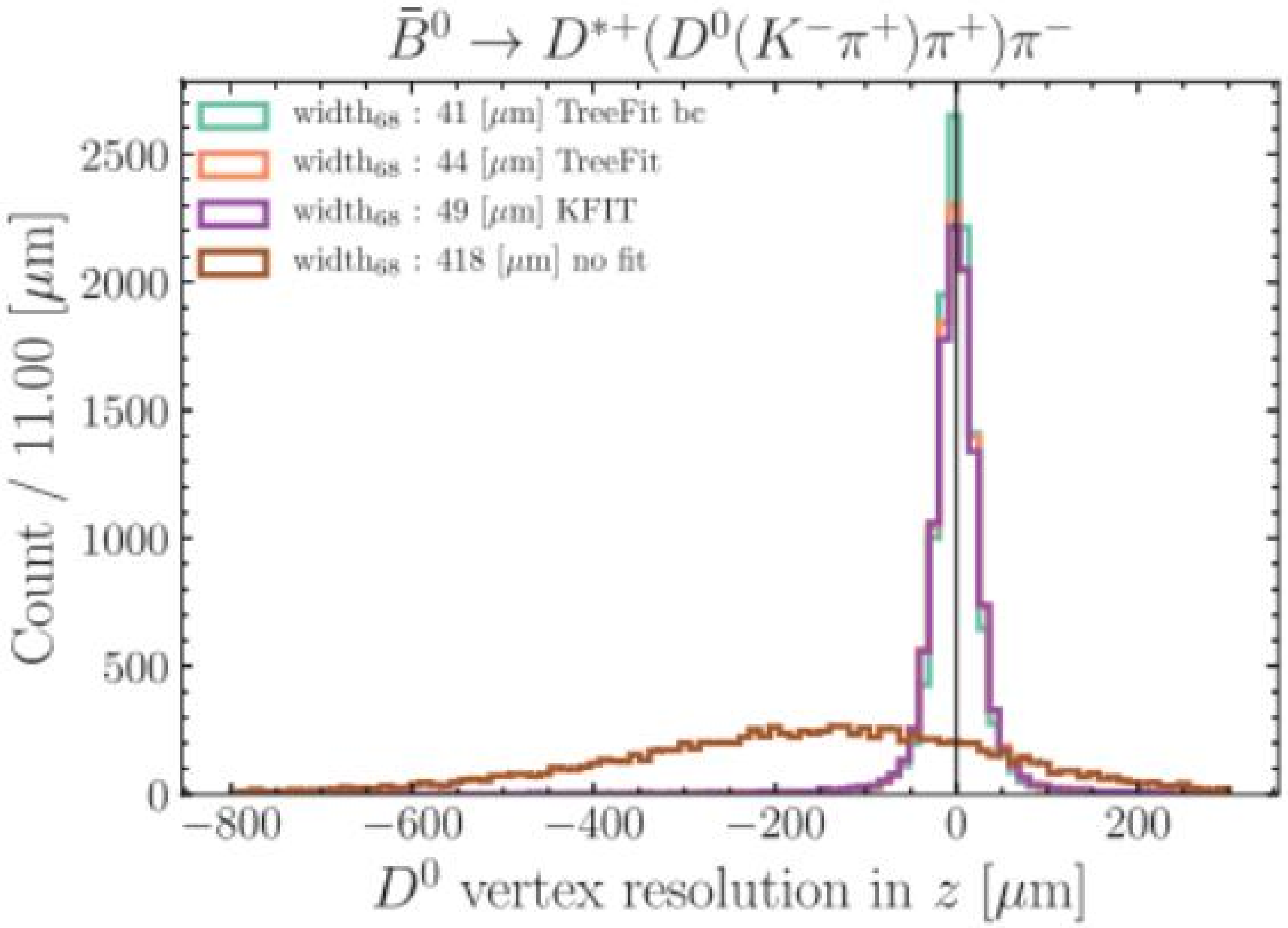
- Summary
  - This report introduces two vertex reconstruction algorithms on STCF: the Lagrange multiplier-based VertexFit and SecondVertexFit algorithms, and the Kalman filter-based GlobalVertexFit algorithm.
  - In this study, we performed vertex fitting using both the Poca point and the First point, and compared their performance. The material effects were found to mainly impact the reconstructed momentum.
  - Based on these tests, we also compared the performance of the GlobalVertexFit and VertexFit algorithms. Results show that GlobalVertexFit works well on STCF and provides better handling of neutral particles compared to VertexFit.

山东大学
SHANDONG UNIVERSITY

Thank you!

汇报人 于明玉 黄性涛 李腾

backup


$\bar{B}^0 \to D^{*+}(D^0(K^-\pi^+)\pi^+)\pi^-$
width$_{68}$: 41 [μm] TreeFit bc
width$_{68}$: 44 [μm] TreeFit
width$_{68}$: 49 [μm] KFIT
width$_{68}$: 418 [μm] no fit
Count / 11.00 [μm]
2500
2000
1500
1000
500
0
−800
−600
−400
−200
0
200
D^0 vertex resolution in z [μm]

研究背景与意义

研究方法

研究进展

总结与展望

backup

backup

- lam： The fraction of particles decaying within each detector layer

[0,	30)	1969893	40.87%
[30,	36)	286961	5.95%
[36,	98)	1744768	36.20%
[98,	160)	599260	12.43%
[160,	200)	125439	2.60%
[200,	850)	93173	1.93%

SHANDONG UNIVERSITY

研究背景与意义

研究方法

研究进展

总结与展望

backup

backup

➢ Global Vertex Fit Strategy for Neutral Particles

$$(\underbrace{p_x, p_y, p_z}_{p}, \underbrace{p_x, p_y, p_z}_{\gamma}, \underbrace{x, y, z, \theta, p_x, p_y, p_z, E}_{\Sigma^+})$$

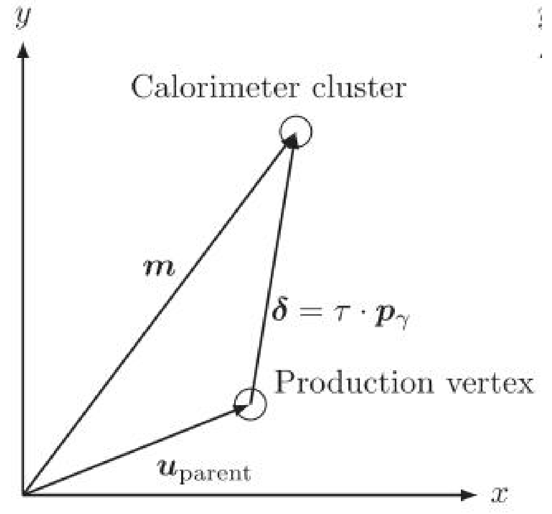
- Constraint Application Order

  ①track

Update the momentum and vertex position of charged particles based on reconstructed track information

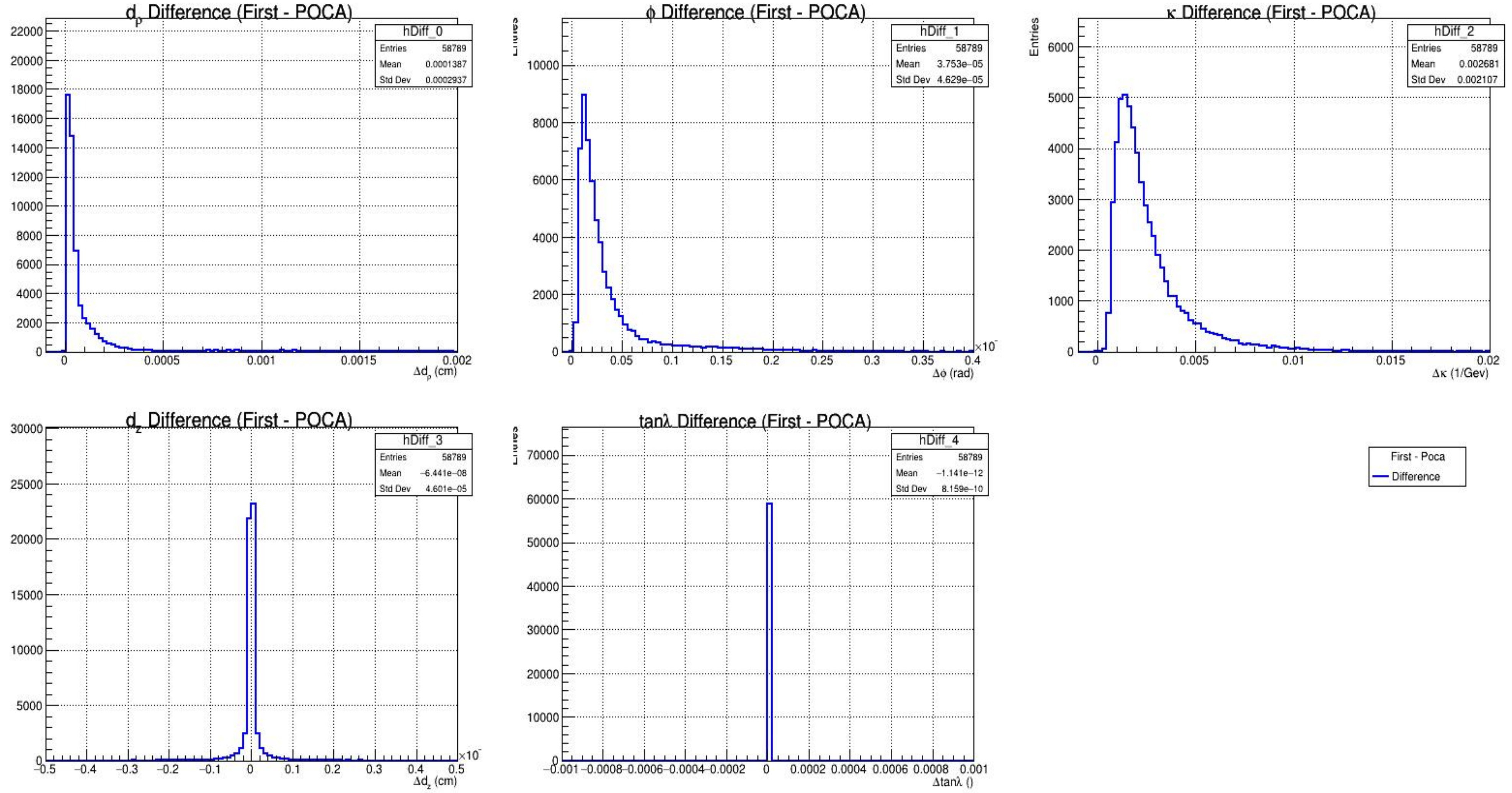
  ②photo

  Update the photon momentum using the vertex information refined by the track update.。



backup

- Difference of Helix Parameters(First - Poca)



P

backup